Chapter 2—Introduction to Probability

MULTIPLE CHOICE

1. Which of the following is not a valid representation of a probability?
   a. 35%
   b. 0
   c. 1.04
   d. 3/8

   ANS: C PTS: 1 TOP: Introduction

2. A list of all possible outcomes of an experiment is called
   a. the sample space.
   b. the sample point.
   c. the experimental outcome.
   d. the likelihood set.

   ANS: A PTS: 1 TOP: Sample space

3. Which of the following is not a proper sample space when all undergraduates at a university are considered?
   a. S = {in-state, out-of-state}
   b. S = {freshmen, sophomores}
   c. S = {age under 21, age 21 or over}
   d. S = {a major within business, no business major}

   ANS: B PTS: 1 TOP: Sample space

4. In the set of all past due accounts, let the event A mean the account is between 31 and 60 days past due and the event B mean the account is that of a new customer. The complement of A is
   a. all new customers.
   b. all accounts fewer than 31 or more than 60 days past due.
   c. all accounts from new customers and all accounts that are from 31 to 60 days past due.
   d. all new customers whose accounts are between 31 and 60 days past due.

   ANS: B PTS: 1 TOP: Complement of an event

5. In the set of all past due accounts, let the event A mean the account is between 31 and 60 days past due and the event B mean the account is that of a new customer. The union of A and B is
   a. all new customers.
   b. all accounts fewer than 31 or more than 60 days past due.
   c. all accounts from new customers and all accounts that are from 31 to 60 days past due.
   d. all new customers whose accounts are between 31 and 60 days past due.

   ANS: C PTS: 1 TOP: Addition law

6. In the set of all past due accounts, let the event A mean the account is between 31 and 60 days past due and the event B mean the account is that of a new customer. The intersection of A and B is
   a. all new customers.
   b. all accounts fewer than 31 or more than 60 days past due.
   c. all accounts from new customers and all accounts that are from 31 to 60 days past due.
   d. all new customers whose accounts are between 31 and 60 days past due.

ANS: D PTS: 1 TOP: Addition law

7. The probability of an event
   a. is the sum of the probabilities of the sample points in the event.
   b. is the product of the probabilities of the sample points in the event.
   c. is the maximum of the probabilities of the sample points in the event.
   d. is the minimum of the probabilities of the sample points in the event.

ANS: A PTS: 1 TOP: Events and their probabilities

8. If $P(A \cap B) = 0$
   a. A and B are independent events.
   b. $P(A) + P(B) = 1$
   c. A and B are mutually exclusive events.
   d. either $P(A) = 0$ or $P(B) = 0$.

ANS: C PTS: 1 TOP: Mutually exclusive events

9. If $P(A|B) = .4$, then
   a. $P(B|A) = .6$
   b. $P(A)*P(B) = .4$
   c. $P(A) / P(B) = .4$
   d. None of the alternatives is correct.

ANS: D PTS: 1 TOP: Conditional probability

10. If $P(A|B) = .2$ and $P(B^c) = .6$, then $P(B|A)$
    a. is .8
    b. is .12
    c. is .33
    d. cannot be determined.

ANS: D PTS: 1 TOP: Bayes' Theorem

11. A method of assigning probabilities that assumes the experimental outcomes are equally likely is referred to as the
    a. objective method
    b. classical method
    c. subjective method
    d. experimental method

ANS: B PTS: 1 TOP: Assigning probabilities

12. When the results of experimentation or historical data are used to assign probability values, the method used to assign probabilities is referred to as the
    a. relative frequency method
    b. subjective method
    c. classical method
    d. posterior method

ANS: A PTS: 1 TOP: Assigning probabilities

13. A method of assigning probabilities based upon judgment is referred to as the
    a. relative method
    b. probability method
    c. classical method

d. None of the alternatives is correct.

ANS: D PTS: 1 TOP: Assigning probabilities

14. The union of events A and B is the event containing
    a. all the sample points common to both A and B
    b. all the sample points belonging to A or B
    c. all the sample points belonging to A or B or both
    d. all the sample points belonging to A or B, but not both

ANS: C PTS: 1 TOP: Addition law

15. If P(A) = 0.38, P(B) = 0.83, and $P(A \cap B) = 0.27$; then $P(A \cup B) =$
    a. 1.21
    b. 0.94
    c. 0.72
    d. 1.48

ANS: B PTS: 1 TOP: Addition law

16. When the conclusions based upon the aggregated crosstabulation can be completely reversed if we look at the unaggregated data, the occurrence is known as
    a. reverse correlation
    b. inferential statistics
    c. Simpson's paradox
    d. disaggregation

ANS: C PTS: 1 TOP: Simpson's paradox

17. Before drawing any conclusions about the relationship between two variables shown in a crosstabulation, you should
    a. investigate whether any hidden variables could affect the conclusions
    b. construct a scatter diagram and find the trendline
    c. develop a relative frequency distribution
    d. construct an ogive for each of the variables

ANS: A PTS: 1 TOP: Simpson's paradox

18. Revised probabilities of events based on additional information are
    a. joint probabilities
    b. posterior probabilities
    c. marginal probabilities
    d. complementary probabilities

ANS: B PTS: 1 TOP: Bayes' Theorem

19. The probability of an intersection of two events is computed using the
    a. addition law
    b. subtraction law
    c. multiplication law
    d. division law

ANS: C PTS: 1 TOP: Multiplication law

20. Of the last 100 customers entering a computer shop, 25 have purchased a computer. If the classical method for computing probability is used, the probability that the next customer will purchase a computer is
    a. 0.25
    b. 0.50
    c. 0.75
    d. 1.00

    ANS: B PTS: 1 TOP: Classical method

TRUE/FALSE

1. Two events that are independent cannot be mutually exclusive.

   ANS: F PTS: 1 TOP: Basic relationships of probability

2. A joint probability can have a value greater than 1.

   ANS: F PTS: 1 TOP: Introduction

3. The intersection of A and A^c is the entire sample space.

   ANS: F PTS: 1 TOP: Basic relationships of probability

4. If 50 of 250 people contacted make a donation to the city symphony, then the relative frequency method assigns a probability of .2 to the outcome of making a donation.

   ANS: T PTS: 1 TOP: Relative frequency method

5. An automobile dealership is waiting to take delivery of nine new cars. Today, anywhere from zero to all nine cars might be delivered. It is appropriate to use the classical method to assign a probability of 1/10 to each of the possible numbers that could be delivered.

   ANS: F PTS: 1 TOP: Classical method

6. When assigning subjective probabilities, use experience, intuition, and any available data.

   ANS: T PTS: 1 TOP: Subjective method

7. $P(A \cap B) \geq P(A)$

   ANS: F PTS: 1 TOP: Addition law

8. If $P(A|B) = .4$ and $P(B) = .6$, then $P(A \cap B) = .667$.

   ANS: F PTS: 1 TOP: Conditional probability

9. Bayes' theorem provides a way to transform prior probabilities into posterior probabilities.

   ANS: T PTS: 1 TOP: Bayes' Theorem

10. If $P(A \cup B) = P(A) + P(B)$, then A and B are mutually exclusive.

ANS: T PTS: 1 TOP: Addition law

11. If A and B are mutually exclusive events, then P(A | B) = 0.

ANS: T PTS: 1 TOP: Mutually exclusive events

12. If A and B are independent events with P(A) = 0.1 and P(B) = 0.5, then $P(A \cup B) = .6$.

ANS: F PTS: 1 TOP: Multiplication law for independent events

13. A graphical device used for enumerating sample points in a multiple-step experiment is a Venn diagram.

ANS: F PTS: 1 TOP: Tree diagram

14. A posterior probability is a conditional probability.

ANS: T PTS: 1 TOP: Bayes' Theorem

15. If A and B are independent events, then $P(A \cap B) = P(A)P(B)$.

ANS: T PTS: 1 TOP: Multiplication law for independent events

16. Two events that are mutually exclusive cannot be independent.

ANS: T PTS: 1 TOP: Basic relationships of probability

17. P(A|B) = P(B|A) for all events A and B.

ANS: F PTS: 1 TOP: Conditional probability

18. P(A|B) = 1 − P(B|A) for all events A and B.

ANS: F PTS: 1 TOP: Conditional probability

19. $P(A|B) = P(A^C|B)$ for all events A and B.

ANS: T PTS: 1 TOP: Conditional probability

20. $P(A|B) + P(A|B^C) = 1$ for all events A and B.

ANS: F PTS: 1 TOP: Conditional probability

SHORT ANSWER

1. Compare these two descriptions of probability: 1) a measure of the degree of uncertainty associated with an event, and 2) a measure of your degree of belief that an event will happen.

ANS:
Answer not provided.

PTS: 1 TOP: Introduction

2. Explain the difference between mutually exclusive and independent events. Can a pair of events be both mutually exclusive and independent?

   ANS:
   Answer not provided.

   PTS: 1 TOP: Multiplication law

3. Use a tree diagram, labeled with appropriate notation, to illustrate Bayes' theorem.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Bayes' Theorem

4. Discuss the problems inherent in using words such as "likely," "possibly," or "probably" to convey degree of belief.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Introduction

5. Draw a Venn diagram and label appropriately to show events A, B, their complements, intersection, and union.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Basic relationships of probability

6. Describe four experiments and list the experimental outcomes associated with each one.

   ANS:
   Answer not provided.

   PTS: 1 TOP: Experiments and the sample space

PROBLEM

1. A market study taken at a local sporting goods store showed that of 20 people questioned, 6 owned tents, 10 owned sleeping bags, 8 owned camping stoves, 4 owned both tents and camping stoves, and 4 owned both sleeping bags and camping stoves.

   Let: Event A = owns a tent
   Event B = owns a sleeping bag
   Event C = owns a camping stove

   and let the sample space be the 20 people questioned.
   a. Find P(A), P(B), P(C), P(A $\cap$ C), P(B $\cap$ C).
   b. Are the events A and C mutually exclusive? Explain briefly.
   c. Are the events B and C independent events? Explain briefly.
   d. If a person questioned owns a tent, what is the probability he also owns a camping stove?

e. If two people questioned own a tent, a sleeping bag, and a camping stove, how many own only a camping stove? In this case is it possible for 3 people to own both a tent and a sleeping bag, but not a camping stove?

ANS:

a. P(A) = .3; P(B) = .5; P(C) = .4; P(A $\cap$ B) = .2; P(B $\cap$ C) = .2
b. Events B and C are not mutually exclusive because there are people (4 people) who both own a tent and a camping stove.
c. Since P(B $\cap$ C) = .2 and P(B)P(C) = (.5)(.4) = .2, then these events are independent.
d. .667
e. Two people own only a camping stove; no, it is not possible

PTS: 1 TOP: Basic relationships of probability

2. An accounting firm has noticed that of the companies it audits, 85% show no inventory shortages, 10% show small inventory shortages and 5% show large inventory shortages. The firm has devised a new accounting test for which it believes the following probabilities hold:

P(company will pass test | no shortage) = .90
P(company will pass test | small shortage) = .50
P(company will pass test | large shortage) = .20

a. If a company being audited fails this test, what is the probability of a large or small inventory shortage?
b. If a company being audited passes this test, what is the probability of no inventory shortage?

ANS:

a. .515
b. .927

PTS: 1 TOP: Conditional probability

3. An investment advisor recommends the purchase of stock shares in Infomatics, Inc. He has made the following predictions:

P(Stock goes up 20% | Rise in GDP) = .6
P(Stock goes up 20% | Level GDP) = .5
P(Stock goes up 20% | Fall in GDP) = .4

An economist has predicted that the probability of a rise in the GDP is 30%, whereas the probability of a fall in the GDP is 40%.
a. What is the probability that the stock will go up 20%?
b. We have been informed that the stock has gone up 20%. What is the probability of a rise or fall in the GDP?

ANS:

a. .49
b. .367 + .327 = .694

PTS: 1 TOP: Conditional probability

4. Global Airlines operates two types of jet planes: jumbo and ordinary. On jumbo jets, 25% of the passengers are on business while on ordinary jets 30% of the passengers are on business. Of Global's air fleet, 40% of its capacity is provided on jumbo jets. (Hint: The 25% and 30% values are conditional probabilities stated as percentages.)
   a. What is the probability a randomly chosen business customer flying with Global is on a jumbo jet?
   b. What is the probability a randomly chosen non-business customer flying with Global is on an ordinary jet?

ANS:

a. .357
b. .583

PTS: 1 TOP: Conditional probability

5. The following probability model describes the number of snow storms for Washington, D.C. for a given year:

Number of Storms	0	1	2	3	4	5	6
Probability	.25	.33	.24	.11	.04	.02	.01

The probability of 7 or more snowstorms in a year is 0.
   a. What is the probability of more than 2 but less than 5 snowstorms?
   b. Given this a particularly cold year (in which 2 snowstorms have already been observed), what is the conditional probability that 4 or more snowstorms will be observed?
   c. If at the beginning of winter there is a snowfall, what is the probability of at least one more snowstorm before winter is over?

ANS:

a. .15
b. .167
c. .56

PTS: 1 TOP: Basic relationships of probability

6. Safety Insurance Company has compiled the following statistics. For any one year period:

P(accident | male driver under 25) = .22
P(accident | male driver over 25) = .15
P(accident | female driver under 25 = .16
P(accident | female driver over 25) = .14

The percentage of Safety's policyholders in each category are:

Male Under 25	20%
Male Over 25	40%
Female Under 25	10%
Female Over 25	30%

a. What is the probability that a randomly selected policyholder will have an accident within the next year?
b. Given that a driver has an accident, what is the probability that the driver is a male over 25?
c. Given that a driver has no accident, what is the probability the driver is a female?
d. Does knowing the fact that a driver has had no accidents give us a great deal of information regarding the driver's sex?

ANS:

a. .162
b. .37
c. .408
d. no

PTS: 1 TOP: Conditional probability

7. Mini Car Motors offers its luxury car in three colors: gold, silver and blue. The vice president of advertising is interested in the order of popularity of the color choices by customers during the first month of sales.
a. How many sample points are there in this experiment?
b. If the event A = gold is the most popular color, list the outcome(s) in event A.
c. If the event B = blue is the least popular color, list the outcome(s) in $A \cap B$.
d. List the outcome(s) in $A \cap B^c$.

ANS:

a. 6
b. {(G,S,B), (G,B,S)}
c. {(G,S,B)}
d. {(G,B,S)}

PTS: 1 TOP: Sample space

8. Higbee Manufacturing Corp. has recently received 5 cases of a certain part from one of its suppliers. The defect rate for the parts is normally 5%, but the supplier has just notified Higbee that one of the cases shipped to them has been made on a misaligned machine that has a defect rate of 97%. So the plant manager selects a case at random and tests a part.
a. What is the probability that the part is defective?
b. Suppose the part is defective, what is the probability that this is from the case made on the misaligned machine?
c. After finding that the first part was defective, suppose a second part from the case is tested. However, this part is found to be good. Using the revised probabilities from part (b) compute the new probability of these parts being from the defective case.
d. Do you think you would obtain the same posterior probabilities as in part (c) if the first part was not found to be defective but the second part was?
e. Suppose, because of other evidence, the plant manager was 80% certain this case was the one made on the misaligned machine. How would your answer to part (b) change?

ANS:

a. .234
b. .829

c. .133
d. yes
e. .987

PTS: 1 TOP: Bayes' Theorem

9. A package of candy contains 12 brown, 5 red, and 8 green candies. You grab three pieces from the package. Give the sample space of colors you could get. Order is not important.

ANS:
Order is not implied: S = {BBB, RRR, GGG, BBR, BBG, RRB, RRG, GGB, GGR, BRG}

PTS: 1 TOP: Sample space

10. There are two more assignments in a class before its end, and if you get an A on at least one of them, you will get an A for the semester. Your subjective assessment of your performance is

Event	Probability
A on paper and A on exam	.25
A on paper only	.10
A on exam only	.30
A on neither	.35

a. What is the probability of getting an A on the paper?
b. What is the probability of getting an A on the exam?
c. What is the probability of getting an A in the course?
d. Are the grades on the assignments independent?

ANS:

a. .35
b. .55
c. .65
d. No

PTS: 1 TOP: Basic relationships of probability

11. A mail order company tracks the number of returns it receives each day. Information for the last 50 days shows

Number of returns	Number of days
0 - 99	6
100 - 199	20
200 - 299	15
300 or more	9

a. How many sample points are there?
b. List and assign probabilities to sample points.
c. What procedure was used to assign these probabilities?

ANS:

a. 4

b. P(0 - 99 returns) = .12
P(100 - 199 returns) = .40
P(200 - 299 returns) = .30
P(300 or more returns) = .18
c. Relative frequency method

PTS: 1 TOP: Relative frequency method

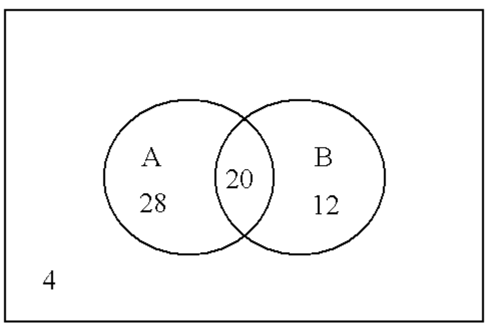
12. Super Cola sales breakdown as 80% regular soda and 20% diet soda. While 60% of the regular soda is purchased by men, only 30% of the diet soda is purchased by men. If a woman purchases Super Cola, what is the probability that it is a diet soda?

ANS:
.30435

PTS: 1 TOP: Conditional probability

13. A food distributor carries 64 varieties of salad dressing. Appleton Markets stocks 48 of these flavors. Beacon Stores carries 32 of them. The probability that a flavor will be carried by Appleton or Beacon is 15/16. Use a Venn diagram to find the probability a flavor is carried by both Appleton and Beacon.

ANS:
The Venn diagram is



and $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 6/8 + 4/8 - 15/16 = 5/16 = .3125$

PTS: 1 TOP: Addition law

14. Through a telephone survey, a low-interest bank credit card is offered to 400 households. The responses are as tabled.

	Income ≤ \$60,000	Income > \$60,000
Accept offer	40	30
Reject offer	210	120

a. Develop a joint probability table and show the marginal probabilities.
b. What is the probability of a household whose income exceeds \$60,000 and who rejects the offer?
c. If income is ≤ \$60,000, what is the probability the offer will be accepted?
d. If the offer is accepted, what is the probability that income exceeds \$60,000?

ANS:

a.

	Income ≤ $60,000	Income > $60,000	Total
Accept offer	.100	.075	.175
Reject offer	.525	.300	.825
Total	.625	.375	1.000

b. .3
c. .16
d. .4286

PTS: 1 TOP: Conditional probability

15. A medical research project examined the relationship between a subject's weight and recovery time from a surgical procedure, as shown in the table below.

	Underweight	Normal weight	Overweight
Less than 3 days	6	15	3
3 to 7 days	30	95	20
Over 7 days	14	40	27

a. Use relative frequency to develop a joint probability table to show the marginal probabilities.
b. What is the probability a patient will recover in fewer than 3 days?
c. Given that recovery takes over 7 days, what is the probability the patient is overweight?

ANS:

a.

	Underweight	Normal weight	Overweight	Total
Less than 3 days	.024	.06	.012	.096
3 to 7 days	.120	.38	.080	.580
Over 7 days	.056	.16	.108	.324
Total	.200	.60	.200	1.00

b. .096
c. 27/81 = .33

PTS: 1 TOP: Conditional probability

16. To better track its patients, a hospital's neighborhood medical center has gathered this information.

	New patient (N)	Existing patient (E)
Scheduled appointment (A)	10	10
Walk-in (W)	12	18

a. Develop a joint probability table. Include the marginal probabilities.
b. Find the conditional probabilities:
P(A|N), P(A|E), P(W|N), P(W|E), P(N|A), P(E|A), P(N|W), P(E|W)

ANS:

a.

	New patient (N)	Existing patient (E)	Total
Scheduled appointment (A)	.20	.20	.40
Walk-in (W)	.24	.36	.60
Total	.44	.56	1.00

b. P(A|N) = .4545
P(A|E) = .3571
P(W|N) = .5454
P(W|E) = .6429
P(N|A) = .5
P(E|A) = .5
P(N|W) = .4
P(E|W) = .6

PTS: 1 TOP: Conditional probability

17. The Ambell Company uses batteries from two different manufacturers. Historically, 60% of the batteries are from manufacturer 1, and 90% of these batteries last for over 40 hours. Only 75% of the batteries from manufacturer 2 last for over 40 hours. A battery in a critical tool fails at 32 hours. What is the probability it was from manufacturer 2?

ANS:
.625

PTS: 1 TOP: Bayes' Theorem

18. It is estimated that 3% of the athletes competing in a large tournament are users of an illegal drug to enhance performance. The test for this drug is 90% accurate. What is the probability that an athlete who tests positive is actually a user?

ANS:
.2177

PTS: 1 TOP: Bayes' Theorem

19. Thirty-five percent of the students who enroll in a statistics course go to the statistics laboratory on a regular basis. Past data indicates that 40% of those students who use the lab on a regular basis make a grade of B or better. On the other hand, 10% of students who do not go to the lab on a regular basis make a grade of B or better. If a particular student made an A, determine the probability that she or he used the lab on a regular basis.

ANS:
0.6829

PTS: 1 TOP: Conditional probability

20. In a recent survey in a Statistics class, it was determined that only 60% of the students attend class on Fridays. From past data it was noted that 98% of those who went to class on Fridays pass the course, while only 20% of those who did not go to class on Fridays passed the course.
a. What percentage of students is expected to pass the course?
b. Given that a person passes the course, what is the probability that he/she attended classes on Fridays?

ANS:
a. 66.8%
b. 0.88

PTS: 1 TOP: Conditional probability

21. An applicant has applied for positions at Company A and Company B. The probability of getting an offer from Company A is 0.4, and the probability of getting an offer from Company B is 0.3. Assuming that the two job offers are independent of each other, what is the probability that
    a. the applicant gets an offer from both companies?
    b. the applicant will get at least one offer?
    c. the applicant will not be given an offer from either company?
    d. Company A does not offer the applicant a job, but Company B does?

    ANS:
    a. 0.12
    b. 0.58
    c. 0.42
    d. 0.18

    PTS: 1 TOP: Multiplication law

22. A corporation has 15,000 employees. Sixty-two percent of the employees are male. Twenty-three percent of the employees earn more than $30,000 a year. Eighteen percent of the employees are male and earn more than $30,000 a year.
    a. If an employee is taken at random, what is the probability that the employee is male?
    b. If an employee is taken at random, what is the probability that the employee earns more than $30,000 a year?
    c. If an employee is taken at random, what is the probability that the employee is male and earns more than $30,000 a year?
    d. If an employee is taken at random, what is the probability that the employee is male or earns more than $30,000 a year or both?
    e. The employee taken at random turns out to be male. Compute the probability that he earns more than $30,000 a year.
    f. Are being male and earning more than $30,000 a year independent?

    ANS:
    a. 0.62
    b. 0.23
    c. 0.18
    d. 0.67
    e. 0.2903
    f. No

    PTS: 1 TOP: Conditional probability