

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

1) How many players are there in the weighted voting system  $[34 : 10, 7, 4, 4, 3, 3, 3, 1, 1]$ ?

- A) 36
- B) 9
- C) 10
- D) 38
- E) none of these

Answer: B

2) How many players are there in the weighted voting system  $[20 : 7, 5, 4, 4, 2, 2, 2, 1, 1]$ ?

- A) 10
- B) 28
- C) 9
- D) 20
- E) none of these

Answer: C

3) What is the quota in the weighted voting system  $[34 : 10, 7, 4, 4, 3, 3, 3, 1, 1]$ ?

- A) 36
- B) 34
- C) 9
- D) 10
- E) none of these

Answer: B

4) What is the quota in the weighted voting system  $[20 : 7, 5, 4, 4, 2, 2, 2, 1, 1]$ ?

- A) 10
- B) 20
- C) 9
- D) 28
- E) none of these

Answer: B

5) The total number of votes in the weighted voting system  $[34 : 10, 7, 4, 4, 3, 3, 3, 1, 1]$  is

- A) 34.
- B) 10.
- C) 9.
- D) 36.
- E) none of these

Answer: D

6) The total number of votes in the weighted voting system  $[20 : 7, 5, 4, 4, 2, 2, 2, 1, 1]$  is

- A) 20.
- B) 48.
- C) 28.
- D) 10.
- E) none of these

Answer: C

- 7) In the weighted voting system  $[q : 10, 8, 6]$ , a strict majority of the votes is needed to pass a motion. The value of the quota  $q$  is
- A) 11.
  - B) 13.
  - C) 10.
  - D) 12.
  - E) none of these

Answer: B

- 8) In the weighted voting system  $[q : 10, 8, 4]$ , a strict majority of the votes is needed to pass a motion. The value of the quota  $q$  is
- A) 12.
  - B) 13.
  - C) 11.
  - D) 10.
  - E) none of these

Answer: A

- 9) In the weighted voting system  $[q : 20, 19, 16, 2, 1, 1]$ , a two-thirds majority of the votes is needed to pass a motion. The value of the quota  $q$  is
- A) 40.
  - B) 7.
  - C) 59.
  - D) 20.
  - E) none of these

Answer: A

- 10) In the weighted voting system  $[q : 10, 9, 8, 1, 1]$ , a two-thirds majority of the votes is needed to pass a motion. The value of the quota  $q$  is
- A) 7.
  - B) 29.
  - C) 20.
  - D) 19.
  - E) none of these

Answer: C

- 11) In the weighted voting system  $[q : 12, 10, 5, 1, 1]$ , the smallest possible value that the quota  $q$  can take is
- A) 29.
  - B) 15.
  - C) 12.
  - D) 14.
  - E) none of these

Answer: B

- 12) In the weighted voting system  $[q : 12, 10, 5, 1, 1]$ , the largest possible value that the quota  $q$  can take is
- A) 29.
  - B) 30.
  - C) 22.
  - D) 15.
  - E) none of these

Answer: A

13) In the weighted voting system  $[q : 10, 8, 5]$ , the smallest possible value that the quota  $q$  can take is

- A) 12.
- B) 11.
- C) 13.
- D) 23.
- E) none of these

Answer: A

14) In the weighted voting system  $[q : 24, 12, 8, 4, 2]$ , the smallest possible value that the quota  $q$  can take is

- A) 25.
- B) 36.
- C) 24.
- D) 50.
- E) none of these

Answer: E

15) In the weighted voting system  $[q : 22, 12, 8, 4]$ , the smallest possible value that the quota  $q$  can take is

- A) 46.
- B) 31.
- C) 23.
- D) 22.
- E) none of these

Answer: E

16) In the weighted voting system  $[q : 24, 12, 8, 4, 2]$ , the largest value that the quota  $q$  can take is

- A) 25.
- B) 24.
- C) 50.
- D) 36.
- E) none of these

Answer: C

17) In the weighted voting system  $[30 : 24, 12, 8, 4, 2]$ , the minimum percentage of votes needed to pass a motion is

- A) 61%.
- B) 60%.
- C) 50%.
- D) 30%.
- E) 51%.

Answer: B

18) In the weighted voting system  $[9 : 11, 4, 2]$ ,

- A)  $P_1$  is a dictator.
- B)  $P_1$  has veto power but is not a dictator.
- C) there are no dictators.
- D) every player is a dictator.
- E) none of these

Answer: A

- 19) In the weighted voting system  $[12 : 13, 7, 2]$ ,  
A) every player is a dictator.  
B)  $P_1$  has veto power but is not a dictator.  
C) there are no dictators  
D)  $P_1$  is a dictator.  
E) none of these

Answer: D

- 20) In the weighted voting system  $[12 : 11, 5, 5]$ ,  
A) every player has veto power.  
B)  $P_1$  is a dictator.  
C) no player has veto power.  
D)  $P_1$  has veto power but is not a dictator.  
E) none of these

Answer: D

- 21) In the weighted voting system  $[13 : 12, 7, 2]$ ,  
A)  $P_1$  has veto power but is not a dictator.  
B)  $P_1$  is a dictator.  
C) no player has veto power.  
D) every player has veto power.  
E) none of these

Answer: A

- 22) In the weighted voting system  $[14 : 7, 7, 6]$ ,  
A)  $P_1$  and  $P_2$  have equal power,  $P_3$  is a dummy.  
B)  $P_1$  and  $P_2$  have equal power,  $P_3$  is not a dummy.  
C)  $P_1$  has all the power,  $P_2$  and  $P_3$  are dummies.  
D) all three players have equal power.  
E) none of these

Answer: A

- 23) In the weighted voting system  $[100 : 50, 50, 48]$ ,  
A) all three players have equal power.  
B)  $P_1$  and  $P_2$  have equal power,  $P_3$  is not a dummy.  
C)  $P_1$  and  $P_2$  have equal power,  $P_3$  is a dummy.  
D)  $P_1$  has all the power,  $P_2$  and  $P_3$  are dummies.  
E) none of these

Answer: C

- 24) In the weighted voting system  $[10 : 5, 4, 2]$ ,  
A)  $P_1$  has veto power,  $P_3$  is a dummy.  
B)  $P_1$  and  $P_2$  have veto power,  $P_3$  is a dummy.  
C) all three players have veto power.  
D) no player has veto power.  
E) none of these

Answer: C

- 25) In the weighted voting system  $[11 : 5, 4, 2]$ ,
- A)  $P_1$  and  $P_2$  have veto power,  $P_3$  is a dummy.
  - B) only  $P_1$  has veto power.
  - C) no player has veto power.
  - D) all three players have veto power.
  - E) none of these

Answer: D

- 26) In the weighted voting system  $[q : 24, 12, 8, 4, 2]$ , what is the smallest possible value of the quota  $q$  for which  $P_5$  is a dummy?
- A) 27
  - B) 30
  - C) 29
  - D) 24
  - E) none of these

Answer: A

- 27) In the weighted voting system  $[q : 24, 12, 8, 4, 2]$ , what is the smallest possible value of the quota  $q$  for which  $P_4$  and  $P_5$  are dummies?
- A) 29
  - B) 27
  - C) 24
  - D) 31
  - E) none of these

Answer: D

- 28) In the weighted voting system  $[q : 24, 12, 6, 3]$ , what is the largest possible value of the quota  $q$  for which  $P_4$  is a dummy?
- A) 40
  - B) 42
  - C) 39
  - D) 41
  - E) none of these

Answer: B

- 29) What is the smallest value of  $w$  for which  $P_4$  is not a dummy in the weighted voting system  $[30 : 24, 12, 6, w]$ ?
- A) 3
  - B) 6
  - C) 5
  - D) 1
  - E) none of these

Answer: B

- 30) In the weighted voting system  $[q : 7, 5, 3, 2, 1]$ , every player has veto power. The value of the quota  $q$  is
- A) 17.
  - B) 15.
  - C) 10.
  - D) 18.
  - E) none of these

Answer: D

- 31) In the weighted voting system  $[q : 6, 5, 4, 3, 2, 1]$ , every player has veto power. The value of the quota  $q$  is
- A) 11.
  - B) 21.
  - C) 20.
  - D) 22.
  - E) none of these

Answer: B

- 32) In the weighted voting system  $[q : 7, 5, 3, 2, 1]$ , no player has veto power. The largest possible value that the quota  $q$  can take is
- A) 17.
  - B) 9.
  - C) 11.
  - D) 10.
  - E) none of these

Answer: C

- 33) In the weighted voting system  $[q : 6, 5, 4, 3, 2, 1]$ , no player has veto power. The largest possible value that the quota  $q$  can take is
- A) 15.
  - B) 11.
  - C) 13.
  - D) 17.
  - E) none of these

Answer: A

- 34) A board is made up of two women (W) and three men (M). In order to pass a motion, three of the five including at least one woman must vote "yes". Which of the following weighted voting systems represent this situation?
- A)  $[6 : 3, 3, 2, 2, 2]$
  - B)  $[3 : 1, 1, 1, 1, 1]$
  - C)  $[7 : 3, 3, 2, 2, 2]$
  - D)  $[8 : 3, 3, 2, 2, 2]$
  - E) none of these

Answer: C

A committee consists of six members (A, B, C, D, E, and F). A has veto power; B, C, D, and E each have one vote. F is a nonvoting member. For a motion to pass it must have the support of A plus at least two additional voting members.

35) A weighted system that could represent this situation is

- A)  $[5 : 3, 1, 1, 1, 1, 0]$ .
- B)  $[6 : 3, 1, 1, 1, 1, 0]$ .
- C)  $[6 : 5, 1, 1, 1, 1, 0]$ .
- D)  $[4 : 2, 1, 1, 1, 1, 0]$ .
- E) none of these

Answer: A

**Solve the problem.**

36) A player whose weight is bigger than the weight of every other player

- A) is a dictator.
- B) is a critical player in every winning coalition.
- C) has veto power.
- D) is a dummy.
- E) none of these

Answer: E

37) Consider the generic weighted voting system  $[q : w_1, w_2, \dots, w_N]$ . Which of the following mathematical statements is equivalent to saying that  $P_1$  is a dictator?

- A)  $w_1 \geq q$
- B)  $w_2 + w_3 + \dots + w_N < q$  and  $w_1 < q$
- C)  $w_1 > w_2$
- D)  $w_1 > q$
- E) none of these

Answer: A

38) Consider the generic weighted voting system  $[q : w_1, w_2, \dots, w_N]$ . Which of the following mathematical statements is equivalent to saying that  $P_1$  has veto power?

- A)  $w_2 + w_3 + \dots + w_N < q$  and  $w_1 < q$
- B)  $w_1 > q$
- C)  $w_1 \geq q$
- D)  $w_1 > w_2$
- E) none of these

Answer: A

**Refer to the weighted voting system  $[35 : 32, 15, 10, 3]$  and the Banzhaf definition of power. (The four players are  $P_1, P_2, P_3$ , and  $P_4$ .)**

39) The weight of the coalition  $\{P_2, P_3, P_4\}$  is

- A) 28.
- B) 57.
- C) 25.
- D) 60.
- E) none of these

Answer: A

40) The winning coalitions are:

- A) all coalitions with two or more players.
- B) all coalitions with two or more players, one of which is  $P_1$ .
- C) all coalitions with three or more players.
- D) all coalitions.
- E) none of these

Answer: B

41) The number of winning coalitions is

- A) 24.
- B) 7.
- C) 8.
- D) 15.
- E) none of these

Answer: B

42) Which players in the coalition  $\{P_1, P_3\}$  are critical?

- A)  $P_3$  only
- B)  $P_1$  only
- C)  $P_1$  and  $P_3$
- D) None of the players
- E) none of these

Answer: C

43) Which players in the coalition  $\{P_1, P_3, P_4\}$  are critical?

- A)  $P_1$  only
- B) None of the players
- C)  $P_1$  and  $P_3$  only
- D) All three players
- E) none of these

Answer: A

44) The Banzhaf power distribution of the weighted voting system is

- A)  $P_1: 75\%$ ;  $P_2: 8\frac{1}{3}\%$ ;  $P_3: 8\frac{1}{3}\%$ ;  $P_4: 8\frac{1}{3}\%$ .
- B)  $P_1: 70\%$ ;  $P_2: 10\%$ ;  $P_3: 10\%$ ;  $P_4: 10\%$ .
- C)  $P_1: 60\%$ ;  $P_2: 20\%$ ;  $P_3: 10\%$ ;  $P_4: 10\%$ .
- D)  $P_1: 40\%$ ;  $P_2: 20\%$ ;  $P_3: 20\%$ ;  $P_4: 20\%$ .
- E) none of these

Answer: B



Refer to the weighted voting system  $[25 : 22, 12, 6, 3]$  and the Banzhaf definition of power. (The four players are  $P_1, P_2, P_3$ , and  $P_4$ .)

45) The weight of the coalition  $\{P_2, P_3, P_4\}$  is

- A) 25.
- B) 40.
- C) 22.
- D) 21.
- E) none of these

Answer: D

46) The winning coalitions are

- A) all coalitions with three or more players.
- B) all coalitions with two or more players.
- C) all coalitions with two or more players, one of which is  $P_1$ .
- D) all coalitions.
- E) none of these

Answer: C

47) The number of winning coalitions is

- A) 8.
- B) 7.
- C) 15.
- D) 1.
- E) none of these

Answer: B

48) Which players in the coalition  $\{P_1, P_2\}$  are critical?

- A)  $P_1$  only
- B)  $P_1$  and  $P_2$
- C)  $P_2$  only
- D) None of the players
- E) none of these

Answer: B

49) Which players in the coalition  $\{P_1, P_3, P_4\}$  are critical?

- A) All three players
- B)  $P_1$  only
- C)  $P_1$  and  $P_3$  only
- D) None of the players
- E) none of these

Answer: B

50) The Banzhaf power distribution of the weighted voting system is

- A)  $P_1$ : 70%;  $P_2$ : 10%;  $P_3$ : 10%;  $P_4$ : 10%.
- B)  $P_1$ : 60%;  $P_2$ : 20%;  $P_3$ : 10%;  $P_4$ : 10%.
- C)  $P_1$ : 40%;  $P_2$ : 20%;  $P_3$ : 20%;  $P_4$ : 20%.
- D)  $P_1$ : 25%;  $P_2$ : 25%;  $P_3$ : 25%;  $P_4$ : 25%.
- E) none of these

Answer: A

**Refer to the weighted voting system  $[14 : 5, 5, 4, 4]$  and the Banzhaf definition of power. (The four players are  $P_1, P_2, P_3$ , and  $P_4$ .)**

51) What is the weight of the coalition  $\{P_2, P_3, P_4\}$ ?

- A) 13
- B) 14
- C) 8
- D) 12
- E) none of these

Answer: A

52) Which players in the coalition  $\{P_1, P_2, P_3, P_4\}$  are critical?

- A)  $P_1$  and  $P_2$
- B)  $P_1$  only
- C) None of the players
- D) All four players
- E) none of these

Answer: A

53) What is the total number of winning coalitions?

- A) 1
- B) 5
- C) 3
- D) 15
- E) none of these

Answer: C

54) The Banzhaf power distribution of the weighted voting system is

- A)  $P_1$ : 40%;  $P_2$ : 40%;  $P_3$ : 10%;  $P_4$ : 10%.
- B)  $P_1$ : 25%;  $P_2$ : 25%;  $P_3$ : 25%;  $P_4$ : 25%.
- C)  $P_1$ : 37.5%;  $P_2$ : 37.5%;  $P_3$ : 12.5%;  $P_4$ : 12.5%.
- D)  $P_1$ : 40%;  $P_2$ : 30%;  $P_3$ : 20%;  $P_4$ : 10%.
- E) none of these

Answer: C

**Refer to the weighted voting system  $[12 : 5, 5, 2, 2]$  and the Banzhaf definition of power. (The four players are  $P_1, P_2, P_3$ , and  $P_4$ .)**

55) What is the weight of the coalition  $\{P_2, P_3, P_4\}$ ?

- A) 6
- B) 9
- C) 12
- D) 10
- E) none of these

Answer: B

56) Which players in the coalition  $\{P_1, P_2, P_3, P_4\}$  are critical?

- A) All four players
- B)  $P_1$  and  $P_2$
- C)  $P_1$  only
- D) None of the players
- E) none of these

Answer: B

57) What is the total number of winning coalitions?

- A) 15
- B) 3
- C) 5
- D) 1
- E) none of these

Answer: B

58) The Banzhaf power distribution of the weighted voting system is

- A)  $P_1$ : 25%;  $P_2$ : 25%;  $P_3$ : 25%;  $P_4$ : 25%.
- B)  $P_1$ : 37.5%;  $P_2$ : 37.5%;  $P_3$ : 12.5%;  $P_4$ : 12.5%.
- C)  $P_1$ : 40%;  $P_2$ : 40%;  $P_3$ : 10%;  $P_4$ : 10%.
- D)  $P_1$ : 40%;  $P_2$ : 30%;  $P_3$ : 20%;  $P_4$ : 10%.
- E) none of these

Answer: B

**Refer to the weighted voting system  $[25 : 16, 8, 6, 3]$  and the Banzhaf definition of power. (The four players are  $P_1, P_2, P_3$  and  $P_4$ .)**

59) The weight of the coalition  $\{P_1, P_2, P_3, P_4\}$  is

- A) 33.
- B) 17.
- C) 34.
- D) 39.
- E) none of these

Answer: A

60) Which players are critical in the coalition  $\{P_1, P_3, P_4\}$ ?

- A) All three players
- B)  $P_1$  and  $P_3$  only
- C) None of the players
- D)  $P_1$  only
- E) none of these

Answer: A

61) Which players are critical in the coalition  $\{P_1, P_2, P_3, P_4\}$ ?

- A)  $P_1$  and  $P_2$  only
- B)  $P_1$  only
- C) All of the players
- D) None of the players
- E) none of these

Answer: B

62) Which players have veto power?

- A) None of the players
- B)  $P_1$  only
- C) All of the players
- D)  $P_1$  and  $P_2$  only
- E) none of these

Answer: B

63) The winning coalitions are

- A) all coalitions with three or more players, one of which is  $P_1$ .
- B) all coalitions with two or more players.
- C) all coalitions with three or more players.
- D) all coalitions with  $P_1$  in it.
- E) none of these

Answer: A

64) The number of winning coalitions is

- A) 15.
- B) 3.
- C) 4.
- D) 5.
- E) none of these

Answer: C

65) The Banzhaf power distribution of the weighted voting system is

- A)  $P_1$ : 40%;  $P_2$ : 20%;  $P_3$ : 20%;  $P_4$ : 20%.
- B)  $P_1$ : 25%;  $P_2$ : 25%;  $P_3$ : 25%;  $P_4$ : 25%.
- C)  $P_1$ : 60%;  $P_2$ : 20%;  $P_3$ : 10%;  $P_4$ : 10%.
- D)  $P_1$ : 50%;  $P_2$ : 30%;  $P_3$ : 10%;  $P_4$ : 10%.
- E) none of these

Answer: A

Refer to the weighted voting system  $[25 : 16, 9, 9, 7]$  and the Banzhaf definition of power. (The four players are  $P_1, P_2, P_3$  and  $P_4$ .)

66) The weight of the coalition  $\{P_1, P_3, P_4\}$  is

- A) 32.
- B) 43.
- C) 41.
- D) 25.
- E) none of these

Answer: A

67) Which players are critical in the coalition  $\{P_1, P_3, P_4\}$ ?

- A)  $P_1$  and  $P_3$  only
- B) None of the players
- C)  $P_1$  only
- D) All three players
- E) none of these

Answer: A

68) Which players are critical in the coalition  $\{P_1, P_2, P_3, P_4\}$ ?

- A)  $P_1, P_2$ , and  $P_3$  only
- B)  $P_1$  only
- C) All of the players
- D)  $P_1$  and  $P_2$  only
- E) None of the players

Answer: E

69) In this weighted voting system, which players have veto power?

- A)  $P_1, P_2$ , and  $P_3$  only
- B) All of the players
- C)  $P_1$  only
- D)  $P_1$  and  $P_2$  only
- E) None of the players

Answer: E

70) The winning coalitions are:

- A)  $\{P_1, P_2\}$ ,  $\{P_1, P_3\}$ , and all coalitions with three or more players.
- B) all coalitions with two or more players.
- C) all coalitions with three or more players.
- D) all coalitions with  $P_1$  in it.
- E) none of these

Answer: A

71) The number of winning coalitions is

- A) 8.
- B) 5.
- C) 7.
- D) 15.
- E) none of these

Answer: C

72) The Banzhaf power distribution of the weighted voting system is

- A)  $P_1: \frac{1}{2}; P_2: \frac{1}{5}; P_3: \frac{1}{5}; P_4: \frac{1}{10}$ .
- B)  $P_1: \frac{7}{12}; P_2: \frac{1}{6}; P_3: \frac{1}{6}; P_4: \frac{1}{12}$ .
- C)  $P_1: \frac{1}{3}; P_2: \frac{1}{4}; P_3: \frac{1}{4}; P_4: \frac{1}{6}$ .
- D)  $P_1: \frac{5}{12}; P_2: \frac{1}{4}; P_3: \frac{1}{4}; P_4: \frac{1}{12}$ .
- E) none of these

Answer: D

**Refer to the weighted voting system  $[26 : 16, 9, 9, 7]$  and the Banzhaf definition of power. (The four players are  $P_1, P_2, P_3$  and  $P_4$ .)**

73) Which players are critical in the coalition  $\{P_1, P_3, P_4\}$ ?

- A)  $P_1$  and  $P_3$  only
- B) All three players
- C)  $P_1$  only
- D) None of the players
- E) none of these

Answer: B

74) Which players are critical in the coalition  $\{P_1, P_2, P_3, P_4\}$ ?

- A)  $P_1$  and  $P_2$  only
- B)  $P_1$  only
- C)  $P_1, P_2$  and  $P_3$  only
- D) All of the players
- E) None of the players

Answer: B

75) In this weighted voting system, which players have veto power?

- A) All of the players
- B)  $P_1, P_2$  and  $P_3$  only
- C)  $P_1$  only
- D)  $P_1$  and  $P_2$  only
- E) None of the players

Answer: C

76) The winning coalitions are

- A) all coalitions with three or more players.
- B) all coalitions with three or more players except for  $\{P_2, P_3, P_4\}$ .
- C) only the grand coalition.
- D) all coalitions with two or more players except for  $\{P_3, P_4\}$ .
- E) none of these

Answer: B

77) The number of losing coalitions is

- A) 11.
- B) 4.
- C) 14.
- D) 1.
- E) none of these

Answer: A

78) The Banzhaf power distribution of the weighted voting system is

- A)  $P_1: \frac{2}{5}; P_2: \frac{1}{5}; P_3: \frac{1}{5}; P_4: \frac{1}{5}$ .
- B)  $P_1: \frac{1}{2}; P_2: \frac{1}{6}; P_3: \frac{1}{6}; P_4: \frac{1}{6}$ .
- C)  $P_1: \frac{1}{2}; P_2: \frac{3}{10}; P_3: \frac{1}{10}; P_4: \frac{1}{10}$ .
- D)  $P_1: \frac{5}{12}; P_2: \frac{1}{4}; P_3: \frac{1}{4}; P_4: \frac{1}{12}$ .
- E) none of these

Answer: A

**Solve the problem.**

79) The Banzhaf power index of player  $P_4$  in the weighted voting system  $[10 : 3, 3, 3, 2]$  is

- A)  $\frac{1}{12}$
- B) 0
- C)  $\frac{1}{4}$
- D)  $\frac{1}{6}$
- E) none of these

Answer: C

**Refer to the weighted voting system  $[22 : 10, 8, 6, 4, 2]$  and the Banzhaf definition of power. (The five players will be called  $P_1, P_2, P_3, P_4$ , and  $P_5$ .)**

80) The number of coalitions is

- A) 63.
- B) 15.
- C) 31.
- D) 120.
- E) none of these

Answer: C

81) The number of coalitions having exactly two players is

- A) 1
- B) 20
- C) 10
- D) 5
- E) none of these

Answer: C

82) The number of winning coalitions having exactly two players is

- A) 5
- B) 10
- C) 1
- D) 0
- E) none of these

Answer: D

83) The number of coalitions having exactly three players is

- A) 20
- B) 10
- C) 1
- D) 5
- E) none of these

Answer: B

84) The number of winning coalitions having exactly three players is

- A) 5
- B) 2
- C) 10
- D) 1
- E) none of these

Answer: B

85) The number of coalitions having exactly four players is

- A) 10
- B) 20
- C) 5
- D) 1
- E) none of these

Answer: C

86) The number of winning coalitions having exactly four players is

- A) 2
- B) 1
- C) 3
- D) 4
- E) none of these

Answer: D



- 87) In this weighted voting system,
- A)  $P_3$  has three times as much power as  $P_4$ .
  - B)  $P_3$  has twice as much power as  $P_4$ .
  - C)  $P_3$  has four times as much power as  $P_5$ .
  - D)  $P_3$  and  $P_4$  have the same power.
  - E) none of these

Answer: D

- 88) In this weighted voting system, giving any individual player one more vote has the effect of
- A) giving that player no more power.
  - B) giving that player  $1/31$  more power.
  - C) giving that player  $1/19$  more power.
  - D) giving that player  $1/5$  more power.
  - E) none of these

Answer: A

**Refer to the weighted voting system  $[24 : 10, 8, 6, 4, 2]$  and the Banzhaf definition of power. (The five players will be called  $P_1, P_2, P_3, P_4$ , and  $P_5$ .)**

- 89) The number of coalitions is
- A) 31.
  - B) 15.
  - C) 63.
  - D) 120.
  - E) none of these

Answer: A

- 90) The number of winning coalitions is
- A) 3.
  - B) 10.
  - C) 5.
  - D) 15.
  - E) none of these

Answer: C

- 91) In this voting system,
- A)  $P_3$  and  $P_4$  have the same power.
  - B)  $P_3$  has twice as much power as  $P_4$ .
  - C)  $P_3$  has four times as much power as  $P_5$ .
  - D)  $P_3$  has three times as much power as  $P_4$ .
  - E) none of these

Answer: D

**A committee consists of six members (A, B, C, D, E, and F). A has veto power; B, C, D, and E each have one vote. F is a nonvoting member. For a motion to pass it must have the support of A plus at least two additional voting members.**

92) Which of the following is not a winning coalition?

- A) {B, C, D, E}
- B) {A, B, C, D}
- C) {A, B, E}
- D) {A, C, D, E}
- E) none of these

Answer: A

93) Which are the critical players in the coalition {A, B, D}?

- A) B only
- B) A, B, and D
- C) D only
- D) A only
- E) none of these

Answer: B

94) The Banzhaf power index of player A is

- A)  $\frac{7}{11}$
- B)  $\frac{1}{3}$
- C)  $\frac{5}{11}$
- D)  $\frac{11}{23}$

E) none of these

Answer: D

**A committee consists of five members (A, B, C, D, and E). A and B have veto power; C, D, and E each have one vote. For a motion to pass it must have the support of both A and B plus at least one additional member.**

95) Which of the following is not a winning coalition?

- A) {B, C, D, E}
- B) {A, B, C, D}
- C) {A, B, C, D, E}
- D) {A, B, E}
- E) none of these

Answer: A

96) Which are the critical players in the coalition {A, B, D, E}?

- A) E only
- B) D only
- C) A and B only
- D) A, B, D and E
- E) none of these

Answer: C

97) The Banzhaf power distribution of the committee is

A) A:  $\frac{1}{5}$ ; B:  $\frac{1}{5}$ ; C:  $\frac{1}{5}$ ; D:  $\frac{1}{5}$ ; E:  $\frac{1}{5}$ .

B) A:  $\frac{7}{17}$ ; B:  $\frac{7}{17}$ ; C:  $\frac{1}{17}$ ; D:  $\frac{1}{17}$ ; E:  $\frac{1}{17}$ .

C) A:  $\frac{7}{20}$ ; B:  $\frac{7}{20}$ ; C:  $\frac{1}{10}$ ; D:  $\frac{1}{10}$ ; E:  $\frac{1}{10}$ .

D) A:  $\frac{7}{20}$ ; B:  $\frac{7}{20}$ ; C:  $\frac{3}{20}$ ; D:  $\frac{1}{10}$ ; E:  $\frac{1}{20}$ .

E) none of these

Answer: B

**Solve the problem.**

98) Consider the generic weighted voting system  $[q : w_1, w_2, w_3, w_4, w_5]$ . Suppose that the winning coalitions are exactly those having 3 or more players. Compute the Banzhaf power index of player  $P_1$ .

A)  $\frac{1}{5}$

B)  $\frac{1}{10}$

C) 0

D)  $\frac{1}{2}$

E) none of these

Answer: A

99) Consider the generic weighted voting system  $[q : w_1, w_2, w_3, w_4, w_5]$ . Suppose that the winning coalitions are  $\{P_1, P_2, P_3\}$ ,  $\{P_1, P_2, P_3, P_4\}$ ,  $\{P_1, P_2, P_3, P_5\}$ , and  $\{P_1, P_2, P_3, P_4, P_5\}$ . Find the Banzhaf power distribution.

A)  $P_1: \frac{1}{5}, P_2: \frac{1}{5}, P_3: \frac{1}{5}, P_4: \frac{1}{5}, P_5: \frac{1}{5}$

B)  $P_1: \frac{5}{16}, P_2: \frac{5}{16}, P_3: \frac{1}{4}, P_4: \frac{1}{16}, P_5: \frac{1}{16}$

C)  $P_1: \frac{1}{4}, P_2: \frac{1}{4}, P_3: \frac{1}{4}, P_4: \frac{1}{8}, P_5: \frac{1}{8}$

D)  $P_1: \frac{1}{3}, P_2: \frac{1}{3}, P_3: \frac{1}{3}, P_4: 0, P_5: 0$

E) none of these

Answer: D

100) In the weighted voting system  $[21 : 12, 8, 6, 3, 2]$ , the total number of possible coalitions is

A) 23.

B) 32.

C) 31.

D) 63.

E) none of these

Answer: C

- 101) In the weighted voting system  $[21 : 10, 8, 5, 3, 2]$ , the total number of possible coalitions is
- A) 31.
  - B) 16.
  - C) 63.
  - D) 32.
  - E) none of these

Answer: A

**Refer to the weighted voting system  $[10 : 7, 5, 4]$  and the Shapley–Shubik definition of power. (The three players are  $P_1$ ,  $P_2$ , and  $P_3$ .)**

- 102) Which player in the sequential coalition  $\langle P_1, P_2, P_3 \rangle$  is pivotal?
- A)  $P_2$
  - B)  $P_1$
  - C)  $P_3$
  - D) All three players
  - E) none of these

Answer: A

- 103) Which player in the sequential coalition  $\langle P_3, P_2, P_1 \rangle$  is pivotal?
- A)  $P_1$
  - B)  $P_3$
  - C)  $P_2$
  - D) All three players
  - E) none of these

Answer: A

- 104) In how many sequential coalitions is  $P_2$  the pivotal player?
- A) 6
  - B) 0
  - C) 2
  - D) 1
  - E) none of these

Answer: D

- 105) The Shapley–Shubik power distribution of the weighted voting system is
- A)  $P_1: \frac{1}{2}; P_2: \frac{1}{2}; P_3: 0$ .
  - B)  $P_1: \frac{2}{3}; P_2: \frac{1}{6}; P_3: \frac{1}{6}$ .
  - C)  $P_1: \frac{1}{3}; P_2: \frac{1}{3}; P_3: \frac{1}{3}$ .
  - D)  $P_1: \frac{1}{2}; P_2: \frac{1}{3}; P_3: \frac{1}{6}$ .
  - E) none of these

Answer: B

Refer to the weighted voting system  $[8: 6, 3, 2]$  and the Shapley–Shubik definition of power. (The three players are  $P_1$ ,  $P_2$ , and  $P_3$ .)

106) Which player in the sequential coalition  $\langle P_1, P_2, P_3 \rangle$  is pivotal?

- A)  $P_3$
- B)  $P_2$
- C)  $P_1$
- D) All three players
- E) none of these

Answer: B

107) Which player in the sequential coalition  $\langle P_3, P_2, P_1 \rangle$  is pivotal?

- A)  $P_2$
- B)  $P_3$
- C)  $P_1$
- D) All three players
- E) none of these

Answer: C

108) In how many sequential coalitions is  $P_2$  the pivotal player?

- A) 0
- B) 1
- C) 6
- D) 2
- E) none of these

Answer: B

109) The Shapley–Shubik power distribution of the weighted voting system is

- A)  $P_1: \frac{1}{2}; P_2: \frac{1}{3}; P_3: \frac{1}{6}$ .
- B)  $P_1: \frac{2}{3}; P_2: \frac{1}{6}; P_3: \frac{1}{6}$ .
- C)  $P_1: \frac{1}{3}; P_2: \frac{1}{3}; P_3: \frac{1}{3}$ .
- D)  $P_1: \frac{1}{2}; P_2: \frac{1}{2}; P_3: 0$ .
- E) none of these

Answer: B

Refer to the weighted voting system  $[10 : 7, 6, 4]$  and the Shapley–Shubik definition of power. (The three players are  $P_1$ ,  $P_2$ , and  $P_3$ .)

110) Which player in the sequential coalition  $\langle P_3, P_2, P_1 \rangle$  is pivotal?

- A)  $P_2$
- B)  $P_1$
- C)  $P_3$
- D) All three players
- E) none of these

Answer: A

111) In how many sequential coalitions is  $P_2$  the pivotal player?

- A) 1
- B) 0
- C) 2
- D) 6
- E) none of these

Answer: C

112) The Shapley–Shubik power distribution of the weighted voting system is

- A)  $P_1: \frac{1}{2}; P_2: \frac{1}{3}; P_3: \frac{1}{6}$ .
- B)  $P_1: \frac{1}{2}; P_2: \frac{1}{2}; P_3: 0$ .
- C)  $P_1: \frac{2}{3}; P_2: \frac{1}{6}; P_3: \frac{1}{6}$ .
- D)  $P_1: \frac{1}{3}; P_2: \frac{1}{3}; P_3: \frac{1}{3}$ .
- E) none of these

Answer: D

Refer to the weighted voting system  $[9 : 4, 3, 2, 1]$  and the Shapley–Shubik definition of power. (The four players will be called  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$ .)

113) The number of sequential coalitions is

- A) 31.
- B) 24.
- C) 16.
- D) 6.
- E) none of these

Answer: B

114) The Shapley–Shubik power index of player  $P_4$  is

- A)  $\frac{1}{10}$ .
- B)  $\frac{1}{9}$ .
- C) 0.
- D)  $\frac{1}{4}$ .
- E) none of these

Answer: C

115) The number of sequential coalitions is

- A) 6.
- B) 16.
- C) 32.
- D) 24.
- E) none of these

Answer: D

116) Which player in the sequential coalition  $\langle P_1, P_2, P_3, P_4 \rangle$  is pivotal?

- A)  $P_2$
- B)  $P_3$
- C)  $P_1$
- D)  $P_4$
- E) none of these

Answer: B

117) Which player in the sequential coalition  $\langle P_2, P_3, P_4, P_1 \rangle$  is pivotal?

- A)  $P_1$
- B)  $P_2$
- C)  $P_3$
- D)  $P_4$
- E) none of these

Answer: A

118) In how many sequential coalitions is player  $P_4$  pivotal?

- A) 6
- B) 1
- C) 0
- D) 2
- E) none of these

Answer: C

119) The Shapley–Shubik power distribution of the weighted voting system is

- A)  $P_1: \frac{1}{4}; P_2: \frac{1}{4}; P_3: \frac{1}{4}; P_4: \frac{1}{4}$ .
- B)  $P_1: \frac{5}{12}; P_2: \frac{1}{3}; P_3: \frac{5}{24}; P_4: \frac{1}{24}$ .
- C)  $P_1: \frac{1}{3}; P_2: \frac{1}{3}; P_3: \frac{1}{3}; P_4: 0$ .
- D)  $P_1: \frac{2}{3}; P_2: \frac{1}{6}; P_3: \frac{1}{6}; P_4: 0$ .

E) none of these

Answer: C

**Refer to the weighted voting system [22 : 10, 8, 6, 4, 2] and the Shapley–Shubik definition of power. (The five players will be called  $P_1, P_2, P_3, P_4$ , and  $P_5$ .)**

120) In how many sequential coalitions is player  $P_5$  pivotal?

- A) 12
- B) 6
- C) 36
- D) 24
- E) none of these

Answer: B

121) If player  $P_5$  is pivotal in a sequential coalition, which player does not appear before  $P_5$ ?

- A)  $P_5$  is never pivotal
- B)  $P_2$
- C)  $P_3$
- D)  $P_4$
- E)  $P_1$

Answer: B

122) In how many sequential coalitions is player  $P_4$  pivotal?

- A) 16
- B) 10
- C) 24
- D) 4
- E) none of these

Answer: A



123) The Shapley–Shubik power index of player  $P_5$  is

A)  $\frac{1}{10}$ .

B)  $\frac{1}{20}$ .

C)  $\frac{3}{10}$ .

D)  $\frac{1}{5}$ .

E) none of these

Answer: B

**Refer to the weighted voting system  $[26 : 10, 8, 6, 4, 2]$  and the Shapley–Shubik definition of power. (The five players will be called  $P_1, P_2, P_3, P_4$ , and  $P_5$ .)**

124) In how many sequential coalitions is player  $P_5$  pivotal?

A) 36

B) 12

C) 24

D) 6

E) none of these

Answer: D

125) In how many sequential coalitions is player  $P_1$  pivotal?

A) 24

B) 36

C) 6

D) 12

E) none of these

Answer: B

126) The Shapley–Shubik power index of player  $P_5$  is

A)  $\frac{1}{5}$ .

B)  $\frac{3}{10}$ .

C)  $\frac{1}{20}$ .

D)  $\frac{1}{10}$ .

E) none of these

Answer: C

**Solve the problem.**

127) The Shapley-Shubik power index of player  $P_4$  in the weighted voting system  $[10 : 3, 3, 3, 2]$  is

- A) 0
- B)  $\frac{1}{12}$
- C)  $\frac{1}{4}$
- D)  $\frac{1}{6}$
- E) none of these

Answer: C

128) Which of the following is not a possible Shapley-Shubik power index for a player in a weighted voting system with three players?

- A) 0
- B)  $\frac{1}{3}$
- C)  $\frac{1}{4}$
- D)  $\frac{1}{2}$
- E) none of these

Answer: C

129) Which of the following is not a possible Shapley-Shubik power index for a player in a weighted voting system with four players?

- A)  $\frac{1}{4}$
- B)  $\frac{1}{6}$
- C)  $\frac{1}{3}$
- D)  $\frac{1}{5}$
- E) none of these

Answer: D

130) Consider the generic weighted voting system  $[q : w_1, w_2, w_3, w_4]$ . Suppose that  $w_2 = w_3$ ,  $P_1$  is a pivotal player 12 times and  $P_2$  is pivotal 4 times. What is the Shapley-Shubik power index of  $P_4$ ?

- A)  $\frac{1}{2}$
- B)  $\frac{1}{6}$
- C)  $\frac{1}{3}$
- D) 0
- E) Can't be determined from the given information.

Answer: B

- 131) In the weighted voting system  $[17 : 10, 7, 5, 3, 1]$ , the total number of possible sequential coalitions involving all five players is
- A) 31.
  - B) 720.
  - C) 24.
  - D) 120.
  - E) none of these

Answer: D

- 132) In the weighted voting system  $[21 : 10, 8, 5, 3, 2]$ , the total number of possible sequential coalitions involving all five players is
- A) 720.
  - B) 16.
  - C) 120.
  - D) 24.
  - E) none of these

Answer: C

- 133)  $\frac{100!}{98!} =$
- A) 9,900
  - B) 199
  - C) 100
  - D) 2
  - E) none of these

Answer: A

- 134)  $\frac{200!}{198!} =$
- A) 399
  - B) 200
  - C) 2
  - D) 39,800
  - E) none of these

Answer: D

- 135)  $99! + 100! =$
- A)  $2 \times 100! - 100$
  - B)  $199!$
  - C)  $101 \times 99!$
  - D)  $2 \times 99! + 100$
  - E) none of these

Answer: C

- 136)  $199! + 200! =$
- A)  $2 \times 199! + 200$
  - B)  $2 \times 200! - 200$
  - C)  $201 \times 199!$
  - D)  $399!$
  - E) none of these

Answer: C

137)  $300! - 299! =$

- A)  $299^2 \times 298!$
- B) 1
- C)  $300 - 2 \times 299!$
- D)  $299 \times 299!$
- E) none of these

Answer: D

138) A, B, C, D, E and F are the starting six players on a hockey team. The coach must choose a set of honorary "captains" for the last game of the season – it can be any number from one to all six. How many different possibilities are there?

- A) 720
- B) 63
- C) 35
- D) 6
- E) none of these

Answer: B

139) A, B, C, D, and E are the starting five players in a basketball team. The coach must choose a set of honorary "captains" for the last game of the season – it can be any number from one to all five. How many different possibilities are there?

- A) 31
- B) 24
- C) 120
- D) 5
- E) none of these

Answer: A

140) The Tasmania State University football team has 11 starting players on their offense. The coach must select an order in which they will be introduced for the last game of the season. How many different possibilities are there?

- A)  $11!$
- B)  $11^2$
- C)  $2^{11} - 1$
- D) 11
- E) none of these

Answer: A

141) A weighted voting system has 100 players. How many coalitions of size 99 are possible?

- A) 100
- B) 50
- C) 99
- D) 1
- E) none of these

Answer: A

142) A weighted voting system has 100 players. How many coalitions of size 98 are possible?

- A) 98
- B) 4900
- C) 8900
- D) 4950
- E) none of these

Answer: D

143) A weighted voting system has 100 players. How many sequential coalitions in which  $P_{50}$  is listed first are possible?

- A)  $50!$
- B)  $100! - 1$
- C)  $\frac{100!}{50}$
- D)  $99!$
- E) none of these

Answer: D

144) Consider the generic weighted voting system  $[q : w_1, w_2, w_3, w_4, w_5]$ . Suppose that the winning coalitions are exactly those having 3 or more players. Which players are critical in the coalition  $\{P_1, P_2, P_4, P_5\}$ ?

- A)  $P_1, P_2, P_4, P_5$
- B)  $P_1$  only
- C)  $P_1, P_2$ , and  $P_4$
- D) None of the players
- E) none of these

Answer: D

145) Consider the generic weighted voting system  $\{q: w_1, w_2, w_3, w_4, w_5\}$ . Suppose that the winning coalitions are  $\{P_1, P_2, P_3\}$ ,  $\{P_1, P_2, P_3, P_4\}$ ,  $\{P_1, P_2, P_3, P_5\}$ , and  $\{P_1, P_2, P_3, P_4, P_5\}$ . Which players are critical in the grand coalition  $\{P_1, P_2, P_3, P_4, P_5\}$ ?

- A)  $P_1, P_2$ , and  $P_3$
- B)  $P_1, P_2, P_3$ , and  $P_4$
- C)  $P_1$  only
- D) None of the players
- E) none of these

Answer: A

146) Consider the generic weighted voting system  $\{q: w_1, w_2, w_3, w_4, w_5\}$ . Suppose that the winning coalitions are  $\{P_1, P_2, P_3\}$ ,  $\{P_1, P_2, P_3, P_4\}$ ,  $\{P_1, P_2, P_3, P_5\}$ , and  $\{P_1, P_2, P_3, P_4, P_5\}$ . Which players have veto power?

- A)  $P_1, P_2$ , and  $P_3$
- B)  $P_1$  only
- C)  $P_1, P_2, P_3$ , and  $P_4$
- D) None of the players
- E) All of the players

Answer: A

- 147) In any weighted voting system having  $N$  players, what is the minimum number of winning coalitions possible?
- A) 0
  - B)  $N$
  - C) 1
  - D)  $N - 1$
  - E) none of these

Answer: C

- 148) In any weighted voting system having  $N$  players, what is the maximum number of players that can have veto power?
- A)  $N$
  - B)  $N - 1$
  - C) 2
  - D) 1
  - E) none of these

Answer: A

- 149) Two weighted voting systems are equivalent if they have the same number of players and exactly the same winning coalitions. Which of the following weighted voting systems are equivalent to  $[5 : 3, 2, 1, 1]$ ?
- A)  $[9 : 5, 4, 3, 1]$
  - B)  $[8 : 4, 3, 2, 1]$
  - C)  $[6 : 3, 2, 1, 1]$
  - D)  $[10 : 6, 5, 4, 2]$
  - E)  $[7 : 3, 2, 1, 1]$

Answer: A