Chapter 2
Right Triangle Trigonometry

2.1 Definition II: Right Triangle Trigonometry

EVEN SOLUTIONS

2. Using Definition II and Figure 8, we would refer to a as the side opposite A, b as the side adjacent to A, and c as the hypotenuse.

4. **a**. cosine (ii) **b**. cosecant (iii) **c**. cotangent (i)

6. Using the Pythagorean Theorem, first find a:

$$\begin{aligned} a^2+8^2&=17^2\\ a^2+64&=289\\ a^2&=225\\ a&=15 \end{aligned}$$

Using a = 15, b = 8, and c = 17, write the six trigonometric functions of A:

$$\sin A=\frac{a}{c}=\frac{15}{17} \qquad \cos A=\frac{b}{c}=\frac{8}{17} \qquad \tan A=\frac{a}{b}=\frac{15}{8}$$

$$\csc A=\frac{c}{a}=\frac{17}{15} \qquad \sec A=\frac{c}{b}=\frac{17}{8} \qquad \cot A=\frac{b}{a}=\frac{8}{15}$$

8. Using the Pythagorean Theorem, first find c:

$$\begin{aligned} 5^2+2^2&=c^2\\ 25+4&=c^2\\ c^2&=29\\ c&=\sqrt{29} \end{aligned}$$

Using a = 5, b = 2, and $c=\sqrt{29}$, write the six trigonometric functions of A:

$$\sin A=\frac{a}{c}=\frac{5}{\sqrt{29}}=\frac{5\sqrt{29}}{29} \qquad \cos A=\frac{b}{c}=\frac{2}{\sqrt{29}}=\frac{2\sqrt{29}}{29} \qquad \tan A=\frac{a}{b}=\frac{5}{2}$$

$$\csc A=\frac{c}{a}=\frac{\sqrt{29}}{5} \qquad \sec A=\frac{c}{b}=\frac{\sqrt{29}}{2} \qquad \cot A=\frac{b}{a}=\frac{2}{5}$$

10. Using the Pythagorean Theorem, first find c:

$$\begin{aligned} 5^2+\left(\sqrt{11}\right)^2&=c^2\\ 25+11&=c^2\\ c^2&=36\\ c&=6 \end{aligned}$$

Using $a = 5$, $b = \sqrt{11}$, and $c = 6$, write the six trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{5}{6} \qquad \cos A = \frac{b}{c} = \frac{\sqrt{11}}{6} \qquad \tan A = \frac{a}{b} = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\csc A = \frac{c}{a} = \frac{6}{5} \qquad \sec A = \frac{c}{b} = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11} \qquad \cot A = \frac{b}{a} = \frac{\sqrt{11}}{5}$$

12. Using the Pythagorean Theorem, first find a:

$$a^2 + 3^2 = 4^2$$
$$a^2 + 9 = 16$$
$$a^2 = 7$$
$$a = \sqrt{7}$$

Using $a = \sqrt{7}$, $b = 3$, and $c = 4$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{\sqrt{7}}{4} \qquad \cos A = \frac{b}{c} = \frac{3}{4} \qquad \tan A = \frac{a}{b} = \frac{\sqrt{7}}{3}$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{3}{4} \qquad \cos B = \sin A = \frac{\sqrt{7}}{4} \qquad \tan B = \cot A = \frac{b}{a} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

14. Using the Pythagorean Theorem, first find c:

$$3^2 + 1^2 = c^2$$
$$9 + 1 = c^2$$
$$c^2 = 10$$
$$c = \sqrt{10}$$

Using $a = 3$, $b = 1$, and $c = \sqrt{10}$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} \qquad \cos A = \frac{b}{c} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \qquad \tan A = \frac{a}{b} = \frac{3}{1} = 3$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \qquad \cos B = \sin A = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} \qquad \tan B = \cot A = \frac{b}{a} = \frac{1}{3}$$

16. Using the Pythagorean Theorem, first find c:

$$1^2 + \left(\sqrt{5}\right)^2 = c^2$$
$$1 + 5 = c^2$$
$$c^2 = 6$$
$$c = \sqrt{6}$$

Using $a = 1$, $b = \sqrt{5}$, and $c = \sqrt{6}$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} \qquad \cos A = \frac{b}{c} = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6} \qquad \tan A = \frac{a}{b} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6} \qquad \cos B = \sin A = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} \qquad \tan B = \cot A = \frac{b}{a} = \sqrt{5}$$

18. Using the Pythagorean Theorem, first find c:

$$x^2 + x^2 = c^2$$
$$c^2 = 2x^2$$
$$c = \sqrt{2}\,x$$

Using $a = x$, $b = x$, and $c = \sqrt{2}\,x$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{x}{\sqrt{2}\,x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos A = \frac{b}{c} = \frac{x}{\sqrt{2}\,x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan A = \frac{a}{b} = \frac{x}{x} = 1$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos B = \sin A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan B = \cot A = \frac{b}{a} = \frac{x}{x} = 1$$

20. The coordinates of point B are $B(8,6)$. Using the Pythagorean Theorem, first find c:

$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$c^2 = 100$$
$$c = 10$$

Using $a = 6$, $b = 8$, and $c = 10$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5} \qquad \cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5} \qquad \tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$$

22. Since $b \le c$, $\frac{c}{b} \ge 1$. Since $\sec\theta = \frac{c}{b} \ge 1$, it is impossible for $\sec\theta = \frac{1}{2}$.

24. Since $b \le c$, $\frac{c}{b} \ge 1$ and can be as large as possible. Since $\sec\theta = \frac{c}{b}$, $\sec\theta$ can be as large as possible.

26. Using the Cofunction Theorem, $\cos 70° = \sin 20°$.

28. Using the Cofunction Theorem, $\cot 22° = \tan 68°$.

30. Using the Cofunction Theorem, $\csc y = \sec(90° - y)$.

32. Using the Cofunction Theorem, $\sin(90° - y) = \cos y$.

34. Complete the table, using the ratio identity $\sec x = \frac{1}{\cos x}$:

x	$\cos x$	$\sec x$
0°	1	1
30°	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{2}{\sqrt{2}} = \sqrt{2}$
60°	$\frac{1}{2}$	2
90°	0	undefined

36. Simplifying the expression: $5\sin^2 60° = 5\left(\frac{\sqrt{3}}{2}\right)^2 = 5 \cdot \frac{3}{4} = \frac{15}{4}$

38. Simplifying the expression: $\cos^3 60° = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

40. Simplifying the expression: $(\sin 60° + \cos 60°)^2 = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^2 = \left(\frac{\sqrt{3}+1}{2}\right)^2 = \frac{4+2\sqrt{3}}{4} = \frac{2+\sqrt{3}}{2}$

42. Simplifying the expression: $\left(\sin 45° - \cos 45°\right)^2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 = 0^2 = 0$

44. Simplifying the expression: $\tan^2 45° + \tan^2 60° = 1^2 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4$

46. Simplifying the expression: $6\cos x = 6\cos 30° = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$

48. Simplifying the expression: $-2\sin\left(90° - y\right) = -2\sin\left(90° - 45°\right) = -2\sin 45° = -2 \cdot \frac{\sqrt{2}}{2} = -\sqrt{2}$

50. Simplifying the expression: $5\sin 2y = 5\sin\left(2 \cdot 45°\right) = 5\sin 90° = 5 \cdot 1 = 5$

52. Simplifying the expression: $2\cos\left(90° - z\right) = 2\cos\left(90° - 60°\right) = 2\cos 30° = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

54. Finding the exact value: $\csc 30° = \frac{1}{\sin 30°} = \frac{1}{1/2} = 2$

56. Finding the exact value: $\sec 60° = \frac{1}{\cos 60°} = \frac{1}{1/2} = 2$

58. Finding the exact value: $\cot 30° = \frac{\cos 30°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

60. Finding the exact value: $\csc 45° = \frac{1}{\sin 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

62. Finding the exact value: $\sec 90° = \frac{1}{\cos 90°} = \frac{1}{0}$, which is undefined

64. Finding the exact value: $\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$, which is undefined

66. First find a using the Pythagorean Theorem:

$$\begin{aligned} 3.68^2 + b^2 &= 5.93^2 \\ b^2 &= 5.93^2 - 3.68^2 \\ b^2 &= 21.6225 \\ b &= 4.65 \end{aligned}$$

Now find $\sin A$ and $\cos A$:

$\sin A = \frac{a}{c} = \frac{3.68}{5.93} \approx 0.62$ $\qquad$ $\cos A = \frac{b}{c} = \frac{4.65}{5.93} \approx 0.78$

Using the Cofunction Theorem:

$\sin B = \cos A \approx 0.78$ $\qquad$ $\cos B = \sin A \approx 0.62$

68. First find c using the Pythagorean Theorem:

$$\begin{aligned} 13.64^2 + 4.77^2 &= c^2 \\ c^2 &= 208.8025 \\ c &= 14.45 \end{aligned}$$

Now find $\sin A$ and $\cos A$:

$\sin A = \frac{a}{c} = \frac{13.64}{14.45} \approx 0.94$ $\qquad$ $\cos A = \frac{b}{c} = \frac{4.77}{14.45} \approx 0.33$

Using the Cofunction Theorem:

$\sin B = \cos A \approx 0.33$ $\qquad$ $\cos B = \sin A \approx 0.94$

70. Since $CG = CD = 3$, using the Pythagorean Theorem:

$$\begin{aligned}(CG)^2 + (CD)^2 &= (DG)^2\\ 3^2 + 3^2 &= (DG)^2\\ 9 + 9 &= (DG)^2\\ (DG)^2 &= 18\\ DG &= \sqrt{18} = 3\sqrt{2}\end{aligned}$$

Now use the Pythagorean Theorem with ΔDGE:

$$\begin{aligned}(DG)^2 + (GE)^2 &= (DE)^2\\ \left(3\sqrt{2}\right)^2 + 3^2 &= (DE)^2\\ 18 + 9 &= (DE)^2\\ (DE)^2 &= 27\\ DE &= \sqrt{27} = 3\sqrt{3}\end{aligned}$$

Now, let θ represent the angle formed by diagonals *DE* and *DG*. Therefore:

$$\sin\theta = \frac{GE}{DE} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cos\theta = \frac{DG}{DE} = \frac{3\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

72. Let $CG = CD = x$, using the Pythagorean Theorem:

$$\begin{aligned}(CG)^2 + (CD)^2 &= (DG)^2\\ x^2 + x^2 &= (DG)^2\\ (DG)^2 &= 2x^2\\ DG &= \sqrt{2x^2} = \sqrt{2}\,x\end{aligned}$$

Now use the Pythagorean Theorem with ΔDGE:

$$\begin{aligned}(DG)^2 + (GE)^2 &= (DE)^2\\ \left(\sqrt{2}\,x\right)^2 + x^2 &= (DE)^2\\ 2x^2 + x^2 &= (DE)^2\\ (DE)^2 &= 3x^2\\ DE &= \sqrt{3x^2} = \sqrt{3}\,x\end{aligned}$$

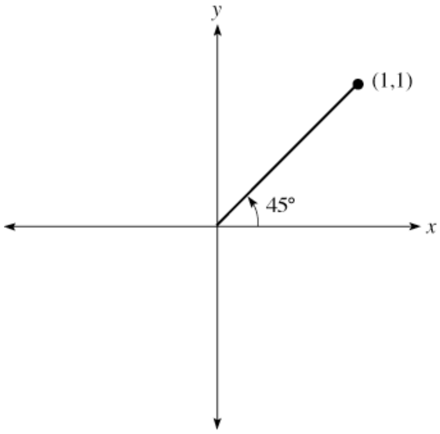
Now, let θ represent the angle formed by diagonals *DE* and *DG*. Therefore:

$$\sin\theta = \frac{GE}{DE} = \frac{x}{\sqrt{3}\,x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cos\theta = \frac{DG}{DE} = \frac{\sqrt{2}\,x}{\sqrt{3}\,x} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

74. Using the distance formula:

$$\begin{aligned}\sqrt{(x-1)^2 + (2-5)^2} &= \left(\sqrt{13}\right)^2\\ (x-1)^2 + 9 &= 13\\ (x-1)^2 &= 4\\ x - 1 &= -2, 2\\ x &= -1, 3\end{aligned}$$

76. A point on the terminal side is $(1,1)$. Drawing the angle in standard position:



78. A coterminal angle to –210° is 150°.

80. Since sin 35° = cos (90° – 35°) = cos 55°, the correct answer is d.

82. Simplifying the expression: $4\cos^2 30° + 2\sin 30° = 4\left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{1}{2}\right) = 4\cdot\frac{3}{4} + 1 = 3 + 1 = 4$. The correct answer is c.

ODD SOLUTIONS

1. triangle measure

3. complement

5.

$$\begin{aligned} a &= \sqrt{c^2 - b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(5)^2 - (3)^2} && \text{Substitute known values} \\ &= \sqrt{25 - 9} && \text{Simplify} \\ &= \sqrt{16} = 4 \end{aligned}$$

$\sin A = \frac{a}{c} = \frac{4}{5}$ $\quad \cot A = \frac{b}{a} = \frac{3}{4}$

$\cos A = \frac{b}{c} = \frac{3}{5}$ $\quad \sec A - \frac{c}{b} = \frac{5}{3}$

$\tan A = \frac{a}{b} = \frac{4}{3}$ $\quad \csc A = \frac{c}{a} = \frac{5}{4}$

7.

$$\begin{aligned} c &= \sqrt{a^2 + b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(2)^2 + (1)^2} && \text{Substitute known values} \\ &= \sqrt{4 + 1} && \text{Simplify} \\ &= \sqrt{5} \end{aligned}$$

$\sin A = \frac{a}{c} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \cot A = \frac{b}{a} = \frac{1}{2}$

$\cos A = \frac{b}{c} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \sec A = \frac{c}{b} = \frac{\sqrt{5}}{1}$

$\tan A = \frac{a}{b} = \frac{2}{1} = 2$ $\quad \csc A = \frac{c}{a} = \frac{\sqrt{5}}{2}$

9.

$$\begin{aligned} c &= \sqrt{a^2 + b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(2)^2 + \left(\sqrt{5}\right)^2} && \text{Substitute known values} \\ &= \sqrt{4 + 5} && \text{Simplify} \\ &= \sqrt{9} = 3 \end{aligned}$$

$\sin A = \frac{a}{c} = \frac{2}{3}$ $\quad \cot A = \frac{b}{a} = \frac{\sqrt{5}}{2}$

$\cos A = \frac{b}{c} = \frac{\sqrt{5}}{3}$ $\quad \sec A = \frac{c}{b} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\tan A = \frac{a}{b} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \csc A = \frac{c}{a} = \frac{3}{2}$

11.

$$\begin{aligned} b &= \sqrt{c^2 - a^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(6)^2 - (5)^2} && \text{Substitute known values} \\ &= \sqrt{36 - 25} && \text{Simplify} \\ &= \sqrt{11} \end{aligned}$$

$\sin A = \frac{a}{c} = \frac{5}{6}$ $\quad \sin B = \frac{b}{c} = \frac{\sqrt{11}}{6}$

$\cos A = \frac{b}{c} = \frac{\sqrt{11}}{6}$ $\quad \cos B = \frac{a}{c} = \frac{5}{6}$

$\tan A = \frac{a}{b} = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$ $\quad \tan B = \frac{b}{a} = \frac{\sqrt{11}}{5}$

13. $c=\sqrt{a^2+b^2}$ Pythagorean Theorem

$=\sqrt{(1)^2+(1)^2}$ Substitute known values

$=\sqrt{1+1}$ Simplify

$=\sqrt{2}$

$\sin A=\frac{a}{c}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$ $\sin B=\frac{b}{c}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\cos A=\frac{b}{c}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$ $\cos B=\frac{a}{c}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\tan A=\frac{a}{b}=\frac{1}{1}=1$ $\tan B=\frac{b}{a}=\frac{1}{1}=1$

15. $b=\sqrt{c^2-a^2}$ Pythagorean Theorem

$=\sqrt{10^2-6^2}$ Substitute known values

$=\sqrt{100-36}$ Simplify

$=\sqrt{64}=8$

$\sin A=\frac{a}{c}=\frac{6}{10}=\frac{3}{5}$ $\sin B=\frac{b}{c}=\frac{8}{10}=\frac{4}{5}$

$\cos A=\frac{b}{c}=\frac{8}{10}=\frac{4}{5}$ $\cos B=\frac{a}{c}=\frac{6}{10}=\frac{3}{5}$

$\tan A=\frac{a}{b}=\frac{6}{8}=\frac{3}{4}$ $\tan B=\frac{b}{a}=\frac{8}{6}=\frac{4}{3}$

17. $a=\sqrt{c^2-b^2}$ Pythagorean Theorem

$=\sqrt{(2x)^2-(x)^2}$ Substitute known values

$=\sqrt{4x^2-x^2}$ Simplify

$=\sqrt{3x^2}$

$=x\sqrt{3}$

$\sin A=\frac{a}{c}=\frac{x\sqrt{3}}{2x}=\frac{\sqrt{3}}{2}$ $\sin B=\frac{b}{c}=\frac{x}{2x}=\frac{1}{2}$

$\cos A=\frac{b}{c}=\frac{x}{2x}=\frac{1}{2}$ $\cos B=\frac{a}{c}=\frac{x\sqrt{3}}{2x}=\frac{\sqrt{3}}{2}$

$\tan A=\frac{a}{b}=\frac{x\sqrt{3}}{x}=\sqrt{3}$ $\tan B=\frac{b}{a}=\frac{x}{x\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

19. The coordinates of B are (4, 3).

$a=3,\quad b=4,\quad c=5$

$\sin A=\frac{a}{c}=\frac{3}{5}$

$\cos A=\frac{b}{c}=\frac{4}{5}$

$\tan A=\frac{a}{b}=\frac{3}{4}$

21. $\cos\theta=\frac{\text{adj side}}{\text{hyp}}=\frac{3}{1}$ For this to be true, the adjacent side would have to be three times larger than the hypotenuse. This is impossible since the hypotenuse is the longest side of a right triangle.

23. $\tan\theta=\frac{\text{opp side}}{\text{adj side}}$ The opposite and adjacent sides can be any number greater than 0. If we choose a very large number for the opposite side and a very small number for the adjacent side, the ratio will approach infinity.

25. $\sin 10^\circ=\cos(90^\circ-10^\circ)=\cos 80^\circ$

27. $\tan 8^\circ=\cot(90^\circ-8^\circ)=\cot 82^\circ$

29. $\sin x^\circ=\cos(90^\circ-x^\circ)$

31. $\tan(90^\circ-x^\circ)=\cot x^\circ$

33. $\csc x=\frac{1}{\sin x}$

$\csc 0^\circ=\frac{1}{0}$ undefined

$\csc 30^\circ=\frac{1}{1/2}=2$

$\csc 45^\circ=\frac{1}{1/\sqrt{2}}=\sqrt{2}$

$\csc 60^\circ=\frac{1}{\sqrt{3}/2}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\csc 90^\circ=\frac{1}{1}=1$

35. $4\sin 30^\circ=4\left(\frac{1}{2}\right)=2$

37. $\left(2\cos 30^\circ\right)^2 = \left[2\left(\frac{\sqrt{3}}{2}\right)\right]^2 = \left(\sqrt{3}\right)^2 = 3$

39. $$\begin{aligned} \sin^2 60^\circ + \cos^2 60^\circ &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \\ &= \frac{3}{4} + \frac{1}{4} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$$

41. $$\begin{aligned} \sin^2 45^\circ - 2\sin 45^\circ \cos 45^\circ + \cos^2 45^\circ &= \left(\frac{\sqrt{2}}{2}\right)^2 - 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)^2 \\ &= \frac{2}{4} - 2\left(\frac{2}{4}\right) + \frac{2}{4} = 0 \end{aligned}$$

43. $$\begin{aligned} \left(\tan 45^\circ + \tan 60^\circ\right)^2 &= \left(1+\sqrt{3}\right)^2 \\ &= \left(1+\sqrt{3}\right)\left(1+\sqrt{3}\right) \\ &= 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3} \end{aligned}$$

45. $$\begin{aligned} 2\sin 30^\circ &= 2\left(\frac{1}{2}\right) \\ &= 1 \end{aligned}$$

47. $$\begin{aligned} 4\cos\left(z - 30^\circ\right) &= 4\cos\left(60^\circ - 30^\circ\right) \\ &= 4\cos 30^\circ \\ &= 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3} \end{aligned}$$

49. $$\begin{aligned} -3\sin 2\left(30^\circ\right) &= -3\sin 60^\circ \\ &= -3\left(\frac{\sqrt{3}}{2}\right) \\ &= -\frac{3\sqrt{3}}{2} \end{aligned}$$

51. $$\begin{aligned} 2\cos\left(3x - 45^\circ\right) &= 2\cos\left(3\cdot 30^\circ - 45^\circ\right) \\ &= 2\cos\left(90^\circ - 45^\circ\right) \\ &= 2\cos 45^\circ \\ &= 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2} \end{aligned}$$

53.

$\sec 30^\circ = \dfrac{1}{\cos 30^\circ}$	Reciprocal identity
$= \dfrac{1}{\sqrt{3}/2}$	Substitute exact value from Table 1
$= \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$	Division of fractions

55. $$\begin{aligned} \csc 60^\circ &= \frac{1}{\sin 60^\circ} \\ &= \frac{1}{\sqrt{3}/2} \\ &= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \end{aligned}$$

57.

$\cot 45^\circ = \dfrac{\cos 45^\circ}{\sin 45^\circ}$	Ratio identity
$= \dfrac{\sqrt{2}/2}{\sqrt{2}/2}$	Substitute values from Table 1
$= 1$	Simplify

59. $\sec 45^\circ = \dfrac{1}{\cos 45^\circ}$

$= \dfrac{1}{1/\sqrt{2}}$

$= \sqrt{2}$

61. $\cot 60^\circ = \dfrac{\cos 60^\circ}{\sin 60^\circ}$ Ratio identity

$= \dfrac{1/2}{\sqrt{3}/2}$ Substitute values from Table 1

$= \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ Simplify

63. $\csc 90^\circ = \dfrac{1}{\sin 90^\circ}$ Reciprocal identity

$= \dfrac{1}{1} = 1$ Substitute values and simplify

65. $a = \sqrt{c^2 - b^2}$

$= \sqrt{(9.62)^2 - (8.88)^2}$

$= \sqrt{13.69}$

$= 3.70$

$\sin A = \dfrac{a}{c} = \dfrac{3.70}{9.62} = 0.38$

$\cos A = \dfrac{b}{c} = \dfrac{8.88}{9.62} = 0.92$

$\sin B = \dfrac{b}{c} = \dfrac{8.88}{9.62} = 0.92$

$\cos B = \dfrac{a}{c} = \dfrac{3.70}{9.62} = 0.38$

67. $c = \sqrt{a^2 + b^2}$

$= \sqrt{(19.44)^2 + (5.67)^2}$

$= \sqrt{410.0625}$

$= 20.25$

$\sin A = \dfrac{a}{c} = \dfrac{19.44}{20.25} = 0.96$

$\cos A = \dfrac{b}{c} = \dfrac{5.67}{20.25} = 0.28$

$\sin B = \dfrac{b}{c} = \dfrac{5.67}{20.25} = 0.28$

$\cos B = \dfrac{a}{c} = \dfrac{19.44}{20.25} = 0.96$

69. $CH = \sqrt{(CD^2) + (DH)^2}$

$= \sqrt{5^2 + 5^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50} = 5\sqrt{2}$

$CF = \sqrt{(CH)^2 + (FH)^2}$

$= \sqrt{\left(5\sqrt{2}\right)^2 + (5)^2}$

$= \sqrt{50 + 25}$

$= \sqrt{75} = 5\sqrt{3}$

$\sin\theta = \dfrac{FH}{CF}$

$= \dfrac{5}{5\sqrt{3}}$

$= \dfrac{\sqrt{3}}{3}$

$\cos\theta = \dfrac{CH}{CF}$

$= \dfrac{5\sqrt{2}}{5\sqrt{3}}$

$= \dfrac{\sqrt{2}}{\sqrt{3}} \; or \; \dfrac{\sqrt{6}}{3}$

71. $CH = \sqrt{(CD^2) + (DH)^2}$

$= \sqrt{x^2 + x^2}$

$= \sqrt{2x^2}$

$= x\sqrt{2}$

$CF = \sqrt{(CH)^2 + (FH)^2}$

$= \sqrt{\left(x\sqrt{2}\right)^2 + x^2}$

$= \sqrt{2x^2 + x^2}$

$= \sqrt{3x^2} = x\sqrt{3}$

$$\sin\theta = \frac{FH}{CF} = \frac{x}{x\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cos\theta = \frac{CH}{CF} = \frac{x\sqrt{2}}{x\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \, or \, \frac{\sqrt{6}}{3}$$

73. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{[3-(-1)]^2 + [-2-(-4)]^2}$ Substitute known values

$= \sqrt{4^2 + 2^2}$ Simplify

$= \sqrt{16+4}$

$= \sqrt{20} = 2\sqrt{5}$

75. The terminal side is the line $y = -x$. Some points in quadrant II on the line $y = -x$ are $(-1,1)$, $(-2,2)$, and $(-3,3)$.

77. $-135^\circ + 360^\circ = 225^\circ$

79. $\sin A = \frac{a}{c} = \frac{16}{20} = \frac{4}{5}$ The answer is c.

81. Statement a is false because $\sin 30^\circ = \frac{1}{2}$.

2.2 Calculators and Trigonometric Functions of an Acute Angle

EVEN SOLUTIONS

2. If $\theta = 7.25°$ in decimal degrees, then the 7 represents the number of degrees, the 2 represents the number of tenths of a degree, and the 5 represents the number of hundredths of a degree.

4. On a calculator, the SIN^{-1}, COS^{-1}, and TAN^{-1} keys allow us to find an angle given the value of a trigonometric function.

6. Adding the angles: $11°41' + 32°16' = 43°57'$

8. Adding the angles: $63°38' + 24°52' = 87°90' = 88°30'$

10. Adding the angles: $77°21' + 26°44' = 104°5'$

12. Subtracting the angles: $90° - 62°25' = 89°60' - 62°25' = 27°35'$

14. Subtracting the angles: $180° - 132°39' = 179°60' - 132°39' = 47°21'$

16. Subtracting the angles: $89°38' - 28°58' = 88°98' - 28°58' = 60°40'$

18. Converting to degrees and minutes: $83.6° = 83° + 0.6° = 83° + 0.6(60') = 83°36'$

20. Converting to degrees and minutes: $78.5° = 78° + 0.5° = 78° + 0.5(60') = 78°30'$

22. Converting to degrees and minutes: $43.85° = 43° + 0.85° = 43° + 0.85(60') = 43°51'$

24. Converting to degrees and minutes: $8.3° = 8° + 0.3° = 8° + 0.3(60') = 8°18'$

26. Converting to decimal degrees: $74°54' = 74° + 54' = 74° + \left(\frac{54}{60}\right)^\circ = 74.9°$

28. Converting to decimal degrees: $21°15' = 21° + 15' = 21° + \left(\frac{15}{60}\right)^\circ = 21.25°$

30. Converting to decimal degrees: $39°10' = 39° + 10' = 39° + \left(\frac{10}{60}\right)^\circ \approx 39.17°$

32. Converting to decimal degrees: $78°37' = 78° + 37' = 78° + \left(\frac{37}{60}\right)^\circ = 78.62°$

34. Calculating the value: $\cos 79.2° \approx 0.1874$

36. Calculating the value: $\sin 4° \approx 0.0698$

38. Calculating the value: $\tan 41.88° \approx 0.8966$

40. Calculating the value: $\cot 29° = \dfrac{1}{\tan 29°} \approx 1.8040$

42. Calculating the value: $\sec 18.7° = \dfrac{1}{\cos 18.7°} \approx 1.0557$

44. Calculating the value: $\csc 77.77° = \dfrac{1}{\sin 77.77°} \approx 1.0232$

46. Calculating the value: $\sin 75°50' = \sin\left(75\dfrac{5}{6}\right)^\circ \approx 0.9696$

48. Calculating the value: $\tan 45°19' = \tan\left(45\dfrac{19}{60}\right)^\circ \approx 1.0111$

50. Calculating the value: $\cos 6°4' = \cos\left(6\dfrac{1}{15}\right)^\circ \approx 0.9944$

52. Calculating the value: $\csc 48°48' = \csc\left(48\dfrac{48}{60}\right)^\circ = \csc 48.8° = \dfrac{1}{\sin 48.8°} \approx 1.3291$

54. Completing the table:

x	$\csc x$	$\sec x$	$\cot x$
0°	error (undefined)	1	error (undefined)
15°	3.8637	1.0353	3.7321
30°	2	1.1547	1.7321
45°	1.4142	1.4142	1
60°	1.1547	2	0.5774
75°	1.0353	3.8637	0.2679
90°	1	error (undefined)	error (undefined)

56. Finding the angle θ: $\theta = \sin^{-1}(0.7139) \approx 45.6°$

58. Finding the angle θ: $\theta = \cos^{-1}(0.0945) \approx 84.6°$

60. Finding the angle θ: $\theta = \tan^{-1}(6.2703) \approx 80.9°$

62. Since $\sec\theta = 8.0101$, $\cos\theta = \dfrac{1}{8.0101}$, so $\theta = \cos^{-1}\left(\dfrac{1}{8.0101}\right) \approx 82.8°$.

64. Since $\csc\theta = 4.2319$, $\sin\theta = \dfrac{1}{4.2319}$, so $\theta = \sin^{-1}\left(\dfrac{1}{4.2319}\right) \approx 13.7°$.

66. Since $\cot\theta = 7.0234$, $\tan\theta = \dfrac{1}{7.0234}$, so $\theta = \tan^{-1}\left(\dfrac{1}{7.0234}\right) \approx 8.1°$.

68. Finding the angle θ: $\theta = \sin^{-1}(0.9459) \approx 71.0672° = 71° + 0.0672(60') = 71°4'$

70. Finding the angle θ: $\theta = \tan^{-1}(2.4652) \approx 67.9202° = 67° + 0.9202(60') = 67°55'$

72. Since $\sec\theta = 1.9102$, $\cos\theta = \dfrac{1}{1.9102}$.

Finding the angle θ: $\theta = \cos^{-1}\left(\dfrac{1}{1.9102}\right) \approx 58.4323° = 58° + 0.4323(60') = 58°26'$

74. Calculating the values: $\sin 13° \approx 0.2250$ and $\cos 77° \approx 0.2250$

76. Calculating the values: $\sec 6.7° \approx 1.0069$ and $\csc 83.3° \approx 1.0069$

78. Calculating the values: $\tan 35°15' = \tan 35.25° \approx 0.7067$ and $\cot 54°45' = \cot 54.75° \approx 0.7067$

80. Calculating the value: $\cos^2 58° + \sin^2 58° = 1$

82. To calculate *B*, $B = \sin^{-1}(4.321)$, which results in an error message. Since, for any angle *B*, $\sin B \le 1$, it is impossible to find an angle *B* such that $\sin B = 4.321$.

84. To calculate $\cot 0°$, we would find $\tan 0° = 0$ then find the reciprocal. This results in an error message. Since $\frac{1}{0}$ is an undefined value, $\cot 0°$ is undefined.

86. **a**. Completing the table:

x	3°	2.5°	2°	1.5°	1°	0.5°	0°
$\cot x$	19.1	22.9	28.6	38.2	57.3	114.6	undefined

b. Completing the table:

x	0.6°	0.5°	0.4°	0.3°	0.2°	0.1°	0°
$\cot x$	95.5	114.6	143.2	191.0	286.5	573.0	undefined

88. Using $\alpha = 36.597°$ and $h = 5$ in the shadow angle formula:

$$\tan\theta = (\sin 36.597°)(\tan(5 \cdot 15°)) \approx 2.2250$$

$$\theta = \tan^{-1}(2.2250) \approx 65.8°$$

90. First find the value of r: $r = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$

Finding the three trigonometric functions using $x = -\sqrt{3}$, $y = 1$, and $r = 2$:

$\sin\theta = \frac{y}{r} = \frac{1}{2}$ $\quad\cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2}$ $\quad\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

92. Let $(-1,-1)$ be a point on the terminal side of $-135°$. First find the value of r: $r = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

Finding the three trigonometric functions using $x = -1$, $y = -1$, and $r = \sqrt{2}$:

$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad\tan\theta = \frac{y}{x} = \frac{-1}{-1} = 1$

94. Since $\tan\theta = -\frac{3}{4}$ and θ terminates in quadrant II (where $x < 0$ and $y > 0$), choose $x = -4$ and $y = 3$. Finding r:

$$r = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Finding the remaining trigonometric functions using $x = -4$, $y = 3$, and $r = 5$:

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\quad\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\quad\cot\theta = \frac{x}{y} = -\frac{4}{3}$

$\csc\theta = \frac{r}{y} = \frac{5}{3}$ $\quad\sec\theta = \frac{r}{x} = -\frac{5}{4}$

96. Since $\sec\theta > 0$, $x > 0$. Thus for $\tan\theta < 0$, we must have $y < 0$. Thus the terminal side of θ lies in quadrant IV.

98. Converting to decimal degrees: $76°36' = 76° + 36' = 76° + \left(\frac{36}{60}\right)^{\circ} = 76.6°$. The correct answer is b.

100. Since $\cot\theta = x$, $\tan\theta = \frac{1}{x}$. Then $\theta = \tan^{-1}\left(\frac{1}{x}\right)$. The correct answer is a.

ODD SOLUTIONS

1. minutes, seconds

3. value, angle

5.
$$\begin{array}{l} \ \ 37°45' \\ +26°24' \\ \ \ 63°69' = 64°9' \text{ since } 60' = 1° \end{array}$$

7.
$$\begin{array}{l} \ \ 51°55' \\ +37°45' \\ \ \ 88°100' = 89°40' \end{array}$$

9.
$$\begin{array}{l} \ \ 61°33' \\ +45°16' \\ \ 106°49' \end{array}$$

11.
$$\begin{array}{lcl} 90° & = & 89°60' \\ -34°12' & & -34°12' \\ & & 55°48' \end{array}$$

13. $\begin{array}{r} 180^\circ \\ -120^\circ 17' \end{array} = \begin{array}{r} 179^\circ 60' \\ -120^\circ 17' \\ \hline 59^\circ 43' \end{array}$ Change 1° to $60'$

15. $\begin{array}{r} 76^\circ 24' \\ -22^\circ 34' \end{array} = \begin{array}{r} 75^\circ 84' \\ -22^\circ 34' \\ \hline 53^\circ 50' \end{array}$

17. $35.4^\circ = 35^\circ + 0.4(60)'$
$= 35^\circ + 24'$
$= 35^\circ 24'$

19. $16.25^\circ = 16^\circ + 0.25(60)'$
$= 16^\circ + 15'$
$= 16^\circ 15'$

21. $92.55^\circ = 92^\circ + 0.55(60)'$
$= 92^\circ + 33'$
$= 92^\circ 33'$

23. $19.9^\circ = 19^\circ + 0.9(60)'$
$= 19^\circ + 54'$
$= 19^\circ 54'$

25. $45^\circ 12' = 45 + \frac{12}{60}$
$= 45.2^\circ$

27. $62^\circ 36' = 62 + \frac{36}{60}$
$= 62.6^\circ$

29. $17^\circ 20' = 17 + \frac{20}{60}$
$= 17.33^\circ$

31. $48^\circ 27' = 48 + \frac{27}{60}$
$= 48.45^\circ$

33. Scientific Calculator: 27.2 $\boxed{\text{sin}}$
Graphing Calculator: $\boxed{\text{sin}}\ \boxed{(}\ 27.2\ \boxed{)}\ \boxed{\textit{ENTER}}$
Answer to 4 places: 0.4571

35. Scientific Calculator: 18 $\boxed{\text{cos}}$
Graphing Calculator: $\boxed{\text{cos}}\ \boxed{(}\ 18\ \boxed{)}\ \boxed{\textit{ENTER}}$
Answer to 4 places: 0.9511

37. Scientific Calculator: 87.32 $\boxed{\text{tan}}$
Graphing Calculator: $\boxed{\text{tan}}\ \boxed{(}\ 87.32\ \boxed{)}\ \boxed{\textit{ENTER}}$
Answer to 4 places: 21.3634

39. $\cot 31^\circ = \frac{1}{\tan 31^\circ}$
Scientific Calculator: 31 $\boxed{\text{tan}}\ \boxed{1/x}$
Graphing Calculator: $\boxed{\text{tan}}\ \boxed{(}\ 31\ \boxed{)}\ \boxed{x^{-1}}\ \boxed{\textit{ENTER}}$
Answer: 1.6643

41. $\sec 48.2^\circ = \frac{1}{\cos 48.2^\circ}$
Scientific Calculator: 48.2 $\boxed{\text{cos}}\ \boxed{1/x}$
Graphing Calculator: $\boxed{\text{cos}}\ \boxed{(}\ 48.2\ \boxed{)}\ \boxed{x^{-1}}$
Answer: 1.5003

43. $\csc 14.15^\circ = \frac{1}{\sin 14.15^\circ}$
Scientific Calculator: 14.15 $\boxed{\text{sin}}\ \boxed{1/x}$
Graphing Calculator: $\boxed{\text{sin}}\ \boxed{(}\ 14.15\ \boxed{)}\ \boxed{x^{-1}}\ \boxed{\textit{ENTER}}$
Answer: 4.0906

45. $24^\circ 30' = 24 + \frac{30}{60} = 24.5^\circ$

Scientific Calculator: 24.5 $\boxed{\cos}$

Graphing Calculator: $\boxed{\cos}\ \boxed{(}\ 24.5\ \boxed{)}\ \boxed{ENTER}$

Answer: 0.9100

47. $42^\circ 15' = 42 + \frac{15}{60}$

$= 42.25^\circ$

Scientific Calculator: 42.25 $\boxed{\tan}$

Graphing Calculator: $\boxed{\tan}\ \boxed{(}\ 42.25\ \boxed{)}\ \boxed{ENTER}$

Answer: 0.9083

49. $56^\circ 40' = 56 + \frac{40}{60} = 56.67^\circ$

Scientific Calculator: 56.67 $\boxed{\sin}$

Graphing Calculator: $\boxed{\sin}\ \boxed{(}\ 56.67\ \boxed{)}\ \boxed{ENTER}$

Answer: 0.8355

51. $45^\circ 54' = 45 + \frac{54}{60}$

$= 45.9^\circ$

$\sec 45.9^\circ = \frac{1}{\cos 45.9^\circ}$

Scientific Calculator: 45.9 $\boxed{\cos}\ \boxed{1/x}$

Graphing Calculator: $\boxed{\cos}\ \boxed{(}\ 45.9\ \boxed{)}\ \boxed{x^{-1}}\ \boxed{ENTER}$

Answer: 1.4370

53. Use your calculator to find the values of the sine, cosine, and tangent of each angle:

x	$\sin x$	$\cos x$	$\tan x$
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	Error (undefined)

55. Scientific Calculator: 0.9770 $\boxed{inv}\ \boxed{\cos}$

Graphing Calculator: $\boxed{2nd}\ \boxed{\cos}\ \boxed{(}\ 0.9770\ \boxed{)}\ \boxed{ENTER}$

Answer: 12.3°

57. Scientific Calculator: 0.6873 $\boxed{inv}\ \boxed{\tan}$

Graphing Calculator: $\boxed{2nd}\ \boxed{\tan}\ \boxed{(}\ 0.6873\ \boxed{)}\ \boxed{ENTER}$

Answer: 34.5°

59. Scientific Calculator: 0.9813 $\boxed{inv}\ \boxed{\sin}$

Graphing Calculator: $\boxed{2nd}\ \boxed{\sin}\ \boxed{(}\ 0.9813\ \boxed{)}\ \boxed{ENTER}$

Answer: 78.9°

61. $\sec\theta = 1.0191$

$\dfrac{1}{\cos\theta} = 1.0191$

$\cos\theta = \dfrac{1}{1.0191}$

Scientific Calculator: $1 \boxed{\div} 1.0191 \boxed{=} \boxed{inv} \boxed{\cos}$

Graphing Calculator: $\boxed{2nd} \boxed{\cos} \boxed{(} 1 \boxed{\div} 1.0191 \boxed{)} \boxed{ENTER}$

Answer: 11.1°

63. $\csc\theta = 1.8214$

$\dfrac{1}{\sin\theta} = 1.8214$

$\sin\theta = \dfrac{1}{1.8214}$

Scientific Calculator: $1 \boxed{\div} 1.8214 \boxed{=} \boxed{inv} \boxed{\sin}$

Graphing Calculator: $\boxed{2nd} \boxed{\sin} \boxed{(} 1 \boxed{\div} 1.8214 \boxed{)} \boxed{ENTER}$

Answer: 33.3°

65. $\cot\theta = 0.6873$

$\dfrac{1}{\tan\theta} = 0.6873$

$\tan\theta = \dfrac{1}{0.6873}$

Scientific Calculator: $1 \boxed{\div} 0.6873 \boxed{=} \boxed{inv} \boxed{\tan}$

Graphing Calculator: $\boxed{2nd} \boxed{\tan} \boxed{(} 1 \boxed{\div} 0.6873 \boxed{)} \boxed{ENTER}$

Answer: 55.5°

67. Scientific Calculator: $0.4112 \boxed{inv} \boxed{\cos}$

Answer in decimal degrees is 65.719°

Convert the decimal part to minutes: $0.719 \boxed{\times} 60 \boxed{=}$

Graphing Calculator: $\boxed{2nd} \boxed{\cos} \boxed{(} 0.4112 \boxed{)} \boxed{2nd} \boxed{APPS} \boxed{\blacktriangleright DMS} \boxed{ENTER}$

Answer: $\theta = 65^\circ 43'$

69. $\cot\theta = 5.5764$

$\dfrac{1}{\tan\theta} = 5.5764$

$\tan\theta = \dfrac{1}{5.5764}$

Scientific Calculator: $1 \boxed{\div} 5.5764 \boxed{=} \boxed{inv} \boxed{\tan}$

Answer in decimal degrees is 10.1666°

Convert the decimal part to minutes: $0.1666 \boxed{\times} 60 \boxed{=}$

Graphing Calculator: $\boxed{2nd} \boxed{\tan} \boxed{(} 5.5764 \boxed{)} \boxed{2nd} \boxed{APPS} \boxed{\blacktriangleright DMS} \boxed{ENTER}$

Answer: $\theta = 10^\circ 10'$

71. $\csc\theta = 7.0683$

$\dfrac{1}{\sin\theta} = 7.0683$

$\sin\theta = \dfrac{1}{7.0683}$

Scientific Calculator: $1 \boxed{\div} 7.0683 \boxed{=} \boxed{inv} \boxed{\sin}$

Answer in decimal degrees is 8.1333°

Convert the decimal part to minutes: $0.133 \boxed{\times} 60 \boxed{=}$

Graphing Calculator: $\boxed{2nd} \boxed{\sin} \boxed{(} 1 \boxed{\div} 7.0683 \boxed{)} \boxed{2nd} \boxed{APPS} \boxed{\blacktriangleright DMS} \boxed{ENTER}$

Answer: $\theta = 8^\circ 8'$

73. Scientific Calculator: $23 \boxed{\sin}$ and $67 \boxed{\cos}$

Graphing Calculator: $\boxed{\sin} \boxed{(} 23 \boxed{)} \boxed{ENTER}$ $\cos \boxed{(} 67 \boxed{)} \boxed{ENTER}$

Both answers should be 0.3907.

75. To calculate $\sec 34.5^\circ = \dfrac{1}{\cos 34.5^\circ}$:

Scientific Calculator: 34.5 $\boxed{\cos}$ $\boxed{1/x}$

Graphing Calculaltor: $\boxed{\cos}$ $\boxed{(}$ 34.5 $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{ENTER}$

To calculate $\csc 55.5^\circ = \dfrac{1}{\sin 55.5^\circ}$:

Scientific Calculator: 55.5 $\boxed{\sin}$ $\boxed{1/x}$

Graphing Calculaltor: $\boxed{\sin}$ $\boxed{(}$ 55.5 $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{ENTER}$

Both answers should be 1.2134.

77. Scientific Calculator: 4.5 $\boxed{\tan}$

Graphing Calculaltor: $\boxed{\tan}$ $\boxed{(}$ 4.5 $\boxed{)}$ $\boxed{ENTER}$

To calculate $\cot 85.5^\circ = \dfrac{1}{\tan 85.5^\circ}$:

Scientific Calculator: 85.5 $\boxed{\tan}$ $\boxed{1/x}$

Graphing Calculaltor: $\boxed{\tan}$ $\boxed{(}$ 85.5 $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{ENTER}$

Both answers should be 0.0787.

79. Scientific Calculator: 37 $\boxed{\cos}$ $\boxed{x^2}$ $\boxed{+}$ 37 $\boxed{\sin}$ $\boxed{x^2}$ $\boxed{=}$

Graphing Calculator: $\boxed{\cos}$ $\boxed{(}$ 37 $\boxed{)}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{\sin}$ $\boxed{(}$ 37 $\boxed{)}$ $\boxed{x^2}$ $\boxed{ENTER}$

Answer should be 1.

81. Scientific Calculator: 1.234 $\boxed{inv}$ $\boxed{\sin}$

Graphing Calculator: $\boxed{2nd}$ $\boxed{\sin}$ 1.234 $\boxed{ENTER}$

You should get an error message. The sine of an angle can never be greater than 1.

83. Scientific Calculator: 90 $\boxed{\tan}$

Graphing Calculaltor: $\boxed{\tan}$ $\boxed{(}$ 90 $\boxed{)}$ $\boxed{ENTER}$

You should get an error message. The tangent of 90° is undefined.

87. $\tan\theta = \sin\alpha\tan\left(h\cdot 15^\circ\right)$ where $\alpha = 35.282^\circ$ and $h = 2$

$$\tan\theta = \sin\left(35.282^\circ\right)\tan\left(2\cdot 15^\circ\right)$$
$$= .333478$$
$$\theta = \tan^{-1}\left(.333478\right)$$
$$\theta = 18.4^\circ$$

89. $(x,y) = (3,-2)$

$x = 3$ and $y = -2$

$$r = \sqrt{3^2 + (-2)^2}$$
$$= \sqrt{9+4} = \sqrt{13}$$

$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{3} = -\frac{2}{3}$$

91. A point on the terminal side of an angle of 90° in standard position is (0, 1), where $x = 0$, $y = 1$, and $r = 1$.

$\sin 90^\circ = \dfrac{y}{r} = \dfrac{1}{1} = 1$

$\cos 90^\circ = \dfrac{x}{r} = \dfrac{0}{1} = 0$

$\tan 90^\circ = \dfrac{y}{x} = \dfrac{1}{0}$ is undefined

93. $\cos\theta = -\dfrac{5}{13}$ and θ is in QIII. In QIII, both x and y are negative.

$\cos\theta = \dfrac{x}{r} = \dfrac{-5}{13}$

$x = -5$ and $r = 13$

$x^2 + y^2 = r^2$

$(-5)^2 + y^2 = 13^2$

$25 + y^2 = 169$

$y^2 = 144$

$y = \pm 12$

$y = -12$ because θ is in QIII

$\sin\theta = \dfrac{y}{r} = \dfrac{-12}{13} = -\dfrac{12}{13}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-12}{-5} = \dfrac{12}{5}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-5}{-12} = \dfrac{5}{12}$

$\sec\theta = \dfrac{r}{x} = \dfrac{13}{-5} = -\dfrac{13}{5}$

$\csc\theta = \dfrac{r}{y} = \dfrac{13}{-12} = -\dfrac{13}{12}$

95. The $\sin\theta$ is positive in QI and QII.
The $\cos\theta$ is negative in QII and QIII.
Therefore, θ must lie in QII.

97.

$$\begin{array}{rcrl} 67^\circ 22' & = & 66^\circ 82' & \text{Change } 1^\circ \text{ to } 60' \\ -34^\circ 30' & & -34^\circ 30' & \\ & & 32^\circ 52' & \end{array}$$

The answer is d.

2.3 Solving Right Triangles

EVEN SOLUTIONS

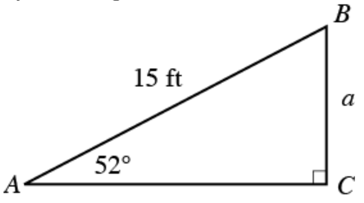
2. If the sides of a triangle are accurate to three significant digits, then angles should be measured to the nearest tenth of a
degree, or the nearest ten minutes.

4. In general, round answers so that the number of significant digits in your answer matches the number of significant digits in the least significant number given in the problem.

6. **a**. three
b. three
c. five
d. three

8. **a**. five
b. five
c. five
d. seven

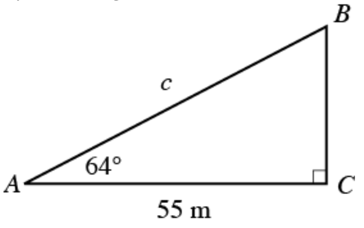
10. Begin by drawing ΔABC:



Therefore:

$$\sin 52° = \frac{a}{15}$$
$$a = 15\sin 52° \approx 12 \text{ ft}$$

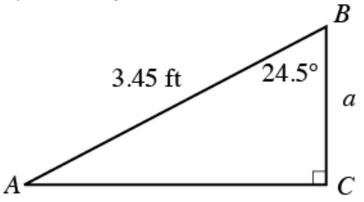
12. Begin by drawing ΔABC:



Therefore:

$$\cos 64° = \frac{55}{c}$$
$$c\cos 64° = 55$$
$$c = \frac{55}{\cos 64°} \approx 125 \text{ m} \approx 130 \text{ m}$$

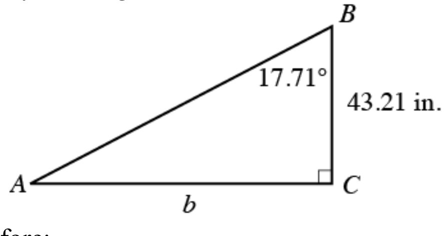
14. Begin by drawing ΔABC:



Therefore:

$$\cos 24.5° = \frac{a}{3.45}$$
$$a = 3.45\cos 24.5° \approx 3.14 \text{ ft}$$

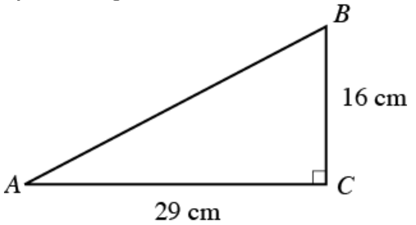
16. Begin by drawing ΔABC:



Therefore:

$$\tan 17.71° = \frac{b}{43.21}$$
$$b = 43.21\tan 17.71° \approx 13.80 \text{ in.}$$

18. Begin by drawing ΔABC:

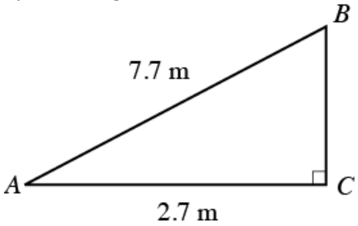


Therefore:

$$\tan A = \frac{16}{29}$$

$$A = \tan^{-1}\left(\frac{16}{29}\right) \approx 29°$$

20. Begin by drawing ΔABC:

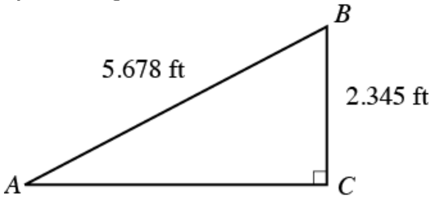


Therefore:

$$\cos A = \frac{2.7}{7.7}$$

$$A = \cos^{-1}\left(\frac{2.7}{7.7}\right) \approx 69°$$

22. Begin by drawing ΔABC:

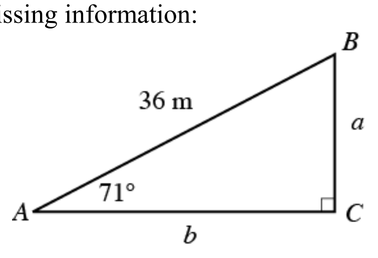


Therefore:

$$\sin A = \frac{2.345}{5.678}$$

$$A = \sin^{-1}\left(\frac{2.345}{5.678}\right) \approx 24.39°$$

24. Begin by drawing ΔABC and label missing information:



Note that $B = 90° - 71° = 19°$. Therefore:

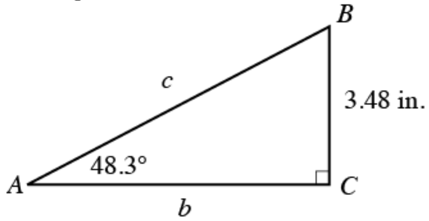
$$\sin 71° = \frac{a}{36}$$

$$a = 36\sin 71° \approx 34 \text{ m}$$

$$\cos 71° = \frac{b}{36}$$

$$b = 36\cos 71° \approx 12 \text{ m}$$

26. Begin by drawing ΔABC and label missing information:

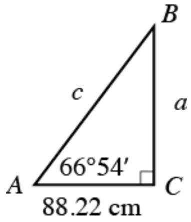


Note that $B = 90° - 48.3° = 41.7°$. Therefore:

$$\sin 48.3° = \frac{3.48}{c}$$
$$c \sin 48.3° = 3.48$$
$$c = \frac{3.48}{\sin 48.3°} \approx 4.66 \text{ in.}$$

$$\cos 48.3° = \frac{b}{4.66}$$
$$b = 4.66 \cos 48.3° \approx 3.10 \text{ in.}$$

28. Begin by drawing ΔABC and label missing information:

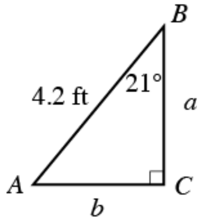


Note that $B = 90° - 66°54' = 89°60' - 66°54' = 23°6'$. So:

$$\cos 66°54' = \frac{88.22}{c}$$
$$c \cos 66°54' = 88.22$$
$$c = \frac{88.22}{\cos 66°54'} \approx 224.9 \text{ cm}$$

$$\tan 66°54' = \frac{a}{88.22}$$
$$a = 88.22 \tan 66°54' \approx 206.8 \text{ cm}$$

30. Begin by drawing ΔABC and label missing information:

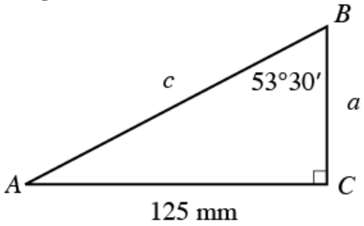


Note that $A = 90° - 21° = 69°$. Therefore:

$$\cos 69° = \frac{b}{4.2}$$
$$b = 4.2 \cos 69° \approx 1.5 \text{ ft}$$

$$\sin 69° = \frac{a}{4.2}$$
$$a = 4.2 \sin 69° \approx 3.9 \text{ ft}$$

32. Begin by drawing ΔABC and label missing information:

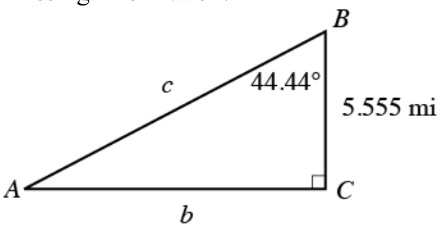


Note that $A = 90° - 53°30' = 89°60' - 53°30' = 36°30'$. So:

$$\cos 36°30' = \frac{125}{c}$$
$$c\cos 36°30' = 125$$
$$c = \frac{125}{\cos 36°30'} \approx 156 \text{ mm}$$

$$\tan 36°30' = \frac{a}{125}$$
$$a = 125\tan 36°30' \approx 92.5 \text{ mm}$$

34. Begin by drawing ΔABC and label missing information:

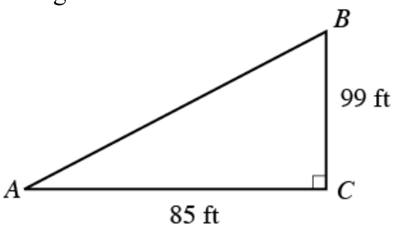


Note that $A = 90° - 44.44° = 45.56°$. Therefore:

$$\sin 45.56° = \frac{5.555}{c}$$
$$c\sin 45.56° = 5.555$$
$$c = \frac{5.555}{\sin 45.56°} \approx 7.780 \text{ mi}$$

$$\tan 45.56° = \frac{5.555}{b}$$
$$b\tan 45.56° = 5.555$$
$$b = \frac{5.555}{\tan 45.56°} \approx 5.447 \text{ mi}$$

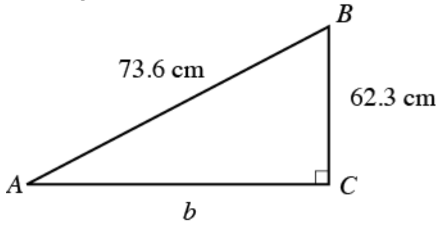
36. Begin by drawing ΔABC and label missing information:



Therefore: $c = \sqrt{85^2 + 99^2} \approx 130 \text{ ft}$. Finding the angles:

$$\tan A = \frac{99}{85}$$
$$A = \tan^{-1}\left(\frac{99}{85}\right) \approx 49°$$
$$B = 90° - 49° = 41°$$

38. Begin by drawing ΔABC and label missing information:



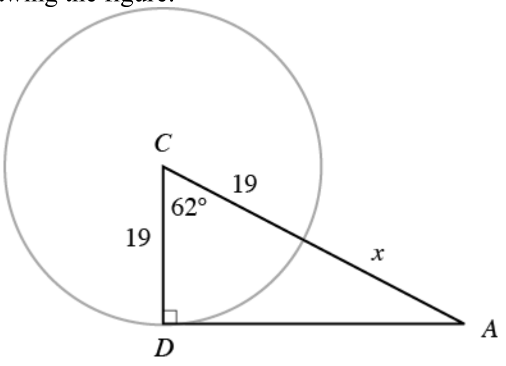
Therefore: $b = \sqrt{73.6^2 - 62.3^2} \approx 39.2 \text{ cm}$. Finding the angles:

$$\sin A = \frac{62.3}{73.6}$$
$$A = \sin^{-1}\left(\frac{62.3}{73.6}\right) \approx 57.8°$$
$$B = 90° - 57.8° = 32.2°$$

40. Since the right triangle is a 45°–45°–90° triangle, its height is 2.0. Therefore:

$$\tan A = \frac{2.0}{3.0}$$
$$A = \tan^{-1}\left(\frac{2.0}{3.0}\right) \approx 34°$$

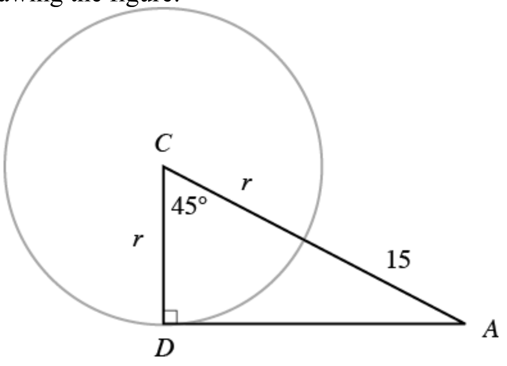
42. Re-drawing the figure:



Using the right triangle:

$$\cos 62° = \frac{19}{19+x}$$
$$(19+x)\cos 62° = 19$$
$$19+x = \frac{19}{\cos 62°}$$
$$x = \frac{19}{\cos 62°} - 19 \approx 21$$

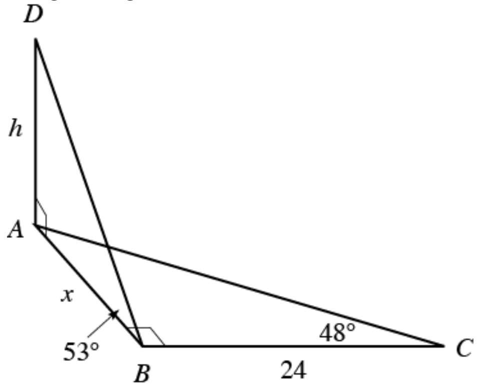
44. Re-drawing the figure:



Using the right triangle:

$$\cos 45° = \frac{r}{r+15}$$
$$(r+15)\cos 45° = r$$
$$r\cos 45° + 15\cos 45° = r$$
$$15\cos 45° = r - r\cos 45°$$
$$15\cos 45° = r(1-\cos 45°)$$
$$r = \frac{15\cos 45°}{1-\cos 45°}$$
$$r = \frac{15\left(\frac{\sqrt{2}}{2}\right)}{1-\frac{\sqrt{2}}{2}} = \frac{15\sqrt{2}}{2-\sqrt{2}} \approx 36$$

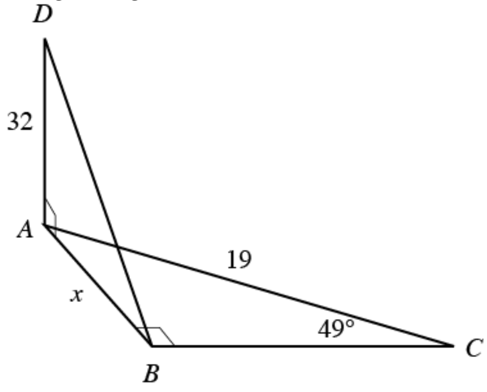
46. Re-drawing the figure:



First find x:

$$\tan 48° = \frac{x}{24}$$
$$x = 24\tan 48° \approx 27$$

48. Re-drawing the figure:



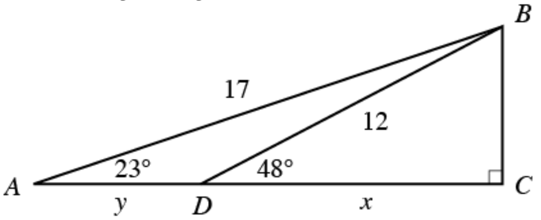
First find x:

$$\sin 49° = \frac{x}{19}$$
$$x = 19\sin 49° \approx 14$$

Now find h:

$$\tan 53° = \frac{h}{47}$$
$$h = 27\tan 53° \approx 35$$

50. Re-drawing the figure:



First find x:

$$\cos 48° = \frac{x}{12}$$
$$x = 12\cos 48° \approx 8.0$$

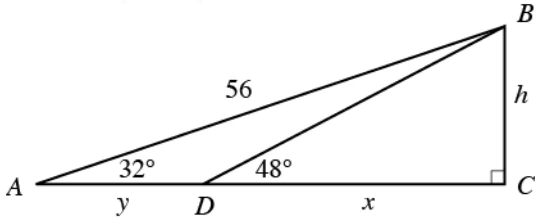
Now find y:

$$\cos 23° = \frac{8+y}{17}$$
$$8+y = 17\cos 23°$$
$$y = 17\cos 23° - 8 \approx 7.6$$

Now find $\angle ABD$:

$$\tan \angle ABD = \frac{32}{14}$$
$$\angle ABD = \tan^{-1}\left(\frac{32}{14}\right) \approx 66°$$

52. Re-drawing the figure:



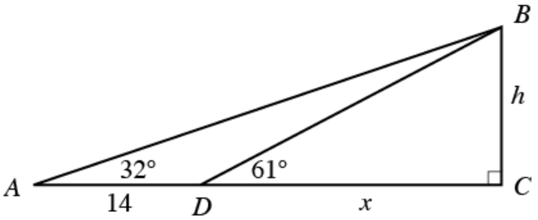
First find h:

$$\sin 32° = \frac{h}{56}$$
$$h = 56\sin 32° \approx 30$$

Now find x:

$$\tan 48° = \frac{30}{x}$$
$$x\tan 48° = 30$$
$$x = \frac{30}{\tan 48°} \approx 27$$

54. Re-drawing the figure:



Note that $\tan 61° = \dfrac{h}{x}$, so $h = x\tan 61°$. Also note that $\tan 32° = \dfrac{h}{14+x}$, so $h = (14+x)\tan 32°$. Setting these two expressions equal:

$$x\tan 61° = (14+x)\tan 32°$$
$$x\tan 61° = 14\tan 32° + x\tan 32°$$
$$x\tan 61° - x\tan 32° = 14\tan 32°$$
$$x(\tan 61° - \tan 32°) = 14\tan 32°$$
$$x = \frac{14\tan 32°}{\tan 61° - \tan 32°} \approx 7.4$$

56. Since $GC = CD = 3.00$, using the Pythagorean Theorem: $GD = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$. Therefore:

$$\tan \angle GDE = \frac{GE}{GD} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$
$$\angle GDE = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 35.3°$$

58. First find $\angle CAB$:

$$\tan(\angle CAB) = \frac{66}{54}$$

$$\angle CAB = \tan^{-1}\left(\frac{66}{54}\right) \approx 50.71°$$

Now find $\angle EAB$:

$$\tan(\angle EAB) = \frac{78}{54}$$

$$\angle EAB = \tan^{-1}\left(\frac{78}{54}\right) \approx 55.30°$$

Therefore: $\angle CAE = \angle EAB - \angle CAB = 55.30° - 50.71° \approx 4.6°$

60. Let O represent the center of the goal.

First find $\angle OAD$:

$$\tan(\angle OAD) = \frac{6}{54}$$

$$\angle OAD = \tan^{-1}\left(\frac{6}{54}\right) \approx 6.34°$$

Now find $\angle OAF$:

$$\tan(\angle OAF) = \frac{12}{54}$$

$$\angle OAF = \tan^{-1}\left(\frac{12}{54}\right) \approx 12.53°$$

Therefore: $\angle DAF = \angle OAF - \angle OAD = 12.53° - 6.34° \approx 6.2°$

Since $\angle CAE$ is also 6.2°, the sum of the angles is 12.4°.

62. Since 12.4° is much greater than 4.6°, the chance of scoring is much better from the center than from the corner of the penalty area.

64. From Example 5, we have:

$$\cos 135° = \frac{139 - h}{125}$$

$$-\frac{1}{\sqrt{2}} = \frac{139 - h}{125}$$

$$-\frac{125}{\sqrt{2}} = 139 - h$$

$$h = 139 + \frac{125}{\sqrt{2}} \approx 227.4$$

The rider is approximately 230 ft above the ground.

66. First note that the distance from the ground to the low point of Colossus is 174 – 165 = 9 ft. The radius is 82.5 ft. Since $x + h = 82.5 + 9 = 91.5$, $x = 91.5 - h$. Therefore:

$$\cos\theta = \frac{x}{82.5} = \frac{91.5 - h}{82.5}$$

$$91.5 - h = 82.5\cos\theta$$

$$h = 91.5 - 82.5\cos\theta$$

a. Substituting $\theta = 150°$: $h = 91.5 - 82.5\cos 150° \approx 163$ ft

b. Substituting $\theta = 240°$: $h = 91.5 - 82.5\cos 240° \approx 133$ ft

c. Substituting $\theta = 315°$: $h = 91.5 - 82.5\cos 315° \approx 33.2$ ft

68. First note that the distance from the ground to the low point of the High Roller is 550 – 520 = 30 ft. The radius is 260 ft. Since $x + h = 260 + 30 = 290$, $x = 290 - h$. Therefore:

$$\cos\theta = \frac{x}{260} = \frac{290 - h}{260}$$

$$290 - h = 260\cos\theta$$

$$h = 290 - 260\cos\theta$$

Substituting $\theta = 110°$: $h = 290 - 260\cos 110° \approx 379 \text{ ft} \approx 380 \text{ ft}$

70. Entering the functions $Y_1 = -\frac{7}{640}(X - 80)^2 + 70$ and $Y_2 = \tan^{-1}\left(\frac{Y_1}{X}\right)$, complete the table:

X	10	5	1	0.5	0.1	0.01
Y_1	16.4063	8.4766	1.7391	0.8723	0.1749	0.0175
Y_2	58.6°	59.5°	60.1°	60.2°	60.2°	60.3°

Based on these results, it appears the angle between the cannon and the horizontal is approximately 60.3°.

72. Since $\csc B = 5$, $\sin B = \frac{1}{5}$, so $\sin^2 B = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$.

74. Finding $\cos^2 A$: $\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$

So $\cos A = \pm\frac{\sqrt{7}}{4}$. Since A terminates in quadrant II, where $x < 0$, $\cos A < 0$. Thus $\cos A = -\frac{\sqrt{7}}{4}$.

76. First find $\sin\theta$ (note $\sin\theta < 0$ since θ terminates in quadrant IV):

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{5}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Now find the other four trigonometric ratios using $x = 1$, $y = -2$, and $r = \sqrt{5}$:

$$\sec\theta = \frac{r}{x} = \sqrt{5} \qquad \csc\theta = \frac{r}{y} = -\frac{\sqrt{5}}{2} \qquad \tan\theta = \frac{y}{x} = -2 \qquad \cot\theta = \frac{x}{y} = -\frac{1}{2}$$

78. Since $\csc\theta = -2$, $\sin\theta = -\frac{1}{2}$. Now find $\cos\theta$ (note that $\cos\theta < 0$ since θ terminates in quadrant III):

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(-\frac{1}{2}\right)^2} = -\sqrt{1-\frac{1}{4}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

Now find the other three trigonometric ratios using $x = -\sqrt{3}$, $y = -1$, and $r = 2$:

$$\sec\theta = \frac{r}{x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \qquad \tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cot\theta = \frac{x}{y} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

80. Finding side b:

$$\cos A = \frac{b}{c}$$
$$\cos 58° = \frac{b}{15}$$
$$b = 15\cos 58° \approx 7.9 \text{ ft}$$

The correct answer is c.

82. Let x represent the height of the rider above the center of the wheel (which is 51.5 feet above the ground). Since the point of the rider is 50° above the horizontal, we have:

$$\sin 50° = \frac{h-51.5}{45}$$
$$h - 51.5 = 45\sin 50°$$
$$h = 51.5 + 45\sin 50° \approx 86 \text{ ft}$$

The correct answer is c.

ODD SOLUTIONS

1. left, right, first nonzero, not

3. sides, angles

5. **a.** 2 **b.** 3 **c.** 2 **d.** 2

7. **a.** 4 **b.** 6 **c.** 4 **d.** 4

9.

$\cos 42^\circ = \frac{b}{89}$	Cosine relationship
$b = 89\cos 42^\circ$	Multiply both sides by 89
$= 89(0.7431)$	Substitute value for $\cos 42^\circ$
$= 66\,\text{cm}$	Answer rounded to 2 significant digits

11. $\sin 34^\circ = \dfrac{22}{c}$ Sine relationship

$c \sin 34^\circ = 22$ Multiply both sides by c

$c = \dfrac{22}{\sin 34^\circ}$ Divide both sides by $\sin 34^\circ$

$c = \dfrac{22}{0.5592}$ Substitute value for $\sin 34^\circ$

$c = 39\,\text{m}$ Answer rounded to 2 significant digits

13. $\sin 16.9^\circ = \dfrac{b}{7.55}$ Sine relationship

$b = 7.55 \sin 16.9^\circ$ Multiply both sides by 7.55

$= 7.55(0.2907)$ Substitute value for $\cos 24.5^\circ$

$= 2.19\,\text{cm}$ Answer rounded to 3 significant digits

15. $\tan 55.33^\circ = \dfrac{12.34}{a}$ Tangent relationship

$a \tan 55.33^\circ = 12.34$ Multiply both sides by a

$a = \dfrac{12.34}{\tan 55.33^\circ}$ Divide both sides by $\tan 55.33^\circ$

$a = \dfrac{12.34}{1.4458}$ Substitute value for $\tan 55.33^\circ$

$a = 8.535\ \text{yd}$ Answer rounded to 4 significant digits

17. $\tan B = \dfrac{32.4}{42.3}$ Tangent relationship

$= 0.7659$ Divide

$B = \tan^{-1}(0.7659)$ Use calculator to find angle

$B = 37.5^\circ$ Answer rounded to the nearest tenth of a degree

19. $\sin B = \dfrac{9.8}{12}$ Sine relationship

$= 0.8166$ Divide

$B = \cos^{-1}(0.8166)$ Use calculator to find angle

$= 55^\circ$ Answer rounded to the nearest degree

21. $\cos B = \dfrac{23.32}{45.54}$ Cosine relationship

$= 0.5120$ Divide 23.32 by 45.54

$B = \cos^{-1}(0.5120)$ Use calculator to find angle

$= 59.20^\circ$ Answer rounded to the nearest hundredth of a degree

23. First, we find $\angle B$: $\angle B = 90^\circ - \angle A = 90^\circ - 25^\circ = 65^\circ$

Next, we find side a:

$\sin 25^\circ = \dfrac{a}{24}$ Sine relationship

$a = 24 \sin 25^\circ$ Multiply both sides by 24

$a = 10\,\text{m}$ Answer rounded to 2 significant digits

Last, we find side b:

$\cos 25^\circ = \dfrac{b}{24}$ Cosine relationship

$b = 24 \sin 25^\circ$ Multiply both sides by 24

$b = 22\,\text{m}$ Answer rounded to 2 significant digits

25. First, we find $\angle B$: $\angle B = 90^\circ - \angle A$

$= 90^\circ - 32.6^\circ = 57.4^\circ$

Next, we find side *c*:

$\sin 32.6^\circ = \dfrac{43.4}{c}$ Sine relationship

$c = \dfrac{43.4}{\sin 32.6^\circ}$ Multiply both sides by *c* then divide by $\sin 32.6^\circ$

$= 80.6\text{ in}$ Answer rounded to 3 significant digits

Last, we find side *b*:

$\tan 57.4^\circ = \dfrac{b}{43.4}$ Tangent relationship

$b = 43.4 \tan 57.4^\circ$ Multiply both sides by 43.4

$= 67.9\text{ in}$ Answer rounded to 2 significant digits

27. First, we find $\angle B$: $\angle B = 90^\circ - \angle A$

$= 90^\circ - 10^\circ 42'$

$= 79^\circ 18'$

Next, we find side *a*:

$\tan 10^\circ 42' = \dfrac{a}{5.932}$ Tangent relationship

$\tan 10.7^\circ = \dfrac{a}{5.932}$ Change angle to decimal degrees

$a = 5.932 \tan 10.7^\circ$ Multiply both sides by 5.932

$a = 1.121\text{cm}$ Answer rounded to 4 significant digits

Last, we find side *c*:

$\cos 10.7^\circ = \dfrac{5.932}{c}$ Cosine relationship

$c = \dfrac{5.932}{\cos 10.7^\circ}$ Multiply both sides by *c* then divide by $\cos 10.7^\circ$

$c = 6.037\text{ cm}$ Answer rounded to 4 significant digits

29. First, we find $\angle A$: $\angle A = 90^\circ - 76^\circ$

$= 14^\circ$

Next, we find side *a*:

$\cos 76^\circ = \dfrac{a}{5.8}$ Cosine relationship

$a = 5.8 \cos 76^\circ$ Multiply both sides by 5.8

$= 1.4\text{ ft}$ Answer rounded to 2 significant digits

Last, we find side *b*:

$\sin 76^\circ = \dfrac{b}{5.8}$ Sine relationship

$b = 5.8 \sin 76^\circ = 5.6\text{ ft}$ Multiply both sides by 5.8 and round to 2 significant digits

31. First, we find $\angle A$: $\angle A = 90^\circ - \angle B$

$= 90^\circ - 26^\circ 30'$

$= 63^\circ 30'$

Next, we find side *a*:

$\tan 26^\circ 30' = \dfrac{324}{a}$	Tangent relationship
$\tan 26.5^\circ = \dfrac{324}{a}$	Change angle to decimal degrees
$a = \dfrac{324}{\tan 26.5^\circ}$	Multiply both sides by *a* then divide by $\tan 26.5^\circ$
$a = 650$ mm	Answer rounded to 3 significant digits

Last, we find side *c*:

$\sin 26.5^\circ = \dfrac{324}{c}$	Sine relationship
$c = \dfrac{324}{\sin 26.5^\circ}$	Multiply both sides by *c* then divide by $\sin 26.5^\circ$
$= 726$ mm	Answer rounded to 3 significant digits

33. First, we find $\angle A$: $\angle A = 90^\circ - 23.45^\circ$

$= 66.55^\circ$

Next, we find side *b*:

$\tan 23.45^\circ = \dfrac{b}{5.432}$	Tangent relationship
$b = 5.432 \tan 23.45^\circ$	Multiply both sides by 5.432
$= 2.356$ mi	Answer rounded to 4 significant digits

Last, we find side *c*:

$\cos 23.45^\circ = \dfrac{5.432}{c}$	Cosine relationship
$c = \dfrac{5.432}{\cos 23.45^\circ}$	Multiply both sides by *c* and then divide by $\cos 23.45^\circ$
$= 5.921$ mi	Answer rounded to 4 significant digits

35. First, we find $\angle A$:

$\tan A = \dfrac{37}{87}$	Tangent relationship
$= 0.4253$	Divide 37 by 87
$A = \tan^{-1}(0.4253)$	Use calculator to find angle
$= 23^\circ$	Answer rounded to nearest degree

Next, we find $\angle B$:

$\angle B = 90^\circ - \angle A = 90^\circ - 23^\circ = 67^\circ$

Last, we find *c*:

$c^2 = 37^2 + 87^2$	Pythagorean Theorem
$= 1369 + 7569$	Simplify
$= 8938$	Simplify
$c = \pm 95$	Take square root of both sides
$= 95$ ft	*c* must be positive

37. First, we find $\angle A$:

$\cos A = \frac{377.3}{588.5}$	Cosine relationship
$= 0.6411$	Divide
$A = \cos^{-1}(0.6411)$	Use calculator to find angle
$= 50.12^\circ$	Answer rounded to nearest hundredth of a degree

Next, we find $\angle B$: $\angle B = 90^\circ - \angle A$

$= 90^\circ - 50.12^\circ$

$= 39.88^\circ$

Last, we find side *a*:

$a^2 + 377.3^2 = 588.5^2$	Pythagorean Theorem
$a^2 = 203{,}976.96$	Subtract and simplify
$a = \pm 451.6$	Take square root of both sides
$= 451.6$ in	*a* must be positive

39. Using ΔBCD, we find *BD*:

$\sin 30^\circ = \frac{BD}{6.0}$	Sine relationship
$BD = 6.0 \sin 30^\circ$	Multiply both sides by 6
$= 3$	Exact answer

Next, we find $\angle A$

$\sin A = \frac{3}{4.0}$	Sine relationship
$= 0.75$	Divide
$A = \sin^{-1}(0.75)$	Use calculator to find angle
$A = 49^\circ$	Answer rounded to the nearest degree

41.

$\sin 31^\circ = \frac{12}{x+12}$	Sine relationship
$(x+12)\sin 31^\circ = 12$	Multiply both sides by $x + 12$
$x + 12 = \frac{12}{\sin 31^\circ}$	Divide both sides by $\sin 31^\circ$
$x = \frac{12}{\sin 31^\circ} - 12 = 11$	Subtract 12 from both sides and round to 2 significant digits

43.

$\cos 65^\circ = \frac{r}{r+22}$	Sine relationship
$r = (r+22)\cos 65^\circ$	Multiply both side by $r + 22$
$r = r\cos 65^\circ + 22\cos 65^\circ$	Use distributive property
$r - r\cos 65^\circ = 22\cos 65^\circ$	Subtract $r \cos 65^\circ$ from both sides
$r(1 - \cos 65^\circ) = 22\cos 65^\circ$	Factor left side
$r = \frac{22\cos 65^\circ}{1 - \cos 65^\circ}$	Divide both sides by $1 - \cos 65^\circ$
$= 16$	Answer rounded to 2 significant digits

45. Using ΔABC, we find side x:

$\tan 62^\circ = \dfrac{x}{42}$	Tangent relationship
$x = 42\tan 62^\circ$	Multiply both sides by 42
$= 79$	Answer rounded to 2 significant digits

Next, using ΔABD, we find side h:

$\tan 27^\circ = \dfrac{h}{x}$	Tangent relationship
$= \dfrac{h}{79}$	Substitute value for x
$h = 79\tan 27^\circ$	Multiply both sides by 79
$h = 40$	Answer rounded to 2 significant digits

47. Using ΔABC, we find side x:

$\sin 41^\circ = \dfrac{x}{32}$	Sine relationship
$x = 32\sin 41^\circ$	Multiply both sides by 32
$= 21$	Answer rounded to 2 significant digits

Next, using ΔABD, we find $\angle ABD$:

$\tan \angle ABD = \dfrac{h}{x}$	Tangent relationship
$= \dfrac{19}{21}$	Substitute known values
$= 0.9047$	Divide 19 by 21
$\angle ABD = \tan^{-1}(0.9047)$	Use calculator to find angle
$\angle ABD = 42^\circ$	Answer rounded to the nearest degree

49. Using ΔBCD, we find side *x:*

$\cos 58^\circ = \dfrac{x}{14}$	Cosine relationship
$x = 14\cos 58^\circ$	Multiply both sides by 14
$x = 7.4$	Answer rounded to 2 significant digits

Next, using ΔABC, we find y:

$\cos 41^\circ = \dfrac{x+y}{18}$	Cosine relationship
$x + y = 18\cos 41^\circ$	Multiply both sides by 18
$x + y = 13.58$	Evaluate right side
$7.4 + y = 13.58$	Substitute value for x
$y = 6.18 \approx 6.2$	Subtract 7.4 from both sides and round to 2 significant digits

51. Using ΔABC, we find side *h*:

$\sin 41^\circ = \dfrac{h}{28}$	Sine relationship
$h = 28\sin 41^\circ$	Multiply both sides by 28
$= 18$	Answer rounded to 2 significant digits

Next, using ΔBCD, we find side *x*:

$\tan 58^\circ = \dfrac{h}{x}$	Tangent relationship
$\tan 58^\circ = \dfrac{18}{x}$	Substitute value found for *h*
$x = \dfrac{18}{\tan 58^\circ} = 11$	Solve for *x* and round to 2 significant digits

53. Since *h* is in both ΔABC and ΔBCD, we will solve for *h* in the two triangles:

In ΔBCD, $\tan 57^\circ = \dfrac{h}{x}$	Tangent relationship
$h = x\tan 57^\circ$	Multiply both sides by *x*
In ΔABC, $\tan 43^\circ = \dfrac{h}{x+y}$	Tangent relationship
$h = (x+y)\tan 43^\circ$	Multiply both sides by *x* + *y*
$h = (x+11)\tan 43^\circ$	Substitute value for *y*
Therefore, $x\tan 57^\circ = (x+11)\tan 43^\circ$	Property of equality
$x\tan 57^\circ = x\tan 43 + 11\tan 43^\circ$	Distribution Property
$x\tan 57^\circ - x\tan 43^\circ = 11\tan 43^\circ$	Subtract $x\ \tan 43^\circ$ from both sides
$x(\tan 57^\circ - \tan 43^\circ) = 11\tan 43^\circ$	Factor left side
$x = \dfrac{11\tan 43^\circ}{\tan 57^\circ - \tan 43^\circ} = 17$	Divide both sides by $\tan 57^\circ - \tan 43^\circ$

55. From Problem 69 in Problem Set 2.1, we found that

$$\sin\theta = \frac{1}{\sqrt{3}} = 0.5774$$

$$\theta = \sin^{-1}(0.5774) = 35.3^\circ$$

57. From Problem 69 in Problem Set 2.1, we found that

$$\cos\theta = \frac{\sqrt{2}}{\sqrt{3}} = 0.8165$$

$$\theta = \cos^{-1}(0.8165) = 35.3^\circ$$

59. We know that $EC = DF = 6\text{ ft}$, *EB* = 78 ft, *CB* = 72 ft, *DB* = 60 ft, and $\angle FAB = 45^\circ$.

$$\tan\angle EAB = \frac{78}{54} \qquad \tan\angle CAB = \frac{72}{54} \qquad \tan\angle DAB = \frac{60}{54}$$

$$\angle EAB = \tan^{-1}\frac{78}{54} \qquad \angle CAB = \tan^{-1}\frac{72}{54} \qquad \angle DAB = \tan^{-1}\frac{60}{54}$$

$$\angle EAB = 55.3^\circ \qquad \angle CAB = 53.1^\circ \qquad \angle DAB = 48.0^\circ$$

$$\angle EAC = \angle EAB - \angle CAB = 55.3^\circ - 53.1^\circ = 2.2^\circ$$

and

$$\angle DAF = \angle DAB - \angle FAB = 48.0^\circ - 45^\circ = 3.0^\circ$$

Therefore, the sum of the angles is 5.2°.

63. $\cos 120^\circ = \dfrac{x}{125} = \dfrac{139-h}{125}$

$125\cos 120^\circ = 139 - h$

$h = 139 - 125\cos 120^\circ$ Solve for h

$= 201.5$

$= 200\text{ ft}$ Round to 2 significant digits

65. $r = 98.5$

a. $h = 12 + 98.5 + x$

$\cos 60.0^\circ = \dfrac{x}{98.5}$

$x = 98.5\cos 60.0^\circ$

$= 49.25$

$h = 12 + 98.5 + 49.25$

$= 159.8 \approx 160\text{ ft}$

b. $h = 12 + 98.5 + x$

$\cos 30.0^\circ = \dfrac{x}{98.5}$

$x = 98.5\cos 30.0^\circ$

$= 85.3$

$h = 12 + 98.5 + 85.3$

$= 195.8 \approx 196\text{ ft}$

c. $r + 12 = 98.5 + 12 = 110.5$

$h = 110.5 - x$

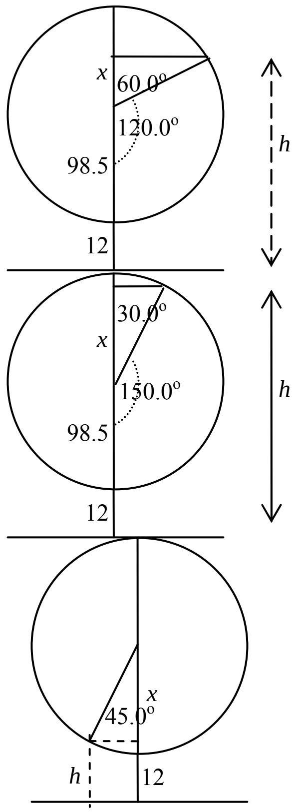
$\cos 45.0^\circ = \dfrac{x}{98.5}$

$x = 98.5\cos 45.0^\circ$

$= 69.7$

$h = 110.5 - 69.7$

$= 40.8\text{ ft}$



67. The radius of the London Eye is $\dfrac{135}{2} = 67.5$.

$\cos\theta = \dfrac{67.5 - 44.5}{67.5}$

$= \dfrac{23}{67.5}$

$\theta = \cos^{-1}(0.6592)$

$\theta = 70.1^\circ$

71. $\sec\theta = 2$

$\cos\theta = \dfrac{1}{\sec\theta}$ Reciprocal identity

$= \dfrac{1}{2}$ Substitute known value

$\cos^2\theta = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ Square both sides

73. $\cos\theta = -\sqrt{1-\sin^2\theta}$ Pythagorean identity, θ in QIII

$$= -\sqrt{1-\left(-\frac{2}{3}\right)^2} = -\sqrt{1-\frac{4}{9}}$$

$$= -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$$

75. $\cos\theta = -\sqrt{1-\sin^2\theta}$ Pythagorean identity, θ in QII

$$= -\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$$ Substitute known value

$$= -\sqrt{1-\frac{3}{4}}$$ Simplify

$$= -\sqrt{\frac{1}{4}} = -\frac{1}{2}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ Ratio identity

$= \frac{\sqrt{3}/2}{-1/2}$ Substitute known values

$= -\sqrt{3}$ Simplify

$\csc\theta = \frac{1}{\sin\theta}$ Reciprocal identity

$= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{1}{\cos\theta}$ Reciprocal identity

$= \frac{1}{-1/2} = -2$

$\cot\theta = \frac{1}{\tan\theta}$ Reciprocal identity

$= \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

77. $\cos\theta = \frac{1}{\sec\theta}$ Reciprocal identity

$= -\frac{1}{2}$ Substitute known values

$\sin\theta = -\sqrt{1-\cos^2\theta}$ Pythagorean identity, θ in QIII

$= -\sqrt{1-\left(-\frac{1}{2}\right)^2}$ Substitute value for $\cos\theta$

$= -\sqrt{1-\frac{1}{4}} = -\sqrt{\frac{3}{4}}$ Simplify

$= -\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ Ratio identity

$= \frac{-\sqrt{3}/2}{-1/2}$ Substitute values

$= \sqrt{3}$

$\cot\theta = \frac{1}{\tan\theta}$ Reciprocal identity

$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc\theta = \frac{1}{\sin\theta}$ Reciprocal identity

$= \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

81. $\tan B = \frac{35}{58}$

$B = 31^\circ$

The answer is b.

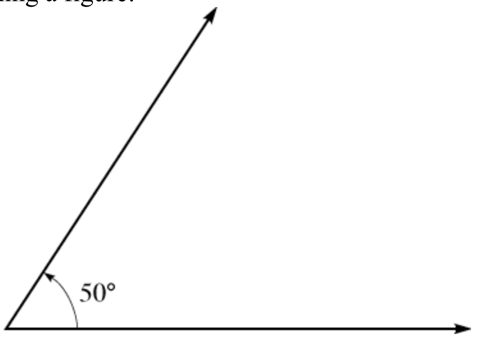
2.4 Applications

EVEN SOLUTIONS

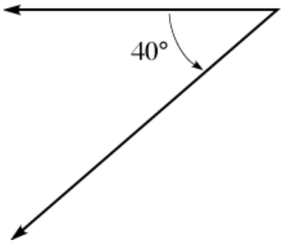
2. If an observer positioned at the vertex of an angle views an object in the direction of the non-horizontal side of the angle, then this side is called the line of sight of the observer.

4. The bearing of a line is always measured as an angle from the north or south rotating toward the east or west.

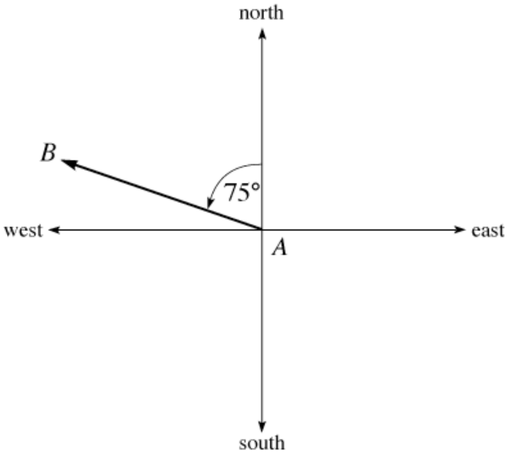
6. Sketching a figure:



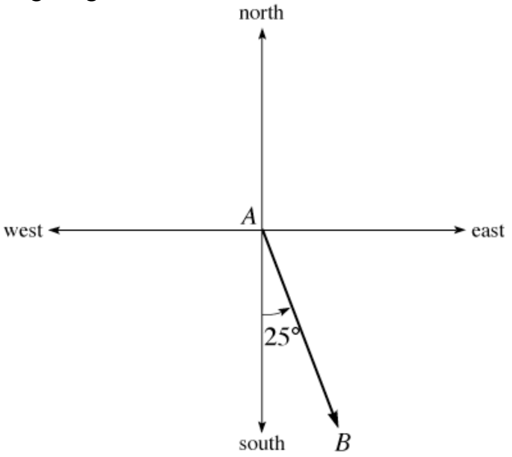
8. Sketching a figure:



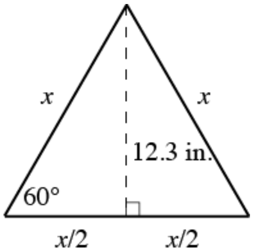
10. Sketching a figure:



12. Sketching a figure:



14. Call x the length of each side. Note the figure:

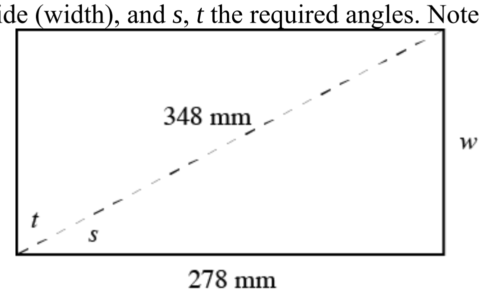


Therefore:

$$\sin 60° = \frac{12.3}{x}$$
$$x \sin 60° = 12.3$$
$$x = \frac{12.3}{\sin 60°} \approx 14.2$$

The length of each side is 14.2 in.

16. Call w the length of the shorter side (width), and s, t the required angles. Note the figure:



Find w using the Pythagorean Theorem:

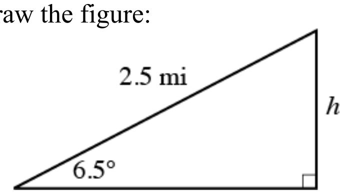
$$278^2 + w^2 = 348^2$$
$$w^2 = 348^2 - 278^2 = 43820$$
$$w \approx 209$$

Now find angles s and t:

$$\cos s = \frac{278}{348}$$
$$s = \cos^{-1}\left(\frac{278}{348}\right) \approx 37.0°$$
$$t = 90° - 37.0° = 53.0°$$

The shorter side is 209 mm, and the two angles are 37.0° and 53.0°.

18. Let h represent the height of the hill. Draw the figure:

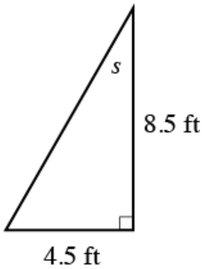


Therefore:

$$\sin 6.5° = \frac{h}{2.5}$$
$$h = 2.5 \sin 6.5° \approx 0.28$$

The hill is approximately 0.28 mi high, which is approximately 1,480 feet.

20. Let s represent the angle between the ladder and the wall. Draw the figure:

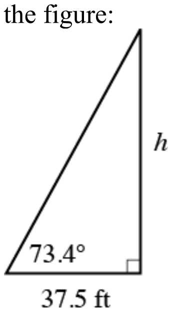


Therefore:

$$\tan s = \frac{4.5}{8.5}$$
$$s = \tan^{-1}\left(\frac{4.5}{8.5}\right) \approx 28°$$

The angle between the ladder and the wall is approximately 28°.

22. Let h represent the height of the building. Draw the figure:

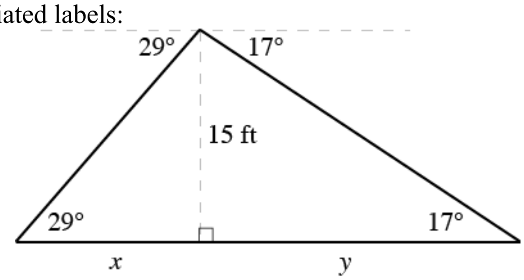


Therefore:

$$\tan 73.4° = \frac{h}{37.5}$$
$$h = 37.5 \tan 73.4° \approx 126$$

The height of the building is approximately 126 feet.

24. Draw the figure with the associated labels:



The sum $x+y$ represents the width of the sand pile. Using the two triangles:

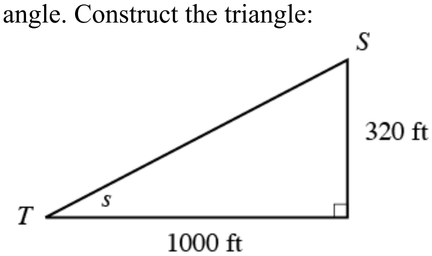
$$\tan 29° = \frac{15}{x} \qquad \tan 17° = \frac{15}{y}$$
$$x \tan 29° = 15 \qquad y \tan 17° = 15$$
$$x = \frac{15}{\tan 29°} \approx 27.1 \qquad y = \frac{15}{\tan 17°} \approx 49.1$$

The width of the sand pile is therefore 27.1 + 49.1 ≈ 76 feet.

26. **a**. First note that $\frac{5}{8}\text{ in.} = \frac{5}{8} \cdot 1600 = 1000\text{ ft}$, which is the horizontal distance between Stacey and Travis.

b. There are 8 contour intervals between Stacey and Travis, which corresponds to a vertical distance of $8 \cdot 40 = 320\text{ ft}$.

c. Let s represent the elevation angle. Construct the triangle:

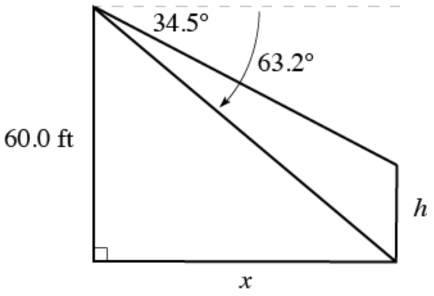


Therefore:

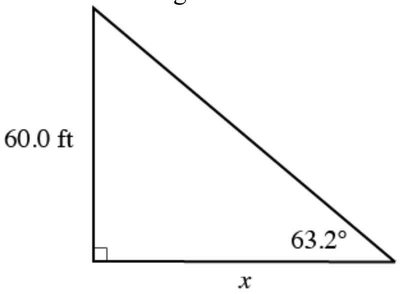
$$\tan s = \frac{320}{1000} = 0.32$$
$$s = \tan^{-1}(0.32) \approx 17.7°$$

The elevation angle from Travis to Stacey is approximately 17.7°.

28. Construct the figure:



First consider the triangle:



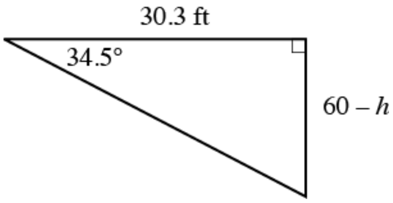
Therefore:

$$\tan 63.2° = \frac{60.0}{x}$$

$$x \tan 63.2° = 60.0$$

$$x = \frac{60.0}{\tan 63.2°} \approx 30.3 \text{ ft}$$

Now consider the triangle:



Therefore:

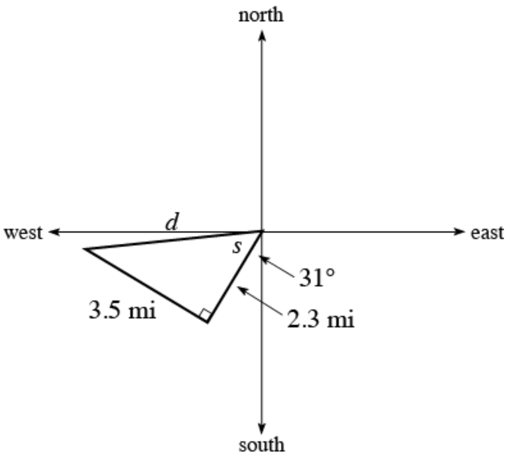
$$\tan 34.5° = \frac{60 - h}{30.3}$$

$$60 - h = 30.3 \tan 34.5°$$

$$h = 60 - 30.3 \tan 34.5° \approx 39.2 \text{ ft}$$

The building next door is approximately 39.2 feet tall.

30. Construct the figure:



To find his distance from the starting point, use the Pythagorean Theorem:

$$d^2 = 3.5^2 + 2.3^2 = 17.54$$

$$d = \sqrt{17.54} \approx 4.2 \text{ mi}$$

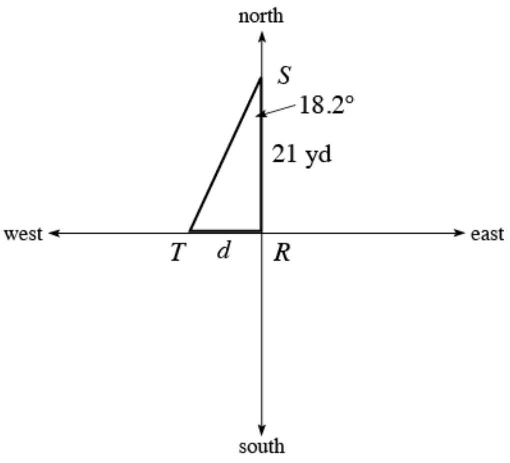
Now find angle s:

$$\tan s = \frac{3.5}{2.3}$$

$$s = \tan^{-1}\left(\frac{3.5}{2.3}\right) \approx 57°$$

His bearing is S 88° W.

32. Construct the figure:

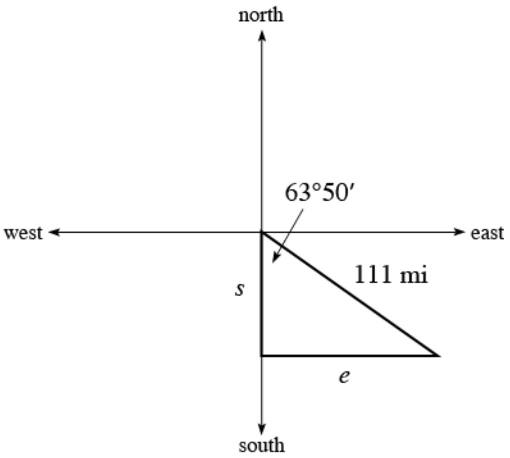


Therefore:

$$\tan 18.2° = \frac{d}{21.0}$$
$$d = 21.0 \tan 18.2° \approx 6.90$$

The distance from the tree to the rock is 6.90 yards.

34. Construct the figure:

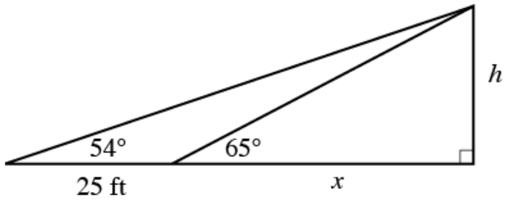


Therefore:

$$\sin 63°50' = \frac{e}{111} \qquad \cos 63°50' = \frac{s}{111}$$
$$e = 111 \sin 63°50' \approx 99.6 \text{ mi} \qquad s = 111 \cos 63°50' \approx 48.9 \text{ mi}$$

The boat travels 48.9 mi south and 99.6 mi east.

36. Draw the figure:



From the smaller triangle:

$$\tan 65° = \frac{h}{x}$$
$$x \tan 65° = h$$
$$x = \frac{h}{\tan 65°}$$

From the larger triangle:

$$\tan 54° = \frac{h}{25+x}$$
$$(25+x)\tan 54° = h$$
$$25+x = \frac{h}{\tan 54°}$$
$$x = \frac{h}{\tan 54°} - 25$$

Setting these two expressions equal:

$$\frac{h}{\tan 65°} = \frac{h}{\tan 54°} - 25$$
$$h\cot 65° = h\cot 54° - 25$$
$$h\cot 65° - h\cot 54° = -25$$
$$h(\cot 65° - \cot 54°) = -25$$
$$h = \frac{25}{\cot 54° - \cot 65°} \approx 96$$

The height of the obelisk is approximately 96 feet.

38. First find the length CB:

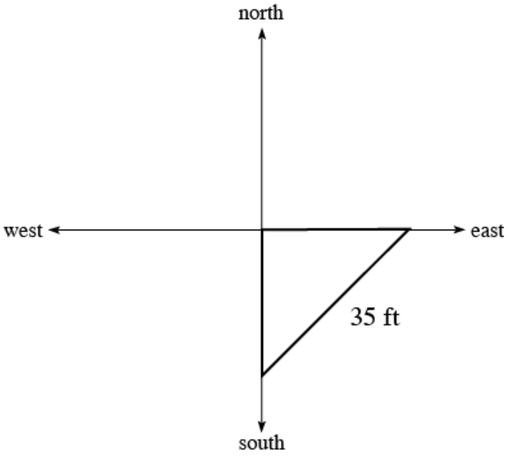
$$\tan 12.3° = \frac{426}{CB}$$
$$CB \tan 12.3° = 426$$
$$CB = \frac{426}{\tan 12.3°} \approx 1,954 \text{ ft}$$

Therefore:

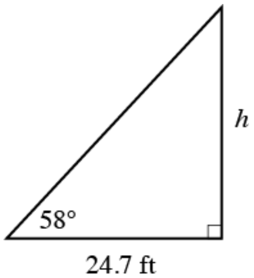
$$\sin \angle BCA = \frac{AB}{CB}$$
$$\sin 57.5° = \frac{AB}{1954}$$
$$AB = 1954 \sin 57.5° \approx 1,650 \text{ ft}$$

A rescue boat at A will have to travel approximately 1,650 feet to reach any survivors at point B.

40. Construct a figure:



First note that each person is $\frac{35}{\sqrt{2}}$ ft ≈ 24.7 ft from the base of the tree. Let h represent the height of the tree. Now construct the triangle:



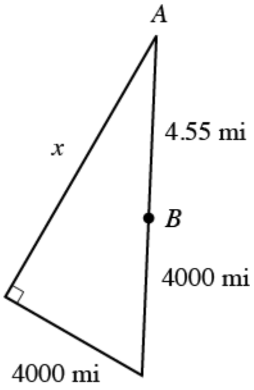
Therefore:

$$\tan 58° = \frac{h}{24.7}$$

$$h = 24.7 \tan 58° \approx 40 \text{ ft}$$

The tree is approximately 27 feet tall.

42. Construct the figure (not drawn to scale):



Using the Pythagorean Theorem:

$$x^2 + 3960^2 = 3964.55^2$$

$$x^2 + 15,681,600 = 15,717,656.7$$

$$x^2 = 36,056.7$$

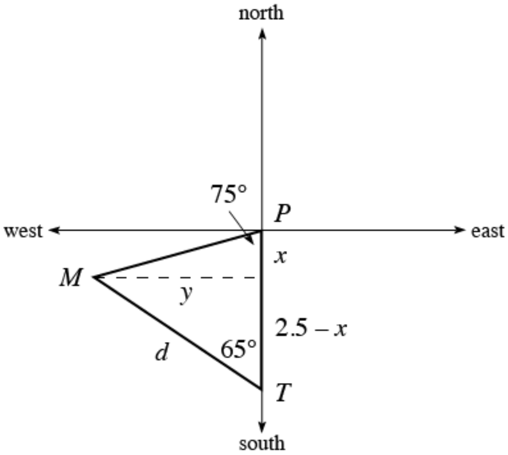
$$x \approx 190$$

The plane is 190 miles from the horizon. Now find angle A:

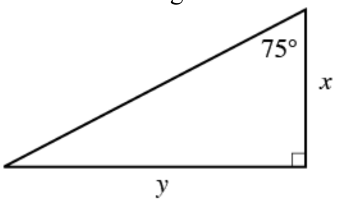
$$\sin A = \frac{3960}{3964.55}$$

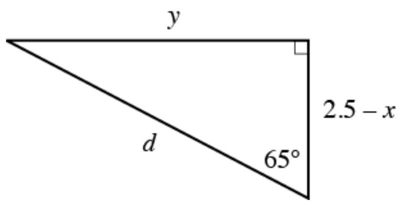
$$A = \sin^{-1}\left(\frac{3960}{3964.55}\right) \approx 87.3°$$

44. Construct the figure:



Consider the two triangles:





Therefore:

$$\tan 75° = \frac{y}{x}$$
$$y = x\tan 75°$$

$$\tan 65° = \frac{y}{2.5 - x}$$
$$y = (2.5 - x)\tan 65°$$

Setting these two expressions equal:

$$\begin{aligned}(2.5 - x)\tan 65° &= x\tan 75° \\ 2.5\tan 65° - x\tan 65° &= x\tan 75° \\ 2.5\tan 65° &= x\tan 75° + x\tan 65° \\ 2.5\tan 65° &= x\left(\tan 75° + \tan 65°\right) \\ x &= \frac{2.5\tan 65°}{\tan 75° + \tan 65°} \approx 0.91\end{aligned}$$

Therefore:

$$\begin{aligned}\cos 65° &= \frac{2.5 - x}{d} = \frac{2.5 - 0.91}{d} = \frac{1.59}{d} \\ d\cos 65° &= 1.59 \\ d &= \frac{1.59}{\cos 65°} \approx 3.8 \text{ mi}\end{aligned}$$

Tim is approximately 3.8 miles from the missile when it is launched.

46. Note that $\sin\theta_1 = \frac{1}{\sqrt{2}}, \sin\theta_2 = \frac{1}{\sqrt{3}}, \sin\theta_3 = \frac{1}{\sqrt{4}}$, ... , thus $\sin\theta_n = \frac{1}{\sqrt{n+1}}$.

48. **a**. Let x represent the height that is illuminated on the floor. Then:

$$\tan 84° = \frac{4}{x}$$
$$x = \frac{4}{\tan 84°} \approx 0.42$$

The illuminated area is then: $(0.42)(6.5) \approx 2.7 \text{ ft}^2$.

b. Following the procedure from part **a**:

$$\tan 37° = \frac{4}{x}$$
$$x = \frac{4}{\tan 37°} \approx 5.31$$

The illuminated area is then: $(5.31)(6.5) \approx 34.5 \text{ ft}^2$. The area is much larger on the winter day.

50. Simplifying: $\frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \sin\theta \cdot \frac{\sin\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

52. Working from the left side: $\cos\theta\csc\theta\tan\theta = \cos\theta \cdot \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = 1$

54. Working from the left side: $(1-\cos\theta)(1+\cos\theta) = 1-\cos\theta+\cos\theta-\cos^2\theta = 1-\cos^2\theta = \sin^2\theta$

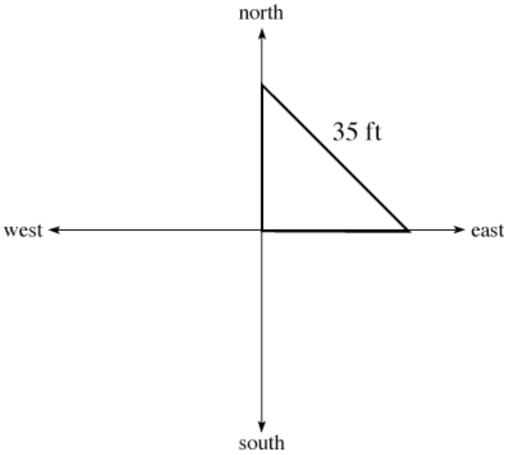
56. Working from the left side: $1 - \frac{\cos\theta}{\sec\theta} = 1 - \frac{\cos\theta}{1/\cos\theta} = 1-\cos^2\theta = \sin^2\theta$

58. Let h represent the height of the flagpole. Then:

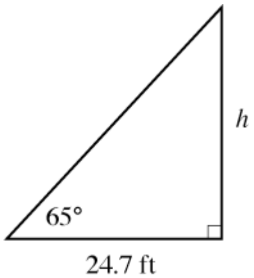
$$\tan 74.3° = \frac{h}{22.5}$$
$$h = 22.5\tan 74.3° \approx 80.0$$

The flagpole is 80.0 feet tall. The correct answer is d.

60. Construct a figure:



First note that each person is $\frac{35}{\sqrt{2}} \text{ ft} \approx 24.7 \text{ ft}$ from the base of the tree. Let h represent the height of the tree. Now construct the triangle:



Therefore:

$$\tan 65° = \frac{h}{24.7}$$
$$h = 24.7\tan 65° \approx 53 \text{ ft}$$

The tree is approximately 53 feet tall. The correct answer is a.

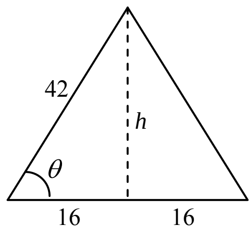
ODD SOLUTIONS

1. elevation, depression **3.** north-south

For problems 5 through 11, see diagrams in textbook answer section.

13. To find the height, *h,* we can use the Pythagorean Theorem:

$$h^2+(16)^2=(42)^2$$
$$h^2+256=1,764$$
$$h^2=1,508$$
$$h=\pm\sqrt{1,508}=39\text{cm}$$



To find angle θ, we can use the cosine ratio:

$$\cos\theta=\frac{16}{42}$$
$$\theta=\cos^{-1}\left(\frac{16}{42}\right)=68^\circ$$

The height is 39 cm and the two equal angles are 68°.

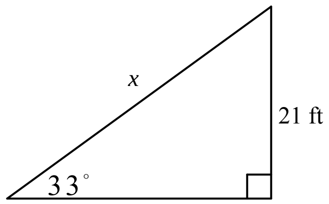
15. Consider the right triangle with sides of 25.3 cm and 5.2 cm (one-half of the diameter):

$$\tan\theta=\frac{25.3}{5.2}$$
$$=4.8654$$
$$\theta=\tan^{-1}(4.8654)$$
$$=78.4^\circ$$

The angle the side makes with the base is 78.4°.

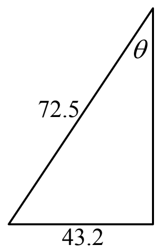
17. To find the length of the escalator, *x*, we use the sine ratio:

$$\sin 33^\circ=\frac{21}{x}$$
$$x=\frac{21}{\sin 33^\circ}$$
$$=\frac{21}{0.5446}=39\text{ ft}$$



The length of the escalator is 39 feet.

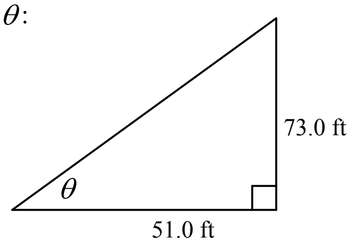
19.
$$\sin\theta=\frac{43.2}{72.5}$$
$$=0.5959$$
$$\theta=\sin^{-1}(0.5959)$$
$$=36.6^\circ$$



The angle the rope makes with the pole is 36.6°

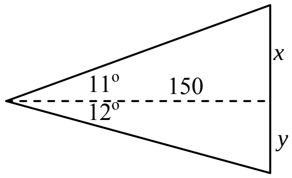
21. We use the tangent ratio to find the angle of elevation to the sun, θ:

$$\tan\theta=\frac{73.0}{51.0}$$
$$=1.4313$$
$$\theta=\tan^{-1}(1.4313)$$
$$=55.1^\circ$$



The angle of elevation to the sun is 55.1°.

23. $\tan 11^\circ = \dfrac{x}{150}$



$$x = 150\tan 11^\circ = 29\text{ cm}$$

$\tan 12^\circ = \dfrac{y}{150}$

$$y = 150\tan 12^\circ$$
$$= 32\text{ cm}$$

The vertical dimension of the mirror is $x + y$ or 61 cm.

25. **a.** horizontal distance = 0.50(1,600) = 800 ft

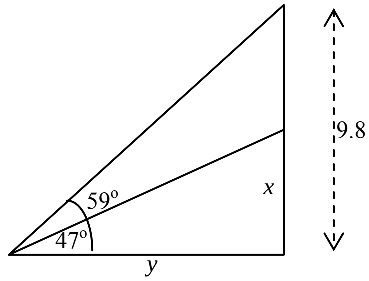
b. vertical distance = (number of contour intervals)(40)

$$= 5(40)$$
$$= 200 \text{ ft}$$

c. $\tan\theta = \dfrac{\textit{vertical distance}}{\textit{horizontal distance}}$

$$= \frac{200}{800}$$
$$= 0.25$$
$$\theta = \tan^{-1}(0.25)$$
$$= 14^\circ$$

27. $\tan 59^\circ = \dfrac{9.8}{y}$



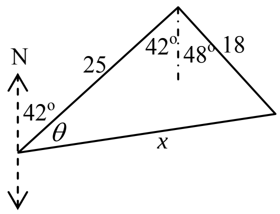
$$y = \frac{9.8}{\tan 59^\circ} = \frac{9.8}{1.6643} = 5.9$$

$\tan 47^\circ = \dfrac{9.8}{\tan 59^\circ}$

$$x = y\tan 47^\circ$$
$$= 5.9(1.0724) = 6.3 \text{ ft}$$

The vertical dimension of the door is 6.3 feet.

29. We use the Pythagorean Theorem to find the distance *x:*



$$x^2 = 25^2 + 18^2$$
$$= 625 + 324$$
$$= 949$$
$$x = 31 \text{ mi}$$

We use the tangent relationship to find angle θ:

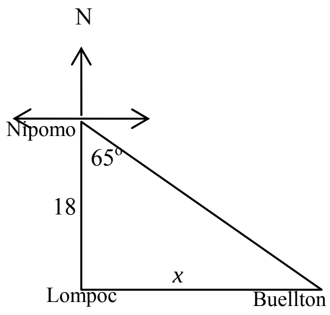
$$\tan\theta = \frac{18}{25}$$
$$= 0.72$$
$$\theta = \tan^{-1}(0.72)$$
$$= 36^\circ$$

To find the bearing we add $42^\circ + 36^\circ = 78^\circ$. The boat is 31 miles from the harbor entrance and its bearing is N 78° E.

31. $\tan 65^\circ = \dfrac{x}{18}$

$$\begin{aligned} x &= 18\tan 65^\circ \\ &= 18(2.1445) \\ &= 39\text{ mi} \end{aligned}$$

The distance from Lompoc to Buellton is 39 miles.



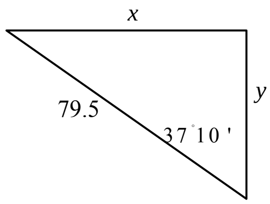
33. We will call the west distance, x and the north distance, y:

$\sin 37^\circ\,10' = \dfrac{x}{79.5}$ $\qquad\qquad$ $\cos 37^\circ 10' = \dfrac{y}{79.5}$

$x = 79.5\sin 37^\circ 10'$ $\qquad\qquad$ $y = 79.5\cos 37^\circ 10'$

$= 48.0\text{ mi}$ $\qquad\qquad$ $= 63.4\text{ mi}$

The boat has traveled 48.0 miles west and 63.4 miles north.



35. In ΔABC, $\tan 42.17^\circ = \dfrac{h}{x+33}$

$$h = (x+33)\tan 42.17^\circ$$

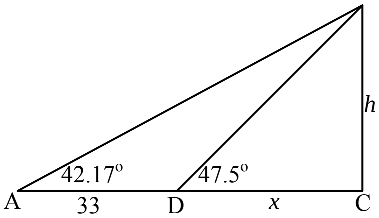
In ΔBCD, $\tan 47.5^\circ = \dfrac{h}{x}$

$$h = x\tan 47.5^\circ$$

Therefore, $x\tan 47.5^\circ = (x+33)\tan 42.17^\circ$

$$\begin{aligned} x\tan 47.5^\circ &= x\tan 42.17^\circ + 33\ \tan 42.17^\circ \\ x\tan 47.5^\circ - x\tan 42.17^\circ &= 33\ \tan 42.17^\circ \\ x\left(\tan 47.5^\circ - \tan 42.17^\circ\right) &= 33\ \tan 42.17^\circ \\ x &= \frac{33\tan 42.17^\circ}{\tan 47.5^\circ - \tan 42.17^\circ} = 161\text{ ft} \end{aligned}$$

The person at point A is 161 feet from the base of the antenna.



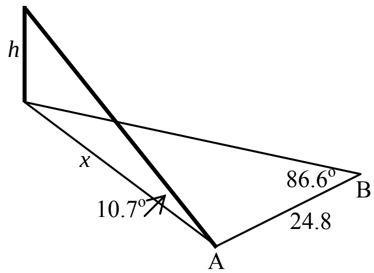
37. $\tan 86.6^\circ = \dfrac{x}{24.8}$

$$\begin{aligned} x &= 24.8\tan 86.6^\circ \\ &= 24.8(16.8319) \\ &= 417.431 \end{aligned}$$

$\tan 10.7^\circ = \dfrac{h}{x}$

$$\begin{aligned} h &= x\tan 10.7^\circ \\ &= (417.431)(0.18895) \\ &= 78.9\text{ ft} \end{aligned}$$

The tree is 78.9 feet high.



39. First, we will find each person's distance from the pole, x, using the Pythagorean Theorem:

$$\begin{aligned} x^2+x^2 &= 25^2 \\ 2x^2 &= 625 \\ x^2 &= 312.5 \\ x &= 17.678 \text{ ft} \end{aligned}$$

Next, we will find the height of the pole, h, using the tangent relationship:

$$\begin{aligned} \tan 56^\circ &= \frac{h}{17.678} \\ h &= 17.678 \tan 56^\circ \\ &= 26 \text{ ft} \end{aligned}$$

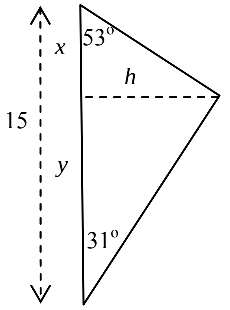
The height of the pole is 26 feet.

41.

$$\begin{aligned} \sin 76.6^\circ &= \frac{r}{r+112} \\ r &= (r+112)\sin 76.6^\circ \\ r &= r\sin 76.6^\circ + 112\sin 76.6^\circ \\ r - r\sin 76.6^\circ &= 112\sin 76.6^\circ \\ r\left(1-\sin 76.6^\circ\right) &= 112\sin 76.6^\circ \\ r &= \frac{112\sin 76.6^\circ}{1-\sin 76.6^\circ} \\ &= \frac{112(0.9728)}{1-0.9728} \\ &= \frac{108.9509}{0.02722} \\ &= 4{,}000 \text{ mi} \end{aligned}$$

The radius of the earth is 4,000 miles.

43. We want to find x and y in terms of h



$$\tan 53^\circ = \frac{h}{x} \qquad \tan 31^\circ = \frac{h}{y}$$

$$x\tan 53^\circ = h \qquad y\tan 31^\circ = h$$

$$x = \frac{h}{\tan 53^\circ} \qquad y = \frac{h}{\tan 31^\circ}$$

We know that $x + y = 15$. Therefore,

$$\begin{aligned} \frac{h}{\tan 53^\circ} + \frac{h}{\tan 31^\circ} &= 15 \\ h\left(\frac{1}{\tan 53^\circ} + \frac{1}{\tan 31^\circ}\right) &= 15 \\ h(0.7536 + 1.6643) &= 15 \\ 2.4179h &= 15 \\ h &= \frac{15}{2.4179} = 6.2 \text{ mi} \end{aligned}$$

The ship is 6.2 miles from the shore.

45. $\tan\theta_1 = \frac{1}{1}$ $\tan\theta_2 = \frac{1}{\sqrt{2}}$ $\tan\theta_3 = \frac{1}{\sqrt{3}}$

$= 1$ $= 0.7071$ $= 0.5774$

$\theta_1 = \tan^{-1}(1)$ $\theta_2 = \tan^{-1}(0.7071)$ $\theta_3 = \tan^{-1}(0.5774)$

$\theta_1 = 45.00^\circ$ $\theta_2 = 35.26^\circ$ $\theta_3 = 30.00^\circ$

49.
$$\begin{aligned}(\sin\theta - \cos\theta)^2 &= (\sin\theta - \cos\theta)(\sin\theta - \cos\theta)\\ &= \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta\\ &= \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta\\ &= 1 - 2\sin\theta\cos\theta\end{aligned}$$

51.

$\sin\theta\cot\theta = \sin\theta\cdot\frac{\cos\theta}{\sin\theta}$	Ratio identity
$= \frac{\sin\theta\cos\theta}{\sin\theta}$	Multiplication of fractions
$= \cos\theta$	Division of common factor

53.

$\frac{\sec\theta}{\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$	Reciprocal and ratio identity
$= \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}$	Division of fractions
$= \frac{1}{\sin\theta}$	Multiplication of fractions and divide common factor
$= \csc\theta$	

55.

$\sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$	Reciprocal identity
$= \frac{1}{\cos\theta} - \cos\theta\cdot\frac{\cos\theta}{\cos\theta}$	L.C.D. is $\cos\theta$
$= \frac{1-\cos^2\theta}{\cos\theta}$	Subtraction of fractions
$= \frac{\sin^2\theta}{\cos\theta}$	Pythagorean identity

2.5 Vectors: A Geometric Approach

EVEN SOLUTIONS

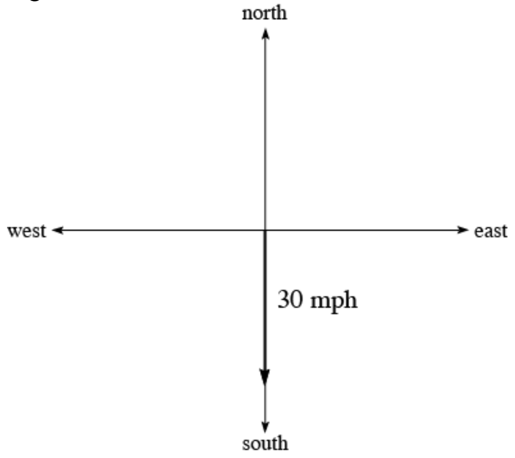
2. Two vectors are equivalent if they have the same magnitude and direction.

4. A vector is in standard position if the tail of the vector is placed at the origin of a rectangular coordinate system.

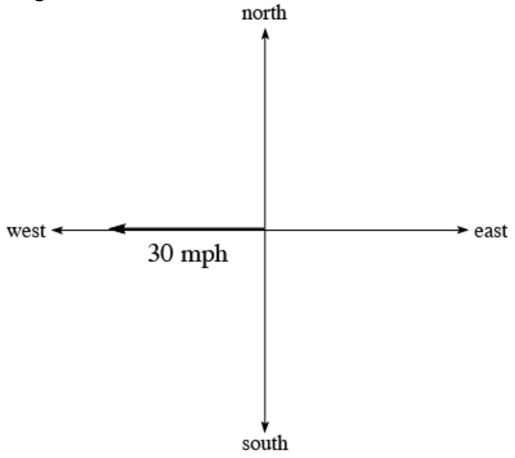
6. If **V** makes and angle θ with the positive x-axis when in standard position, then $|\mathbf{V}_x| = |\mathbf{V}|\cos\theta$ and $|\mathbf{V}_y| = |\mathbf{V}|\sin\theta$.

8. If a constant force **F** is applied to an object and moves the object in a straight line a distance d at an angle θ with the force, then the work performed by the force is found by multiplying $|\mathbf{F}|\cos\theta$ and d.

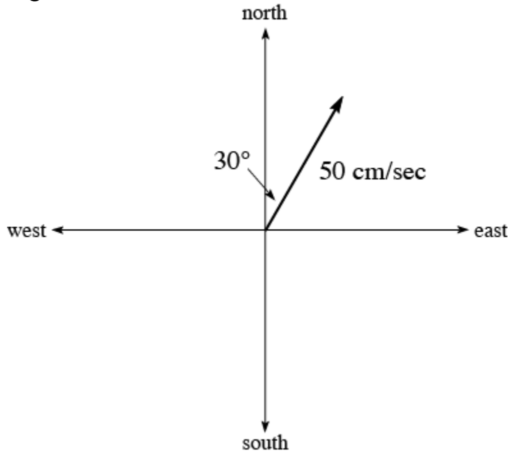
10. Sketching the vector:



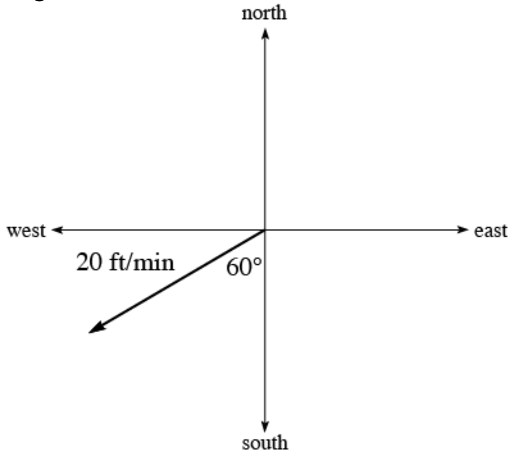
12. Sketching the vector:



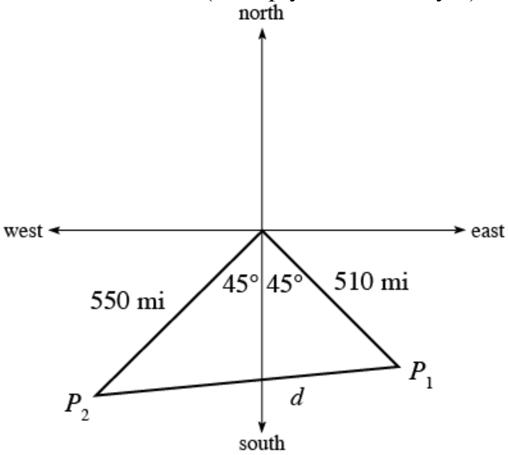
14. Sketching the vector:



16. Sketching the vector:



18. Construct the figure for their position after 2 hours (multiply their rates by 2):

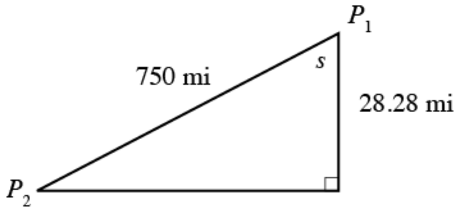


Using the Pythagorean Theorem: $d=\sqrt{550^2+510^2}\approx 750$ miles

To find the bearing from P_1 to P_2, first find the vertical (north-south) change in their positions. This is given by:

$$550\sin 45°-510\sin 45°\approx 28.28 \text{ miles}$$

Construct the triangle:



Therefore:

$$\cos s=\frac{28.28}{750}$$

$$s=\cos^{-1}\left(\frac{28.28}{750}\right)\approx 87.8°$$

The bearing from P_1 to P_2 is S 87.8° W.

20. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x|=17.6\cos 72.6°\approx 5.26$$

$$|\mathbf{V}_y|=17.6\sin 72.6°\approx 16.8$$

22. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x|=383\cos 12°20'\approx 374$$

$$|\mathbf{V}_y|=383\sin 12°20'\approx 81.8$$

24. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

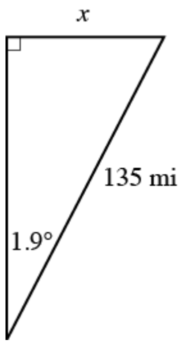
$$|\mathbf{V}_x|=84\cos 90°=0$$

$$|\mathbf{V}_y|=84\sin 90°=84$$

26. Using the Pythagorean Theorem: $|\mathbf{V}|=\sqrt{54.2^2+14.5^2}\approx 56.1$

28. Using the Pythagorean Theorem: $|\mathbf{V}|=\sqrt{2.2^2+8.8^2}\approx 9.1$

30. Construct the triangle:



Therefore:

$$\sin 1.9° = \frac{x}{135}$$
$$x = 135\sin 1.9° \approx 4.48 \text{ miles}$$

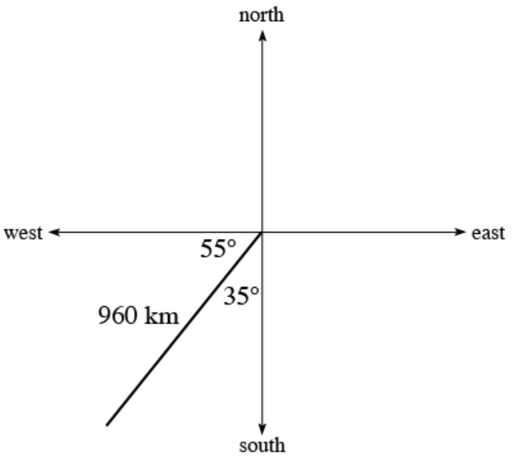
The plane will be approximately 4.48 miles off course.

32. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$\left|\mathbf{V}_x\right| = 1{,}800\cos 55° = 1{,}032\,\tfrac{\text{ft}}{\text{sec}} \approx 1{,}000\,\tfrac{\text{ft}}{\text{sec}}$$
$$\left|\mathbf{V}_y\right| = 1{,}800\sin 55° \approx 1{,}474\,\tfrac{\text{ft}}{\text{sec}} \approx 1{,}500\,\tfrac{\text{ft}}{\text{sec}}$$

34. The horizontal distance traveled is $1.5 \cdot 1{,}032 = 1{,}548 \text{ feet} \approx 1{,}500 \text{ feet}$.

36. Draw the figure corresponding to t = 3 hours:



The west and south distances are given by:

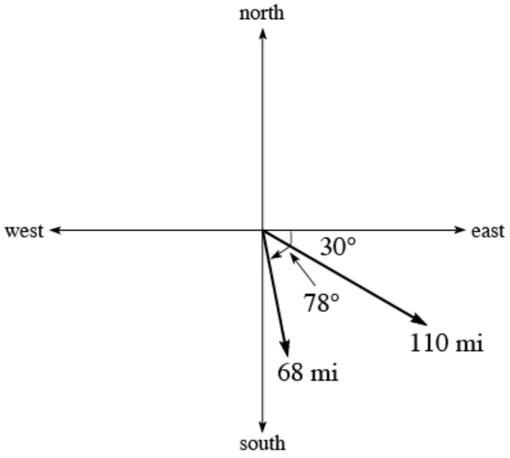
west: $960\cos 55° \approx 550 \text{ km}$ south: $960\sin 55° \approx 790 \text{ km}$

38. Using the Pythagorean Theorem: $\left|\mathbf{V}\right| = \sqrt{16.5^2 + 24.3^2} \approx 29.4$ ft/sec

The elevation angle is given by:

$$\tan\theta = \frac{24.3}{16.5}$$
$$\theta = \tan^{-1}\left(\frac{24.3}{16.5}\right) \approx 55.8°$$

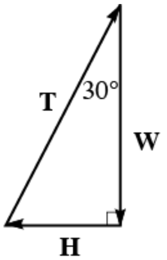
40. Construct the figure:



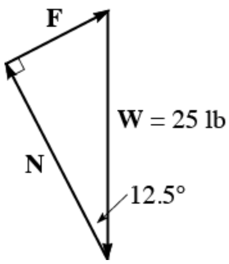
The total distance south and east is given by:

east: $68\cos 78° + 110\cos 30° \approx 110$ miles south: $68\sin 78° + 110\sin 30° \approx 120$ miles

42. The corresponding force diagram would be:



44. The corresponding force diagram would be:

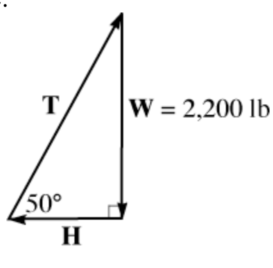


Therefore:

$$\sin 12.5° = \frac{|\mathbf{F}|}{25.0} \qquad\qquad \cos 12.5° = \frac{|\mathbf{N}|}{25.0}$$

$$|\mathbf{F}| = 25.0\sin 12.5° \approx 5.41 \text{ lb} \qquad\qquad |\mathbf{N}| = 25.0\cos 12.5° \approx 24.4 \text{ lb}$$

46. The corresponding force diagram would be:



Therefore:

$$\sin 50° = \frac{2{,}200}{|\mathbf{T}|} \qquad \tan 50° = \frac{2{,}200}{|\mathbf{H}|}$$

$$|\mathbf{T}|\sin 50° = 2{,}200 \qquad |\mathbf{H}|\tan 50° = 2{,}200$$

$$|\mathbf{T}| = \frac{2{,}200}{\sin 50°} \approx 2{,}900 \text{ lb} \qquad |\mathbf{H}| = \frac{2{,}200}{\tan 50°} \approx 1{,}800 \text{ lb}$$

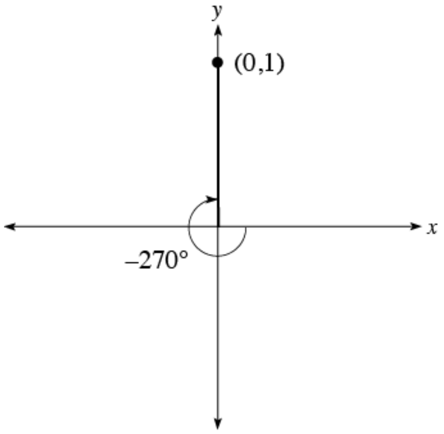
48. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}|\cos 35° = 15\cos 35° \text{ lb}$

The work is then given by: $\text{Work} = (15\cos 35°)(52) \approx 640 \text{ ft-lb}$

50. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}|\cos 15° = 85\cos 15° \text{ lb}$

The work is then given by: $\text{Work} = (85\cos 15°)(110) \approx 9{,}000 \text{ ft-lb}$

52. Drawing the angle in standard position:



Since $r = 1$, $\sin(-270°) = 1$, $\cos(-270°) = 0$, and $\tan(-270°)$ is undefined.

54. Choose $(-1,1)$ as a point on the terminal side of θ. Then $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$. Therefore:

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{x}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

56. Since $\cos\theta = \frac{x}{r} = -\frac{3}{5} = -\frac{6}{10}$, choose $x = -6$ and $r = 10$. Now find y:

$$\begin{aligned}(-6)^2 + y^2 &= 10^2 \\ 36 + y^2 &= 100 \\ y^2 &= 64 \\ y &= \pm 8\end{aligned}$$

58. Using the Pythagorean Theorem: $|\mathbf{V}| = \sqrt{9.6^2 + 2.3^2} \approx 9.9$

Finding the angle:

$$\tan\theta = \frac{2.3}{9.6}$$

$$\theta = \tan^{-1}\left(\frac{2.3}{9.6}\right) \approx 13°$$

The correct answer is c.

60. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}|\cos 35° = 28\cos 35° \text{ lb}$

The work is then given by: $\text{Work} = (28\cos 35°)(150) \approx 3{,}400 \text{ ft-lb}$

The correct answer is b.

ODD SOLUTIONS

1. scalar, vector

3. resultant, diagonal

5. horizontal, component, vertical, component

7. zero, static equilibrium

For problems 9 through 15, see textbook answer section for diagrams.

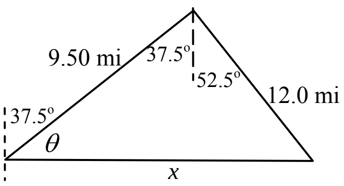
17. The first hour, the distance traveled is

(9.50 mph)(1 hr) = 9.50 miles

The next hour and a half, the distance traveled is

(8.00 mph)(1.5 hr) = 12.0 miles

We will use the Pythagorean Theorem to find *x*:



$$x^2 = 9.50^2 + 12.0^2$$
$$x^2 = 234.25$$
$$x = 15.3 \text{ mi}$$

We will use the tangent ratio to find θ and then add 37.5°:

$$\tan\theta = \frac{12.0}{9.50} = 1.2632$$
$$\theta = \tan^{-1}(1.2632) = 51.6^\circ \qquad 51.6^\circ + 37.5^\circ = 89.1^\circ$$

The balloon is 15.3 miles from its starting point. The bearing is N 89.1° E.

19. $|V_x| = |V|\cos\theta = 13.8\cos 24.2^\circ = 12.6$ $\qquad$ $|V_y| = |V|\sin\theta = 13.8\sin 24.2^\circ = 5.66$

21. $|V_x| = |V|\cos\theta = 425 \ \cos 36^\circ 10' = 425 \ \cos 36.17^\circ = 425(0.8073) = 343$ $\qquad$ $|V_y| = |V|\sin\theta = 425 \ \sin 36^\circ 10' = 425 \ \sin 36.17^\circ = 425(0.5901) = 251$

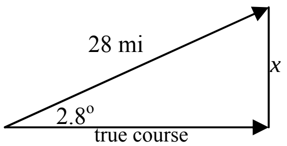
23. $|V_x| = |V|\cos\theta = 64 \cos 0^\circ = 64(1) = 64$ $\qquad$ $|V_y| = |V|\sin\theta = 64 \sin 0^\circ = 64(0) = 0$

25.
$$|V| = \sqrt{|V_x|^2 + |V_y|^2} = \sqrt{(35.0)^2 + (26.0)^2} = \sqrt{1{,}225 + 676} = \sqrt{1{,}901} = 43.6$$

27.
$$|V| = \sqrt{|V_x|^2 + |V_y|^2} = \sqrt{(4.5)^2 + (3.8)^2} = \sqrt{20.25 + 14.44} = \sqrt{34.69} = 5.9$$

29. To find the distance, *x*, the plane has flown off its course, we can use the sine ratio:

$$\sin 2.8^\circ = \frac{x}{28}$$
$$x = 28\sin 2.8^\circ = 1.37 \text{ miles}$$



31. $\left|V_x\right| = \left|V\right|\cos\theta$ $\left|V_y\right| = \left|V\right|\sin\theta$

$= 1{,}200\cos 45^\circ$ $= 1{,}200\sin 45^\circ$

$= 1200(0.7071)$ $= 1200(0.7071)$

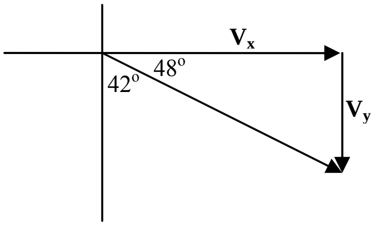
$= 850$ feet per second $= 850$ feet per second

33. In 3 seconds, the bullet travels 3(850 ft/sec) = 2,550 ft.

35. $\left|V_x\right| = 130\cos 48^\circ$ $\left|V_y\right| = 130\sin 48^\circ$

$= 87$ $= 97$

The ship has traveled 97 km south and 87 km east.



37. We are given that $\left|V_x\right| = 35.0$ and $\left|V_y\right| = 15.0$

$\left|V\right| = \sqrt{\left|V_x\right|^2 + \left|V_y\right|^2}$ $\tan\theta = \dfrac{\left|V_y\right|}{\left|V_x\right|}$

$= \sqrt{(35.0)^2 + (15.0)^2}$ $= \dfrac{15.0}{35.0}$

$= \sqrt{1{,}225 + 225}$ $= 0.4285$

$= \sqrt{1{,}450}$

$= 38.1$ feet per second $\theta = 23.2^\circ$

Therefore, the velocity of the arrow is 38.1 feet per second at an elevation of 23.2°.

39. To find the total distance traveled north, we must find the sum of $\left|V_y\right|$ and $\left|W_y\right|$ and to find the total distance traveled east, we must find the sum of $\left|V_x\right|$ and $\left|W_x\right|$.

We are given that $\left|V\right|$ is 170 mi. at an angle of inclination of $90^\circ - 18^\circ$ or 72° and also that $\left|W\right|$ is 120 mi. at an angle of inclination of $90^\circ - 49^\circ$ or 41°

$\left|V_x\right| = \left|V\right|\cos\theta_1$ $\left|V_y\right| = \left|V\right|\sin\theta_1$

$= 170\cos 72^\circ$ $= 170\sin 72^\circ$

$= 170(0.3090)$ $= 170(0.9510)$

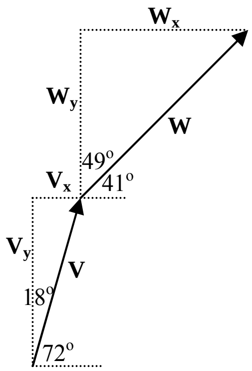
$= 53$ mi $= 162$ mi

$\left|W_x\right| = \left|W\right|\cos\theta_2$ $\left|W_y\right| = \left|W\right|\sin\theta_2$

$= 120\cos 41^\circ$ $= 120\sin 41^\circ$

$= 120(0.7547)$ $= 120(0.6560)$

$= 90$ mi $= 80$ mi



$\left|V_y\right| + \left|W_y\right| = 162 + 80 = 242$ and $\left|V_x\right| + \left|W_x\right| = 53 + 90 = 143$

The total distance north is 240 miles and the total distance east is 140 miles, rounded to 2 significant digits.

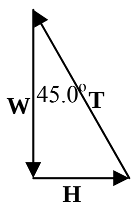
41. $\left|W\right| = 42.0$

$\cos 45.0^\circ = \dfrac{\left|W\right|}{\left|T\right|}$ $\tan 45.0^\circ = \dfrac{\left|H\right|}{\left|W\right|}$

$\left|T\right| = \dfrac{\left|W\right|}{\cos 45.0^\circ}$ $\left|H\right| = \left|W\right|\tan 45.0^\circ$

$= \dfrac{42.0}{\cos 45.0^\circ}$ $= 42.0\tan 45.0^\circ$

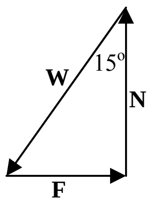
$= 59.4$ lb. $= 42.0$ lb.



43. We are given that $|W| = 8.0$

$$\cos 15^\circ = \frac{|N|}{|W|} \qquad\qquad \sin 15^\circ = \frac{|F|}{|W|}$$

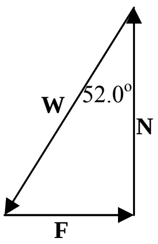
$$\begin{aligned} |N| &= |W|\cos 15^\circ \\ &= 8.0(0.9659) \\ &= 7.7 \text{ pounds} \end{aligned} \qquad\qquad \begin{aligned} |F| &= |W|\sin 15^\circ \\ &= 8.0(0.2588) \\ &= 2.1 \text{ pounds} \end{aligned}$$



45. $|W| = 42.0$

$$\sin 52.0^\circ = \frac{|F|}{|W|}$$

$$\begin{aligned} |F| &= |W|\sin 52.0^\circ \\ &= 42.0 \sin 52.0^\circ \\ &= 33.1 \text{ lb} \end{aligned}$$



47. $\theta = 20^\circ, |F| = 40$ lb, and $d = 75$ ft

$$\begin{aligned} |F_x| &= |F|\cos\theta \\ &= 41\cos 20^\circ \end{aligned} \qquad\qquad \begin{aligned} \text{Work} &= |F_x| \cdot d \\ &= (41\cos 20^\circ)(75) \\ &= 2900 \text{ ft - lb.} \end{aligned}$$

49. $\theta = 30^\circ, |F| = 25$ lb, and $d = 350$ ft

$$\begin{aligned} |F_x| &= |F|\cos\theta \\ &= 25\cos 30^\circ \end{aligned} \qquad\qquad \begin{aligned} \text{Work} &= |F_x| \cdot d \\ &= (25\cos 30^\circ)(350) \\ &= 7,600 \text{ ft - lb.} \end{aligned}$$

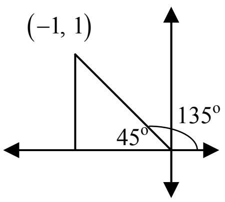
51. $(x, y) = (-1, 1)$

$x = -1, y = 1$ and $r = \sqrt{2}$

$$\sin 135^\circ = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 135^\circ = \frac{x}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\tan 135^\circ = \frac{y}{x} = \frac{1}{-1} = -1$$



53. A point on the line $y = 2x$ in quadrant I is (1, 2). $x = 1, y = 2$, and $r = \sqrt{1^2 + 2^2} = \sqrt{5}$

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \qquad\qquad \cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

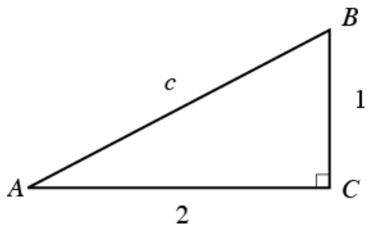
55. $\sin\theta = \dfrac{y}{r} = \dfrac{-4}{5} = \dfrac{-8}{10}$

$y = -8$ and $r = 10$

$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-8)^2 &= 10^2 \\ x^2 + 64 &= 100 \\ x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

Chapter 2 Test

1. First draw the triangle:

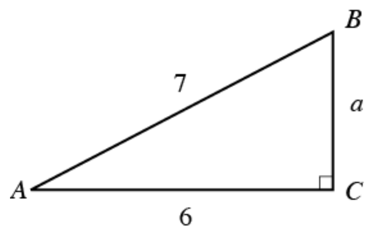


Note that $c=\sqrt{2^2+1^2}=\sqrt{5}$. Therefore:

$$\sin A=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5} \qquad \cos A=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5} \qquad \tan A=\frac{1}{2}$$

$$\sin B=\cos A=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5} \qquad \cos B=\sin A=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5} \qquad \tan B=\cot A=\frac{2}{1}=2$$

2. First draw the triangle:

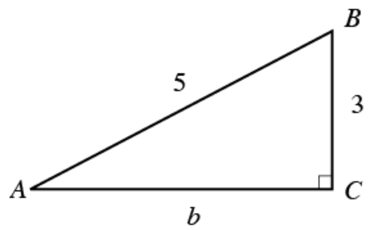


Note that $a=\sqrt{7^2-6^2}=\sqrt{13}$. Therefore:

$$\sin A=\frac{\sqrt{13}}{7} \qquad \cos A=\frac{6}{7} \qquad \tan A=\frac{\sqrt{13}}{6}$$

$$\sin B=\cos A=\frac{6}{7} \qquad \cos B=\sin A=\frac{\sqrt{13}}{7} \qquad \tan B=\cot A=\frac{6}{\sqrt{13}}=\frac{6\sqrt{13}}{13}$$

3. First draw the triangle:



Note that $b=\sqrt{5^2-3^2}=\sqrt{16}=4$. Therefore:

$$\sin A=\frac{3}{5} \qquad \cos A=\frac{4}{5} \qquad \tan A=\frac{3}{4}$$

$$\sin B=\cos A=\frac{4}{5} \qquad \cos B=\sin A=\frac{3}{5} \qquad \tan B=\cot A=\frac{4}{3}$$

4. Since $y\le r$, $\frac{y}{r}\le 1$. Therefore $\sin\theta=\frac{y}{r}\le 1$, so it is impossible for $\sin\theta=2$.

5. $\sin 14°=\cos(90°-14°)=\cos 76°$

6. Simplifying: $\sin^2 45°+\cos^2 60°=\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{1}{2}\right)^2=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

7. Simplifying: $\tan 45°+\cot 45°=1+1=2$

8. Simplifying: $\sin^2 60° - \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} - \frac{3}{4} = 0$

9. Simplifying: $\frac{1}{\csc 30°} = \sin 30° = \frac{1}{2}$

10. Adding: $48°31' + 24°52' = 72°83' = 73°23'$

11. Converting to degrees and minutes: $73.2° = 73° + 0.2° = 73° + 0.2(60') = 73°12'$

12. Converting to decimal degrees: $2°48' = 2° + 48' = 2° + \left(\frac{48}{60}\right)^\circ = 2.8°$

13. Calculating the value: $\sin 24°20' = \sin\left(24\frac{1}{3}\right)^\circ \approx 0.4120$

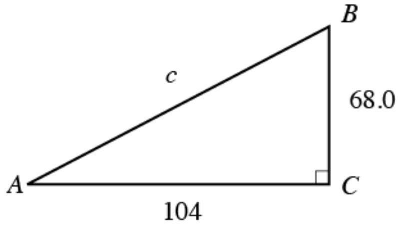
14. Calculating the value: $\cos 48.3° \approx 0.6652$

15. Calculating the value: $\cot 71°20' = \cot\left(71\frac{1}{3}\right)^\circ = \frac{1}{\tan\left(71\frac{1}{3}\right)^\circ} \approx 0.3378$

16. Since $\sin\theta = 0.6459$, $\theta = \sin^{-1}(0.6459) \approx 40.2°$.

17. Since $\sec\theta = 1.923$, $\cos\theta = \frac{1}{1.923}$, so $\theta = \cos^{-1}\left(\frac{1}{1.923}\right) \approx 58.7°$.

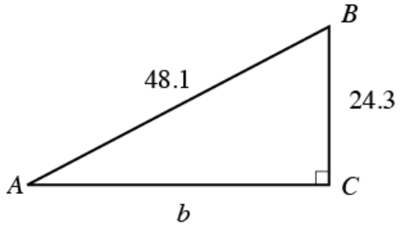
18. First sketch the triangle:



Using the Pythagorean Theorem: $c = \sqrt{104^2 + 68^2} \approx 124$. Therefore:

$$\tan A = \frac{68}{104}$$
$$A = \tan^{-1}\left(\frac{68}{104}\right) \approx 33.2°$$
$$B = 90° - 33.2° = 56.8°$$

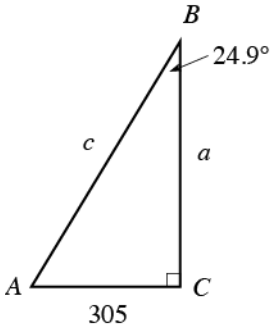
19. First sketch the triangle:



Using the Pythagorean Theorem: $b = \sqrt{48.1^2 - 24.3^2} \approx 41.5$. Therefore:

$$\sin A = \frac{24.3}{48.1}$$
$$A = \sin^{-1}\left(\frac{24.3}{48.1}\right) \approx 30.3°$$
$$B = 90° - 30.3° = 59.7°$$

20. First sketch the triangle:

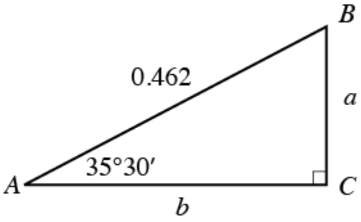


Note that $A = 90° - 24.9° = 65.1°$. Therefore:

$$\tan 65.1° = \frac{a}{305}$$
$$a = 305 \tan 65.1° \approx 657$$

$$\cos 65.1° = \frac{305}{c}$$
$$c \cos 65.1° = 305$$
$$c = \frac{305}{\cos 65.1°} \approx 724$$

21. First sketch the triangle:

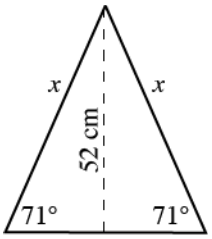


Note that $B = 90° - 35°30' = 89°60' - 35°30' = 54°30'$. Also:

$$\sin 35.5° = \frac{a}{0.462}$$
$$a = 0.462 \sin 35.5° \approx 0.268$$

$$\cos 35.5° = \frac{b}{0.462}$$
$$b = 0.462 \cos 35.5° \approx 0.376$$

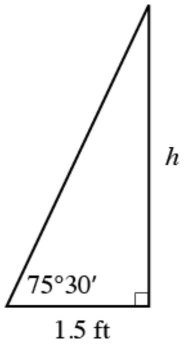
22. First sketch the triangle:



Therefore:

$$\sin 71° = \frac{52}{x}$$
$$x \sin 71° = 52$$
$$x = \frac{52}{\sin 71°} \approx 55 \text{ cm}$$

23. Sketch the figure:

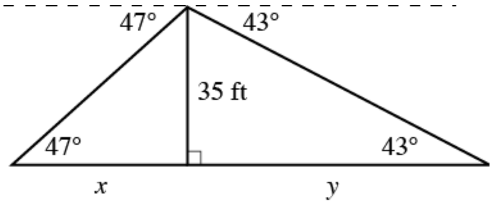


Therefore:

$$\tan 75.5° = \frac{h}{1.5}$$
$$h = 1.5 \tan 75.5° \approx 5.8$$

The post is approximately 5.8 feet tall.

24. Draw the figure:



Therefore:

$$\tan 47° = \frac{35}{x} \qquad \tan 43° = \frac{35}{y}$$
$$x \tan 47° = 35 \qquad y \tan 43° = 35$$
$$x = \frac{35}{\tan 47°} \approx 32.64 \text{ feet} \qquad y = \frac{35}{\tan 43°} \approx 37.53 \text{ feet}$$

The stakes are 32.6 + 37.5 ≈ 70 feet apart.

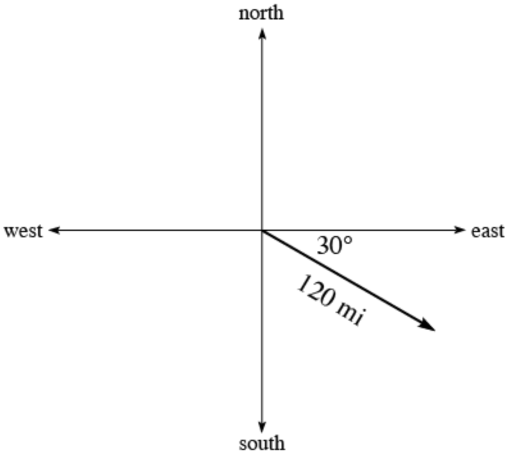
25. Let θ represent the required angle. Then:

$$\tan\theta = \frac{31}{11}$$
$$\theta = \tan^{-1}\left(\frac{31}{11}\right) \approx 70°$$

26. The magnitudes are given by:

$$\left|\mathbf{V}_x\right| = 850 \cos 52° \approx 523 \text{ ft/sec} \approx 520 \text{ ft/sec}$$
$$\left|\mathbf{V}_y\right| = 850 \sin 52° \approx 670 \text{ ft/sec}$$

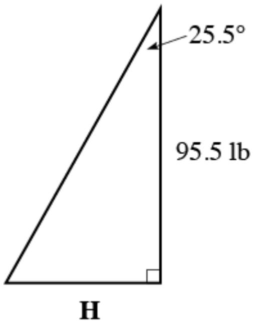
27. Draw the figure:



Therefore the distances the ship has traveled are:

east: $120\cos 30° \approx 100$ miles   south: $120\sin 30° = 60$ miles

28. Drawing the figure:

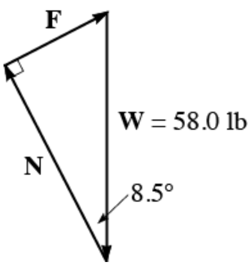


Now find the magnitude of **H**:

$$\tan 25.5° = \frac{|\mathbf{H}|}{95.5}$$
$$|\mathbf{H}| = 95.5\tan 25.5° \approx 45.6$$

Kelly must push horizontally with a force of 45.6 lb.

29. The corresponding force diagram would be:



Now find the magnitude of **F**:

$$\sin 8.5° = \frac{|\mathbf{F}|}{58.0}$$
$$|\mathbf{F}| = 58.0\sin 8.5° \approx 8.57 \text{ lb}$$

Tyler must push with a force of 8.57 lb.

30. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}|\cos 40° = 44\cos 40°$ lb

The work is then given by: $\text{Work} = (44\cos 40°)(85) \approx 2{,}865 \text{ ft-lb} \approx 2{,}900 \text{ ft-lb ft-lb}$