

Solutions Manual<sup>©</sup>  
to accompany  
*System Dynamics*, Third Edition  
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Solutions to Problems in Chapter Two

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**2.1** a) Nonlinear because of the  $y\ddot{y}$  term. b) Nonlinear because of the  $\sin y$  term. c) Nonlinear because of the  $\sqrt{y}$  term. d) Variable coefficient, but Linear. e) Nonlinear because of the  $\sin y$  term. f) Variable coefficient, but linear.

**2.2** a)

$$4 \int_2^x dx = 3 \int_0^t t dt$$

$$x(t) = 2 + \frac{3}{8}t^2$$

b)

$$5 \int_3^x dx = 2 \int_0^t e^{-4t} dt$$

$$x(t) = 3.1 - 0.1e^{-4t}$$

c) Let  $v = \dot{x}$ .

$$3 \int_7^v dv = 5 \int_0^t t dt$$

$$v(t) = \frac{dx}{dt} = 7 + \frac{5}{6}t^2$$

$$\int_2^x dx = \int_0^t \left(7 + \frac{5}{6}t^2\right) dt$$

$$x(t) = 2 + 7t + \frac{5}{18}t^3$$

d) Let  $v = \dot{x}$ .

$$4 \int_2^v dv = 7 \int_0^t e^{-2t} dt$$

$$v(t) = \frac{23}{8} - \frac{7}{8}e^{-2t}$$

$$\int_4^x dx = \int_0^t \left(\frac{23}{8} - \frac{7}{8}e^{-2t}\right) dt$$

$$x(t) = \frac{57}{16} + \frac{23}{8}t + \frac{7}{16}e^{-2t}$$

e)  $\dot{x} = C_1$ , but  $\ddot{x}(0) = 5$ , so  $C_1 = 5$ .  $x = 5t + C_2$ , but  $x(0) = 2$ , so  $C_2 = 2$ . Thus  $x = 5t + 2$ .

**2.3** a)

$$\int_3^x \frac{dx}{25 - 5x^2} = \int_0^t dt = t$$
$$\int_3^x \frac{dx}{25 - 5x^2} = \frac{\sqrt{5}}{25} \left[ \operatorname{arctanh} \left( \frac{\sqrt{5}x}{5} \right) - \operatorname{arctanh} \left( \frac{3\sqrt{5}}{5} \right) \right] = t$$

Let

$$C = \operatorname{arctanh} \left( \frac{3\sqrt{5}}{5} \right)$$

Solve for  $x$  to obtain

$$x = \sqrt{5} \tanh(5\sqrt{5}t + C)$$

b)

$$\int_{10}^x \frac{dx}{36 + 4x^2} = \int_0^t dt = t$$
$$\frac{1}{12} \tan^{-1} \frac{x}{3} \Big|_{10}^x = t$$
$$x(t) = 3 \tan(12t + C) \quad C = \tan^{-1} \frac{10}{3}$$

c)

$$\int_4^x \frac{x dx}{5x + 25} = \int_0^t dt$$
$$\frac{x}{5} - \ln(x + 5) \Big|_4^x = \frac{x}{5} - \ln(x + 5) - \frac{4}{5} + \ln 9 = t$$
$$x - 5 \ln(x + 5) = 5t + 4 - 5 \ln 9$$

So a closed form solution does not exist.

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Problem 2.3 continued:

d)

$$\int_5^x \frac{dx}{x} = -2 \int_0^t e^{-4t} dt$$

$$\ln x|_5^x = \frac{1}{2} (e^{-4t} - 1)$$

$$\ln \frac{x}{5} = \frac{1}{2} (e^{-4t} - 1)$$

$$x(t) = \frac{5}{\sqrt{e}} e^{\frac{1}{2}e^{-4t}}$$

**2.4** From the transform definition, we have

$$\mathcal{L}[mt] = \lim_{T \rightarrow \infty} \left[ \int_0^T mte^{-st} dt \right] = m \lim_{T \rightarrow \infty} \left[ \int_0^T te^{-st} dt \right]$$

The method of *integration by parts* states that

$$\int_0^T u dv = uv|_0^T - \int_0^T v du$$

Choosing  $u = t$  and  $dv = e^{-st} dt$ , we have  $du = dt$ ,  $v = -e^{-st}/s$ , and

$$\begin{aligned} \mathcal{L}[mt] &= m \lim_{T \rightarrow \infty} \left[ \int_0^T te^{-st} dt \right] = m \lim_{T \rightarrow \infty} \left[ t \frac{e^{-st}}{-s} \Big|_0^T - \int_0^T \frac{e^{-st}}{-s} dt \right] \\ &= m \lim_{T \rightarrow \infty} \left[ t \frac{e^{-st}}{-s} \Big|_0^T - \frac{e^{-st}}{(-s)^2} \Big|_0^T \right] = m \lim_{T \rightarrow \infty} \left[ \frac{Te^{-sT}}{-s} - 0 - \frac{e^{-sT}}{(-s)^2} + \frac{e^0}{(-s)^2} \right] \\ &= \frac{m}{s^2} \end{aligned}$$

because, if we choose the real part of  $s$  to be positive, then

$$\lim_{T \rightarrow \infty} Te^{-sT} = 0$$

**2.5** From the transform definition, we have

$$\mathcal{L}[t^2] = \lim_{T \rightarrow \infty} \left[ \int_0^T t^2 e^{-st} dt \right]$$

The method of *integration by parts* states that

$$\int_0^T u dv = uv \Big|_0^T - \int_0^T v du$$

Choosing  $u = t^2$  and  $dv = e^{-st} dt$ , we have  $du = 2t dt$ ,  $v = -e^{-st}/s$ , and

$$\begin{aligned} \mathcal{L}[t^2] &= \lim_{T \rightarrow \infty} \left[ \int_0^T t^2 e^{-st} dt \right] = \lim_{T \rightarrow \infty} \left[ t^2 \frac{e^{-st}}{-s} \Big|_0^T - \int_0^T \frac{e^{-st}}{-s} 2t dt \right] \\ &= \lim_{T \rightarrow \infty} \left[ -T^2 \frac{e^{-sT}}{s} + \frac{2}{s} \int_0^T t e^{-st} dt \right] = \lim_{T \rightarrow \infty} \left[ -T^2 \frac{e^{-sT}}{s} \right] + \frac{2}{s} \left( \frac{1}{s^2} \right) \\ &= \frac{2}{s^3} \end{aligned}$$

because, if we choose the real part of  $s$  to be positive, then,

$$\lim_{T \rightarrow \infty} T^2 e^{-sT} = 0$$

2.6 a)

$$X(s) = \frac{10}{s} + \frac{2}{s^3}$$

b)

$$X(s) = \frac{6}{(s+5)^2} + \frac{1}{s+3}$$

c) From Property 8,

$$X(s) = -\frac{dY(s)}{ds}$$

where  $y(t) = e^{-3t} \sin 5t$ . Thus

$$Y(s) = \frac{5}{(s+3)^2 + 5^2} = \frac{5}{s^2 + 6s + 34}$$

$$\frac{dY(s)}{ds} = -\frac{10s + 30}{(s^2 + 6s + 34)^2}$$

Thus

$$X(s) = \frac{10s + 30}{(s^2 + 6s + 34)^2}$$

d)  $X(s) = e^{-5s}G(s)$ , where  $g(t) = t$ . Thus  $G(s) = 1/s^2$  and

$$X(s) = \frac{e^{-5s}}{s^2}$$



## 2.7

$$f(t) = 5u_s(t) - 7u_s(t - 6) + 2u_s(t - 14)$$

Thus

$$F(s) = \frac{5}{s} - 7\frac{e^{-6s}}{s} + 2\frac{e^{-14s}}{s}$$

2.8 a)

$$2 \sin 3t$$

b)

$$4 \cos 2t + \frac{5}{2} \sin 2t$$

c)

$$2e^{-2t} \sin 3t$$

d)

$$\frac{5}{3} - \frac{5e^{-3t}}{3}$$

e)

$$\frac{5e^{-3t}}{2} - \frac{5e^{-7t}}{2}$$

f)

$$\frac{e^{-3t}}{2} + \frac{3e^{-7t}}{2}$$

**2.9** a)

$$5 \cos(3t)$$

b)

$$e^{3t} - e^{-3t}$$

c)

$$5 - 15te^{-3t} - 5e^{-3t}$$

d)

$$\frac{2}{13} - \frac{2e^{-2t} \left( \cos 3t + \frac{2 \sin 3t}{3} \right)}{13}$$

e)

$$5 - 5 \cos 2t$$

f)

$$5t \sin 2t$$

2.10 a)

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{5}{3s+7} = \frac{5}{3}$$

$$x(\infty) = \lim_{s \rightarrow 0} s \frac{5}{3s+7} = 0$$

b)

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{10}{3s^2+7s+4} = 0$$

$$x(\infty) = \lim_{s \rightarrow 0} s \frac{10}{3s^2+7s+4} = 0$$

2.11 a)

$$X(s) = \frac{3}{2} \left( \frac{1}{s} - \frac{1}{s+4} \right)$$

$$x(t) = \frac{3}{2} (1 - e^{-4t})$$

b)

$$X(s) = \frac{5}{3} \frac{1}{s} + \frac{31}{3} \frac{1}{s+3}$$

$$x(t) = \frac{5}{3} + \frac{31}{3} e^{-3t}$$

c)

$$X(s) = -\frac{1}{3} \frac{1}{s+2} + \frac{13}{3} \frac{1}{s+5}$$

$$x(t) = -\frac{1}{3} e^{-2t} + \frac{13}{3} e^{-5t}$$

d)

$$X(s) = \frac{5/2}{s^2(s+4)} = \frac{5}{8} \frac{1}{s^2} - \frac{5}{32} \frac{1}{s} + \frac{5}{32} \frac{1}{s+4}$$

$$x(t) = \frac{5}{8} t - \frac{5}{32} + \frac{5}{32} e^{-4t}$$

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Problem 2.11 continued:

e)

$$X(s) = \frac{2}{5} \frac{1}{s^2} + \frac{13}{25} \frac{1}{s} - \frac{13}{25} \frac{1}{s+5}$$
$$x(t) = \frac{2}{5}t + \frac{13}{25} - \frac{13}{25}e^{-5t}$$

f)

$$X(s) = -\frac{31}{4} \frac{1}{(s+3)^2} + \frac{79}{16} \frac{1}{s+3} - \frac{79}{16} \frac{1}{s+7}$$
$$x(t) = -\frac{31}{4}te^{-3t} + \frac{79}{16}e^{-3t} - \frac{79}{16}e^{-7t}$$

**2.12 a)**

$$X(s) = \frac{7s+2}{(s+3)^2+5^2} = C_1 \frac{5}{(s+3)^2+5^2} + C_2 \frac{s+3}{(s+3)^2+5^2}$$

or

$$X(s) = -\frac{19}{5} \frac{5}{(s+3)^2+5^2} + 7 \frac{s+3}{(s+3)^2+5^2}$$

$$x(t) = -\frac{19}{5} e^{-3t} \sin 5t + 7 e^{-3t} \cos 5t$$

b)

$$X(s) = \frac{4s+3}{s[(s+3)^2+5^2]} = \frac{C_1}{s} + C_2 \frac{5}{(s+3)^2+5^2} + C_3 \frac{s+3}{(s+3)^2+5^2}$$

or

$$X(s) = \frac{3}{34} \frac{1}{s} + \frac{127}{170} \frac{5}{(s+3)^2+5^2} - \frac{3}{34} \frac{s+3}{(s+3)^2+5^2}$$

$$x(t) = \frac{3}{34} + \frac{127}{170} e^{-3t} \sin 5t - \frac{3}{34} e^{-3t} \cos 5t$$

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Problem 2.12 continued:

c)

$$\begin{aligned} X(s) &= \frac{4s+9}{[(s+3)^2+5^2][(s+2)^2+4^2]} \\ &= C_1 \frac{5}{(s+3)^2+5^2} + C_2 \frac{s+3}{(s+3)^2+5^2} + C_3 \frac{4}{(s+2)^2+4^2} + C_4 \frac{s+2}{(s+2)^2+4^2} \end{aligned}$$

or

$$\begin{aligned} X(s) &= -\frac{44}{205} \frac{5}{(s+3)^2+5^2} - \frac{19}{82} \frac{s+3}{(s+3)^2+5^2} \\ &\quad + \frac{69}{328} \frac{4}{(s+2)^2+4^2} + \frac{19}{82} \frac{s+2}{(s+2)^2+4^2} \end{aligned}$$

$$x(t) = -\frac{44}{205} e^{-3t} \sin 5t - \frac{19}{82} e^{-3t} \cos 5t + \frac{69}{328} e^{-2t} \sin 4t + \frac{19}{82} e^{-2t} \cos 4t$$

d)

$$\begin{aligned} X(s) &= 2.625 \frac{1}{s+2} - 18.75 \frac{1}{s+4} + 21.125 \frac{1}{s+6} \\ x(t) &= 2.625 e^{-2t} - 18.75 e^{-4t} + 21.125 e^{-6t} \end{aligned}$$



2.13 a)  $\dot{x} = 7t/5$

$$\int_3^x dx = \frac{7}{5} \int_0^t t \, dt$$

$$x(t) = \frac{7}{10}t^2 + 3$$

b)  $\dot{x} = 3e^{-5t}/4$

$$\int_4^x dx = \frac{3}{4} \int_0^t e^{-5t} \, dt$$

$$x(t) = \frac{3}{20} (1 - e^{-5t}) + 4$$

c)  $\ddot{x} = 4t/7$

$$\dot{x}(t) - \dot{x}(0) = \frac{4}{7} \int_0^t t \, dt$$

$$\dot{x}(t) = \frac{4}{14}t^2 + 5$$

$$\int_3^x dx = \int_0^t \left( \frac{4}{14}t^2 + 5 \right) dt$$

$$x(t) = \frac{4}{42}t^3 + 5t + 3$$

d)  $\ddot{x} = 8e^{-4t}/3$

$$\dot{x}(t) - \dot{x}(0) = \frac{8}{3} \int_0^t e^{-4t} \, dt$$

$$\dot{x}(t) = \frac{17}{3} - \frac{8}{12}e^{-4t}$$

$$\int_3^x dx = \int_0^t \left( \frac{17}{3} - \frac{8}{12}e^{-4t} \right) dt$$

$$x(t) = \frac{17}{3}t + \frac{1}{6}e^{-4t} + \frac{17}{6}$$

**2.14** a) The root is  $-7/5$  and the form is  $x(t) = Ce^{-7t/5}$ . With  $x(0) = 4$ ,  $C = 4$  and  $x(t) = 4e^{-7t/5}$

b) The root is  $-7/5$  and the form is  $x(t) = C_1e^{-7t/5} + C_2$ . At steady state,  $x = 15/7 = C_2$ . With  $x(0) = 0$ ,  $C_1 = -15/7$ . Thus

$$x(t) = \frac{15}{7} \left(1 - e^{-7t/5}\right)$$

c) The root is  $-7/5$  and the form is  $x(t) = C_1e^{-7t/5} + C_2$ . At steady state,  $x = 15/7 = C_2$ . With  $x(0) = 4$ ,  $C_1 = 13/7$ . Thus

$$x(t) = \frac{13}{7} \left(1 + e^{-7t/5}\right)$$

d)

$$sX(s) - x(0) + 7X(s) = \frac{4}{s^2}$$

$$X(s) = \frac{5s^2 + 4}{s^2(s + 7)} = \frac{4}{7s^2} - \frac{4}{49} + \frac{249}{49}e^{-7t}$$

$$x(t) = \frac{4}{7}t - \frac{4}{49} + \frac{249}{49}e^{-7t}$$

**2.15** a) The roots are  $-7$  and  $-3$ . The form is

$$x(t) = C_1 e^{-7t} + C_2 e^{-3t}$$

Evaluating  $C_1$  and  $C_2$  for the initial conditions gives

$$x(t) = -\frac{9}{4}e^{-7t} + \frac{25}{4}e^{-3t}$$

b) The roots are  $-7$  and  $-7$ . The form is

$$x(t) = C_1 e^{-7t} + C_2 t e^{-7t}$$

Evaluating  $C_1$  and  $C_2$  for the initial conditions gives

$$x(t) = e^{-7t} + 10t e^{-7t}$$

c) The roots are  $-7 \pm 3j$ . The form is

$$x(t) = C_1 e^{-7t} \sin 3t + C_2 e^{-7t} \cos 3t$$

Evaluating  $C_1$  and  $C_2$  for the initial conditions gives

$$x(t) = \frac{20}{3}e^{-7t} \sin 3t + 4e^{-7t} \cos 3t$$

2.16 a)

$$x = 6e^{-2t} - 3e^{-5t} + 2$$

b)

$$x = \frac{18e^{-2t}}{5} + \frac{76te^{-2t}}{5} + \frac{7}{5}$$

c)

$$x = 3 \sin 4t - 4 \cos 4t + 9$$

d)

$$x = 3 \cos 5te^{-3t} + \frac{16 \sin 5te^{-3t}}{5} + 2$$

**2.17** a) The roots are  $-3$  and  $-7$ . The form is

$$x(t) = C_1 e^{-3t} + C_2 e^{-7t} + C_3$$

At steady state,  $x = 5/63$  so  $C_3 = 5/63$ . Evaluating  $C_1$  and  $C_2$  for the initial conditions gives

$$x(t) = -\frac{5}{36}e^{-3t} + \frac{5}{84}e^{-7t} + \frac{5}{63}$$

b) The roots are  $-7$  and  $-7$ . The form is

$$x(t) = C_1 e^{-7t} + C_2 t e^{-7t} + C_3$$

At steady state,  $x = 98/49 = 2$  so  $C_3 = 2$ . Evaluating  $C_1$  and  $C_2$  for the initial conditions gives

$$x(t) = -2e^{-7t} - 14te^{-7t} + 2$$

c) The roots are  $-7 \pm 3j$ . The form is

$$x(t) = C_1 e^{-7t} \sin 3t + C_2 e^{-7t} \cos 3t + C_3$$

At steady state,  $x = 174/58 = 3$  so  $C_3 = 3$ . Evaluating  $C_1$  and  $C_2$  for the initial conditions gives

$$x(t) = -7e^{-7t} \sin 3t - 3e^{-7t} \cos 3t + 3$$

2.18 a)

$$X(s) = \frac{60}{s^2 + 8s + 12}$$

$$x = 15 e^{-2t} - 15 e^{-6t}$$

b)

$$X(s) = \frac{288}{s^2 + 12s + 144}$$

$$x = 16\sqrt{3}e^{-6t} \sin 6\sqrt{3}t$$

c)

$$X(s) = \frac{147}{s^2 + 49}$$

$$x = 21 \sin 7t$$

d)

$$X(s) = \frac{170}{s^2 + 14s + 85}$$

$$x = \frac{85 e^{-7t} \sin 6t}{3}$$

2.19 a)

$$\frac{6}{s(s+5)} = \frac{6}{5s} - \frac{6}{5} \frac{1}{s+5}$$

$$x(t) = \frac{6}{5} (1 - e^{-5t})$$

b)

$$\frac{4}{(s+3)(s+8)} = \frac{4}{5} \frac{1}{s+3} - \frac{4}{5} \frac{1}{s+8}$$

$$x(t) = \frac{4}{5} (e^{-3t} - e^{-8t})$$

c)

$$\frac{8s+5}{2s^2+20s+48} = \frac{1}{2} \frac{8s+5}{(s+4)(s+6)} = -\frac{27}{4} \frac{1}{s+4} + \frac{43}{4} \frac{1}{s+6}$$

$$x(t) = -\frac{27}{4} e^{-4t} + \frac{43}{4} e^{-6t}$$

d) The roots are  $s = -4 \pm 10j$ .

$$\begin{aligned} \frac{4s+13}{s^2+8s+116} + \frac{4s+13}{(s+4)^2+10^2} &= C_1 \frac{10}{(s+4)^2+10^2} + C_2 \frac{s+4}{(s+4)^2+10^2} \\ &= -\frac{3}{10} \frac{10}{(s+4)^2+10^2} + 4 \frac{s+4}{(s+4)^2+10^2} \end{aligned}$$

$$x(t) = -\frac{3}{10} e^{-4t} \sin 10t + 4e^{-4t} \cos 10t$$

2.20 a)

$$\frac{3s+2}{s^2(s+10)} = \frac{1}{5} \frac{1}{s^2} + \frac{7}{25} \frac{1}{s} - \frac{7}{25} \frac{1}{s+10}$$

$$x(t) = \frac{1}{5}t + \frac{7}{25} \left(1 - e^{-10t}\right)$$

b)

$$\frac{5}{(s+4)^2(s+1)} = -\frac{15}{9} \frac{1}{(s+4)^2} - \frac{5}{9} \frac{1}{s+4} + \frac{5}{9} \frac{1}{s+1}$$

$$x(t) = -\frac{15}{9}te^{-4t} - \frac{5}{9}e^{-4t} + \frac{5}{9}e^{-t}$$

c)

$$\frac{s^2+3s+5}{s^3(s+2)} = \frac{5}{2} \frac{1}{s^3} + \frac{1}{4} \frac{1}{s^2} + \frac{3}{8} \frac{1}{s} - \frac{3}{8} \frac{1}{s+2}$$

$$x(t) = \frac{5}{4}t^2 + \frac{1}{4}t + \frac{3}{8} - \frac{3}{8}e^{-2t}$$

d)

$$\frac{s^3+s+6}{s^4(s+2)} = 3 \frac{1}{s^4} - \frac{1}{s^3} + \frac{1}{2} \frac{1}{s^2} + \frac{1}{4} \frac{1}{s} - \frac{1}{4} \frac{1}{s+2}$$

$$x(t) = \frac{1}{2}t^3 - \frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{4} - \frac{1}{4}e^{-2t}$$



2.21 a)

$$5[sX(s) - 2] + 3X(s) = \frac{10}{s} + \frac{2}{s^3}$$

$$X(s) = \frac{10s^3 + 10s^2 + 2}{5s^3(s+3)} = \frac{2s^3 + 2s^2 + 2/5}{s^3(s+3/5)} = \frac{2}{3} \frac{1}{s^3} - \frac{10}{9} \frac{1}{s^2} + \frac{140}{9} \frac{1}{s} - \frac{86}{27} \frac{1}{s+3/5}$$

$$x(t) = \frac{1}{3}t^2 - \frac{10}{9}t + \frac{140}{27} - \frac{86}{27}e^{-3t/5}$$

b)

$$4[sX(s) - 5] + 7X(s) = \frac{6}{(s+5)^2} + \frac{1}{s+3}$$

$$\begin{aligned} X(s) &= \frac{1}{4} \frac{20s^3 + 261s^2 + 1116s + 1543}{(s+5)^2(s+7/4)(s+3)} \\ &= \frac{1}{4} \left[ -\frac{24}{13} \frac{1}{(s+5)^2} - \frac{96}{169} \frac{1}{s+5} + \frac{18056}{845} \frac{1}{s+7/4} - \frac{4}{5} \frac{1}{s+3} \right] \end{aligned}$$

$$x(t) = -\frac{6}{13}te^{-5t} - \frac{24}{169}e^{-5t} + \frac{4514}{845}e^{-7t/4} - \frac{1}{5}e^{-3t}$$

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Problem 2.21 continued:

c) This simple-looking problem actually requires quite a lot of algebra to find the solution, and thus it serves as a good motivating example of the convenience of using MATLAB. The algebraic complexity is due to a pair of repeated complex roots.

First obtain the transform of the forcing function. Let  $f(t) = te^{-3t} \sin 5t$ . From Property 8,

$$F(s) = -\frac{dY(s)}{ds}$$

where  $y(t) = e^{-3t} \sin 5t$ . Thus

$$Y(s) = \frac{5}{(s+3)^2 + 5^2} = \frac{5}{s^2 + 6s + 34}$$

$$\frac{dY(s)}{ds} = -\frac{10s + 30}{(s^2 + 6s + 34)^2}$$

Thus

$$F(s) = \frac{10s + 30}{(s^2 + 6s + 34)^2} \quad (1)$$

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Problem 2.21 continued:

Using the same technique, we find that the transform of  $te^{-3t} \cos 5t$  is

$$\frac{2s^2 + 12s + 18}{(s^2 + 6s + 34)^2} - \frac{1}{s^2 + 6s + 34} \quad (2)$$

This fact will be useful in finding the forced response.

From the differential equation,

$$4[s^2 X(s) - 10s + 2] + 3X(s) = F(s) = \frac{10s + 30}{(s^2 + 6s + 34)^2}$$

Solve for  $X(s)$ .

$$X(s) = \frac{40s - 8}{4s^2 + 3} + \frac{10s + 30}{[(s + 3)^2 + 25]^2(4s^2 + 3)}$$

The free response is given by the first fraction, and is

$$x_{\text{free}}(t) = -\frac{4}{\sqrt{3}} \sin \frac{\sqrt{3}}{2}t + 10 \cos \frac{\sqrt{3}}{2}t = -2.3094 \sin 0.866t + 10 \cos 0.866t \quad (3)$$

The forced response is given by the second fraction, which can be expressed as

$$\frac{2.5s + 7.5}{[(s + 3)^2 + 25]^2(s^2 + 3/4)} \quad (4)$$

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Problem 2.21 continued:

The roots of this are  $s = \pm j\sqrt{3}/2$  and the repeated pair  $s = -3 \pm 5j$ . Thus, referring to (1), (2), and (3), we see that the form of the forced response will be

$$\begin{aligned} x_{\text{forced}}(t) &= C_1 t e^{-3t} \sin 5t + C_2 t e^{-3t} \cos 5t \\ &+ C_3 e^{-3t} \sin 5t + C_4 e^{-3t} \cos 5t \\ &+ C_5 \sin \frac{\sqrt{3}}{2} t + C_6 \cos \frac{\sqrt{3}}{2} t \quad (5) \end{aligned}$$

The forced response can be obtained several ways. 1) You can substitute the form (5) into the differential equation and use the initial conditions to obtain equations for the  $C_i$  coefficients. 2) You can use (1) and (2) to create a partial fraction expansion of (4) in terms of the complex factors. 3) You can perform an expansion in terms of the six roots, of the form

$$\begin{aligned} \frac{A_1}{(s+3+5j)^2} &+ \frac{A_2}{s+3+5j} + \frac{A_3}{(s+3-5j)^2} + \frac{A_4}{s+3-5j} \\ &+ \frac{\sqrt{3}A_5/2}{s^2+3/4} + \frac{A_6 s}{s^2+3/4} \end{aligned}$$

4) You can use the MATLAB **residue** function.

The solution for the forced response is

$$\begin{aligned} x_{\text{forced}}(t) &= -0.0034 t e^{-3t} \sin 5t + 0.0066 t e^{-3t} \cos 5t \\ &- 0.0026 e^{-3t} \sin 5t + 2.308 \times 10^{-4} e^{-3t} \cos 5t \\ &+ 0.00796 \sin 0.866t - 2.308 \times 10^{-4} \cos 0.866t \end{aligned}$$

The initial condition  $\dot{x}(0) = 0$  is not exactly satisfied by this expression because of the limited number of digits used to display it.

**2.22** The denominator roots are  $s = -3$  and  $s = -5$ , which are distinct. Factor the denominator so that the highest coefficients of  $s$  in each factor are unity:

$$X(s) = \frac{7s + 4}{2s^2 + 16s + 30} = \frac{1}{2} \left[ \frac{7s + 4}{(s + 3)(s + 5)} \right]$$

The partial-fraction expansion has the form

$$X(s) = \frac{1}{2} \left[ \frac{7s + 4}{(s + 3)(s + 5)} \right] = \frac{C_1}{s + 3} + \frac{C_2}{s + 5}$$

Using the coefficient formula, we obtain

$$C_1 = \lim_{s \rightarrow -3} \left[ (s + 3) \frac{7s + 4}{2(s + 3)(s + 5)} \right] = \lim_{s \rightarrow -3} \left[ \frac{7s + 4}{2(s + 5)} \right] = -\frac{17}{4}$$

$$C_2 = \lim_{s \rightarrow -5} \left[ (s + 5) \frac{7s + 4}{2(s + 3)(s + 5)} \right] = \lim_{s \rightarrow -5} \left[ \frac{7s + 4}{2(s + 3)} \right] = \frac{31}{4}$$

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Problem 2.22 continued:

Using the LCD method we have

$$\begin{aligned}\frac{1}{2} \frac{7s+4}{(s+3)(s+5)} &= \frac{C_1}{s+3} + \frac{C_2}{s+5} = \frac{C_1(s+5) + C_2(s+3)}{(s+3)(s+5)} \\ &= \frac{(C_1 + C_2)s + 5C_1 + 3C_2}{(s+3)(s+5)}\end{aligned}$$

Comparing numerators, we see that  $C_1 + C_2 = 7/2$  and  $5C_1 + 3C_2 = 4/2 = 2$ , which give  $C_1 = -17/4$  and  $C_2 = 31/4$ .

The inverse transform is

$$x(t) = C_1 e^{-3t} + C_2 e^{-5t} = -\frac{17}{4} e^{-3t} + \frac{31}{4} e^{-5t}$$

In this example the LCD method requires more algebra, including the solution of two equations for the two unknowns  $C_1$  and  $C_2$ .

**2.23** a) The roots are  $-3$  and  $-5$ . The form of the free response is

$$x(t) = A_1 e^{-3t} + A_2 e^{-5t}$$

Evaluating this with the given initial conditions gives

$$x(t) = 27e^{-3t} - 17e^{-5t}$$

The steady-state solution is  $x_{ss} = 30/15 = 2$ . Thus the form of the forced response is

$$x(t) = 2 + B_1 e^{-3t} + B_2 e^{-5t}$$

Evaluating this with zero initial conditions gives

$$x(t) = 2 - 5e^{-3t} + 3e^{-5t}$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 2 + 22e^{-3t} - 14e^{-5t}$$

The transient response consists of the two exponential terms.

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Problem 2.23 continued:

b) The roots are  $-5$  and  $-5$ . The form of the free response is

$$x(t) = A_1 e^{-5t} + A_2 t e^{-5t}$$

Evaluating this with the given initial conditions gives

$$x(t) = e^{-5t} + 9t e^{-5t}$$

The steady-state solution is  $x_{ss} = 75/25 = 3$ . Thus the form of the forced response is

$$x(t) = 3 + B_1 e^{-5t} + B_2 t e^{-5t}$$

Evaluating this with zero initial conditions gives

$$x(t) = 3 - 3e^{-5t} - 15t e^{-5t}$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 3 - 2e^{-5t} - 6t e^{-5t}$$

The transient response consists of the two exponential terms.

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Problem 2.23 continued:

c) The roots are  $\pm 5j$ . The form of the free response is

$$x(t) = A_1 \sin 5t + A_2 \cos 5t$$

Evaluating this with the given initial conditions gives

$$x(t) = \frac{4}{5} \sin 5t + 10 \cos 5t$$

The form of the forced response is

$$x(t) = B_1 + B_2 \sin 5t + B_3 \cos 5t$$

Thus the entire forced response is the steady-state forced response. There is no transient forced response. Evaluating this function with zero initial conditions shows that  $B_2 = 0$  and  $B_3 = -B_1$ . Thus

$$x(t) = B_1 - B_1 \cos 5t$$

Substituting this into the differential equation shows that  $B_1 = 4$  and the forced response is

$$x(t) = 4 - 4 \cos 5t$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 4 + 6 \cos 5t + \frac{4}{5} \sin 5t$$

The entire response is the steady-state response. There is no transient response.

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Problem 2.23 continued:

d) The roots are  $-4 \pm 7j$ . The form of the free response is

$$x(t) = A_1 e^{-4t} \sin 7t + A_2 e^{-4t} \cos 7t$$

Evaluating this with the given initial conditions gives

$$x(t) = \frac{44}{7} e^{-4t} \sin 7t + 10 e^{-4t} \cos 7t$$

The form of the forced response is

$$x(t) = B_1 + B_2 e^{-4t} \sin 7t + B_3 e^{-4t} \cos 7t$$

The steady-state solution is  $x_{ss} = 130/65 = 2$ . Thus  $B_1 = 2$ . Evaluating this function with zero initial conditions shows that  $B_2 = -8/7$  and  $B_3 = -2$ . Thus the forced response is

$$x(t) = 2 - \frac{8}{7} e^{-4t} \sin 7t - 2 e^{-4t} \cos 7t$$

The total response is the sum of the free and the forced response. It is

$$x(t) = 2 + \frac{36}{7} e^{-4t} \sin 7t + 8 e^{-4t} \cos 7t$$

The transient response consists of the two exponential terms.

- 2.24** a) The root is  $s = 5/3$ , which is positive. So the model is unstable.  
b) The roots are  $s = 5$  and  $-2$ , one of which is positive. So the model is unstable.  
c) The roots are  $s = 3 \pm 5j$ , whose real part is positive. So the model is unstable.  
d) The root is  $s = 0$ , so the model is neutrally stable.  
e) The roots are  $s = \pm 2j$ , whose real part is zero. So the model is neutrally stable.  
f) The roots are  $s = 0$  and  $-5$ , one of which is zero and the other is negative. So the model is neutrally stable.

**2.25** a) The system is stable if both of its roots are real and negative or if the roots are complex with negative real parts. Assuming that  $m \neq 0$ , we can divide the characteristic equation by  $m$  to obtain

$$s^2 + \frac{c}{m}s + \frac{k}{m} = s^2 + as + b = 0$$

where  $a = c/m$  and  $b = k/m$ . The roots are given by the quadratic formula:

$$s = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

(continued on the next page)

Problem 2.25 continued:

Thus the condition that  $m$ ,  $c$ , and  $k$  have the same sign is equivalent to  $a > 0$  and  $b > 0$ . There are three cases to be considered:

1. Complex roots ( $a^2 - 4b < 0$ ). In this case the real part of both roots is  $-a/2$  and is negative if  $a > 0$ .
2. Repeated, real roots ( $a^2 - 4b = 0$ ). In this case both roots are  $-a/2$  and are negative if  $a > 0$ .
3. Distinct, real roots ( $a^2 - 4b > 0$ ). Let the two roots be denoted  $r_1$  and  $r_2$ . We can factor the characteristic equation as  $s^2 + as + b = (s - r_1)(s - r_2) = 0$ . Expanding this gives

$$(s - r_1)(s - r_2) = s^2 - (r_1 + r_2)s + r_1r_2 = 0$$

Comparing the two forms shows that

$$r_1r_2 = b \quad (1) \quad \text{and} \quad r_1 + r_2 = -a \quad (2)$$

If  $b > 0$ , condition (1) shows that both roots have the same sign. If  $a < 0$ , condition (2) shows that the roots must be negative. Therefore, if the roots are distinct and real, the roots will be negative if  $a > 0$  and  $b > 0$ .

b) Neutral stability occurs if either 1) both roots are imaginary or 2) one root is zero while the other root is negative. Imaginary roots occur when  $a = 0$  (the roots are  $s = \pm\sqrt{b}$ ). In this case the free response is a constant-amplitude oscillation. Case 2 occurs when  $b = 0$  and  $a > 0$  (the roots are  $s = 0$  and  $s = -a$ ). In this case the free response decays to a non-zero constant.

**2.26** a)  $\tau = 5$

b)  $\tau = 4$

c)  $\tau = 3$

d) The roots is  $s = 3/8$ , so the model is unstable, so no time constant is defined.

**2.27** a) The root is  $s = -4/13$ , so the model is stable, and  $x_{ss} = 16/4 = 4$ . Since  $\tau = 13/4$ , it takes about  $4\tau = 13$  to reach steady state.

b) The root is  $s = -4/13$ , so the model is stable, and  $x_{ss} = 16/4 = 4$ . Since  $\tau = 13/4$ , it takes about  $4\tau = 13$  to reach steady state.

c) The root is  $s = 7/15$ , so the model is unstable, and no steady state exists.

**2.28** 1)

$$X(s) = \frac{s+1}{4s+1} \frac{5}{s} = \frac{1}{4} \frac{s+1}{s+1/4} \frac{5}{s} = \frac{C_1}{s} + \frac{C_2}{s+1/4}$$

$C_1 = 5$ ,  $C_2 = -15/4$ , so

$$x(t) = 5 - \frac{15}{4}e^{-t/4}$$

2)

$$X(s) = \frac{1}{4s+1} \frac{5}{s} = \frac{1}{4} \frac{1}{s+1/4} \frac{5}{s} = \frac{C_1}{s} + \frac{C_2}{s+1/4}$$

$C_1 = 5$ ,  $C_2 = -5$ , so

$$x(t) = 5 - 5e^{-t/4}$$



**2.29**

$$3[sX(s) - 4] + X(s) = 6$$

$$X(s) = \frac{6}{s + 1/3}$$

$$x(t) = 6e^{-t/3}$$

**2.30 a)**

$$\zeta = \frac{4}{2\sqrt{40}} = \frac{\sqrt{10}}{10} \quad \omega_n = \sqrt{\frac{40}{1}} = 2\sqrt{10}$$
$$s = -2 \pm 6j$$

so  $\tau = 1/2$  and  $\omega_d = 6$ .

b)

$$s = 1 \pm 4.7958j$$

So the model is oscillatory but unstable, and thus  $\zeta$  and  $\tau$  are not defined.

$$\omega_n = \sqrt{\frac{24}{1}} = 2\sqrt{6} \quad \omega_d = 4.7958$$

c)

$$\zeta = \frac{20}{2\sqrt{100}} = 1$$
$$s = -10, -10$$

so  $\tau = 1/10$ . Since the roots are real, the response is not oscillatory, and  $\omega_n$  and  $\omega_d$  have no meaning.

d) The root is  $s = -10$ , so  $\tau = 1/10$ . Since the model is first order,  $\zeta$ ,  $\omega_n$  and  $\omega_d$  have no meaning.

**2.31** a) The roots are

$$s = \frac{-10d \pm \sqrt{100d^2 - 4(29)d^2}}{2} = (-5 \pm 2j) d$$

So if  $d > 0$ , the real part is negative, and the system is stable.

b)

$$\zeta = \frac{10d}{2\sqrt{29}d^2} = \frac{10}{2\sqrt{29}} < 1$$

So the free response is always oscillatory.

**2.32** a)

$$\frac{X(s)}{F(s)} = \frac{15}{5s + 7}$$

The root is  $s = -7/5$ .

b)

$$\frac{X(s)}{F(s)} = \frac{5}{3s^2 + 30s + 63}$$

The roots are  $s = -7$  and  $s = -3$ .

c)

$$\frac{X(s)}{F(s)} = \frac{4}{s^2 + 10s + 21}$$

The roots are  $s = -7$  and  $s = -3$ .

d)

$$\frac{X(s)}{F(s)} = \frac{7}{s^2 + 14s + 49}$$

The roots are  $s = -7$  and  $s = -7$ .

e)

$$\frac{X(s)}{F(s)} = \frac{6s + 4}{s^2 + 14s + 58}$$

The roots are  $s = -7 \pm 3j$ .

f)

$$\frac{X(s)}{F(s)} = \frac{4s + 15}{5s + 7}$$

The root is  $s = -7/5$ .

**2.33** Transform each equation using zero initial conditions.

$$3sX(s) = Y(s)$$

$$sY(s) = F(s) - 3Y(s) - 15X(s)$$

Solve for  $X(s)/F(s)$  and  $Y(s)/F(s)$ .

$$\frac{X(s)}{F(s)} = \frac{1}{3s^2 + 9s + 15}$$

$$\frac{Y(s)}{F(s)} = \frac{3s}{3s^2 + 9s + 15}$$

**2.34** Transform each equation using zero initial conditions.

$$sX(s) = -2X(s) + 5Y(s)$$

$$sY(s) = F(s) - 6Y(s) - 4X(s)$$

Solve for  $X(s)/F(s)$  and  $Y(s)/F(s)$ .

$$\frac{X(s)}{F(s)} = \frac{5}{s^2 + 8s + 32}$$

$$\frac{Y(s)}{F(s)} = \frac{s + 2}{s^2 + 8s + 32}$$

**2.35** a) Transform both equations to obtain  $4sX(s) = Y(s)$  and  $s(Y(s) = F(s) - 3Y(s) - 12X(s))$ . Eliminate  $X(s)$  to obtain

$$\frac{Y(s)}{F(s)} = \frac{s}{s^2 + 3s + 3}$$

Use  $Y(s) = 4sX(s)$  to eliminate  $Y(s)$ .

$$\frac{Y(s)}{F(s)} = \frac{1}{4} \frac{1}{s^2 + 3s + 3}$$

b) The roots are

$$s = \frac{-3 \pm \sqrt{3}}{2}$$

Thus

$$\tau = \frac{2}{3} \quad \zeta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\omega_n = \sqrt{3} \quad \omega_d = \frac{\sqrt{3}}{2}$$

c) The response oscillates with a frequency of  $\omega_d = \sqrt{3}/2$  and essentially disappears for  $t > 4\tau = 8/3$ .

d) With  $F(s) = 1/s$ ,

$$X(s) = \frac{1}{4} \frac{1}{s(s^2 + 3s + 3)} = \frac{1}{4} \frac{1}{s[(s + \frac{3}{2})^2 + \frac{3}{4}]}$$

or

$$X(s) = \frac{C_1(s + \frac{3}{2}) + C_2 \frac{\sqrt{3}}{2}}{(s + \frac{3}{2})^2 + \frac{3}{4}} + \frac{C_3}{s}$$

where  $C_1 = -C_3 = -1/12$  and  $C_2 = -\sqrt{3}/12$ . Thus

$$x(t) = e^{-3t/2} \left( -\frac{1}{12} \cos \frac{\sqrt{3}}{2}t - \frac{\sqrt{3}}{12} \sin \frac{\sqrt{3}}{2}t \right) + \frac{1}{12}$$

**2.36** a) Transform both equations to obtain

$$4sX(s) = -4X(s) + 2Y(s) + F(s)$$

$$sY(s) = -9Y(s) - 5X(s) + G(s)$$

These can be solved using Cramer's rule to obtain

$$\frac{X(s)}{F(s)} = \frac{s + 9}{4s^2 + 40s + 46}$$

$$\frac{X(s)}{G(s)} = \frac{2}{4s^2 + 40s + 46}$$

b) The roots are  $s = -1.3258$  and  $s = -8.6742$ . The time constants are  $\tau = 0.7543$  and  $\tau = 0.1153$ . The response does not oscillate.

c) The free response is governed by the dominant time constant, which is  $\tau = 0.7543$ . The response is essentially zero for  $t > 4\tau = 3.0172$ .



**2.37 a)**

$$7[sX(s) - 3] + 5X(s) = 4$$

$$X(s) = \frac{25}{7s + 5} = \frac{25/7}{s + 5/7}$$

$$x(t) = \frac{25}{7}e^{-5t/7}$$

Note that this gives  $x(0+) = 25/7$ . From the initial value theorem

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{25/7}{s + 5/7} = \frac{25}{7}$$

which is not the same as  $x(0-)$ .

b)

$$(3s^2 + 30s + 63)X(s) = 5$$

$$X(s) = \frac{5}{3s^2 + 30s + 63} = \frac{5/3}{s^2 + 10s + 21} = \frac{5}{12} \frac{1}{s + 3} - \frac{5}{12} \frac{1}{s + 7}$$

$$x(t) = \frac{5}{12} (e^{-3t} - e^{-7t})$$

From the initial value theorem

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{5/3}{s^2 + 10s + 21} = 0$$

which is the same as  $x(0-)$ . Also

$$\dot{x}(0+) = \lim_{s \rightarrow \infty} s^2 \frac{5/3}{s^2 + 10s + 21} = \frac{5}{3}$$

which is not the same as  $\dot{x}(0-)$ .

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Problem 2.37 continued:

c)

$$s^2X(s) - 2s - 3 + 14[sX(s) - 2] + 49X(s) = 3$$

$$X(s) = \frac{2s + 34}{s^2 + 14s + 49} = 20 \frac{1}{(s + 7)^2} + 2 \frac{1}{s + 7}$$

$$x(t) = 20te^{-7t} + 2e^{-7t}$$

From the initial value theorem

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{2s + 34}{s^2 + 14s + 49} = 2$$

which is the same as  $x(0-)$ . However, the initial value theorem is invalid for computing  $\dot{x}(0+)$  and gives an undefined result because the orders of the numerator and denominator of  $sX(s)$  are equal.

d)

$$s^2X(s) - 4s - 7 + 14[sX(s) - 4] + 58X(s) = 4$$

$$X(s) = \frac{4s + 67}{s^2 + 14s + 58} = \frac{4s + 67}{(s + 7)^2 + 3^2} = 13 \frac{3}{(s + 7)^2 + 3^2} + 4 \frac{s + 7}{(s + 7)^2 + 3^2}$$

$$x(t) = 13e^{-7t} \sin 3t + 4e^{-7t} \cos 3t$$

From the initial value theorem

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{4s + 67}{s^2 + 14s + 58} = 4$$

which is the same as  $x(0-)$ . However, the initial value theorem is invalid for computing  $\dot{x}(0+)$  and gives an undefined result because the order of the numerator of  $sX(s)$  is greater than the denominator.

2.38 a)

$$7[sX(s) - 3] + 5X(s) = 4s\frac{1}{s} = 4$$

$$X(s) = \frac{25}{7s + 5} = \frac{25/7}{s + 5/7}$$

$$x(t) = \frac{25}{7}e^{-5t/7}$$

From the initial value theorem

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{25/7}{s + 5/7} = \frac{25}{7}$$

which is not the same as  $x(0-)$ .

b)

$$7[sX(s) - 3] + 5X(s) = 4s\frac{1}{s} + \frac{6}{s}$$

$$X(s) = \frac{25s + 6}{s(7s + 5)} = \frac{1}{7} \frac{25s + 6}{s(s + 5/7)} = \frac{6}{5} \frac{1}{s} + \frac{83}{35} \frac{1}{s + 5/7}$$

$$x(t) = \frac{6}{5} + \frac{83}{35}e^{-5t/7}$$

which gives  $x(0+) = 25/7$ , which is not the same as  $x(0-)$ . However, the initial value theorem is invalid for computing  $x(0+)$  and gives an undefined result because the orders of the numerator and denominator of  $X(s)$  are equal.

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Problem 2.38 continued:

c)

$$3[s^2X(s) - 2s - 3] + 30[sX(s) - 2] + 63X(s) = 4s\frac{1}{s} = 4$$

$$X(s) = \frac{1}{3} \frac{6s + 73}{(s + 3)(s + 7)} = \frac{55}{12} \frac{1}{s + 3} - \frac{31}{12} \frac{1}{s + 7}$$

$$x(t) = \frac{55}{12}e^{-3t} - \frac{31}{12}e^{-7t}$$

This gives  $x(0) = 2$ , which is the same as  $x(0-)$ , and  $\dot{x}(0) = 13/2$ , which is not the same as  $\dot{x}(0-)$ .

From the initial value theorem

$$x(0+) = \lim_{s \rightarrow \infty} s \frac{1}{3} \frac{6s + 73}{(s + 3)(s + 7)} = 2$$

which is the same as  $x(0-)$ . However, the initial value theorem is invalid for computing  $\dot{x}(0+)$  and gives an undefined result because the order of the numerator of  $sX(s)$  is greater than the denominator.

(continued on the next page)

Problem 2.38 continued:

d)

$$3[s^2X(s) - 4s - 7] + 30[sX(s) - 4] + 63X(s) = 4s\frac{1}{s} + \frac{6}{s}$$
$$X(s) = \frac{1}{3} \frac{12s^2 + 145s + 6}{s(s^2 + 10s + 21)} = 0.0952\frac{1}{s} + 8.9167\frac{1}{s+3} - 5.0119\frac{1}{s+7}$$
$$x(t) = 0.0952 + 8.9167e^{-3t} - 5.0119e^{-7t}$$

This gives  $x(0) = 4$ , which is the same as  $x(0-)$ , and  $\dot{x}(0) = 8.3332$ , which is not the same as  $\dot{x}(0-)$ .

The initial value theorem gives  $x(0+) = 4$  but is invalid for computing  $\dot{x}(0+)$  because the orders of the numerator and denominator of  $sX(s)$  are equal.

**2.39** Transform each equation.

$$3[sX(s) - 5] = Y(s)$$

$$sY(s) - 10 = \frac{4}{s} - 3Y(s) - 15X(s)$$

Solve for  $X(s)$  and  $Y(s)$ .

$$X(s) = \frac{15s^2 + 55s + 4}{3s^3 + 9s^2 + 15s} = \frac{1}{3} \frac{15s^2 + 55s + 4}{s(s^2 + 3s + 5)}$$

$$Y(s) = \frac{30s - 213}{3s^2 + 9s + 15} = \frac{1}{3} \frac{30s - 213}{s^2 + 3s + 5}$$

The denominator roots are  $s = -1.5 \pm 1.658j$ . Thus

$$X(s) = \frac{C_1}{s} + \frac{1}{3} \left[ C_1 \frac{1.658}{(s + 1.5)^2 + 2.75} + C_2 \frac{s + 1.5}{(s + 1.5)^2 + 2.75} \right]$$

and

$$x(t) = \frac{1}{4} + \frac{1}{165} e^{-3t/2} \left[ 781 \cos \left( \frac{\sqrt{11}}{2} t \right) + 313 \sqrt{11} \sin \left( \frac{\sqrt{11}}{2} t \right) \right]$$

Also,

$$Y(s) = C_1 \frac{1.658}{(s + 1.5)^2 + 2.75} + C_2 \frac{s + 1.5}{(s + 1.5)^2 + 2.75}$$

and

$$y(t) = \frac{2}{11} e^{-3t/2} \left[ 55 \cos \left( \frac{\sqrt{11}}{2} t \right) - 86 \sqrt{11} \sin \left( \frac{\sqrt{11}}{2} t \right) \right]$$

**2.40** Transform each equation.

$$sX(s) - 5 = -2X(s) + 5Y(s)$$

$$sY(s) - 2 = -6Y(s) - 4X(s) + \frac{10}{s}$$

Solve for  $X(s)$  and  $Y(s)$ .

$$X(s) = \frac{5s^2 + 40s + 50}{s^3 + 8s^2 + 32s}$$

$$Y(s) = \frac{2s^2 - 6s + 20}{s^3 + 8s^2 + 32s}$$

The denominator roots are  $s = 0$  and  $s = -4 \pm 4j$ . Thus

$$\begin{aligned} X(s) &= \frac{C_1}{s} + C_2 \frac{4}{(s+4)^2 + 4^2} + C_3 \frac{s+4}{(s+4)^2 + 4^2} \\ &= \frac{25}{16s} + \frac{55}{16} \frac{4}{(s+4)^2 + 4^2} + \frac{55}{16} \frac{s+4}{(s+4)^2 + 4^2} \\ x(t) &= \frac{25}{16} + \frac{55}{16} e^{-4t} \sin 4t + \frac{55}{16} e^{-4t} \cos 4t \end{aligned}$$

Also,

$$\begin{aligned} Y(s) &= \frac{C_1}{s} + C_2 \frac{4}{(s+4)^2 + 4^2} + C_3 \frac{s+4}{(s+4)^2 + 4^2} \\ &= \frac{5}{8s} - \frac{33}{8} \frac{4}{(s+4)^2 + 4^2} + \frac{11}{8} \frac{s+4}{(s+4)^2 + 4^2} \\ y(t) &= \frac{5}{8} - \frac{33}{8} e^{-4t} \sin 4t + \frac{11}{8} e^{-4t} \cos 4t \end{aligned}$$

**2.41** Transforming both sides of the equation we obtain

$$s^2Y(s) - sy(0) - \dot{y}(0) + Y(s) = \frac{1}{s+1}$$

which gives

$$Y(s) = \frac{(s+1)[sy(0) + \dot{y}(0)] + 1}{(s+1)(s^2+1)} = \frac{s^2y(0) + [y(0) + \dot{y}(0)] + \dot{y}(0) + 1}{(s+1)(s^2+1)}$$

This can be expanded as follows.

$$Y(s) = C_1 \frac{1}{s+1} + C_2 \frac{1}{s^2+1} + C_3 \frac{s}{s^2+1}$$

We find the coefficients following the usual procedure and obtain  $C_1 = 1/2$ ,  $C_2 = \dot{y}(0) + 1/2$ , and  $C_3 = y(0) - 1/2$ . Thus the solution is

$$y(t) = \frac{1}{2}e^{-t} + \left[\dot{y}(0) + \frac{1}{2}\right] \sin t + \left[y(0) - \frac{1}{2}\right] \cos t$$

(continued on the next page)



Problem 2.41 continued:

Because the initial values can be arbitrary, the general form of the solution is

$$y(t) = \frac{1}{2}e^{-t} + A_1 \sin t + A_2 \cos t \quad (1)$$

This form can be used to obtain a solution for cases where  $y(t)$  or  $\dot{y}(t)$  are specified at points other than  $t = 0$ . For example, suppose we are given that  $y(0) = 5/2$  and  $y(\pi/2) = 3$ . Then evaluation of equation (1) at  $t = 0$  and at  $t = \pi/2$  gives

$$y(0) = \frac{1}{2} + A_2 = \frac{5}{2} \quad y\left(\frac{\pi}{2}\right) = \frac{1}{2}e^{-\pi/2} + A_1 = 3$$

The solution of these two equations is  $A_1 = 3 - e^{-\pi/2}/2 = 2.896$  and  $A_2 = 2$ , and the solution of the differential equation is

$$y(t) = \frac{1}{2}e^{-t} + 2.896 \sin t + 2 \cos t$$

**2.42** (a) For nonzero initial conditions, the transform gives

$$s^2 X(s) - sx(0) + \dot{x}(0) + 4X(s) = \frac{3}{s^2}$$

or

$$X(s) = \frac{s^3 x(0) + s^2 \dot{x}(0) + 3}{s^2(s^2 + 4)} = \frac{C_1}{s^2} + \frac{C_2}{s} + C_3 \frac{2}{s^2 + 4} + C_4 \frac{s}{s^2 + 4}$$

The solution form is thus

$$x(t) = C_1 t + C_2 + C_3 \sin 2t + C_4 \cos 2t$$

which can be used even if the boundary conditions are not specified at  $t = 0$ .

(b) The form from part (a) satisfies the differential equation if  $C_1 = 3/4$  and  $C_2 = 0$ . From  $x(0) = 10$ , we obtain  $C_4 = 10$ . From  $x(5) = 30$ , we obtain  $C_3 = -63.675$ . Thus

$$x(t) = \frac{3}{4}t - 63.675 \sin 2t + 10 \cos 2t$$

**2.43** The denominator roots are  $s = -3 \pm 5j$  and  $s = \pm 6j$ . Thus we can express  $X(s)$  as follows.

$$X(s) = \frac{30}{[(s+3)^2 + 5^2](s^2 + 6^2)}$$

which can be expressed as the sum of terms that are proportional to entries 8 through 11 in Table 2.2.1.

$$X(s) = C_1 \frac{5}{(s+3)^2 + 5^2} + C_2 \frac{s+3}{(s+3)^2 + 5^2} + C_3 \frac{6}{s^2 + 6^2} + C_4 \frac{s}{s^2 + 6^2} \quad (1)$$

We can obtain the coefficients by noting that  $X(s)$  can be written as

$$X(s) = \frac{5C_1(s^2 + 6^2) + C_2(s+3)(s^2 + 6^2) + 6C_3[(s+3)^2 + 5^2] + C_4s[(s+3)^2 + 5^2]}{[(s+3)^2 + 5^2](s^2 + 6^2)} \quad (2)$$

Comparing the numerators of equations (1) and (2), and collecting powers of  $s$ , we see that

$$\begin{aligned} (C_2 + C_4)s^3 + (5C_1 + 3C_2 + 6C_3 + 6C_4)s^2 + (36C_2 + 36C_3 + 34C_4)s \\ + 180C_1 + 108C_2 + 204C_3 = 30 \end{aligned}$$

or

$$\begin{aligned} C_2 + C_4 &= 0 & 5C_1 + 3C_2 + 6C_3 + 6C_4 &= 0 \\ 36C_2 + 36C_3 + 34C_4 &= 0 & 180C_1 + 108C_2 + 204C_3 &= 30 \end{aligned}$$

These are four equations in four unknowns. Note that the first equation gives  $C_4 = -C_2$ . Thus we can easily eliminate  $C_4$  from the equations and obtain a set of three equations in three unknowns. The solution is  $C_1 = 6/65$ ,  $C_2 = 9/65$ , and  $C_3 = -1/130$ , and  $C_4 = -9/65$ .

(continued on the next page)

Problem 2.43 continued:

The inverse transform is

$$\begin{aligned}x(t) &= C_1 e^{-3t} \sin 5t + C_2 e^{-3t} \cos 5t + C_3 \sin 6t + C_4 \cos 6t \\&= \frac{6}{65} e^{-3t} \sin 5t + \frac{9}{65} e^{-3t} \cos 5t - \frac{1}{130} \sin 6t - \frac{9}{65} \cos 6t\end{aligned}$$

**2.44** Transform the equation.

$$(s^2 + 12s + 40)X(s) = 3\frac{5}{s^2 + 25}$$

The characteristic roots are  $s = -6 \pm 2j$ . Thus

$$\begin{aligned} X(s) &= \frac{15}{(s^2 + 25)(s^2 + 12s + 40)} \\ &= C_1 \frac{5}{s^2 + 25} + C_2 \frac{s}{s^2 + 25} + C_3 \frac{2}{(s + 6)^2 + 4} + C_4 \frac{s + 6}{(s + 6)^2 + 4} \end{aligned}$$

or

$$X(s) = \frac{1}{85} \frac{5}{s^2 + 25} - \frac{4}{85} \frac{s}{s^2 + 25} + \frac{19}{170} \frac{2}{(s + 6)^2 + 4} + \frac{4}{85} \frac{s + 6}{(s + 6)^2 + 4}$$

Thus

$$x(t) = \frac{1}{85} \sin 5t - \frac{4}{85} \cos 5t + \frac{19}{170} e^{-6t} \sin 2t + \frac{4}{85} e^{-6t} \cos 2t$$

**2.45** From the text example, the form  $A \sin(\omega t + \phi)$  has the transform

$$A \frac{s \sin \phi + \omega \cos \phi}{s^2 + \omega^2}$$

For this problem,  $\omega = 5$ . Comparing numerators gives

$$A(s \sin \phi + 5 \cos \phi) = 4s + 9$$

Thus

$$A \sin \phi = 4 \quad 5A \cos \phi = 9$$

With  $A > 0$ ,  $\phi$  is seen to be in the first quadrant.

$$\phi = \tan^{-1} \frac{\sin \phi}{\cos \phi} = \tan^{-1} \frac{4/A}{9/5A} = \tan^{-1} \frac{20}{9} = 1.148 \text{ rad}$$

Because  $\sin^2 \phi + \cos^2 \phi = 1$ ,

$$\left(\frac{4}{A}\right)^2 + \left(\frac{9}{5A}\right)^2 = 1$$

which gives  $A = 4.386$ . Thus

$$x(t) = 4.386 \sin(5t + 1.148)$$

**2.46** Taking the transform of both sides of the equation and noting that both initial conditions are zero, we obtain

$$s^2 X(s) + 6sX(s) + 34X(s) = 5 \frac{6}{s^2 + 6^2}$$

Solve for  $X(s)$ .

$$X(s) = \frac{30}{(s^2 + 6s + 34)(s^2 + 6^2)}$$

The inverse transform is

$$x(t) = \frac{6}{65} e^{-3t} \sin 5t + \frac{9}{65} e^{-3t} \cos 5t - \frac{1}{130} \sin 6t - \frac{9}{65} \cos 6t$$

**2.47** Transform the equation.

$$(s^2 + 12s + 40)X(s) = \frac{10}{s}$$

or, since the characteristic roots are  $s = -6 \pm 2j$ ,

$$X(s) = \frac{10}{s[(s+6)^2 + 2^2]} \quad (1)$$

From the text example, the form  $Ae^{-at} \sin(\omega t + \phi)$  has the transform

$$A \frac{s \sin \phi + a \sin \phi + \omega \cos \phi}{(s+a)^2 + \omega^2}$$

For this problem,  $a = 6$  and  $\omega = 2$ . Thus

$$X(s) = \frac{10}{s[(s+6)^2 + 2^2]} = \frac{C_1}{s} + C_2 \frac{s \sin \phi + 6 \sin \phi + 2 \cos \phi}{(s+6)^2 + 2^2}$$

or

$$X(s) = \frac{C_1(s^2 + 12s + 40) + C_2 s^2 \sin \phi + 6C_2 s \sin \phi + 2C_2 s \cos \phi}{s[(s+6)^2 + 2^2]} \quad (2)$$

(continued on the next page)



Problem 2.47 continued:

Collecting terms and comparing the numerators of equations (1) and (2), we have

$$(C_1 + C_2 \sin \phi)s^2 + (12C_1 + 6C_2 \sin \phi + 2C_2 \cos \phi)s + 40C_1 = 10$$

Thus comparing terms, we see that  $C_1 = 1/4$  and

$$\frac{1}{4} + C_2 \sin \phi = 0$$

$$3 + 6C_2 \sin \phi + 2C_2 \cos \phi = 0$$

So

$$C_2 \sin \phi = -\frac{1}{4} \quad C_2 \cos \phi = -\frac{3}{4}$$

Thus  $\phi$  is in the third quadrant and

$$\phi = \tan^{-1} \frac{-1/4}{-3/4} = 0.322 + \pi = 3.463 \text{ rad}$$

Because  $\sin^2 \phi + \cos^2 \phi = 1$ ,

$$\left(\frac{1}{4C_2}\right)^2 + \left(\frac{3}{4C_2}\right)^2 = 1$$

which gives  $C_2 = 0.791$ . Thus

$$x(t) = \frac{1}{4} + 0.791e^{-6t} \sin(2t + 3.463)$$

**2.48** Transform the equation.

$$X(s) = \frac{F(s)}{s^2 + 8s + 1}$$

Thus

$$F(s) - X(s) = F(s) - \frac{F(s)}{s^2 + 8s + 1} = \frac{s^2 + 8s}{s^2 + 8s + 1} F(s)$$

Because  $F(s) = 6/s^2$ ,

$$F(s) - X(s) = \frac{s^2 + 8s}{s^2 + 8s + 1} \frac{6}{s^2} = \frac{s + 8}{s^2 + 8s + 1} \frac{6}{s}$$

From the final value theorem,

$$f_{ss} - x_{ss} = \lim_{s \rightarrow 0} s[F(s) - X(s)] = \lim_{s \rightarrow 0} s \frac{s + 8}{s^2 + 8s + 1} \frac{6}{s} = 8$$

**2.49** The roots are  $s = -2$  and  $-4$ . Thus

$$X(s) = \frac{1 - e^{-3s}}{(s+2)(s+4)}$$

Let

$$F(s) = \frac{1}{(s+2)(s+4)} = \frac{1}{2} \left( \frac{1}{s+2} - \frac{1}{s+4} \right)$$

so

$$f(t) = \frac{1}{2} (e^{-2t} - e^{-4t})$$

From Property 6 of the Laplace transform,

$$x(t) = \frac{1}{2} (e^{-2t} - e^{-4t}) - \frac{1}{2} [e^{-2(t-3)} - e^{-4(t-3)}] u_s(t-3)$$

**2.50**

$$f(t) = \frac{C}{D}tu_s(t) - \frac{2C}{D}(t-D)u_s(t-D) + \frac{C}{D}(t-2D)u_s(t-2D)$$

From Property 6 of the Laplace transform,

$$F(s) = \frac{C}{Ds^2} - \frac{2C}{Ds^2}e^{-Ds} + \frac{C}{Ds^2}e^{-2Ds} = \frac{C}{Ds^2} \left(1 - 2e^{-Ds} + e^{-2Ds}\right)$$

**2.51**

$$f(t) = \frac{C}{D}tu_s(t) - \frac{C}{D}(t-D)u_s(t-D) - Cu_s(t-D)$$

From Property 6 of the Laplace transform,

$$F(s) = \frac{C}{Ds^2} - \frac{C}{Ds^2}e^{-Ds} - \frac{C}{s}e^{-Ds}$$

**2.52**

$$f(t) = Mu_s(t) - 2Mu_s(t - T) + Mu_s(t - 2T)$$

From Property 6,

$$F(s) = \frac{M}{s} - \frac{2M}{s}e^{-Ts} + \frac{M}{s}e^{-2Ts}$$

**2.53**

$$P(t) = 3u_s(t) - 3u_s(t - 5)$$

From Property 6,

$$P(s) = \frac{3}{s} - \frac{3}{s}e^{-5s}$$

$$X(s) = \frac{P(s)}{4s + 1} = \frac{3(1 - e^{-5s})}{s(4s + 1)} = \frac{3}{4} \frac{1 - e^{-5s}}{s(s + 1/4)}$$

Let

$$F(s) = \frac{3}{4} \frac{1}{s(s + 1/4)} = 3 \left( \frac{1}{s} - \frac{1}{s + 1/4} \right)$$

Then

$$f(t) = 3 \left( 1 - e^{-t/4} \right)$$

Since

$$X(s) = F(s) \left( 1 - e^{-5s} \right)$$

we have

$$x(t) = f(t) - f(t - 5)u_s(t - 5) = 3 \left( 1 - e^{-t/4} \right) - 3 \left[ 1 - e^{-(t-5)/4} \right] u_s(t - 5)$$

**2.54** Let

$$f(t) = t + \frac{t^3}{3} + \frac{2t^5}{15}$$

Then

$$F(s) = \frac{1}{s^2} + \frac{2}{s^4} + \frac{16}{s^6} = \frac{s^4 + 2s^2 + 16}{s^6}$$

From the differential equation,

$$\begin{aligned} X(s) &= \frac{F(s)}{s+1} = \frac{s^4 + 2s^2 + 16}{s^6(s+1)} \\ &= \frac{16}{s^6} - \frac{16}{s^5} + \frac{18}{s^4} - \frac{18}{s^3} + \frac{19}{s^2} - \frac{19}{s} + \frac{19}{s+1} \end{aligned}$$

Thus

$$x(t) = \frac{2}{15}t^5 - \frac{2}{3}t^4 + 3t^3 - 9t^2 + 19t - 19 + 19e^{-t}$$

On a plot of this and the solution obtained from the lower-order approximation, the two solutions are practically indistinguishable.



**2.55** From the derivative property of the Laplace transform, we know that

$$\mathcal{L}[\dot{x}(t)] = \int_0^\infty \dot{x}(t)e^{-st} dt = sX(s) - x(0)$$

Therefore

$$\begin{aligned} \lim_{s \rightarrow \infty} [sX(s)] &= \lim_{s \rightarrow \infty} \left[ x(0) + \int_0^\infty \dot{x}(t)e^{-st} dt \right] \\ &= \lim_{s \rightarrow \infty} x(0) + \lim_{s \rightarrow \infty} \left\{ \lim_{\epsilon \rightarrow 0+} \left[ \int_0^\epsilon \dot{x}(t)e^{-st} dt \right] \right\} + \lim_{\epsilon \rightarrow 0+} \left\{ \int_0^\epsilon \lim_{s \rightarrow \infty} [\dot{x}(t)e^{-st} dt] \right\} \end{aligned}$$

The limits on  $\epsilon$  and  $s$  can be interchanged because  $s$  is independent of  $t$ . Within the interval  $[0, 0+]$ ,  $e^{-st} = 1$ , and so

$$\begin{aligned} \lim_{s \rightarrow \infty} [sX(s)] &= x(0) + \lim_{s \rightarrow \infty} \left\{ \lim_{\epsilon \rightarrow 0+} \left[ \int_0^\epsilon \dot{x}(t) dt \right] \right\} + \lim_{\epsilon \rightarrow 0+} \left\{ \int_0^\epsilon \lim_{s \rightarrow \infty} [\dot{x}(t)e^{-st} dt] \right\} \\ &= x(0) + x(t)|_{t=0}^{t=0+} + 0 = x(0+) \end{aligned}$$

This proves the theorem.

**2.56** From the derivative property of the Laplace transform, we know that

$$\mathcal{L}[\dot{x}(t)] = \int_0^\infty \dot{x}(t)e^{-st} dt = sX(s) - x(0)$$

Therefore,

$$\begin{aligned}\lim_{s \rightarrow 0} [sX(s)] &= \lim_{s \rightarrow 0} x(0) + \lim_{s \rightarrow 0} \left[ \int_0^\infty \dot{x}(t)e^{-st} dt \right] \\ &= x(0) + \int_0^\infty \lim_{s \rightarrow 0} [\dot{x}(t)e^{-st}] dt = x(0) + \int_0^\infty \dot{x}(t) dt\end{aligned}$$

because  $s$  is independent of  $t$  and  $\lim_{s \rightarrow 0} e^{-st} = 1$ . Thus

$$\begin{aligned}\lim_{s \rightarrow 0} [sX(s)] &= x(0) + \lim_{T \rightarrow \infty} \left[ \int_0^T \dot{x}(t) dt \right] = x(0) + \lim_{T \rightarrow \infty} [x(t)|_{t=0}^{t=T}] \\ &= x(0) + \lim_{T \rightarrow \infty} x(T) - x(0) = \lim_{T \rightarrow \infty} x(T) = \lim_{t \rightarrow \infty} x(t)\end{aligned}$$

This proves the theorem.

**2.57** Let

$$g(t) = \int_0^t x(t) dt$$

Then

$$\mathcal{L} \left[ \int_0^t x(t) dt \right] = \mathcal{L}[g(t)] = \int_0^t g(t) e^{-st} dt$$

To use integration by parts we define  $u = g$  and  $dv = e^{-st} dt$ , which give  $du = dg = x(t) dt$  and  $v = -e^{-st}/s$ . Thus

$$\begin{aligned} \int_0^t g(t) e^{-st} dt &= \frac{g(t) e^{-st}}{-s} \Big|_{t=0}^{t=\infty} - \int_0^\infty \frac{e^{-st}}{-s} x(t) dt \\ &= 0 + \frac{g(0)}{s} + \frac{1}{s} \int_0^\infty x(t) e^{-st} dt = \frac{g(0)}{s} + \frac{X(s)}{s} \\ &= \frac{1}{s} \int_0^\infty x(t) dt \Big|_{t=0} + \frac{X(s)}{s} \end{aligned}$$

This proves the property.

If there is an impulse in  $x(t)$  at  $t = 0$ , then  $g(0)$  equals the strength of the impulse. If there is no impulse at  $t = 0$ , then  $g(0) = 0$ .

2.58 a)

$$[r,p,k] = \text{residue}([8,5],[2,20,48])$$

The result is  $r = [10.7500, -6.7500]$ ,  $p = [-6.0000, -4.0000]$ , and  $k = [ ]$ . The solution is

$$x(t) = 10.75e^{-6t} - 6.75e^{-4t}$$

b)

$$[r,p,k] = \text{residue}([4,13],[1,8,116])$$

The result is  $r = [2.0000 - 0.1500i, 2.0000 + 0.1500i]$ ,  $p = [-4.0000 + 10.0000i, -4.0000 - 10.0000i]$ , and  $k = [ ]$ . The solution is

$$x(t) = (2 - 0.15j)e^{(-4+10j)t} + (2 + 0.15j)e^{(-4-10j)t}$$

The solution is

$$x(t) = 2e^{-4t}(2 \cos 10t + 0.15 \sin 10t)$$

c)

$$[r,p,k] = \text{residue}([3,2],[1,10,0,0])$$

The result is  $r = [-0.2800, 0.2800, 0.2000]$ ,  $p = [-10, 0, 0]$ , and  $k = [ ]$ . The solution is

$$x(t) = -0.28e^{-10t} + 0.28 + 0.2t$$

(continued on the next page)

Problem 2.58 continued:

d)

$$[r,p,k] = \text{residue}([1,0,1,6],[1,2,0,0,0,0])$$

The result is  $r = [-0.2500, 0.2500, 0.5000, -1.0000, 3.0000]$ ,  $p = [-2, 0, 0, 0, 0]$ , and  $k = []$ . The solution is

$$x(t) = -0.25e^{-2t} + 0.25 + 0.5t - \frac{1}{2}t^2 + \frac{1}{2}t^3$$

e)

$$[r,p,k] = \text{residue}([4,3],[1,6,34,0])$$

The result is  $r = [-0.0441 - 0.3735i, -0.0441 + 0.3735i, 0.0882]$ ,  $p = [-3.0000 + 5.0000i, -3.0000 - 5.0000i, 0]$ , and  $k = []$ . The solution is

$$x(t) = (-0.0441 - 0.3735j)e^{(-3+5j)t} + (-0.0441 + 0.3735j)e^{(-3-5j)t} + 0.0882$$

The solution is

$$x(t) = 2e^{-3t}(-0.0441 \cos 5t + 0.3735 \sin 5t) + 0.0882$$

(continued on the next page)

Problem 2.58 continued:

f)

`[r,p,k] = residue([5,3,7],[1,12,44,48])`

The result is  $\mathbf{r} = [21.1250 \ -18.7500 \ 2.6250]$ ,  $\mathbf{p} = [-6, -4, -2]$ , and  $\mathbf{k} = []$ . The solution is

$$x(t) = 21.125e^{-6t} - 18.75e^{-4t} + 2.625e^{-2t}$$

2.59 a)

$$[r,p,k] = \text{residue}(5, \text{conv}([1,8,16],[1,1]))$$

The result is  $r = [-0.5556, -1.6667, 0.5556]$ ,  $p = [-4.0000, -4.0000, -1.0000]$ ,  $k = [ ]$ . The solution is

$$x(t) = -0.5556e^{-4t} - 1.6667te^{-4t} + 0.5556e^{-t}$$

b)

$$[r,p,k] = \text{residue}([4,9], \text{conv}([1,6,34],[1,4,20]))$$

The result is  $r = [-0.1159 + 0.1073i, -0.1159 - 0.1073i, 0.1159 - 0.1052i, 0.1159 + 0.1052i]$ ,  $p = [-3.0000 + 5.0000i, -3.0000 - 5.0000i, -2.0000 + 4.0000i, -2.0000 - 4.0000i]$ , and  $k = [ ]$ . The solution is

$$\begin{aligned} x(t) &= (-0.1159 + 0.1073j)e^{(-3+5j)t} + (-0.1159 - 0.1073j)e^{(-3-5j)t} \\ &+ (0.1159 - 0.1052j)e^{(-2+4j)t} + (0.1159 + 0.1052j)e^{(-2-4j)t} \end{aligned}$$

The solution is

$$x(t) = 2e^{-3t}(-0.1159 \cos 5t - 0.1073 \sin 5t) + 2e^{-2t}(0.1159 \cos 4t + 0.1052 \sin 4t)$$

**2.60** a)

```
sys = tf(1,[3,21,30]);  
step(sys)
```

b)

```
sys = tf(1,[5,20, 65]);  
step(sys)
```

c)

```
sys = tf([3,2],[4,32,60]);  
step(sys)
```



**2.61** a)

```
sys = tf(1,[3,21,30]);  
impulse(sys)
```

b)

```
sys = tf(1,[5,20, 65]);  
impulse(sys)
```

**2.62**

```
sys = tf(5,[3,21,30]);  
impulse(sys)
```

## 2.63

```
sys = tf(5,[3,21,30]);  
step(sys)
```

2.64 a)

```
sys = tf(1,[3,21,30]);  
t = [0:0.001:1.5];  
f = 5*t;  
[x,t] = lsim(sys,f,t);  
plot(t,x)
```

b)

```
sys = tf(1,[5,20,65]);  
t = [0:0.001:1.5];  
f = 5*t;  
[x,t] = lsim(sys,f,t);  
plot(t,x)
```

c)

```
sys = tf([3,2],[4,32,60]);  
t = [0:0.001:1.5];  
f = 5*t;  
[x,t] = lsim(sys,f,t);  
plot(t,x)
```

**2.65 a)**

```
sys = tf(1,[3,21,30]);  
t = [0:0.001:6];  
f = 6*cos(3*t);  
[x,t] = lsim(sys,f,t);  
plot(t,x)
```

b)

```
sys = tf(1,[5,20,65]);  
t = [0:0.001:6];  
f = 6*cos(3*t);  
[x,t] = lsim(sys,f,t);  
plot(t,x)
```

c)

```
sys = tf([3,2],[4,32,60]);  
t = [0:0.001:6];  
f = 6*cos(3*t);  
[x,t] = lsim(sys,f,t);  
plot(t,x)
```