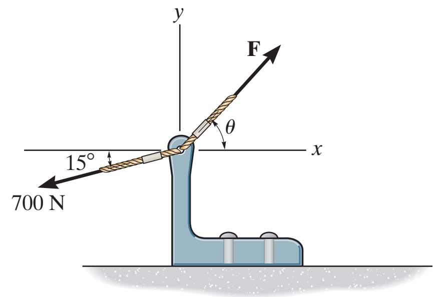
2–1.

If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45°}$$

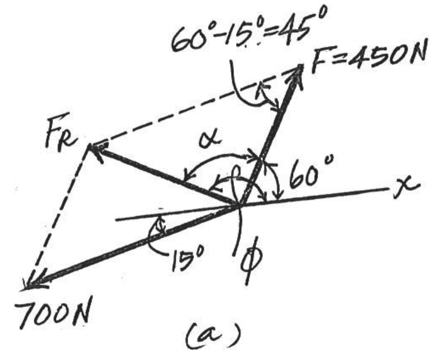
$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

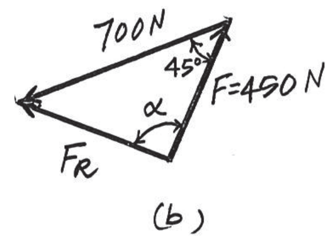
This yields

$$\frac{\sin\alpha}{700} = \frac{\sin 45°}{497.01} \qquad \alpha = 95.19°$$

Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60° = 95.19° + 60° = 155° \qquad \textbf{Ans.}$$

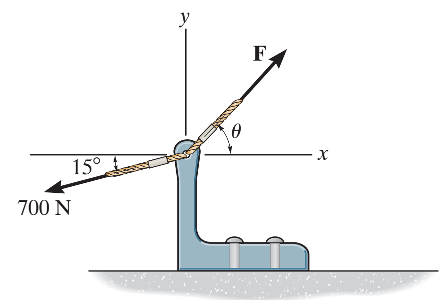




Ans:
$F_R = 497$ N
$\phi = 155°$

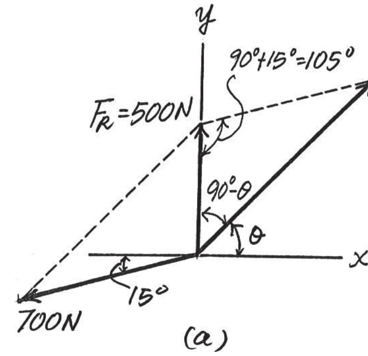
2–2.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



Applying the law of cosines to Fig. *b*,

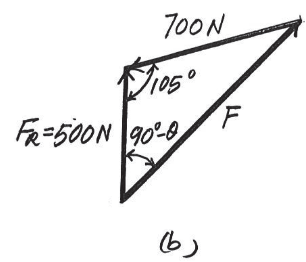
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105°}$$

$$= 959.78 \text{ N} = 960 \text{ N} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b*, and using this result, yields

$$\frac{\sin(90° + \theta)}{700} = \frac{\sin 105°}{959.78}$$

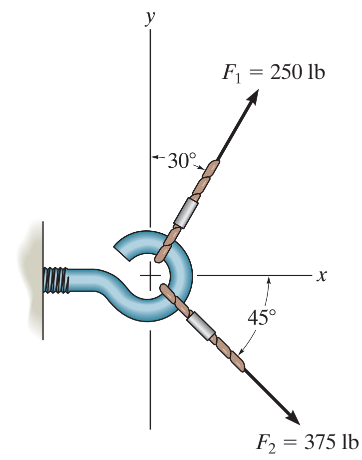
$$\theta = 45.2° \qquad \textbf{Ans.}$$



Ans:
$F = 960$ N
$\theta = 45.2°$

2–3.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



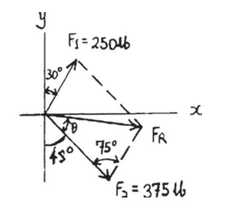
SOLUTION

$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb}$$ **Ans.**

$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin \theta}$$

$$\theta = 37.89^\circ$$

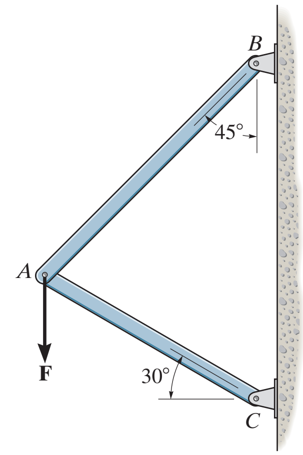
$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ$$ **Ans.**



Ans:
$F_R = 393$ lb
$\phi = 353^\circ$

***2–4.**

Determine the magnitudes of the two components of **F** directed along members AB and AC. Set $F = 500$ N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

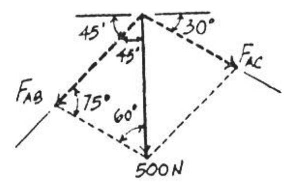
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{500}{\sin 75^\circ}$$

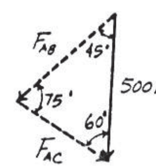
$$F_{AB} = 448 \text{ N}$$ **Ans.**

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{500}{\sin 75^\circ}$$

$$F_{AC} = 366 \text{ N}$$ **Ans.**



(a)

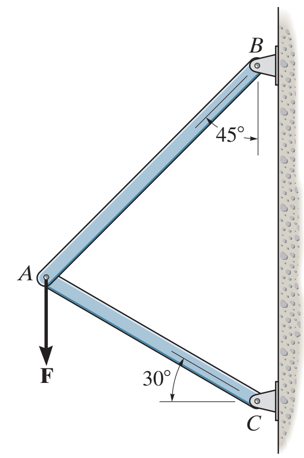


(b)

Ans:
$F_{AB} = 448$ N
$F_{AC} = 366$ N

2–5.

Solve Prob. 2–4 with $F = 350$ lb.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

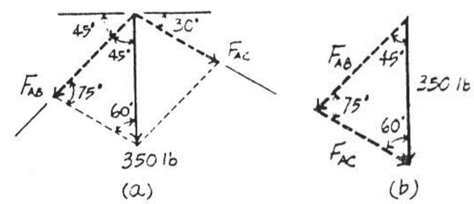
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

$$F_{AB} = 314 \text{ lb}$$ **Ans.**

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

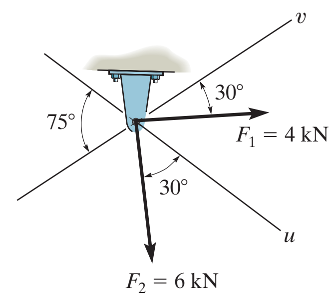
$$F_{AC} = 256 \text{ lb}$$ **Ans.**



Ans:
$F_{AB} = 314$ lb
$F_{AC} = 256$ lb

2–6.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying Law of cosines by referring to Fig. *b*,

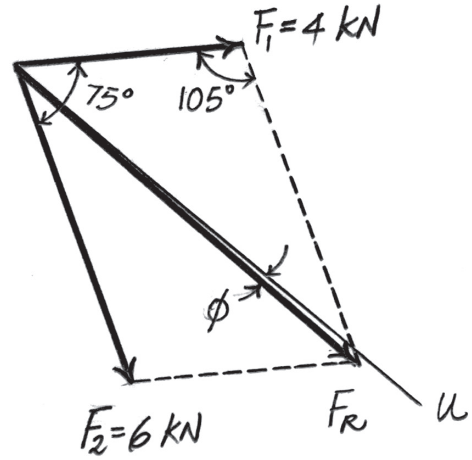
$$F_R = \sqrt{4^2 + 6^2 - 2(4)(6)\cos 105^\circ} = 8.026 \text{ kN} = 8.03 \text{ kN} \qquad \textbf{Ans.}$$

Using this result to apply Law of sines, Fig. *b*,

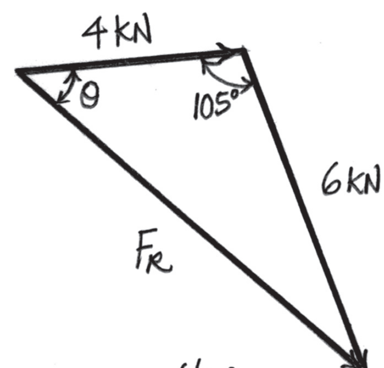
$$\frac{\sin\theta}{6} = \frac{\sin 105^\circ}{8.026}; \qquad \theta = 46.22^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured clockwise from the positive u axis is

$$\phi = 46.22^\circ - 45^\circ = 1.22^\circ \qquad \textbf{Ans.}$$



(a)

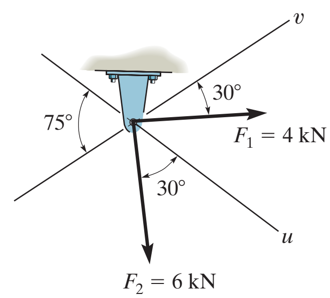


(b)

Ans:
$F_R = 8.03$ kN
$\phi = 1.22^\circ$

2–7.

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

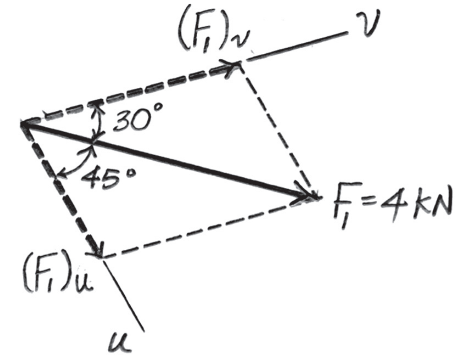


SOLUTION

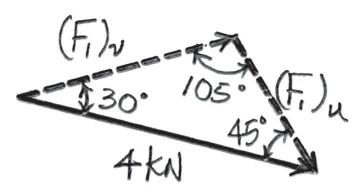
Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the sines law by referring to Fig. *b*.

$$\frac{(F_1)_v}{\sin 45°} = \frac{4}{\sin 105°}; \qquad (F_1)_v = 2.928 \text{ kN} = 2.93 \text{ kN} \qquad \textbf{Ans.}$$

$$\frac{(F_1)_u}{\sin 30°} = \frac{4}{\sin 105°}; \qquad (F_1)_u = 2.071 \text{ kN} = 2.07 \text{ kN} \qquad \textbf{Ans.}$$



(a)

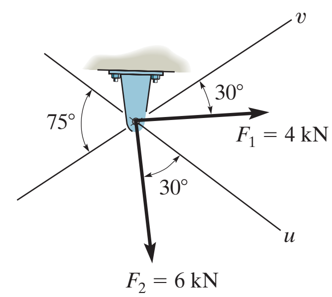


(b)

Ans:
$(F_1)_v = 2.93$ kN
$(F_1)_u = 2.07$ kN

***2–8.**

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.

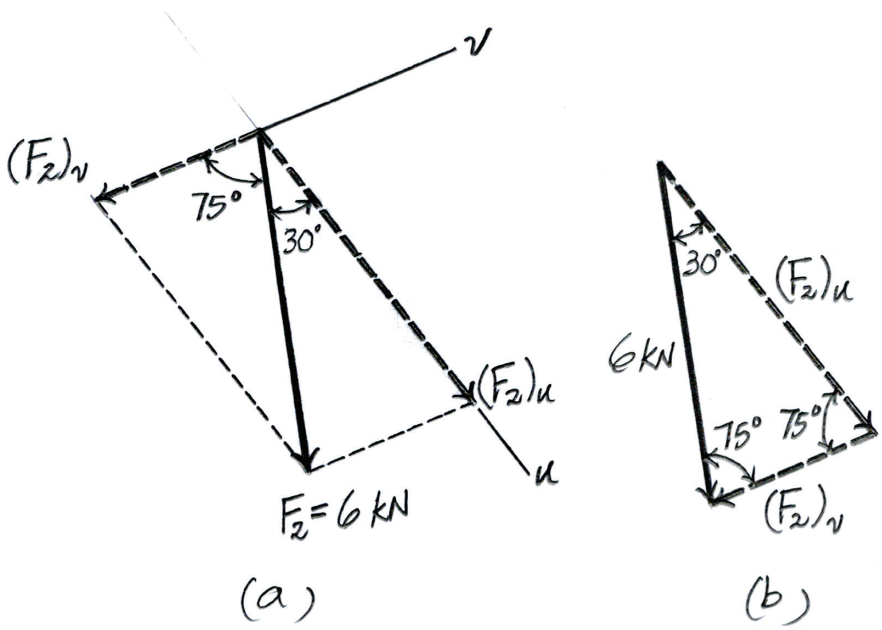


SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the sines law of referring to Fig. *b*,

$$\frac{(F_2)_u}{\sin 75°} = \frac{6}{\sin 75°}; \qquad (F_2)_u = 6.00 \text{ kN} \qquad \textbf{Ans.}$$

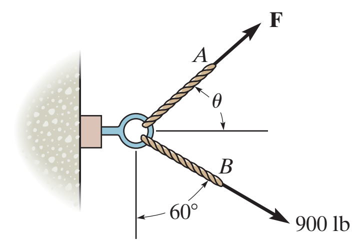
$$\frac{(F_2)_v}{\sin 30°} = \frac{6}{\sin 75°}; \qquad (F_2)_v = 3.106 \text{ kN} = 3.11 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$(F_2)_u = 6.00$ kN
$(F_2)_v = 3.11$ kN

2–9.

If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope *A* and the corresponding angle θ.



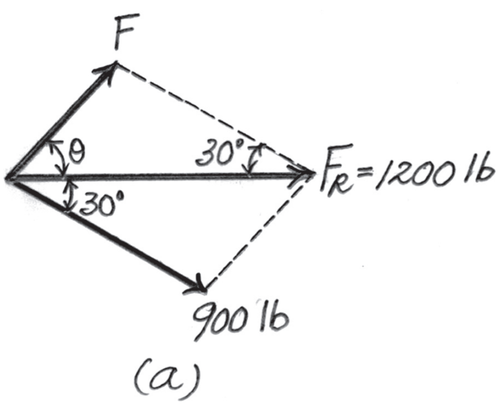
SOLUTION

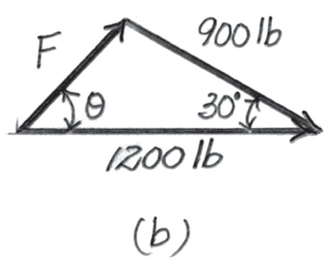
Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the law of cosines by referring to Fig. *b*,

$$F = \sqrt{900^2 + 1200^2 - 2(900)(1200)\cos 30^\circ} = 615.94 \text{ lb} = 616 \text{ lb} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin\theta}{900} = \frac{\sin 30^\circ}{615.94}; \qquad \theta = 46.94^\circ = 46.9^\circ \qquad \textbf{Ans.}$$

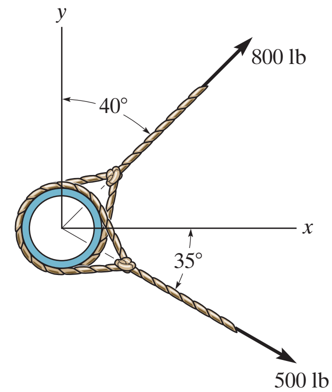




Ans:
$F = 616$ lb
$\theta = 46.9^\circ$

2–10.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the law of cosines by referring to Fig. *b*,

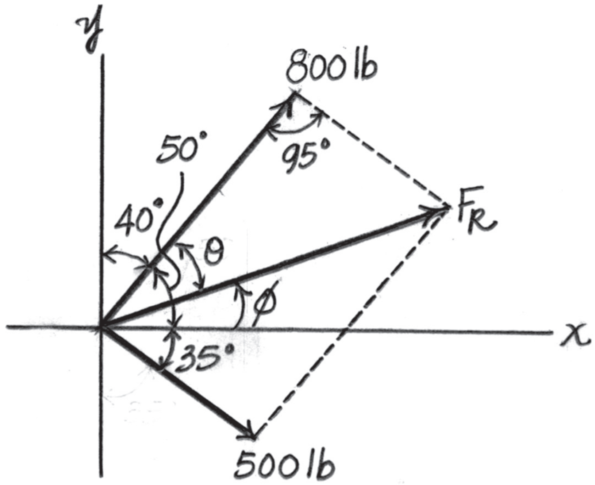
$$F_R = \sqrt{800^2 + 500^2 - 2(800)(500)\cos 95^\circ} = 979.66 \text{ lb} = 980 \text{ lb} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. *b*,

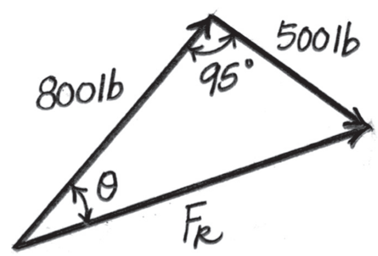
$$\frac{\sin\theta}{500} = \frac{\sin 95^\circ}{979.66}; \qquad \theta = 30.56^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\phi = 50^\circ - 30.56^\circ = 19.44^\circ = 19.4^\circ \qquad \textbf{Ans.}$$



(a)

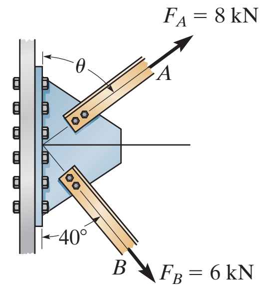


(b)

Ans:
$F_R = 980$ lb
$\phi = 19.4^\circ$

2–11.

If $\theta = 60°$, determine the magnitude of the resultant and its direction measured clockwise from the horizontal.



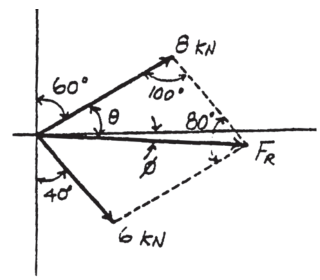
SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans.}$$



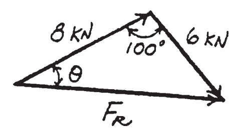
(a)

The angle θ can be determined using law of sines (Fig. *b*).

$$\frac{\sin\theta}{6} = \frac{\sin 100°}{10.80}$$

$$\sin\theta = 0.5470$$

$$\theta = 33.16°$$



(b)

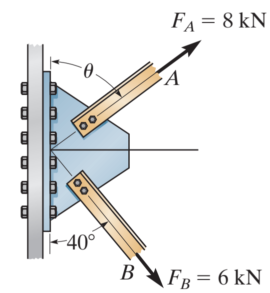
Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16° - 30° = 3.16° \qquad \textbf{Ans.}$$

Ans:
$F_R = 10.8$ kN
$\phi = 3.16°$

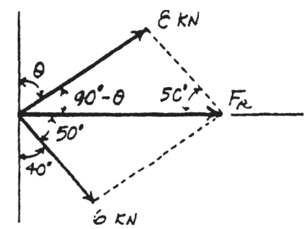
***2–12.**

Determine the angle θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.



Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

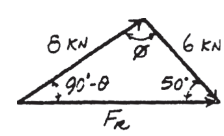
$$\sin(90^\circ - \theta) = 0.5745$$

$$\theta = 54.93^\circ = 54.9^\circ \qquad \textbf{Ans.}$$

From the triangle, $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93^\circ}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$

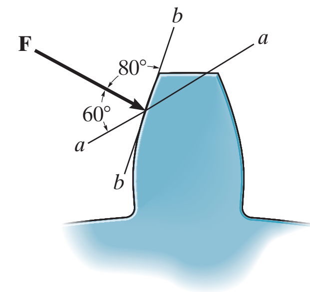


(b)

Ans:
$\theta = 54.9^\circ$
$F_R = 10.4$ kN

2–13.

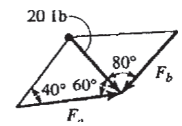
The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines *aa* and *bb*.



SOLUTION

$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \qquad F_a = 30.6 \text{ lb}$$ **Ans.**

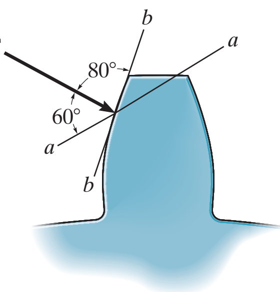
$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.9 \text{ lb}$$ **Ans.**



Ans:
$F_a = 30.6$ lb
$F_b = 26.9$ lb

2–14.

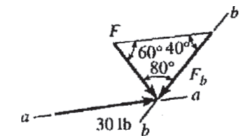
The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



SOLUTION

$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6 \text{ lb} \qquad \textbf{Ans.}$$

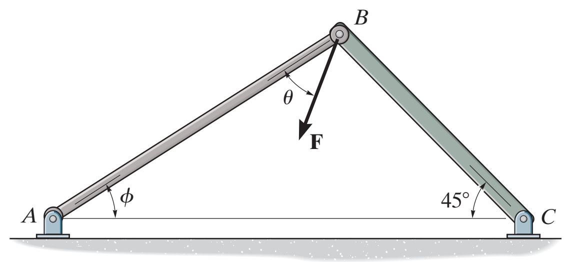
$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$F = 19.6$ lb
$F_b = 26.4$ lb

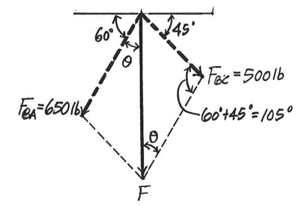
2–15.

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. a and b, respectively.

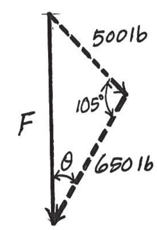


(a)

Applying the law of cosines to Fig. b,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

$$= 916.91 \text{ lb} = 917 \text{ lb} \qquad \textbf{Ans.}$$



(b)

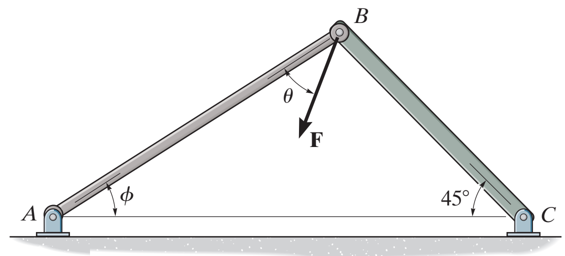
Using this result and applying the law of sines to Fig. b yields

$$\frac{\sin\theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8° \qquad \textbf{Ans.}$$

Ans:
$F = 917$ lb
$\theta = 31.8°$

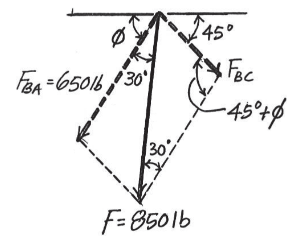
***2–16.**

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ ($0° \le \phi \le 45°$) and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)

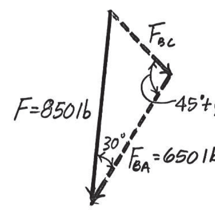
Applying the law of cosines to Fig. *b*,

$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30°}$$

$$= 433.64 \text{ lb} = 434 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the sine law to Fig. *b* yields

$$\frac{\sin(45° + \phi)}{850} = \frac{\sin 30°}{433.64} \qquad \phi = 33.5° \qquad \textbf{Ans.}$$

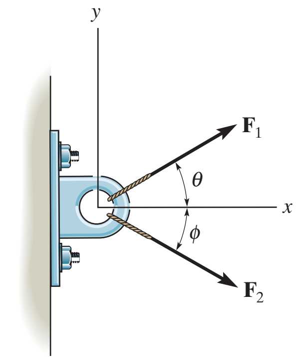


(b)

Ans:
$F_{BC} = 434$ lb
$\phi = 33.5°$

2–17.

If $F_1 = 30$ lb and $F_2 = 40$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 60$ lb.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.
Trigonometry. Applying the law of cosine by referring to Fig. *b*,

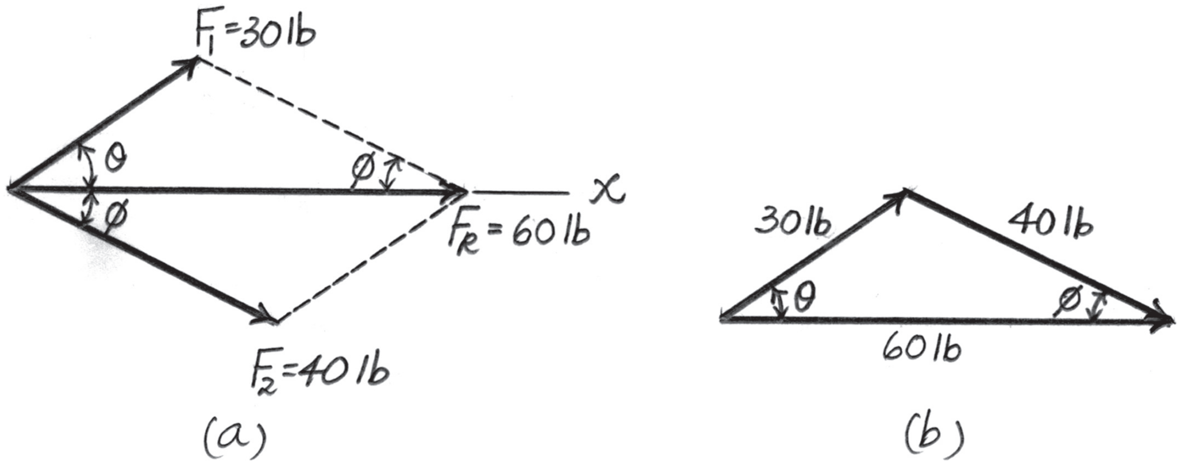
$$40^2 = 30^2 + 60^2 - 2(30)(60)\cos\theta$$

$$\theta = 36.34° = 36.3° \qquad \textbf{Ans.}$$

And

$$30^2 = 40^2 + 60^2 - 2(40)(60)\cos\phi$$

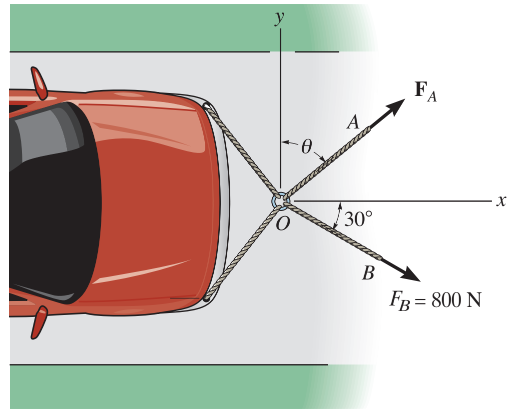
$$\phi = 26.38° = 26.4° \qquad \textbf{Ans.}$$



Ans:
$\theta = 36.3°$
$\phi = 26.4°$

2–18.

Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



SOLUTION

$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = F_A \sin\theta + 800 \cos 30° = 1250$

$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = F_A \cos\theta - 800 \sin 30° = 0$

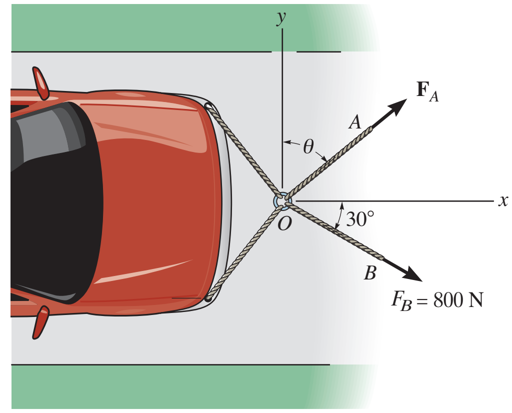
$\theta = 54.3°$ **Ans.**

$F_A = 686 \text{ N}$ **Ans.**

Ans:
$\theta = 54.3°$
$F_A = 686 \text{ N}$

2–19.

Determine the magnitude of the resultant force acting on the ring at O if $F_A = 750$ N and $\theta = 45^\circ$. What is its direction, measured counterclockwise from the positive x axis?



SOLUTION

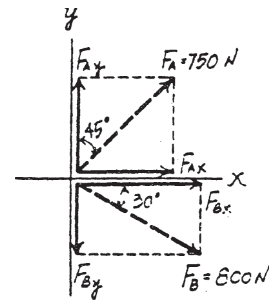
Scalar Notation: Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 750 \sin 45^\circ + 800 \cos 30^\circ$$

$$= 1223.15 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 750 \cos 45^\circ - 800 \sin 30^\circ$$

$$= 130.33 \text{ N} \uparrow$$



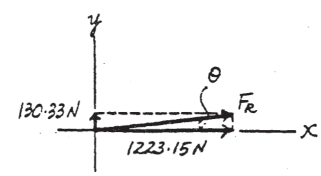
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \qquad \textbf{Ans.}$$

The directional angle θ measured counterclockwise from positive x axis is

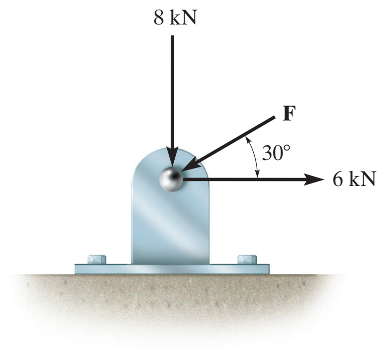
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08^\circ \qquad \textbf{Ans.}$$



Ans:
$F_R = 1.23$ kN
$\theta = 6.08^\circ$

***2–20.**

Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?



SOLUTION

Parallelogram Law. The parallelogram laws of addition for 6 kN and 8 kN and then their resultant F' and F are shown in Figs. *a* and *b*, respectively. In order for F_R to be minimum, it must act perpendicular to **F**.

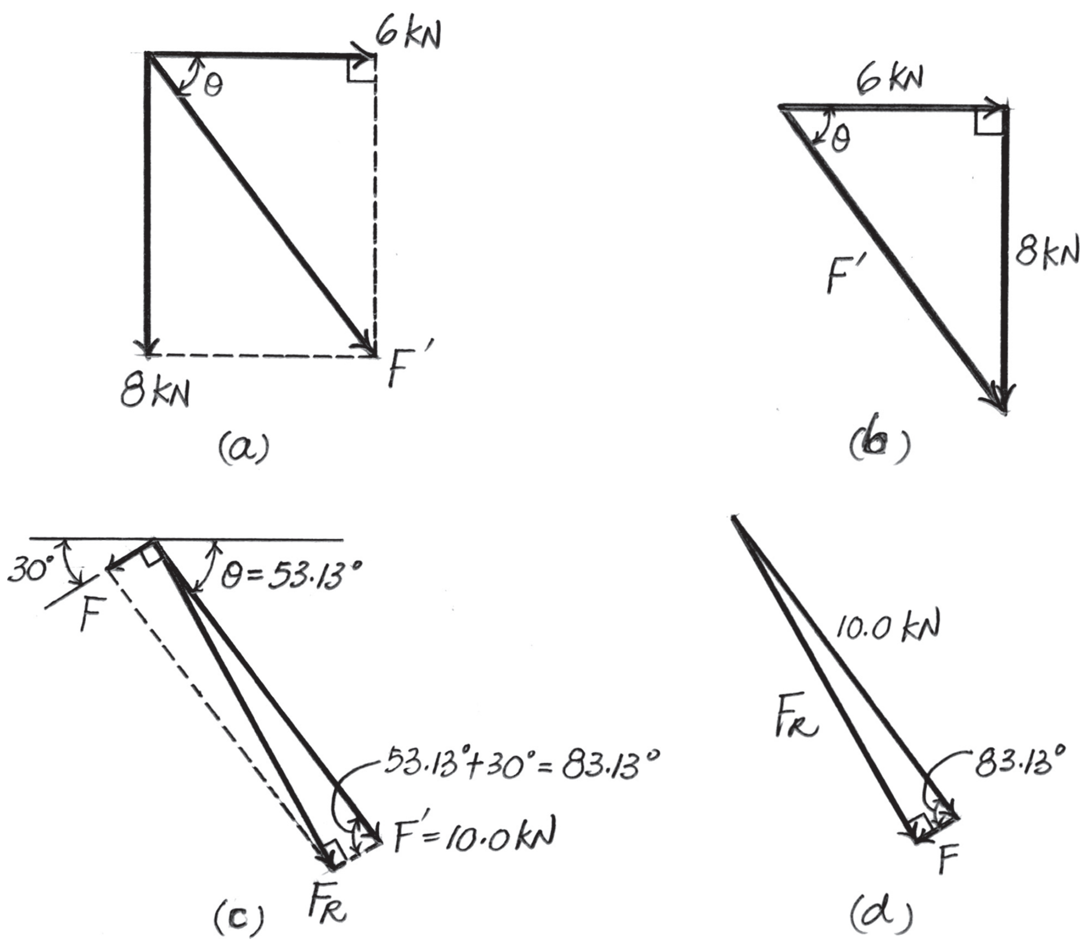
Trigonometry. Referring to Fig. *b*,

$$F' = \sqrt{6^2 + 8^2} = 10.0\text{ kN} \qquad \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°.$$

Referring to Figs. *c* and *d*,

$$F_R = 10.0 \sin 83.13° = 9.928\text{ kN} = 9.93\text{ kN} \qquad \textbf{Ans.}$$

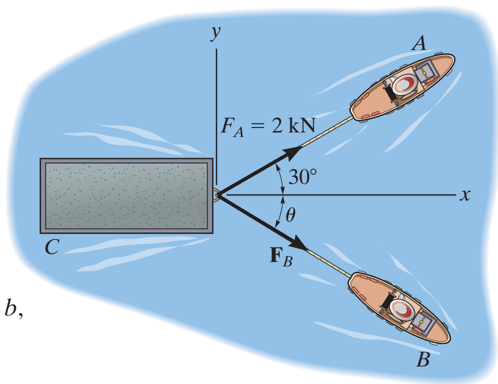
$$F = 10.0 \cos 83.13° = 1.196\text{ kN} = 1.20\text{ kN} \qquad \textbf{Ans.}$$



Ans:
$F_R = 9.93$ kN
$F = 1.20$ kN

2–21.

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.



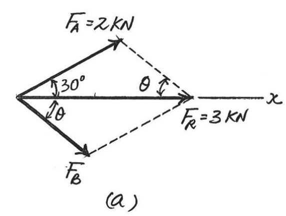
SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

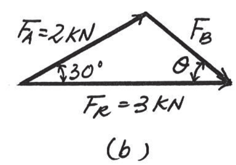
$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

$$= 1.615\text{kN} = 1.61 \text{ kN} \qquad \textbf{Ans.}$$



Using this result and applying the law of sines to Fig. *b* yields

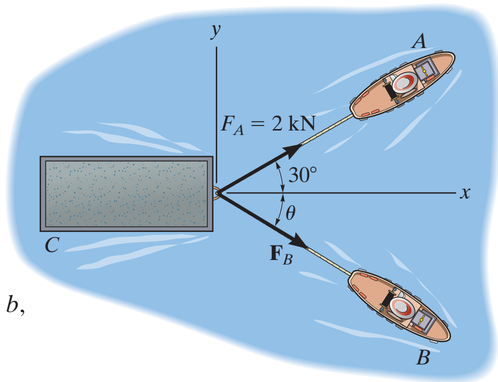
$$\frac{\sin\theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ \qquad \textbf{Ans.}$$



Ans:
$F_B = 1.61$ kN
$\theta = 38.3^\circ$

2–22.

If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

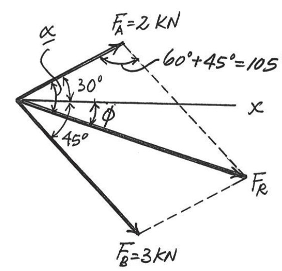
$$= 4.013 \text{ kN} = 4.01 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b* yields

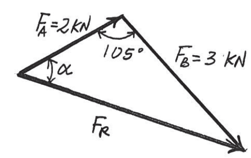
$$\frac{\sin \alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\phi = \alpha - 30° = 46.22° - 30° = 16.2° \qquad \textbf{Ans.}$$



(a)

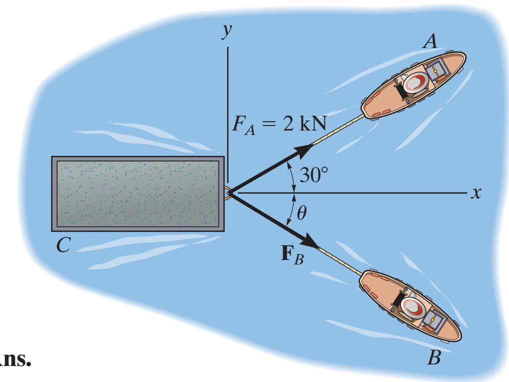


(b)

Ans:
$F_R = 4.01$ kN
$\phi = 16.2°$

2–23.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and $\mathbf{F}_B$ is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.



SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

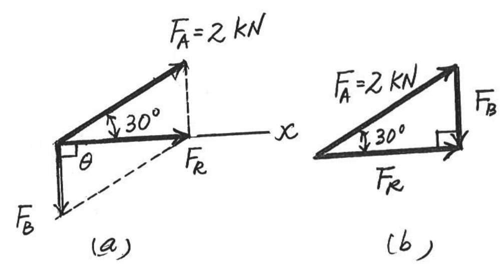
$$\theta = 90^\circ \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

By applying simple trigonometry to Fig. *b*,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN} \qquad \textbf{Ans.}$$

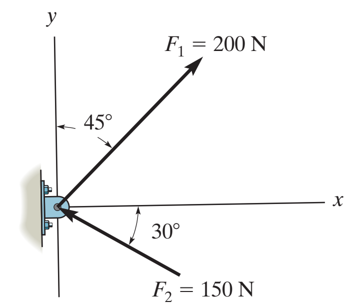
$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$\theta = 90^\circ$
$F_B = 1$ kN
$F_R = 1.73$ kN

***2–24.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 \sin 45^\circ - 150 \cos 30^\circ = 11.518 \text{ N} \rightarrow$$

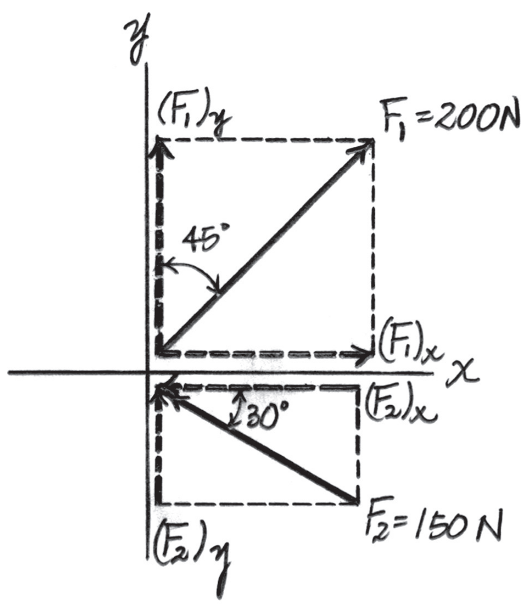
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 200 \cos 45^\circ + 150 \sin 30^\circ = 216.42 \text{ N} \uparrow$$

Referring to Fig. b, the magnitude of the resultant force F_R is

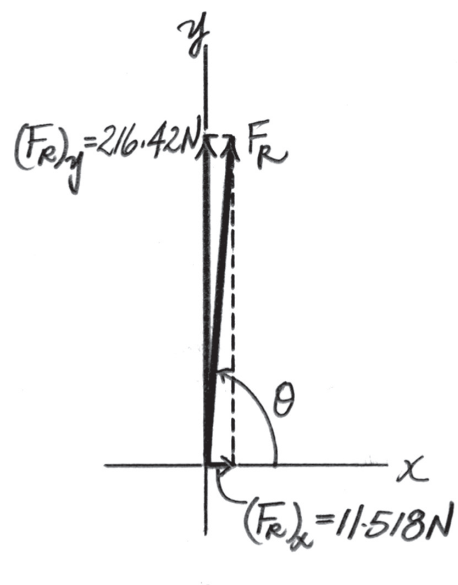
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{11.518^2 + 216.42^2} = 216.73 \text{ N} = 217 \text{ N} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{216.42}{11.518}\right) = 86.95^\circ = 87.0^\circ \qquad \textbf{Ans.}$$



(a)

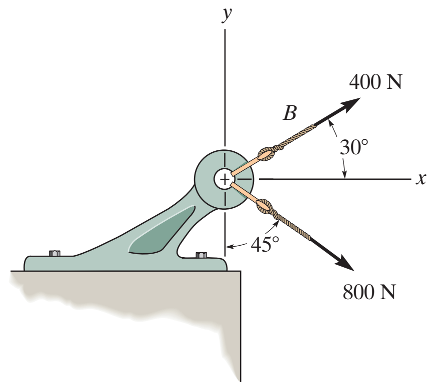


(b)

Ans:
$F_R = 217$ N
$\theta = 87.0^\circ$

2–25.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 400 \cos 30° + 800 \sin 45° = 912.10 \text{ N} \rightarrow$$

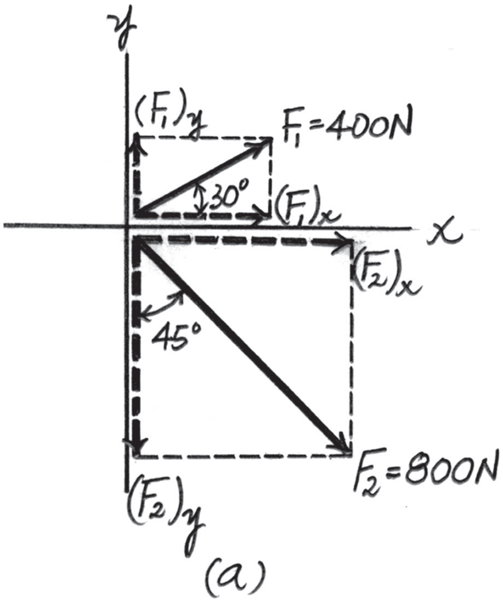
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 400 \sin 30° - 800 \cos 45° = -365.69 \text{ N} = 365.69 \text{ N}\downarrow$$

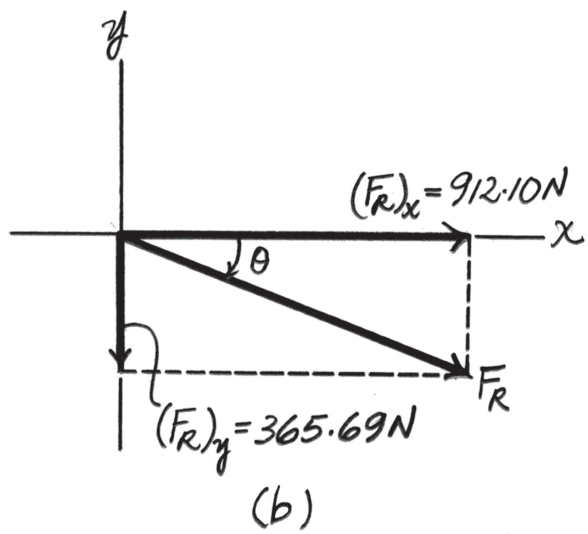
Referring to Fig. b, the magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{912.10^2 + 365.69^2} = 982.67 \text{ N} = 983 \text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{365.69}{912.10}\right) = 21.84° = 21.8° \qquad \textbf{Ans.}$$

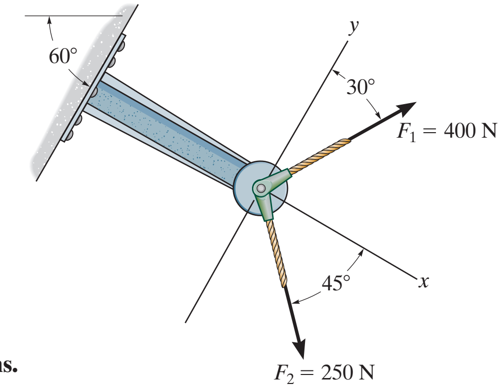




Ans:
$F_R = 983$ N
$\theta = 21.8°$

2–26.

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



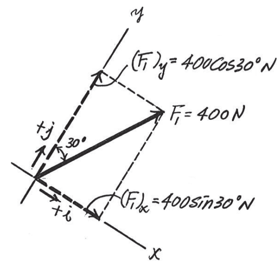
SOLUTION

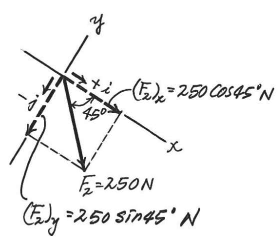
$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\} \text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\} \text{ N}$$

$$= \{177\mathbf{i} - 177\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$

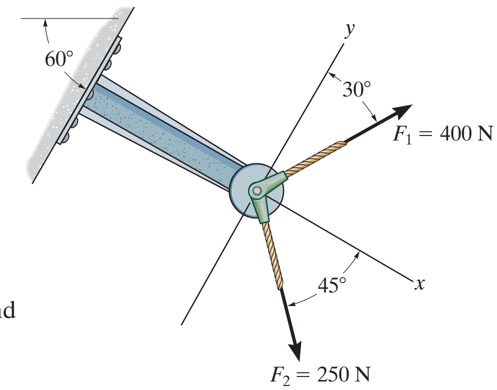




Ans:
$\mathbf{F}_1 = \{200\mathbf{i} + 346\mathbf{j}\}$ N
$\mathbf{F}_2 = \{177\mathbf{i} - 177\mathbf{j}\}$ N

2–27.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$$(F_1)_x = 400 \sin 30° = 200 \text{ N} \qquad (F_1)_y = 400 \cos 30° = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45° = 176.78 \text{ N} \qquad (F_2)_y = 250 \sin 45° = 176.78 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 + 176.78 = 376.78 \text{ N}$$

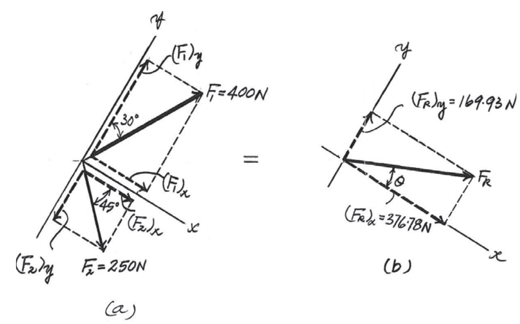
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. b, measured counterclockwise from the positive axis, is

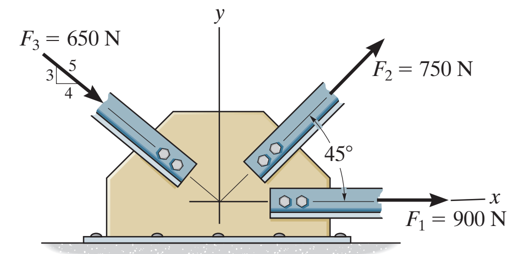
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{169.63}{376.78}\right) = 24.2° \qquad \textbf{Ans.}$$



Ans:
$F_R = 413$ N
$\theta = 24.2°$

***2–28.**

Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

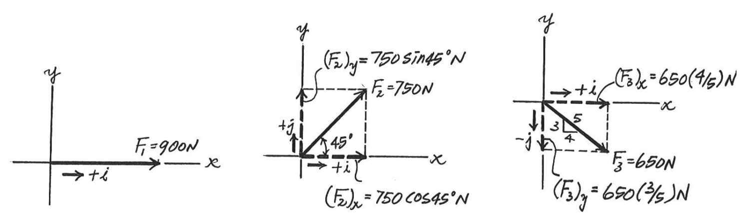


SOLUTION

$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = \{750\cos 45°(+\mathbf{i}) + 750\sin 45°(+\mathbf{j})\}\text{ N}$$
$$= \{530\mathbf{i} + 530\mathbf{j}\}\text{ N}$$ **Ans.**

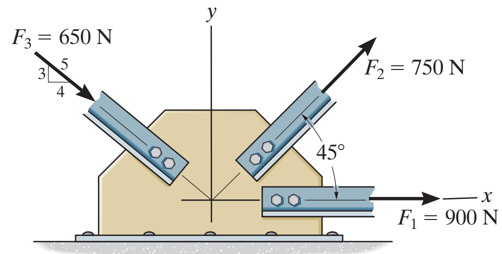
$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\text{ N}$$
$$= \{520\,\mathbf{i} - 390\mathbf{j})\}\text{ N}$$ **Ans.**



Ans:
$\mathbf{F}_1 = \{900\mathbf{i}\}$ N
$\mathbf{F}_2 = \{530\mathbf{i} + 530\mathbf{j}\}$ N
$\mathbf{F}_3 = \{520\mathbf{i} - 390\mathbf{j}\}$ N

2–29.

Determine the magnitude of the resultant force acting on the gusset plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900\text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750\cos 45° = 530.33\text{ N} \qquad (F_2)_y = 750\sin 45° = 530.33\text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520\text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33\text{ N} \rightarrow$$

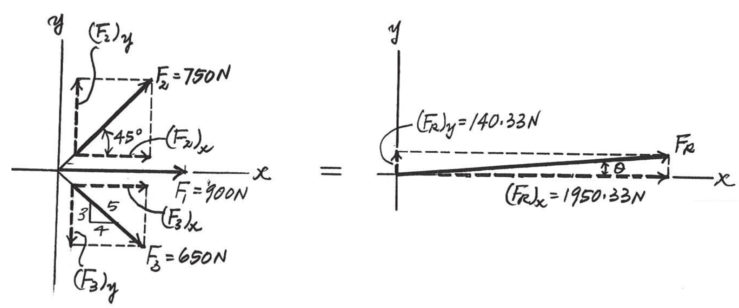
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33\text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2} = 1955\text{ N} = 1.96\text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \quad \textbf{Ans.}$$

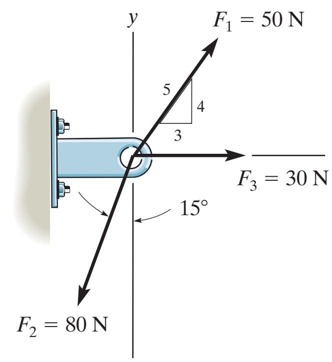


(a)

Ans:
$F_R = 1.96$ kN
$\theta = 4.12°$

2–30.

Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



SOLUTION

Cartesian Notation. Referring to Fig. *a*,

$$\mathbf{F}_1 = (F_1)_x\,\mathbf{i} + (F_1)_y\,\mathbf{j} = 50\left(\frac{3}{5}\right)\mathbf{i} + 50\left(\frac{4}{5}\right)\mathbf{j} = \{30\,\mathbf{i} + 40\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -(F_2)_x\,\mathbf{i} - (F_2)_y\,\mathbf{j} = -80\sin 15°\,\mathbf{i} - 80\cos 15°\,\mathbf{j}$$

$$= \{-20.71\,\mathbf{i} - 77.27\,\mathbf{j}\}\text{ N}$$

$$= \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$F_3 = (F_3)_x\,\mathbf{i} = \{30\,\mathbf{i}\} \qquad \textbf{Ans.}$$

Thus, the resultant force is

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (30\mathbf{i} + 40\,\mathbf{j}) + (-20.71\mathbf{i} - 77.27\mathbf{j}) + 30\mathbf{i}$$

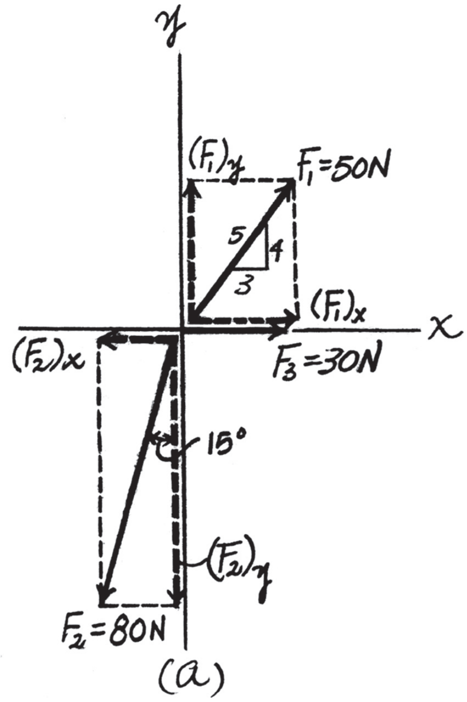
$$= \{39.29\,\mathbf{i} - 37.27\,\mathbf{j}\}\text{ N}$$

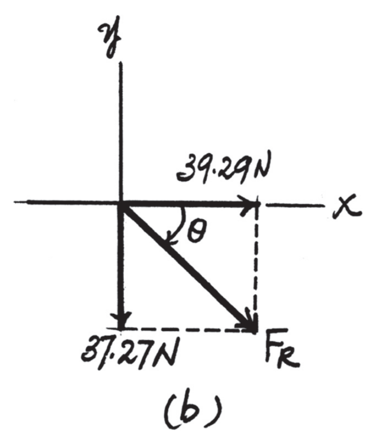
Referring to Fig. *b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{39.29^2 + 37.27^2} = 54.16\text{ N} = 54.2\text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left(\frac{37.27}{39.29}\right) = 43.49° = 43.5° \qquad \textbf{Ans.}$$

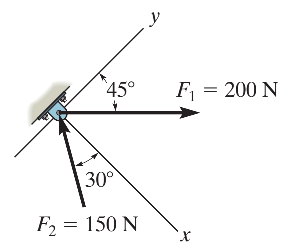




Ans:
$\mathbf{F}_1 = \{30\mathbf{i} + 40\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}$ N
$F_3 = \{30\,\mathbf{i}\}$
$F_R = 54.2$ N
$\theta = 43.5°$

2–31.

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

$F_{1x} = 200 \sin 45° = 141 \text{ N}$ **Ans.**

$F_{1y} = 200 \cos 45° = 141 \text{ N}$ **Ans.**

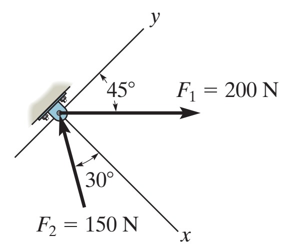
$F_{2x} = -150 \cos 30° = -130 \text{ N}$ **Ans.**

$F_{2y} = 150 \sin 30° = 75 \text{ N}$ **Ans.**

Ans:
$F_{1x} = 141$ N
$F_{1y} = 141$ N
$F_{2x} = -130$ N
$F_{2y} = 75$ N

***2–32.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$+\searrow F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -150 \cos 30° + 200 \sin 45° = 11.518 \text{ N}$

$\nearrow + F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 150 \sin 30° + 200 \cos 45° = 216.421 \text{ N}$

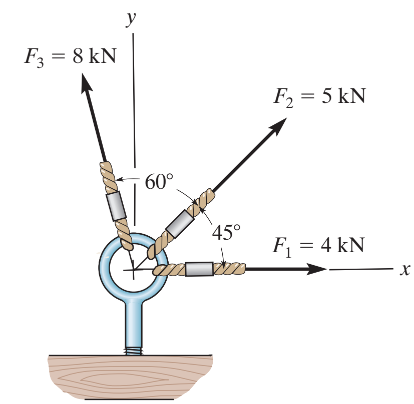
$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{216.421}{11.518}\right) = 87.0°$ **Ans.**

Ans:
$F_R = 217$ N
$\theta = 87.0°$

2–33.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

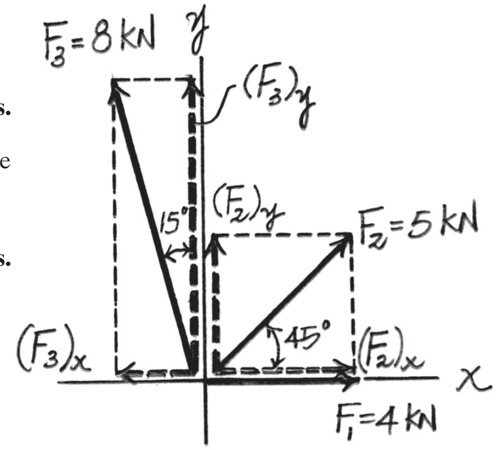


SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 4 + 5\cos 45° - 8\sin 15° = 5.465 \text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 5\sin 45° + 8\cos 15° = 11.263 \text{ kN} \uparrow$$



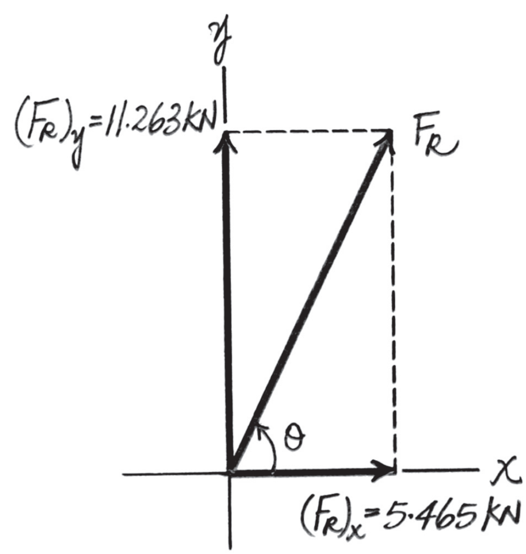
(a)

By referring to Fig. b, the magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5.465^2 + 11.263^2} = 12.52 \text{ kN} = 12.5 \text{ kN} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{11.263}{5.465}\right) = 64.12° = 64.1° \qquad \textbf{Ans.}$$

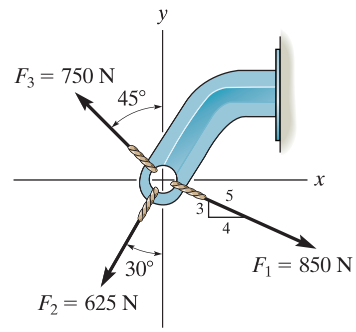


(b)

Ans:
$F_R = 12.5$ kN
$\theta = 64.1°$

2–34.

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = \frac{4}{5}(850)\,\mathbf{i} - \frac{3}{5}(850)\,\mathbf{j}$$

$$= \{680\,\mathbf{i} - 510\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -625\sin 30^\circ\,\mathbf{i} - 625\cos 30^\circ\,\mathbf{j}$$

$$= \{-312\,\mathbf{i} - 541\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

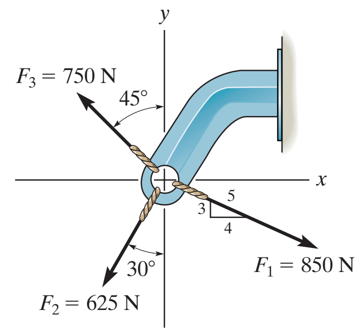
$$\mathbf{F}_3 = -750\sin 45^\circ\,\mathbf{i} + 750\cos 45^\circ\,\mathbf{j}$$

$$= \{-530\,\mathbf{i} + 530\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{680\mathbf{i} - 510\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-312\mathbf{i} - 541\mathbf{j}\}$ N
$\mathbf{F}_3 = \{-530\mathbf{i} + 530\mathbf{j}\}$ N

2–35.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

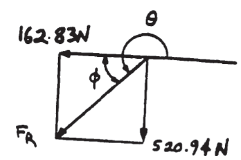
$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.83 \text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.94 \text{ N}$

$$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546 \text{ N}$$ **Ans.**

$$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64°$$

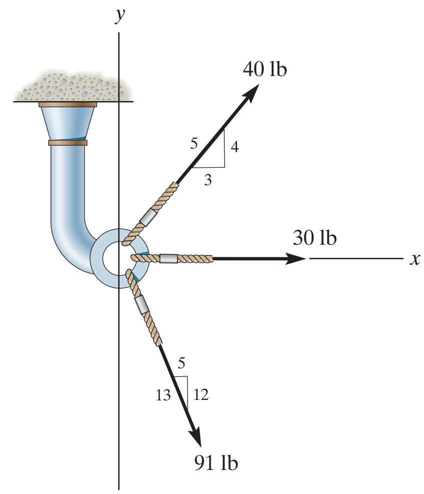
$$\theta = 180° + 72.64° = 253°$$ **Ans.**



Ans:
$F_R = 546$ N
$\theta = 253°$

***2–36.**

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 40\left(\frac{3}{5}\right) + 91\left(\frac{5}{13}\right) + 30 = 89 \text{ lb} \rightarrow$$

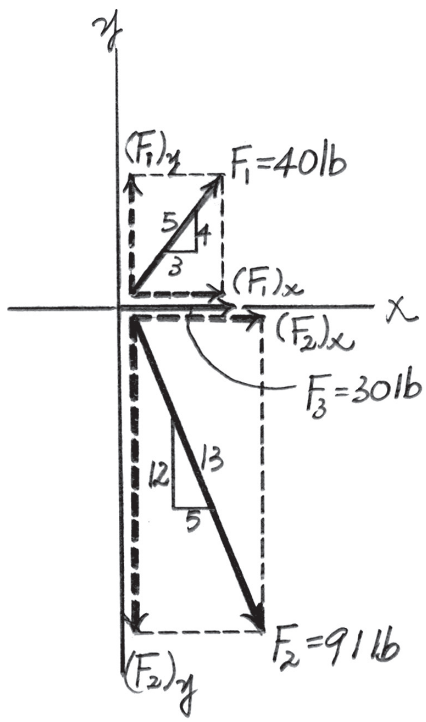
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 40\left(\frac{4}{5}\right) - 91\left(\frac{12}{13}\right) = -52 \text{ lb} = 52 \text{ lb}\downarrow$$

By referring to Fig. b, the magnitude of resultant force is

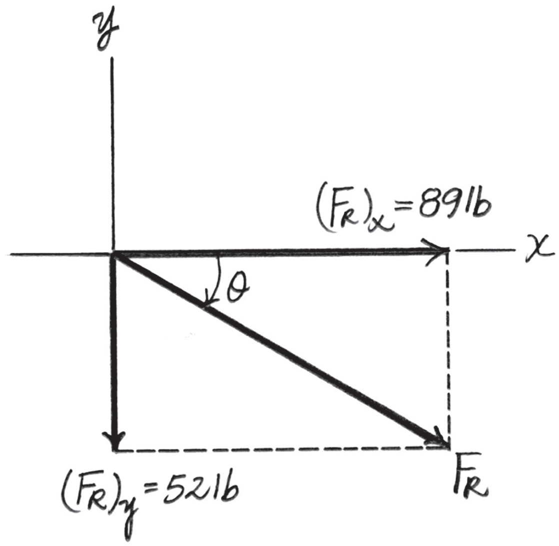
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{89^2 + 52^2} = 103.08 \text{ lb} = 103 \text{ lb} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{52}{89}\right) = 30.30° = 30.3° \qquad \textbf{Ans.}$$



(a)

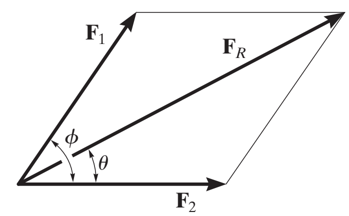


(b)

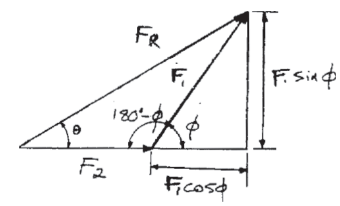
Ans:
$F_R = 103$ lb
$\theta = 30.3°$

2–37.

Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.



SOLUTION



$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180° - \phi)$$

Since $\cos(180° - \phi) = -\cos\phi$,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$$ **Ans.**

From the figure,

$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$$ **Ans.**

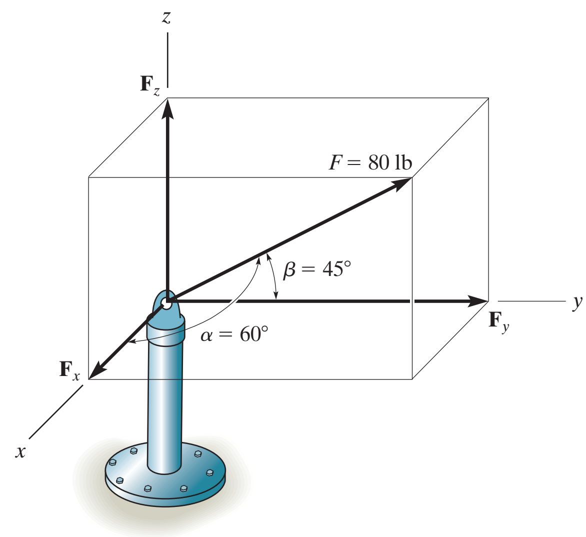
Ans:

$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$

$\theta = \tan^{-1}\left(\dfrac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$

2–38.

The force **F** has a magnitude of 80 lb. Determine the magnitudes of the *x, y, z* components of **F**.



SOLUTION

$1 = \cos^2 60° + \cos^2 45° + \cos^2 \gamma$

Solving for the positive root, $\gamma = 60°$

$F_x = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

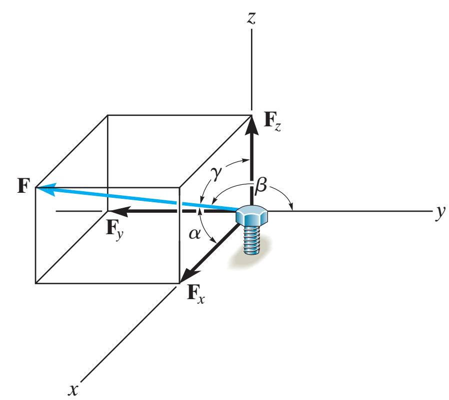
$F_y = 80 \cos 45° = 56.6 \text{ lb}$ **Ans.**

$F_z = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

Ans:
$F_x = 40.0$ lb
$F_y = 56.6$ lb
$F_z = 40.0$ lb

2–39.

The bolt is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60°$ and $\gamma = 45°$, determine the magnitudes of its components.



SOLUTION

$$\cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$\beta = 120°$$

$F_x = |80 \cos 60°| = 40 \text{ N}$ **Ans.**

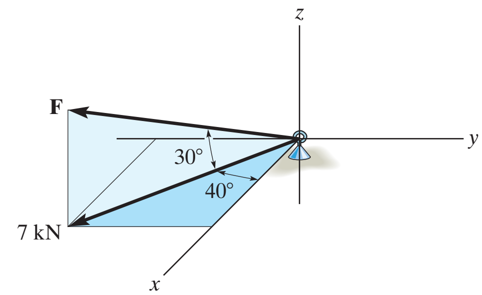
$F_y = |80 \cos 120°| = 40 \text{ N}$ **Ans.**

$F_z = |80 \cos 45°| = 56.6 \text{ N}$ **Ans.**

Ans:
$F_x = 40$ N
$F_y = 40$ N
$F_z = 56.6$ N

***2–40.**

Determine the magnitude and coordinate direction angles of the force **F** acting on the support. The component of **F** in the x–y plane is 7 kN.



SOLUTION

Coordinate Direction Angles. The unit vector of **F** is

$$\mathbf{u}_F = \cos 30^\circ \cos 40^\circ \mathbf{i} - \cos 30^\circ \sin 40^\circ \mathbf{j} + \sin 30^\circ \mathbf{k}$$

$$= \{0.6634\mathbf{i} - 0.5567\mathbf{j} + 0.5\,\mathbf{k}\}$$

Thus,

$\cos\alpha = 0.6634;\quad \alpha = 48.44^\circ = 48.4^\circ$ **Ans.**

$\cos\beta = -0.5567;\quad \beta = 123.83^\circ = 124^\circ$ **Ans.**

$\cos\gamma = 0.5;\quad \gamma = 60^\circ$ **Ans.**

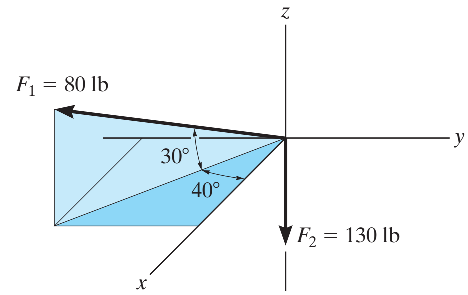
The magnitude of **F** can be determined from

$F\cos 30^\circ = 7;\quad F = 8.083\text{ kN} = 8.08\text{ kN}$ **Ans.**

Ans:
$\alpha = 48.4^\circ$
$\beta = 124^\circ$
$\gamma = 60^\circ$
$F = 8.08\text{ kN}$

2–41.

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40°\mathbf{i} - 80 \cos 30° \sin 40°\mathbf{j} + 80 \sin 30°\mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

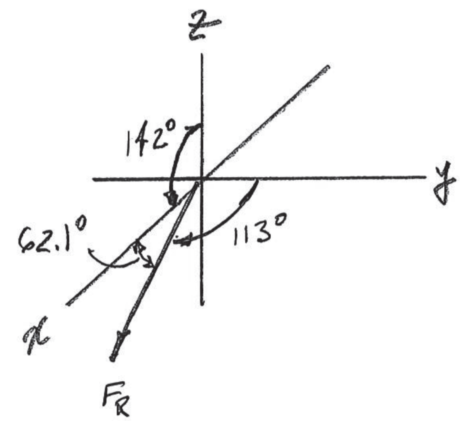
$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb

$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans.**

$\alpha = \cos^{-1}\left(\frac{53.1}{113.6}\right) = 62.1°$ **Ans.**

$\beta = \cos^{-1}\left(\frac{-44.5}{113.6}\right) = 113°$ **Ans.**

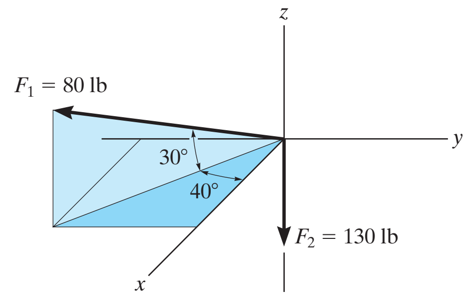
$\gamma = \cos^{-1}\left(\frac{-90.0}{113.6}\right) = 142°$ **Ans.**



Ans:
$F_R = 114$ lb
$\alpha = 62.1°$
$\beta = 113°$
$\gamma = 142°$

2–42.

Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$ **Ans.**

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb **Ans.**

$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

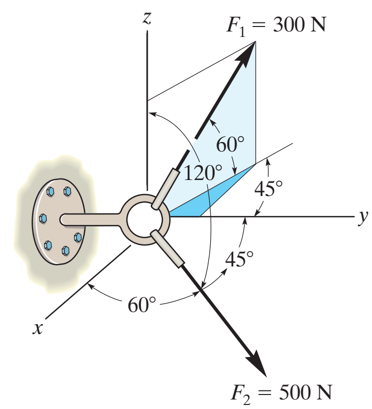
$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$ **Ans.**

Ans:
$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb
$\alpha_1 = 48.4°$
$\beta_1 = 124°$
$\gamma_1 = 60°$
$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb
$\alpha_2 = 90°$
$\beta_2 = 90°$
$\gamma_2 = 180°$

2–43.

Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$\mathbf{F}_1 = 300(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$

$= \{-106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k}\}$ N

$= \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_2 = 500(\cos 60^\circ \mathbf{i} + \cos 45^\circ \mathbf{j} + \cos 120^\circ \mathbf{k})$

$= \{250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}\}$ N

$= \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= -106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k} + 250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}$

$= 143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}$

$= \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$ N **Ans.**

$F_R = \sqrt{143.93^2 + 459.62^2 + 9.81^2} = 481.73 \text{ N} = 482 \text{ N}$ **Ans.**

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}}{481.73} = 0.2988\mathbf{i} + 0.9541\mathbf{j} + 0.02036\mathbf{k}$$

$\cos\alpha = 0.2988$ $\alpha = 72.6^\circ$ **Ans.**

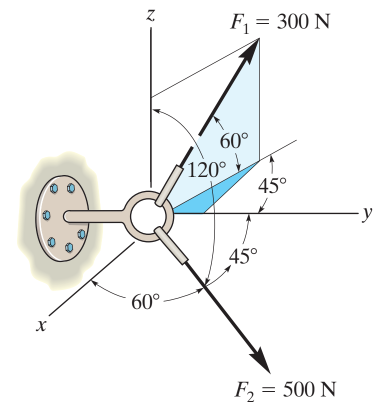
$\cos\beta = 0.9541$ $\beta = 17.4^\circ$ **Ans.**

$\cos\gamma = 0.02036$ $\gamma = 88.8^\circ$ **Ans.**

Ans:
$\mathbf{F}_1 = \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N
$\mathbf{F}_2 = \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$ N
$\mathbf{F}_R = \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$ N
$F_R = 482$ N
$\alpha = 72.6^\circ$
$\beta = 17.4^\circ$
$\gamma = 88.8^\circ$

***2–44.**

Determine the coordinate direction angles of $\mathbf{F}_1$.



SOLUTION

$$\mathbf{F}_1 = 300(-\cos 60^\circ \sin 45^\circ\,\mathbf{i} + \cos 60^\circ \cos 45^\circ\,\mathbf{j} + \sin 60^\circ\,\mathbf{k})$$

$$= \{-106.07\,\mathbf{i} + 106.07\,\mathbf{j} + 259.81\,\mathbf{k}\}\text{ N}$$

$$= \{-106\,\mathbf{i} + 106\,\mathbf{j} + 260\,\mathbf{k}\}\text{ N}$$

$$\mathbf{u}_1 = \frac{\mathbf{F}_1}{300} = -0.3536\,\mathbf{i} + 0.3536\,\mathbf{j} + 0.8660\,\mathbf{k}$$

$\alpha_1 = \cos^{-1}(-0.3536) = 111^\circ$ **Ans.**

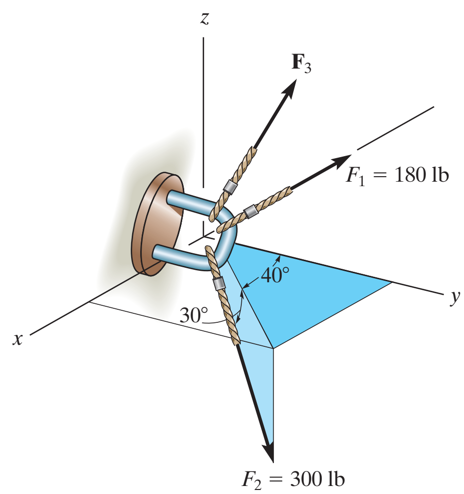
$\beta_1 = \cos^{-1}(0.3536) = 69.3^\circ$ **Ans.**

$\gamma_1 = \cos^{-1}(0.8660) = 30.0^\circ$ **Ans.**

Ans:
$\alpha_1 = 111^\circ$
$\beta_1 = 69.3^\circ$
$\gamma_1 = 30.0^\circ$

2–45.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600 lb.



SOLUTION

$F_{Rx} = \Sigma F_x\ ; \qquad 0 = -180 + 300 \cos 30° \sin 40° + F_3 \cos \alpha$

$F_{Ry} = \Sigma F_y\ ; \qquad 600 = 300 \cos 30° \cos 40° + F_3 \cos \beta$

$F_{Rz} = \Sigma F_z\ ; \qquad 0 = -300 \sin 30° + F_3 \cos \gamma$

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

Solving:

$F_3 = 428$ lb **Ans.**

$\alpha = 88.3°$ **Ans.**

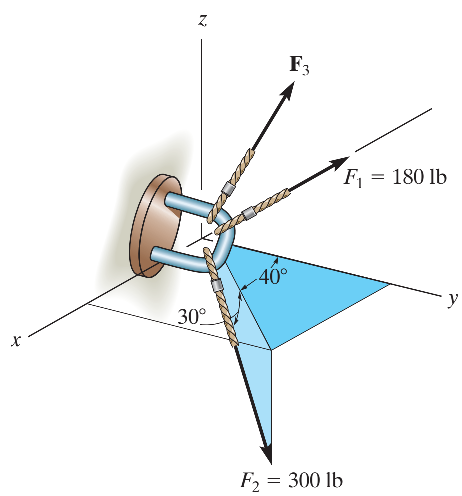
$\beta = 20.6°$ **Ans.**

$\gamma = 69.5°$ **Ans.**

Ans:
$F_3 = 428$ lb
$\alpha = 88.3°$
$\beta = 20.6°$
$\gamma = 69.5°$

2–46.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



SOLUTION

$F_{Rx} = \Sigma F_x; \quad 0 = -180 + 300 \cos 30° \sin 40° + F_3 \cos \alpha$

$F_{Ry} = \Sigma F_y; \quad 0 = 300 \cos 30° \cos 40° + F_3 \cos \beta$

$F_{Rz} = \Sigma F_z; \quad 0 = -300 \sin 30° + F_3 \cos \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Solving:

$F_3 = 250$ lb **Ans.**

$\alpha = 87.0°$ **Ans.**

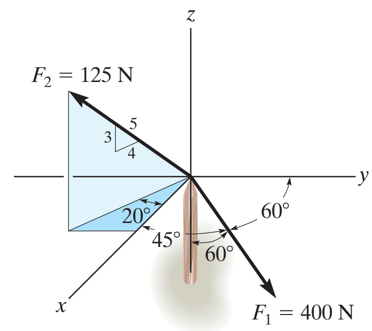
$\beta = 143°$ **Ans.**

$\gamma = 53.1°$ **Ans.**

Ans:
$F_3 = 250$ lb
$\alpha = 87.0°$
$\beta = 143°$
$\gamma = 53.1°$

2–47.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 400\,(\cos 45°\mathbf{i} + \cos 60°\mathbf{j} - \cos 60°\mathbf{k}) = \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 125\left[\frac{4}{5}(\cos 20°)\mathbf{i} - \frac{4}{5}(\sin 20°)\mathbf{j} + \frac{3}{5}\mathbf{k}\right] = \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\} + \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$

$$= \{376.81\mathbf{i} + 165.80\mathbf{j} - 125.00\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{376.81^2 + 165.80^2 + (-125.00)^2}$$

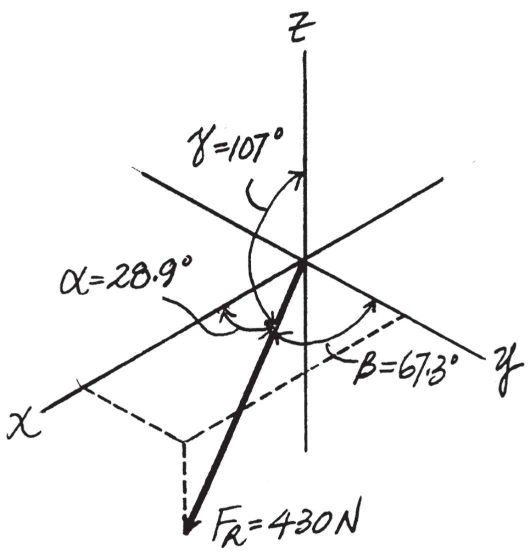
$$= 430.23\text{ N} = 430\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{376.81}{430.23}; \qquad \alpha = 28.86° = 28.9° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{165.80}{430.23}; \qquad \beta = 67.33° = 67.3° \qquad \textbf{Ans.}$$

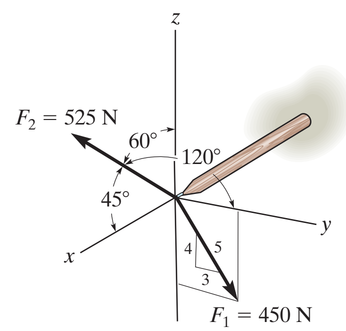
$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-125.00}{430.23}; \qquad \gamma = 106.89° = 107° \qquad \textbf{Ans.}$$



Ans:
$F_R = 430$ N
$\alpha = 28.9°$
$\beta = 67.3°$
$\gamma = 107°$

***2–48.**

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 450\left(\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}\right) = \{270\mathbf{j} - 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 525\,(\cos 45°\mathbf{i} + \cos 120°\mathbf{j} + \cos 60°\mathbf{k}) = \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}\text{ N}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= \{270\mathbf{j} - 360\mathbf{k}\} + \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}$$
$$= \{371.23\mathbf{i} + 7.50\mathbf{j} - 97.5\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

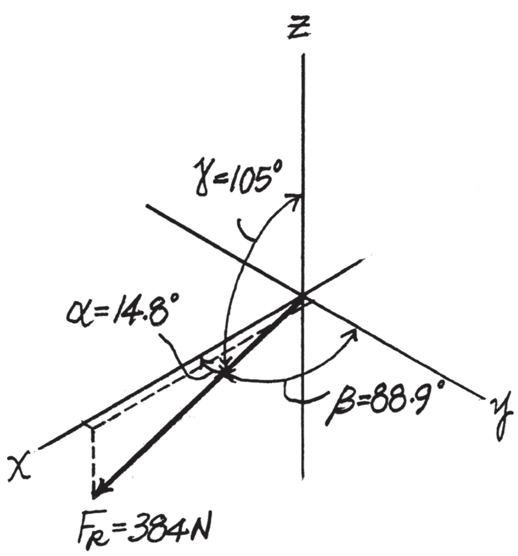
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{371.23^2 + 7.50^2 + (-97.5)^2}$$
$$= 383.89\text{ N} = 384\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{371.23}{383.89}; \qquad \alpha = 14.76° = 14.8° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{7.50}{383.89}; \qquad \beta = 88.88° = 88.9° \qquad \textbf{Ans.}$$

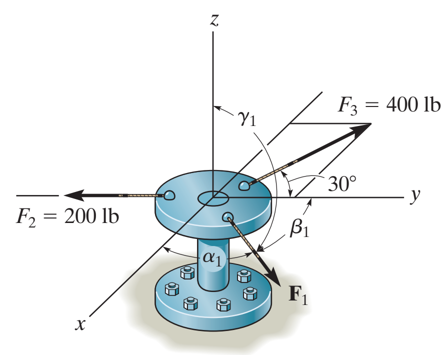
$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-97.5}{383.89}; \qquad \gamma = 104.71° = 105° \qquad \textbf{Ans.}$$



Ans:
$F_R = 384$ N
$\alpha = 14.8°$
$\beta = 88.9°$
$\gamma = 105°$

2–49.

Determine the magnitude and coordinate direction angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant of the three forces acting on the bracket is $\mathbf{F}_R = \{-350\mathbf{k}\}$ lb.



SOLUTION

$$\mathbf{F}_1 = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

$$\mathbf{F}_2 = -200\,\mathbf{j}$$

$$\mathbf{F}_3 = -400\sin 30^\circ\,\mathbf{i} + 400\cos 30^\circ\,\mathbf{j}$$

$$= -200\,\mathbf{i} + 346.4\,\mathbf{j}$$

$$\mathbf{F}_R = \Sigma\mathbf{F}$$

$$-350\,\mathbf{k} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k} - 200\,\mathbf{j} - 200\,\mathbf{i} + 346.4\,\mathbf{j}$$

$$0 = F_x - 200\,; \qquad F_x = 200 \text{ lb}$$

$$0 = F_y - 200 + 346.4\,; \qquad F_y = -146.4 \text{ lb}$$

$$F_z = -350 \text{ lb}$$

$$F_1 = \sqrt{(200)^2 + (-146.4)^2 + (-350)^2}$$

$$F_1 = 425.9 \text{ lb} = 429 \text{ lb}$$ **Ans.**

$$\alpha_1 = \cos^{-1}\left(\frac{200}{428.9}\right) = 62.2^\circ$$ **Ans.**

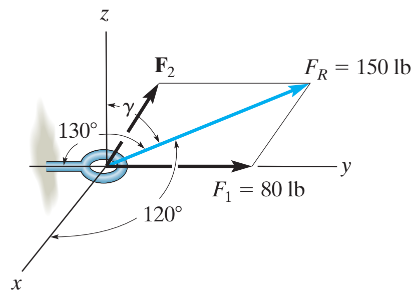
$$\beta_1 = \cos^{-1}\left(\frac{-146.4}{428.9}\right) = 110^\circ$$ **Ans.**

$$\gamma_1 = \cos^{-1}\left(\frac{-350}{428.9}\right) = 145^\circ$$ **Ans.**

Ans:
$F_1 = 429$ lb
$\alpha_1 = 62.2^\circ$
$\beta_1 = 110^\circ$
$\gamma_1 = 145^\circ$

2–50.

If the resultant force $\mathbf{F}_R$ has a magnitude of 150 lb and the coordinate direction angles shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_R$, γ can be determined from

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 120° + \cos^2 50° + \cos^2\gamma = 1$$

$$\cos\gamma = \pm 0.5804$$

Here $\gamma < 90°$, then

$$\gamma = 54.52°$$

Thus

$$\mathbf{F}_R = 150(\cos 120°\mathbf{i} + \cos 50°\mathbf{j} + \cos 54.52°\mathbf{k})$$

$$= \{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Also

$$\mathbf{F}_1 = \{80\mathbf{j}\}\text{ lb}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\} = \{80\mathbf{j}\} + \mathbf{F}_2$$

$$F_2 = \{-75.0\mathbf{i} + 16.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Thus, the magnitude of $\mathbf{F}_2$ is

$$F_2 = \sqrt{(F_2)_x + (F_2)_y + (F_2)_z} = \sqrt{(-75.0)^2 + 16.42^2 + 87.05^2}$$

$$= 116.07\text{ lb} = 116\text{ lb} \qquad \textbf{Ans.}$$

And its coordinate direction angles are

$$\cos\alpha_2 = \frac{(F_2)_x}{F_2} = \frac{-75.0}{116.07}; \qquad \alpha_2 = 130.25° = 130° \qquad \textbf{Ans.}$$

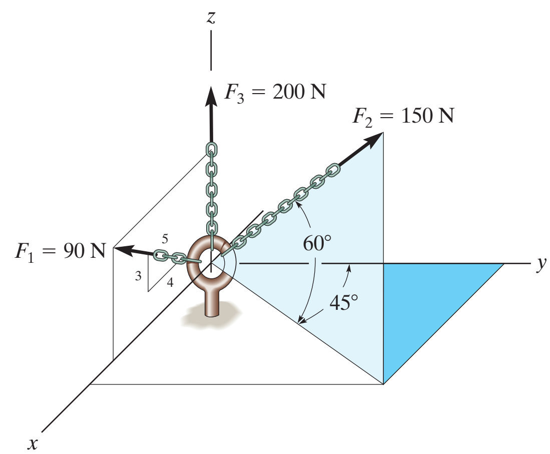
$$\cos\beta_2 = \frac{(F_2)_y}{F_2} = \frac{16.42}{116.07}; \qquad \beta_2 = 81.87° = 81.9° \qquad \textbf{Ans.}$$

$$\cos\gamma_2 = \frac{(F_2)_z}{F_2} = \frac{87.05}{116.07}; \qquad \gamma_2 = 41.41° = 41.4° \qquad \textbf{Ans.}$$

Ans:
$F_2 = 116$ lb
$\alpha_2 = 130°$
$\beta_2 = 81.9°$
$\gamma_2 = 41.4°$

2–51.

Express each force as a Cartesian vector.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 150\,(\cos 60^\circ \sin 45^\circ\mathbf{i} + \cos 60^\circ \cos 45^\circ\mathbf{j} + \sin 60^\circ\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

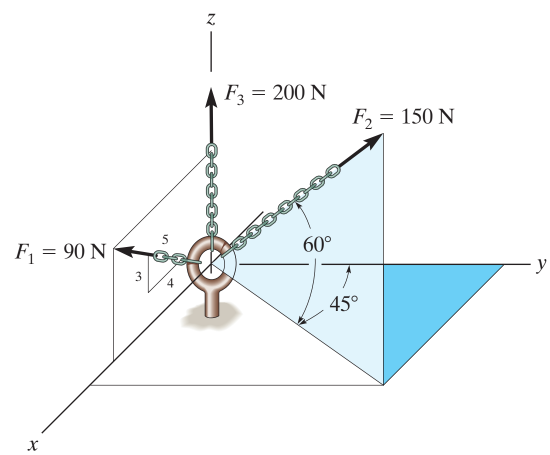
$$= \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_3 = \{200\,\mathbf{k}\} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{72.0\mathbf{i} + 54.0\mathbf{k}\}$ N
$\mathbf{F}_2 = \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\}$ N
$\mathbf{F}_3 = \{200\,\mathbf{k}\}$

***2–52.**

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 150\,(\cos 60°\sin 45°\mathbf{i} + \cos 60°\cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

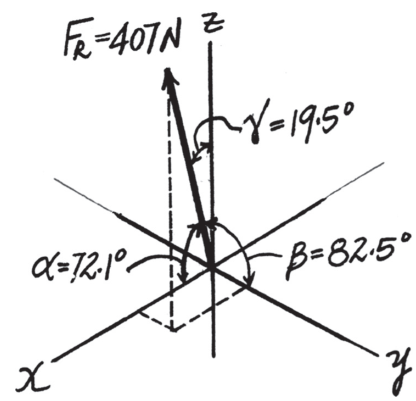
$$\mathbf{F}_3 = \{200\text{ k}\}\text{ N}$$

Resultant Force.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (72.0\mathbf{i} + 54.0\mathbf{k}) + (53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}) + (200\mathbf{k})$$

$$= \{125.03\mathbf{i} + 53.03\mathbf{j} + 383.90\}\text{ N}$$



The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{125.03^2 + 53.03^2 + 383.90^2}$$

$$= 407.22\text{ N} = 407\text{ N} \qquad \textbf{Ans.}$$

And the coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{125.03}{407.22};\qquad \alpha = 72.12° = 72.1° \qquad \textbf{Ans.}$$

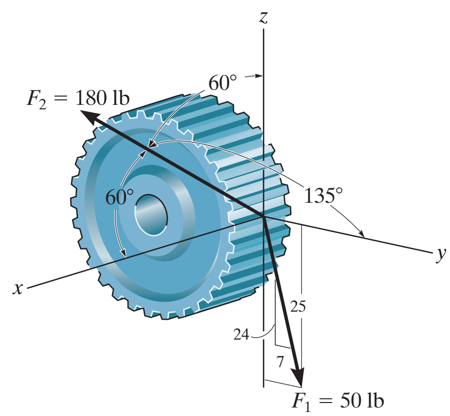
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{53.03}{407.22};\qquad \beta = 82.52° = 82.5° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{383.90}{407.22};\qquad \gamma = 19.48° = 19.5° \qquad \textbf{Ans.}$$

Ans:
$F_R = 407$ N
$\alpha = 72.1°$
$\beta = 82.5°$
$\gamma = 19.5°$

2–53.

The spur gear is subjected to the two forces. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\}\text{ lb}$$ **Ans.**

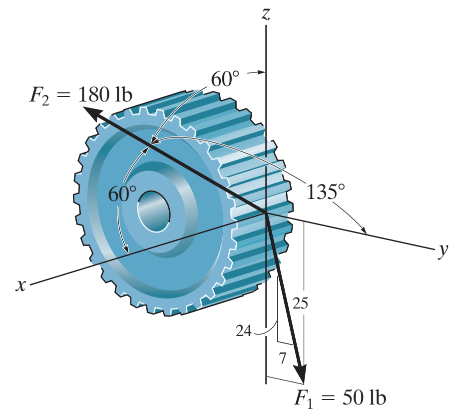
$$\mathbf{F}_2 = 180\cos 60^\circ\mathbf{i} + 180\cos 135^\circ\mathbf{j} + 180\cos 60^\circ\mathbf{k}$$

$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{14.0\mathbf{j} - 48.0\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}$ lb

2–54.

The spur gear is subjected to the two forces. Determine the resultant of the two forces and express the result as a Cartesian vector.



SOLUTION

$$F_{Rx} = 180\cos 60° = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180\cos 135° = -113$$

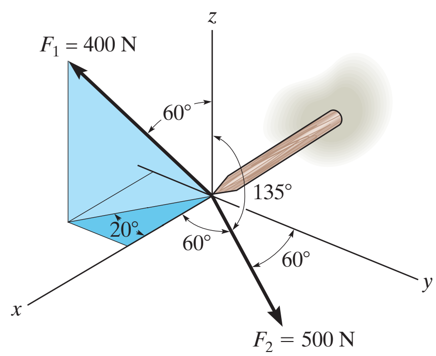
$$F_{Rz} = -\frac{24}{25}(50) + 180\cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}$ lb

2–55.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

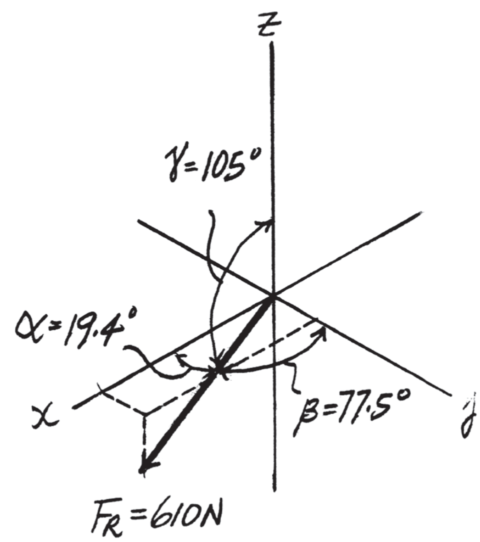
Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 400\,(\sin 60°\cos 20°\mathbf{i} - \sin 60°\sin 20°\mathbf{j} + \cos 60°\mathbf{k})$$

$$= \{325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 500\,(\cos 60°\mathbf{i} + \cos 60°\mathbf{j} + \cos 135°\mathbf{k})$$

$$= \{250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k}\}\text{ N}$$



Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}) + (250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k})$$

$$= \{575.52\mathbf{i} + 131.52\mathbf{j} - 153.55\,\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{575.52^2 + 131.52^2 + (-153.55)^2}$$

$$= 610.00\text{ N} = 610\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{575.52}{610.00} \qquad \alpha = 19.36° = 19.4° \qquad \textbf{Ans.}$$

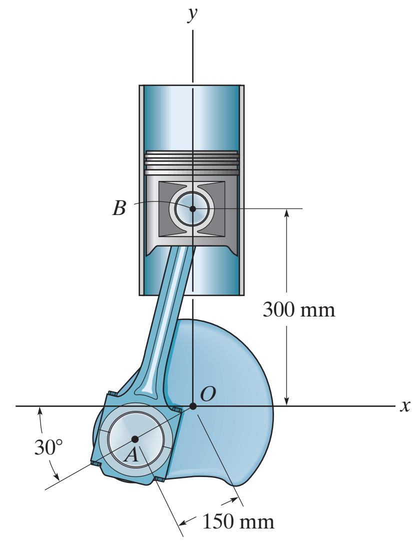
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{131.52}{610.00} \qquad \beta = 77.549° = 77.5° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-153.55}{610.00} \qquad \gamma = 104.58° = 105° \qquad \textbf{Ans.}$$

Ans:
$F_R = 610$ N
$\alpha = 19.4°$
$\beta = 77.5°$
$\gamma = 105°$

***2–56.**

Determine the length of the connecting rod AB by first formulating a position vector from A to B and then determining its magnitude.



SOLUTION

Position Vector. The coordinates of points A and B are $A(-150\cos 30°, -150\sin 30°)$ mm and $B(0, 300)$ mm respectively. Then

$$\mathbf{r}_{AB} = [0 - (-150\cos 30°)]\mathbf{i} + [300 - (-150\sin 30°)]\mathbf{j}$$

$$= \{129.90\mathbf{i} + 375\mathbf{j}\}\text{ mm}$$

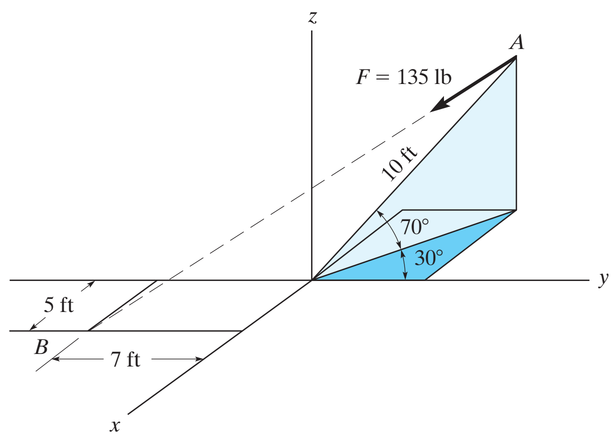
Thus, the magnitude of $\mathbf{r}_{AB}$ is

$$r_{AB} = \sqrt{129.90^2 + 375^2} = 396.86\text{ mm} = 397\text{ mm}$$ **Ans.**

Ans:
$r_{AB} = 397$ mm

2–57.

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



SOLUTION

$$\mathbf{r}_{AB} = (5 + 10\cos 70° \sin 30°)\mathbf{i}$$
$$+ (-7 - 10\cos 70° \cos 30°)\mathbf{j} - 10\sin 70°\mathbf{k}$$

$$\mathbf{r}_{AB} = \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(6.710)^2 + (-9.962)^2 + (-9.397)^2} = 15.25$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = (0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k})$$

$$\mathbf{F} = 135\mathbf{u}_{AB} = (59.40\mathbf{i} - 88.18\mathbf{j} - 83.18\mathbf{k})$$

$$= \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{59.40}{135}\right) = 63.9°$$ **Ans.**

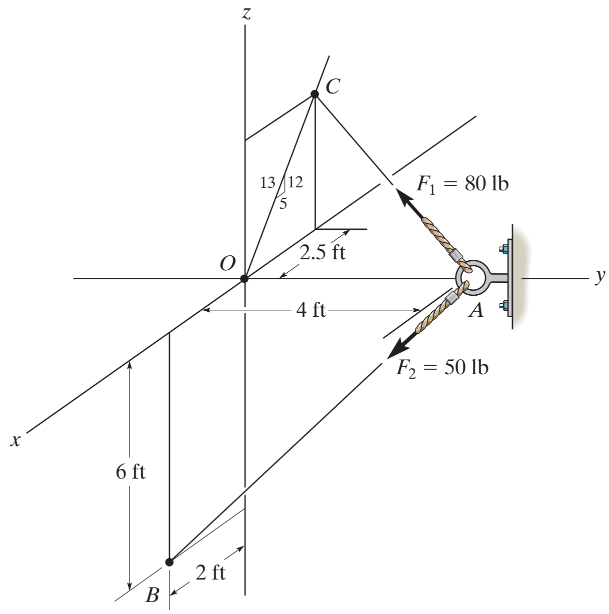
$$\beta = \cos^{-1}\left(\frac{-88.18}{135}\right) = 131°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-83.18}{135}\right) = 128°$$ **Ans.**

Ans:
$\mathbf{F} = \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}$ lb
$\alpha = 63.9°$
$\beta = 131°$
$\gamma = 128°$

2–58.

Express each force as a Cartesian vector, and then determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{r}_{AC} = \left\{-2.5\,\mathbf{i} - 4\,\mathbf{j} + \frac{12}{5}(2.5)\,\mathbf{k}\right\}\text{ ft}$$

$$\mathbf{F}_1 = 80\text{ lb}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = -26.20\,\mathbf{i} - 41.93\,\mathbf{j} + 62.89\,\mathbf{k}$$

$$= \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{r}_{AB} = \{2\,\mathbf{i} - 4\,\mathbf{j} - 6\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_2 = 50\text{ lb}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 13.36\,\mathbf{i} - 26.73\,\mathbf{j} - 40.09\,\mathbf{k}$$

$$= \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -12.84\,\mathbf{i} - 68.65\,\mathbf{j} + 22.80\,\mathbf{k}$$

$$= \{-12.8\,\mathbf{i} - 68.7\,\mathbf{j} + 22.8\,\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{(-12.84)^2\,(-68.65)^2 + (22.80)^2} = 73.47 = 73.5\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-12.84}{73.47}\right) = 100^\circ$$ **Ans.**

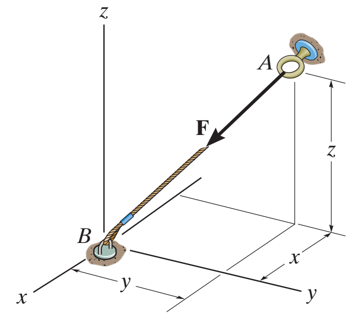
$$\beta = \cos^{-1}\left(\frac{-68.65}{73.47}\right) = 159^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{22.80}{73.47}\right) = 71.9^\circ$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}$ lb
$F_R = 73.5$ lb
$\alpha = 100^\circ$
$\beta = 159^\circ$
$\gamma = 71.9^\circ$

2–59.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - x]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= -x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = -0.5623\mathbf{i} \quad 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$\frac{x}{9} = -0.5623 \qquad x = -5.06 \text{ m} \qquad \textbf{Ans.}$$

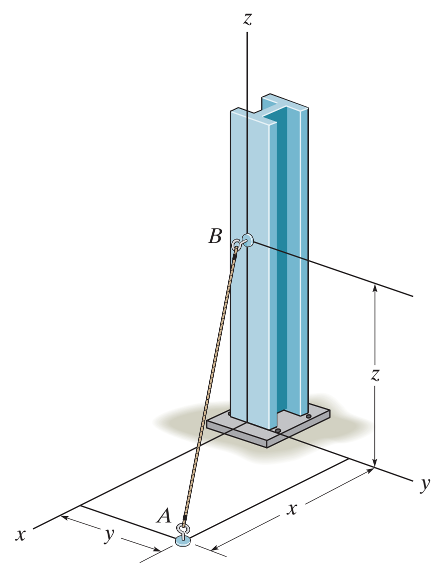
$$\frac{-y}{9} = -0.4016 \qquad y = 3.61 \text{ m} \qquad \textbf{Ans.}$$

$$\frac{-z}{9} = 0.7229 \qquad z = 6.51 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$x = -5.06$ m
$y = 3.61$ m
$z = 6.51$ m

***2–60.**

The 8-m-long cable is anchored to the ground at A. If $x = 4$ m and $y = 2$ m, determine the coordinate z to the highest point of attachment along the column.



SOLUTION

$$\mathbf{r} = \{4\mathbf{i} + 2\mathbf{j} + z\mathbf{k}\} \text{ m}$$

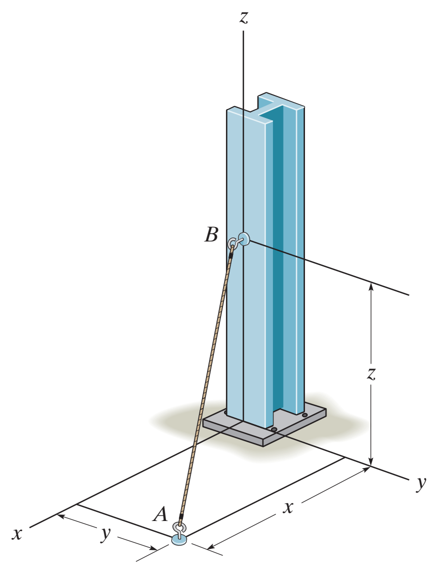
$$r = \sqrt{(4)^2 + (2)^2 + (z)^2} = 8$$

$$z = 6.63 \text{ m}$$ **Ans.**

Ans:
6.63 m

2–61.

The 8-m-long cable is anchored to the ground at A. If $z = 5$ m, determine the location $+x, +y$ of the support at A. Choose a value such that $x = y$.



SOLUTION

$$\mathbf{r} = \{x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}\}\text{ m}$$

$$r = \sqrt{(x)^2 + (y)^2 + (5)^2} = 8$$

$x = y$, thus

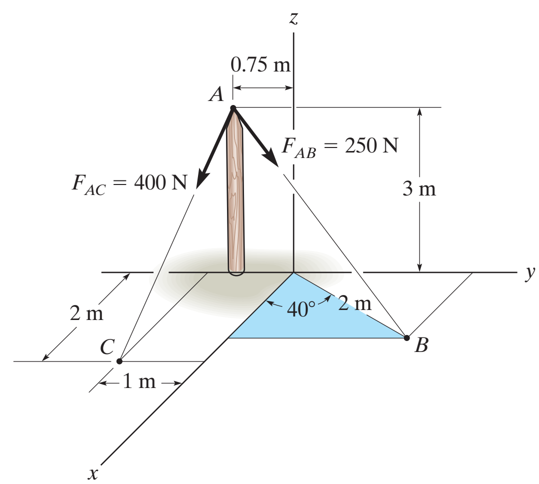
$$2x^2 = 8^2 - 5^2$$

$$x = y = 4.42\text{ m}$$ **Ans.**

Ans:
$x = y = 4.42$ m

2–62.

Express each of the forces in Cartesian vector form and then determine the magnitude and coordinate direction angles of the resultant force



SOLUTION

Unit Vectors. The coordinates for points A, B and C are $(0, -0.75, 3)$ m, $B(2\cos 40°, 2\sin 40°, 0)$ m and $C(2, -1, 0)$ m, respectively.

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(2\cos 40° - 0)\mathbf{i} + [2\sin 40° - (-0.75)]\mathbf{j} + (0 - 3)\mathbf{k}}{\sqrt{(2\cos 40° - 0)^2 + [2\sin 40° - (-0.75)]^2 + (0 - 3)^2}}$$

$$= 0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(2 - 0)\mathbf{i} + [-1 - (-0.75)]\mathbf{j} + (0 - 3)\mathbf{k}}{\sqrt{(2 - 0)^2 + [-1 - (-0.75)]^2 + (0 - 3)^2}}$$

$$= 0.5534\mathbf{i} - 0.0692\mathbf{j} - 0.8301\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 250\,(0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k})$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\}\text{ N}$$

$$= \{97.3\mathbf{i} + 129\mathbf{j} - 191\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 400\,(0.5534\mathbf{i} - 0.06917\mathbf{j} - 0.8301\mathbf{k})$$

$$= \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}\text{ N}$$

$$= \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}\text{ N}$$ **Ans.**

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\} + \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}$$

$$= \{318.67\mathbf{i} + 101.63\mathbf{j} - 522.58\,\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{318.67^2 + 101.63^2 + (-522.58)^2}$$

$$= 620.46\text{ N} = 620\text{ N}$$

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{318.67}{620.46}; \qquad \alpha = 59.10° = 59.1°$$ **Ans.**

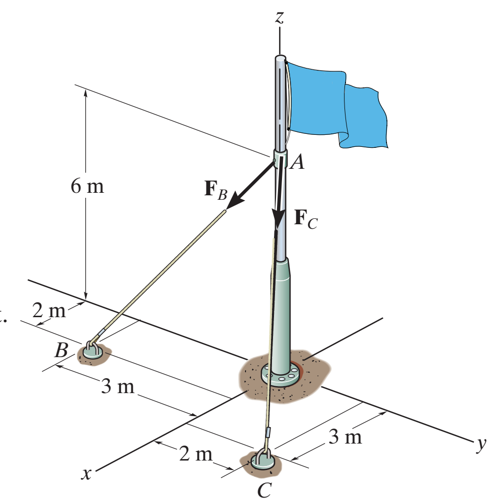
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{101.63}{620.46}; \qquad \beta = 80.57° = 80.6°$$ **Ans.**

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-522.58}{620.46}; \qquad \gamma = 147.38° = 147°$$ **Ans.**

Ans:
$\mathbf{F}_{AB} = \{97.3\mathbf{i} - 129\mathbf{j} - 191\mathbf{k}\}$ N
$\mathbf{F}_{AC} = \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}$ N
$F_R = 620$ N
$\alpha = 59.1°$
$\beta = 80.6°$
$\gamma = 147°$

2–63.

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 560\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 700\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k})$$

$$= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

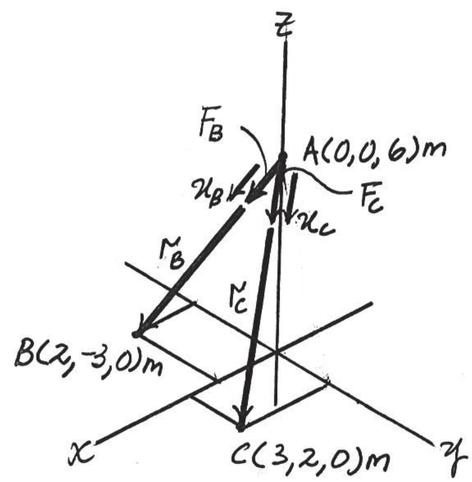
$$= \sqrt{(460)^2 + (-40)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN} \quad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9^\circ \quad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0^\circ \quad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157^\circ \quad \textbf{Ans.}$$

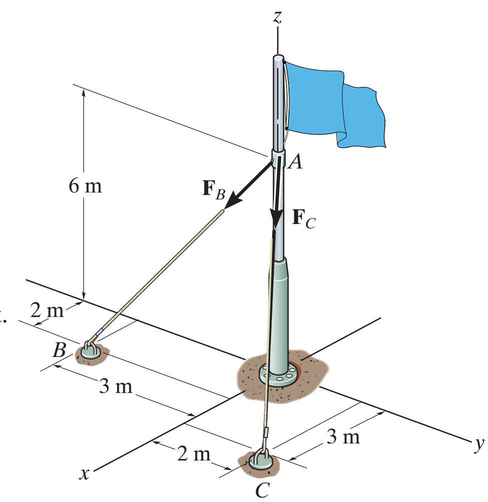


(a)

Ans:
$F_R = 1.17$ kN
$\alpha = 66.9^\circ$
$\beta = 92.0^\circ$
$\gamma = 157^\circ$

***2–64.**

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

Resultant Force:

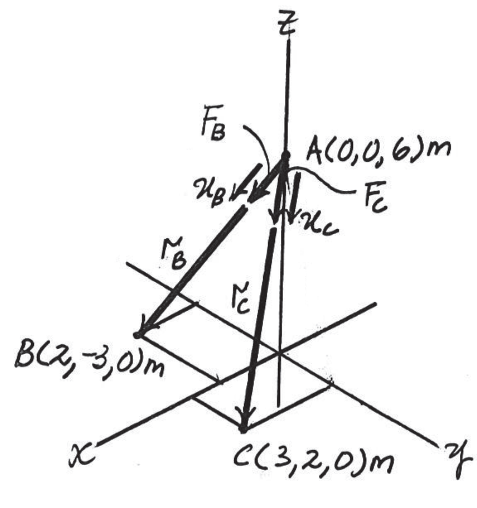
$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$

$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN} \qquad \textbf{Ans.}$$



(a)

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0° \qquad \textbf{Ans.}$$

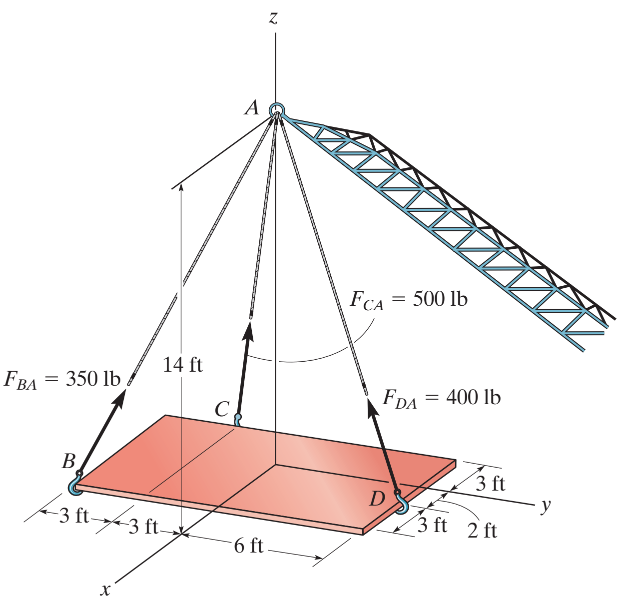
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157° \qquad \textbf{Ans.}$$

Ans:
$F_R = 1.17$ kN
$\alpha = 68.0°$
$\beta = 96.8°$
$\gamma = 157°$

2–65.

The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_{BA} = 350\left(\frac{\mathbf{r}_{BA}}{r_{BA}}\right) = 350\left(-\frac{5}{16.031}\mathbf{i} + \frac{6}{16.031}\mathbf{j} + \frac{14}{16.031}\mathbf{k}\right)$$

$$= \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_{CA} = 500\left(\frac{\mathbf{r}_{CA}}{r_{CA}}\right) = 500\left(\frac{3}{14.629}\mathbf{i} + \frac{3}{14.629}\mathbf{j} + \frac{14}{14.629}\mathbf{k}\right)$$

$$= \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}\text{ lb}$$ **Ans.**

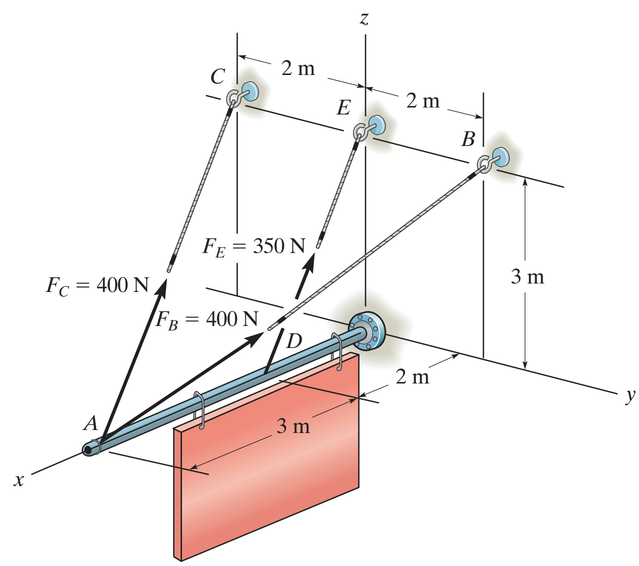
$$\mathbf{F}_{DA} = 400\left(\frac{\mathbf{r}_{DA}}{r_{DA}}\right) = 400\left(-\frac{2}{15.362}\mathbf{i} - \frac{6}{15.362}\mathbf{j} + \frac{14}{15.362}\mathbf{k}\right)$$

$$= \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_{BA} = \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}$ lb
$\mathbf{F}_{CA} = \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}$ lb
$\mathbf{F}_{DA} = \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}$ lb

2–66.

Represent each cable force as a Cartesian vector.



SOLUTION

$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$ m

$r_C = \sqrt{(-5)^2 + (-2)^2 + 3^2} = \sqrt{38}$ m

$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}$ m

$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}$ m

$\mathbf{r}_E = (0 - 2)\mathbf{i} + (0 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}\}$ m

$r_E = \sqrt{(-2)^2 + 0^2 + 3^2} = \sqrt{13}$ m

$$\mathbf{F} = F_{\mathbf{u}} = F\left(\frac{\mathbf{r}}{r}\right)$$

$$\mathbf{F}_C = 400\left(\frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}\text{ N}$$ **Ans.**

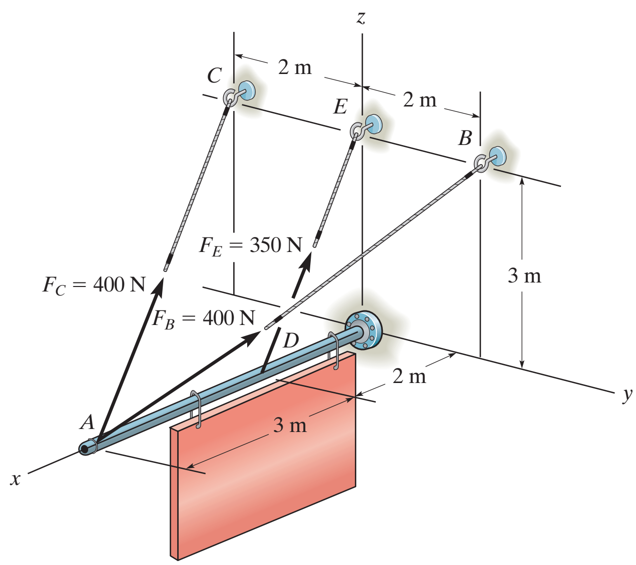
$$\mathbf{F}_B = 400\left(\frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_E = 350\left(\frac{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}}{\sqrt{13}}\right) = \{-194\mathbf{i} + 291\mathbf{k}\}\text{ N}$$ **Ans.**

Ans:
$\mathbf{F}_C = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_B = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_E = \{-194\mathbf{i} + 291\mathbf{k}\}$ N

2–67.

Determine the magnitude and coordinate direction angles of the resultant force of the two forces acting at point A.



SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + (3)^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_C = 400\left(\frac{\mathbf{r}_C}{r_C}\right) = 400\left(\frac{(-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_C = (-324.4428\mathbf{i} - 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_B = 400\left(\frac{\mathbf{r}_B}{r_B}\right) = 400\left(\frac{(-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_B = (-324.443\mathbf{i} + 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_C + \mathbf{F}_B = (-648.89\mathbf{i} + 389.33\mathbf{k})$$

$$F_R = \sqrt{(-648.89)^2 + (389.33)^2 + 0^2} = 756.7242$$

$F_R = 757$ N **Ans.**

$\alpha = \cos^{-1}\left(\frac{-648.89}{756.7242}\right) = 149.03 = 149°$ **Ans.**

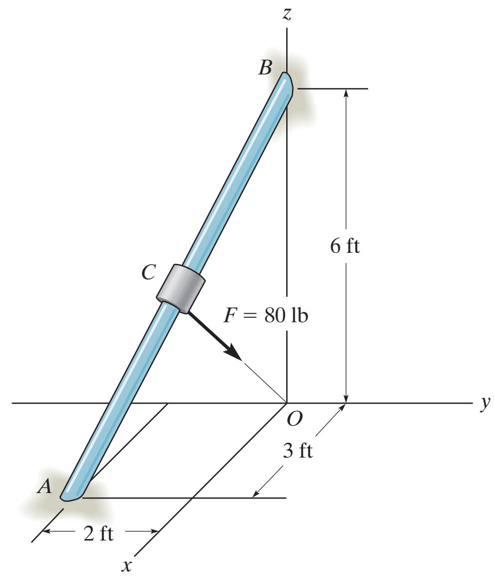
$\beta = \cos^{-1}\left(\frac{0}{756.7242}\right) = 90.0°$ **Ans.**

$\gamma = \cos^{-1}\left(\frac{389.33}{756.7242}\right) = 59.036 = 59.0°$ **Ans.**

Ans:
$F_R = 757$ N
$\alpha = 149°$
$\beta = 90.0°$
$\gamma = 59.0°$

***2–68.**

The force **F** has a magnitude of 80 lb and acts at the midpoint *C* of the rod. Express this force as a Cartesian vector.



SOLUTION

$\mathbf{r}_{AB} = (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$

$\mathbf{r}_{CB} = \frac{1}{2}\mathbf{r}_{AB} = (-1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k})$

$\mathbf{r}_{CO} = \mathbf{r}_{BO} + \mathbf{r}_{CB}$

$= -6\mathbf{k} - 1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k}$

$= -1.5\mathbf{i} + 1\mathbf{j} - 3\mathbf{k}$

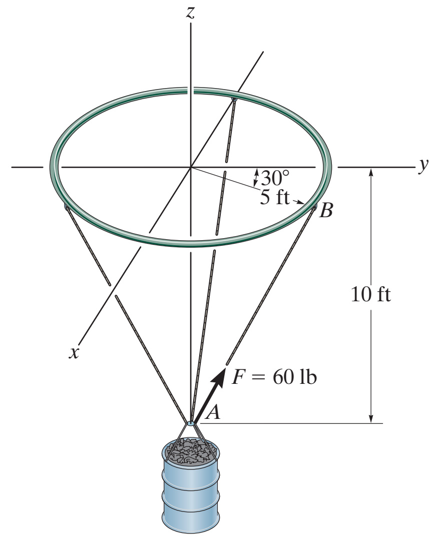
$r_{CO} = 3.5$

$F = 80\left(\frac{\mathbf{r}_{CO}}{r_{CO}}\right) = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}$ lb **Ans.**

Ans:
$F = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}$ lb

2–69.

The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector.



SOLUTION

Unit Vector: First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

$B\,(5 \sin 30°, 5 \cos 30°, 0)\text{ ft} = B\,(2.50, 4.330, 0)\text{ ft}$

Then

$$\mathbf{r}_{AB} = \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\}\text{ ft}$$

$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

Force Vector:

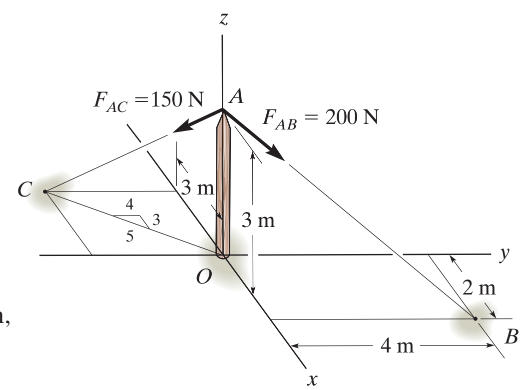
$$\mathbf{F} = F\mathbf{u}_{AB} = 60\,\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\}\text{ lb}$$

$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F} = \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\text{ lb}$

2–70.

Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



SOLUTION

Unit Vector. The coordinates for points A, B and C are $A(0, 0, 3)$ m, $B(2, 4, 0)$ m, and $C(-3, -4, 0)$ m, respectively.

$$\mathbf{r}_{AB} = (2 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = \{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 4^2 + (-3)^2}} = \frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}$$

$$\mathbf{r}_{AC} = (-3 - 0)\mathbf{i} + (-4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (-3)^2}} = -\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 200\left(\frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}\right)$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 150\left(-\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}\right)$$

$$= \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}\text{ N}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} + \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}$$

$$= \{-2.896\mathbf{i} + 45.66\mathbf{j} - 188.59\,\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(-2.896)^2 + 45.66^2 + (-188.59)^2}$$

$$= 194.06\text{ N} = 194\text{ N}$$ **Ans.**

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{-2.896}{194.06}; \quad \alpha = 90.86^\circ = 90.9^\circ$$ **Ans.**

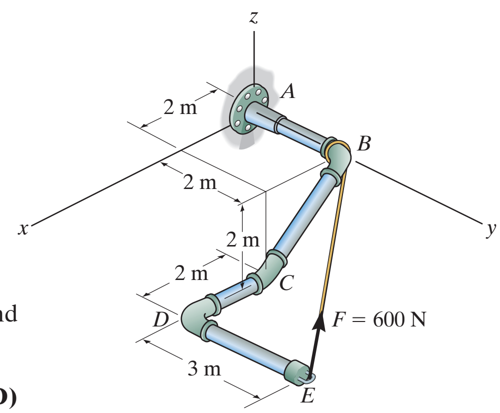
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{45.66}{194.06}; \quad \beta = 76.39^\circ = 76.4^\circ$$ **Ans.**

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-188.59}{194.06}; \quad \gamma = 166.36^\circ = 166^\circ$$ **Ans.**

Ans:
$F_R = 194$ N
$\alpha = 90.9^\circ$
$\beta = 76.4^\circ$
$\gamma = 166^\circ$

2–71.

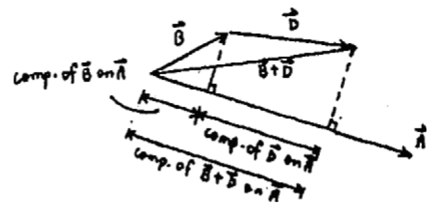
Given the three vectors **A**, **B**, and **D**, show that **A** · (**B** + **D**) = (**A** · **B**) + (**A** · **D**).



SOLUTION

Since the component of (**B** + **D**) is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \textbf{(QED)}$$



Also,

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

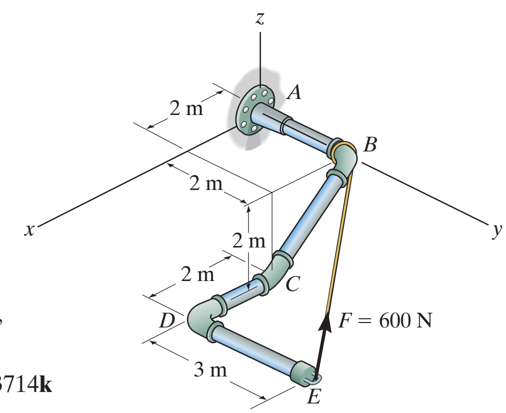
$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \textbf{(QED)}$$

***2–72.**

Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600\left(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}\right) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the component of **F** parallel to segment DE of the pipe assembly is

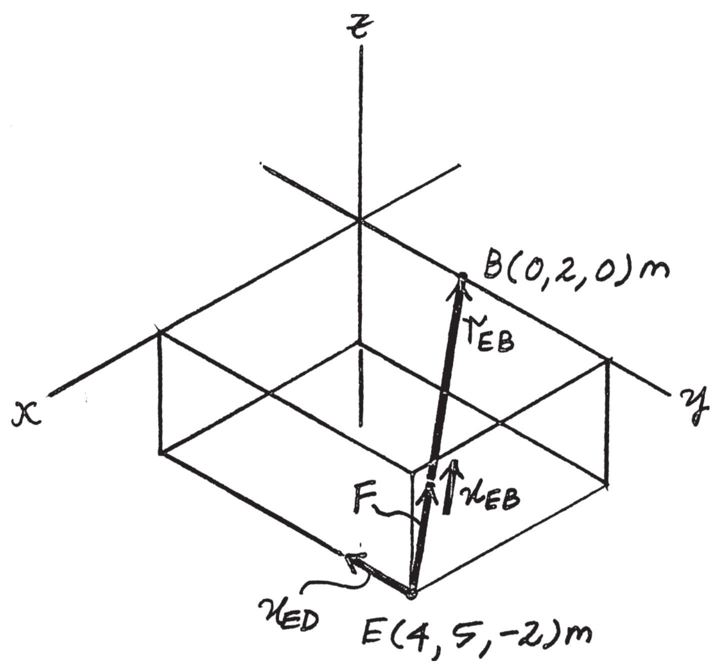
$$(F_{ED})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{ED} = \left(-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}\right)\cdot\left(-\mathbf{j}\right)$$

$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$

$$= 334.25 = 334\text{ N}$$ **Ans.**

The component of **F** perpendicular to segment DE of the pipe assembly is

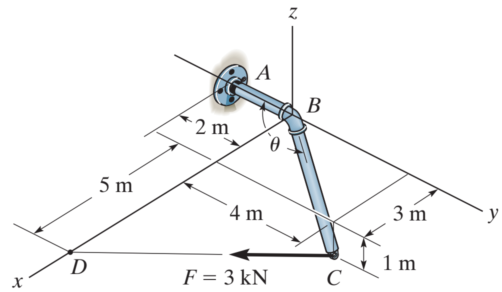
$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}{}^2} = \sqrt{600^2 - 334.25^2} = 498\text{ N}$$ **Ans.**



Ans:
$(F_{ED})_{||} = 334$ N
$(F_{ED})_{\perp} = 498$ N

2–73.

Determine the angle θ between BA and BC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(0, -2, 0)$ m, $B(0, 0, 0)$ m and $C(3, 4, -1)$ m respectively. Thus, the unit vectors along BA and BC are

$$\mathbf{u}_{BA} = -\mathbf{j} \qquad \mathbf{u}_{BE} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{\mathbf{4}}{\sqrt{\mathbf{26}}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

The Angle θ Between BA and BC.

$$\mathbf{u}_{BA}\,\mathbf{u}_{BC} = (-\mathbf{j}) \cdot \left(\frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}\right)$$

$$= (-1)\left(\frac{4}{\sqrt{26}}\right) = -\frac{4}{\sqrt{26}}$$

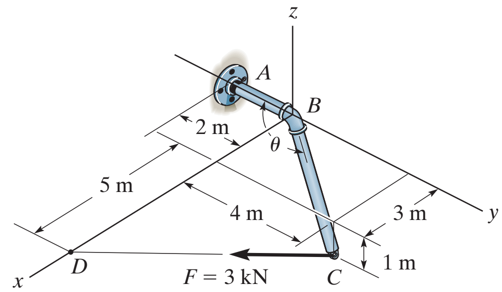
Then

$$\theta = \cos^{-1}(\mathbf{u}_{BA} \cdot \mathbf{u}_{BC}) = \cos^{-1}\left(-\frac{4}{\sqrt{26}}\right) = 141.67^\circ = 142^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 142^\circ$

2–74.

Determine the magnitude of the projected component of the 3 kN force acting along axis BC of the pipe.



SOLUTION

Unit Vectors. Here, the coordinates of points B, C and D are $B\,(0, 0, 0)$ m, $C(3, 4, -1)$ m and $D(8, 0, 0)$. Thus the unit vectors along BC and CD are

$$\mathbf{u}_{BC} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

$$\mathbf{u}_{CD} = \frac{(8-3)\,\mathbf{i} + (0-4)\,\mathbf{j} + [0-(-1)]\,\mathbf{k}}{\sqrt{(8-3)^2 + (0-4)^2 + [0-(-1)]^2}} = \frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}$$

Force Vector. For **F**,

$$\mathbf{F} = F\mathbf{u}_{CD} = 3\left(\frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}\right)$$

$$= \left(\frac{15}{\sqrt{42}}\,\mathbf{i} - \frac{12}{\sqrt{42}}\,\mathbf{j} + \frac{3}{\sqrt{42}}\,\mathbf{k}\right)\text{ kN}$$

Projected Component of F. Along BC, it is

$$\left|(F_{BC})\right| = \left|\mathbf{F}\cdot\mathbf{u}_{BC}\right| = \left|\left(\frac{15}{\sqrt{42}}\mathbf{i} - \frac{12}{\sqrt{42}}\mathbf{j} + \frac{3}{\sqrt{42}}\mathbf{k}\right)\cdot\left(\frac{3}{\sqrt{26}}\mathbf{i} + \frac{4}{\sqrt{26}}\mathbf{j} - \frac{1}{\sqrt{26}}\mathbf{k}\right)\right|$$

$$= \left|\left(\frac{15}{\sqrt{42}}\right)\left(\frac{3}{\sqrt{26}}\right) + \left(-\frac{12}{\sqrt{42}}\right)\left(\frac{4}{\sqrt{26}}\right) + \frac{3}{\sqrt{42}}\left(-\frac{1}{\sqrt{26}}\right)\right|$$

$$= \left|-\frac{6}{\sqrt{1092}}\right| = \left|-0.1816\text{ kN}\right| = 0.182\text{ kN} \qquad \textbf{Ans.}$$

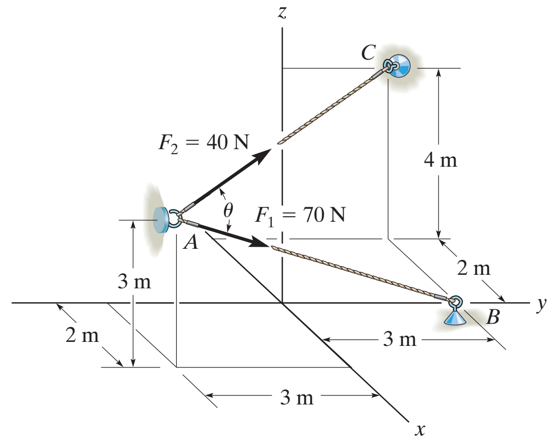
The negative signs indicate that this component points in the direction opposite to that of $\mathbf{u}_{BC}$.

Ans:

$\left|(F_{BC})\right| = 0.182$ kN

2–75.

Determine the angle θ between the two cables.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$ m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m, respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

The Angle θ Between AB and AC.

$$\mathbf{u}_{AB}\cdot\mathbf{u}_{AC} = \left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right)\cdot\left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= \left(-\frac{2}{7}\right)\left(-\frac{4}{\sqrt{53}}\right) + \frac{6}{7}\left(\frac{6}{\sqrt{53}}\right) + \left(-\frac{3}{7}\right)\left(\frac{1}{\sqrt{53}}\right)$$

$$= \frac{41}{7\sqrt{53}}$$

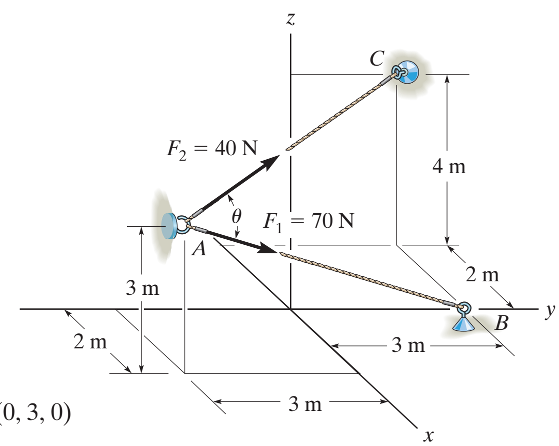
Then

$$\theta = \cos^{-1}\left(\mathbf{u}_{AB}\cdot\mathbf{u}_{AC}\right) = \cos^{-1}\left(\frac{41}{7\sqrt{53}}\right) = 36.43^\circ = 36.4^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 36.4^\circ$

***2–76.**

Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m, respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

Force Vector, For $\mathbf{F}_1$,

$$\mathbf{F}_1 = F_1\,\mathbf{u}_{AB} = 70\left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) = \{-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}\}\text{ N}$$

Projected Component of $\mathbf{F}_1$. Along AC, it is

$$(F_1)_{AC} = \mathbf{F}_1 \cdot \mathbf{u}_{AC} = (-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= (-20)\left(-\frac{4}{\sqrt{53}}\right) + 60\left(\frac{6}{\sqrt{53}}\right) + (-30)\left(\frac{1}{\sqrt{53}}\right)$$

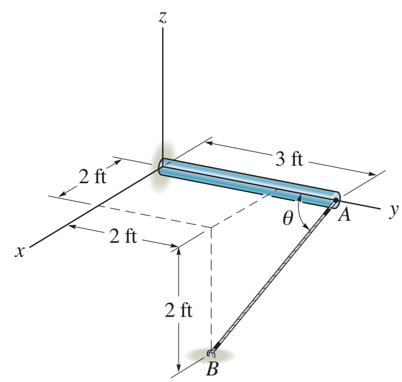
$$= 56.32\text{ N} = 56.3\text{ N}$$ **Ans.**

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{AC}$.

Ans:
$(F_1)_{AC} = 56.3$ N

2–77.

Determine the angle θ between the pole and the wire AB.



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{(2 - 0)\mathbf{i} + (2 - 3)\mathbf{j} + (-2 - 0)\mathbf{k}\}\text{ ft}$$

$$= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00\text{ ft} \qquad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00\text{ ft}$$

The Angles Between Two Vectors θ: The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k})$$

$$= 0(2) + (-3)(-1) + 0(-2)$$

$$= 3$$

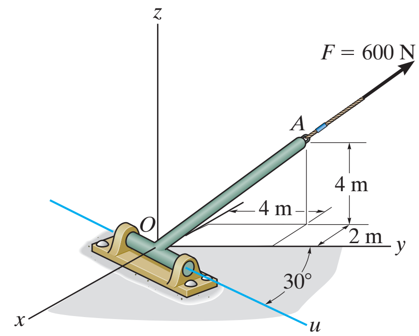
Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5° \qquad \textbf{Ans.}$$

Ans:
$\theta = 70.5°$

2–78.

Determine the magnitude of the projection of the force along the u axis.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_u$ must be determined first. From Fig. a,

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-2-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (4-0)^2 + (4-0)^2}} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_u = \sin 30°\mathbf{i} + \cos 30°\mathbf{j}$$

Thus, the force vectors **F** is given by

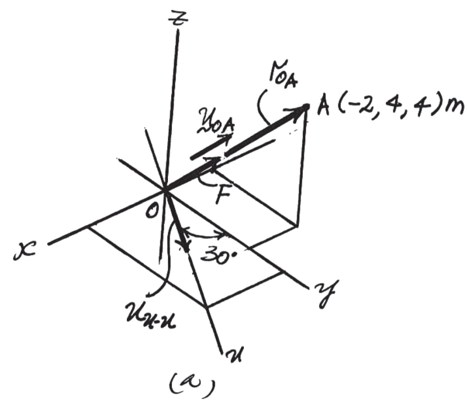
$$\mathbf{F} = F\,\mathbf{u}_{OA} = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

Vector Dot Product: The magnitude of the projected component of **F** along the u axis is

$$\mathbf{F}_u = F \cdot \mathbf{u}_u = (-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}) \cdot (\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= (-200)(\sin 30°) + 400(\cos 30°) + 400(0)$$

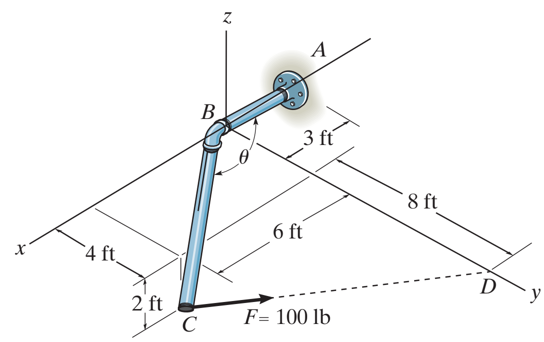
$$= 246\text{ N}$$ **Ans.**



Ans:
$\mathbf{F}_u = 246$ N

2–79.

Determine the magnitude of the projected component of the 100-lb force acting along the axis *BC* of the pipe.



SOLUTION

$$\vec{\gamma}_{BC} = \{6\hat{i} + 4\hat{j} - 2\hat{k}\} \text{ ft}$$

$$\vec{F} = 100 \frac{\{-6\hat{i} + 8\hat{j} + 2\hat{k}\}}{\sqrt{(-6)^2 + 8^2 + 2^2}}$$

$$= \{-58.83\hat{i} + 78.45\hat{j} + 19.61\hat{k}\} \text{ lb}$$

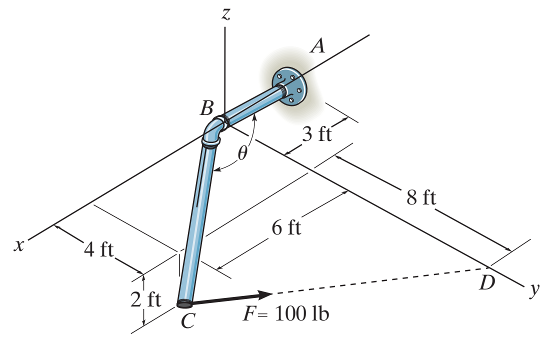
$$F_p = \vec{F} \cdot \vec{\mu}_{BC} = \vec{F} \cdot \frac{\vec{\gamma}_{BC}}{|\vec{\gamma}_{BC}|} = \frac{-78.45}{7.483} = -10.48$$

$F_p = 10.5$ lb **Ans.**

Ans:
$F_p = 10.5$ lb

***2–80.**

Determine the angle θ between pipe segments BA and BC.



SOLUTION

$$\vec{\gamma}_{BC} = \{6\hat{i} + 4\hat{j} - 2\hat{k}\}\text{ ft}$$

$$\vec{\gamma}_{BA} = \{-3\hat{i}\}\text{ ft}$$

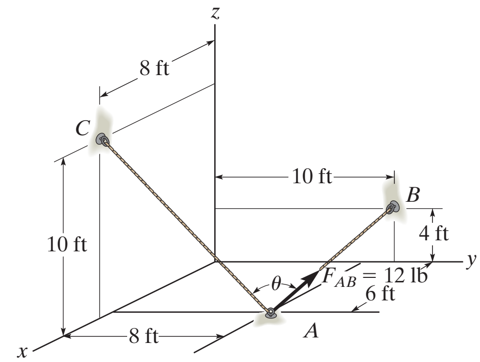
$$\theta = \cos^{-1}\left(\frac{\vec{\gamma}_{BC} \cdot \vec{\gamma}_{BA}}{|\vec{\gamma}_{BC}||\vec{\gamma}_{BA}|}\right) = \cos^{-1}\left(\frac{-18}{22.45}\right)$$

$\theta = 143°$ **Ans.**

Ans:
$\theta = 143°$

2–81.

Determine the angle θ between the two cables.



SOLUTION

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC}\cdot\mathbf{r}_{AB}}{r_{AC}\,r_{AB}}\right)$$

$$= \cos^{-1}\left[\frac{(2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k})\cdot(-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k})}{\sqrt{2^2 + (-8)^2 + 10^2}\,\sqrt{(-6)^2 + 2^2 + 4^2}}\right]$$

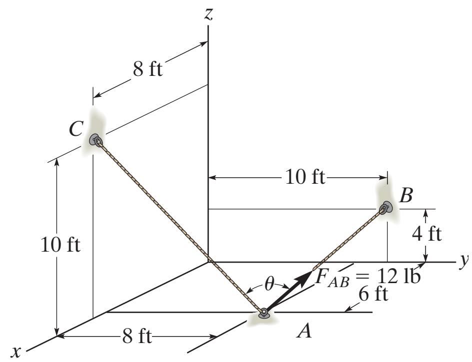
$$= \cos^{-1}\left(\frac{12}{96.99}\right)$$

$\theta = 82.9^\circ$ **Ans.**

Ans:
$\theta = 82.9^\circ$

2–82.

Determine the projected component of the force acting in the direction of cable *AC*. Express the result as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AC} = \{2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_{AB} = 12\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 12\left(-\frac{6}{7.483}\mathbf{i} + \frac{2}{7.483}\mathbf{j} + \frac{4}{7.483}\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{-9.621\,\mathbf{i} + 3.207\,\mathbf{j} + 6.414\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{u}_{AC} = \frac{2}{12.961}\mathbf{i} - \frac{8}{12.961}\mathbf{j} + \frac{10}{12.961}\mathbf{k}$$

$$\text{Proj } F_{AB} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} = -9.621\left(\frac{2}{12.961}\right) + 3.207\left(-\frac{8}{12.961}\right) + 6.414\left(\frac{10}{12.961}\right)$$

$$= 1.4846$$

$$\text{Proj } \mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AC}$$

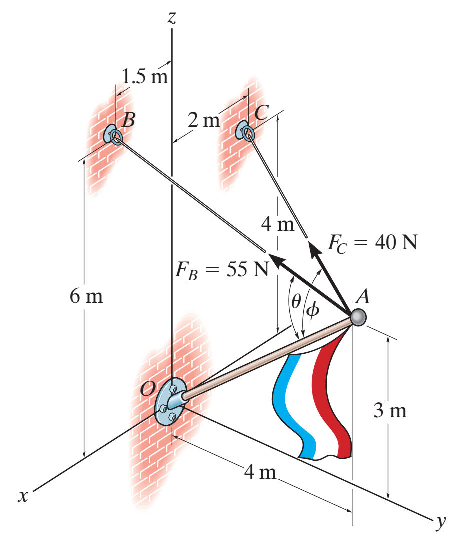
$$\text{Proj } \mathbf{F}_{AB} = (1.4846)\left[\frac{2}{12.962}\mathbf{i} - \frac{8}{12.962}\mathbf{j} + \frac{10}{12.962}\mathbf{k}\right]$$

$$\text{Proj } \mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
Proj $\mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}$ lb

2–83.

Determine the angles θ and ϕ between the flag pole and the cables AB and AC.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m ; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$ m

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4^\circ$$ **Ans.**

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

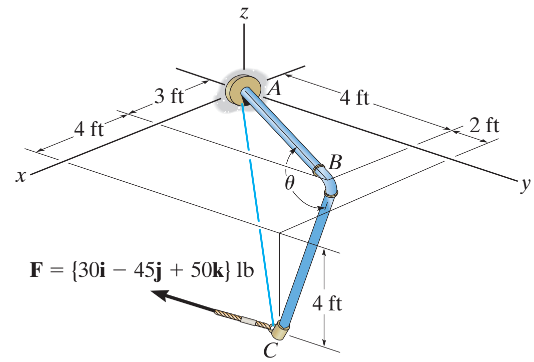
$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4^\circ$$ **Ans.**

Ans:
$\theta = 74.4^\circ$
$\phi = 55.4^\circ$

***2–84.**

Determine the magnitudes of the components of **F** acting along and perpendicular to segment *BC* of the pipe assembly.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to segment *BC* of the pipe assembly is

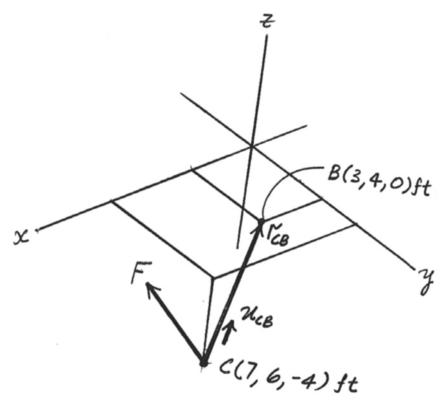
$$(F_{BC})_{\text{pa}} = \mathbf{F}\cdot\mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

$$= 28.33\text{ lb} = 28.3\text{ lb} \qquad \textbf{Ans.}$$

The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment *BC* of the pipe assembly can be determined from

$$(F_{BC})_{\text{per}} = \sqrt{F^2 - (F_{BC})_{\text{pa}}{}^2} = \sqrt{5425 - 28.33^2} = 68.0\text{ lb} \qquad \textbf{Ans.}$$

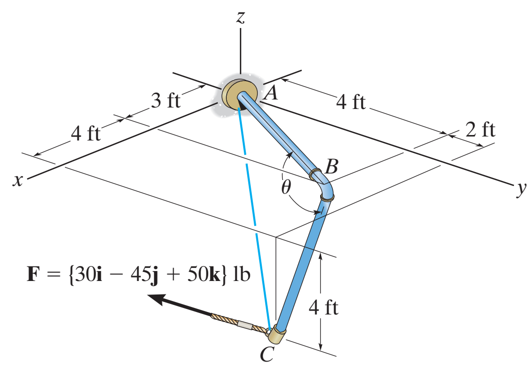


(a)

Ans:
$(F_{BC})_{||} = 28.3$ lb
$(F_{BC})_{\perp} = 68.0$ lb

2–85.

Determine the magnitude of the projected component of **F** along line *AC*. Express this component as a Cartesian vector.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

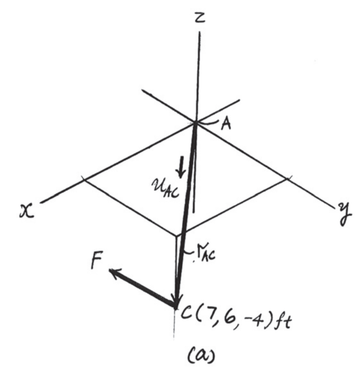
$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line *AC* is

$$F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k}) \cdot (0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$

$$= 25.87 \text{ lb}$$ **Ans.**



Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

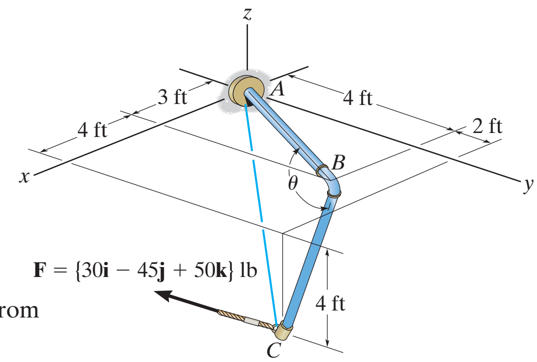
$$\mathrm{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$ **Ans.**

Ans:
$F_{AC} = 25.87$ lb
$F_{AC} = \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\}$ lb

2–86.

Determine the angle θ between the pipe segments BA and BC.

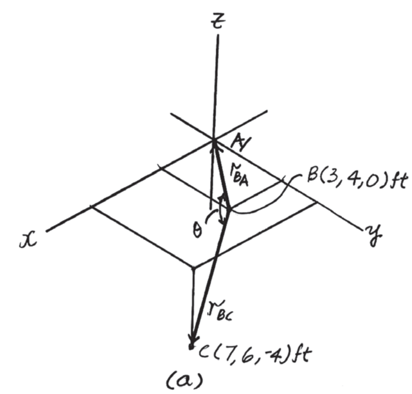


SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{BA} = (0 - 3)\mathbf{i} + (0 - 4)\mathbf{j} + (0 - 0)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{BC} = (7 - 3)\mathbf{i} + (6 - 4)\mathbf{j} + (-4 - 0)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\text{ ft}$$



The magnitude of $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ are

$$r_{BA} = \sqrt{(-3)^2 + (-4)^2} = 5\text{ ft}$$

$$r_{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6\text{ ft}$$

Vector Dot Product:

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-3\mathbf{i} - 4\mathbf{j}) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$
$$= (-3)(4) + (-4)(2) + 0(-4)$$
$$= -20\text{ ft}^2$$

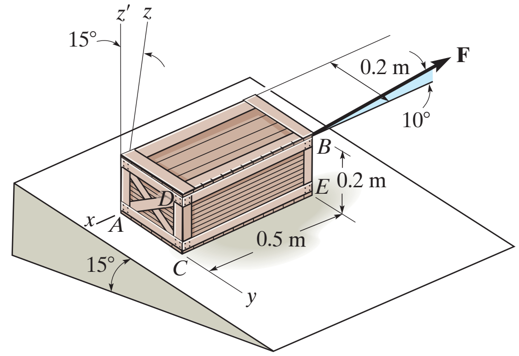
Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA}\, r_{BC}}\right) = \cos^{-1}\left[\frac{-20}{5(6)}\right] = 132^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 132^\circ$

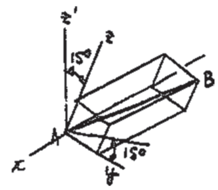
2–87.

If the force $F = 100$ N lies in the plane $DBEC$, which is parallel to the x–z plane, and makes an angle of 10° with the extended line DB as shown, determine the angle that **F** makes with the diagonal AB of the crate.



SOLUTION

Use the x, y, z axes.



$$\mathbf{u}_{AB} = \left(\frac{-0.5\mathbf{i} + 0.2\mathbf{j} + 0.2\mathbf{k}}{0.57446}\right)$$

$$= -0.8704\mathbf{i} + 0.3482\mathbf{j} + 0.3482\mathbf{k}$$

$$\mathbf{F} = -100\cos 10°\mathbf{i} + 100\sin 10°\mathbf{k}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{F}\cdot\mathbf{u}_{AB}}{F\,u_{AB}}\right)$$

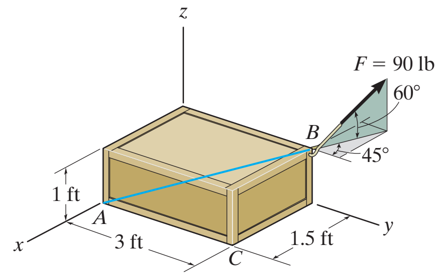
$$= \cos^{-1}\left(\frac{-100\,(\cos 10°)(-0.8704) + 0 + 100\sin 10°\,(0.3482)}{100(1)}\right)$$

$$= \cos^{-1}(0.9176) = 23.4°$$ **Ans.**

Ans:
$\theta = 23.4°$

***2–88.**

Determine the magnitudes of the components of the force acting parallel and perpendicular to diagonal AB of the crate.



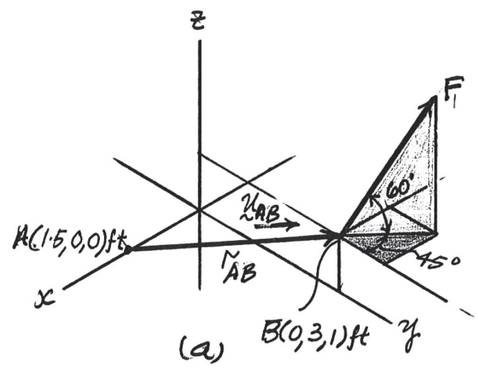
SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{F} = 90(-\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\}\ \text{lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0 - 1.5)\mathbf{i} + (3 - 0)\mathbf{j} + (1 - 0)\mathbf{k}}{\sqrt{(0 - 1.5)^2 + (3 - 0)^2 + (1 - 0)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



Vector Dot Product: The magnitude of the projected component of **F** parallel to the diagonal AB is

$$[(F)_{AB}]_{pa} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

$$= 63.18\ \text{lb} = 63.2\ \text{lb}$$ **Ans.**

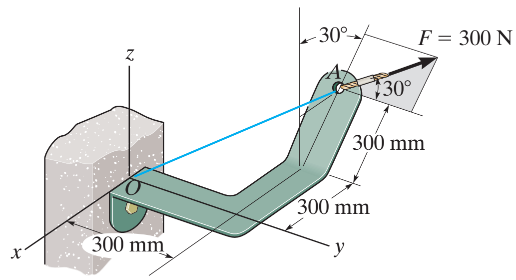
The magnitude of the component **F** perpendicular to the diagonal AB is

$$[(F)_{AB}]_{per} = \sqrt{F^2 - [(F)_{AB}]_{pa}^{\,2}} = \sqrt{90^2 - 63.18^2} = 64.1\ \text{lb}$$ **Ans.**

Ans:
$[(F)_{AB}]_{\parallel} = 63.2$ lb
$[(F)_{AB}]_{\perp} = 64.1$ lb

2–89.

Determine the magnitudes of the projected components of the force acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. a,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{j}$$

$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260 \text{ N}$$

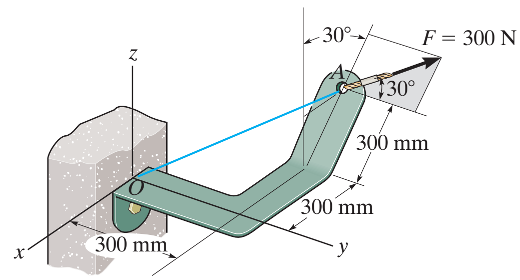
The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

$$F_x = 75 \text{ N}, \qquad F_y = 260 \text{ N}$$ **Ans.**

Ans:
$F_x = 75$ N
$F_y = 260$ N

2–90.

Determine the magnitude of the projected component of the force acting along line OA.



SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a,

$$\mathbf{F} = (-300 \sin 30° \sin 30°\mathbf{i} + 300 \cos 30°\mathbf{j} + 300 \sin 30° \cos 30°\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

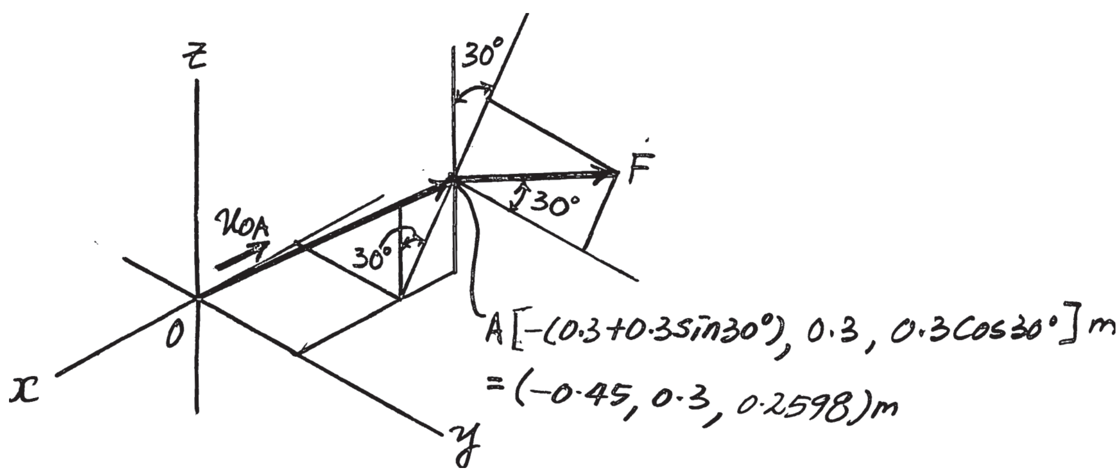
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line OA is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \left(-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}\right)$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

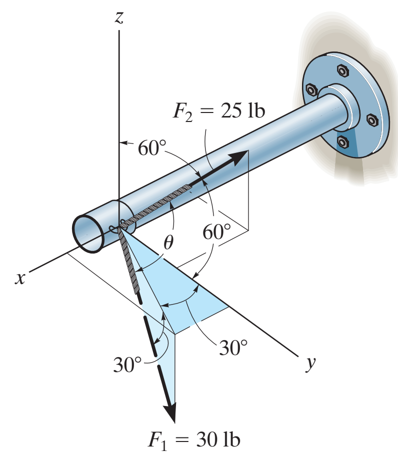
$$= 242 \text{ N}$$ **Ans.**



Ans:
$F_{OA} = 242$ N

2–91.

Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R\mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k})\text{ lb}$$

$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\}\text{ lb}$$

Unit Vector: One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

$$= -5.44\text{ lb}$$

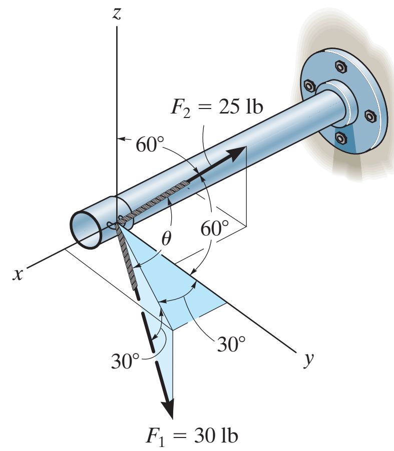
Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

Ans:
The magnitude is $(F_1)_{F_2} = 5.44$ lb

***2–92.**

Determine the angle θ between the two forces.



SOLUTION

Unit Vectors:

$$\mathbf{u}_{F_1} = \cos 30^\circ \sin 30^\circ \mathbf{i} + \cos 30^\circ \cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

$$= -0.1812$$

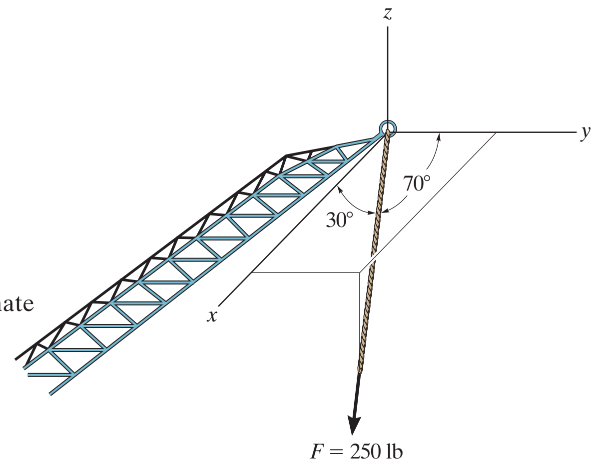
Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100^\circ$$ **Ans.**

Ans:
$\theta = 100^\circ$

***R2–4.**

The cable exerts a force of 250 lb on the crane boom as shown. Express this force as a Cartesian vector.



SOLUTION

Cartesian Vector Notation: With $\alpha = 30°$ and $\beta = 70°$, the third coordinate direction angle γ can be determined using Eq. 2–8.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 30° + \cos^2 70° + \cos^2 \gamma = 1$$

$$\cos \gamma = \pm 0.3647$$

$$\gamma = 68.61° \text{ or } 111.39°$$

By inspection, $\gamma = 111.39°$ since the force **F** is directed in negative octant.

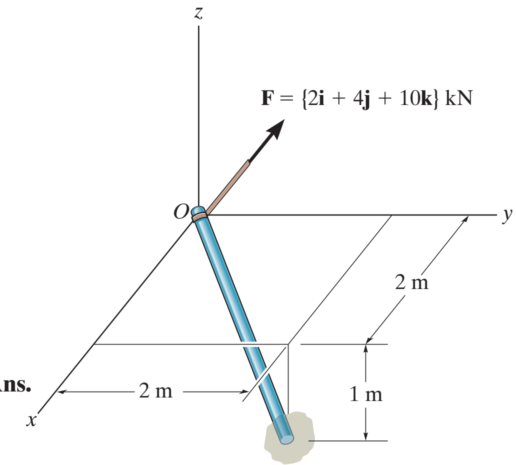
$$\mathbf{F} = 250\{\cos 30°\mathbf{i} + \cos 70°\mathbf{j} + \cos 111.39°\} \text{ lb}$$

$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\} \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F} = \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\}$ lb

***R2–8.**

Determine the projection of the force **F** along the pole.



SOLUTION

$$\text{Proj } F = \mathbf{F}\cdot\mathbf{u}_a = (2\,\mathbf{i} + 4\,\mathbf{j} + 10\,\mathbf{k})\cdot\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right)$$

$$\text{Proj } F = 0.667 \text{ kN}$$ **Ans.**

Ans:
$F = 0.667$ kN