Chapter 2
Introduction to Optimization & Linear Programming

1. If an LP model has more than one optimal solution it has an ***infinite*** number of alternate optimal solutions. In Figure 2.8, the two extreme points at (122, 78) and (174, 0) are alternate optimal solutions, but there are an infinite number of alternate optimal solutions along the edge connecting these extreme points. This is true of all LP models with alternate optimal solutions.

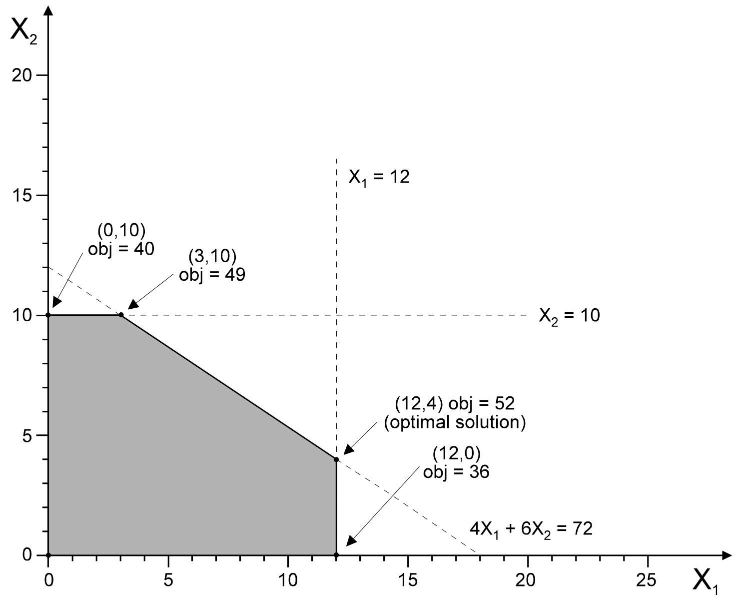
2. There is no guarantee that the optimal solution to an LP problem will occur at an integer-valued extreme point of the feasible region. (An exception to this general rule is discussed in Chapter 5 on networks).

3. We can graph an inequality as if they were an equality because the condition imposed by the equality corresponds to the boundary line (or most extreme case) of the inequality.

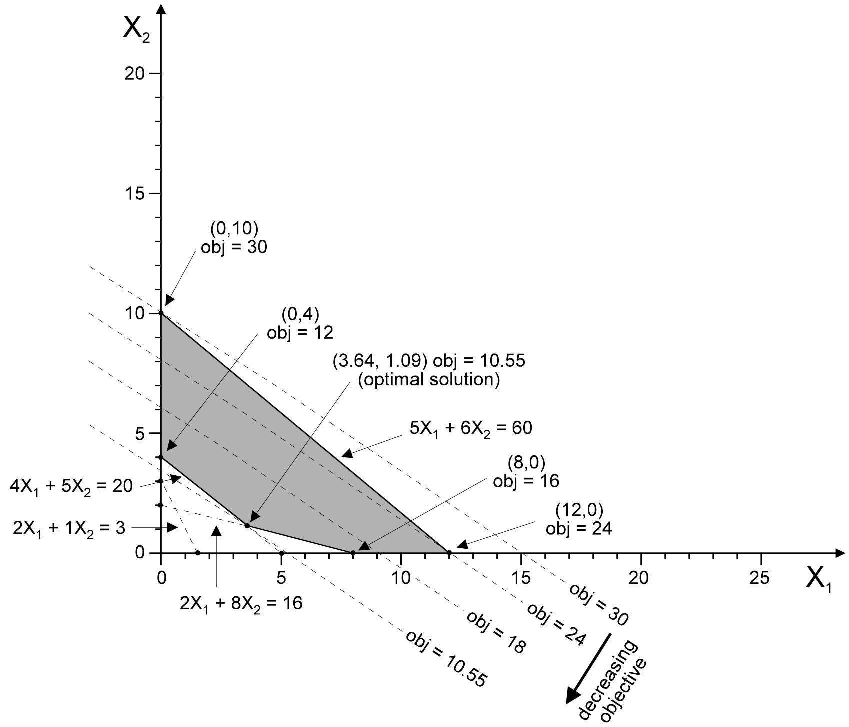
4. The objectives are equivalent. For any values of X_1 and X_2, the absolute value of the objectives are the same. Thus, maximizing the value of the first objective is equivalent to minimizing the value of the second objective.

5. a. linear
   b. nonlinear
   c. linear, can be re-written as: $4 X_1 - .3333 X_2 = 75$
   d. linear, can be re-written as: $2.1 X_1 + 1.1 X_2 - 3.9 X_3 \leq 0$
   e. nonlinear

6.

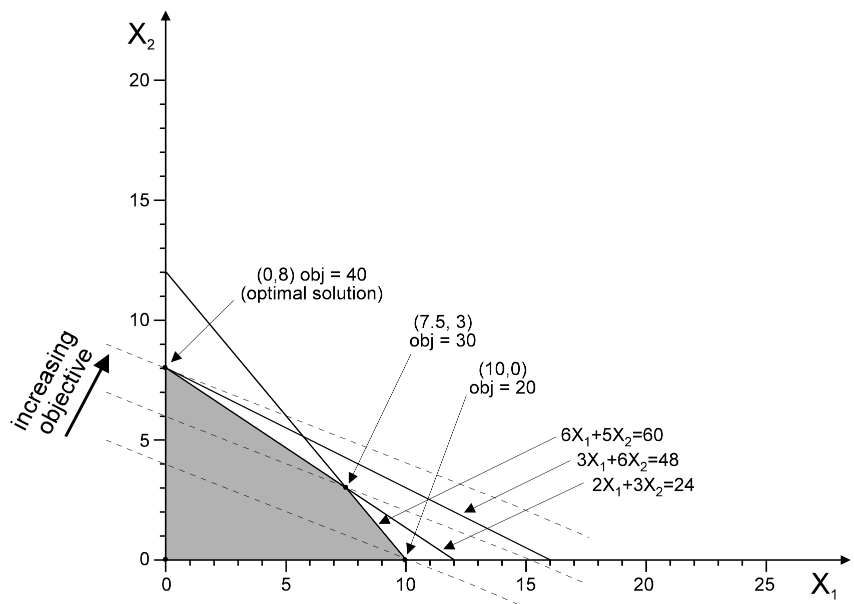


7.



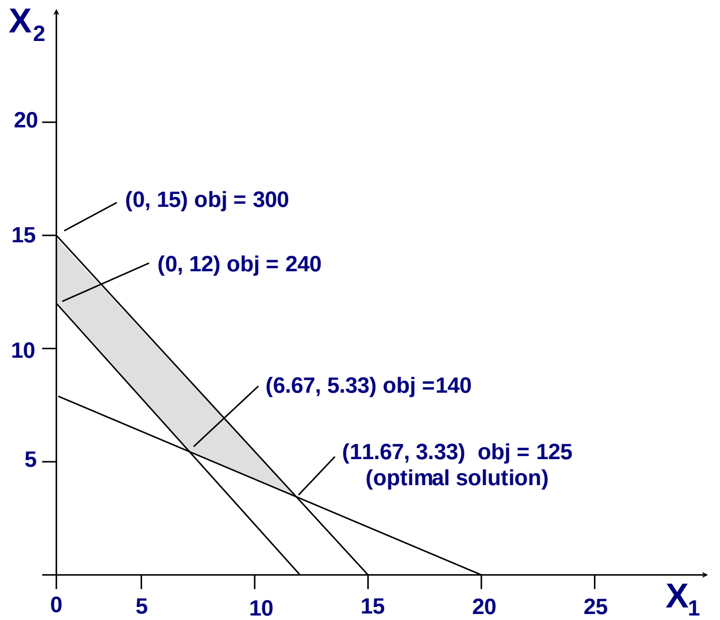
X2
20
15
10
5
0
(0,10)
obj = 30
(0,4)
obj = 12
(3.64, 1.09) obj = 10.55
(optimal solution)
5X1 + 6X2 = 60
(8,0)
obj = 16
(12,0)
obj = 24
4X1 + 5X2 = 20
2X1 + 1X2 = 3
2X1 + 8X2 = 16
0
5
10
15
20
25
X1
obj = 30
obj = 24
obj = 18
obj = 10.55
decreasing
objective

8.

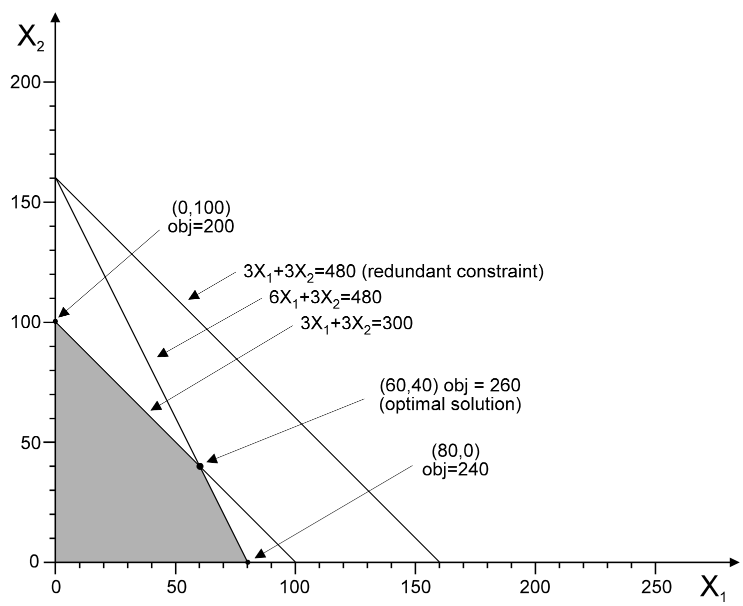


X2
20
15
10
5
0
(0,8) obj = 40
(optimal solution)
(7.5, 3)
obj = 30
(10,0)
obj = 20
increasing
objective
6X1+5X2=60
3X1+6X2=48
2X1+3X2=24
0
5
10
15
20
25
X1

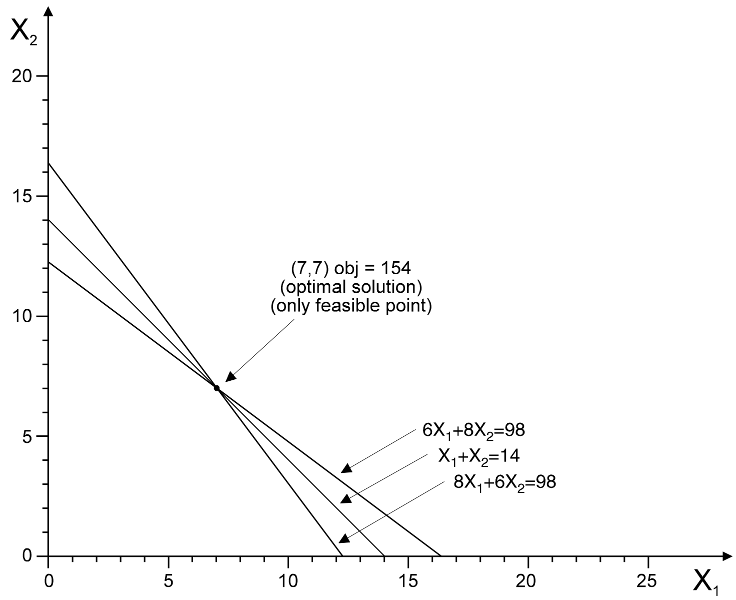
9.


X2
20
15
10
5
0
5
10
15
20
25
X1
(0, 15) obj = 300
(0, 12) obj = 240
(6.67, 5.33) obj =140
(11.67, 3.33) obj = 125
(optimal solution)

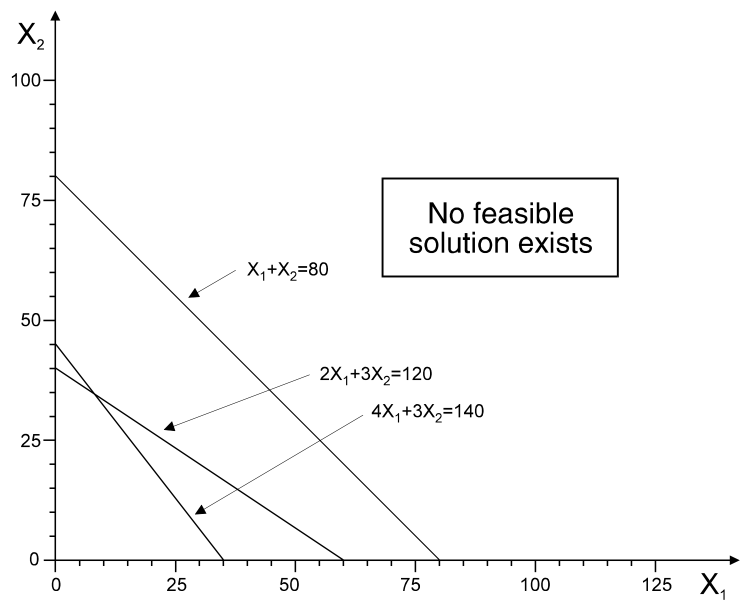
10.


X2
200
150
100
50
0
0
50
100
150
200
250
X1
(0,100)
obj=200
3X1+3X2=480 (redundant constraint)
6X1+3X2=480
3X1+3X2=300
(60,40) obj = 260
(optimal solution)
(80,0)
obj=240

11.



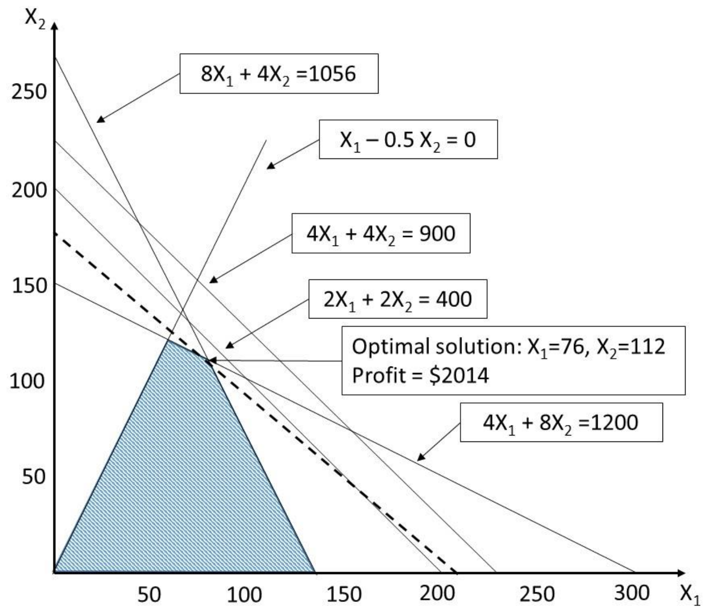
12.



13. X_1 = number of His chairs to produce, X_2 = number of Hers chairs to produce

MAX $10\,X_1 + 12\,X_2$

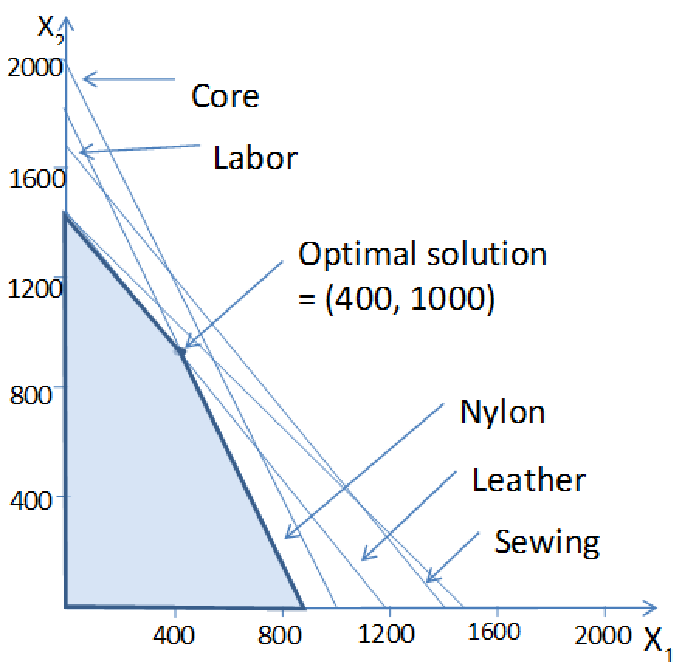
ST
$4\,X_1 + 8\,X_2 \leq 1200$
$8\,X_1 + 4\,X_2 \leq 1056$
$2\,X_1 + 2\,X_2 \leq 400$
$4\,X_1 + 4\,X_2 \leq 900$
$1\,X_1 + 0.5\,X_2 \geq 0$
$X_1, X_2 \geq 0$



14. X_1 = number of softballs to produce, X_2 = number of baseballs to produce

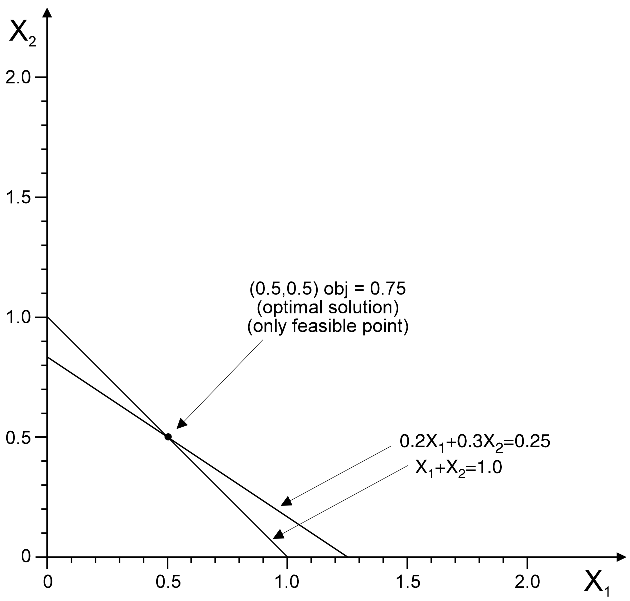
MAX $6 X_1 + 4.5 X_2$

ST
$5X_1 + 4 X_2 \leq 6000$
$6 X_1 + 3 X_2 \leq 5400$
$4 X_1 + 2 X_2 \leq 4000$
$2.5 X_1 + 2 X_2 \leq 3500$
$1 X_1 + 1 X_2 \leq 1500$
$X_1, X_2 \geq 0$



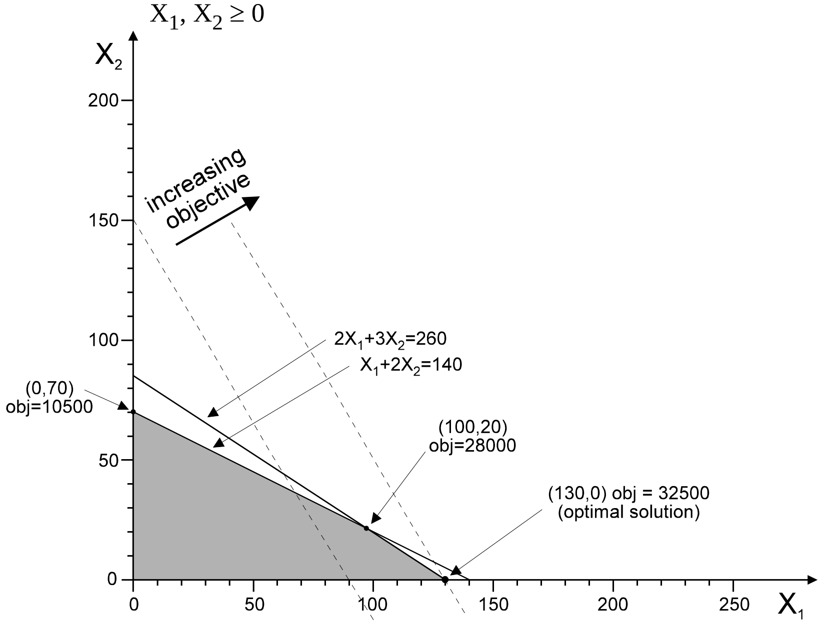
15. X_1 = proportion of beef in the mix, X_2 = proportion of pork in the mix

MIN $.85 X_1 + .65 X_2$
ST $1X_1 + 1 X_2 = 1$
$0.2 X_1 + 0.3 X_2 \le 0.25$
$X_1, X_2 \ge 0$



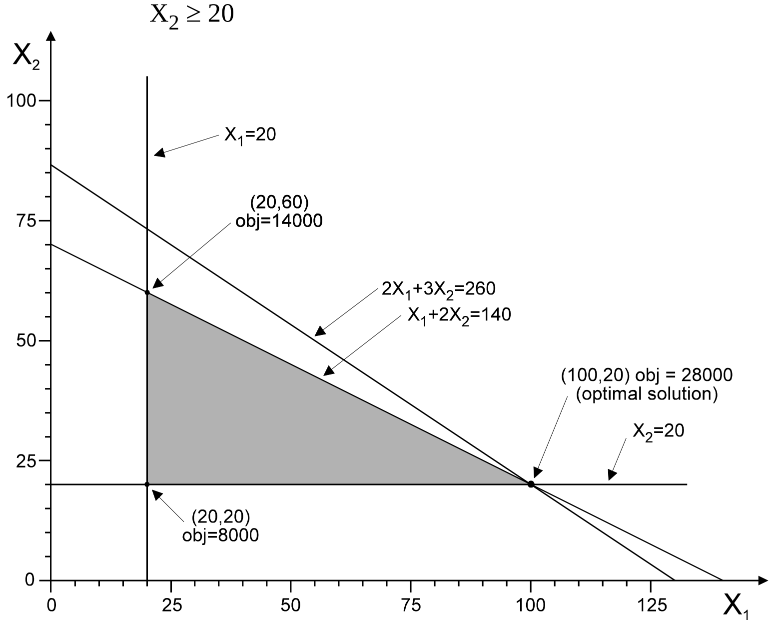
16. X_1 = number of generators, X_2 = number of alternators

MAX $250 X_1 + 150 X_2$
ST $2 X_1 + 3 X_2 \le 260$
$1 X_1 + 2 X_2 \le 140$
$X_1, X_2 \ge 0$



17. X_1 = number of generators, X_2 = number of alternators

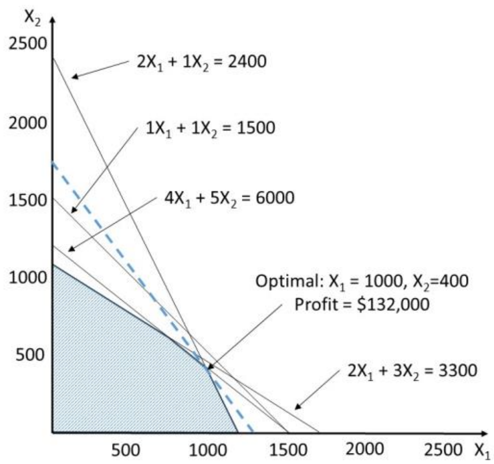
MAX	$250\ X_1 + 150\ X_2$
ST	$2\ X_1 + 3\ X_2 \leq 260$
	$1\ X_1 + 2\ X_2 \leq 140$
	$X_1 \geq 20$
	$X_2 \geq 20$



d. No, the feasible region would not increase so the solution would not change -- you'd just have extra (unused) wiring capacity.

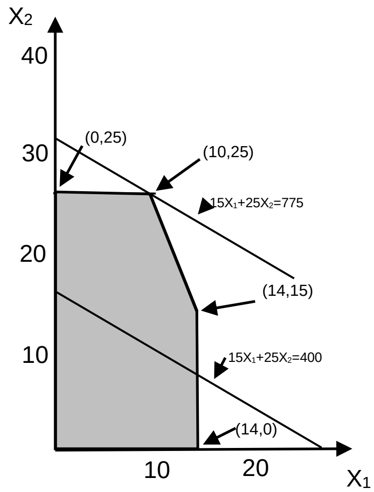
18. X_1 = number of propane grills to produce, X_2 = number of electric grills to produce

MAX	$100\ X_1 + 80\ X_2$
ST	$2\ X_1 + 1\ X_2 \leq 2400$
	$4\ X_1 + 5\ X_2 \leq 6000$
	$2\ X_1 + 3\ X_2 \leq 3300$
	$1\ X_1 + 1\ X_2 \leq 1500$
	$X_1, X_2 \geq 0$



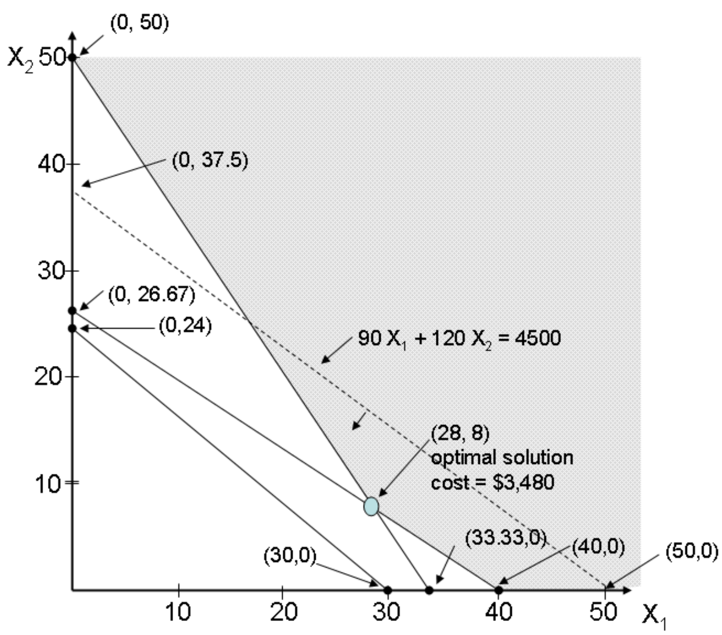
19. X_1 = # of TV spots, X_2 = # of magazine ads

MAX	$15 X_1 + 25 X_2$	(profit)
ST	$5 X_1 + 2 X_2 \le 100$	(ad budget)
	$5 X_1 + 0 X_2 \le 70$	(TV limit)
	$0 X_1 + 2 X_2 \le 50$	(magazine limit)
	$X_1, X_2 \ge 0$	



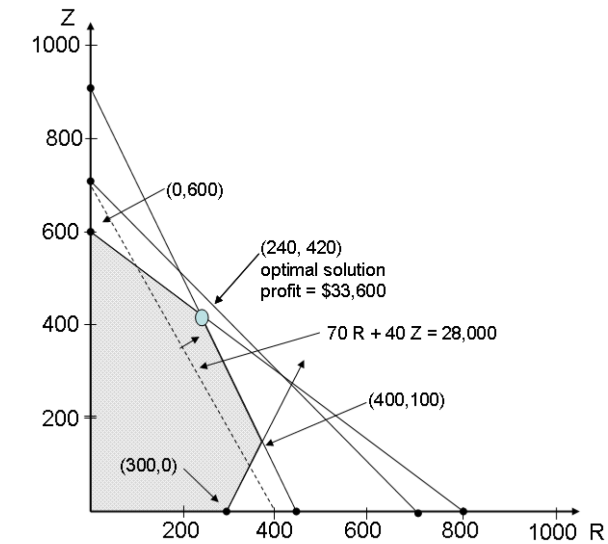
20. X_1 = tons of ore purchased from mine 1, X_2 = tons of ore purchased from mine 2

MIN	$90 X_1 + 120 X_2$	(cost)
ST	$0.2 X_1 + 0.3 X_2 \ge 8$	(copper)
	$0.2 X_1 + 0.25 X_2 \ge 6$	(zinc)
	$0.15 X_1 + 0.1 X_2 \ge 5$	(magnesium)
	$X_1, X_2 \ge 0$	



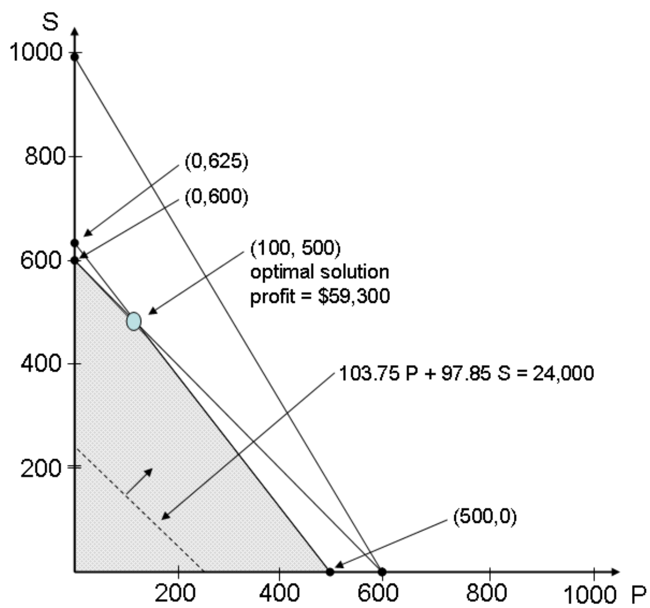
21. R = number of Razors produced, Z = number of Zoomers produced

MAX 70 R + 40 Z
ST R + Z ≤ 700
R – Z ≤ 300
2 R + 1 Z ≤ 900
3 R + 4 Z ≤ 2400
R, Z ≥ 0



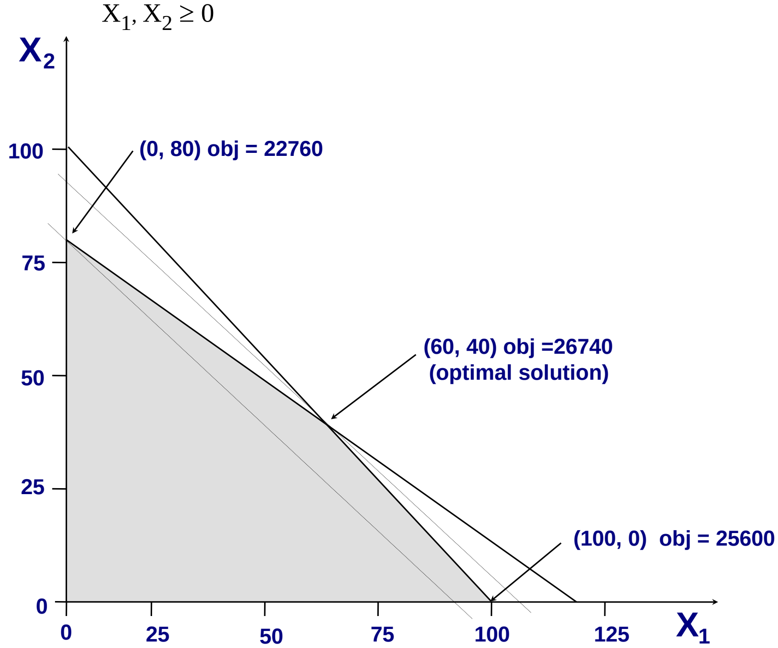
22. P = number of Presidential desks produced, S = number of Senator desks produced

MAX 103.75 P + 97.85 S
ST 30 P + 24 S ≤ 15,000
1 P + 1 S ≤ 600
5 P + 3 S ≤ 3000
P, S ≥ 0



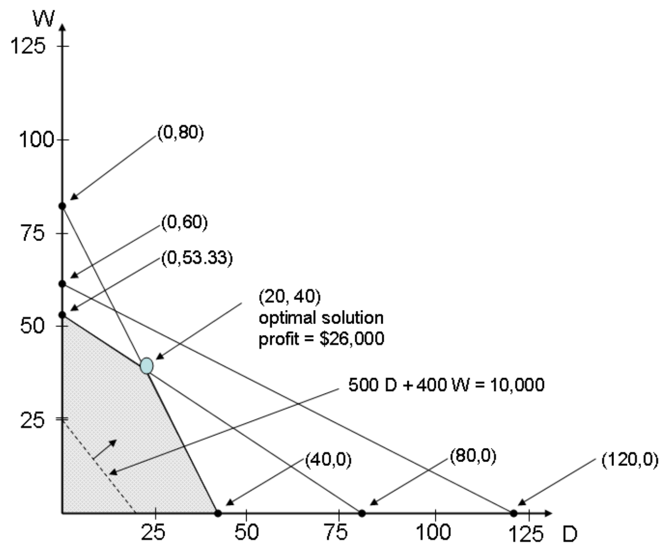
23. X_1 = acres planted in watermelons, X_2 = acres planted in cantaloupes

MAX $256 X_1 + 284.5 X_2$
ST $50 X_1 + 75 X_2 \leq 6000$
$X_1 + X_2 \leq 100$
$X_1, X_2 \geq 0$



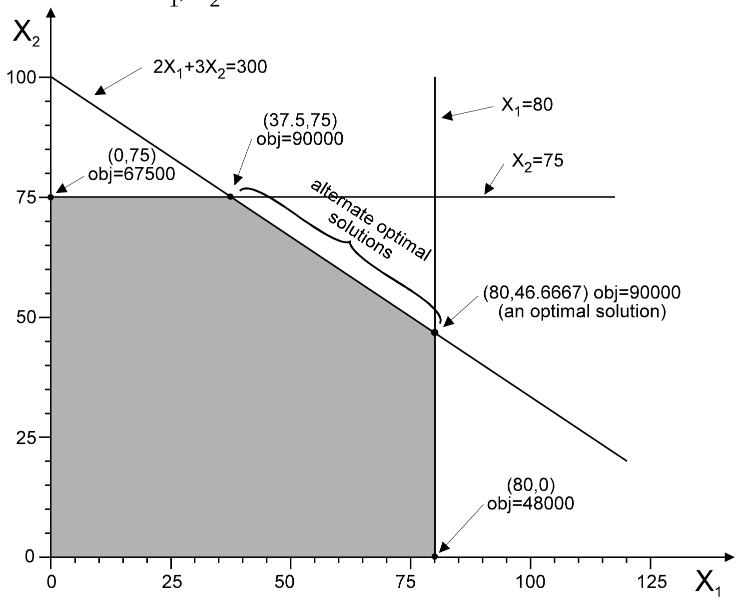
24. D = number of doors produced, W = number of windows produced

MAX $500 D + 400 W$
ST $1 D + 0.5 W \leq 40$
$0.5 D + 0.75 W \leq 40$
$0.5 D + 1 W \leq 60$
$D, W \geq 0$



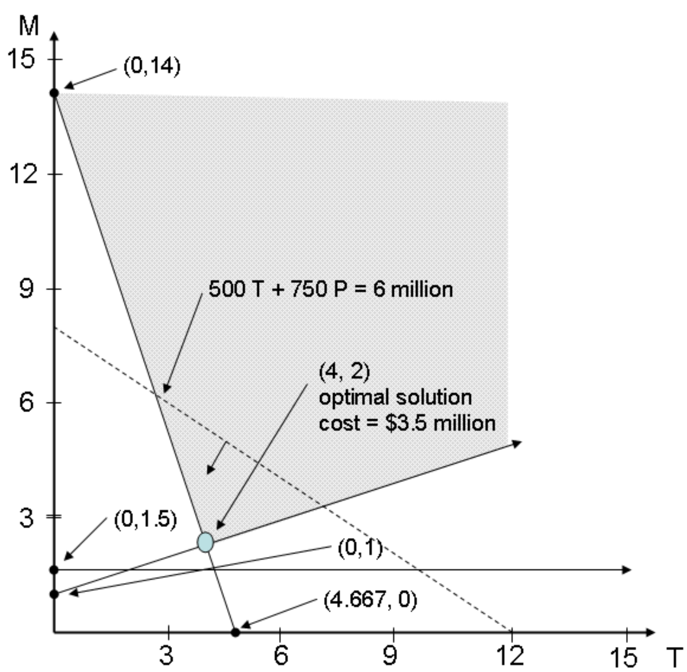
25. X_1 = number of desktop computers, X_2 = number of laptop computers

MAX	$600\ X_1 + 900\ X_2$
ST	$2\ X_1 + 3\ X_2 \le 300$
	$X_1 \le 80$
	$X_2 \le 75$
	$X_1, X_2 \ge 0$



26. T= number of TV ads to run, M = number of magazine ads to run

MIN	$500\ T + 750\ P$
ST	$3T + 1P \ge 14$
	$-1T + 4P \ge 4$
	$0T + 2P \ge 3$
	$T, P \ge 0$



hangin'

1. 200 pumps, 1566 labor hours, 2712 feet of tubing.

2. Pumps are a binding constraint and should be increased to 207, if possible. This would increase profits by $1,400 to $67,500.

3. Labor is a binding constraint and should be increased to 1800, if possible. This would increase profits by $3,900 to $70,000.

4. Tubing is a non-binding constraint. They've already got more than they can use and don't need any more.

5. 9 to 8: profit increases by $3,050
   8 to 7: profit increases by $850
   7 to 6: profit increases by $0

6. 6 to 5: profit increases by $975
   5 to 4: profit increases by $585
   4 to 3: profit increases by $390

7. 12 to 11: profit increases by $0
   11 to 10: profit increases by $0
   10 to 9: profit increases by $0

8. 16 to 15: profit increases by $0
   15 to 14: profit increases by $0
   14 to 13: profit increases by $0

9. The profit on Aqua-Spas can vary between $300 and $450 without changing the optimal solution.

10. The profit on Hydro-Luxes can vary between $233.33 and $350 without changing the optimal solution.