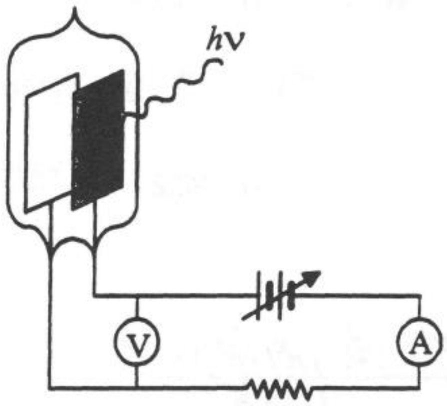
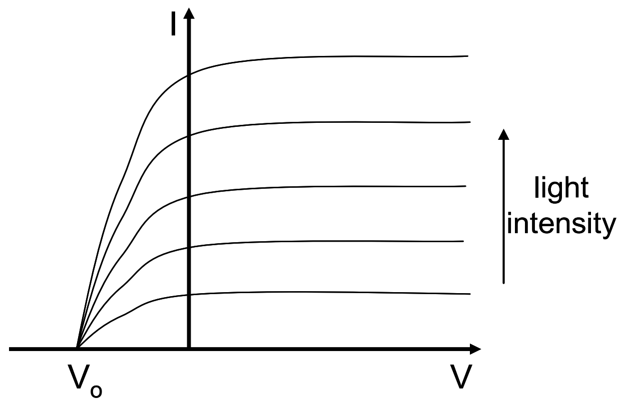
Chapter 2 Solutions

Prob. 2.1

(a&b) Sketch a vacuum tube device. Graph photocurrent I versus retarding voltage V for several light intensities.





Note that V_o remains same for all intensities.

(c) Find retarding potential.

λ=2440A=0.244μm $\qquad$ Φ=4.09eV

$$V_o = h\nu - \Phi = \frac{1.24\text{eV}\cdot\mu\text{m}}{\lambda(\mu\text{m})} - \Phi = \frac{1.24\text{eV}\cdot\mu\text{m}}{0.244\mu\text{m}} - 4.09\text{eV} = 5.08\text{eV} - 4.09\text{eV} \approx 1\text{eV}$$

Prob. 2.2

Show third Bohr postulate equates to integer number of DeBroglie waves fitting within circumference of a Bohr circular orbit.

$$r_n = \frac{4\pi\epsilon_o n^2\hbar^2}{mq^2} \text{ and } \frac{q^2}{4\pi\epsilon_o r^2} = \frac{mv^2}{r} \text{ and } p_\theta = mvr$$

$$r_n = \frac{4\pi\epsilon_o n^2\hbar^2}{mq^2} = \frac{n^2\hbar^2}{mr_B{}^2}\cdot\frac{4\pi\epsilon_o r_n{}^2}{q^2} = \frac{n^2\hbar^2}{mr_n{}^2}\cdot\frac{r_n}{mv^2} = \frac{n^2\hbar^2}{m^2v^2r_n}$$

$$m^2v^2r_n{}^2 = n^2\hbar^2$$

$$mvr_n = n\hbar$$

$p_\theta = n\hbar$ is the third Bohr postulate

Prob. 2.3

(a) Find generic equation for Lyman, Balmer, and Paschen series.

$$\Delta E = \frac{hc}{\lambda} = \frac{mq^4}{32\pi^2 \epsilon_o^{\,2} n_1^{\,2} \hbar^2} - \frac{mq^4}{32\pi^2 \epsilon_o^{\,2} n_2^{\,2} \hbar^2}$$

$$\frac{hc}{\lambda} = \frac{mq^4 (n_2^{\,2} - n_1^{\,2})}{32 \epsilon_o^{\,2} n_1^{\,2} n_2^{\,2} \hbar^2 \pi^2} = \frac{mq^4 (n_2^{\,2} - n_1^{\,2})}{8 \epsilon_o^{\,2} n_1^{\,2} n_2^{\,2} h^2}$$

$$\lambda = \frac{8 \epsilon_o^{\,2} n_1^{\,2} n_2^{\,2} h^2 \cdot hc}{mq^4 (n_2^{\,2} - n_1^{\,2})} = \frac{8 \varepsilon_o^{\,2} h^3 c}{mq^4} \cdot \frac{n_1^{\,2} n_2^{\,2}}{n_2^{\,2} - n_1^{\,2}}$$

$$\lambda = \frac{8(8.85 \cdot 10^{-12} \,\tfrac{F}{m})^2 \cdot (6.63 \cdot 10^{-34}\,J{\cdot}s)^3 \cdot 2.998 \cdot 10^8 \,\tfrac{m}{s}}{9.11 \cdot 10^{-31} kg \cdot (1.60 \cdot 10^{-19} C)^4} \cdot \frac{n_1^{\,2} n_2^{\,2}}{n_2^{\,2} - n_1^{\,2}}$$

$$\lambda = 9.11 \cdot 10^8 m \cdot \frac{n_1^{\,2} n_2^{\,2}}{n_2^{\,2} - n_1^{\,2}} = 9.11\text{Å} \cdot \frac{n_1^{\,2} n_2^{\,2}}{n_2^{\,2} - n_1^{\,2}}$$

n_1=1 for Lyman, 2 for Balmer, and 3 for Paschen

(b) Plot wavelength versus n for Lyman, Balmer, and Paschen series.

LYMAN SERIES				
n	n^2	n^2-1	n^2/(n^2-1)	911*n^2/(n^2-1)
2	4	3	1.33	1215
3	9	8	1.13	1025
4	16	15	1.07	972
5	25	24	1.04	949

LYMAN LIMIT 911Å

BALMER SERIES				
n	n^2	n^2-4	4n^2/(n^2-4)	911*4*n^2/(n^2-4)
3	9	5	7.20	6559
4	16	12	5.33	4859
5	25	21	4.76	4338
6	36	32	4.50	4100
7	49	45	4.36	3968

BALMER LIMIT 3644Å

PASCHEN SERIES				
n	n^2	n^2-9	9*n^2/(n^2-9)	911*9*n^2/(n^2-9)
4	16	7	20.57	18741
5	25	16	14.06	12811
6	36	27	12.00	10932
7	49	40	11.03	10044
8	64	55	10.47	9541
9	81	72	10.13	9224
10	100	91	9.89	9010

PASCHEN LIMIT 8199Å

Prob. 2.4

(a) Find Δp_x for Δx=1Å.

$$\Delta p_x \cdot \Delta x = \frac{h}{4\pi} \rightarrow \Delta p_x = \frac{h}{4\pi \cdot \Delta x} = \frac{6.63 \cdot 10^{-34} \mathrm{J \cdot s}}{4\pi \cdot 10^{-10} \mathrm{m}} = 5.03 \cdot 10^{-25} \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

(b) Find Δt for ΔE=1eV.

$$\Delta E \cdot \Delta t = \frac{h}{4\pi} \rightarrow \Delta t = \frac{h}{4\pi \cdot \Delta E} = \frac{4.14 \cdot 10^{-15} \mathrm{eV \cdot s}}{4\pi \cdot 1\mathrm{eV}} = 3.30 \cdot 10^{-16} \mathrm{s}$$

Prob. 2.5

Find wavelength of 100eV and 12keV electrons. Comment on electron microscopes compared to visible light microscopes.

$$E = \tfrac{1}{2} m v^2 \rightarrow v = \sqrt{\frac{2 \cdot E}{m}}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2 \cdot E \cdot m}} = \frac{6.63 \cdot 10^{-34} \mathrm{J \cdot s}}{\sqrt{2 \cdot 9.11 \cdot 10^{-31} \mathrm{kg}}} \cdot E^{-\frac{1}{2}} = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m}$$

For 100eV,

$$\lambda = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = (100\mathrm{eV} \cdot 1.602 \cdot 10^{-19} \tfrac{\mathrm{J}}{\mathrm{eV}})^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = 1.23 \cdot 10^{-10} \mathrm{m} = 1.23\text{Å}$$

For 12keV,

$$\lambda = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = (1.2 \cdot 10^{4} \mathrm{eV} \cdot 1.602 \cdot 10^{-19} \tfrac{\mathrm{J}}{\mathrm{eV}})^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = 1.12 \cdot 10^{-11} \mathrm{m} = 0.112\text{Å}$$

The resolution on a visible microscope is dependent on the wavelength of the light which is around 5000Å; so, the much smaller electron wavelengths provide much better resolution.

Prob. 2.6

Which of the following could NOT possibly be wave functions ***and why*?** Assume 1-D in each case. (Here i= imaginary number, C is a normalization constant)

A) Ψ (x) = C for all x.

B) Ψ (x) = C for values of x between 2 and 8 cm, and Ψ (x) = 3.5 C for values of x between 5 and 10 cm. Ψ (x) is zero everywhere else.

C) Ψ (x) = i C for x= 5 cm, and linearly goes down to zero at x= 2 and x = 10 cm from this peak value, and is zero for all other x.

If any of these are valid wavefunctions, calculate C for those case(s). What potential energy for x $\leq$ 2 and x $\geq$ 10 is consistent with this?

A) For a wavefunction $\Psi(x)$, we know $P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=1$

$$P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=c^2\int_{-\infty}^{\infty}dx \;\rightarrow P=\begin{cases}0 & c=0\\ \infty & c\neq 0\end{cases} \Rightarrow \Psi(x) \text{ cannot be a wave function}$$

B) For $5\le x\le 8$, $\Psi(x)$ has two values, C and 3.5C. For $c\neq 0$, $\Psi(x)$ is not a function and for $c=0$: $P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=0 \Rightarrow \Psi(x)$ cannot be a wave function.

C) $$\Psi(x)=\begin{cases}\frac{iC}{3}(x\text{-}2) & 2\le x\le 5\\ -\frac{iC}{5}(x\text{-}10) & 5\le x\le 10\end{cases}$$

$$P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=\int_{2}^{5}\frac{c^2}{9}(x\text{-}2)^2\,dx+\int_{5}^{10}\frac{c^2}{25}(x\text{-}10)^2\,dx$$

$$=\frac{c^2}{3\times 9}(x\text{-}2)^3\Big]_2^5+\frac{c^2}{3\times 25}(x\text{-}10)^3\Big]_5^{10}$$

$$=c^2\left[\frac{27}{27}+\frac{125}{3\times 25}\right]=\frac{8c^2}{3}$$

$$P=1 \;\Rightarrow \frac{8c^2}{3}=1 \;\Rightarrow c=0.612 \;\rightarrow \Psi(x) \text{ can be a wave function}$$

Since $\Psi(x)=0$ for $x\le 2$ and $x\ge 10$, the potential energy should be infinite in these two regions.

Prob. 2.7

A particle is described in 1D by a wavefunction:
$\Psi = Be^{-2x}$ for $x \geq 0$ and Ce^{+4x} for $x<0$, and B and C are real constants. Calculate B and C to make Ψ a valid wavefunction. Where is the particle most likely to be?

A valid wavefunction must be continuous, and normalized.

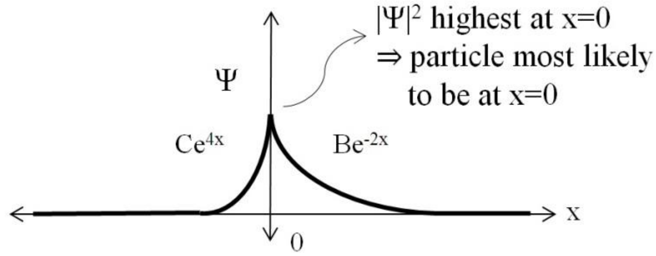
For $\Psi(0) = C = B$

To normalize Ψ, $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$

$$\int_{-\infty}^{0} C^2 e^{8x} dx + \int_{0}^{\infty} C^2 e^{-4x} dx = 1$$

$$\frac{C^2}{8}\left[e^{8x}\right]_{-\infty}^{0} + C^2\left(\frac{-1}{4}\right)\left[e^{-4x}\right]_{0}^{\infty} = 1$$

$$\frac{C^2}{8} + \frac{C^2}{4} = 1 \Rightarrow C = \sqrt{\frac{8}{3}}$$



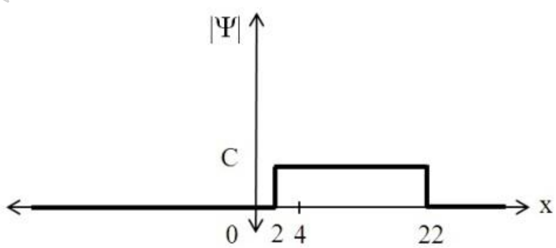
Prob. 2.8

The electron wavefunction is Ce^{ikx} between x=2 and 22 cm, and zero everywhere else. What is the value of C? What is the probability of finding the electron between x=0 and 4 cm?

$$\Psi = Ce^{ikx}$$

$$\int_{2}^{22} \Psi^* \Psi dx = C^2(20) = 1 \Rightarrow C = \frac{1}{\sqrt{20}} \text{cm}^{-1}$$

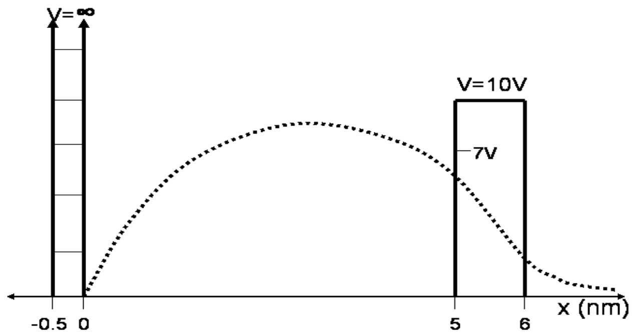
$$\text{Probability} = \int_{0}^{4} |\Psi|^2 dx = \left(\frac{1}{\sqrt{20}}\right)^2 (2) = \frac{1}{10}$$



Prob. 2.9

Find the probability of finding an electron at x<0. Is the probability of finding an electron at x>0 zero or non-zero? Is the classical probability of finding an electron at x>6 zero or non?

The energy barrier at x=0 is infinite; so, there is zero probability of finding an electron at x<0 ($|\psi|^2=0$). However, it is possible for electrons to tunnel through the barrier at 5<x<6; so, the probability of finding an electron at x>6 would be quantum mechanically greater than zero ($|\psi|^2>0$) and classical mechanically zero.



Prob. 2.10

Find $4\cdot p_x^{\,2}+2\cdot p_z^{\,2}+7mE$ *for* $\Psi(x,y,z,t) = A\cdot e^{j(10\cdot x+3\cdot y\text{-}4\cdot t)}$.

$$\left\langle p_x^{\,2}\right\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(\frac{\hbar}{j}\frac{\partial}{\partial x}\right)^2 A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dx}{\int_{-\infty}^{\infty} |A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dx} = 100\cdot\hbar^2$$

$$\left\langle p_z^{\,2}\right\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(\frac{\hbar}{j}\frac{\partial}{\partial z}\right)^2 A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dz}{\int_{-\infty}^{\infty} |A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dz} = 0$$

$$\left\langle E\right\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(-\frac{\hbar}{j}\frac{\partial}{\partial t}\right) A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dt}{\int_{-\infty}^{\infty} |A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dt} = 4\cdot\hbar$$

$$4\cdot p_x^{\,2}+2\cdot p_z^{\,2}+7mE = 400\hbar^2+28(9.11\cdot 10^{-31}\text{kg})\,\hbar$$

Prob. 2.11

Find the uncertainty in position (Δx) and momentum (Δp).

$$\Psi(x,t)=\sqrt{\frac{2}{L}}\cdot\sin\left(\frac{\pi x}{L}\right)\cdot e^{-2\pi jEt/h}\quad\text{and}\quad\int_0^L\Psi^*\cdot\Psi dx=1$$

$$\langle x\rangle=\int_0^L\Psi^*\cdot x\cdot\Psi dx=\frac{2}{L}\int_0^L x\cdot\sin^2\left(\frac{\pi x}{L}\right)dx=0.5L\ \ \text{(from problem note)}$$

$$\langle x^2\rangle=\int_0^L\Psi^*\cdot x\cdot\Psi dx=\frac{2}{L}\int_0^L x^2\cdot\sin^2\left(\frac{\pi x}{L}\right)dx=0.28L^2\ \ \text{(from problem note)}$$

$$\Delta x=\sqrt{\langle x^2\rangle-\langle x\rangle^2}=\sqrt{0.28L^2-(0.5L)^2}=0.17L$$

$$\Delta p\geq\frac{h}{4\pi\cdot\Delta x}=0.47\cdot\frac{h}{L}$$

Prob. 2.12

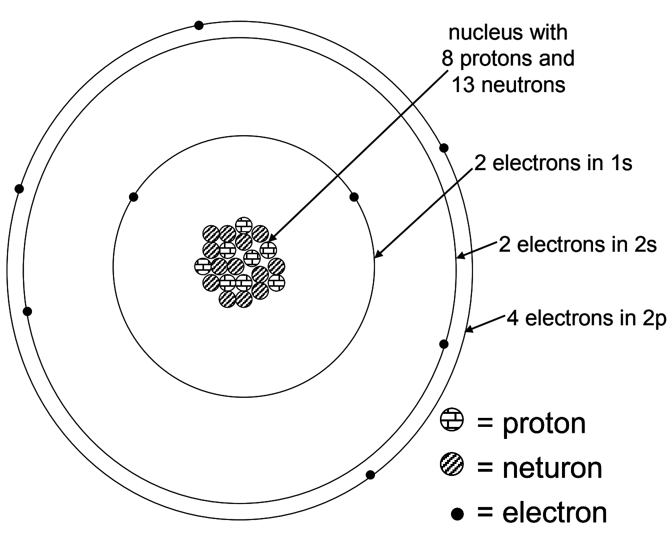
Calculate the first three energy levels for a 10Å quantum well with infinite walls.

$$E_n=\frac{n^2\cdot\pi^2\cdot\hbar^2}{2\cdot m\cdot L^2}=\frac{(6.63\cdot10^{-34})^2}{8\cdot9.11\cdot10^{-31}\cdot(10^{-9})^2}\cdot n^2=6.03\cdot10^{-20}\cdot n^2$$

$$E_1=6.03\cdot10^{-20}J=0.377eV$$
$$E_2=4\cdot0.377eV=1.508eV$$
$$E_3=9\cdot0.377eV=3.393eV$$

Prob. 2.13

Show schematic of atom with $1s^2 2s^2 2p^4$ and atomic weight 21. Comment on its reactivity.



This atom is chemically reactive because the outer 2p shell is not full. It will tend to try to add two electrons to that outer shell.