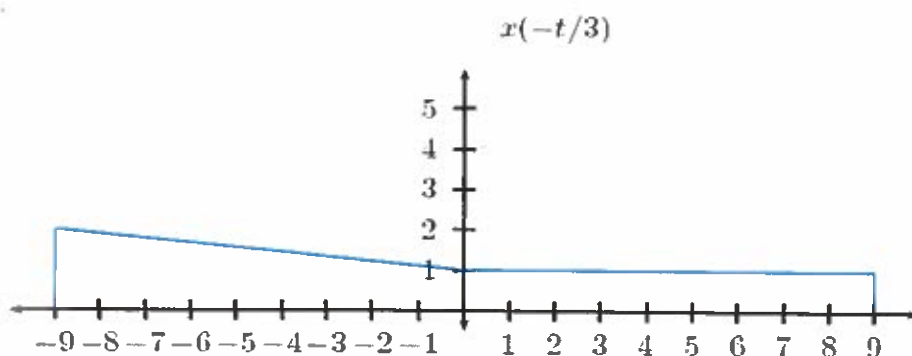


Chapter 2 solutions

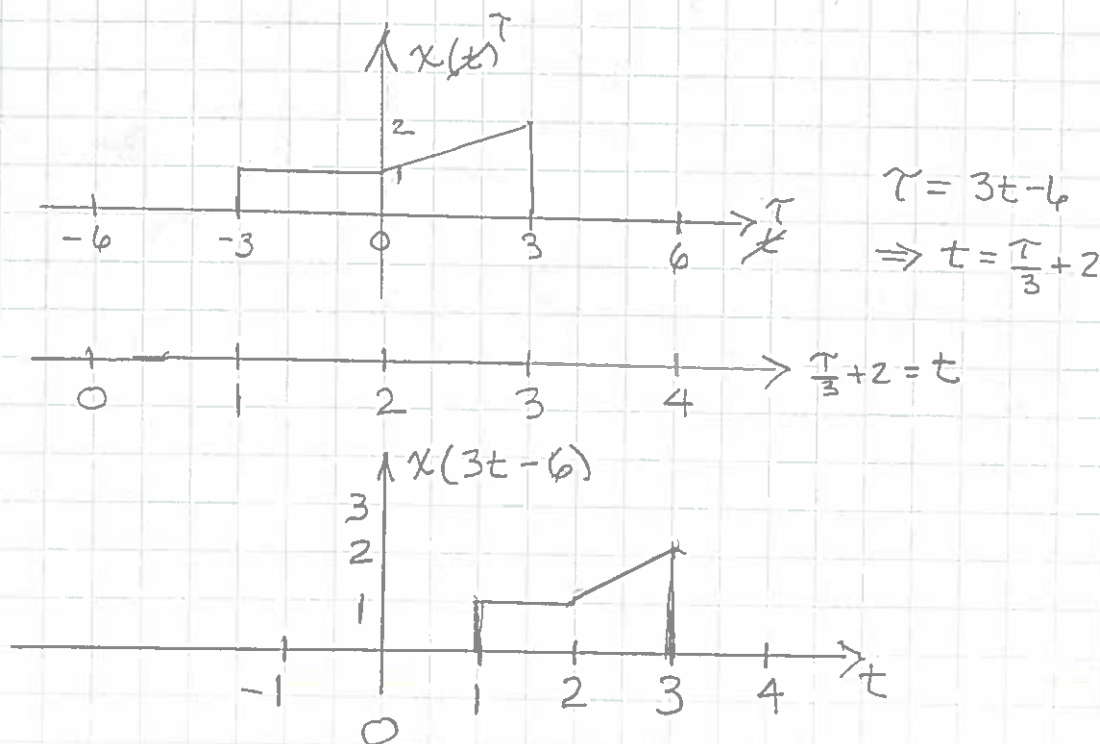
2.1

(a)

(i)

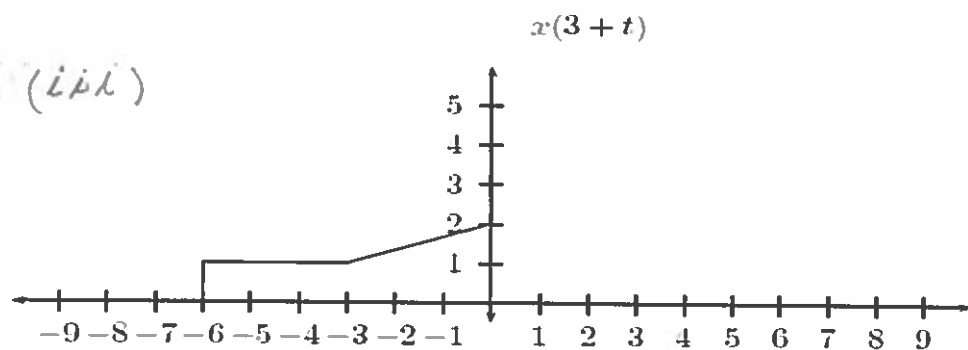


(ii)

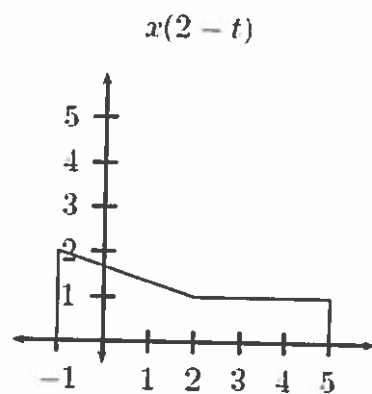


PROBLEM 2.1(a) continued

$x(t)$

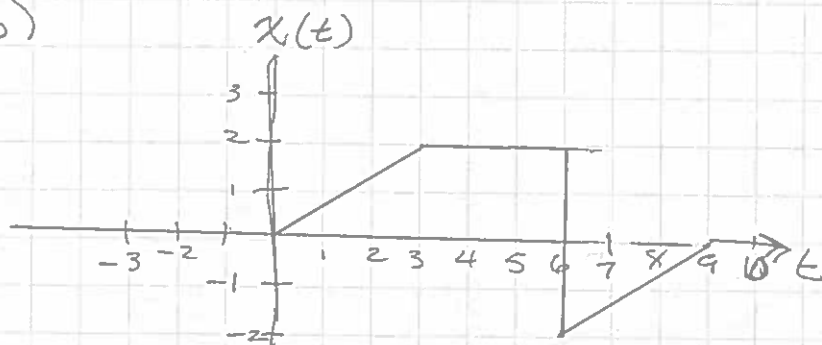


(iv)

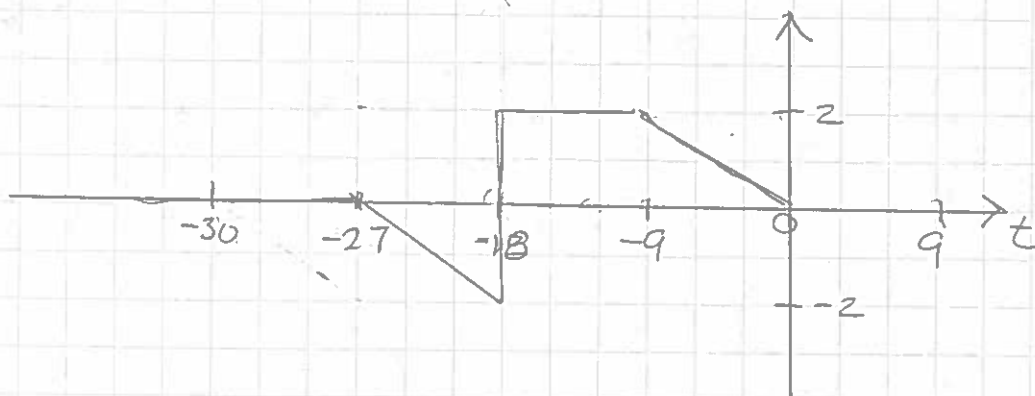


Solutions

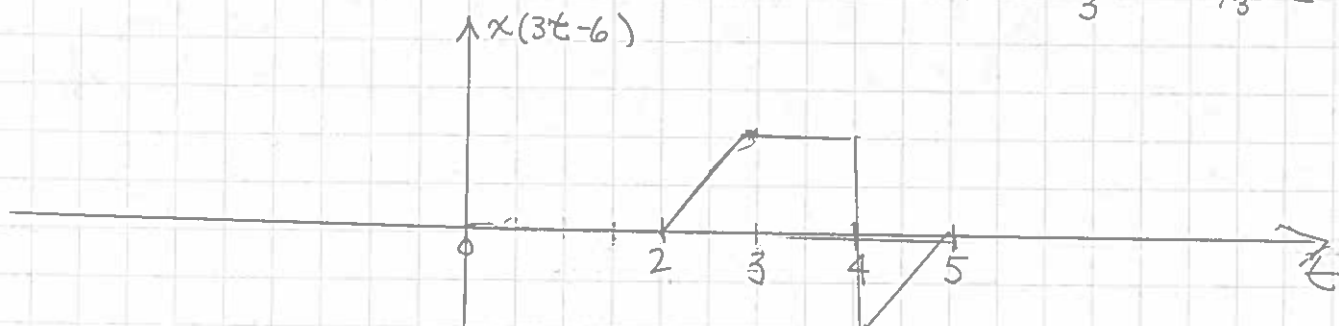
2.1(b)



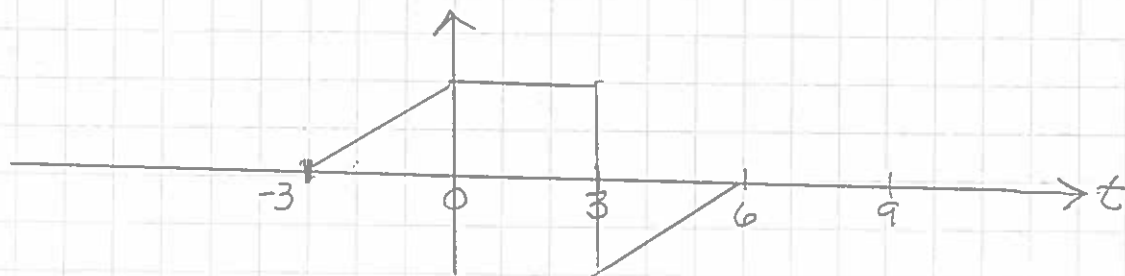
(i) $x(-t/3) = x(\tau) \Rightarrow \tau = -t/3 \Rightarrow t = -3\tau$



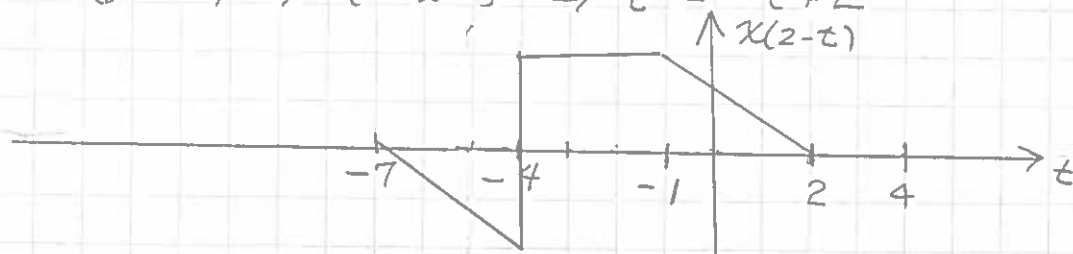
(ii) $x(3t-6) = x(\tau) \Rightarrow \tau = 3t-6 \Rightarrow t = \frac{\tau+6}{3} = \tau/3 + 2$



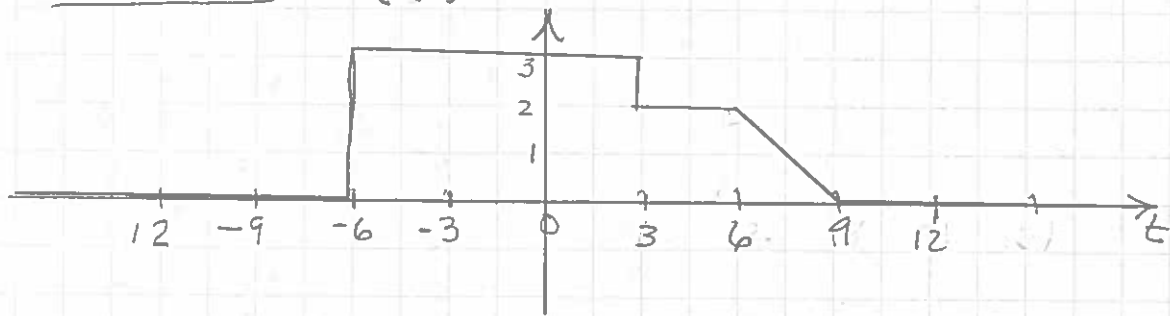
(iii) $x(3+t) = x(\tau) \Rightarrow \tau = t+3 \Rightarrow t = \tau-3$



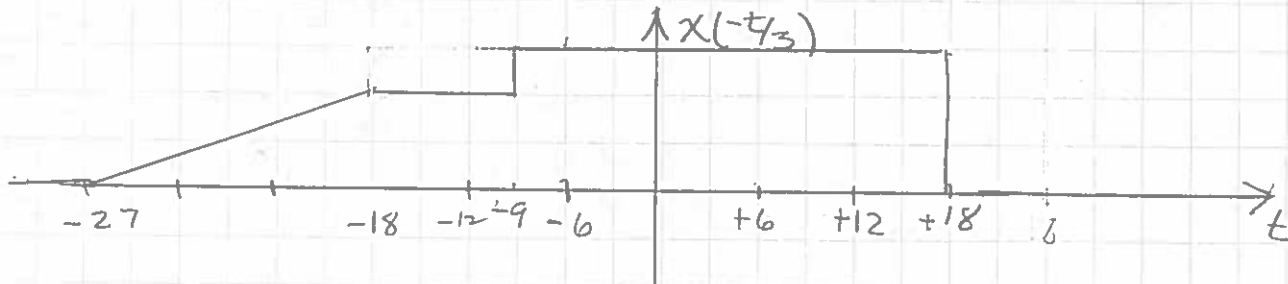
(iv) $x(2-t) \Rightarrow \tau = 2-t \Rightarrow t = -\tau+2$



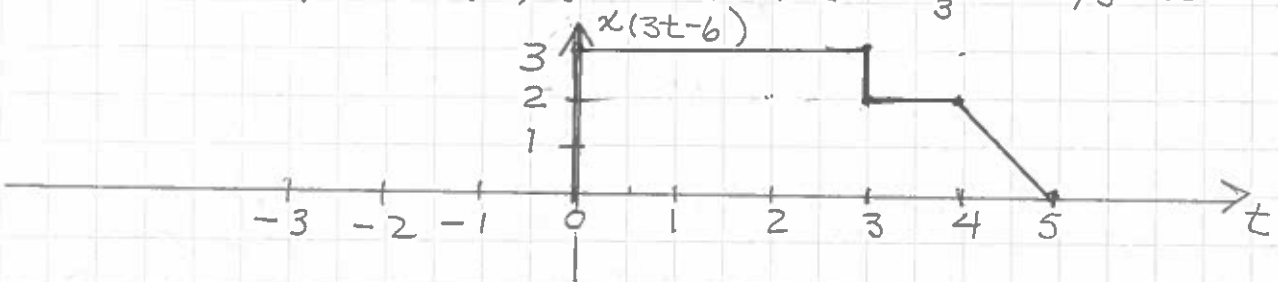
Problem 2.1(c)



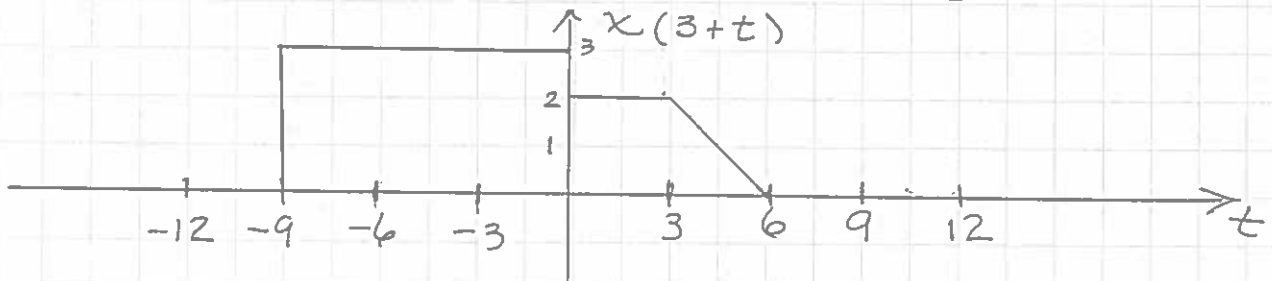
(i) $x(-t/3) = x(\tau) \Rightarrow \tau = -t/3 \Rightarrow t = -3\tau$



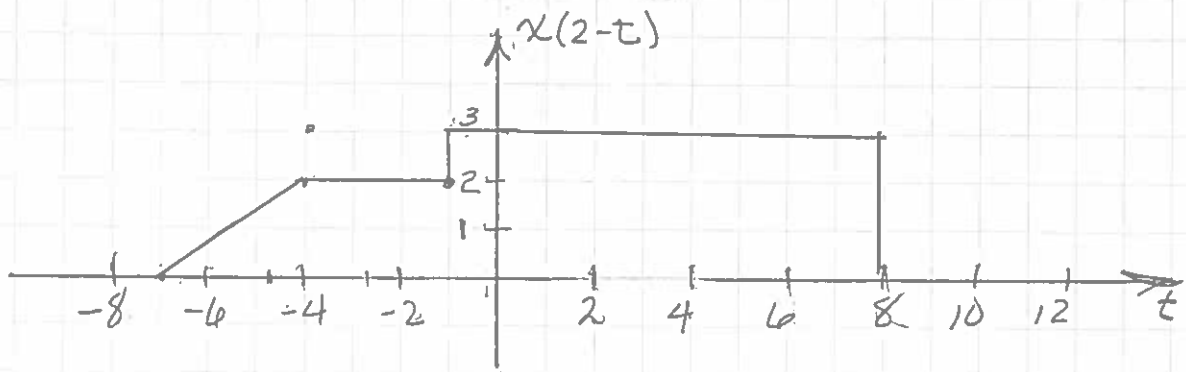
(ii) $x(3t-6) = x(\tau) \Rightarrow \tau = 3t-6 \Rightarrow t = \frac{\tau+6}{3} = \tau/3 + 2$



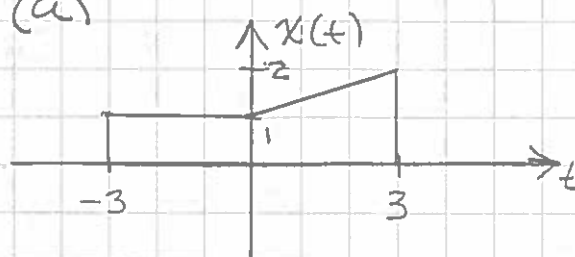
(iii) $x(3+t) = x(\tau) \Rightarrow \tau = 3+t \Rightarrow t = \tau-3$



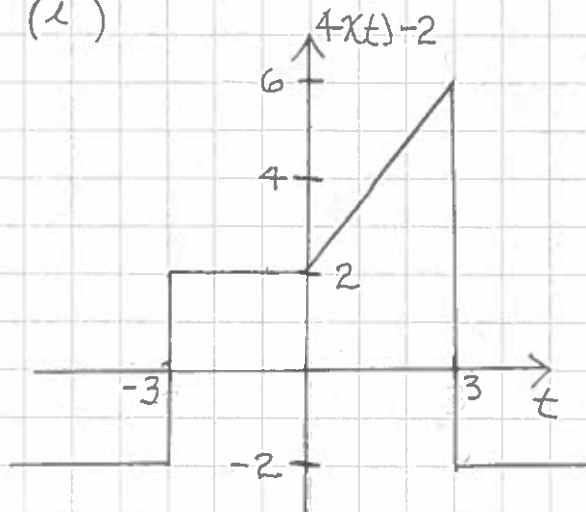
(iv) $x(2-t) = x(\tau) \Rightarrow \tau = 2-t \Rightarrow t = -\tau+2$



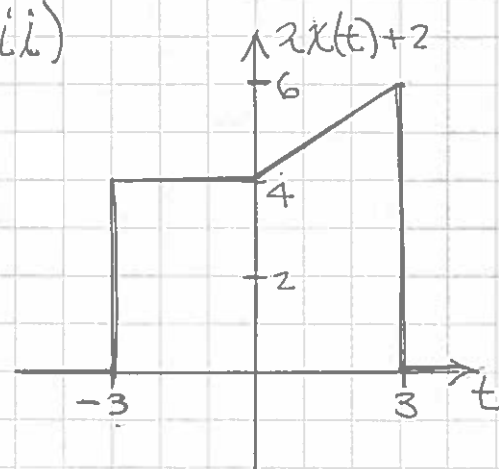
PROBLEM 2.2 (a)



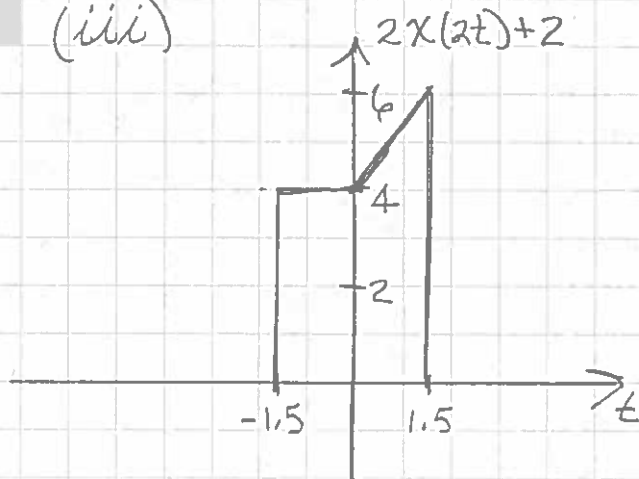
(i)



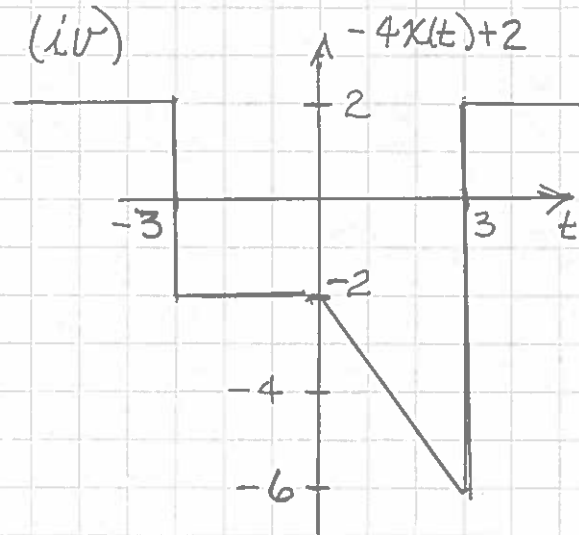
(ii)



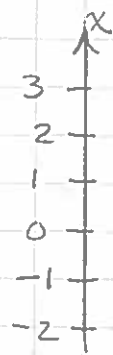
(iii)



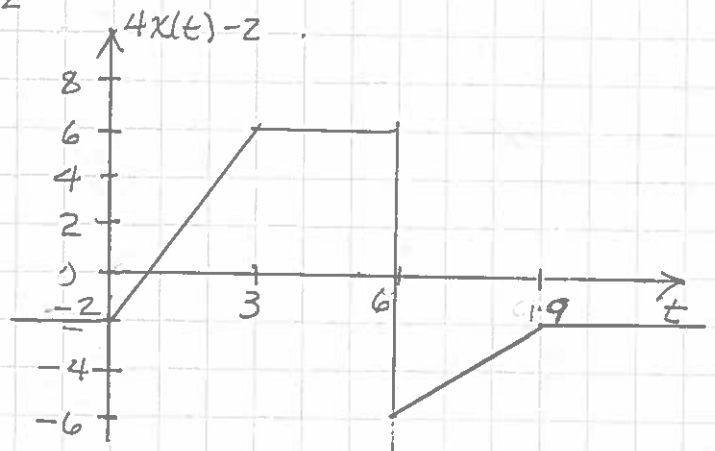
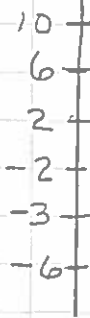
(iv)



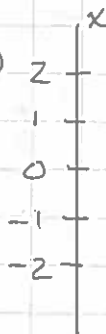
2.2(b)(i)



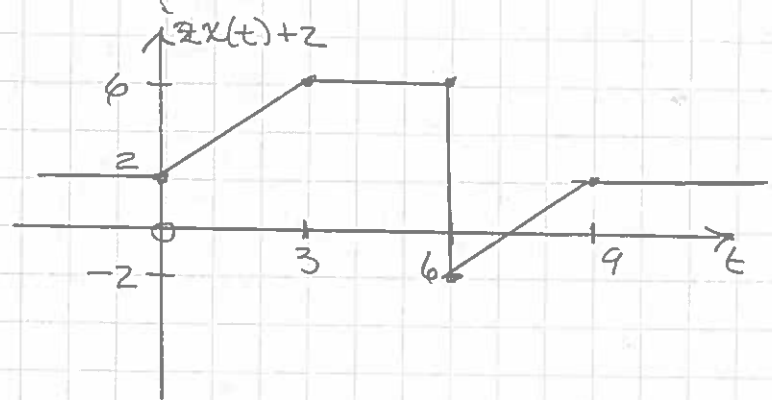
$$y = 4x - 2$$



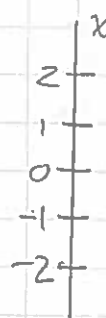
(ii)



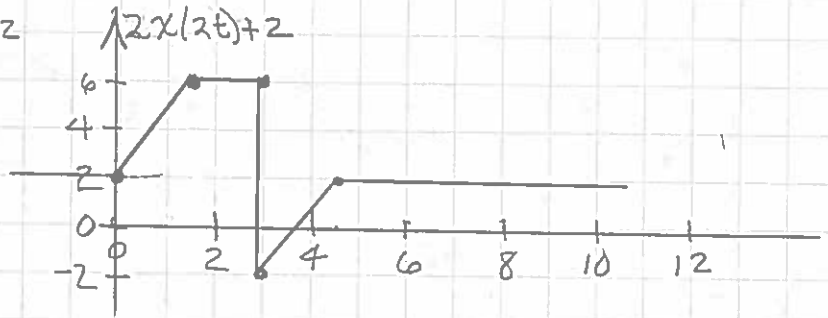
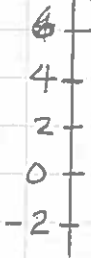
$$y = 2x + 2$$



(iii) $y(\tau) = 2x(2t) + 2 \Rightarrow \tau = 2t \Rightarrow t = \tau/2$



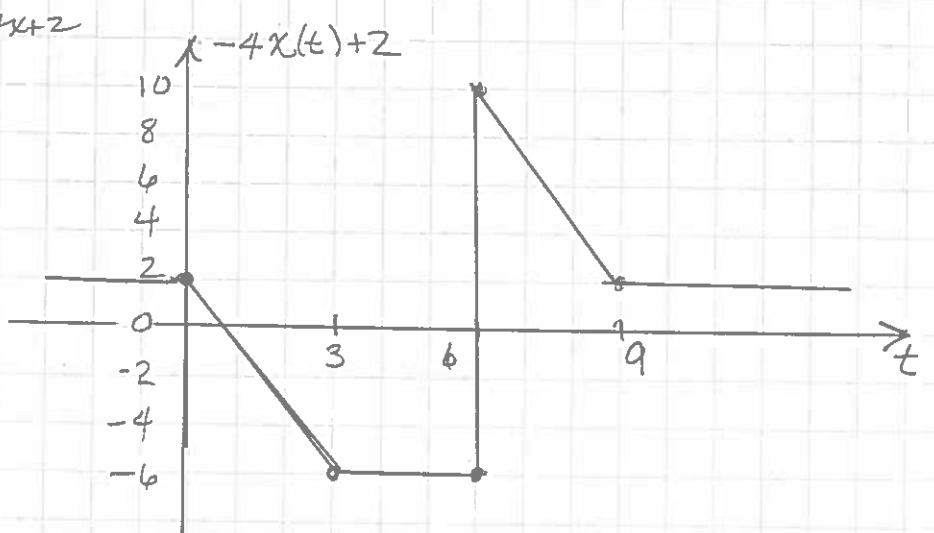
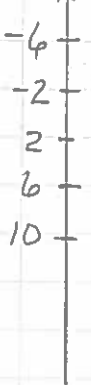
$$y = 2x + 2$$



(iv) $y(\tau) = -4x(t) + 2 \Rightarrow \tau = t$

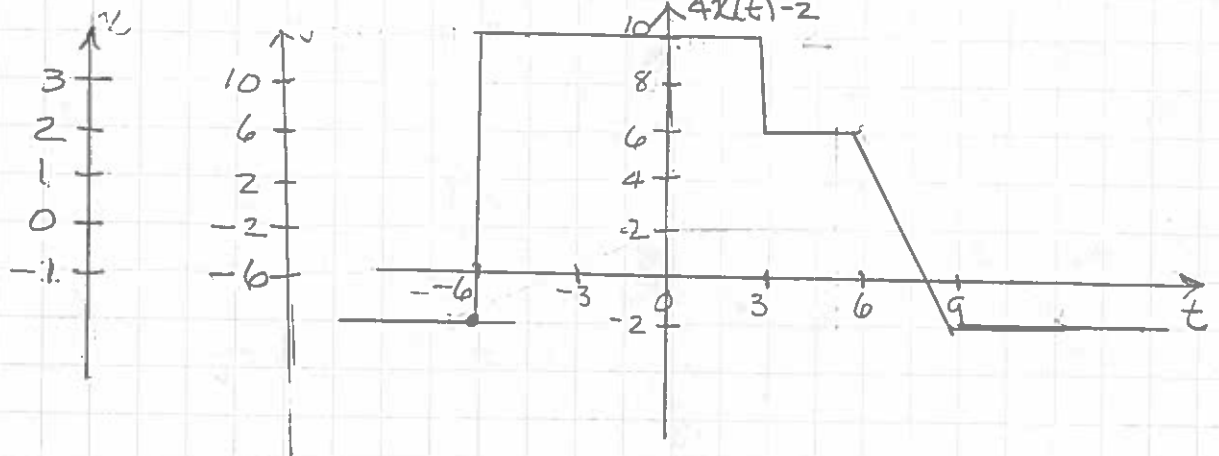


$$y = -4x + 2$$

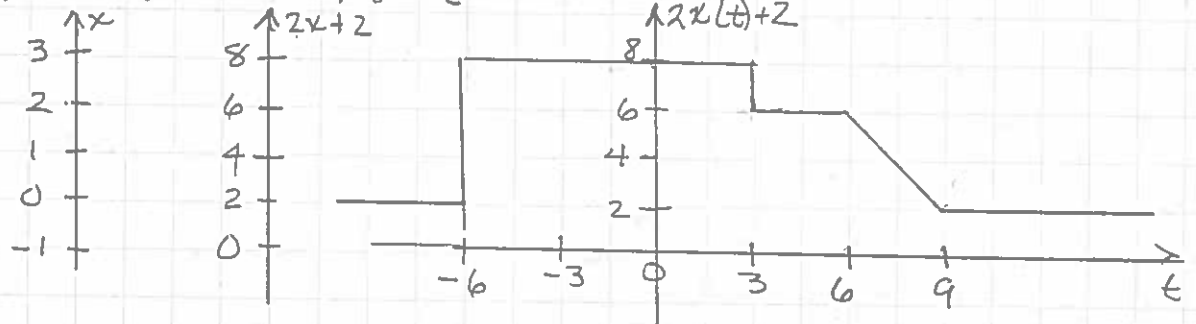


P2.2(c)

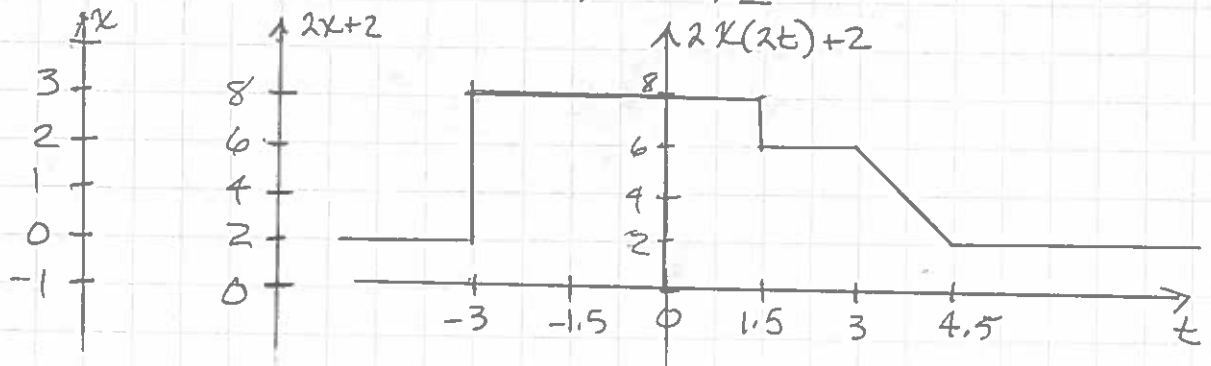
(i) $y(\tau) = 4x(t) - 2, \Rightarrow \tau = t$



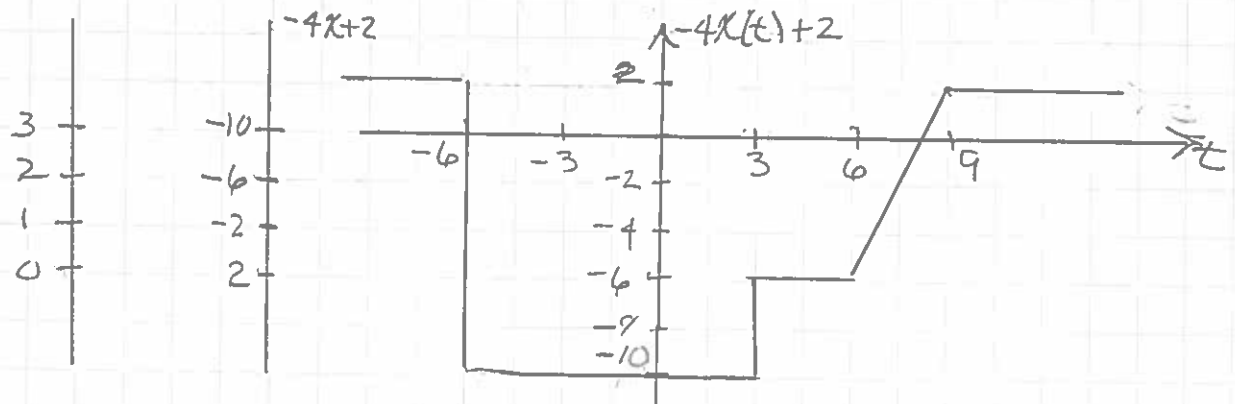
(ii) $y(\tau) = 2x(t) + 2, \Rightarrow \tau = t$



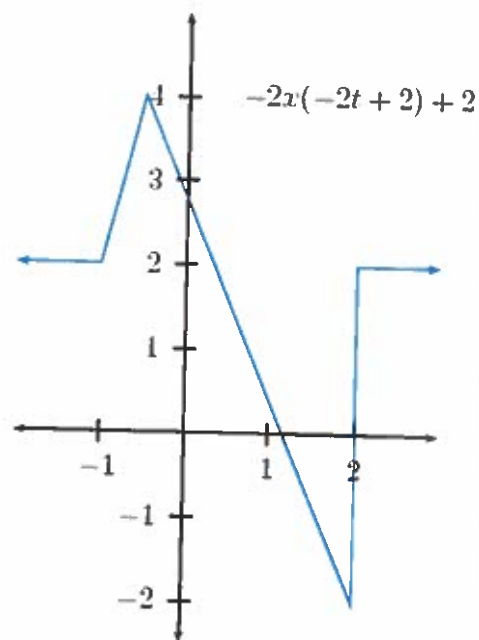
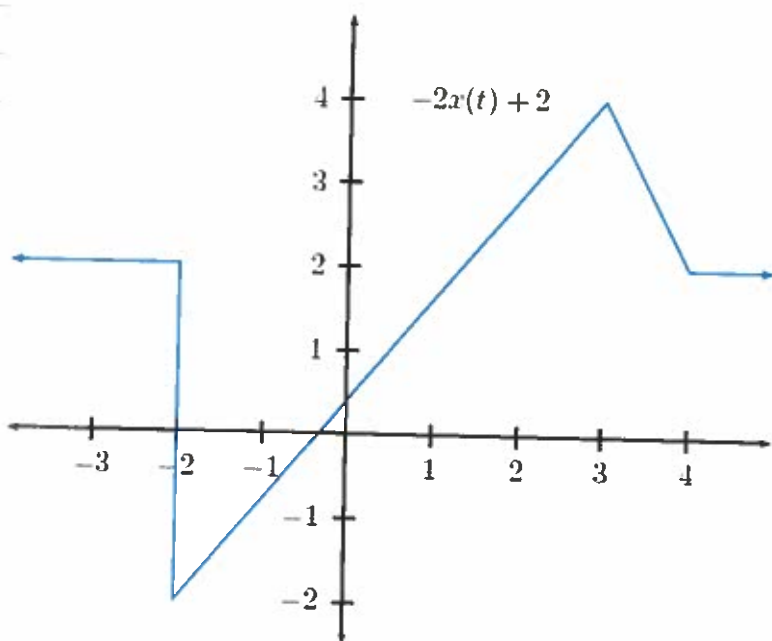
(iii) $y(\tau) = 2x(2t) + 2, \Rightarrow \tau = 2t \Rightarrow t = \tau/2$



(iv) $y(\tau) = -4x(t) + 2, \Rightarrow \tau = t$



PROBLEM 2.3



(a)

$$y(t) = -2(x(-2t + 2)) + 2$$

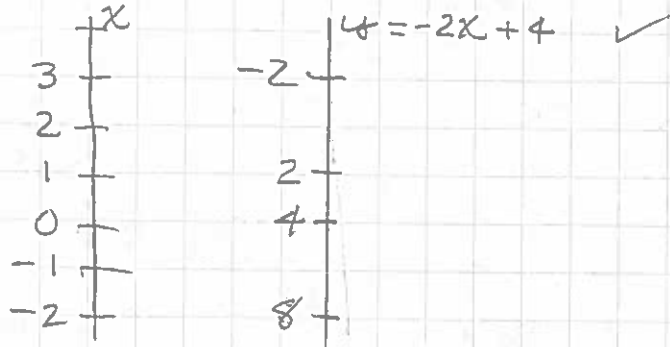
(b)

t	$y(t)$	$-2t + 2$	$-2(x(-2t - 1)) + 2$
-0.5	4	3	4
-1	2	4	2
1	0.4	0	0.4

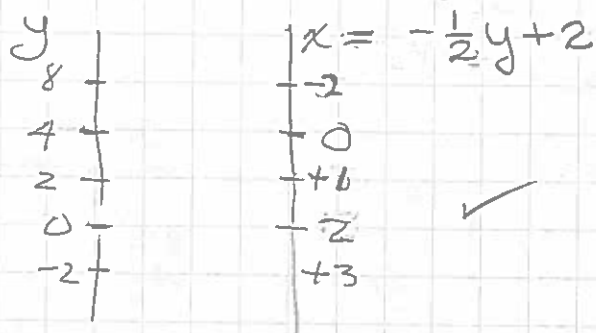
Solution P 2.4

(a) $y(t) = -2x(-t/2 + 2) + 4$

$\Rightarrow \tau = -t/2 + 2 \Rightarrow t = -2\tau + 4$



(b) $x(t) = -\frac{1}{2}y(2t+4)+2 \Rightarrow \tau = -2t+4$
 $\Rightarrow t = -\frac{1}{2}\tau + 2$

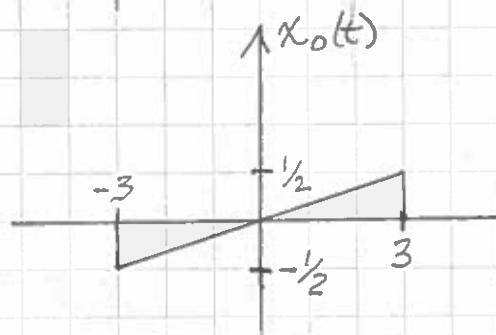
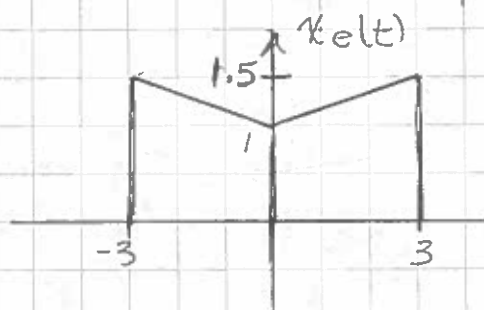


Problem 2.5

$$(a) \quad x_e(t) = \frac{1}{2} [x(t) + x(-t)] \quad (2.13)$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)] \quad (2.14)$$

t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
> 3	0	0	0	0
3	2	1	$\frac{3}{2}$	$\frac{1}{2}$
1.5	1.5	1	1.25	0.25
0	1	1	0	0
-1.5		1.5	1.25	-0.25
-3		2	$\frac{3}{2}$	$-\frac{1}{2}$
< -3	0	0	0	0

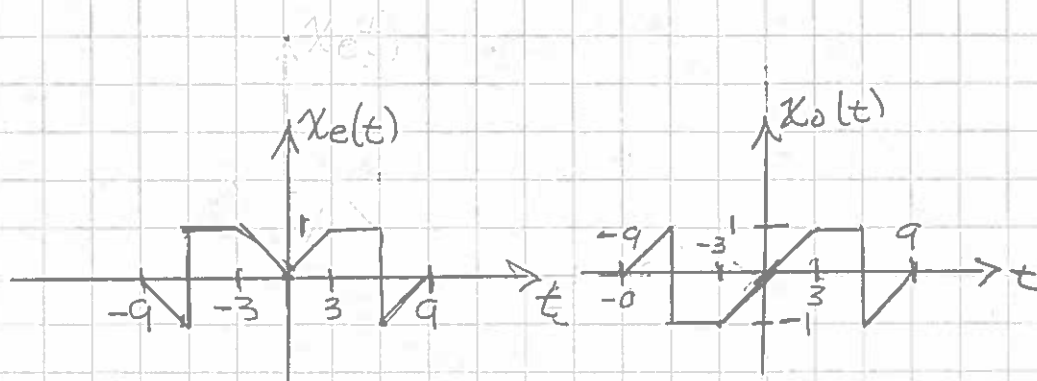


verify $x_o(t) + x_e(t) = x(t)$

Problem 2.5

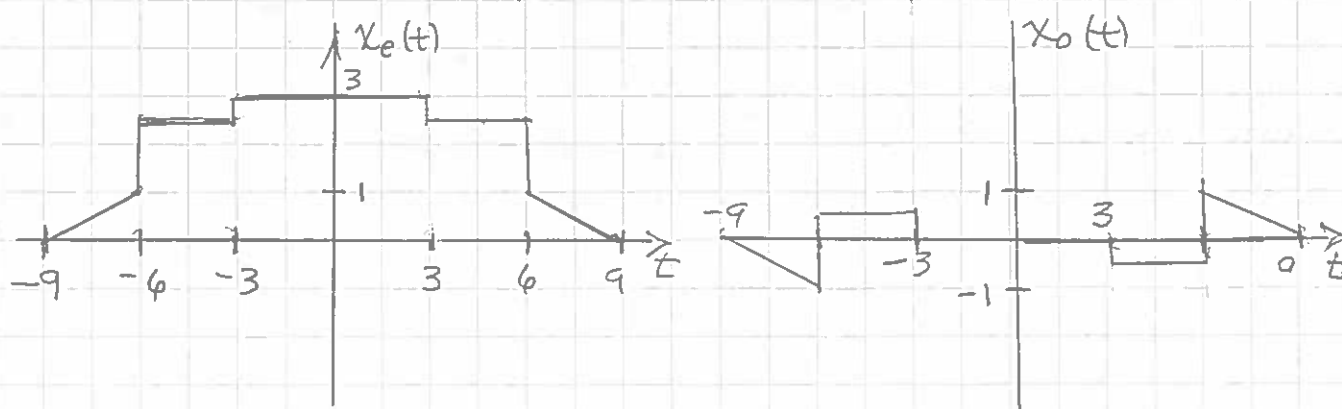
(b)

t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
>9	0	0	0	0
6^+	-2	0	-1	-1
6^-	2	0	-1	-1
3	2	0	1	1
0	0	0	0	0
-3	0	2	1	-1
-6^+	0	-2	-1	1



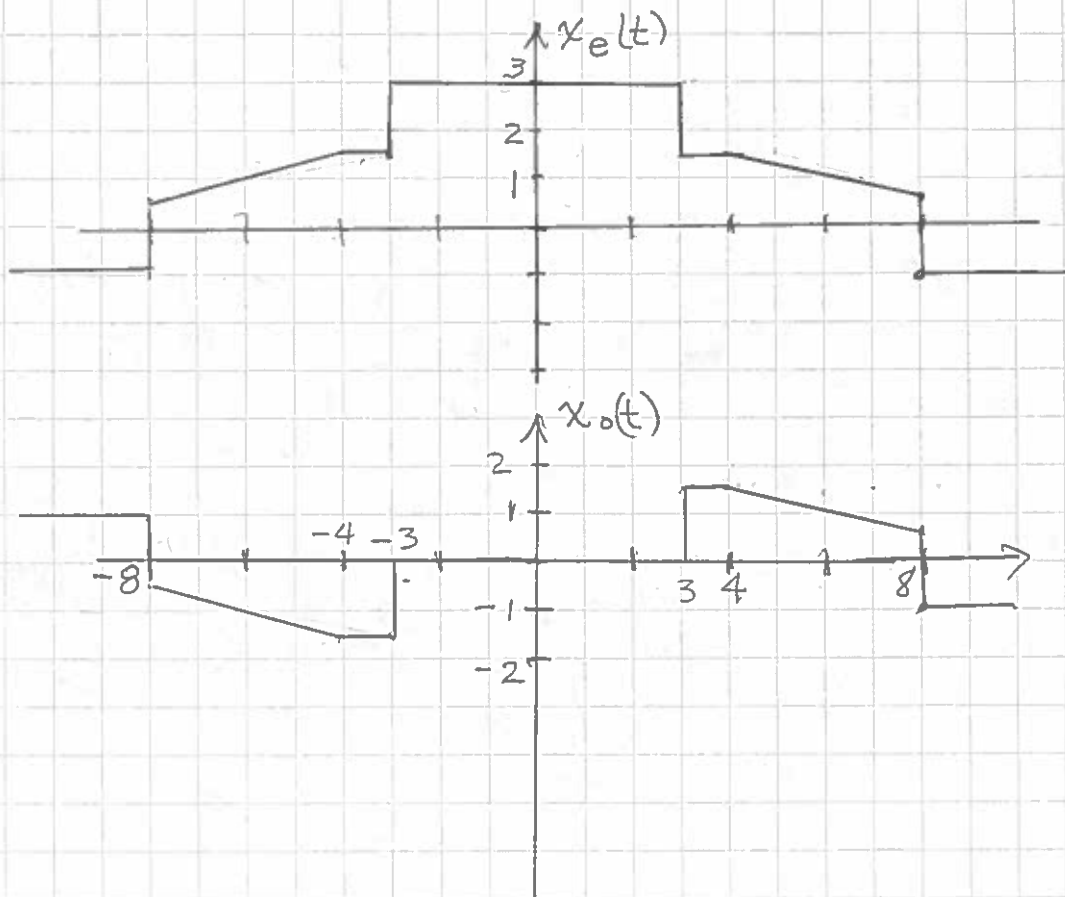
(c)

t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
>9	0	0	0	0
6^+	2	0	1	1
6^-	2	0	2.5	-0.5
3	3	0	3	0
0	3	0	3	0
-3^+	3	3	3	0
-6^+	3	2	2.5	0.5
-6^-	0	2	1	-1



Problem 2.5 (d)

t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
$\geq 8^+$	-2	0	-1	-1
8^-	1	0	0.5	0.5
4	3	0	1.5	1.5
3^+	3	0	1.5	1.5
3^-	3	3	3	0
-3^+	3	3	3	0
-3^-	0	3	1.5	-1.5
-4	0	3	1.5	-1.5
-8^+	0	1	0.5	-0.5
$\leq 8^-$	0	-2	-1	+1



Problem 2.6

$$(a) \quad x(t) = -4t \Rightarrow x(-t) = 4t$$

$$x(t) = -x(-t) \quad \therefore \underline{x(t) \text{ is odd}}$$

$$(b) \quad x(t) = e^{-|t|} \Rightarrow x(-t) = e^{-|-t|} = e^{-|t|}$$

$$x(t) = x(-t) \quad \therefore \underline{x(t) \text{ is even}}$$

$$(c) \quad x(t) = 5 \cos 3t \Rightarrow x(-t) = 5 \cos 3(-t)$$

$$= 5 \cos -3t = 5 \cos 3t$$

$$x(t) = x(-t) \quad \therefore \underline{x(t) \text{ is even}}$$

$$(d) \quad x(t) = \sin\left(3t + \frac{3\pi}{2}\right) = \sin\left(3\left[t + \frac{\pi}{2}\right]\right)$$

$$x(t) = -\cos(3t)$$

$$x(t) = x(-t) \quad \therefore \underline{x(t) \text{ is even}}$$

$$(e) \quad x(t) = u(t) - u(-t) \Rightarrow x(-t) = u(-t) - u(t)$$

$$= -[u(t) - u(-t)]$$

$$x(t) = -x(-t) \quad \therefore \underline{x(t) \text{ is odd}}$$

$$(f) \quad x(t) = -u(t-1) + u(-t-1)$$

$$x(-t) = -u(-t-1) + u(t-1)$$

$$x(-t) = -x(t), \quad \therefore \underline{x(t) \text{ is odd}}$$

PROBLEM 2.7

$$(a) \int_{-T}^T x_o(t) dt = \int_{-T}^0 x_o(t) dt + \int_0^T x_o(t) dt \quad ; \quad x_o(t) = -x_o(-t)$$

$$\therefore \int_{-T}^0 x_o(t) dt = - \int_{-T}^0 x_o(-t) dt \Big|_{t=-T} = \int_{-T}^0 x_o(\tau) d\tau = - \int_0^T x_o(\tau) d\tau$$

$$\therefore \int_{-T}^T x_o(t) dt = 0$$

$$(b) \int_{-T}^T x(t) dt = \int_{-T}^T [x_e(t) + x_o(t)] dt = \int_{-T}^T x_e(t) dt$$

$$\text{and } A_x = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x_e(t) d\tau$$

(c) $x_o(0) = -x_o(-0) = -x_o(0)$. The only number with $a = -a$ is $a = 0$ so this implies $x_o(0) = 0$.
 $x(0) = x_e(0) + x_o(0) = x_e(0)$.

PROBLEM 2.8

(a) Let $z(t)$ be the sum of two even functions $x_1(t)$ and $x_2(t)$. To show that $z(t)$ is even, we need to show that $z(t) = z(-t)$ for all t . This is easy to show, since $z(t) = x_1(t) + x_2(t)$ and $z(-t) = x_1(-t) + x_2(-t)$ (since to get $z(-t)$ we just plug in $-t$ everywhere for t , which amounts to just plugging in $-t$ in $x_1(t)$ and $x_2(t)$). Now since $x_1(t)$ and $x_2(t)$ are even, by definition $x_1(t) = x_1(-t)$ and $x_2(t) = x_2(-t)$ so $x_1(t) + x_2(t) = x_1(-t) + x_2(-t)$ so $z(t) = z(-t)$.

(b) Let $x_1(t)$ and $x_2(t)$ be two odd functions. Then $x_1(-t) + x_2(-t) = -x_1(t) + (-x_2(t)) = -(x_1(t) + x_2(t))$ which shows that $x_1(t) + x_2(t)$ is odd.

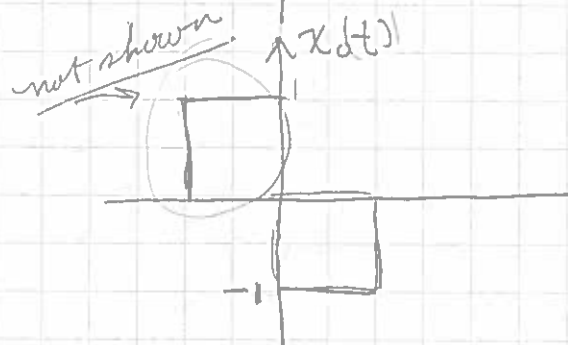
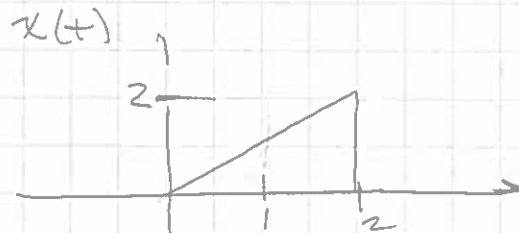
(c) Let $z(t) = x_1(t) + x_2(t)$ as in part a, where now $x_1(-t) = x_1(t)$ and $x_2(-t) = -x_2(t)$. We need to show that $z(t) \neq z(-t)$, $z(t) \neq -z(-t)$. Consider that $z(-t) = x_1(-t) + x_2(-t) = x_1(t) - x_2(t)$. In order to have $z(t)$ be even, we would therefore need to have $x_1(t) + x_2(t) = x_1(t) - x_2(t)$ for all t , which is equivalent to having $x_2(t) = -x_2(t)$ for all t , which is not possible for nonzero $x_2(t)$. Similarly, in order to have $z(t)$ be odd, we would need to have $z(t) = -z(-t) \implies x_1(t) + x_2(t) = x_2(t) - x_1(t)$, which is not possible for nonzero $x_1(t)$. So the sum of an even and odd function must be neither even nor odd.

(d) Let $z(t) = x_1(t)x_2(t)$ where $x_1(t) = x_1(-t)$ and $x_2(t) = x_2(-t)$. Then $z(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = z(t)$ which shows that $z(t)$ is even.

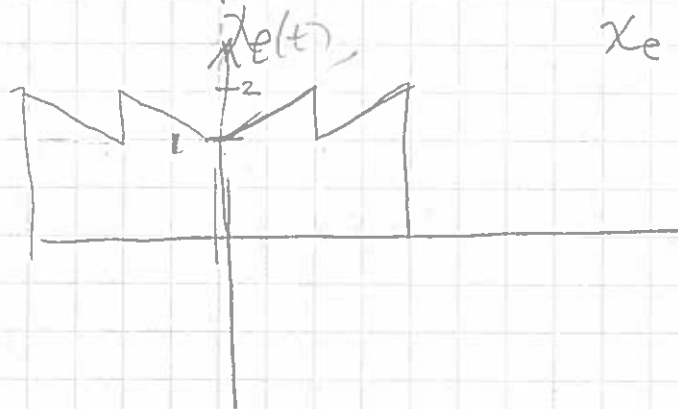
(e) Let $z(t) = x_1(t)x_2(t)$, where $x_1(t) = -x_1(-t)$ and $x_2(t) = -x_2(-t)$. Clearly $z(t)$ is even because $z(-t) = x_1(-t)x_2(-t) = (-x_1(t))(-x_2(t)) = x_1(t)x_2(t) = z(t)$, which is the definition of evenness.

(f) Let $z(t) = x_1(t)x_2(t)$, where $x_1(t) = -x_1(-t)$ and $x_2(t) = x_2(-t)$. Clearly $z(t)$ is odd because $z(-t) = x_1(-t)x_2(-t) = (-x_1(t))x_2(t) = -x_1(t)x_2(t) = -z(t)$, which is the definition of oddness.

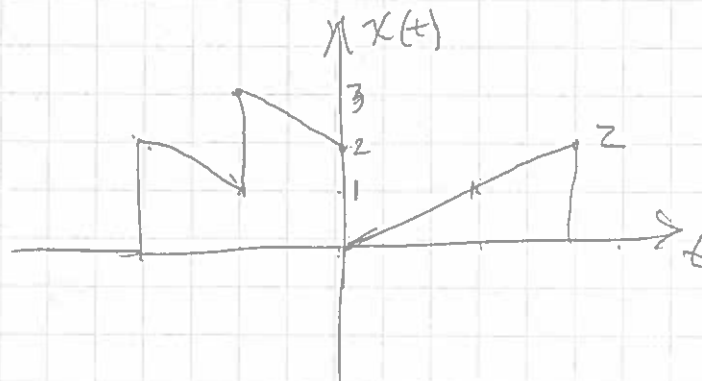
P 2.9



$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



$$x_e = \frac{x(t) + x(-t)}{2}$$



$$x(t) = x_o(t) + x_e(t)$$

PROBLEM 2.10

(a) $\sin(t) = \sin(t + n2\pi)$ for any integer n , so $7\sin(3t) = 7\sin(3t + n2\pi) = 7\sin(3(t + n\frac{2\pi}{3}))$; therefore $x(t)$ is periodic with fundamental period $T_0 = \frac{2\pi}{3}$ and fundamental frequency $\omega_0 = \frac{2\pi}{T_0} = 3$.

(b) $\sin(8(t + \frac{2\pi}{8}) + 30) = \sin(8t + 2\pi + 30) = \sin(8t + 30)$.
 $\omega_0 = 8$ and $T_0 = \frac{2\pi}{8} = \frac{\pi}{4}$.

(c) $e^{jt} = \cos(t) + j\sin(t)$ is periodic with fundamental period 2π , so e^{j2t} is periodic with fundamental period $\frac{2\pi}{2} = \pi$, and fundamental frequency $\omega_0 = 2$.

(d) $\cos 2t + \sin 5t$

$$T_1 = \frac{2\pi}{2} = \pi, T_2 = \frac{2\pi}{5} \Rightarrow \frac{T_1}{T_2} = \frac{\pi}{2\pi/5} = 5/2 \text{ ratio of integers}$$

$$T_0 = k_0 T_1 \therefore T_0 = 2\pi(\text{s}), (\text{periodic}) \quad k_0 = 2$$

$$(e) e^{-j(10t + \pi/3)} = e^{-j\pi/3} e^{-j10t} = (\cos \pi/3 - j \sin \pi/3) e^{-j10t} \\ = (0.5 + j0.866) e^{-j10t}$$

$$T_0 = \frac{2\pi}{10} = \pi/5 (\text{s}), \text{ periodic}$$

$$(f) e^{j15t} - e^{j20t}$$

e^{j15t} & e^{j20t} are both periodic

$$T_1 = \frac{2\pi}{15}, T_2 = \frac{2\pi}{20} \Rightarrow \frac{T_1}{T_2} = \frac{2/15}{2/20} = \frac{20}{15} \text{ ratio of integers} \\ \therefore \text{periodic}$$

$$\frac{20}{15} = \frac{4}{3} \Rightarrow k_0 = 3 \therefore T_0 = 3T_1 = \frac{2\pi}{5} \text{ s}$$

$$\frac{2\pi}{\frac{2\pi}{5}} = 3, \quad \frac{2\pi}{\frac{2\pi}{20}} = 4 \quad \checkmark$$

P 2, 11

(a) $x(t) = \cos 3t + \sin 5t$

(b) $x(t) = \cos t + \sin \pi t$

(c) $x(t) = \cos 3t + \sin 9t$

(d) $x(t) = \cos 3\pi t + \sin 4\pi t + \cos(5t)$

(e) $x(t) = \cos 4\pi t + \sin 8\pi t + e^{j5\pi t}$

(f) $x(t) = \cos(8t + 30^\circ) + e^{j2t} + \sin(3\pi t)$

Solution

(a) $T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{5}$, $\frac{T_1}{T_2} = \frac{\frac{2\pi}{3}}{\frac{2\pi}{5}} = \frac{5}{3}$ ✓
 $T_0 = 3T_1 = 2\pi$ = periodic

(b) $T_1 = \frac{2\pi}{1}$, $T_2 = \frac{2\pi}{\pi} = 1$, $\frac{T_1}{T_2} = 2\pi$ not a ratio of integers
 $\therefore$ not periodic

(c) $T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{9}$, $\frac{T_1}{T_2} = \frac{9}{3} = 3/1 \Rightarrow$ ratio of integers
 $T_0 = \frac{2\pi}{3}$, periodic

(d) $T_1 = \frac{2\pi}{3\pi} = \frac{2}{3}$, $T_2 = \frac{2\pi}{4\pi} = \frac{1}{2}$, $T_3 = \frac{2\pi}{5}$
 $\frac{T_1}{T_2} = \frac{2/3}{1/2} = \frac{4}{3}$, $\frac{T_1}{T_3} = \frac{2/3}{2\pi/5} = \frac{10}{6\pi} = \frac{5}{3\pi} \leftarrow$ NOT A RATIO OF INTEGERS
 $\therefore$ SUM IS NOT PERIODIC

(e) $T_1 = \frac{2\pi}{4\pi} = \frac{1}{2}$, $T_2 = \frac{2\pi}{8\pi} = \frac{1}{4}$, $T_3 = \frac{2\pi}{5\pi} = \frac{2}{5}$
 $\frac{T_1}{T_2} = \frac{1/2}{1/4} = \frac{2}{1}$, $\frac{T_1}{T_3} = \frac{1/2}{2/5} = \frac{5}{4}$ both ratios of integers
 $\therefore$ sum periodic
 LCM of denominators = $4 \times 2 = 8 = k_0$

$T_0 = 8T_1 = 4$
 (f) $T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{2}$, $T_3 = \frac{2\pi}{3\pi}$, $\frac{T_1}{T_3} = \frac{2\pi/3}{2/3} = \pi$ not rational
 $\therefore$ sum not periodic

2.12

(a) $x(t) = 5 \sin(15t - 60^\circ) + 2 \sin(7t)$

$x_1(t) = 5 \sin(15t - 60^\circ)$ is periodic $\omega_1 = 15 \text{ rad/s}$
 $x_2(t) = 2 \sin(7t)$ " " $\omega_2 = 7 \text{ rad/s}$

$T_1 = \frac{2\pi}{15}$, $T_2 = \frac{2\pi}{7}$ $\frac{T_1}{T_2} = \frac{7}{15}$ ratio of integer
 $\therefore$ Sum is periodic
 $k_0 = 15 \Rightarrow T_0 = 15T_1 = \underline{2\pi}$

(b) $x_1(t) = 5 \sin 5t$ is periodic $\omega_1 = 5$

$x_2(t) = 5e^{-15t}$ is periodic $\omega_2 = 15$
 $x_3(t) = \sin 7t$ $\omega_3 = 7$

$T_1 = \frac{2\pi}{5}$, $T_2 = \frac{2\pi}{15}$, $T_3 = \frac{2\pi}{7}$

$\frac{T_1}{T_2} = \frac{15}{5} = 3$, $\frac{T_1}{T_3} = \frac{7}{5}$ lcm denom = 5

$T_0 = 5T_1 = 2\pi$

(c) $x_1(t)$ is periodic

$T_1 = \frac{2\pi}{\pi} = 2$

x_2 is not periodic

$T_2 = \frac{2\pi}{3}$

$\frac{T_1}{T_2} = \frac{2}{2\pi/3} = \frac{3}{\pi}$ not rational $\therefore$ Sum not periodic

$\cos 4\pi t$ is periodic w/ $T_1 = \frac{2\pi}{4\pi} = 1/2$

(d) $\sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t+n/2}{0.2}\right)$ is periodic with $T_1 = 0.5$

$4 \sin\left(\frac{5\pi}{7}t + \pi/4\right)$ is periodic w/ $T_2 = \frac{2\pi}{5\pi/7} = \frac{14}{5}$

$\frac{T_1}{T_2} = \frac{1/2}{14/5} = \frac{5}{28}$ $\Rightarrow k_0 = 28$, $T_0 = 28T_1 = \underline{14}$

PROBLEM 2.13

(a) For $x_1(t) + x_2(t)$ to be periodic we need some number T such that $x_1(t+T) + x_2(t+T) = x_1(t) + x_2(t)$ for all t . This can only be true if $x_1(t+T) = x_1(t)$ and $x_2(t+T) = x_2(t)$, which can only be true if $T = k_1 T_1$ and $T = k_2 T_2$ (T is an integer multiple of both the periods). So we need there to be some integers k_1 and k_2 such that $k_1 T_1 = k_2 T_2 \implies \frac{T_1}{T_2} = \frac{k_2}{k_1}$.

(b) Put $\frac{k_2}{k_1}$ in its most reduced form $\frac{n}{m}$ by canceling any common terms in the numerator and denominator; then $T_0 = nT_2 = mT_1$.

Problem 2.14

(a)

```
>> syms t
>> xa=5*exp(-t/2);
>> ezplot(xa), grid
```

(c)

```
>> syms t
>> xc=5*exp(t/2);
>> ezplot(xc), grid
```

(e)

```
>> syms t
>> xe=5*(1-exp(-2*t));
>> ezplot(xe), grid
```

(g)

```
>> syms t
>> xg=5*exp(-2*t)*2*sin(2*t);
>> ezplot(xg), grid
```

(b)

```
>> syms t
>> xb=5*exp(-2*t);
>> ezplot(xb), grid
```

(d)

```
>> syms t
>> xd=5*(1-exp(-t/2));
>> ezplot(xd), grid
```

(f)

```
>> syms t
>> xf=5*2*sin(2*t);
>> ezplot(xf), grid
```

(h)

```
>> syms t
>> xh=5*exp(-0.5*t)*2*sin(2*t);
>> ezplot(xh), grid
```

Problem 2.15

(a)

$$\begin{aligned}\cos(\theta + \phi) &= \operatorname{Re}\{e^{j(\theta + \phi)}\} = \operatorname{Re}\{e^{j\theta} e^{j\phi}\} \\ &= \operatorname{Re}\{(\cos\theta + j\sin\theta)(\cos\phi + j\sin\phi)\} \\ &= \operatorname{Re}\{\cos\theta\cos\phi + j\sin\theta\cos\phi \\ &\quad + j\cos\theta\sin\phi - \sin\theta\sin\phi\} \\ &= \cos\theta\cos\phi - \sin\theta\sin\phi\end{aligned}$$

(b)

$$\begin{aligned}\sin(\theta + \phi) &= \operatorname{Im}\{e^{j(\theta + \phi)}\} = \operatorname{Im}\{e^{j\theta} e^{j\phi}\} \\ &= \operatorname{Im}\{(\cos\theta + j\sin\theta)(\cos\phi + j\sin\phi)\} \\ &= \operatorname{Im}\{\cos\theta\cos\phi + j\sin\theta\cos\phi \\ &\quad + j\cos\theta\sin\phi - \sin\theta\sin\phi\} \\ &= \cos\theta\sin\phi + \sin\theta\cos\phi\end{aligned}$$

(c)

$$\begin{aligned}\cos\theta\cos\phi &= \operatorname{Re}\left\{e^{j\theta} \frac{e^{j\phi} + e^{-j\phi}}{2}\right\} = \operatorname{Re}\left\{\frac{e^{j(\theta + \phi)} + e^{j(\theta - \phi)}}{2}\right\} \\ &= \operatorname{Re}\left\{\frac{e^{j(\theta + \phi)}}{2} + \frac{e^{j(\theta - \phi)}}{2}\right\} = \frac{\cos(\theta + \phi)}{2} + \frac{\cos(\theta - \phi)}{2}\end{aligned}$$

(d)

$$\begin{aligned}\sin\theta\cos\phi &= \operatorname{Im}\left\{e^{j\theta} \frac{e^{j\phi} + e^{-j\phi}}{2}\right\} \\ &= \operatorname{Im}\left\{\frac{e^{j(\theta + \phi)} + e^{j(\theta - \phi)}}{2}\right\} \\ &= \frac{1}{2}[\sin(\theta + \phi) + \sin(\theta - \phi)]\end{aligned}$$

PROBLEM 2.16

(a) $x(t) = 3\cos(2t) + \sin(2t)$

$$\begin{aligned}x(t) &= \frac{3}{2}(e^{j2t} + e^{-j2t}) + \frac{1}{2j}(e^{j2t} - e^{-j2t}) \\ &= \left(\frac{3}{2} - j\frac{1}{2}\right)e^{j2t} + \left(\frac{3}{2} + j\frac{1}{2}\right)e^{-j2t} \\ &= \frac{\sqrt{10}}{2} \tan^{-1}\left(\frac{1}{3}\right) e^{j2t} + \frac{\sqrt{10}}{2} \tan^{-1}\left(\frac{1}{3}\right) e^{-j2t}\end{aligned}$$

$$\tan^{-1}\left(\frac{1}{3}\right) = -0.32 \text{ rad}$$

$$\tan^{-1}\left(\frac{1}{3}\right) = +0.32 \text{ rad}$$

$$\begin{aligned}x(t) &= \frac{\sqrt{10}}{2} e^{-j0.32} e^{j2t} + \frac{\sqrt{10}}{2} e^{+j0.32} e^{-j2t} \\ &= \sqrt{10} \frac{e^{j(2t - 0.32)} + e^{-j(2t - 0.32)}}{2}\end{aligned}$$

$$x(t) = \sqrt{10} \cos(2t - 0.32 \text{ rad})$$

$$= 3.16 \cos(2t - 18.33^\circ)$$

Problem 2.1.6 (Continued)

$$\begin{aligned} (b) \quad x(t) &= 4 \cos(4\pi t) + 3 \sin(4\pi t) \\ &= \frac{4}{2} \left(\frac{e^{j4\pi t} + e^{-j4\pi t}}{2} \right) + 3 \left(\frac{e^{j4\pi t} - e^{-j4\pi t}}{2j} \right) \\ &= (2 - 3/2j) e^{j4\pi t} + (2 + 3/2j) e^{-j4\pi t} \end{aligned}$$

$$\begin{aligned} (2 - 3/2j) &= \sqrt{(2)^2 + (3/2)^2} \angle \tan^{-1}\left(\frac{-3/2}{2}\right) = \frac{5}{2} \angle \tan^{-1}(-3/4) \\ &= \frac{5}{2} e^{-j0.64} \end{aligned}$$

$$\begin{aligned} x(t) &= \frac{5}{2} e^{-j0.64} e^{j4\pi t} + \frac{5}{2} e^{j0.64} e^{-j4\pi t} \\ &= 5 \frac{e^{j(4\pi t - 0.64)} + e^{-j(4\pi t - 0.64)}}{2} \end{aligned}$$

$$\begin{aligned} x(t) &= 5 \cos(4\pi t - 0.64 \text{ rad}) \\ &\text{or } 5 \cos(4\pi t - 36.87^\circ) \end{aligned}$$

$$\begin{aligned} (c) \quad x(t) &= A \cos(\omega_0 t) + B \sin(\omega_0 t) \\ &= A \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + B \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j2} \\ &= \frac{A - jB}{2} e^{j\omega_0 t} + \frac{A + jB}{2} e^{-j\omega_0 t} \\ &= \sqrt{\frac{A^2 + B^2}{4}} \angle \tan^{-1}\left(\frac{-B}{A}\right) e^{j\omega_0 t} + \sqrt{\frac{A^2 + B^2}{4}} \angle \tan^{-1}\left(\frac{B}{A}\right) e^{-j\omega_0 t} \\ &= \frac{\sqrt{A^2 + B^2}}{2} e^{j \tan^{-1}(-B/A) \omega_0 t} + \frac{\sqrt{A^2 + B^2}}{2} e^{j \tan^{-1}(B/A) \omega_0 t} \\ &\quad \tan^{-1}\left(\frac{B}{A}\right) = -\tan^{-1}\left(-B/A\right) \\ \therefore x(t) &= \frac{\sqrt{A^2 + B^2}}{2} \left(e^{j(\omega_0 t - \tan^{-1}(B/A))} + e^{-j(\omega_0 t - \tan^{-1}(B/A))} \right) \\ &= \sqrt{A^2 + B^2} \cos(\omega_0 t - \tan^{-1}(B/A)) \end{aligned}$$

PROBLEM 2.17

$$\int_{-\infty}^{\infty} \delta(at-b) \sin^2(t-c) dt$$

$$\delta(at-b) = \delta(a(t-b/a)) = \frac{1}{a} \delta(t-b/a)$$

$$\delta(t-b/a) \sin^2(t-c) = \sin^2(b/a-c) \delta(t-b/a)$$

$$\begin{aligned} \therefore \int_{-\infty}^{\infty} \delta(at-b) \sin^2(t-c) dt &= \frac{1}{a} \sin^2(b/a-c) \int_{-\infty}^{\infty} \delta(t-b/a) dt \\ &= \frac{1}{a} \sin^2(b/a-c) \end{aligned}$$

PROBLEM 2.18

$$(a) \quad y(t) = \int_{-\infty}^{\infty} x(\tau) [\delta(\tau+5) - \delta(\tau-5)] d\tau$$

$$x(\tau) \delta(\tau-a) = x(a) \delta(\tau-a)$$

$$\therefore y(t) = x(-5) \int_{-\infty}^{\infty} \delta(\tau+5) d\tau - x(5) \int_{-\infty}^{\infty} \delta(\tau-5) d\tau$$

$$y(t) = x(-5) - x(5)$$

$$(b) \quad y(t) = \frac{1}{2} \int_{-\infty}^{\infty} x(t) e^{j\pi/2 t} \delta(2t-3) dt$$

$$\delta(2t-3) = \frac{1}{2} \delta(t-3/2)$$

$$\frac{1}{2} \delta(t-3/2) x(t) e^{j\pi/2 t} = \frac{1}{2} x(3/2) e^{j3\pi/4}$$

$$\therefore y(t) = \frac{1}{4} x(3/2) e^{j3\pi/4} \int_{-\infty}^{\infty} \delta(t-3/2) d\tau$$

$$y(t) = \frac{1}{4} x(3/2) e^{j3\pi/4}$$

PROBLEM 2.19

$$(a) \text{ Let } \tau = at, \text{ then } \int_{-\infty}^{\infty} \delta(at) dt = \int_{-\infty}^{\infty} \delta(\tau) \frac{d\tau}{a} \\ = \frac{1}{a} \int_{-\infty}^{\infty} \delta(\tau) d\tau \Rightarrow \delta(at) = \frac{1}{a} \delta(t), a > 0$$

$$\text{for } a < 0, at = \tau \Rightarrow -|a|t = \tau \\ \Rightarrow dt = -\frac{d\tau}{|a|}$$

$$\therefore \int_{-\infty}^{\infty} \delta(at) dt = \int_{\infty}^{-\infty} \delta(\tau) \frac{-d\tau}{|a|} = \frac{1}{|a|} \int_{-\infty}^{\infty} \delta(\tau) d\tau$$

$$\therefore \delta(at) = \frac{1}{|a|} \delta(t) \text{ for the general case.}$$

$$(b) \int_{-\infty}^t \delta(\sigma) d\sigma = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases} = u(t)$$

$$\therefore \int_{-\infty}^t \delta(\tau - t_0) d\tau = \begin{cases} 1, & t > t_0 \\ 0, & t < t_0 \end{cases} = u(t - t_0)$$

(c)

Recall the rules about integrating delta functions: $\delta(t)$ is nonzero only at $t = 0$, so $x(t)\delta(t) = x(0)\delta(t)$, and $\int_{-\infty}^{\infty} \delta(t) dt = 1$, so $\int_{-\infty}^{\infty} x(t)\delta(t) dt = \int_{-\infty}^{\infty} x(0)\delta(t) dt = x(0) \int_{-\infty}^{\infty} \delta(t) dt = x(0)$. We can time-shift the delta function: $\delta(t - t_0)$ is nonzero only at $t = t_0$, so $x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$ and $\int_{-\infty}^{\infty} x(t)\delta(t - t_0) dt = x(t_0)$.

$$i) \int_{-\infty}^{\infty} \cos(2t)\delta(t) dt = \cos(2 \cdot 0) \int_{-\infty}^{\infty} \delta(t) dt = 1.$$

ii) $\delta(t - \frac{\pi}{4})$ is a time-shifted version of $\delta(t)$, and is nonzero only at $t = \frac{\pi}{4}$. So:

$$\int_{-\infty}^{\infty} \sin(2t)\delta(t - \frac{\pi}{4}) dt = \int_{-\infty}^{\infty} \sin(2 \cdot \frac{\pi}{4})\delta(t - \frac{\pi}{4}) dt \\ = \sin(\frac{\pi}{2}) \int_{-\infty}^{\infty} \delta(t - \frac{\pi}{4}) dt = \sin(\frac{\pi}{2}) = 1$$

$$2.19 \text{ (c) (iii)} \int_{-\infty}^{\infty} \sin(2t) \delta(t - \pi/6) dt$$

$$= \int_{-\infty}^{\infty} \sin\left(2\pi/6\right) \delta(t - \pi/6) dt = \sin(\pi/3) \int_{-\infty}^{\infty} \delta(t - \pi/6) dt$$

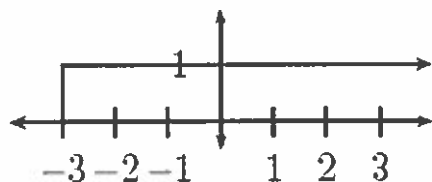
$$= \sin(\pi/3) = 0.866$$

$$\begin{aligned} \text{(iv)} \int_{-\infty}^{\infty} \sin\left[t - \pi/4\right] \delta\left(t - \frac{\pi}{2}\right) dt &= \int_{-\infty}^{\infty} \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right) \delta\left(t - \frac{\pi}{2}\right) dt \\ &= \sin\left(\frac{\pi}{4}\right) \int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{2}\right) dt = 0.707 \end{aligned}$$

$$\begin{aligned} \text{(v)} \int_{-\infty}^{\infty} \sin\left(t - \pi/6\right) \delta\left(2t - 2\pi/3\right) dt &= \int_{-\infty}^{\infty} \sin\left(t - \pi/6\right) \delta\left[2\left(t - \pi/3\right)\right] dt \\ &= \int_{-\infty}^{\infty} \sin\left(\pi/3 - \pi/6\right) \delta\left[2\left(t - \pi/3\right)\right] dt = \frac{1}{2} \sin(\pi/6) = 0.25 \end{aligned}$$

PROBLEM 2.20

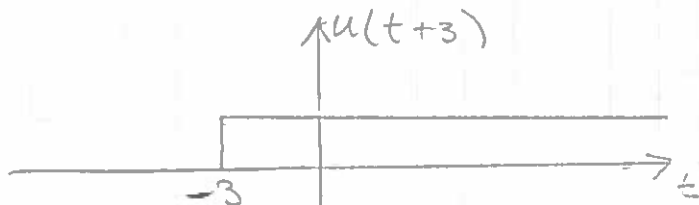
$$(a) u(2t+6) = u(t+3)$$



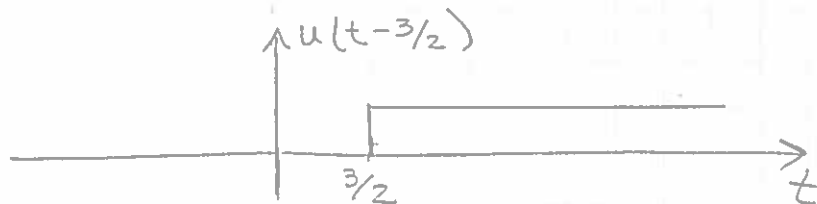
$$(b) u(-3t+6) = u[-3(t-2)] = u(-t+2)$$



$$(c) u(t/3+1) = u(1/3(t+3)) = u(t+3)$$

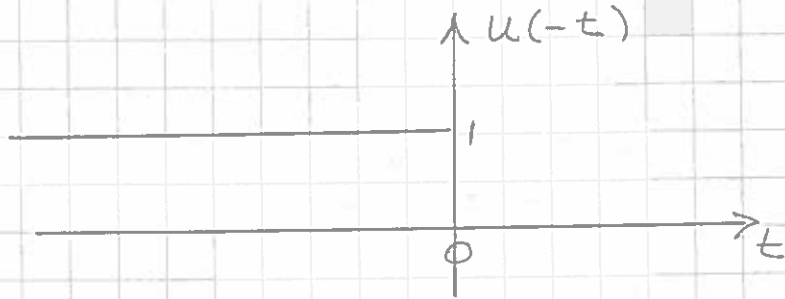


$$(d) u(t/3-1/2) = u[1/3(t-3/2)] = u(t-3/2)$$

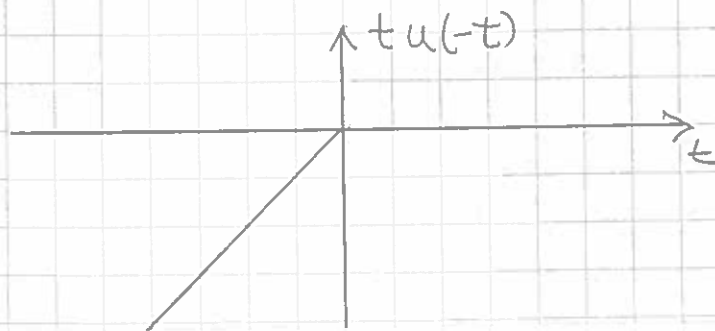


PROBLEM 2.2]

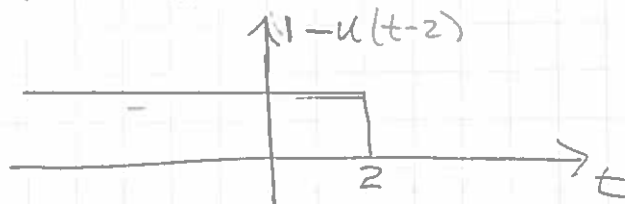
(a) $u(-t) = 1 - u(t)$



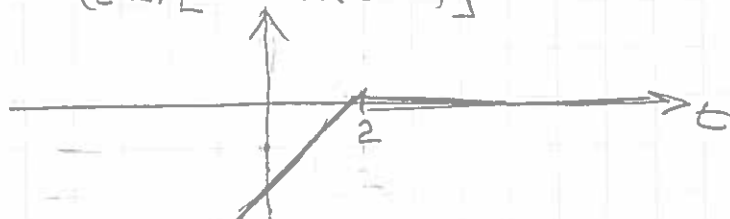
(b) $t u(-t) = t [1 - u(t)]$



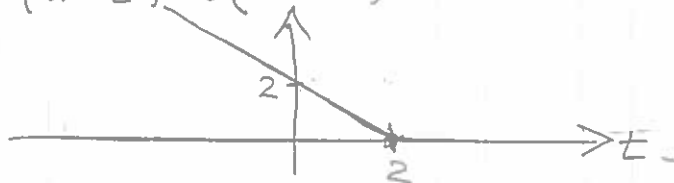
(c) $u(-t+2) = u[-1(t-2)] = 1 - u(t-2)$



(d) $(t-2) u(2-t) = (t-2) u[-1(t-2)]$
 $= (t-2) [1 - u(t-2)]$



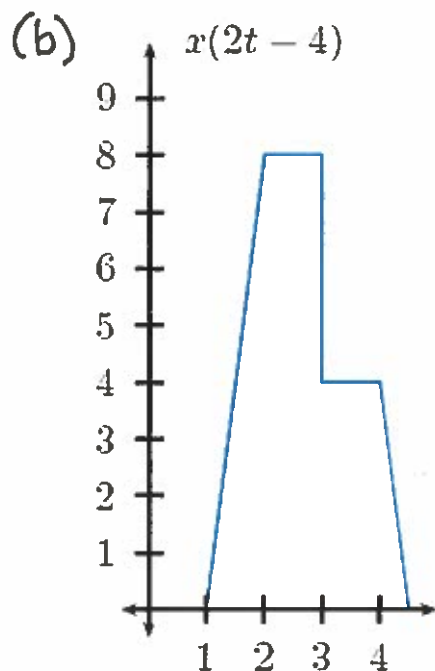
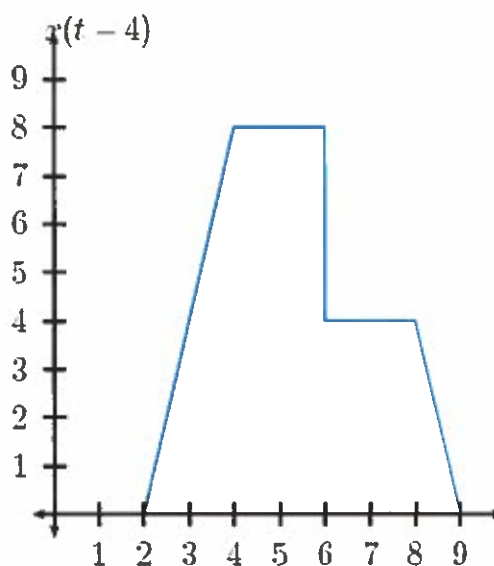
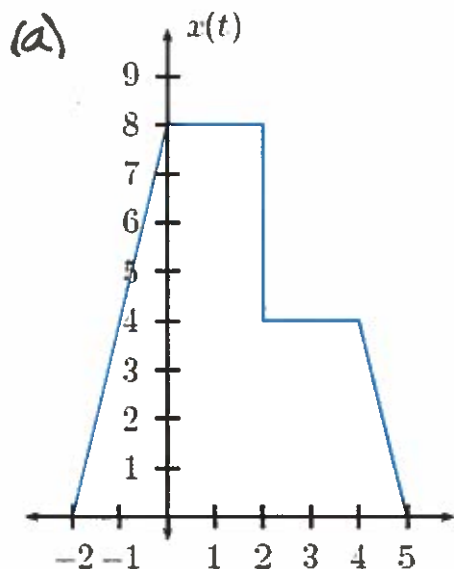
(e) $(2-t) u(-t+2) = (2-t) u(-(t-2)) = (2-t) [1 - u(t-2)]$



PROBLEM 2.22

$$x(2t-4) = 4[(2t-2)u(2t-2) - (2t-4)u(2t-4) - u(2t-6) - (2t-8)u(2t-8) - (2t-9)u(2t-9)]$$

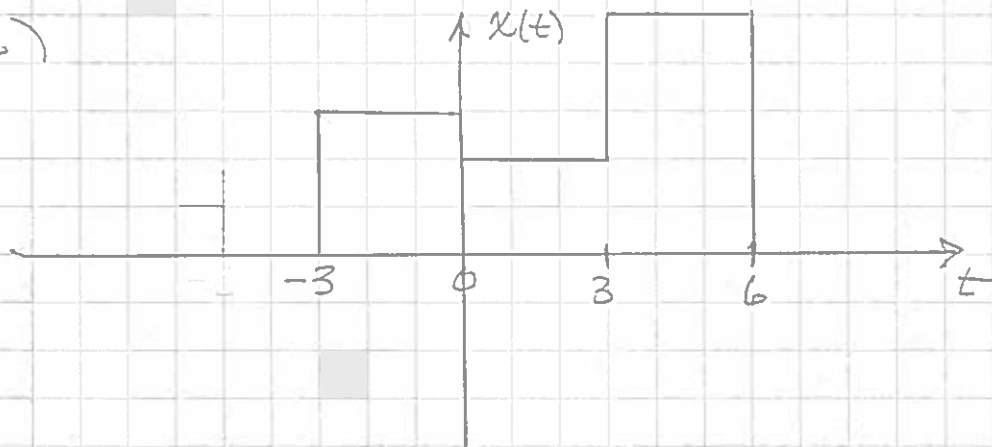
$$= 4[(2t-2)u(t-1) - (2t-4)u(t-2) - u(t-3) - (2t-8)u(t-4) - (2t-9)u(t-4.5)]$$



PROBLEM 2.23

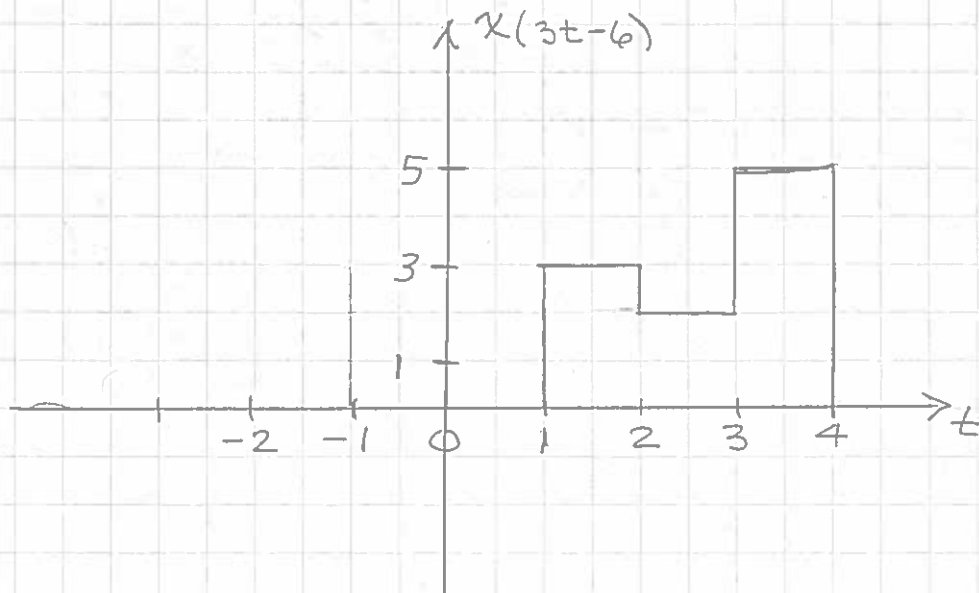
$$x(t) = 3u(t+3) - u(t) + 3u(t-3) - 5u(t-6)$$

(a)



$$(b) \quad x(3t-6) = 3u(3t-6+3) - u(3t-6) + 3u(3t-6-3) - 5u(3t-6-6)$$

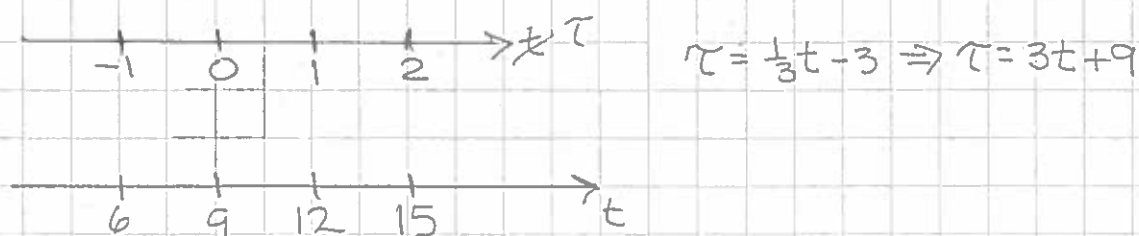
$$\begin{aligned} &= 3u(3t-3) - u(3t-6) + 3u(3t-9) - 5u(3t-12) \\ &= 3u(t-1) - u(t-2) + 3u(t-3) - 5u(t-4) \end{aligned}$$



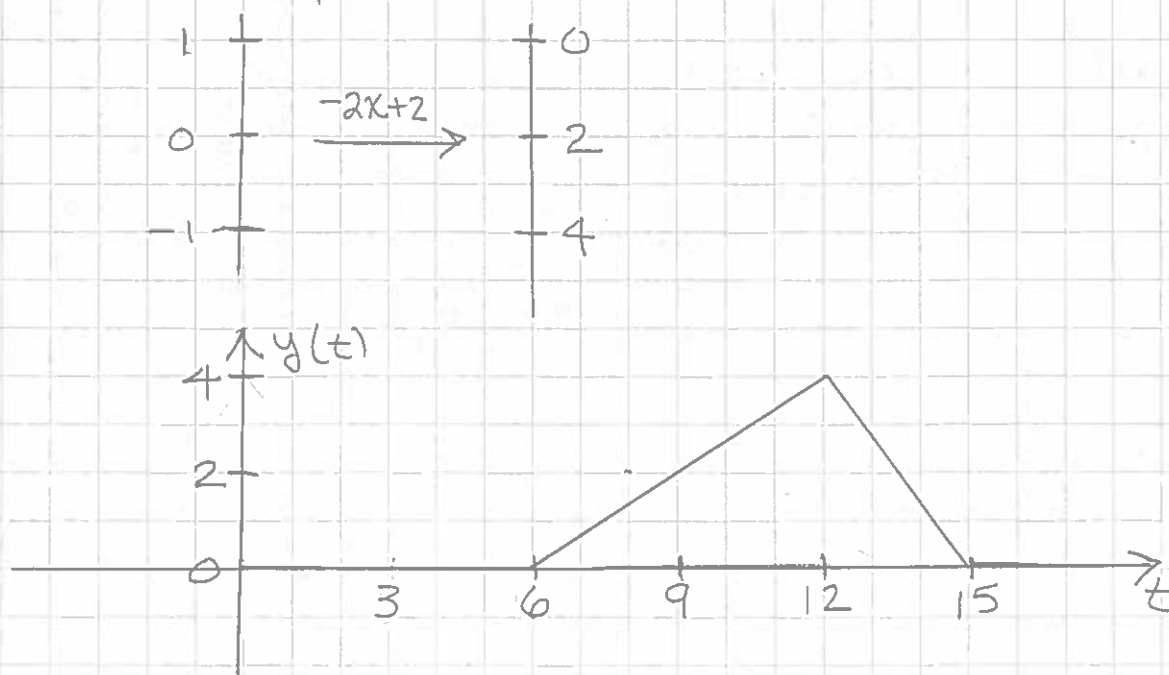
PROBLEM 2.24

$$\begin{aligned} (a) \quad x(t) &= 1 - (t+1)[u(t+1) - u(t-1)] \\ &\quad + [-2 + 2(t-1)][u(t-1) - u(t-2)] \\ &= 1 - (t+1)u(t+1) + 3(t-1)u(t-1) - 2(t-2)u(t-2) \end{aligned}$$

(b) use time transformation



and amplitude transform



PROBLEM 2.25

$$(a) \quad x_1(t) = 2t u(t) - 4(t-1) u(t-1) + 2(t-2) u(t-2)$$

$$(b) \quad t < 0, \quad x_1(t) = 0 \quad \checkmark$$

$$0 < t < 1, \quad x_1(t) = 2t \quad \checkmark$$

$$1 < t < 2, \quad x_1(t) = 2t - 4t + 4 = 4 - 2t \quad \checkmark$$

$$2 < t, \quad x_1(t) = 4 - 2t + 2t - 4 = 0 \quad \checkmark$$

$$(c) \quad x(t) = \sum_{k=-\infty}^{\infty} x_1(t - kT_0) = \sum_{k=-\infty}^{\infty} x_1(t - 2k)$$

PROBLEM 2.26

$$(a) \quad x_1(t) = 3[t u(t) - (t-1) u(t-1)] \\ - 3[(t-2) u(t-2) - (t-3) u(t-3)]$$

$$(t < 0), \quad x_1(t) = 0 \quad \checkmark$$

$$(0 < t < 1), \quad x_1(t) = 3t \quad \checkmark$$

$$(1 < t < 2), \quad x_1(t) = 3t - 3t + 3 = 3 \quad \checkmark$$

$$(2 < t < 3), \quad x_1(t) = 3t - 3t + 3 - 3t + 6 = -3t + 9 \quad \checkmark$$

$$(t > 3), \quad x_1(t) = 3t - 3t + 3 - 3t + 6 + 3t - 9 = 0 \quad \checkmark$$

$$(b) \quad x_2(t) \text{ is periodic with } T_0 = 4$$

$$x_2(t) = \sum_{n=-\infty}^{\infty} x_1(t + 4n)$$

$$= \sum_{n=-\infty}^{\infty} 3 \left[(t+4n) u(t+4n) - (t-1+4n) u(t-1+4n) \right] \\ - 3 \left[(t-2+4n) u(t-2+4n) - (t-3+4n) u(t-3+4n) \right]$$

PROBLEM 2.27

$$(a) \quad y_2(t) = T_2[T_1[x(t)]] \quad , \quad y_3(t) = T_3[T_1[x(t)]]$$

$$y(t) = T_2[T_1[x(t)]] + T_4\{T_3[T_1[x(t)]] + T_5[x(t)]\}$$

$$(b) \quad y(t) = T_3\{T_2[T_1[x(t)]]\} + T_4\{T_2[T_1[x(t)]]\} + T_5[T_1[x(t)]]$$

$$(c) \quad y_3(t) = T_3[T_4[T_5[x(t)]]]$$

$$y_4(t) = T_2[T_4[T_5[x(t)]]]$$

$$y_5(t) = T_1[T_5[x(t)]]$$

$$y(t) = y_3(t) + y_4(t) + y_5(t)$$

$$y(t) = T_3[T_4[T_5[x(t)]]] + T_2[T_4[T_5[x(t)]]]$$

$$= T_1 + T_1[T_5[x(t)]]$$

$$(d) \quad y(t) = y_3(t) \times y_4(t) \times y_5(t)$$

$$y(t) = T_3[T_4[T_5[x(t)]]] \times T_2[T_4[T_5[x(t)]]] \times T_1[T_5[x(t)]]$$

PROBLEM 2.28

$$m(t) = T_2\{x(t) - T_4[y(t)]\}$$

$$y(t) = T_3\{m(t) + T_1[x(t)]\}$$

$$y(t) = T_3\{T_2[x(t) - T_4[y(t)]] + T_1[x(t)]\}$$

PROBLEM 2.29

$$m(t) = T_2\{x(t) - T_3[y(t)] - T_4[y(t)]\}$$

$$y(t) = T_1[m(t)]$$

$$y(t) = T_1\{T_2[x(t) - T_3[y(t)] - T_4[y(t)]]\}$$

PROBLEM 2.30

- (a) (i) The system has memory the output, $y(t)$ depends on inputs $x(t)$ over a period of time.
(ii) The system is not invertible. The input at time t_0 cannot be determined from knowledge of the output at t_0 .
(iii) The system is stable. A bounded input will result in a bounded output.
(iv) The system is time-invariant.
(v) The system is linear.

(b) The system is causal if $y(t_0)$ depends on values of $x(t)$ for $t \leq t_0$ only. Therefore for causality $t_0 + 1 - \alpha \leq t_0 \Rightarrow \alpha \geq 1$.

PROBLEM 2.31

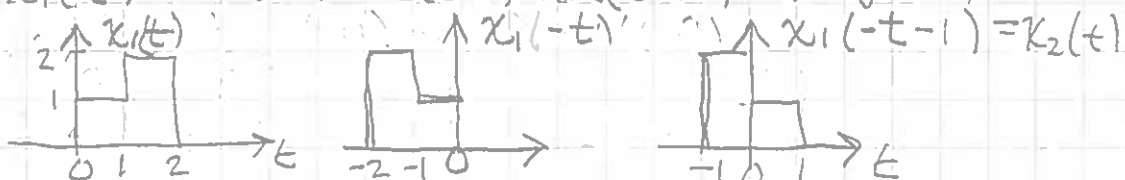
(a) $y(t) = \cos[x(t-2)]$

- (i) The system has memory
(ii) The system is not invertible
(iii) The system is causal
(iv) $|\cos(x)| \leq 1$ for all x . $\therefore$ Stable.
(v) $y(t-t_0) = x(t-t_0-2) \therefore$ time invariant.
(vi) $y(t) = \cos[x_1(t) + x_2(t)] \neq \cos x_1(t) + \cos x_2(t)$,
 $\therefore$ Not linear.

Problem 2.32

$$\begin{aligned} \text{(a)} \quad x_2(t) &= 2u(t+1) - u(t) - u(t-1) \\ &= 2[u(t+1) - u(t)] + [u(t) - u(t-1)] \\ &= 2x_1(t+1) + x_1(t) \\ \therefore y_2(t) &= 2y_1(t+1) + y_1(t) \end{aligned}$$

(b) $x_1(t) = u(t) + u(t-1) - 2u(t-2) \Rightarrow y_1(t)$



$$\therefore y_2(t) = y_1(-t-1) = y_1[-(t+1)]$$

PROBLEM 2.33

(a)

(i) The system is memoryless only if $t_0 = 0$.

(ii) The system is invertible; $x(t) = y(t+t_0)$.

(iii) The system is causal only if $t_0 \geq 0$.

(iv) The system is BIBO stable.

(v) The system is time invariant.

$$x(t-t_0-t_1) \leftrightarrow y(t-t_1)$$

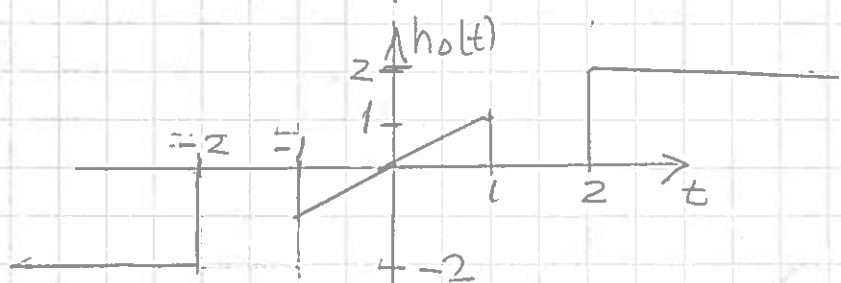
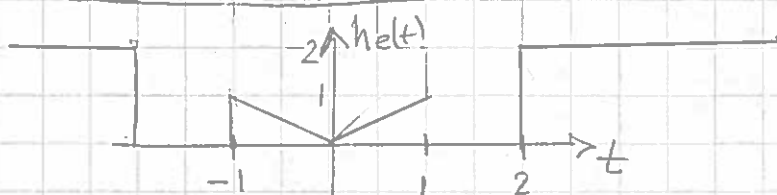
(vi) The system is linear.

$$x_1(t-t_0) \rightarrow y_1(t)$$

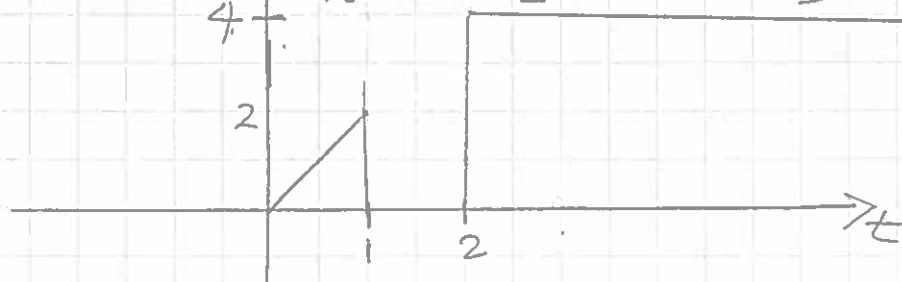
$$x_2(t-t_0) \rightarrow y_2(t)$$

$$ax_1(t-t_0) + bx_2(t-t_0) \rightarrow ay_1(t) + by_2(t).$$

PROBLEM 2.34



$$h(t) = 2t[u(t) - u(t-1)] + 4u(t-2)$$



PROBLEM 2.35

(a) (i) memoryless

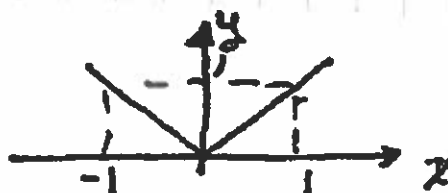
(ii) $y=1$ for $x=\pm 1$, not invertible

(iii) causal

(iv) stable

(v) time invariant

(vi) $|x_1 + x_2| \neq |x_1| + |x_2|$ not linear



$$y = |x|$$

(b) (i) memoryless

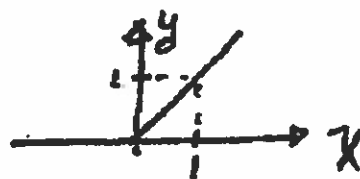
(ii) $y=0$ for $x \leq 0$, not invertible

(iii) causal

(iv) stable

(v) time invariant

(vi) $y|_{x_1=1} \neq y|_{x_1+x_2}$, not linear



(c) (i) Memoryless: $y(t)$ determined by current input.

(ii) Not invertible: $y(t) = 1$ for all $x(t) < -1$.

(iii) Causal.

(iv) Stable: $|y(t)| \leq 1$.

(v) Time invariant.

(vi) Not linear: $y(t) = 1$ for all values of $x(t) < -1$.

(d) (i) Memoryless

(ii) Not invertible: $y(t) = 4$ for all $x(t) > 2$.

(iii) Causal

(iv) Stable: $0 \leq y(t) \leq 4$

(v) Time invariant

(vi) Not linear: $y(t) = 4$ for all $x(t) > 2$