2

MOTION IN ONE DIMENSION

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

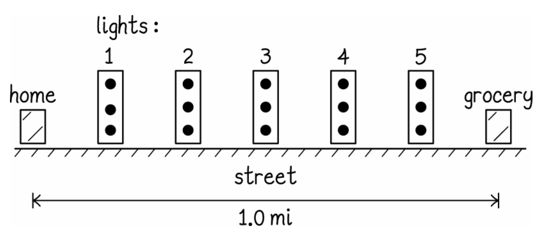
Developing a Feel

1. 10^2 m **2.** 10^{18} m **3.** 10^0 m; 10^1 m **4.** 10^0 s **5.** 10^5 s **6.** 10^2 m **7.** 10^2 m/s **8.** 10^0 m/s **9.** 10^5 s **10.** 10^7
11. 10^1 m/s **12.** 10^{-10} m

Guided Problems

2.2 City driving

1. Getting Started We start by drawing a diagram of the setup:



Between lights, your speed could not possibly be constant, since you have to stop and start at lights. But your average speed has already taken this stopping and starting into account. The average speed (while moving) is the distance between two lights divided by the time required to travel between those two lights. In terms of velocity, we can write $\vec{v}_{av} = \dfrac{\vec{x}_f - \vec{x}_i}{t_f - t_i}$.

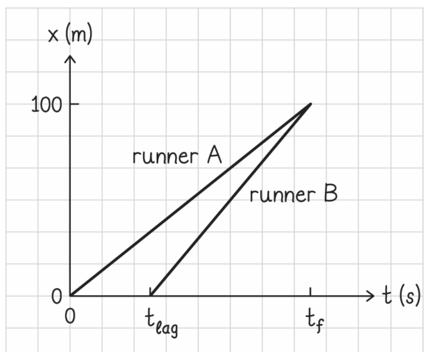
2. Devise Plan We can calculate the total time spent stopped a lights, because we know how much time is spent at each one. We can also easily calculate how much time is spent driving, because we know what distance must be covered, and we know what the average speed is while driving. The sum of these two times will be the total time for the trip. We already know the total distance travelled to get to the store, so knowing the total time will enable us to calculate the average speed using $v_{av} = d/t$. In general, average speeds and average velocities could be completely different. But in this case, the velocity is always in the same direction. In this case, the average speed will just be the magnitude of the average velocity.

3. Execute Plan (*a*) The time spent stopped at lights is $t_{lights} = (5\text{ lights}) \times \dfrac{(1\text{ min})}{\text{light}} \times \dfrac{1\text{ h}}{60\text{ min}} = 0.083\text{ h}.$ The time spent driving is $t_{drive} = \dfrac{d}{v} = \dfrac{1.0\text{ mi}}{20\text{ mi/h}} = 0.050\text{ h}.$ So the total time required for the trip is $t = 0.13\text{ h}.$ (*b*) The distance travelled is 1.0 mi. This is the same as the magnitude of your displacement in this case. Your displacement is 1.0 mi west. Hence the average velocity is $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{x}}{t} = \dfrac{(1.0\text{ mi})\text{ west}}{0.13\text{ h}} = 7.5\text{ mi/h west}.$ (*c*) In this case the average speed is just the magnitude of the average velocity, so $v_{\text{av}} = 7.5\text{ mi/h}.$

4. Evaluate Result The time required for the trip is 0.13 h or about 8 minutes. This is a reasonable time for a trip down the road. The speed and velocities do seem a little slow. But we know the answers must be significantly lower than 20 mi/h, because that is the average speed while moving. Therefore the speed and velocity also fit with expectations.

2.4 Race rematch

1. Getting Started We start by drawing a position vs time plot of the two runners:



We want the two runners to start at different times, but reach the 100 m mark at the same instant.

2. Devise Plan The runners are assumed to move at a constant speed in this problem. Certainly real runners require a few moments to reach their top speed. But after they reach their top speed, they may run at a constant speed to a very good approximation. If the runners move at a constant speed, we can express their positions as

$$x_{\text{f}} = x_{\text{i}} + v_x \Delta t = x_{\text{i}} + v_x (t_{\text{f}} - t_{\text{f}}) \tag{1}$$

We want to determine how long after runner A starts runner B should start. We call this time t_{lag}. We can use the speeds given to us in Worked Problem 2.3. We can set the final positions for the two runners at the final time equal to one another and solve for the lag time.

3. Execute Plan We write the position of each runner in the form of equation (1) and set the two final positions equal. We will call the position of the starting line $x_{\text{i}} = 0,$ and the time at which runner A starts $t = 0.$ To find the final time, let us consider runner A: $t_{\text{f}} = d/v_{\text{A}x}.$

$$x_{\text{Af}} = v_{\text{A}x}(t_{\text{f}}) = x_{\text{Bf}} = v_{\text{B}x}(t_{\text{f}} - t_{\text{lag}})$$

$$t_{\text{lag}} = \left(1 - \frac{v_{\text{A}x}}{v_{\text{B}x}}\right) t_{\text{f}}$$

$$t_{\text{lag}} = \left(1 - \frac{v_{\text{A}x}}{v_{\text{B}x}}\right)\left(\frac{d}{v_{\text{A}x}}\right)$$

$$t_{\text{lag}} = \left(1 - \frac{(8.00\text{ m/s})}{(9.30\text{ m/s})}\right)\left(\frac{100\text{ m}}{8.00\text{ m/s}}\right)$$

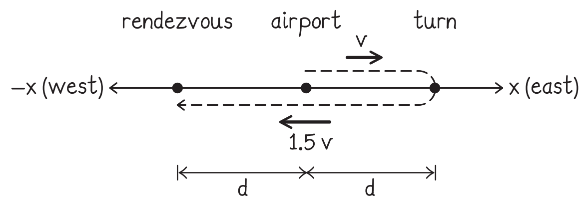
$$t_{\text{lag}} = 1.75\text{ s}$$

Hence, runner B would have to wait 1.75 s before starting, in order for the race to end in a tie.

4. Evaluate Result This is a reasonable time to wait at the beginning of a 100 m race. We can compare this to the answer to Worked Problem 2.3, in which it was shown that a 2.0 s head start resulted in runner A winning by 2.4 m. Clearly, a 2.0 s head start is a little too long to end in a tie. But the result of Worked Problem 2.3 was so close to a tie that we would expect our answer to be fairly close to 2.0 s. Our answer of 1.75 s fits our expectations.

2.6 Wrong way

1. Getting Started We start by making a diagram of the setup. We show the initial velocity vectors of the helicopter pointing eastward, and the final velocities vectors as being larger and pointing westward. The distance between the airport and the turning point is d, as is the distance from the airport to the actual rendezvous point.



2. Devise a Plan This is similar to Worked Problem 2.5 in that it involves constant velocity motion in two separate intervals. The difference is that here the object turns around and heads in the opposite direction, whereas in Worked Problem 2.5, the object merely changed its speed.

Because no numbers are given, we expect an answer that contains given variables v and d. It also seems likely that the answer may be totally in terms of v, based on units (if d were in the expression, we would have no time variable to get that term in units of speed). We know the displacement vectors for each leg of the journey: $\Delta\vec{x} = d\,\hat{i}$ for the eastward leg and $\Delta\vec{x} = -2d\,\hat{i}$ for the westward leg. We can use the displacement and relative velocities to find expressions for the time required for each leg: $t_{\text{east}} = \dfrac{\Delta x}{v_x} = \dfrac{d}{v}$, and $t_{\text{west}} = \dfrac{\Delta x}{v_x} = \dfrac{-2d}{-1.5v} = \dfrac{4}{3}\left(\dfrac{d}{v}\right)$. Now that we have times in terms of the variables given, we can calculate expressions for the average x component of the velocity using $v_{x,\text{av}} = \dfrac{\Delta x}{t}$.

3. Execute Plan (*a*) The average x component of the velocity is given by

$$v_{x,\text{av}} = \frac{\Delta x}{t} = \frac{d-2d}{\dfrac{d}{v}+\dfrac{4}{3}\left(\dfrac{d}{v}\right)} = -\frac{3}{7}v$$

(*b*) The velocity follows immediately from its x component: $\vec{v}_{\text{av}} = -\dfrac{3}{7}v\,\hat{i}$.

4. Evaluate Result Our result matches our expectations in terms of being written in terms of v. We also expect the final velocity to be in the $-x$ direction, because the final position of the helicopter is west of the airport. Finally, we expect the average velocity to have a magnitude that is significantly less than v because the helicopter changes directions. Our answer shows very good agreement with our expectations.

2.8 You're it!

1. Getting Started Child A runs eastward at a high and constant speed. Child B starts out very slowly but increases his speed in the eastward direction. Child C runs in the westward direction at a constant speed; his speed is not as great as Child A's. Child D starts our running east fairly quickly, but slows down. Child E remains stationary the entire time. Child F starts moving westward with the same initial speed as Child C, but Child F slows down almost to a stop.

2. Devise Plan We determine the direction of a child's motion by the sign of the slope on the $x(t)$ graph. If the slope is positive, the child is moving to the east; if the slope is negative, the child is moving to the west. Since the slope also tells us the magnitude of the child's velocity, and the sign tells us the direction, this information completely determines the velocity and speed of a child.

We can recognize a child moving at a constant velocity because the $x(t)$ curve will be a straight line. A child with a changing velocity will appear to have a curved path on the $x(t)$ graph (changing slope). Among the children who move at a constant velocity, the fastest child will have the greatest slope of the $x(t)$ line.

Whenever children pass each other, their position along the east-west direction must momentarily be the same. This corresponds to their $x(t)$ curves crossing each other.

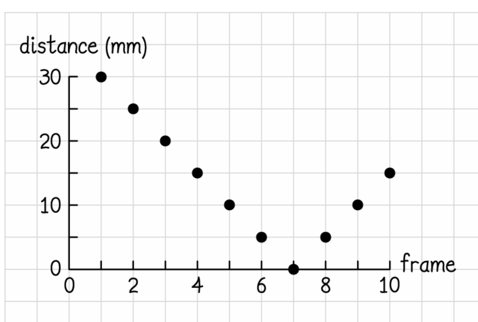
3. Execute Plan (*a*) Children A, B, and D are moving east; children C and F are moving west; child E is staying in the same place. (*b*) Children A, C, and E are all moving with constant velocity. The velocity of child A is positive, the velocity of child C is negative, and the velocity of child E is zero. (*c*) Children B, D, and F are changing their velocities as time passes. Children D and F are slowing down, while child B is speeding up. (*d*) Child A has the highest average speed. Child E has the lowest average speed. (*e*) Child B passes child D.

4. Evaluate Result These results are reasonable descriptions of playing children.

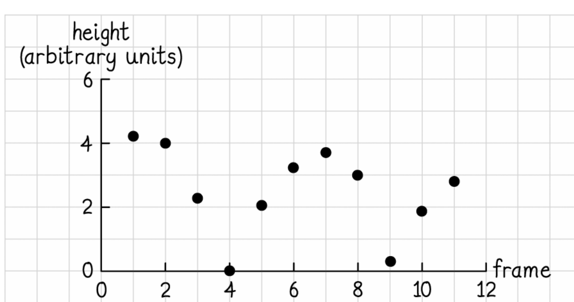
Questions and Problems

2.1. We must know the rate at which the camera took pictures (or equivalently the time interval between successive frames were captured), and something to provide a length scale (either the size of the object in the clip or a known distance between objects in the clip).

2.2.



2.3.



2.4. The object starts out 35 mm from the edge, moving toward the edge. This motion continues for 120 frames, at which point the object is 20 mm from the edge. The object remains in that position for 90 frames, before moving back away from the edge. It recedes back to the original distance (36 mm) between frames 210 and 270. The final motion away from the edge is faster than the initial motion toward the edge, assuming the frames were captured at a constant rate.

2.5. (*a*), (*c*), (*d*), (*e*), and (*f*) could all describe the same motion. All of these graphs show an object initially moving in one direction at a constant speed, then stopping, and then continuing on in the same direction as before at the same speed. They merely disagree on the location of the origin, which direction is positive, and the scales of the axes.

2.6. $\Delta x = x_f - x_i = (0.23\text{ m}) - (6.5\text{m}) = -6.3\text{ m}$

2.7. The distance is 6.4 km, whereas the displacement is zero (your initial and final positions are the same).

2.8. Zero. Since the distance of the race is an integer multiple of the track length $(\ell_{\text{race}} = 5\ell_{\text{track}})$ a runner will end the race at exactly the same position he or she started the race. Hence the displacement is zero.

2.9. (*a*) We could produce infinitely many graphs of the observations. We could put the origin anywhere, choose many possible timescales and guess many possible length scales. (*b*) We could still produce infinitely many graphs. We could still choose any timescale, and guess any length for the dog. (*c*) We could still choose any timescale and could therefore still make infinitely many graphs. (*d*) 2 (one for each choice of the positive direction).

2.10. (*a*) On your graph the person walking will always be somewhere on the positive x axis, whereas your friend's graph will show the person walking on the negative x axis. (*b*) Yes, both are equally good.

2.11. If numerical values of time and distance are converted, but the scale of each axis still uses the same numerical labels (that is, "0.40 m" becomes "0.40 in"), then the curve would be much narrower and much taller. This is just a matter of perspective, though. If the scale of each axis is also converted, so that "0.40 m" becomes "16 in", then the shape of the graph is not changed by the conversion of units.

2.12. (*a*) $\Delta\vec{x} = (x_{\text{f}} - x_{\text{i}})\ \hat{\imath} = ((+5\text{ m}) - (0))\ \hat{\imath} = (+5\text{m})\ \hat{\imath}$ (*b*) The object starts at the origin and moves in the $+x$ direction for 7 m before stopping. When the object starts moving again, it moves 2 m back in the $-x$ direction. Hence the object covers a total distance of 9 m.

2.13. In this case your total displacement in the x direction is made of several smaller displacements: $\Delta x = ((+4) + (-2) + (+1) + (+5) + (-7))\text{ blocks} = +1\text{ block}.$

2.14. Answers may vary depending on the height of the table. In the picture, the difference between the position of the bottom of the leg and the top of the table is $h = (+65\text{ mm}) - (+12\text{ mm}) = (+53\text{ mm}).$ From this information, we can figure out the length scale in this picture. A standard table is approximately 0.75 m tall, though obviously there could be variations. Using this approximate height of 0.75 m, each millimeter of picture would correspond to a real life distance of $(0.75\text{ real m})/(53\text{ picture mm}) = 0.014\text{ real m/picture mm}.$ Then the real length of the table would be given by this factor times the length of the table in the picture: $(0.014\text{ real m/picture mm}) \times ((+99\text{ mm}) - (+14\text{ mm})) = 1.2\text{ m}.$

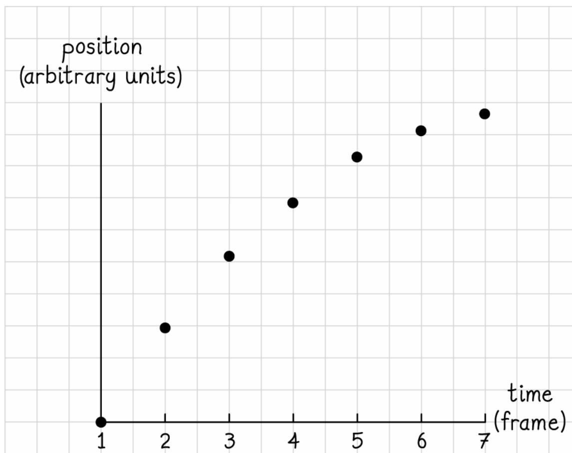
2.15. The swimmer swims in the positive x direction at a constant speed (left sloping leg of curve, increasing x values). She stops briefly (horizontal leg, most probably at end of her lane) and then returns to the starting point (right sloping leg, decreasing x values) at a speed slightly lower than her initial speed (this leg not as steep as left leg).

2.16. They are travelling along an essentially one-dimensional path. Because they travel in opposite directions over the course of the day, there must be an instant when they have the same position.

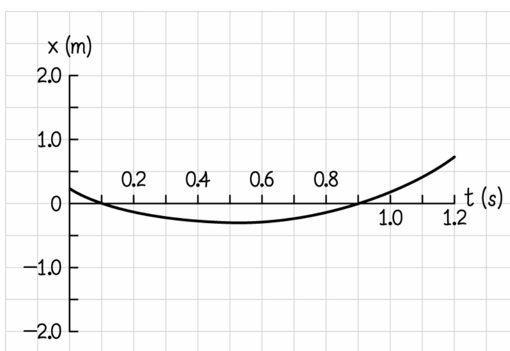
2.17. Interpolation always gives a continuous path, but there is no reason to expect that the path is accurate everywhere. Suppose you are photographing a clock's pendulum at 1.0-s intervals, collecting data to use in a graph showing the pendulum's position as a function of time. Suppose further that the pendulum takes 1.0 s to swing from left to right and back left again. At this swing speed, the pendulum has just enough time between photographs to swing and return to its initial position, so that the photographs make it appear that the pendulum does not move at all. An interpolation of data points collected from the photographs would show a continuous horizontal line on a position-versus-time graph, which is certainly not correct.

2.18. Starting from the earliest possible time, we see that the object first passes through $x = 2.0$ m at the time $t = 20$ s. After that, the object passes through the position $x = 3.0$ m three times: at $t = 30$ s, $t = 60$ s, and $t = 80$ s. Hence there are three correct answers: 10 s, 40 s, and 60 s.

2.19.



2.20. (*a*)



(*b*) We calculate the position of the object at the two times specified and take the difference:

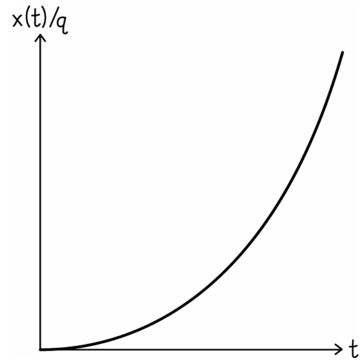
$$x(t = 0.50\ \text{s}) = (+0.20\ \text{m}) + (-2.0\ \text{m/s})(0.50\ \text{s}) + (+2.0\ \text{m/s}^2)(0.50\ \text{s})^2 = -0.30\ \text{m}$$

$$x(t = 0) = (+0.20\ \text{m}) + (-2.0\ \text{m/s})(0) + (+2.0\ \text{m/s}^2)(0)^2 = 0.20\ \text{m}$$

Hence $\Delta\vec{x} = (x_\text{f} - x_\text{i})\ \hat{\imath} = ((-0.30\ \text{m}) - (0.20\ \text{m}))\ \hat{\imath} = (-0.50\ \text{m})\ \hat{\imath}$.

(*c*) The farthest the object ever gets from the origin is its distance from the origin at the end of this time period (at $t = 1.2$ s). $x(t = 1.2\ \text{s}) = (+0.20\ \text{m}) + (-2.0\ \text{m/s})(1.2\ \text{s}) + (+2.0\ \text{m/s}^2)(1.2\ \text{s})^2 = (+0.68)$ m.

2.21. (*a*)



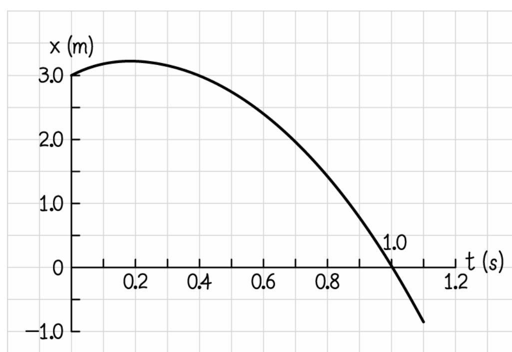
(*b*) $\Delta\vec{x} = (x_f - x_i)\ \hat{i} = (q(3T)^3 - q(T)^3)\ \hat{i} = 26qT^3\ \hat{i}$

2.22. (*a*) $x(t=0) = (+3.0\text{ m}) + (+2.0\text{ m/s})(0) + (-5.0\text{ m/s}^2)(0)^2 = (+3.0)\text{ m}$. (*b*) To find the time at which the position takes its maximum value, we take the derivative of the function with respect to time and solve for the time:

$$\frac{dx(t)}{dt} = q + 2rt = 0$$

$$t = -q/2r = \frac{-(+2.0\text{ m/s})}{2(-5.0\text{ m/s}^2)} = 0.20\text{ s}$$

Hence the time at which the function reaches a maximum value is 0.2 s. (*c*) Inserting the value obtained in part (*b*) into the position function yields $x(t=0.20\text{ s}) = (+3.0\text{ m}) + (+2.0\text{ m/s})(0.20\text{ s}) + (-5.0\text{ m/s}^2)(0.20\text{ s})^2 = (+3.2)\text{ m}$. So the maximum value of the position is 3.2 m.
(*d*)



(*e*) The object starts at the position $x = 3.0\text{ m}$ and travels in the positive x direction with a decreasing speed. The object finally stops momentarily at $t = 0.2\text{ s}$ at $x = 3.2\text{ m}$. The object turns around and accelerates steadily in the $-x$ direction, crossing the time axis at $t = 1.0\text{ s}$. The motion is parabolic. (*f*) After 0.20 s, the object turns around. In the first 0.20 s, the object travels 0.20 m in the $+x$ direction. This initial distance must be added on to the distance the object moves in the $-x$ direction after 0.20 s. For the first interval:

$$x(t=0.50\text{ s}) = (+3.0\text{ m}) + (+2.0\text{ m/s})(0.50\text{ s}) + (-5.0\text{ m/s}^2)(0.50\text{ s})^2 = (+2.75)\text{ m}$$

Which corresponds to 0.20 m in the $+x$ direction and another 0.45 m in the $-x$. Hence the total distance travelled is 0.65 m.

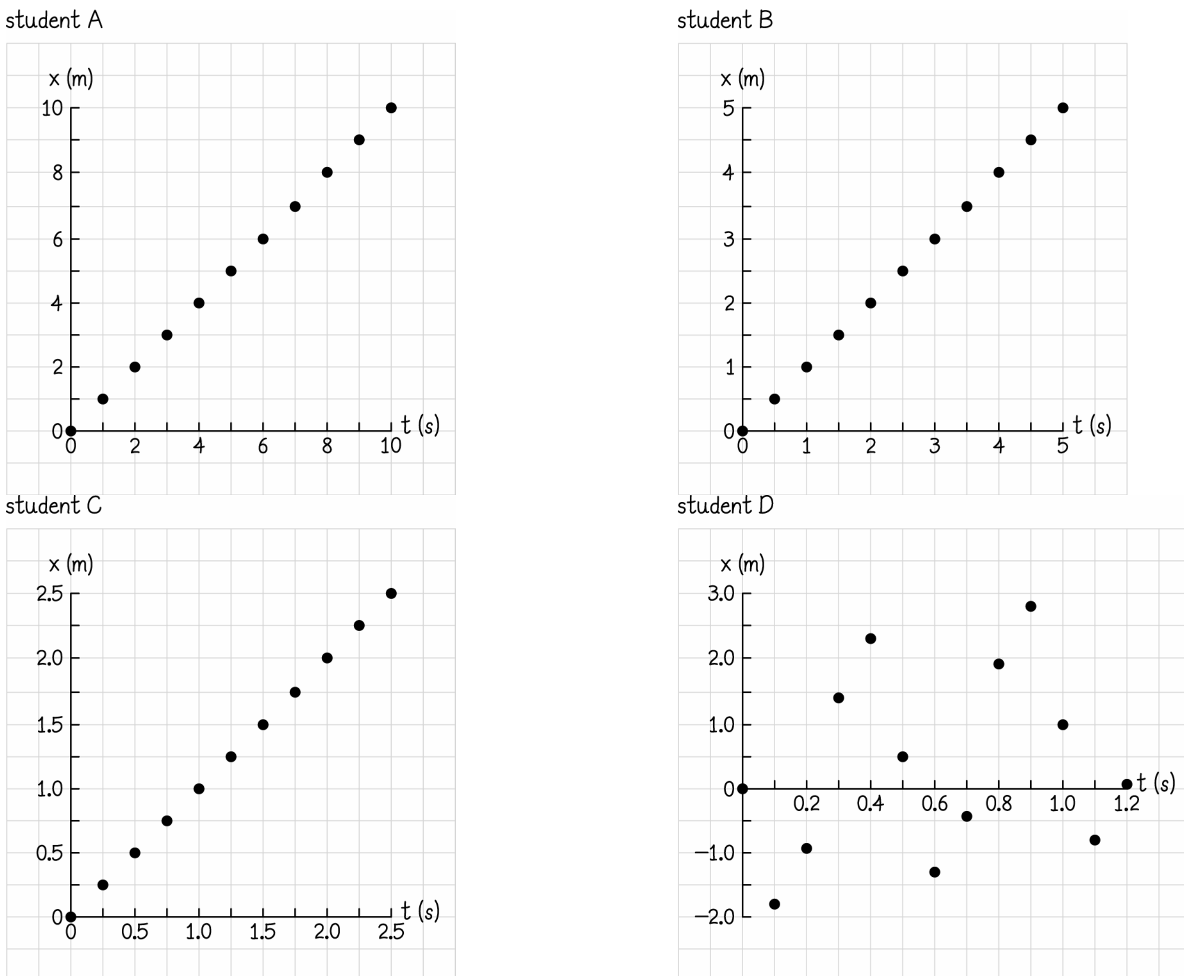
For the second interval:

$$x(t=1.0\text{ s}) = (+3.0\text{ m}) + (+2.0\text{ m/s})(1.0\text{ s}) + (-5.0\text{ m/s}^2)(1.0\text{ s})^2 = (0)\text{ m}$$

This corresponds to an initial distance of 0.20 in the $+x$ direction and another 3.2 m in the $-x$. Hence the total distance travelled is 3.4 m.

On the third interval, there is no change of direction. So in this case the distance is equal to the magnitude of the displacement. Since the position was 2.75 m at 0.50 s, and the final position is at 0.0 m after 1.0 s, the distance travelled is 2.75 m.

2.23.



Students A, B, and C agree that the motion is linear. Only student D has taken data points at time intervals short enough to reveal that the x values keep changing back and forth between positive and negative values.

2.24. A moving car can be filmed from the aircraft. Using the camera's frame rate and the known lengths of the marks, the police can calculate the car's speed.

2.25. In all cases we simply use $\text{speed} = \dfrac{\text{distance}}{\text{time}}$, converting units to SI as needed.

(*a*) $\text{speed} = \dfrac{100\text{ m}}{9.84\text{ s}} = 10.2\text{ m/s}$. (*b*) $\text{speed} = \dfrac{200\text{ m}}{19.32\text{ s}} = 10.4\text{ m/s}$. (*c*) $\text{speed} = \dfrac{400\text{ m}}{43.29\text{ s}} = 9.24\text{ m/s}$. (*d*) Here we first calculate the total time in seconds:

$$t = 3\text{ min} \times \frac{60\text{ s}}{1\text{ min}} + 27.37\text{ s} = 207.37\text{ s}$$

The speed is given as before.

$$\text{speed} = \frac{1500 \text{ m}}{207.37 \text{ s}} = 7.233 \text{ m/s}$$

(*e*) The total time is given by

$$t = 26 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} + 38.08 \text{ s} = 1598.08 \text{ s}$$

The speed is then

$$\text{speed} = \frac{10 \times 10^3 \text{ m}}{1598.08 \text{ s}} = 6.2575 \text{ m/s}$$

(*f*) The total distance is 138,435 ft, which we convert to meters.

$$(1.3844 \times 10^5 \text{ ft}) \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{0.0254 \text{ m}}{\text{in}} = 4.2195 \times 10^4 \text{ m}$$

The total time is

$$t = 2 \text{ h} \times \frac{3600 \text{ s}}{1 \text{ h}} + 6 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} + 50 \text{ s} = 7610 \text{ s}$$

Hence the average speed is

$$\text{speed} = \frac{4.21950 \times 10^4 \text{ m}}{7610 \text{ s}} = 5.545 \text{ m/s}$$

2.26. (*a*) Yes. Speed is a scalar, the direction is irrelevant. (*b*) No. Velocity is a vector. The direction is different and so the velocity must be different.

2.27. (*a*) Because the images were created at equal time intervals, the spacing between adjacent images is a function of the ball's speed. That the spacing between adjacent images has one value in the first five frames and a different value in the final five frames tells you that the ball moved at one speed at the beginning of the motion and at a different speed at the end of the motion. (*b*) The ball had a higher speed in the final frames.

2.28. No. If the speed is constant, average speed and maximum speed are the same. If the speed is not constant, any speeds lower than the maximum contribute to the average and makes average speed lower than maximum speed.

2.29. (*a*) The cars are side by side at an instant just before t_2 and just after t_6. The graph shows that the cars have the same position at these times. (*b*) The cars have the same speed at (or very close to) t_4. Near t_4 there is a time at which the slopes of the two lines are equal, which is equivalent to the cars moving at the same speed.

2.30. The speeds of the pucks appear to the same. The spacing between images in (*a*) is about two-thirds the spacing between images in (*b*). But the frame rate is faster in (*a*) than in (*b*) by exactly two-thirds. Hence the distance per unit time is approximately the same.

2.31. B is closer to C than it is to A. Both segments take the same amount of time. Since the average speed from A to B was faster, the distance covered from A to B will be greater than the distance covered from B to C.

2.32. We find expressions for the time required in each case. Case 1 is travelling half the time at each speed, and case 2 is travelling half the distance at each speed. The distance from the starting point to the finish line *d* is the same in either case. We label the speeds, times, etc with subscripts that refer to the case being considered, and the speed travelled along a particular segment of the journey. For example, $t_{1,35}$ is the time required in case 1 for the segment travelled at 35 m/s.

Case 1: Call the time required in this case t_1.

$$d = v_{25}t_{1,25} + v_{35}t_{1,35} = (v_{25} + v_{35})\frac{t_1}{2} = (30 \text{ m/s})t_1$$

$$t_1 = \frac{d}{(30 \text{ m/s})}$$

Case 2: Call the time required in this case t_2.

$$t_2 = t_{1,25} + t_{1,35} = \frac{d/2}{v_{25}} + \frac{d/2}{v_{35}} = \frac{d}{2}\left(\frac{1}{25\text{ m/s}} + \frac{1}{35\text{ m/s}}\right)$$
$$t_2 = \frac{d}{(29.17\text{ m/s})}$$

Clearly $t_1 < t_2$, meaning that travelling half the time at each speed will yield a shorter time.

2.33. (*a*) The average speed is the distance covered in the total time: $\text{speed}_{\text{av}} = \frac{d}{t} = \frac{d}{t_{\text{there}} + t_{\text{back}}} = \frac{d}{\frac{d/2}{v_{\text{there}}} + \frac{d/2}{v_{\text{back}}}} =$ $2\left(\frac{1}{v_{\text{there}}} + \frac{1}{v_{\text{back}}}\right)^{-1} = 2\left(\frac{1}{10\text{ m/s}} + \frac{1}{16\text{ m/s}}\right)^{-1} = 12\text{ m/s}$ (*b*) He likely treated it as though the cyclist had ridden at each speed for equal times, rather than equal distances, and so just averaged the speeds: (10 m/s + 16 m/s) /2 = 13 m/s.

2.34. (*a*) The total distance that you cover is 2.5 km. The total time over which this trip was made is 100 minutes. The average speed is thus

$$\frac{2.50\times 10^3\text{ m}}{100\text{ min}} \times \frac{1\text{min}}{60\text{ s}} = 0.42\text{ m/s}$$

We could also express this in terms of km/h:

$$\frac{2.50\text{ km}}{100\text{ min}} \times \frac{60\text{min}}{1\text{ h}} = 1.5\text{ km/h}$$

(*b*) Zero. The initial and final positions are the same.

2.35. Given the distance you travel and your speed, we know that your trip requires a time $t = d/v = (500\text{ km})/(100\text{ km/h}) = 5.00\text{ h}$. Your brother leaves half an hour after you, meaning he makes the trip in only 4.50 h. Hence your brother's speed is $d/t = (500\text{ km})/(4.50\text{ h}) = 111\text{ km/h}$.

2.36. (*a*) Let us break the distance to the lake into three segments: the initial 3.0 km that you and your brother travel together, the additional distance that you travel before your brother reaches the lake, and the 5.0 km that remain for you to travel when your brother reaches the lake. Adding these three distances yields $d = (3.0\text{ km}) + \left((100\text{ km/h})(0.333\text{ h})\right) + (5.0\text{ km}) = 41\text{ km}$. (*b*) Your brother required one half hour to reach the lake, so his average speed is $\frac{d}{t} = \frac{(41.3\text{ km})}{(0.5\text{ h})} = 83\text{ km/h}$. (*c*) You have another 5.0 km to travel, at a speed of 100 km/h. The time is given by $t = \frac{d}{v} = \frac{(5.0\text{ km})}{(100\text{ km/h})} = 0.05\text{ h or }3.0\text{ min}$.

2.37. (1) No, because you could walk westward or eastward. (2) Yes, unless you also walk with some displacement north or south.

2.38. (*a*) 3 m (*b*) 3 m/s (*c*) –3 m/s

2.39. (*a*) 3 m (*b*) 3 m/s (*c*) 3 m/s

2.40. (*a*) –5 m for both (*b*) +5 m for both

2.41. (*a*) $A_x\hat{i}$ (*b*) $A_x\hat{i}$ Note that here A_x is a negative number (*c*) $A_x\hat{i}$

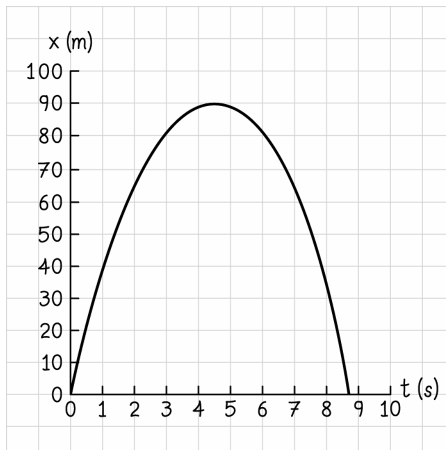
2.42. (*a*) $\vec{A}+\vec{B}=(+3.0\text{ m})\,\hat{\imath}+(-5.0\text{ m})\,\hat{\imath}=(-2.0\text{ m})\,\hat{\imath}$ (*b*) $\vec{A}-\vec{B}=(+3.0\text{ m})\,\hat{\imath}-(-5.0\text{ m})\,\hat{\imath}=(+8.0\text{ m})\,\hat{\imath}$

2.43. (*a*) The tip is 4.0 m above your head, meaning its *x* coordinate is $+4.0\text{m}$. (*b*) Expressed in terms of unit vectors, the position vector is $(+4.0\text{ m})\,\hat{\imath}$.

2.44. (*a*) We find the time using the quadratic equation:

$$t=\frac{-p\pm\sqrt{p^2-4(-q)(-x)}}{2(-q)}=\frac{-(42\text{ m/s})\pm\sqrt{(42\text{ m/s})^2-4(-4.9\text{ m/s}^2)(-20\text{ m})}}{2(-4.9\text{ m/s}^2)}=0.51\text{ s or }8.1\text{s}$$

(*b*) The rocket passes through a height of 20 m on its way up and again on its way down.
(*c*)

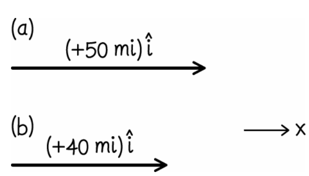


2.45. (*a*) 0.52 m (*b*) 0.8 m (*c*) 0.0 m (*d*) $\Delta\vec{x}=\vec{x}_f-\vec{x}_i=(0.8\text{ m }\hat{\imath})-(0.5\text{ m }\hat{\imath})=0.3\text{ m }\hat{\imath}$ (*e*) $\Delta\vec{x}=\vec{x}_f-\vec{x}_i=(0.0\text{ m }\hat{\imath})-(0.8\text{ m }\hat{\imath})=-0.8\text{ m }\hat{\imath}$ (*f*) $\Delta\vec{x}=\vec{x}_f-\vec{x}_i=(0.0\text{ m }\hat{\imath})-(0.5\text{ m }\hat{\imath})=-0.5\text{ m }\hat{\imath}$ (*g*) The object first travels a distance of 0.45 m in the positive *x* direction, followed by approximately 0.20 in the negative *x* direction. Hence the total distance travelled is 0.65 m (*h*) On this time interval, all motion is in the negative *x* direction. The object moves from an initial position of 0.8 m along the *x* axis to a final position of 0.0 m. Hence the object covers a distance of 0.80 m. (*i*) The object first moves 0.45 m in the positive *x* direction, followed by 1.0 m in the negative *x* direction. To the appropriate number of significant digits, this corresponds to a distance of 1.5 m.

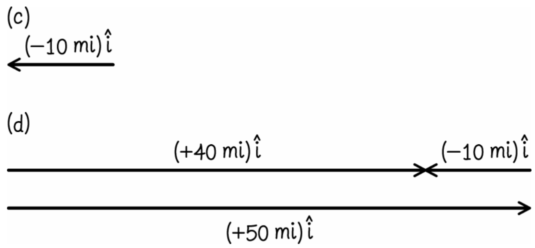
2.46. (*a*) $(+5\text{ m})\,\hat{\imath}+(-2\text{ m})\,\hat{\imath}+(+7\text{ m})\,\hat{\imath}$ (*b*) $(+5\text{ m})\,\hat{\imath}+(-2\text{ m})\,\hat{\imath}+(-7\text{ m})\,\hat{\imath}$ (*c*) $(+5\text{ m})\,\hat{\imath}+(+2\text{ m})\,\hat{\imath}+(-7\text{ m})\,\hat{\imath}$

2.47. $\vec{B}=-\vec{A}/2$

2.48. (*a*) $\vec{x}_{f1}=(+25\text{ mi/h})\,\hat{\imath}(2.0\text{ h})=(+50\text{ mi})\,\hat{\imath}$ (*b*) $\vec{x}_{f2}=\vec{x}_{f1}+(-20\text{ mi/h})\,\hat{\imath}(0.50\text{ h})=(+50\text{ mi})\,\hat{\imath}+(-10\text{ mi})\,\hat{\imath}=(+40\text{ mi})\,\hat{\imath}$ (*c*) $\Delta\vec{x}_2=\vec{x}_{f2}-\vec{x}_{f1}=(-20\text{ mi/h})\,\hat{\imath}(0.50\text{ h})=(-10\text{ mi})\,\hat{\imath}$ (*d*) $\Delta\vec{x}=\vec{x}_{f2}-\vec{x}_{f0}=(+40\text{ mi})\,\hat{\imath}-(0)=(+40\text{ mi})\,\hat{\imath}$
(*e*) You first travelled 50 mi in one direction, followed by 10 mi in the opposite direction. So the total distance travelled is 60 mi.
(*f*)

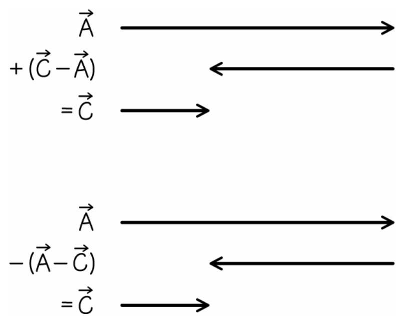


(*g*)



Yes, the values agree.

2.49. (*a*) $\vec{C}-\vec{A}$ (*b*) $\vec{A}-\vec{C}$



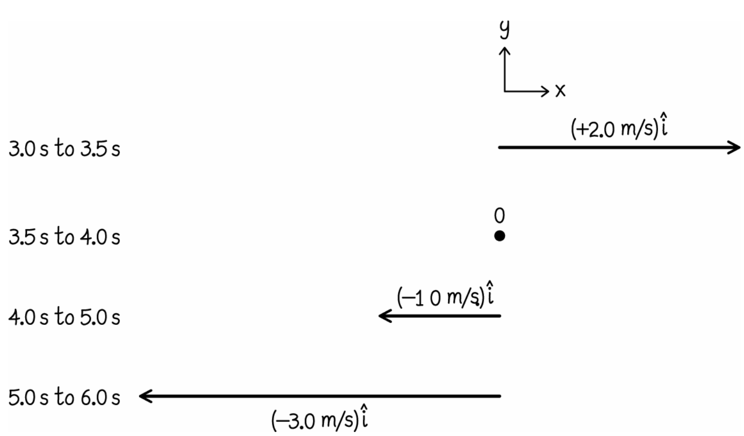
2.50. (*a*) 40 km/h (*b*) scalar (*c*) $d = vt = (40\text{ km/h})(2.0\text{ h}) = (80\text{ km})$. Distance is a scalar. (*d*) 80 km east (*e*) $\vec{v} = (40\text{ km/h})\,\hat{i}$.

2.51. (*a*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_\text{f}-\vec{x}_\text{i}}{\Delta t} = \dfrac{(0.0\text{ m }\hat{i})-(0.50\text{ m }\hat{i})}{1.2\text{ s}} = (-0.42\text{ m/s})\,\hat{i}$ (*b*) $\text{speed}_{\text{av}} = \dfrac{d}{\Delta t} = \dfrac{(1.5\text{ m})}{1.2\text{ s}} = (1.3\text{ m/s})$ (*c*) Some of the time the object was moving in the positive x direction, and sometimes it was moving in the negative x direction. Because velocity takes direction into account, this change of direction reduces the average velocity. In general because average velocity considers only actual distance between initial and final positions, but average speed considers distance traveled between these two positions; average speed is path-dependent, average velocity is not.

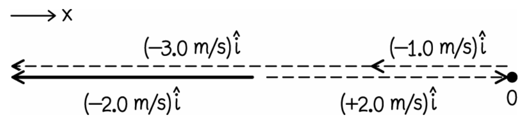
2.52. (*a*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_\text{f}-\vec{x}_\text{i}}{\Delta t} = \dfrac{(3.0\text{ m})\,\hat{i}-(0.0\text{ m})\,\hat{i}}{1.0\text{ s}} = (+3.0\text{ m/s})\,\hat{i}$ (*b*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_\text{f}-\vec{x}_\text{i}}{\Delta t} = \dfrac{(4.0\text{ m }\hat{i})-(0.0\text{ m }\hat{i})}{1.0\text{ s}} = (+1.0\text{ m/s})\,\hat{i}$

(*c*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_\text{f}-\vec{x}_\text{i}}{\Delta t} = \dfrac{(0.0\text{ m }\hat{i})-(3.0\text{ m }\hat{i})}{3.0\text{ s}} = (-1.0\text{ m/s})\,\hat{i}$ (*d*) Starting at time $t = 3.0\text{ s}$, the object first moves 1.0 m in the $+x$ direction, then turns around and moves 4.0 m in the $-x$ direction. Hence the object covers a total distance of 5.0 m between $t = 3.0\text{ s}$ and $t = 6.0\text{ s}$. The average speed is then given by $d/t = (5.0\text{ m})/(3.0\text{ s}) = 1.7\text{ m/s}$.

(*e*)



(*f*) Adding the arrows from part (*e*) shown dashed below, we obtain the bold arrow:



2.53. We refer to the first two thirds of the trip as segment 1 and the last third as segment 2. Starting from the definition of average speed, we write

$$\frac{d}{t}=\frac{d_1+d_2}{t_1+t_2}=\frac{d_1+d_2}{d_1/v_1+d_2/v_2}$$

Solving this expression for v_2 yields

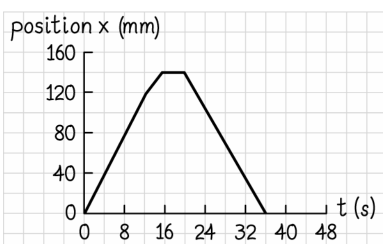
$$v_2=d_2\left(\frac{t(d_1+d_2)}{d}-d_1/v_1\right)^{-1}$$

$$v_2=d_2\left(\frac{(12\,\text{h})((2/3)d+(1/3)d)}{d}-(800\,\text{km})/(108\ \text{km/h})\right)^{-1}$$

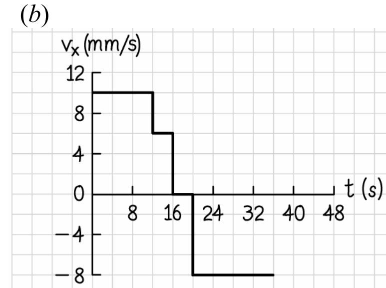
$$v_2=87\ \text{km/h}$$

2.54. We find the final position from the given data using $x_\text{f}=x_\text{i}+v_x t=(-2.073\ \text{m})+(-4.02\ \text{m/s})(0.103\ \text{s})$ $=-2.487\ \text{m}$.

2.55. (*a*)



(*b*)



2.56. The distance covered by you added to the distance covered by your friend must equal four blocks. So we can write: $\Delta x=\Delta x_\text{you}+\Delta x_\text{friend}=v_\text{you}t+v_\text{friend}t.$ Assuming you leave your respective buildings simultaneously, the same amount of time passes for each of you. Solving for this time yields $t=\Delta x/(v_\text{you}+v_\text{friend})$. Since we know both speeds and the total distance covered, we can use this expression to find the distance that you cover:

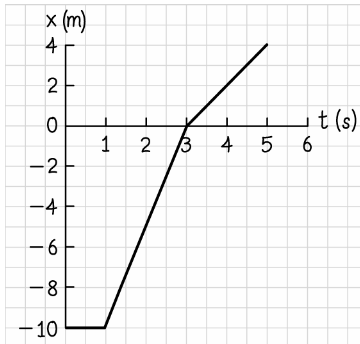
$$\Delta x_\text{you}=v_\text{you}t=v_\text{you}\Delta x/(v_\text{you}+v_\text{friend})=\frac{(1.6\ \text{m/s})(4\ \text{blocks})}{(1.6\ \text{m/s})+(1.2\ \text{m/s})}=2.3\ \text{blocks}$$

Hence the restaurant must be 2.3 blocks from your building.

2.57. Object A has the greater displacement. The area under curve A is fairly large and positive, partly because the velocity is always in the $+x$ direction. The area under curve B appears to be approximately zero, as the velocity is negative for half the time shown and positive for half the time (with approximately equal magnitudes).

2.58. Object B has the greater displacement. The area under curve B is greater than the area under A.

2.59.



2.60. We simply calculate the travel time for each of you and compare. The time required for you to reach the school is given by $t_{\text{you}} = d/v_{\text{you}} = (1.5\times10^4\ \text{m})/(10\ \text{m/s}) = 1.5\times10^3\ \text{s}$. The time required for you friend to reach the school is the time spent riding added to the time spent fixing the tire, so $t_{\text{friend}} = d/v_{\text{friend}} + t_{\text{flat}} = (1.5\times10^4\ \text{m})/(15\ \text{m/s}) + (720\ \text{s}) = 1.7\times10^3\ \text{s}$. Hence, you get to the school first.

2.61. (*a*) Let us call the direction of your initial motion the $+x$ direction. We begin by finding how far from the house you were when you stopped to chat with a neighbor. You position was $x_{\text{neighbor}} = v_x t = (5.0\ \text{m/s})(600\text{s}) = 3.0\times10^3\ \text{m}$. So, after you turn around and make it halfway back to your house (when your friend passes you) you are 1.5×10^3 m from your house. This is the total displacement of your friend for the period of time in question. The total time elapsed depends on when you choose to mark the “beginning of the trip”. Your friend remained at home for three minutes once you started riding. This could be treated as part of her trip (during which she had zero velocity) or one could start timing the trip once she finally starts moving. Here, we treat the trip as though it began at the same time for both of you, and you friend simply maintained zero velocity for three minutes. Then the total time is $\Delta t = \Delta t_{\text{away}} + \Delta t_{\text{neighbor}} + \Delta t_{\text{back}} = (600\ \text{s}) + (300\ \text{s}) + (1500\text{m})/(10\ \text{m/s}) = 1050\ \text{s}$. Then the magnitude of your friend’s average velocity is $|v_{\text{av}}| = |(x_{\text{f}} - x_{\text{i}})|/\Delta t = (1500\ \text{m})/(1050\ \text{s}) = 1.4\ \text{m/s}$. (*b*) You and your friend have the same initial and final positions during this time interval. Hence your average velocity and hers must be the same. Hence the magnitude of her average velocity is also 10 m/s.

2.62. We start by finding the amount of time required for your roommate to arrive: $t = \Delta x/v = (320\ \text{mi})/(60\ \text{mi/h}) = 5\ \text{h, 20 min}$. You wish to arrive after only 4h 50 min. Driving at 70 mi/h means you will have to actually be driving for $t = \Delta x/v = (320\ \text{mi})/(70\ \text{mi/h}) = 4\ \text{h, 34 min}$. So you need 16 minutes less than you have, and can hence afford to stop for as much as 16 minutes.

2.63. The entire time of your trip can be written as $\Delta t = \Delta t_{\text{E}} + \Delta t_{\text{W}} + \Delta t_{\text{N}} + \Delta t_{\text{S}}$, where the subscripts indicate the direction of your run during different segments. Even though your westward return trip is split up, you still have to cross the same distance westward to return home as you initially jogged eastward. Since you jog east and west at 2.0 m/s, these two segments of your trip will take the same amount of time. Similarly, the distances and speeds for the northward and southward segments are the same. Hence we can write $\Delta t = 2\Delta t_{\text{E}} + 2\Delta t_{\text{N}} = 2d_{\text{E}}/v_{\text{E}} + 2d_{\text{N}}/v_{\text{N}}$. Solving for the distance in the northward direction yields

$$d_{\text{N}} = v_{\text{N}}(\Delta t - (2d_{\text{E}}/v_{\text{E}}))/2 = (3.0\ \text{m/s})((2400\text{s}) - 2(2000\ \text{m})\,/\,(2.0\ \text{m/s}))/2 = 6.0\times10^2\ \text{m}$$

Hence you travelled 6.0×10^2 m northward before turning around.

2.64. Position

2.65. The velocity is the rate of change of position, meaning $v_x(t)=\dfrac{dx(t)}{dt}=\dfrac{d}{dt}(bt^{3/2})=\dfrac{3}{2}bt^{1/2}$. Hence

$$v_x(t=1.0\text{ s})=\frac{3}{2}(30.2\text{ m/s}^{3/2})(1.0\text{ s})^{1/2}=45\text{ m/s},\;\; v_x(t=4.0\text{ s})=\frac{3}{2}(30.2\text{ m/s}^{3/2})(4.0\text{ s})^{1/2}=91\text{ m/s}$$

2.66. (*a*) It may be instrumental to first determine whether the direction of the mouse's velocity is always the same on the time interval specified. If the mouse changes direction, it must momentarily stop as its velocity passes from negative to positive or vice-versa. That is equivalent to saying

$$v_x(t_{\text{stop}})=\left.\frac{dx(t)}{dt}\right|_{t_{\text{stop}}}=2pt_{\text{stop}}+q=0$$

This yields $t_{\text{stop}}=-q/2p=-(1.2\text{ m/s})/(2(0.40\text{ m/s}^2))=1.5\text{ s}$. So between $t=0$ and $t=1.0\text{ s}$ the velocity of the mouse is always in the same direction. So here the speed and velocity should have the same magnitude. The average velocity is given by $\vec{v}_{\text{av}}=\dfrac{\vec{x}_{\text{f}}-\vec{x}_{\text{i}}}{\Delta t}=\dfrac{x(t=1.0\text{ s})-x(t=0)}{1.0\text{ s}}\hat{i}=(-0.80\text{ m/s})\,\hat{i}$, and the average speed is thus 0.80 m/s.

(*b*) The velocity can be calculated on this interval in the same way: $\vec{v}_{\text{av}}=\dfrac{\vec{x}_{\text{f}}-\vec{x}_{\text{i}}}{\Delta t}=$ $\dfrac{x(t=4.0\text{ s})-x(t=1.0\text{ s})}{3.0\text{ s}}\hat{i}=\dfrac{(1.6\text{ m})-(-0.80\text{ m})}{3.0\text{ s}}\hat{i}=(+0.80\text{ m/s})\,\hat{i}$.

The average speed is not just the magnitude of the average velocity over this interval, because the velocity changes direction. To find the speed, we must consider that the mouse moves 0.10 m in the $-x$ direction before turning around and travelling 2.5 m in the $+x$ direction. Hence the average speed is $d/t=\big((2.5\text{ m})-(-0.1\text{ m})\big)/(3.0\text{ s})=0.87\text{ m/s}$.

2.67. Initially car A is moving faster than car B, because it passes car B. If car A continued moving faster than 30 m/s eastward the entire time, then there is no way that car B could ever have caught up with car A. Hence car A must have slowed from a velocity greater than 30 m/s eastward to some velocity that is less than 30 m/s eastward. The car cannot make discontinuous jumps in speed/velocity. So to pass smoothly from a velocity greater than 30 m/s eastward to a velocity less than 30 m/s eastward, it must have passed through the velocity of 30 m/s eastward at some point. Note that the speed could have always been greater than 30 m/s, but the velocity could not always have been greater than 30 m/s eastward.

2.68. For all cases, we use that $v_x(t)=\dfrac{dx(t)}{dt}=3pt^2+2qt$

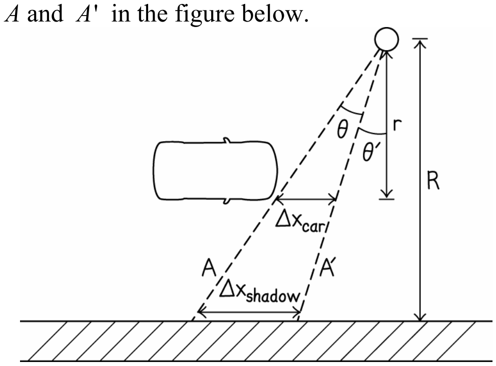
(*a*) $\vec{v}(\text{t}=0)=3(-2.0\text{ m/s}^3)(0)^2+2(+1.0\text{ m/s}^2)(0)=0$

(*b*) $\vec{v}(\text{t}=1.0\text{ s})=3(-2.0\text{ m/s}^3)(1.0\text{ s})^2+2(+1.0\text{ m/s}^2)(1.0\text{ s})=(-4.0\text{m/s})\,\hat{i}$

(*c*) $\vec{v}(\text{t}=1.0\text{ s})=3(-2.0\text{ m/s}^3)(2.0\text{ s})^2+2(+1.0\text{ m/s}^2)(2.0\text{ s})=(-20\text{m/s})\,\hat{i}$

(*d*) $\vec{v}(\text{t}=1.0\text{ s})=3(-2.0\text{ m/s}^3)(3.0\text{ s})^2+2(+1.0\text{ m/s}^2)(3.0\text{ s})=(-48\text{m/s})\,\hat{i}$

2.69. (*a*) The average speed of the shadow's leading edge is greater. (*b*) No, speed of shadow is always greater. To see this, consider the rays of light A and A' in the figure below.



Ray A passes just in front of the front end of the car at time t_0 and ray A' makes the front edge at time $t_0 + dt$. In the short time interval dt, the car moves a distance Δx_{car} and the shadow moves a distance Δx_{shadow}. We can write expressions for each of these distances from the geometry shown in the figure:

$$\Delta x_{\text{car}} = r\tan(\theta) - r\tan(\theta')$$

$$\Delta x_{\text{shadow}} = R\tan(\theta) - R\tan(\theta')$$

We can write the ratio of the speed of the shadow v_{shadow} to the speed of the car v_{car} as

$$\frac{v_{\text{shadow}}}{v_{\text{car}}} = \frac{R(\tan(\theta) - \tan(\theta'))/dt}{r(\tan(\theta) - \tan(\theta'))/dt}$$

We can cancel everything but the relevant distances, provided that $\tan(\theta) - \tan(\theta') \neq 0$. This assumption is true on the interval $-90° < \theta, \theta' < +90°$ (note here that $\theta \neq \theta'$ as long as the speed is not zero, because we are considering a non-zero interval of time). Finally, we have $v_{\text{shadow}} = \frac{R}{r} v_{\text{car}}$ for all positions of the car. This means the speed of the shadow is always greater than the speed of the car. Note that this includes the moment when the car's leading edge is directly between the light and the wall.

2.70. (*a*) $v_{x,\text{av}} = \frac{x_\text{f} - x_\text{i}}{t} = \frac{(p + qt_\text{f}^2) - (p + qt_\text{i}^2)}{t_\text{f} - t_\text{i}} = \frac{(+2.00 \text{ m/s}^2)((3.00 \text{ s})^2 - (2.00 \text{ s})^2)}{(1.00 \text{ s})} = 10.0 \text{ m/s}$

(*b*) $v_{x,\text{av}} = \frac{x_\text{f} - x_\text{i}}{t} = \frac{(p + qt_\text{f}^2) - (p + qt_\text{i}^2)}{t_\text{f} - t_\text{i}} = \frac{(+2.00 \text{ m/s}^2)\left((2.10 \text{ s})^2 - (2.00 \text{ s})^2\right)}{(0.10 \text{ s})} = 8.20 \text{ m/s}$

(*c*) $v_{x,\text{av}} = \frac{x_\text{f} - x_\text{i}}{t} = \frac{(p + qt_\text{f}^2) - (p + qt_\text{i}^2)}{t_\text{f} - t_\text{i}} = \frac{(+2.00 \text{ m/s}^2)((2.01 \text{ s})^2 - (2.00 \text{ s})^2)}{(0.01 \text{ s})} = 8.02 \text{ m/s}$

(*d*) $$\lim_{\Delta t \to 0}(\vec{v}_{\text{av}}) = \lim_{\Delta t \to 0}\left(\frac{\vec{x}(t + \Delta t) - \vec{x}(t)}{(t + \Delta t) - t}\right) = \lim_{\Delta t \to 0}\left(\frac{p + q(t + \Delta t)^2 - (p + q(t)^2)}{\Delta t}\right)\hat{i}$$

$$= \lim_{\Delta t \to 0}\left(\frac{q(t^2 + 2t\Delta t + \Delta t^2) - qt^2}{\Delta t}\right)\hat{i} = \lim_{\Delta t \to 0}(2qt + q\Delta t)\,\hat{i}$$

$$= 2qt\,\hat{i} = 2(2.00 \text{ m/s}^2)(2.00 \text{ s})\,\hat{i} = (+8.00 \text{ m/s})\,\hat{i}$$

(*e*) $v_x(t) = dx/dt = 2qt \Rightarrow v_x(t = 2.00 \text{ s}) = 8.00 \text{ m/s}$

2.71. (*a*) The average speed is the distance divided by time: $\frac{d}{\Delta t} = \frac{58 \text{ km}}{45 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{10^3 \text{ m}}{1 \text{ km}} = 21 \text{ m/s}$

(*b*) $\frac{d}{\Delta t} = \frac{58 \text{ km}}{45 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{1 \text{ mi}}{1.609 \text{ km}} = 48 \text{ mi/h}$

2.72. Yes, if the second place runner had a lower speed throughout most of the race, then the second place runner would be significantly behind the first place runner nearing the finish line. If the second place runner speeds up near the end, she could have a greater instantaneous speed and still fail to catch up with the first place runner.

2.73. (*a*) $x = -6.0 \text{ m}$ (*b*) $\vec{x} = (-6.0 \text{ m})\,\hat{i}$ (*c*) $x = 6.0 \text{ m}$

2.74. We use information about the distance and times for each runner to first calculate the time at which runner Q reaches the 1-km mark. Using that time, we calculate the positions of runners P and R. To do this we will need to know the speed of each runner:

$$v_\text{P} = \frac{d}{t_\text{P}} = \frac{5.000 \text{ km}}{15 \text{ min}} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ min}}{60 \text{ s}} = 5.555 \text{ m/s}$$

$$v_\text{Q} = \frac{d}{t_\text{Q}} = \frac{5.000 \text{ km}}{20 \text{ min}} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ min}}{60 \text{ s}} = 4.167 \text{ m/s}$$

$$v_R = \frac{d}{t_R} = \frac{5.000\text{ km}}{25\text{ min}} \times \frac{10^3\text{ m}}{1\text{ km}} \times \frac{1\text{ min}}{60\text{ s}} = 3.333\text{ m/s}$$

So, the time at which runner Q reaches the 1-km mark is given by $t = \frac{d}{v} = \frac{10^3\text{ m}}{4.167\text{ m/s}} = 240\text{ s}$. The positions of runners P and R after 240 s are $d_P = v_P t = (5.555\text{ m/s})(240\text{ s}) = 1333\text{ m}$, and $d_R = v_R t = (3.333\text{ m/s})(240\text{ s}) = 800\text{ m}$, respectively. Hence runners P and R are $d_P - d_R = 533\text{ m}$ apart as runner Q crosses the 1-km mark.

2.75. The problem only tells us by what distance one runner beats another. The problem says nothing about time, and therefore says nothing about absolute speed. We only know information about relative speeds, and can therefore only give an answer in terms of other variables in the problem. Let Δt_A be the time required for runner A to complete the race. When runner A crosses the finish line, runner B is only at the 90-m mark. Hence the speed of runner B can be written as

$$v_B = \frac{90\text{ m}}{\Delta t_A} = \frac{100\text{ m}}{\Delta t_B}$$

$$\Delta t_B = \frac{100\text{ m}}{90\text{ m}} \Delta t_A$$

Runner C is at the 90-m mark when runner B completes the race. Hence the time required for runner C to finish the 100-m race is

$$v_C = \frac{90\text{ m}}{\Delta t_B} = \frac{100\text{ m}}{\Delta t_C}$$

$$\Delta t_C = \frac{100\text{ m}}{90\text{ m}} \Delta t_B = \left(\frac{100\text{ m}}{90\text{ m}}\right)^2 \Delta t_A$$

The amount of time by which runner A beats runner C is

$$\Delta t_C - \Delta t_A = \left[\left(\frac{100\text{ m}}{90\text{ m}}\right)^2 - 1\right] \Delta t_A = 0.23 \Delta t_A$$

2.76. (*a*) At the instant car B passes the milepost, it is a certain distance behind car A. This distance is given by $v_A \Delta t$. Car B is gaining on car A at a rate equal to the difference between their speeds: $v_{rel} = (v_B - v_A)$. The amount of time required for car B to catch up to car A is the distance separating them divided by the rate at which car B gains on car A: $t = \Delta t + v_A \Delta t/(v_B - v_A) = v_B \Delta t/(v_B - v_A)$. (*b*) The time interval found in part (*a*) began with car B right next to the mile post. So the distance from the milepost at which the two cars meet is equal to the distance travelled by car B in the interval found in (*a*). Hence $d = v_A v_B \Delta t/(v_B - v_A)$.

2.77. (*a*) At the instant your friend crosses the starting line you are ahead by a distance of $d_{you} = v_{you} t = (4.0\text{ m/s})(15\text{ s}) = 60\text{ m}$. Once you increase your speed to 5.0 m/s, your friend is only gaining on you by 1.0 m/s (he is only moving 1.0 m/s faster than you are). So the time required for him to make up the distance between you is $t = \frac{d}{v} = \frac{60\text{ m}}{1.0\text{ m/s}} = 60\text{ s}$. (*b*) The 60 s time interval in part (*a*) began with your friend crossing the starting line. For that entire 60 s interval, your friend was running at a speed of 6.0 m/s. Hence your friend passes you a distance $d = vt = (6.0\text{ m/s})(60\text{ s}) = 3.6 \times 10^2\text{ m}$ from the starting line.

2.78. The entire distance covered is 12 mi. The time required for the driving portion is given by $t = \frac{d}{v} = \frac{10\text{ mi}}{45\text{ mi/h}} = 0.22\text{ h}$. The time spent walking is equal to 0.67 h. Hence the average speed is $v_{av} = \frac{d}{t} = \frac{12\text{ mi}}{0.89\text{ h}} = 14\text{ mi/h}$.

2.79. (*a*) $v_{\text{av}} = \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{c(\sqrt{20\text{ s}} - \sqrt{10\text{ s}})}{10\text{ s}} = (0.131\text{ s}^{-1/2})c,$ and the speed at 15 s is $v(t = 15\text{ s}) = \dfrac{1}{2}ct^{-1/2}\Big|_{t=15\text{ s}} =$ $\dfrac{1}{2}c(15\text{ s})^{-1/2} = (0.129\text{ s}^{-1/2})c.$

Hence the average speed is greater than the instantaneous speed at 15 s. (*b*) This can be answered using intuition. The position is always increasing, but at a smaller and smaller rate. Hence the final speed should be smaller than the speed at all previous times. The average speed must be greater than the speed at 20 s. Again we confirm this explicitly. The average speed is the same as that calculated in part (*a*). The instantaneous speed at 20 s is $v(t = 20\text{ s}) = \dfrac{1}{2}ct^{-1/2}\Big|_{t=20\text{ s}} = \dfrac{1}{2}c(20\text{ s})^{-1/2} = (0.112\text{ s}^{-1/2})c.$ This confirms our argument that the average speed must be greater than the speed after 20 s.

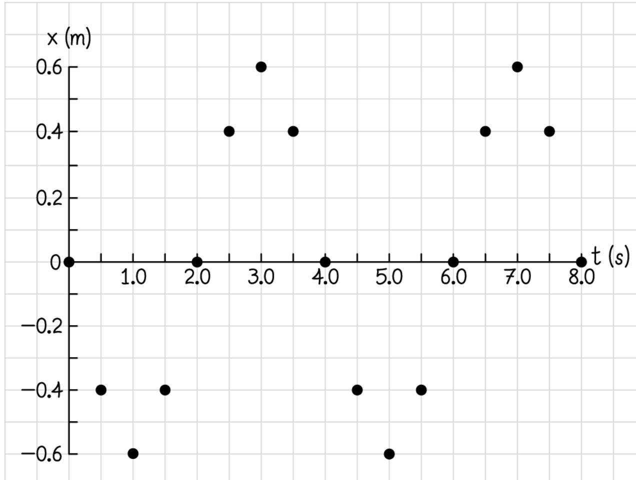
2.80. (*a*) These positions are all polynomials in time. The components of the velocities in the x direction are given by taking the derivative of each with respect to time. Terms that are linear in time will become time-independent. The only way for the velocity to have time dependence is for the position to have terms that are quadratic in time (t^2), or higher orders of time. Hence objects 3 and 4 have time-dependent velocities. (*b*) We solve each equation to find at what time the position is at the origin: Object 1 is never at the origin; it remains stationary at $x = 5$ m. Object 2 is at the origin when $bt + c = 0$ or $t = -c/b = -(-1\text{ m})/(+4\text{ m/s}) = 0.25\text{ s}.$ Object 3 is at the origin when $et^2 + ft = 0 \Rightarrow t = 0$ or $t = -f/e = -(-9\text{ m/s})/(+5\text{ m/s}^2) = 1.8\text{ s}.$ Finally, object 4 is at the origin when $gt^2 + h = 0$ or $t = \pm\sqrt{-h/g} = \pm\sqrt{-(+12\text{ m/s})/(-3\text{ m/s}^2)} = \pm 2\text{ s}.$ Hence, object 4 is at the origin at the earliest time, at $t = -2\text{ s}.$ (*c*) The x component of the object's velocity is given by the time derivative of the position: $v_x(t) = 2gt.$ So $v_x(-1\text{ s}) = 2(-3\text{ m/s}^2)(-1\text{ s}) = +6\text{ m/s}.$ The velocity is $(+6.0\text{ m/s})\,\hat{\imath}.$

2.81. (*a*) For the safe to move upward by a certain distance *d*, the length of rope on either side of the pulley attached to the safe must each decrease by *d*. Hence the mover must take up a length of rope 2*d* for every *d* of vertical rise of the safe. Hence the ratio of the vertical distance the safe moves to the length or rope pulled by the move is 1/2. (*b*) The relationship described in part (*a*) does not change in time. Hence, dividing any distances by times (to obtain speeds) will not affect the fraction found in part (*a*). The ratio is still 1/2.

2.82. The steam rollers approach each other at a combined relative speed of 2.0 m/s, and must cover 100 m between them. Hence, the steam rollers will meet after a time of $t = d/v = (100\text{ m})/(2.0\text{ m/s}) = 50\text{ s}.$ The fly is moving at a constant speed of 2.20 m/s. Hence the fly covers a distance of $d = vt = (2.20\text{ m/s})(50\text{ s}) = 110\text{ m}.$

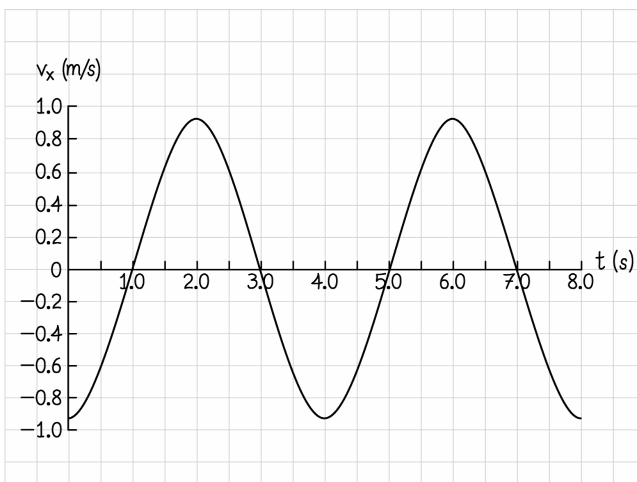
2.83. (*a*) $v_{x,\text{av}} = \dfrac{x_f - x_f}{t_f - t_i} = \dfrac{c(t_f^3 - t_i^3)}{t_f - t_i} = \dfrac{(+0.120\text{ m/s}^3)((1.50\text{ s})^3 - (0.500\text{ s})^3)}{(1.50\text{ s}) - (0.500\text{ s})} = (+0.39\text{ m/s})$ (*b*) $v_{x,\text{av}} = \dfrac{x_f - x_f}{t_f - t_i} =$ $\dfrac{c(t_f^3 - t_i^3)}{t_f - t_i} = \dfrac{(+0.120\text{ m/s}^3)((1.05\text{ s})^3 - (0.950\text{ s})^3)}{(1.05\text{ s}) - (0.950\text{ s})} = (+0.3603\text{ m/s})$ (*c*) Shortening the time interval by another factor of ten would make $t_i = 0.995\text{ s}$ and $t_f = 1.005\text{ s}.$ Then the same process as before yields $v_{x,\text{av}} = \dfrac{x(t = 1.005) - x(t = 0.995)}{0.01\text{ s}} = (+0.360003\text{ m/s})$ whereas $v_x = \dfrac{dx}{dt} = 3ct^2$ such that $v_x(t = 1.0\text{ s}) = 0.36\text{ m/s}.$

2.84. (*a*)



(*b*) These times could be read off of the plot in part (*a*) by looking for points where the slope of a continuous curve would be zero, or by solving $\frac{d}{dt}x(t)=0$. This equation becomes $\frac{d}{dt}(A\cos(pt+q))=-Ap\sin(pt+q)=0$. The sine function takes a value of zero whenever the argument is $n\pi$. Hence $pt+q=n\pi$ or $t=\frac{n\pi-q}{p}=\frac{(n-(1/2))\pi}{\pi/2\text{ s}^{-1}}=2(n-(1/2))\text{ s}$. Hence, the *x* component of the velocity is zero at 1.0 s, 3.0 s, 5.0 s, 7.0 s.

(*c*)



2.85. (*a*) $d/2\Delta t$ (*b*) $2\Delta t$ (*c*) The distance traveled by the runner is an infinite series that requires infinitely many terms to approach *d*, but the time intervals that correspond to these distance intervals also get smaller and smaller in the series. At higher and higher terms in the series, the runner travels almost no distance in each term but does so in a time interval that is almost zero. Because the two effects cancel each other, the runner travels the distance from starting line to finish line in a finite time interval.

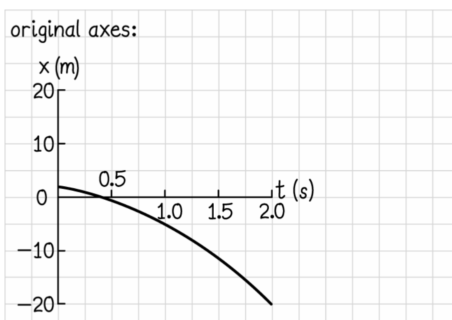
2.86. You start at the first light. The second light won't turn green for 10 s. So if you travel at a speed of $v=d/t=(300\text{ m})/(10\text{ s})=30\text{ m/s}$, you would have to stop at the second light momentarily. Hence there is an upper limit on your speed of 30 m/s. Since there is a 10-s lag between each light turning green, the last light will not turn green until 30 s after the first one turns green. That light will only stay on for 15 s, meaning you have a total of 45 s to get from the first light to through the fourth light. This requires that your speed be at least

$v = d/t = (900\ \text{m})/(45\ \text{s}) = 20\ \text{m/s}$. Hence the only constraints set by the lights are that your speed must satisfy $20\ \text{m/s} < v < 30\ \text{m/s}$. But if you also want to drive as slowly as possible, then $v = 20\ \text{m/s}$ is best.

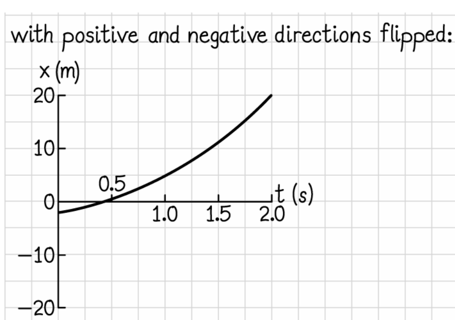
2.87. Since Hare can cover 1.0 mi in 10 minutes, when he falls asleep at after 5 minutes he must be at the half-way point (0.5 mi). This distance would take Tortoise 30 minutes, since he can only cover 1.0 mi in one hour. So 30 minutes into the race, Hare and Tortoise are both at the half-way point, with Hare only 25 minutes into his 40 minute nap, and with tortoise increasing his speed to (5/3) mi/h. From the half-way point, Tortoise will reach the finish line in an additional $t = d/v = (0.50\ \text{mi})/(5/3\ \text{mi/h}) = 0.3\ \text{h}$. Hare will not wake up for another 0.25 h. This means Hare would have to cover the remaining 0.50 mi in just 0.05 h. Hare would have to run at 10 mi/h to win. This is not possible. Hare can only run at 6 mi/h. If Hare ran at that top speed, Hare could only cover $x = vt = (6\ \text{mi/h})(0.05\ \text{h}) = 0.30\ \text{mi}$ before Tortoise crosses the finish line. So, Tortoise wins by 0.2 miles, even with Hare running at his top speed. Even if Hare can push himself a little faster than 6 mi/h, it is not likely that he could suddenly increase his top speed by 67% to reach the required 10 mi/h.

2.88. Runner B will require 13.5 s to complete the race. Runner A has an average speed of $v_{x,\text{av}} = d/t = (100\ \text{m})/(12.0\ \text{s}) = 8.33\ \text{m/s}$. Hence, in the 13.5 s needed by runner B, runner A can cover a distance of $d = vt = (8.33\ \text{m/s})(13.5\ \text{s}) = 112.5\ \text{m}$. Hence, runner A should add an extra 12.5 m to her distance, starting 12.5 m behind the starting line.

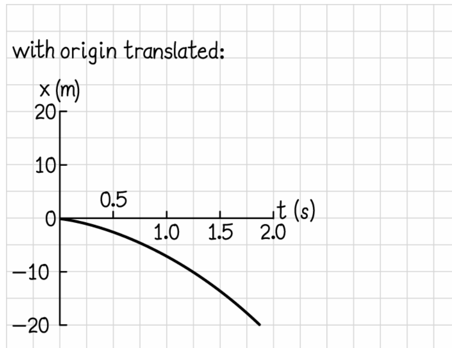
2.89. (*a*) Keeping the coefficients as they are defined in the problem, the position would become $x(t) = -p - qt - rt^2$. (*b*) Before any change in coordinates, the initial position of the car was at $x = (+2.0\ \text{m})$. This must now be subtracted off, such that the car starts at the origin. This could be written as $x(t) = (p - 2.0\ \text{m}) + qt + rt^2$ or simply as $x(t) = qt + rt^2$.
(*c*)



With the positive and negative directions flipped:



With the origin translated:



(*d*) Before axes were altered, the equation was $x(t) = p + qt + rt^2$, such that $x(t = 4.0 \text{ s}) = (+2.0 \text{ m}) + (-3.0 \text{ m/s})(4.0 \text{ s}) + (-4.0 \text{ m/s}^2)(4.0 \text{ s})^2 = -74 \text{ m}$. After the direction of the axis is flipped, everything changes sign, such that the new displacement is +74 m. With the origin shifted, all that changes from the initial setup is that the position is shifted by –2.0 m, such that the position is now –76 m. (*e*) –35 m/s, 35 m/s, –35 m/s (*f*) You have only changed what position you choose to call zero, and which directions you choose to call positive.

2.90. The time required for light to travel from Earth to Mars is $t = d/v = (2.0\times10^{11} \text{ m})/(3.0\times10^8 \text{ m/s}) = 670 \text{ s}$ or just over 11 minutes. This means it would take 22 minutes for an image of a cliff to reach you and for you to send back a "stop" signal. In those 22 minutes, the rover can travel about 44 m. Hence, if the rover is travelling at top speed, you would need to be able to see 44 m ahead of the rover. However, it is probably not advisable to push the limits of this. It is more likely that you would send a command to move 25 m and then stop, rather than letting the rover continue until you specifically send a "stop" signal.

2.91. A loaded truck takes 7 hours to reach the mill, and an empty truck takes 6 hours to return from the mill. First consider your trip to the mill: Because the drivers from the mill should be spaced 30 minutes from the mine, 90 minutes from the mine, [etc.], there should be 6 other trucks already on the road. You must pass all 6 of these, plus any additional trucks that are sent out during your trip. Since your trip lasts 7 hours, 7 additional trucks will be sent out as you drive. This makes a total of 13 empty trucks that you pass along the way. The answer does not change on your return trip. One way to easily see this is that when you leave for the mine, there are 7 trucks already on the road. That is one more than we considered for your trip to the mill. But because you are driving faster now, you will require only 6 hours for the trip, meaning only 6 additional trucks will be sent out as you drive. That is one fewer than we considered on your trip to the mill. Hence you also pass 13 full trucks on the way back.w