For Thought

1. False, since $\{(1, 2), (1, 3)\}$ is not a function.

2. False, since $f(5)$ is not defined. **3.** True

4. False, since a student's exam grade is a function of the student's preparation. If two classmates had the same IQ and only one prepared then the one who prepared will most likely achieve a higher grade.

5. False, since $(x + h)^2 = x^2 + 2xh + h^2$

6. False, since the domain is all real numbers.

7. True **8.** True **9.** True

10. False, since $\left(\frac{3}{8}, 8\right)$ and $\left(\frac{3}{8}, 5\right)$ are two ordered pairs with the same first coordinate and different second coordinates.

2.1 Exercises

1. relation

2. function

3. independent, dependent

4. domain, range

5. difference quotient

6. average rate of change

7. Note, $b = 2\pi a$ is equivalent to $a = \frac{b}{2\pi}$.

Then a is a function of b, and b is a function of a.

8. Note, $b = 2(5 + a)$ is equivalent to $a = \frac{b - 10}{2}$.

So a is a function of b, and b is a function of a.

9. a is a function of b since a given denomination has a unique length. Since a dollar bill and a five-dollar bill have the same length, then b is not a function of a.

10. Since different U.S. coins have different diameters, then a is a function of b and b is a function of a.

11. Since an item has only one price, b is a function of a. Since two items may have the same price, a is not a function of b.

12. a is not a function of b since there may be two students with the same semester grades but different final exams scores. b is not a function of a since there may be identical final exam scores with different semester grades.

13. a is not a function of b since it is possible that two different students can obtain the same final exam score but the times spent on studying are different.

b is not a function of a since it is possible that two different students can spend the same time studying but obtain different final exam scores.

14. a is not a function of b since it is possible that two adult males can have the same shoe size but have different ages.

b is not a function of a since it is possible for two adults with the same age to have different shoe sizes.

15. Since 1 in $\approx$ 2.54 cm, a is a function of b and b is a function of a.

16. Since there is only one cost for mailing a first class letter, then a is a function of b. Since two letters with different weights each under 1/2-ounce cost 34 cents to mail first class, b is not a function of a.

17. No **18.** No **19.** Yes

20. Yes **21.** Yes **22.** No

23. Yes **24.** Yes

25. Not a function since 25 has two different second coordinates. **26.** Yes

27. Not a function since 3 has two different second coordinates.

28. Yes **29.** Yes **30.** Yes

31. Since the ordered pairs in the graph of $y = 3x - 8$ are $(x, 3x - 8)$, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

32. Since the ordered pairs in the graph of $y = x^2 - 3x + 7$ are $(x, x^2 - 3x + 7)$, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

33. Since $y = (x + 9)/3$, the ordered pairs are $(x, (x+9)/3)$. Thus, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

34. Since $y = \sqrt[3]{x}$, the ordered pairs are $(x, \sqrt[3]{x})$. Thus, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

35. Since $y = \pm x$, the ordered pairs are $(x, \pm x)$. Thus, there are two ordered pairs with the same first coordinate and different second coordinates. We do not have a function.

36. Since $y = \pm\sqrt{9 + x^2}$, the ordered pairs are $(x, \pm\sqrt{9 + x^2})$. Thus, there are two ordered pairs with the same first coordinate and different second coordinates. We do not have a function.

37. Since $y = x^2$, the ordered pairs are (x, x^2). Thus, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

38. Since $y = x^3$, the ordered pairs are (x, x^3). Thus, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

39. Since $y = |x| - 2$, the ordered pairs are $(x, |x| - 2)$. Thus, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

40. Since $y = 1 + x^2$, the ordered pairs are $(x, 1 + x^2)$. Thus, there are no two ordered pairs with the same first coordinate and different second coordinates. We have a function.

41. Since $(2, 1)$ and $(2, -1)$ are two ordered pairs with the same first coordinate and different second coordinates, the equation does not define a function.

42. Since $(2, 1)$ and $(2, -1)$ are two ordered pairs with the same first coordinate and different second coordinates, the equation does not define a function.

43. Domain $\{-3, 4, 5\}$, range $\{1, 2, 6\}$

44. Domain $\{1, 2, 3, 4\}$, range $\{2, 4, 8, 16\}$

45. Domain $(-\infty, \infty)$, range $\{4\}$

46. Domain $\{5\}$, range $(-\infty, \infty)$

47. Domain $(-\infty, \infty)$;
since $|x| \geq 0$, the range of $y = |x| + 5$ is $[5, \infty)$

48. Domain $(-\infty, \infty)$;
since $x^2 \geq 0$, the range of $y = x^2 + 8$ is $[8, \infty)$

49. Since $x = |y| - 3 \geq -3$, the domain of $x = |y| - 3$ is $[-3, \infty)$; range $(-\infty, \infty)$

50. Since $\sqrt{y} - 2 \geq -2$, the domain of $x = \sqrt{y} - 2$ is $[-2, \infty)$; Since $\sqrt{y}$ is a real number whenever $y \geq 0$, the range is $[0, \infty)$.

51. Since $\sqrt{x - 4}$ is a real number whenever $x \geq 4$, the domain of $y = \sqrt{x - 4}$ is $[4, \infty)$.

Since $y = \sqrt{x - 4} \geq 0$ for $x \geq 4$, the range is $[0, \infty)$.

52. Since $\sqrt{5 - x}$ is a real number whenever $x \leq 5$, the domain of $y = \sqrt{5 - x}$ is $(-\infty, 5]$.

Since $y = \sqrt{5 - x} \geq 0$ for $x \leq 5$, the range is $[0, \infty)$.

53. Since $x = -y^2 \leq 0$, the domain of $x = -y^2$ is $(-\infty, 0]$; range is $(-\infty, \infty)$;

54. Since $x = -|y| \leq 0$, the domain of $x = -|y|$ is $(-\infty, 0]$; range is $(-\infty, \infty)$;

55. 6 **56.** 5

57. $g(2) = 3(2) + 5 = 11$

58. $g(4) = 3(4) + 5 = 17$

59. Since $(3,8)$ is the ordered pair, one obtains $f(3) = 8$. The answer is $x = 3$.

60. Since $(2,6)$ is the ordered pair, one obtains $f(2) = 6$. The answer is $x = 2$.

61. Solving $3x + 5 = 26$, we find $x = 7$.

62. Solving $3x + 5 = -4$, we find $x = -3$.

63. $f(4) + g(4) = 5 + 17 = 22$

64. $f(3) - g(3) = 8 - 14 = -6$

65. $3a^2 - a$ **66.** $3w^2 - w$

67. $4(a+2)-2 = 4a+6$ **68.** $4(a-5)-2 = 4a-22$

69. $3(x^2 + 2x + 1) - (x + 1) = 3x^2 + 5x + 2$

70. $3(x^2 - 6x + 9) - (x - 3) = 3x^2 - 19x + 30$

71. $4(x + h) - 2 = 4x + 4h - 2$

72. $3(x^2+2xh+h^2)-x-h = 3x^2+6xh+3h^2-x-h$

73. $3(x^2 + 2x + 1) - (x + 1) - 3x^2 + x = 6x + 2$

74. $4(x + 2) - 2 - 4x + 2 = 8$

75. $3(x^2 + 2xh + h^2) - (x + h) - 3x^2 + x = 6xh + 3h^2 - h$

76. $(4x + 4h - 2) - 4x + 2 = 4h$

77. The average rate of change is

$$\frac{8,000 - 20,000}{5} = -\$2,400 \text{ per year.}$$

78. The average rate of change as the number of cubic yards changes from 12 to 30 and from 30 to 60 are

$\dfrac{528 - 240}{30 - 12} = \$16$ per yd^3 and

$\dfrac{948 - 528}{60 - 30} = \$14$ per yd^3, respectively.

79. The average rate of change on $[0, 2]$ is

$\dfrac{h(2) - h(0)}{2 - 0} = \dfrac{0 - 64}{2 - 0} = -32$ ft/sec.

The average rate of change on $[1, 2]$ is

$\dfrac{h(2) - h(1)}{2 - 1} = \dfrac{0 - 48}{2 - 1} = -48$ ft/sec.

The average rate of change on $[1.9, 2]$ is

$\dfrac{h(2) - h(1.9)}{2 - 1.9} = \dfrac{0 - 6.24}{0.1} = -62.4$ ft/sec.

The average rate of change on $[1.99, 2]$ is

$\dfrac{h(2) - h(1.99)}{2 - 1.99} = \dfrac{0 - 0.6384}{0.01} = -63.84$ ft/sec.

The average rate of change on $[1.999, 2]$ is

$\dfrac{h(2) - h(1.999)}{2 - 1.999} = \dfrac{0 - 0.063984}{0.001} = -63.984$ ft/sec.

80. $\dfrac{6 - 70}{2 - 0} = \dfrac{-64}{2} = -32$ ft/sec

81. The average rate of change is $\dfrac{1768 - 1970}{20} = -10.1$ million hectares per year.

82. If 10.1 million hectares are lost each year and since $\dfrac{1970}{10.1} \approx 195$ years, then the forest will be eliminated in the year 2183 ($= 1988 + 195$).

83.

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{4(x+h) - 4x}{h} \\ &= \frac{4h}{h} \\ &= 4\end{aligned}$$

84.

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{2}(x+h) - \frac{1}{2}x}{h} \\ &= \frac{\frac{1}{2}h}{h} \\ &= \frac{1}{2}\end{aligned}$$

85.

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{3(x+h) + 5 - 3x - 5}{h} \\ &= \frac{3h}{h} \\ &= 3\end{aligned}$$

86.

$$\frac{f(x+h)-f(x)}{h} = \frac{-2(x+h)+3+2x-3}{h} = \frac{-2h}{h} = -2$$

87. Let $g(x) = x^2 + x$. Then we obtain

$$\frac{g(x+h)-g(x)}{h} = \frac{(x+h)^2+(x+h)-x^2-x}{h} = \frac{2xh+h^2+h}{h} = 2x+h+1.$$

88. Let $g(x) = x^2 - 2x$. Then we get

$$\frac{g(x+h)-g(x)}{h} = \frac{(x+h)^2-2(x+h)-x^2+2x}{h} = \frac{2xh+h^2-2h}{h} = 2x+h-2.$$

89. Difference quotient is

$$= \frac{-(x+h)^2+(x+h)-2+x^2-x+2}{h} = \frac{-2xh-h^2+h}{h} = -2x-h+1$$

90. Difference quotient is

$$= \frac{(x+h)^2-(x+h)+3-x^2+x-3}{h} = \frac{2xh+h^2-h}{h} = 2x+h-1$$

91. Difference quotient is

$$= \frac{3\sqrt{x+h}-3\sqrt{x}}{h}\cdot\frac{3\sqrt{x+h}+3\sqrt{x}}{3\sqrt{x+h}+3\sqrt{x}} = \frac{9(x+h)-9x}{h(3\sqrt{x+h}+3\sqrt{x})} = \frac{9h}{h(3\sqrt{x+h}+3\sqrt{x})} = \frac{3}{\sqrt{x+h}+\sqrt{x}}$$

92. Difference quotient is

$$= \frac{-2\sqrt{x+h}+2\sqrt{x}}{h}\cdot\frac{-2\sqrt{x+h}-2\sqrt{x}}{-2\sqrt{x+h}-2\sqrt{x}} = \frac{4(x+h)-4x}{h(-2\sqrt{x+h}-2\sqrt{x})} = \frac{4h}{h(-2\sqrt{x+h}-2\sqrt{x})} = \frac{-2}{\sqrt{x+h}+\sqrt{x}}$$

93. Difference quotient is

$$= \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\cdot\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}} = \frac{(x+h+2)-(x+2)}{h(\sqrt{x+h+2}+\sqrt{x+2})} = \frac{h}{h(\sqrt{x+h+2}+\sqrt{x+2})} = \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$$

94. Difference quotient is

$$= \frac{\sqrt{\frac{x+h}{2}}-\sqrt{\frac{x}{2}}}{h}\cdot\frac{\sqrt{\frac{x+h}{2}}+\sqrt{\frac{x}{2}}}{\sqrt{\frac{x+h}{2}}+\sqrt{\frac{x}{2}}} = \frac{\frac{x+h}{2}-\frac{x}{2}}{h\left(\sqrt{\frac{x+h}{2}}+\sqrt{\frac{x}{2}}\right)} = \frac{\frac{h}{2}}{h\left(\sqrt{\frac{x+h}{2}}+\sqrt{\frac{x}{2}}\right)} = \frac{1}{2\left(\sqrt{\frac{x+h}{2}}+\sqrt{\frac{x}{2}}\right)} = \frac{1}{\sqrt{2}\left(\sqrt{x+h}+\sqrt{x}\right)}$$

95. Difference quotient is

$$= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$
$$= \frac{x-(x+h)}{xh(x+h)}$$
$$= \frac{-h}{xh(x+h)}$$
$$= \frac{-1}{x(x+h)}$$

96. Difference quotient is

$$= \frac{\frac{3}{x+h} - \frac{3}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$
$$= \frac{3x-3(x+h)}{xh(x+h)}$$
$$= \frac{-3h}{xh(x+h)}$$
$$= \frac{-3}{x(x+h)}$$

97. Difference quotient is

$$= \frac{\frac{3}{x+h+2} - \frac{3}{x+2}}{h} \cdot \frac{(x+h+2)(x+2)}{(x+h+2)(x+2)}$$
$$= \frac{3(x+2)-3(x+h+2)}{h(x+h+2)(x+2)}$$
$$= \frac{-3h}{h(x+h+2)(x+2)}$$
$$= \frac{-3}{(x+h+2)(x+2)}$$

98. Difference quotient is

$$= \frac{\frac{2}{x+h-1} - \frac{2}{x-1}}{h} \cdot \frac{(x+h-1)(x-1)}{(x+h-1)(x-1)}$$
$$= \frac{2(x-1)-2(x+h-1)}{h(x+h-1)(x-1)}$$
$$= \frac{-2h}{h(x+h-1)(x-1)}$$
$$= \frac{-2}{(x+h-1)(x-1)}$$

99. **a)** $A = s^2$ **b)** $s = \sqrt{A}$ **c)** $s = \dfrac{d\sqrt{2}}{2}$
d) $d = s\sqrt{2}$ **e)** $P = 4s$ **f)** $s = P/4$
g) $A = P^2/16$ **h)** $d = \sqrt{2A}$

100. **a)** $A = \pi r^2$ **b)** $r = \sqrt{\dfrac{A}{\pi}}$ **c)** $C = 2\pi r$
d) $d = 2r$ **e)** $d = \dfrac{C}{\pi}$ **f)** $A = \dfrac{\pi d^2}{4}$
g) $d = 2\sqrt{\dfrac{A}{\pi}}$

101. $C = 50 + 35n$

102. **a)** When $d = 100$ ft, the atmospheric pressure
is $A(100) = .03(100) + 1 = 4$ atm.

b) When $A = 4.9$ atm, the depth is found by solving $4.9 = 0.03d + 1$; the depth is

$$d = \frac{3.9}{0.03} = 130 \text{ ft.}$$

103.

(a) The quantity $C(4) = (0.95)(4) + 5.8 =$ \$9.6 billion represents the amount spent on computers in the year 2004.

(b) By solving $0.95n + 5.8 = 15$, we obtain

$$n = \frac{9.2}{0.95} \approx 10.$$

Thus, spending for computers will be \$15 billion in 2010.

104.

(a) The quantity $E(4)+C(4) = [0.5(4)+1]+ 9.6 =$ \$12.6 billion represents the total amount spent on electronics and computers in the year 2004.

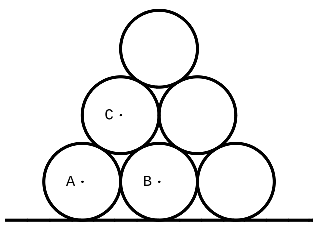
(b) By solving

$$\begin{aligned} (0.5n+1)+(0.95n+5.8) &= 20 \\ 1.45n &= 13.2 \\ n &\approx 9 \end{aligned}$$

we find that the total spending will reach \$20 billion in the year 2009 ($= 2000 + 9$).

(c) The amount spent on computers is growing faster since the slope of $C(n)$ [which is 1] is greater than the slope of $E(n)$ [which is 0.95].

105. Let a be the radius of each circle. Note, triangle $\triangle ABC$ is an equilateral triangle with side $2a$ and height $\sqrt{3}a$.

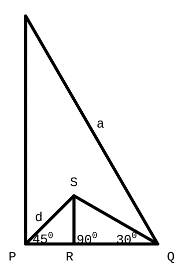


Thus, the height of the circle centered at C from the horizontal line is $\sqrt{3}a + 2a$. Hence, by using a similar reasoning, we obtain that height of the highest circle from the line is

$$2\sqrt{3}a + 2a$$

or equivalently $(2\sqrt{3} + 2)a$.

106. In the triangle below, PS bisects the 90-angle at P and SQ bisects the 60-angle at Q.



In the 45-45-90 triangle $\triangle SPR$, we find

$$PR = SR = \sqrt{2}d/2.$$

And, in the 30-60-90 triangle $\triangle SQR$ we get

$$PQ = \frac{\sqrt{6}}{2}d.$$

Since $PQ = PR + RQ$, we obtain

$$\begin{aligned} \frac{a}{2} &= \frac{\sqrt{2}}{2}d + \frac{\sqrt{6}}{2}d \\ a &= \sqrt{2}d + \sqrt{6}d \\ a &= (\sqrt{6} + \sqrt{2})d \\ d &= \frac{a}{\sqrt{6} + \sqrt{2}} \\ d &= \frac{\sqrt{6} - \sqrt{2}}{4}a. \end{aligned}$$

107. When $x = 18$ and $h = 0.1$, we have

$$\frac{R(18.1) - R(18)}{0.1} = 1,950.$$

The revenue from the concert will increase by approximately \$1,950 if the price of a ticket is raised from \$18 to \$19.

If $x = 22$ and $h = 0.1$, then

$$\frac{R(22.1) - R(22)}{0.1} = -2,050.$$

The revenue from the concert will decrease by approximately \$2,050 if the price of a ticket is raised from \$22 to \$23.

108. When $r = 1.4$ and $h = 0.1$, we obtain

$$\frac{A(1.5) - A(1.4)}{0.1} \approx -16.1$$

The amount of tin needed decreases by approximately 16.1 in.^{2} if the radius increases from 1.4 in. to 2.4 in.

If $r = 2$ and $h = 0.1$, then

$$\frac{A(2.1) - A(2)}{0.1} \approx 8.6$$

The amount of tin needed increases by about 8.6 in.^{2} if the radius increases from 2 in. to 3 in.

111.

$$\begin{aligned} \frac{3}{2}x - \frac{5}{9}x &= \frac{1}{3} - \frac{5}{6} \\ \frac{17}{18}x &= -\frac{1}{2} \\ x &= -\frac{1}{2} \cdot \frac{18}{17} \\ x &= -\frac{9}{17} \end{aligned}$$

112. If m is the number of males, then

$$\begin{aligned} m+\frac{1}{2}m &= 36 \\ \frac{3}{2}m &= 36 \\ m &= (36)\frac{2}{3} \\ x &= 24 \text{ males} \end{aligned}$$

113. $\sqrt{(-4+6)^2+(-3-3)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

114. The slope is $\frac{3-2}{5+1} = \frac{1}{6}$. The line is given by $y = \frac{1}{6}x + b$ for some b. Substitute the coordinates of $(-1,2)$ as follows:

$$\begin{aligned} 2 &= \frac{1}{6}(-1)+b \\ \frac{13}{6} &= b \end{aligned}$$

The line is given by

$$y = \frac{1}{6}x + \frac{13}{6}.$$

115.

$$\begin{aligned} x^2-x-6 &= 36 \\ x^2-x-42 &= 0 \\ (x-7)(x+6) &= 0 \end{aligned}$$

The solution set is $\{-6,7\}$.

116. The inequality is equivalent to

$$-13 < 2x-9 < 13$$

$$-4 < 2x < 22$$

$$-2 < 2x < 11$$

The solution set is $(-2,11)$.

Thinking Outside the Box XXII

$(30+25)^2 = 3025$

2.1 Pop Quiz

1. Yes, since $A = \pi r^2$ where A is the area of a circle with radius r.

2. No, since the ordered pairs $(2,4)$ and $(-2,4)$ have the same first coordinates.

3. No, since the ordered pairs $(1,0)$ and $(-1,0)$ have the same first coordinates.

4. $[1,\infty)$ **5.** $[2,\infty)$ **6.** 9

7. If $2a = 1$, then $a = 1/2$.

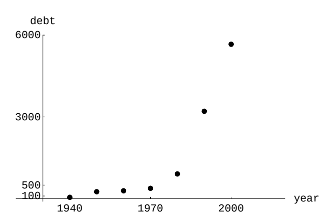
8. $\frac{40-20}{2008-1998} = \$2$ per year

9. The difference quotient is

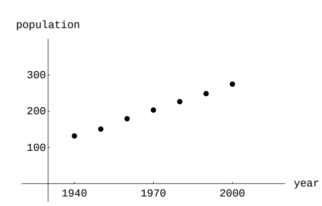
$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^2+3-x^2-3}{h} \\ &= \frac{x^2+2xh+h^2-x^2}{h} \\ &= \frac{2xh+h^2}{h} \\ &= 2x+h \end{aligned}$$

2.1 Linking Concepts

(a) The first graph shows U.S. federal debt D versus year y



and the second graph shows population P (in millions) versus y.



(b) The first table shows the average rates of change for the U.S. federal debt

10 – year period	ave. rate of change
1940 – 50	$\frac{257-51}{10} = 20.6$
1950 – 60	$\frac{291-257}{10} = 3.4$
1960 – 70	$\frac{381-291}{10} = 9.0$
1970 – 80	$\frac{909-381}{10} = 52.8$
1980 – 90	$\frac{3207-909}{10} = 229.8$
1990 – 2000	$\frac{5666-3207}{10} = 245.9$

The second table shows the average rates of change for the U.S. population

10 – year period	ave. rate of change
1940 – 50	$\frac{150.7-131.7}{10} \approx 1.9$
1950 – 60	$\frac{179.3-150.7}{10} \approx 2.9$
1960 – 70	$\frac{203.3-179.3}{10} \approx 2.4$
1970 – 80	$\frac{226.5-203.3}{10} \approx 2.3$
1980 – 90	$\frac{248.7-226.5}{10} \approx 2.2$
1990 – 2000	$\frac{274.8-248.7}{10} \approx 2.6$

(c) The first table shows the difference between consecutive average rates of change for the U.S. federal debt.

10-year periods	difference
1940-50 & 1950-60	$3.4 - 20.6 = -17.2$
1950-60 & 1960-70	$9.0 - 3.4 = 5.6$
1960-70 & 1970-80	$52.8 - 9.0 = 43.8$
1970-80 & 1980-90	$229.8 - 52.8 = 177.0$
1980-90 & 1990-00	$245.9 - 229.8 = 16.1$

The second table shows the difference between consecutive average rates of change for the U.S. population.

10-year periods	difference
1940-50 & 1950-60	$2.9 - 1.9 = 1.0$
1950-60 & 1960-70	$2.4 - 2.9 = -0.5$
1960-70 & 1970-80	$2.3 - 2.4 = -0.1$
1970-80 & 1980-90	$2.2 - 2.3 = -0.1$
1980-90 & 1990-00	$2.6 - 2.2 = 0.4$

(d) For both the U.S. federal debt and population, the average rates of change are all positive.

(e) In part (c), for the federal debt most of the differences are positive and for the population most of the differences are negative.

(f) The U.S. federal debt is growing out of control when compared to the U.S. population. See part (g) for an explanation.

(g) Since most of the differences for the federal debt in part (e) are positive, the federal debts are increasing at an increasing rate. While the U.S. population is increasing at a decreasing rate since most of the differences for population in part (e) are negative.

For Thought

1. True, since the graph is a parabola opening down with vertex at the origin.

2. False, the graph is decreasing.

3. True

4. True, since $f(-4.5) = [-1.5] = -2$.

5. False, since the range is $\{\pm 1\}$.

6. True **7.** True **8.** True

9. False, since the range is the interval $[0, 4]$.

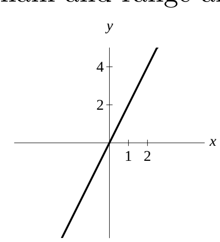
10. True

2.2 Exercises

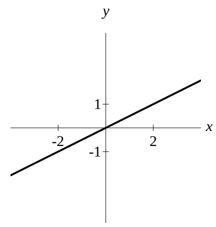
1. parabola

2. piecewise

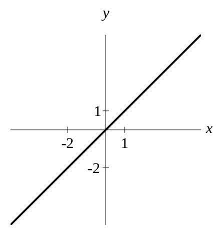
3. Function $y = 2x$ includes the points $(0, 0)$, $(1, 2)$, domain and range are both $(-\infty, \infty)$



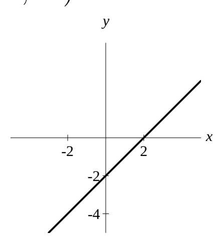
4. Function $x = 2y$ includes the points $(0, 0)$, $(2, 1), (-2, -1)$, domain and range are both $(-\infty, \infty)$



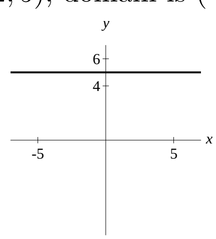
5. Function $x - y = 0$ includes the points $(-1, -1)$, $(0, 0), (1, 1)$, domain and range are both $(-\infty, \infty)$



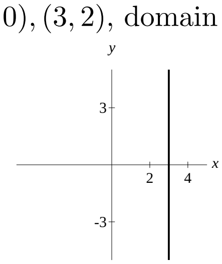
6. Function $x - y = 2$ includes the points $(2, 0)$, $(0, -2), (-2, -4)$, domain and range are both $(-\infty, \infty)$



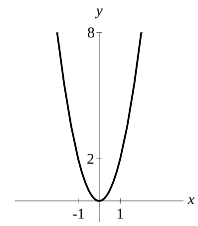
7. Function $y = 5$ includes the points $(0, 5)$, $(\pm 2, 5)$, domain is $(-\infty, \infty)$, range is $\{5\}$



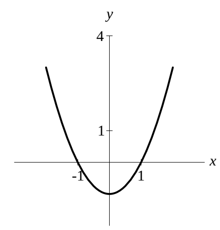
8. $x = 3$ is not a function and includes the points $(3, 0), (3, 2)$, domain is $\{3\}$, range is $(-\infty, \infty)$



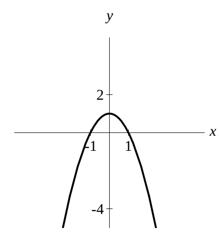
9. Function $y = 2x^2$ includes the points $(0, 0)$, $(\pm 1, 2)$, domain is $(-\infty, \infty)$, range is $[0, \infty)$



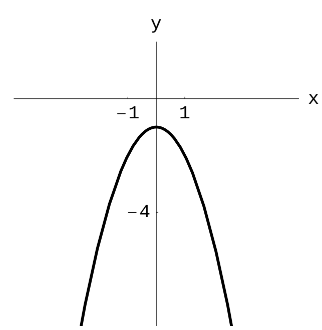
10. Function $y = x^2 - 1$ goes through $(0, -1)$, $(\pm 1, 0)$, domain is $(-\infty, \infty)$, range is $[-1, \infty)$



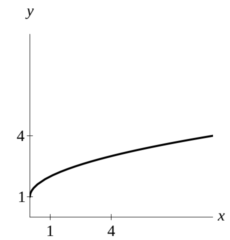
11. Function $y = 1 - x^2$ includes the points $(0, 1)$, $(\pm 1, 0)$, domain is $(-\infty, \infty)$, range is $(-\infty, 1]$



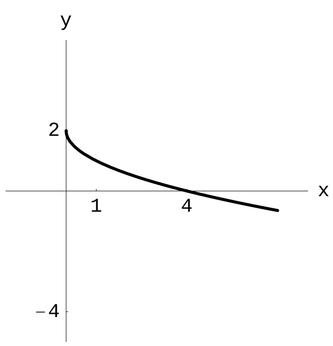
12. Function $y = -1 - x^2$ includes the points $(0, -1)$, $(\pm 1, -2)$, domain is $(-\infty, \infty)$, range is $(-\infty, -1]$



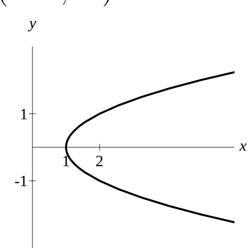
13. Function $y = 1 + \sqrt{x}$ includes the points $(0, 1)$, $(1, 2), (4, 3)$, domain is $[0, \infty)$, range is $[1, \infty)$



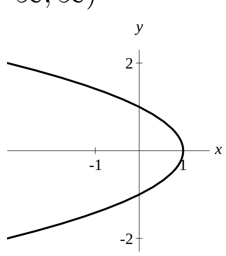
14. Function $y = 2 - \sqrt{x}$ includes the points $(0, 2)$, $(4, 0)$, domain is $[0, \infty)$, range is $(-\infty, 2]$



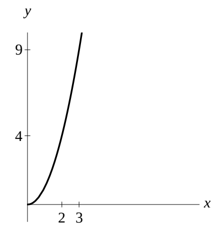
15. $x = y^2 + 1$ is not a function and includes the points $(1, 0), (2, \pm 1)$, domain is $[1, \infty)$, range is $(-\infty, \infty)$



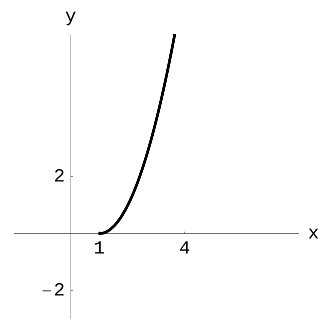
16. $x = 1 - y^2$ is not function and includes the points $(1, 0), (0, \pm 1)$, domain is $(-\infty, 1]$, range is $(-\infty, \infty)$



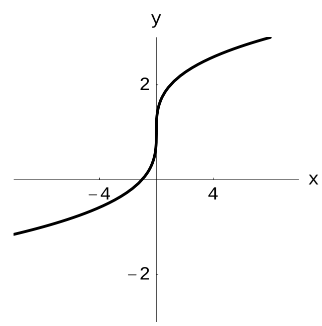
17. Function $x = \sqrt{y}$ goes through $(0, 0), (2, 4), (3, 9)$, domain and range is $[0, \infty)$



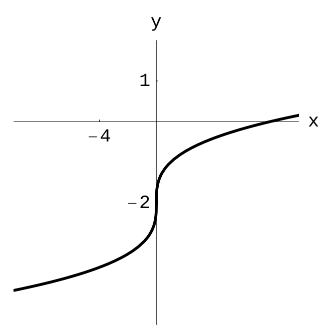
18. Function $x - 1 = \sqrt{y}$ goes through $(1, 0)$, $(3, 4), (4, 9)$, domain $[1, \infty)$, and range $[0, \infty)$,



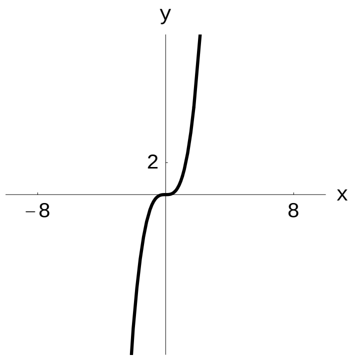
19. Function $y = \sqrt[3]{x} + 1$ goes through $(-1, 0), (1, 2), (8, 3)$, domain $(-\infty, \infty)$, and range $(-\infty, \infty)$



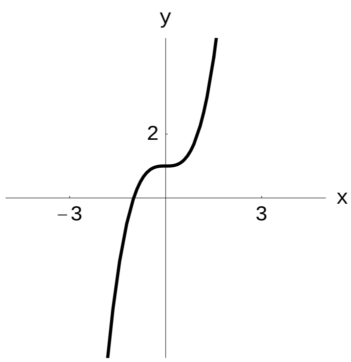
20. Function $y = \sqrt[3]{x} - 2$ goes through $(-1, -3), (1, -1), (8, 0)$, domain $(-\infty, \infty)$, and range $(-\infty, \infty)$



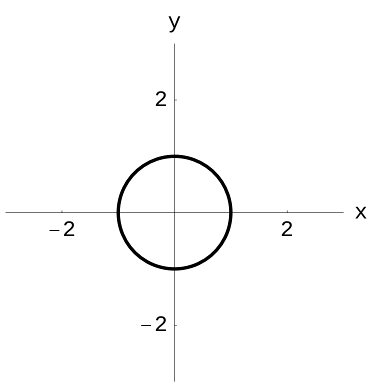
21. Function, $x = \sqrt[3]{y}$ goes through $(0,0), (1,1), (2,8)$, domain $(-\infty, \infty)$, and range $(-\infty, \infty)$



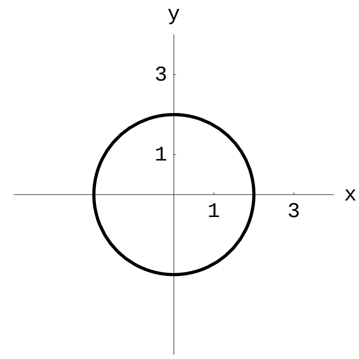
22. Function, $x = \sqrt[3]{y-1}$ goes through $(0,1), (1,2), (-1,0)$, domain $(-\infty, \infty)$, and range $(-\infty, \infty)$



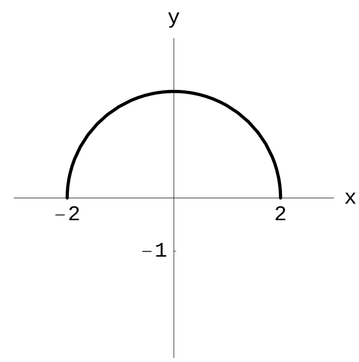
23. Not a function, $y^2 = 1 - x^2$ goes through $(1,0), (0,1), (-1,0)$, domain $[-1,1]$, and range $[-1,1]$



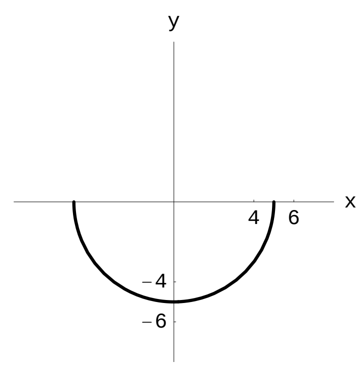
24. Not a function, $x^2 + y^2 = 4$ goes through $(2,0), (0,2), (-2,0)$, domain $[-2,2]$, and range $[-2,2]$



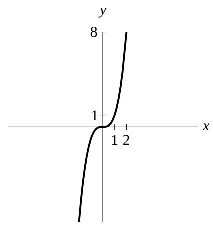
25. Function, $y = \sqrt{1-x^2}$ goes through $(\pm 1, 0), (0,1)$, domain $[-1,1]$, and range $[0,1]$



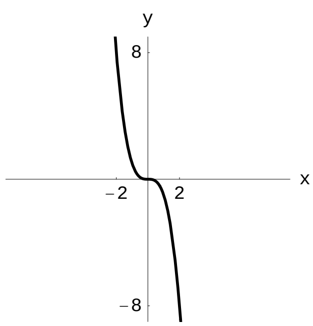
26. Function, $y = -\sqrt{25-x^2}$ goes through $(\pm 5, 0), (0,-5)$, domain $[-5,5]$, and range $[-5,0]$



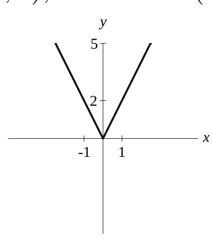
27. Function $y = x^3$ includes the points $(0, 0)$, $(1, 1), (2, 8)$, domain and range are both $(-\infty, \infty)$



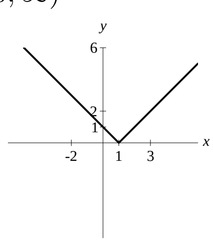
28. Function $y = -x^3$ includes the points $(0, 0)$, $(1, -1), (2, -8)$, domain and range are both $(-\infty, \infty)$



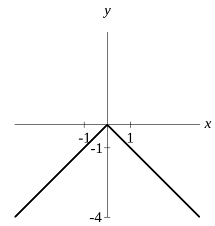
29. Function $y = 2|x|$ includes the points $(0, 0)$, $(\pm 1, 2)$, domain is $(-\infty, \infty)$, range is $[0, \infty)$



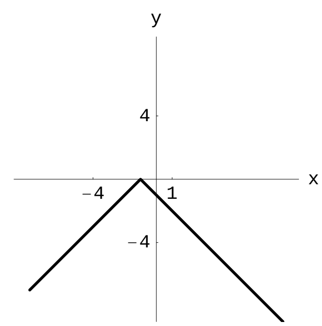
30. Function $y = |x - 1|$ includes the points $(0, 1), (1, 0), (2, 1)$, domain is $(-\infty, \infty)$, range is $[0, \infty)$



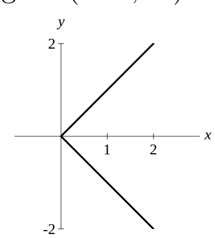
31. Function $y = -|x|$ includes the points $(0, 0)$, $(\pm 1, -1)$, domain is $(-\infty, \infty)$, range is $(-\infty, 0]$



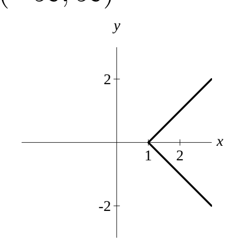
32. Function $y = -|x + 1|$ includes the points $(-1, 0)$, $(0, -1)$, $(-2, -1)$, domain is $(-\infty, \infty)$, range is $(-\infty, 0]$



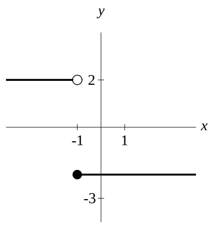
33. Not a function, graph of $x = |y|$ includes the points $(0, 0), (2, 2), (2, -2)$, domain is $[0, \infty)$, range is $(-\infty, \infty)$



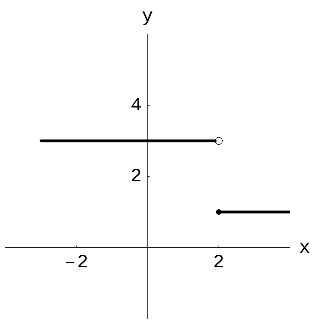
34. $x = |y| + 1$ is not a function and includes the points $(1, 0), (2, \pm 1)$, domain is $[1, \infty)$, range is $(-\infty, \infty)$



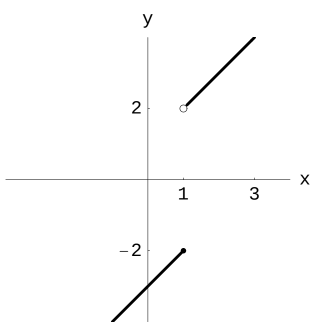
35. Domain is $(-\infty, \infty)$, range is $\{\pm 2\}$, some points are $(-3, -2)$, $(1, -2)$



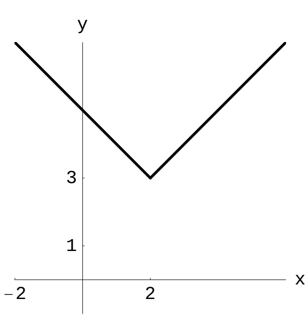
36. Domain is $(-\infty, \infty)$, range is $\{1, 3\}$, some points are $(0, 2)$, $(4, 1)$



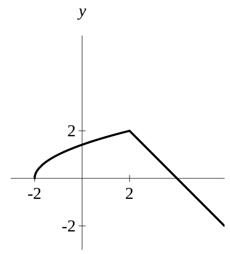
37. Domain is $(-\infty, \infty)$, range is $(-\infty, -2] \cup (2, \infty)$, some points are $(2, 3)$, $(1, -2)$



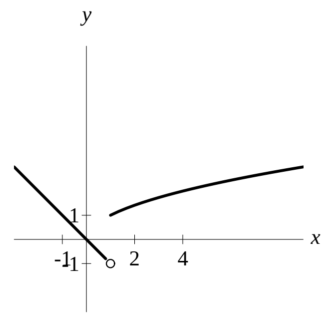
38. Domain is $(-\infty, \infty)$, range is $[3, \infty)$, some points are $(2, 3)$, $(3, 4)$



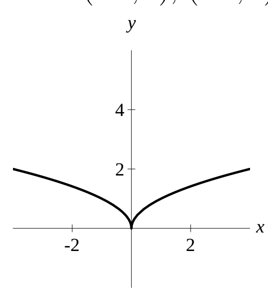
39. Domain is $[-2, \infty)$, range is $(-\infty, 2]$, some points are $(2, 2)$, $(-2, 0)$, $(3, 1)$



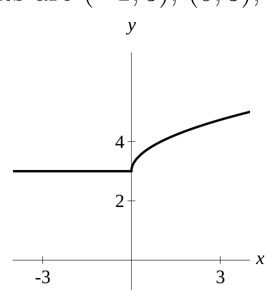
40. Domain is $(-\infty, \infty)$, range is $(-1, \infty)$, some points are $(1, 1)$, $(4, 2)$, $(-1, 1)$



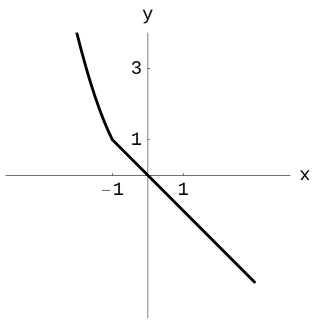
41. Domain is $(-\infty, \infty)$, range is $[0, \infty)$, some points are $(-1, 1)$, $(-4, 2)$, $(4, 2)$



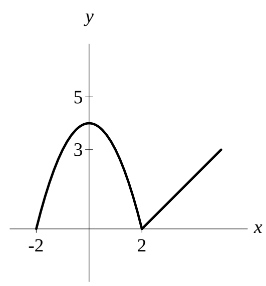
42. Domain is $(-\infty, \infty)$, range is $[3, \infty)$, some points are $(-1, 3)$, $(0, 3)$, $(1, 4)$



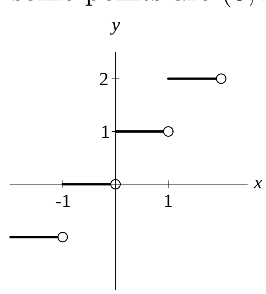
43. Domain is $(-\infty, \infty)$, range is $(-\infty, \infty)$, some points are $(-2, 4)$, $(1, -1)$



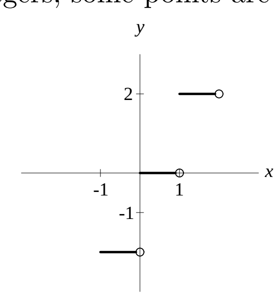
44. Domain is $[-2, \infty)$, range is $[0, \infty)$, some points are $(\pm 2, 0)$, $(3, 1)$



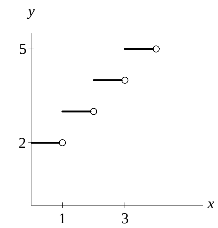
45. Domain is $(-\infty, \infty)$, range is the set of integers, some points are $(0, 1)$, $(1, 2)$, $(1.5, 2)$



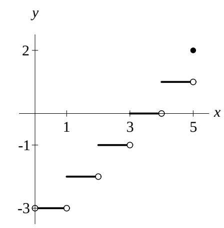
46. Domain is $(-\infty, \infty)$, range is the set of even integers, some points are $(0, 0)$, $(1, 2)$, $(1.5, 2)$



47. Domain $[0, 4)$, range is $\{2, 3, 4, 5\}$, some points are $(0, 2)$, $(1, 3)$, $(1.5, 3)$



48. Domain is $(0, 5]$, range is $\{-3, -2, -1, 0, 1, 2\}$, some points are $(0, -3)$, $(1, -2)$, $(1.5, -2)$



49. **a.** Domain and range are both $(-\infty, \infty)$, decreasing on $(-\infty, \infty)$

b. Domain is $(-\infty, \infty)$, range is $(-\infty, 4]$ increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$

50. **a.** Domain and range are both $(-\infty, \infty)$, increasing on $(-\infty, \infty)$

b. Domain is $(-\infty, \infty)$, range is $[-3, \infty)$ increasing on $(0, \infty)$, decreasing on $(-\infty, 0)$

51. **a.** Domain is $[-2, 6]$, range is $[3, 7]$ increasing on $(-2, 2)$, decreasing on $(2, 6)$

b. Domain $(-\infty, 2]$, range $(-\infty, 3]$, increasing on $(-\infty, -2)$, constant on $(-2, 2)$

52. **a.** Domain is $[0, 6]$, range is $[-4, -1]$ increasing on $(3, 6)$, decreasing on $(0, 3)$

b. Domain $(-\infty, \infty)$, range $[1, \infty)$, increasing on $(3, \infty)$, constant on $(1, 3)$, decreasing on $(-\infty, 1)$

53. **a.** Domain is $(-\infty, \infty)$, range is $[0, \infty)$ increasing on $(0, \infty)$, decreasing on $(-\infty, 0)$

b. Domain and range are both $(-\infty, \infty)$ increasing on $(-2, -2/3)$, decreasing on $(-\infty, -2)$ and $(-2/3, \infty)$

54. **a.** Domain is $[-4, 4]$, range is $[0, 4]$
increasing on $(-4, 0)$, decreasing on $(0, 4)$

b. Domain is $(-\infty, \infty)$, range is $[-2, \infty)$
increasing on $(2, \infty)$, decreasing on $(-\infty, -2)$, constant on $(-2, 2)$

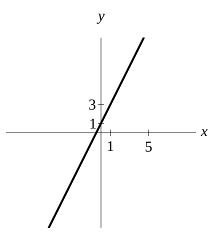
55. **a.** Domain and range are both $(-\infty, \infty)$,
increasing on $(-\infty, \infty)$

b. Domain is $[-2, 5]$, range is $[1, 4]$
increasing on $(1, 2)$, decreasing on $(-2, 1)$, constant on $(2, 5)$

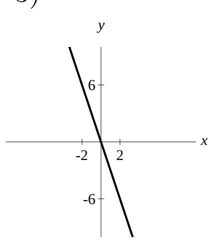
56. **a.** Domain is $(-\infty, \infty)$, range is $(-\infty, 3]$
increasing on $(-\infty, 2)$, decreasing on $(2, \infty)$

b. Domain and range are both $(-\infty, \infty)$,
decreasing on $(-\infty, \infty)$

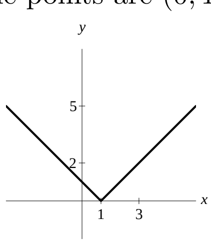
57. Domain and range are both $(-\infty, \infty)$ increasing on $(-\infty, \infty)$, some points are $(0, 1)$, $(1, 3)$



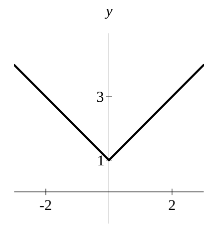
58. Domain and range are both $(-\infty, \infty)$, decreasing on $(-\infty, \infty)$, some points are $(0, 0)$, $(1, -3)$



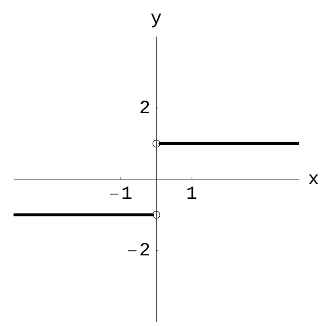
59. Domain is $(-\infty, \infty)$, range is $[0, \infty)$, increasing on $(1, \infty)$, decreasing on $(-\infty, 1)$, some points are $(0, 1)$, $(1, 0)$



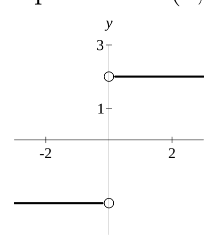
60. Domain is $(-\infty, \infty)$, range is $[1, \infty)$, increasing on $(0, \infty)$, decreasing on $(-\infty, 0)$, some points are $(0, 1)$, $(-1, 2)$



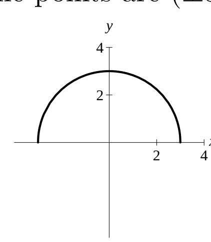
61. Domain is $(-\infty, 0) \cup (0, \infty)$, range is $\{\pm 1\}$, constant on $(-\infty, 0)$ and $(0, \infty)$, some points are $(1, 1)$, $(-1, -1)$



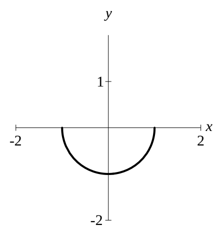
62. Domain is $(-\infty, 0) \cup (0, \infty)$, range is $\{\pm 2\}$, constant on $(-\infty, 0)$ and $(0, \infty)$, some points are $(1, 2)$, $(-1, -2)$



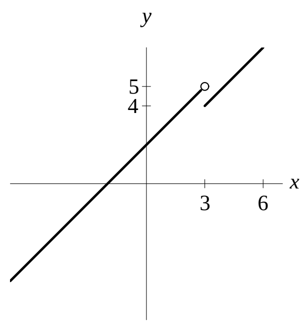
63. Domain is $[-3, 3]$, range is $[0, 3]$, increasing on $(-3, 0)$, decreasing on $(0, 3)$, some points are $(\pm 3, 0)$, $(0, 3)$



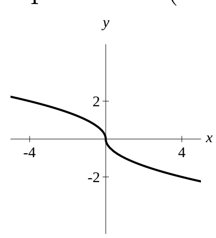
64. Domain is $[-1, 1]$, range is $[-1, 0]$, increasing on $(0, 1)$, decreasing on $(-1, 0)$, some points are $(\pm 1, 0)$, $(0, -1)$



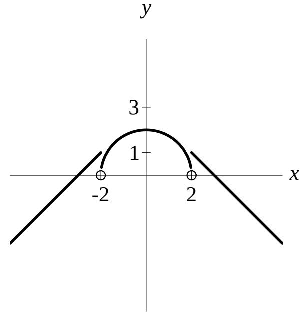
65. Domain and range are both $(-\infty, \infty)$, increasing on $(-\infty, 3)$ and $(3, \infty)$, some points are $(4, 5)$, $(0, 2)$



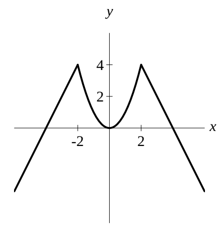
66. Domain and range are both $(-\infty, \infty)$, decreasing on $(-\infty, \infty)$, some points are $(-1, 1)$, $(1, -1)$



67. Domain is $(-\infty, \infty)$, range is $(-\infty, 2]$, increasing on $(-\infty, -2)$ and $(-2, 0)$, decreasing on $(0, 2)$ and $(2, \infty)$, some points are $(-3, 0)$, $(0, 2)$, $(4, -1)$



68. Domain is $(-\infty, \infty)$, range is $(-\infty, 4]$, increasing on $(-\infty, -2)$ and $(0, 2)$, decreasing on $(-2, 0)$ and $(2, \infty)$, some points are $(-3, 2)$, $(0, 0)$, $(3, 2)$



69. $f(x) = \begin{cases} 2 & \text{for} \quad x > -1 \\ -1 & \text{for} \quad x \le -1 \end{cases}$

70. $f(x) = \begin{cases} 3 & \text{for} \quad x \le 1 \\ -2 & \text{for} \quad x > 1 \end{cases}$

71. The line joining $(-1, 1)$ and $(-3, 3)$ is $y = -x$, and the line joining $(-1, -2)$ and $(3, 2)$ is $y = x - 1$. The piecewise function is

$$f(x) = \begin{cases} x - 1 & \text{for} \quad x \ge -1 \\ -x & \text{for} \quad x < -1. \end{cases}$$

72. The line joining $(1, 3)$ and $(-3, -1)$ is $y = x+2$, and the line joining $(1, 1)$ and $(3, -1)$ is $y = 2 - x$. The piecewise function is

$$f(x) = \begin{cases} x + 2 & \text{for} \quad x \le 1 \\ 2 - x & \text{for} \quad x > 1. \end{cases}$$

73. The line joining $(0, -2)$ and $(2, 2)$ is $y = 2x-2$, and the line joining $(0, -2)$ and $(-3, 1)$ is $y = -x - 2$. The piecewise function is

$$f(x) = \begin{cases} 2x - 2 & \text{for} \quad x \ge 0 \\ -x - 2 & \text{for} \quad x < 0. \end{cases}$$

74. The line joining $(1, 3)$ and $(3, 1)$ is $y = 4 - x$, and the line joining $(1, 3)$ and $(-2, -3)$ is $y = 2x + 1$. The piecewise function is

$$f(x) = \begin{cases} 2x + 1 & \text{for} \quad x \le 1 \\ 4 - x & \text{for} \quad x > 1. \end{cases}$$

75. increasing on the interval $[0.83, \infty)$,

decreasing on $(-\infty, 0.83]$

76. increasing on the interval $(-\infty, 0.17]$,

decreasing on $[0.17, \infty)$

77. increasing on $(-\infty, -1]$ and $[1, \infty)$, decreasing on $[-1, 1]$

78. increasing on $[-2.35, 0)$ and $[2.35, \infty)$, decreasing on $(-\infty, -2.35]$ and $(0, 2.35]$

79. increasing on $[-1.73, 0)$ and $[1.73, \infty)$, decreasing on $(-\infty, -1.73]$ and $(0, 1.73]$

80. increasing on $(-\infty, -2.59]$, $[-1.03, 1.03]$, and $[2.59, \infty)$, decreasing on $[-2.59, -1.03]$ and $[1.03, 2.59]$

81. increasing on $[30, 50]$, and $[70, \infty)$, decreasing on $(-\infty, 30]$ and $[50, 70]$

82. increasing on $(-\infty, -50]$, and $[-30, \infty)$, decreasing on $[-50, -30]$

83. c, graph was increasing at first, then suddenly dropped and became constant, then increased slightly

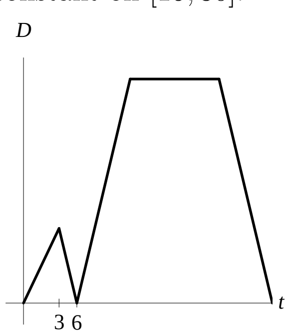
84. a

85. d, graph was decreasing at first, then fluctuated between increases and decreases, then the market increased

86. b

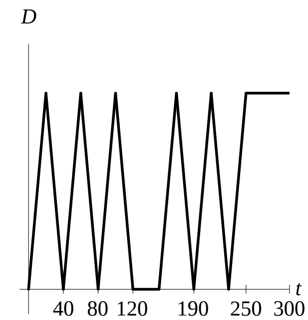
87. The independent variable is time t where t is the number of minutes after 7:45 and the dependent variable is distance D from the holodeck.

D is increasing on the intervals $[0, 3]$ and $[6, 15]$, decreasing on $[3, 6]$ and $[30, 39]$, and constant on $[15, 30]$.



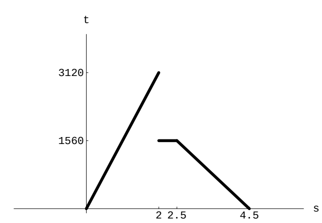
88. Independent variable is time t in seconds, dependent variable is distance D from the pit

D is increasing on the intervals $(0, 20)$, $(40, 60)$, $(80, 100)$, $(150, 170)$, $(190, 210)$, and $(230, 250)$; D is decreasing on the intervals $(20, 40)$, $(60, 80)$, $(100, 120)$, $(170, 190)$, $(210, 230)$; D is constant on $(120, 150)$ and $(250, 300)$.



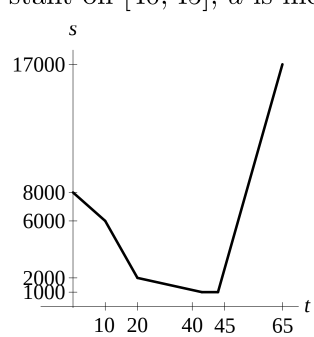
89. Independent variable is time t in years, dependent variable is savings s in dollars

s is increasing on the interval $[0, 2]$; s is constant on $[2, 2.5]$; s is decreasing on $[2.5, 4.5]$.



90. Independent variable is time t in days, dependent variable is the amount, a, (in dollars) in the checking account.

a is decreasing on the interval $[0, 40]$; a is constant on $[40, 45]$; a is increasing on $[45, 65]$.



91. In 1988, there were $M(18) = 565$ million cars. In 2010, it is projected that there will be $M(40) = 800$ million cars.
The average rate of change from 1984 to 1994 is $\frac{M(24) - M(14)}{10} = 14.5$ million cars per year.

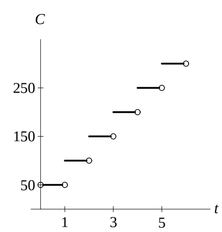
92. In developing countries and Eastern Europe, the average rate of change of motor vehicle ownership is $\frac{M(40) - M(20)}{20} = 6.25$ million vehicles per year.

In developed countries, the average rate of change of motor vehicle ownership from 1990 to 2010 is 10 million vehicles per year (see previous model). Then vehicle ownership is expected to grow faster in developed countries.

93. Constant on $[0, 10^4]$, increasing on $[10^4, \infty)$

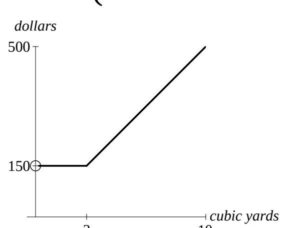
94. Decreasing since a car has a lower mileage at high speeds.

95. The cost is over \$235 for t in $[5, \infty)$.



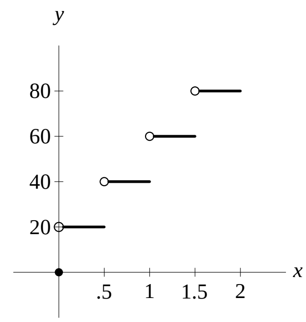
96. If $200 + 37[w/100] < 862$ then $[w/100] < 662/37 \approx 17.9$. Then $w/100 < 18$ and the values of w lie in $(0, 1800)$.

97. $f(x) = \begin{cases} 150 & \text{if} \quad 0 < x < 3 \\ 50x & \text{if} \quad 3 \le x \le 10 \end{cases}$



98. $f(x) = \begin{cases} -4[-x] & \text{if} \quad 0 < x \le 3 \\ 15 & \text{if} \quad 3 < x \le 8 \end{cases}$

99. A mechanic's fee is \$20 for each half-hour of work with any fraction of a half-hour charged as a half-hour. If y is the fee in dollars and x is the number of hours, then $y = -20[-2x]$.



100. For example, choose any a, b satisfying $a < b$ and graph the function defined by

$$f(x) = \begin{cases} 2x & \text{if} \quad x \le a \\ 5x - 3a & \text{if} \quad a < x < b \\ 5b - 3a & \text{if} \quad x \ge b \end{cases}$$

101. Since $x - 2 \ge 0$, the domain is $[2, \infty)$.
Since $\sqrt{x-2}$ is nonnegative, we find $\sqrt{x-2} + 3$ is at least three. Thus, the range is $[3, \infty)$.

102. Since an absolute value is nonnegative, the equation $|13x - 55| = -9$ has no solution. The solution set is $\emptyset$.

103. Note, we have $13x - 55 = 0$.
The solution set is $\{55/13\}$.

104. Rewrite without the absolute values:

$$\begin{aligned} 13x - 55 &= \pm 9 \\ 13x &= 55 \pm 9 = 46, 64 \end{aligned}$$

The solution set is $\left\{\frac{46}{13}, \frac{64}{13}\right\}$.

105. Since we have a perfect square

$$(2x - 5)^2 = 0$$

the solution set is $\{5/2\}$.

106. Applying the quadratic formula to $2w^2 - 5w - 9 = 0$, we find

$$w = \frac{5 \pm \sqrt{25 + 72}}{4}.$$

The solution set is $\left\{\frac{5 \pm \sqrt{97}}{4}\right\}$.

Thinking Outside the Box XXIII

a) Consider the circle $x^2+(y-r)^2=r^2$ where $r>0$. Suppose the circle intersects the parabola $y=x^2$ only at the origin. Substituting $y=x^2$, we obtain

$$\begin{aligned} y+(y-r)^2 &= r^2 \\ y^2+y(1-2r) &= 0 \end{aligned}$$

Thus, $1-2r=0$ since the circle and the parabola has exactly one point of intersection. The radius of the circle is 1/2.

b) Consider the circle $x^2+(y-r)^2=r^2$ where $r>0$. If the circle intersects the parabola $y=ax^2$ only at the origin, then the equation below must have exactly one solution, namely, $y=0$.

$$\begin{aligned} \frac{1}{a}y+(y-r)^2 &= r^2 \\ y^2+y\left(\frac{1}{a}-2r\right) &= 0 \end{aligned}$$

Necessarily, we have $\frac{1}{a}-2r=0$ or $a=\frac{1}{2r}$. Thus, if $r=3$ then $a=1/6$.

2.2 Pop Quiz

1. Domain $[0,\infty)$, range $(-\infty,1]$

2. Domain $[-3,3]$, range $[0,3]$

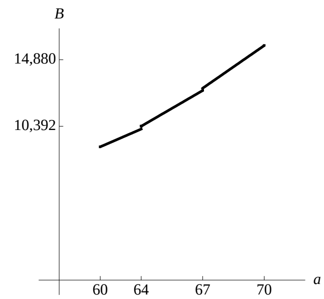
3. Range $[2,\infty)$

4. Increasing on $[0,\infty)$

5. Decreasing on $(-\infty,3]$

2.2 Linking Concepts

(a) A graph of the benefit function is given below.



(b) $B(64)=804(64)-41,064=\$10392$

(c) If $B=\$14,880$, then

$$\begin{aligned} 960a-51,360 &= 14,880 \\ 960a &= 66,240 \\ a &= 69. \end{aligned}$$

At age 69 years, the annual benefit is $14,880.

(d) Since the function is piecewise defined consisting of linear functions, the average rate of change is the slope of the linear function.

Then the average rate of change for ages 62-63 is 600.

The average rate of change for ages 64-66 is 804.

The average rate of change for ages 67-70 is 960.

(e) Yes, the answers to part (d) are the slopes of the three lines in the piecewise function defining the benefit formula.

(f) Note, $B(62)=\$9000$. Then the total amount Bob expects to withdraw is

$$(67.0166(1.00308)^{62}-62)\$9000 \approx \$171,800.$$

(g) Note, $B(70)=\$15,840$. Then the total amount Bill expects to withdraw is

$$(67.0166(1.00308)^{70}-70)\$15,840 \approx \$207,700.$$

For Thought

1. False, it is a reflection in the y-axis.

2. True **3.** False, rather it is a left translation.

4. True **5.** True

6. False, the down shift should come after the reflection. **7.** True

8. False, since their domains are different.

9. True **10.** True

2.3 Exercises

1. rigid

2. nonrigid

3. parabola

4. translation

5. reflection

6. identity

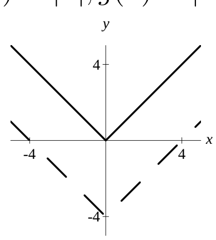
7. linear

8. constant

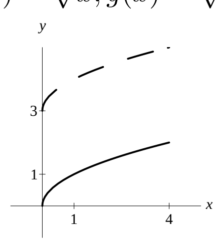
9. odd

10. even

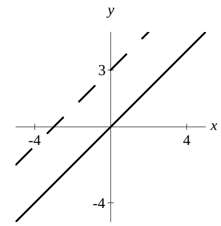
11. $f(x) = |x|, g(x) = |x| - 4$



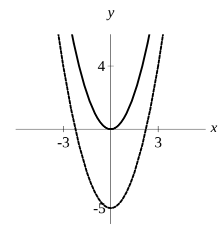
12. $f(x) = \sqrt{x}, g(x) = \sqrt{x} + 3$



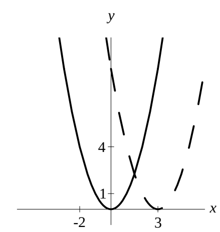
13. $f(x) = x, g(x) = x + 3$



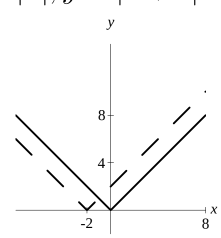
14. $f(x) = x^2, g(x) = x^2 - 5$



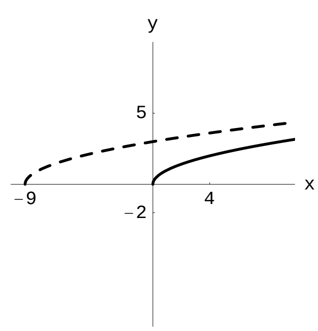
15. $y = x^2, y = (x - 3)^2$



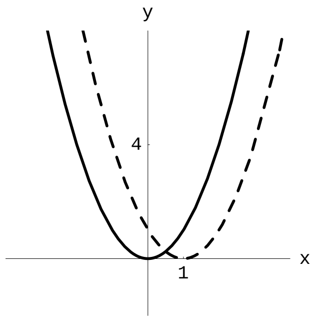
16. $y = |x|, y = |x + 2|$



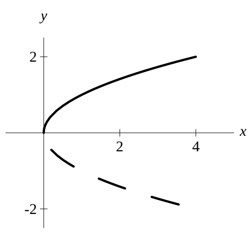
17. $y = \sqrt{x}$, $y = \sqrt{x + 9}$



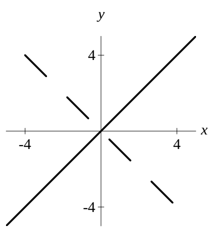
18. $y = x^2$, $y = (x-1)^2$



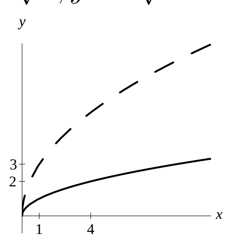
19. $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x}$



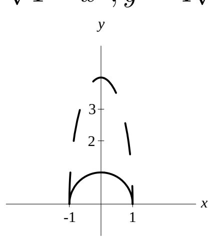
20. $f(x) = x, g(x) = -x$



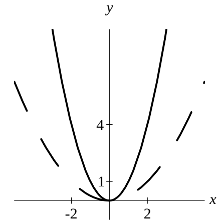
21. $y = \sqrt{x}, y = 3\sqrt{x}$



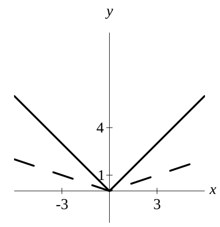
22. $y = \sqrt{1-x^2}, y = 4\sqrt{1-x^2}$



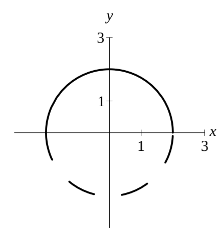
23. $y = x^2, y = \dfrac{1}{4}x^2$



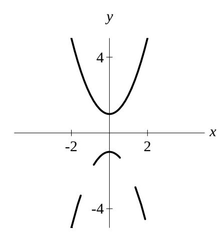
24. $y = |x|, y = \dfrac{1}{3}|x|$



25. $y = \sqrt{4-x^2}, y = -\sqrt{4-x^2}$



26. $f(x) = x^2 + 1, g(x) = -x^2 - 1$



27. g **28.** h **29.** b **30.** d

31. c **32.** a **33.** f **34.** e

35. $y = \sqrt{x} + 2$ **36.** $y = \sqrt{x} - 3$

37. $y = (x-5)^2$ **38.** $y = (x+7)^2$

39. $y = (x-10)^2 + 4$

40. $y = \sqrt{x+5} - 12$

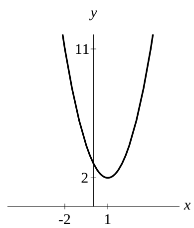
41. $y = -(3\sqrt{x} + 5)$ or $y = -3\sqrt{x} - 5$

42. $y = -\left((x-13)^2 - 6\right)$ or $y = -(x-13)^2 + 6$

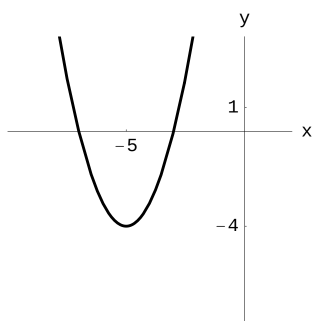
43. $y = -3|x-7| + 9$

44. $y = -2(x+6) - 8$ or $y = -2x - 20$

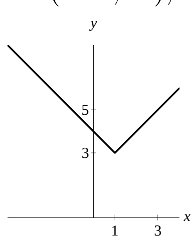
45. $y = (x-1)^2 + 2$; right by 1, up by 2, domain $(-\infty, \infty)$, range $[2, \infty)$



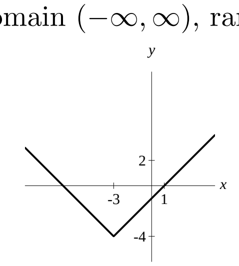
46. $y = (x+5)^2 - 4$; left by 5, down by 4, domain $(-\infty, \infty)$, range $[-4, \infty)$



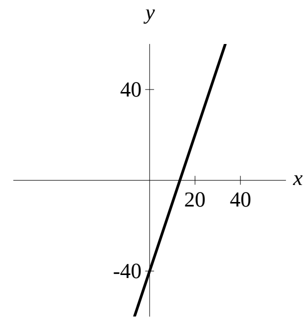
47. $y = |x-1| + 3$; right by 1, up by 3 domain $(-\infty, \infty)$, range $[3, \infty)$



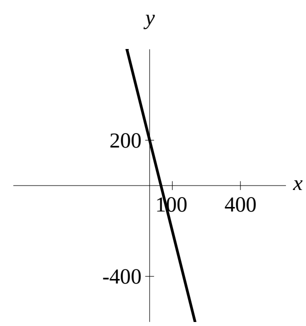
48. $y = |x+3| - 4$; left by 3, down by 4 domain $(-\infty, \infty)$, range $[-4, \infty)$



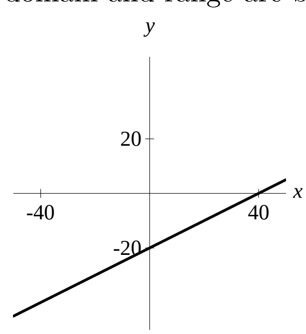
49. $y = 3x - 40$,
domain and range are both $(-\infty, \infty)$



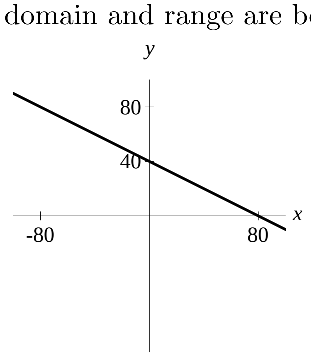
50. $y = -4x + 200$,
domain and range are both $(-\infty, \infty)$



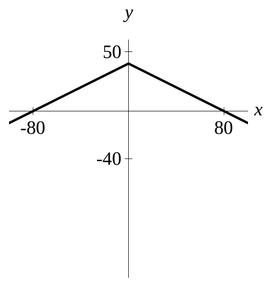
51. $y = \frac{1}{2}x - 20$,
domain and range are both $(-\infty, \infty)$



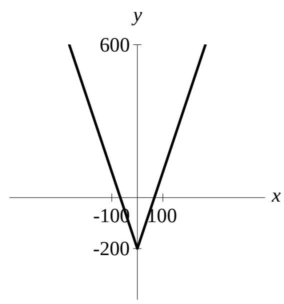
52. $y = -\frac{1}{2}x + 40$,
domain and range are both $(-\infty, \infty)$



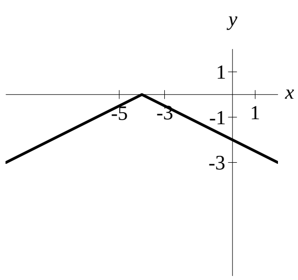
53. $y = -\frac{1}{2}|x| + 40$, shrink by $1/2$,
reflect about x-axis, up by 40,
domain $(-\infty, \infty)$, range $(-\infty, 40]$



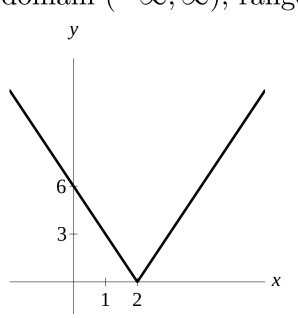
54. $y = 3|x| - 200$, stretch by 3, down by 200,
domain $(-\infty, \infty)$, range $[-200, \infty)$



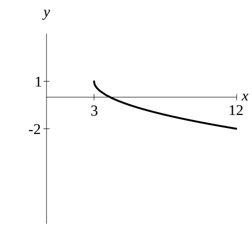
55. $y = -\frac{1}{2}|x + 4|$, left by 4,
reflect about x-axis, shrink by $1/2$,
domain $(-\infty, \infty)$, range $(-\infty, 0]$



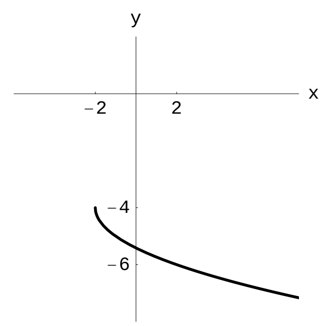
56. $y = 3|x - 2|$, right by 2, stretch by 3,
domain $(-\infty, \infty)$, range $[0, \infty)$



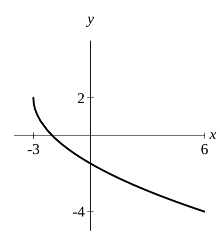
57. $y = -\sqrt{x - 3} + 1$, right by 3,
reflect about x-axis, up by 1,
domain $[3, \infty)$, range $(-\infty, 1]$



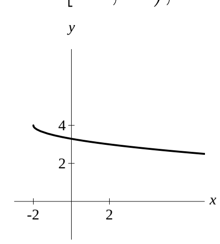
58. $y = -\sqrt{x + 2} - 4$, left by 2,
reflect about x-axis, down by 4,
domain $[-2, \infty)$, range $(-\infty, -4]$



59. $y = -2\sqrt{x + 3} + 2$, left by 3, stretch by 2,
reflect about x-axis, up by 2,
domain $[-3, \infty)$, range $(-\infty, 2]$



60. $y = -\frac{1}{2}\sqrt{x + 2} + 4$, left by 2, shrink by $1/2$,
reflect about x-axis, up by 4,
domain $[-2, \infty)$, range $(-\infty, 4]$



61. Symmetric about y-axis, even function
since $f(-x) = f(x)$

62. Symmetric about y-axis, even function since $f(-x) = f(x)$

63. No symmetry, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

64. Symmetric about the origin, odd function since $f(-x) = -f(x)$

65. Symmetric about $x = -3$, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

66. Symmetric about $x = 1$, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

67. Symmetry about $x = 2$, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

68. Symmetric about the y-axis, even function since $f(-x) = f(x)$

69. Symmetric about the origin, odd function since $f(-x) = -f(x)$

70. Symmetric about the origin, odd function since $f(-x) = -f(x)$

71. No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

72. No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

73. No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

74. Symmetric about the y-axis, even function since $f(-x) = f(x)$

75. Symmetric about the y-axis, even function since $f(-x) = f(x)$

76. Symmetric about the y-axis, even function since $f(-x) = f(x)$

77. No symmetry, not an even or odd function since $f(-x) = -f(x)$ and $f(-x) \neq -f(x)$

78. Symmetric about the y-axis, even function since $f(-x) = f(x)$

79. Symmetric about the y-axis, even function since $f(-x) = f(x)$

80. Symmetric about the y-axis, even function since $f(-x) = f(x)$

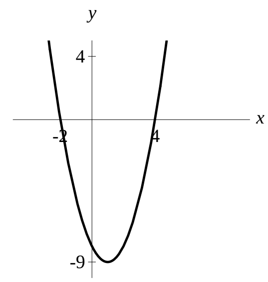
81. e **82.** a **83.** g **84.** h

85. b **86.** d **87.** c **88.** f

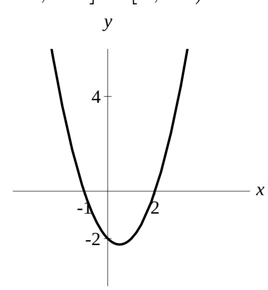
89. $(-\infty, -1] \cup [1, \infty)$ **90.** $\left(-\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}\right)$

91. $(-\infty, -1) \cup (5, \infty)$ **92.** $[-3, 1]$

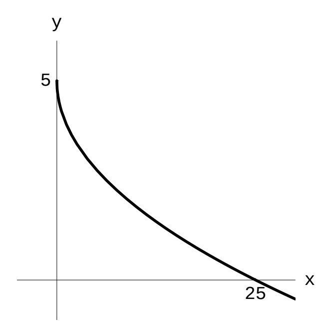
93. Using the graph of $y = (x-1)^2 - 9$, we find that the solution is $(-2, 4)$.



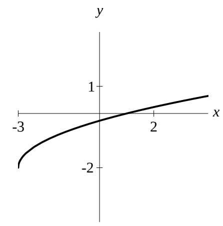
94. Graph of $y = \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$ shows solution is $(-\infty, -1] \cup [2, \infty)$



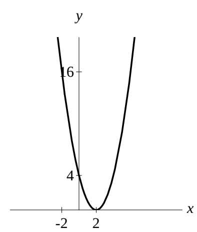
95. From the graph of $y = 5 - \sqrt{x}$, we find that the solution is $[0, 25]$.



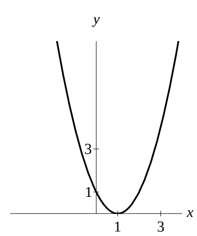
96. Graph of $y = \sqrt{x+3} - 2$ shows solution is $[1, \infty)$



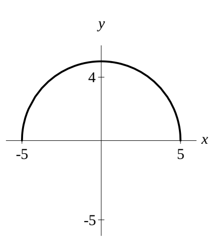
97. Note, the points of intersection of $y = 3$ and $y = (x-2)^2$ are $(2 \pm \sqrt{3}, 3)$. The solution set of $(x-2)^2 > 3$ is $\left(-\infty, 2-\sqrt{3}\right) \cup \left(2+\sqrt{3}, \infty\right)$.



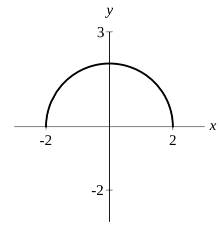
98. The points of intersection of $y = 4$ and $y = (x-1)^2$ are $(-1, 4)$ and $(3, 4)$. The solution set of $(x-1)^2 < 4$ is the interval $(-1, 3)$.



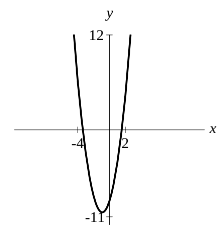
99. From the graph of $y = \sqrt{25 - x^2}$, we conclude that the solution is $(-5, 5)$.



100. Graph of $y = \sqrt{4 - x^2}$ shows solution is $[-2, 2]$

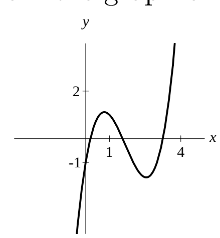


101. From the graph of $y = \sqrt{3}x^2 + \pi x - 9$,



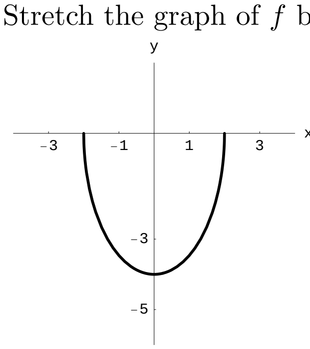
we observe that the solution set of $\sqrt{3}x^2 + \pi x - 9 < 0$ is $(-3.36, 1.55)$.

102. From the graph of $y = x^3 - 5x^2 + 6x - 1$,

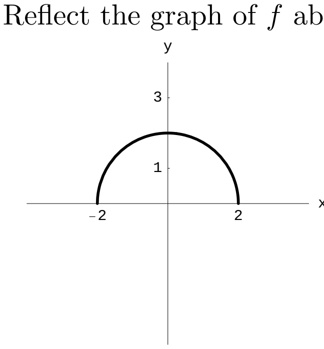


the solution set of $x^3 - 5x^2 + 6x - 1 > 0$ is $(0.20, 1.55) \cup (3.25, \infty)$.

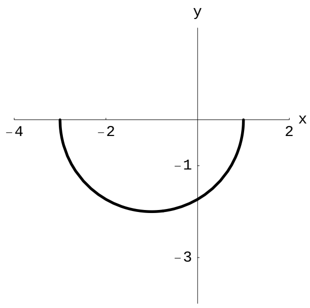
103. a. Stretch the graph of f by a factor of 2.



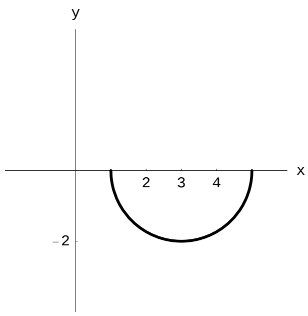
b. Reflect the graph of f about the x-axis.



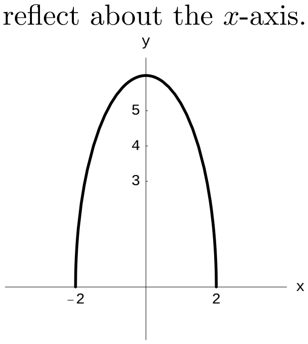
c. Translate the graph of f to the left by 1-unit.



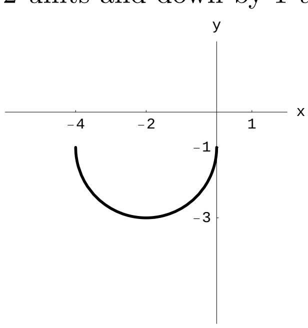
d. Translate the graph of f to the right by 3-units.



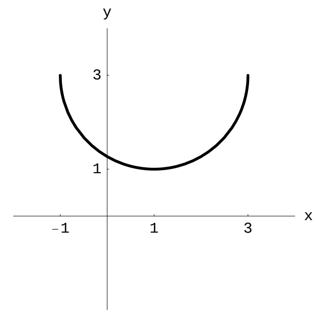
e. Stretch the graph of f by a factor of 3 and reflect about the x-axis.



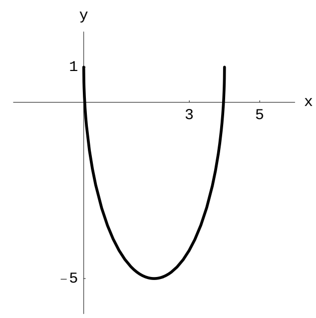
f. Translate the graph of f to the left by 2-units and down by 1-unit.



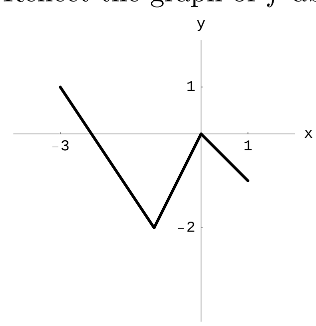
g. Translate the graph of f to the right by 1-unit and up by 3-units.



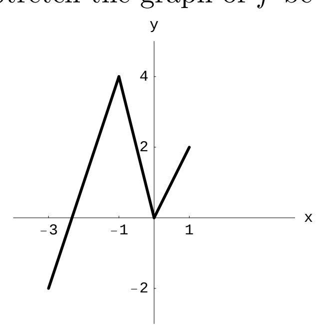
h. Translate the graph of f to the right by 2-units, stretch by a factor of 3, and up by 1-unit.



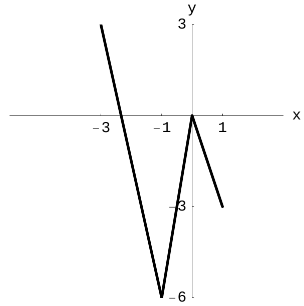
104. a. Reflect the graph of f about the x-axis.



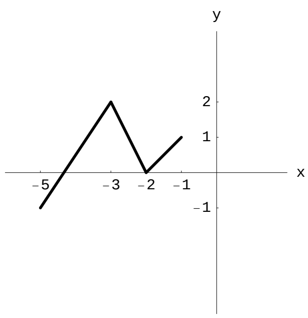
b. Stretch the graph of f be a factor of 2.



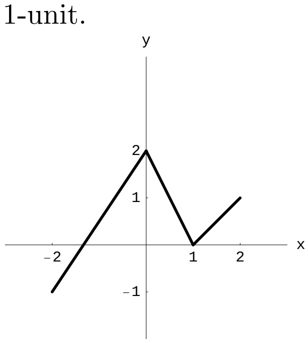
c. Stretch the graph of f by a factor of 3 and reflect about the x-axis.



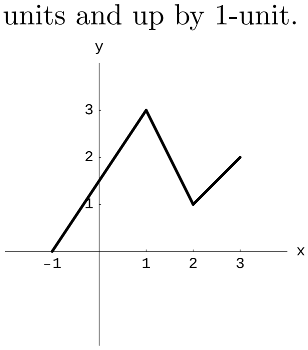
d. Translate the graph of f to the left by 2-units.



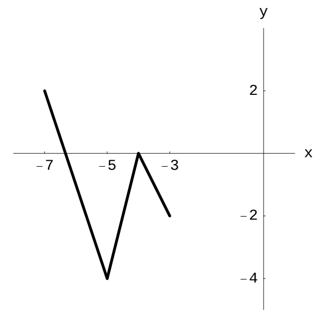
e. Translate the graph of f to the right by 1-unit.



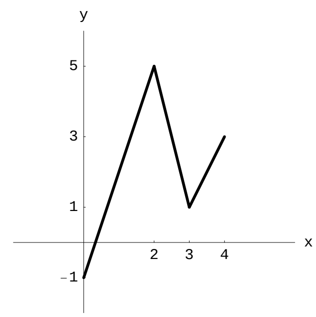
f. Translate the graph of f to the right by 2-units and up by 1-unit.



g. Translate the graph of f to the left by 4-units and reflect about the x-axis.



h. Translate the graph of f to the right by 3-units, stretch by a factor of 2, and up by 1-unit.



105. $N(x) = x + 2000$

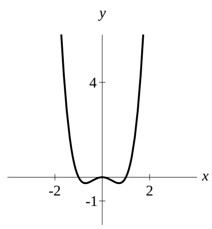
106. $N(x) = 1.05x + 3000$. Yes, if the merit increase is followed by the cost of living raise then the new salary becomes higher and is $N'(x) = 1.05(x + 3000) = 1.05x + 3150$.

107. If inflation rate is less than 50%, then $1 - \sqrt{x} < \dfrac{1}{2}$. This simplifies to $\dfrac{1}{2} < \sqrt{x}$. After squaring we have $\dfrac{1}{4} < x$ and so $x > 25\%$.

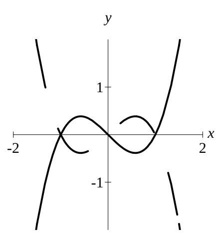
108. If production is at least 28 windows, then $1.75\sqrt{x} \geq 28$. They need at least $x = \left(\dfrac{28}{1.75}\right)^2 = 256$ hrs.

109.

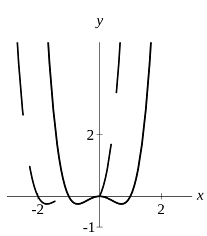
(a) Both functions are even functions and the graphs are identical



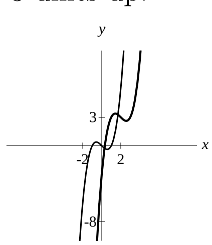
(b) One graph is a reflection of the other about the y-axis. Both functions are odd functions.



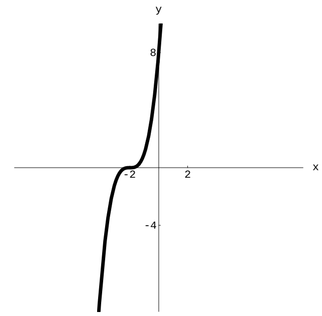
(c) The second graph is obtained by shifting the first one to the left by 1 unit.



(d) The second graph is obtained by translating the first one to the right by 2 units and 3 units up.



110. The graph of $y = x^3 + 6x^2 + 12x + 8$ or equivalently $y = (x+2)^3$ can be obtained by shifting the graph of $y = x^3$ to the right by 2 units.



111. If $f(x) = x^2 + 1$, then

$$\begin{aligned} f(a+2) - f(a) &= \\ (a+2)^2 + 1 - a^2 - 1 &= \\ a^2 + 4a + 4 - a^2 &= \\ 4a + 4 & \end{aligned}$$

112. The graph is a lower semicircle of radius 6 with center at the origin. Then the domain is $[-6, 6]$ and the range is $[-6, 0]$.

113. Note $|x| \geq 2/3$ is equivalent to $x \geq 2/3$ or $x \leq -2/3$. Then the solution set is

$$(-\infty, -2/3] \cup [2/3, \infty).$$

114. $i^{83} = \left(i^4\right)^{20} i^3 = i^3 = -i$

115.

$$\begin{aligned} ay + by - cy &= -2 \\ y &= \frac{-2}{a+b-c} \end{aligned}$$

116. $\dfrac{-2+i}{4+2i} \cdot \dfrac{4-2i}{4-2i} = \dfrac{-6+8i}{20} = -\dfrac{3}{10} + \dfrac{2}{5}i$

Thinking Outside the Box XXIV

Note, if $(x, x+h)$ is such an ordered pair then the average is $x + h/2$. Since the average is not a whole number, then $h = 1$. Thus, the ordered pairs are $(4, 5)$, $(49, 50)$, $(499, 500)$, and $(4999, 5000)$.

2.3 Pop Quiz

1. $y = \sqrt{x} + 8$

2. $y = (x-9)^2$

3. $y = (-x)^3$ or $y = -x^3$

4. Domain $[1, \infty)$, range $(-\infty, 5]$

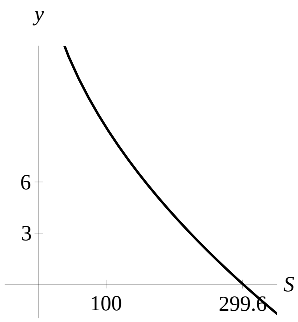
5. $y = -3(x-6)^2 + 4$

6. Even function

2.3 Linking Concepts

(a) From the graph of

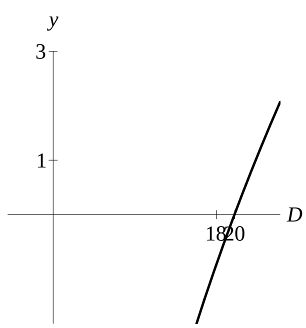
$$y = 16.96 + 9.8(17.67)^{1/3} - 20.85 - 1.25\sqrt{S}$$



we obtain S must lie in $(0, 299.58]$.

(b) From the graph of

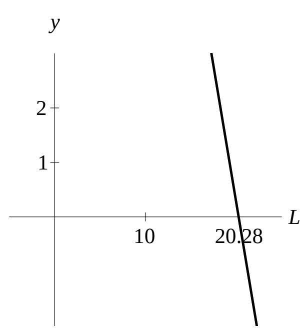
$$y = 16.96 + 9.8D^{1/3} - 21.45 - 1.25\sqrt{312.54}$$



we conclude D must lie in $[19.97, \infty)$.

(c) Given a portion of the graph of

$$y = 16.96 + 9.8(17.26)^{1/3} - L - 1.25\sqrt{310.28}$$



it follows that L must lie in $(0, 20.27]$.

(d) The variables with negative coefficients have maximum values. And, the variable D (with a positive coefficient) has a minimum value.

For Thought

1. False, since $f + g$ has an empty domain.

2. True **3.** True **4.** True

5. True, since $A = P^2/16$. **6.** True

7. False, since $(f \circ g)(x) = \sqrt{x-2}$ **8.** True

9. False, since $(h \circ g)(x) = x^2 - 9$.

10. True, since x belongs to the domain if $\sqrt{x-2}$ is a real number, i.e., if $x \geq 2$.

2.4 Exercises

1. sum

2. composition

3. $-1 + 2 = 1$ **4.** $6 + 0 = 6$

5. $-5 - 6 = -11$ **6.** $42 - (-9) = 51$

7. $(-4) \cdot 2 = -8$ **8.** $(-3) \cdot 0 = 0$

9. $1/12$ **10.** 12

11. $(a-3) + (a^2 - a) = a^2 - 3$

12. $(b-3) - (b^2 - b) = 2b - 3 - b^2$

13. $(a-3)(a^2 - a) = a^3 - 4a^2 + 3a$

14. $(b-3)/(b^2 - b)$

15. $f + g = \{(-3, 1+2), (2, 0+6)\} = \{(-3, 3), (2, 6)\}$, domain $\{-3, 2\}$

16. $f + h = \{(2, 0+4)\} = \{(2, 4)\}$, domain $\{2\}$

17. $f - g = \{(-3, 1-2), (2, 0-6)\} = \{(-3, -1), (2, -6)\}$, domain $\{-3, 2\}$

18. $f - h = \{(2, 0-4)\} = \{(2, -4)\}$, domain $\{2\}$

19. $f \cdot g = \{(-3, 1 \cdot 2), (2, 0 \cdot 6) = \{(-3, 2), (2, 0)\}$, domain $\{-3, 2\}$

20. $f \cdot h = \{(2, 0 \cdot 4)\} = \{(2, 0)\}$, domain $\{2\}$

21. $g/f = \{(-3, 2/1)\} = \{(-3, 2)\}$, domain $\{-3\}$

22. $f/g = \{(-3, 1/2), (2, 0/6) = \{(-3, 1/2), (2, 0)\}$, domain $\{-3, 2\}$

23. $(f + g)(x) = \sqrt{x} + x - 4$, domain is $[0, \infty)$

24. $(f + h)(x) = \sqrt{x} + \dfrac{1}{x - 2}$, domain is $[0, 2) \cup (2, \infty)$

25. $(f - h)(x) = \sqrt{x} - \dfrac{1}{x - 2}$, domain is $[0, 2) \cup (2, \infty)$

26. $(h - g)(x) = \dfrac{1}{x - 2} - x + 4$, domain is $(-\infty, 2) \cup (2, \infty)$

27. $(g \cdot h)(x) = \dfrac{x - 4}{x - 2}$, domain is $(-\infty, 2) \cup (2, \infty)$

28. $(f \cdot h)(x) = \dfrac{\sqrt{x}}{x - 2}$, domain is $[0, 2) \cup (2, \infty)$

29. $\left(\dfrac{g}{f}\right)(x) = \dfrac{x - 4}{\sqrt{x}}$, domain is $(0, \infty)$

30. $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x}}{x - 4}$, domain is $[0, 4) \cup (4, \infty)$

31. $\{(-3, 0), (1, 0), (4, 4)\}$ **32.** $\{(-3, 2), (0, 0)\}$

33. $\{(1, 4)\}$ **34.** $\{(-3, 0)\}$

35. $\{(-3, 4), (1, 4)\}$ **36.** $\{(2, 0)\}$

37. $f(2) = 5$ **38.** $g(-4) = 17$

39. $f(2) = 5$ **40.** $h(-22) = -7$

41. $f(20.2721) = 59.8163$

42. $\approx g(-2.95667) \approx 9.742$

43. $(g \circ h \circ f)(2) = (g \circ h)(5) = g(2) = 5$

44. $(h \circ f \circ g)(3) = (h \circ f)(10) = h(29) = 10$

45. $(f \circ g \circ h)(2) = (f \circ g)(1) = f(2) = 5$

46. $(h \circ g \circ f)(0) = (h \circ g)(-1) = h(2) = 1$

47. $(f \circ h)(a) = f\left(\dfrac{a + 1}{3}\right) = 3\left(\dfrac{a + 1}{3}\right) - 1 = (a + 1) - 1 = a$

48. $(h \circ f)(w) = h(3w - 1) = \dfrac{(3w - 1) + 1}{3} = \dfrac{3w}{3} = w$

49. $(f \circ g)(t) = f(t^2 + 1) = 3(t^2 + 1) - 1 = 3t^2 + 2$

50. $(g \circ f)(m) = g(3m - 1) = (3m - 1)^2 + 1 = 9m^2 - 6m + 2$

51. $(f \circ g)(x) = \sqrt{x} - 2$, domain $[0, \infty)$

52. $(g \circ f)(x) = \sqrt{x - 2}$, domain $[2, \infty)$

53. $(f \circ h)(x) = \dfrac{1}{x} - 2$, domain $(-\infty, 0) \cup (0, \infty)$

54. $(h \circ f)(x) = \dfrac{1}{x - 2}$, domain $(-\infty, 2) \cup (2, \infty)$

55. $(h \circ g)(x) = \dfrac{1}{\sqrt{x}}$, domain $(0, \infty)$

56. $(g \circ h)(x) = \sqrt{\dfrac{1}{x}} = \dfrac{1}{\sqrt{x}}$, domain $(0, \infty)$

57. $(f \circ f)(x) = (x - 2) - 2 = x - 4$, domain $(-\infty, \infty)$

58. $(g \circ g)(x) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$, domain $[0, \infty)$

59. $(h \circ g \circ f)(x) = h(\sqrt{x - 2}) = \dfrac{1}{\sqrt{x - 2}}$, domain $(2, \infty)$

60. $(f \circ g \circ h)(x) = f\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{\sqrt{x}} - 2$, domain $(0, \infty)$

61. $(h \circ f \circ g)(x) = h(\sqrt{x} - 2) = \dfrac{1}{\sqrt{x} - 2}$, domain $(0, 4) \cup (4, \infty)$

62. $(g \circ h \circ f)(x) = g\left(\frac{1}{x-2}\right) = \frac{1}{\sqrt{x-2}}$,

domain $(2, \infty)$

63. $F = g \circ h$ **64.** $G = g \circ f$

65. $H = h \circ g$ **66.** $M = f \circ g$

67. $N = h \circ g \circ f$ **68.** $R = f \circ g \circ h$

69. $P = g \circ f \circ g$ **70.** $C = h \circ g \circ h$

71. $S = g \circ g$ **72.** $T = h \circ h$

73. If $g(x) = x^3$ and $h(x) = x - 2$, then

$$(h \circ g)(x) = g(x) - 2 = x^3 - 2 = f(x).$$

74. If $g(x) = x - 2$ and $h(x) = x^3$, then

$$(h \circ g)(x) = g(x)^3 = (x-2)^3.$$

75. If $g(x) = x + 5$ and $h(x) = \sqrt{x}$, then

$$(h \circ g)(x) = \sqrt{g(x)} = \sqrt{x+5} = f(x).$$

76. If $g(x) = \sqrt{x}$ and $h(x) = x + 5$, then

$$(h \circ g)(x) = g(x) + 5 = \sqrt{x} + 5 = f(x).$$

77. If $g(x) = 3x - 1$ and $h(x) = \sqrt{x}$, then

$$(h \circ g)(x) = \sqrt{g(x)} = \sqrt{3x-1} = f(x).$$

78. If $g(x) = \sqrt{x}$ and $h(x) = 3x - 1$, then

$$(h \circ g)(x) = 3g(x) - 1 = 3\sqrt{x} - 1 = f(x).$$

79. If $g(x) = |x|$ and $h(x) = 4x + 5$, then

$$(h \circ g)(x) = 4g(x) + 5 = 4|x| + 5 = f(x).$$

80. If $g(x) = 4x + 5$ and $h(x) = |x|$, then

$$(h \circ g)(x) = |g(x)| = |4x+5| = f(x).$$

81. $y = 2(3x+1) - 3 = 6x - 1$

82. $y = -4(-3x-2) - 1 = 12x + 7$

83. $y = (x^2 + 6x + 9) - 2 = x^2 + 6x + 7$

84. $y = 3(x^2 - 2x + 1) - 3 = 3x^2 - 6x$

85. $y = 3 \cdot \frac{x+1}{3} - 1 = x + 1 - 1 = x$

86. $y = 2\left(\frac{1}{2}x - \frac{5}{2}\right) + 5 = x - 5 + 5 = x$

87. Since $m = n - 4$ and $y = m^2$, $y = (n-4)^2$.

88. Since $u = t + 9$ and $v = \frac{u}{3}$, $v = \frac{t+9}{3}$.

89. Since $w = x + 16$, $z = \sqrt{w}$, and $y = \frac{z}{8}$,

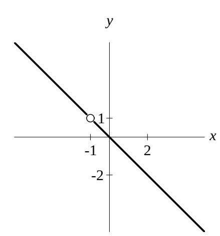
we obtain $y = \frac{\sqrt{x+16}}{8}$.

90. Since $a = b^3$, $c = a + 25$, and $d = \sqrt{c}$, we have $d = \sqrt{b^3 + 25}$.

91. After multiplying y by $\frac{x+1}{x+1}$ we have

$$y = \frac{\frac{x-1}{x+1} + 1}{\frac{x-1}{x+1} - 1} = \frac{(x-1)+(x+1)}{(x-1)-(x+1)} = -x$$

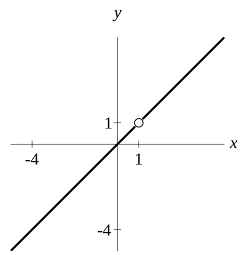
The domain of the original function is $(-\infty, -1) \cup (-1, \infty)$ while the domain of the simplified function is $(-\infty, \infty)$. The two functions are not the same.



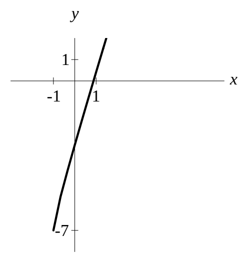
92. After multiplying y by $\frac{x-1}{x-1}$ we have

$$y = \frac{\frac{3x+1}{x-1} + 1}{\frac{3x+1}{x-1} - 3} = \frac{(3x+1)+(x-1)}{(3x+1)-3(x-1)} = x$$

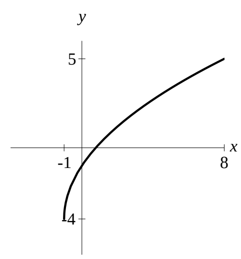
The domain of the original function is $(-\infty, 1) \cup (1, \infty)$ while the domain of the simplified function is $(-\infty, \infty)$. The two functions are not the same.



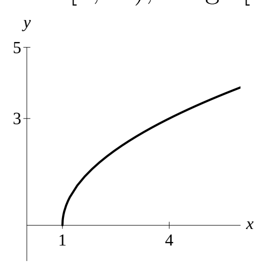
93. Domain $[-1, \infty)$, range $[-7, \infty)$



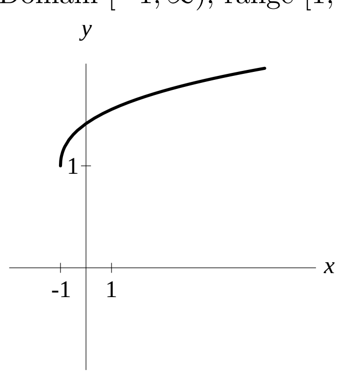
94. Domain $[-1, \infty)$, range $[-4, \infty)$



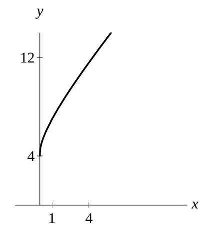
95. Domain $[1, \infty)$, range $[0, \infty)$



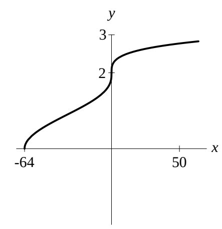
96. Domain $[-1, \infty)$, range $[1, \infty)$



97. Domain $[0, \infty)$, range $[4, \infty)$



98. Domain $[-64, \infty)$, range $[0, \infty)$



99. $P(x) = 68x - (40x + 200) = 28x - 200.$

Since $200/28 \approx 7.1$, the profit is positive when the number of trimmers satisfies $x \geq 8$.

100. $P(x) = (3000x - 20x^2) - (600x + 4000) = -20x^2 + 2400x - 4000.$

101. $A = d^2/2$ **102.** $P = 4\sqrt{A}$

103. $(f \circ f)(x) = 0.899x$ and $(f \circ f \circ f)(x) = 0.852x$ are the amounts of forest land at the start of 2002 and 2003, respectively.

104. $(f \circ f)(x) = 2(2x) = 4x$ and $(f \circ f \circ f)(x) = 8x$ are the values of x dollars invested in bonds after 24 and 36 years, respectively.

105. Total cost is $(T \circ C)(x) = 1.05(1.20x) = 1.26x$.

106. **a)** The slope of the linear function is

$$\frac{3200 - 2200}{30 - 20} = 100.$$

Using $C(x) - 2200 = 100(x - 20)$, the cost before taxes is $C(x) = 100x + 200$.

b) $T(x) = 1.09x$

c) Total cost of x pallets with tax included is $(T \circ C)(x) = 1.09(100x + 200) = 109x + 218.$

107. Note, $D = \frac{d/2240}{x} = \frac{d/2240}{L^3/100^3} = \frac{100^3 d}{2240L^3}$
$= \frac{100^3(26000)}{2240L^3} = \frac{100^4(26)}{224L^3} = \frac{100^4(13)}{112L^3}$.
Expressing D as a function of L, we write $D = \frac{(13)100^4}{112L^3}$ or $D = \frac{1.16 \times 10^7}{L^3}$.

108. Note, $S = \frac{6500}{(d/64)^{2/3}} = \frac{6500}{d^{2/3}/16} = \frac{16(6500)}{d^{2/3}}$.
Expressing S as a function of d, we obtain $S = \frac{16(6500)}{d^{2/3}}$ or $S = 104,000d^{-2/3}$.

109. The area of a semicircle with radius $s/2$ is $(1/2)\pi(s/2)^2 = \pi s^2/8$. The area of the square is s^2. The area of the window is

$$W = s^2 + \frac{\pi s^2}{8} = \frac{(8+\pi)s^2}{8}.$$

110. The area of the square is $A = s^2$ and the area of the semicircle is $S = \frac{1}{2}\pi\left(\frac{s}{2}\right)^2$.
Then $s^2 = \frac{8S}{\pi}$ and $A = \frac{8S}{\pi}$.

111. Form a right triangle with two sides of length s and a hypotenuse of length d. By the Pythagorean Theorem, we obtain

$$d^2 = s^2 + s^2.$$

Solving for s, we have $s = \frac{d\sqrt{2}}{2}$.

112. Construct an equilateral triangle where the length of one side is $d/2$ and the altitude is $p/2$. By the Pythagorean Theorem,

$$(p/2)^2 + (d/4)^2 = (d/2)^2.$$

Solving for p, we get $p = \frac{\sqrt{3}}{2}d$.

113. If a coat is on sale at 25% off and there is an additional 10% off, then the coat will cost $0.90(.75x) = 0.675x$ where x is the regular price. Thus, the discount sale is 32.5% off and not 35% off.

114. Addition and multiplication of functions are commutative, i.e., $(f+g)(x) = (g+f)(x)$ and $(f \cdot g)(x) = (g \cdot f)(x)$. Addition, multiplication, and composition of functions are associative, i.e.,

$$Q((f+g)+h)(x) = (f+(g+h))(x),$$
$$((f \cdot g) \cdot h)(x) = (f \cdot (g \cdot h))(x),$$

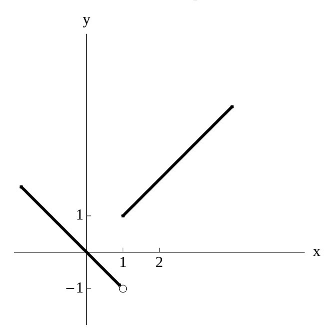
and

$$((f \circ g) \circ h)(x) = (f \circ (g \circ h))(x).$$

115. The difference quotient is

$$\frac{1 + \frac{3}{x+h} - 1 - \frac{3}{x}}{h} = \frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \frac{3x - 3x - 3h}{xh(x+h)} = \frac{-3}{x(x+h)} =$$

116. From the graph below, we conclude the domain is $(-\infty, \infty)$, the range is $(-1, \infty)$, and increasing on $[1, \infty)$.



117. Since $x - 3 \geq 0$, the domain is $[3, \infty)$. Since $-5\sqrt{x-3}$ has range $(-\infty, 0]$, the range of $f(x) = -5\sqrt{x-3} + 2$ is $(-\infty, 2]$.

118. By the square root property, we find

$$(x-3)^2 = \frac{3}{2}$$
$$x - 3 = \pm\frac{\sqrt{6}}{2}$$
$$x = \frac{6 \pm \sqrt{6}}{2}.$$

The solution set is $\left\{\frac{6 \pm \sqrt{6}}{2}\right\}$.

119. Since $5x > -1$, the solution set is $(-1/5, \infty)$.

120. Since $y = \frac{1}{3}x - \frac{7}{9}$, the slope is $\frac{1}{3}$.

Thinking Outside the Box

XXV. $1 - (0.7)(0.7)^2(0.7)^3(0.7)^4 \approx 0.97175$ or 97.2%

XXVI. Since $20 = 3(3) + 11(1)$ and $1 = 3(4) + 11(-1)$, all integers at least 20 can be expressed in the form $3x + 11y$.

Note, there are no whole numbers x and y that satisfy $19 = 3x + 11y$. Thus, 19 is the largest whole number N that cannot be expressed in the form $3x + 11y$.

2.4 Pop Quiz

1. $A = \pi r^2 = \pi(d/2)^2$ or $A = \frac{\pi d^2}{4}$

2. $(f + g)(3) = 9 + 1 = 10$

3. $(f \cdot g)(4) = 16 \cdot 2 = 32$

4. $(f \circ g)(5) = f(3) = 9$

5. $\{(4, 8 + 9)\} = \{(4, 17)\}$

6. Since $(n \circ m)(1) = n(3) = 5$, we find $\{(1, 5)\}$.

7. Since $(h + j)(x) = x^2 + \sqrt{x+2}$, the domain is $[-2, \infty)$.

8. Since $(h \circ j)(x) = \left(\sqrt{x+2}\right)^2$, the domain is $[-2, \infty)$.

9. Since $(j \circ h)(x) = \sqrt{x^2+2}$, the domain is $(-\infty, \infty)$.

2.4 Linking Concepts

a) Let n be the number of beads in one foot and suppose n is also the number of spaces. Since the sum of the diameters of the notches and the spaces is 1 foot, we have $1 = (2n)\frac{d}{12}$.

Solving for n, we obtain $n = \frac{6}{d}$.

b) If c is the area in square inches of a cross section of a bead, then $c = \frac{1}{2}\pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{8}$.

Thus, $c = \frac{\pi d^2}{8}$ sq in.

c) Note, a parallel bead is a half-circular cylinder. Since the length of a bead is 12 inches and the area of a cross section of a bead is known (as in part (b)), the volume of a bead is given by

$v_1 = c \cdot 12 = \frac{\pi d^2}{8}(12) = \frac{3\pi d^2}{2}$ cubic inches.

d) The volume v_2 of glue on 1 ft^2 of floor is $v_2 = v_1 \cdot n$. Thus, $v_2 = v_1 \cdot n = \frac{3\pi d^2}{2} \cdot \frac{6}{d} = 9\pi d$ cubic inches.

e) Note, $1\text{ft}^3 = 1728\text{in}^3$ and 1 gal $= \frac{1728}{7.5}$ cubic inches. Let A be the number of square inches one gallon of glue will cover. Then

$$A = \frac{1728}{7.5} \div v_2 = \frac{1728}{7.5} \div (9\pi d).$$

Hence, $A = \frac{25.6}{\pi d}$ square feet.

f) For the square notch whose side is d inches in length, we find the corresponding values of n, c, v_1, v_2, and A as in parts (a)-(e).

i) $n = \frac{6}{d}$.

ii) $c = d^2$ square inches

iii) $v_1 = c \cdot 12 = 12d^2$ cubic inches

iv) $v_2 = v_1 \cdot n = 12d^2 \cdot \frac{6}{d} = 72d$ cubic inches

v) As in part e), we have

$$A = \frac{1728}{7.5} \div v_2 = \frac{1728}{7.5} \div (72d).$$

Thus, $A = \frac{3.2}{d}$ square feet.

For Thought

1. False, since the inverse function is $\{(3, 2), (5, 5)\}$.

2. False, since it is not one-to-one.

3. False, $g^{-1}(x)$ does not exist since g is not one-to-one.

4. True

5. False, a function that fails the horizontal line test has no inverse.

6. False, since it fails the horizontal line test.

7. False, since $f^{-1}(x) = \left(\frac{x}{3}\right)^2 + 2$ where $x \geq 0$.

8. False, $f^{-1}(x)$ does not exist since f is not one-to-one.

9. False, since $y = |x|$ is V-shaped and the horizontal line test fails.

10. True

2.5 Exercises

1. one-to-one

2. invertible

3. inverse

4. symmetric

5. Yes, since all second coordinates are distinct.

6. Yes, since all second coordinates are distinct.

7. No, since there are repeated second coordinates such as $(-1, 1)$ and $(1, 1)$.

8. No, since there are repeated second coordinates as in $(3, 2)$ and $(5, 2)$.

9. No, since there are repeated second coordinates such as $(1, 99)$ and $(5, 99)$.

10. No, since there are repeated second coordinates as in $(-1, 9)$ and $(1, 9)$.

11. Not one-to-one **12.** Not one-to-one

13. One-to-one **14.** One-to-one

15. Not one-to-one **16.** One-to-one

17. One-to-one; since the graph of $y = 2x-3$ shows $y = 2x - 3$ is an increasing function, the Horizontal Line Test implies $y = 2x - 3$ is one-to-one.

18. One-to-one; since the graph of $y = 4x-9$ shows $y = 4x - 9$ is an increasing function, the Horizontal Line Test implies $y = 4x - 9$ is one-to-one.

19. One-to-one; for if $q(x_1) = q(x_2)$ then

$$\begin{aligned}
\frac{1-x_1}{x_1-5} &= \frac{1-x_2}{x_2-5} \\
(1-x_1)(x_2-5) &= (1-x_2)(x_1-5) \\
x_2 - 5 - x_1x_2 + 5x_1 &= x_1 - 5 - x_2x_1 + 5x_2 \\
x_2 + 5x_1 &= x_1 + 5x_2 \\
4(x_1 - x_2) &= 0 \\
x_1 - x_2 &= 0.
\end{aligned}$$

Thus, if $q(x_1) = q(x_2)$ then $x_1 = x_2$. Hence, q is one-to-one.

20. One-to-one; for if $g(x_1) = g(x_2)$ then

$$\begin{aligned}
\frac{x_1+2}{x_1-3} &= \frac{x_2+2}{x_2-3} \\
(x_1+2)(x_2-3) &= (x_2+2)(x_1-3) \\
x_1x_2 - 3x_1 + 2x_2 - 6 &= x_1x_2 - 3x_2 + 2x_1 - 6 \\
-3x_1 + 2x_2 &= -3x_2 + 2x_1 \\
5(x_2 - x_1) &= 0 \\
x_2 - x_1 &= 0.
\end{aligned}$$

Thus, if $g(x_1) = g(x_2)$ then $x_1 = x_2$. Hence, g is one-to-one.

21. Not one-to-one for $p(-2) = p(0) = 1$.

22. Not one-to-one for $r(0) = r(2) = 2$.

23. Not one-to-one for $w(1) = w(-1) = 4$.

24. Not one-to-one for $v(1) = v(-1) = 1$.

25. One-to-one; for if $k(x_1) = k(x_2)$ then

$$\begin{aligned}
\sqrt[3]{x_1+9} &= \sqrt[3]{x_2+9} \\
(\sqrt[3]{x_1+9})^3 &= (\sqrt[3]{x_2+9})^3 \\
x_1 + 9 &= x_2 + 9 \\
x_1 &= x_2.
\end{aligned}$$

Thus, if $k(x_1) = k(x_2)$ then $x_1 = x_2$.
Hence, k is one-to-one.

26. One-to-one; for if $t(x_1) = t(x_2)$ then

$$\begin{aligned} \sqrt{x_1+3} &= \sqrt{x_2+3} \\ (\sqrt{x_1+3})^2 &= (\sqrt{x_2+3})^2 \\ x_1+3 &= x_2+3 \\ x_1 &= x_2. \end{aligned}$$

Thus, if $t(x_1) = t(x_2)$ then $x_1 = x_2$.
Hence, t is one-to-one.

27. Invertible, $\{(3,9),(2,2)\}$

28. Invertible, $\{(5,4),(6,5)\}$

29. Not invertible

30. Not invertible

31. Invertible, $\{(3,3),(2,2),(4,4),(7,7)\}$

32. Invertible, $\{(1,1),(4,2),(16,4),(49,7)\}$

33. Not invertible

34. Not invertible

35. Not invertible, there can be two different items with the same price.

36. Not invertible since the number of years (given as a whole number) cannot determine the number of days since your birth.

37. Invertible, since the playing time is a function of the length of the VCR tape.

38. Invertible, since 1.6 km $\approx$ 1 mile

39. Invertible, assuming that cost is simply a multiple of the number of days. If cost includes extra charges, then the function may not be invertible.

40. Invertible, since the interest is uniquely determined by the number of days.

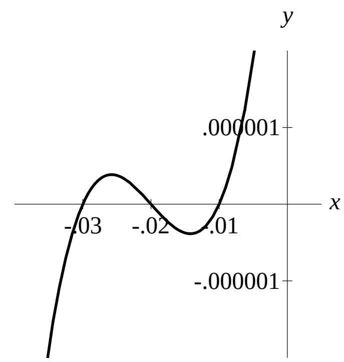
41. $f^{-1} = \{(1,2),(5,3)\}$, $f^{-1}(5) = 3$, $(f^{-1} \circ f)(2) = 2$

42. $f^{-1} = \{(5,-1),(0,0),(6,2)\}$, $f^{-1}(5) = -1$, $(f^{-1} \circ f)(2) = 2$

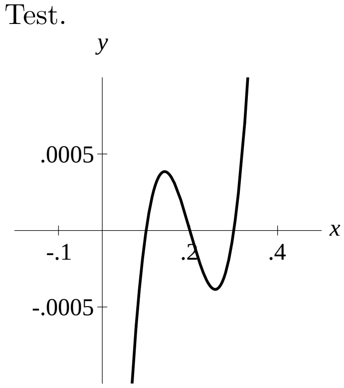
43. $f^{-1} = \{(-3,-3),(5,0),(-7,2)\}$, $f^{-1}(5) = 0$, $(f^{-1} \circ f)(2) = 2$

44. $f^{-1} = \{(5,3.2),(1.99,2)\}$, $f^{-1}(5) = 3.2$, $(f^{-1} \circ f)(2) = 2$

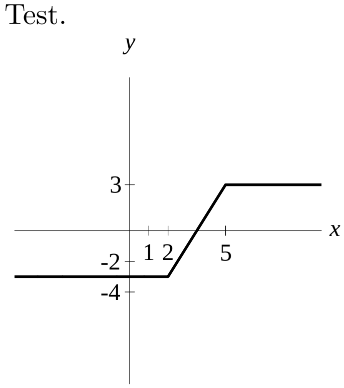
45. Not invertible since it fails the Horizontal Line Test.



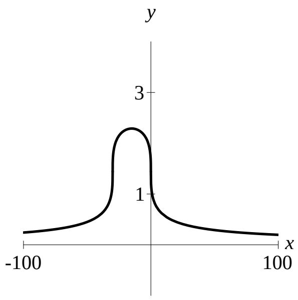
46. Not invertible since it fails the Horizontal Line Test.



47. Not invertible since it fails the Horizontal Line Test.



48. Not invertible since it fails the Horizontal Line Test.



49. **a)** $f(x)$ is the composition of multiplying x by 5, then subtracting 1.
Reversing the operations, the inverse is $f^{-1}(x) = \dfrac{x-1}{5}$

b) $f(x)$ is the composition of multiplying x by 3, then subtracting 88.
Reversing the operations, the inverse is $f^{-1}(x) = \dfrac{x+88}{3}$

c) $f^{-1}(x) = (x+7)/3$

d) $f^{-1}(x) = \dfrac{x-4}{-3}$

e) $f^{-1}(x) = 2(x+9) = 2x+18$

f) $f^{-1}(x) = -x$

g) $f(x)$ is the composition of taking the cube root of x, then subtracting 9.
Reversing the operations, the inverse is $f^{-1}(x) = (x+9)^3$

h) $f(x)$ is the composition of cubing x, multiplying the result by 3, then subtracting 7.
Reversing the operations, the inverse is $f^{-1}(x) = \sqrt[3]{\dfrac{x+7}{3}}$

i) $f(x)$ is the composition of subtracting 1 from x, taking the cube root of the result, then adding 5.
Reversing the operations, the inverse is $f^{-1}(x) = (x-5)^3 + 1$

j) $f(x)$ is the composition of subtracting 7 from x, taking the cube root of the result, then multiplying by 2.
Reversing the operations, the inverse is $f^{-1}(x) = \left(\dfrac{x}{2}\right)^3 + 7$

50. **a)** $f(x)$ is the operation of dividing x by 2. Reversing the operation, the inverse is $f^{-1}(x) = 2x$

b) $f(x)$ is the operation of adding 99 to x. Reversing the operation, the inverse is $f^{-1}(x) = x - 99$

c) $f(x)$ is the composition of multiplying x by 5, then adding 1.
Reversing the operations, the inverse is $f^{-1}(x) = (x-1)/5$

d) $f(x)$ is the composition of multiplying x by -2, then adding 5.
Reversing the operations, the inverse is $f^{-1}(x) = \dfrac{x-5}{-2}$

e) $f(x)$ is the composition of dividing x by 3, then adding 6. Reversing the operations, the inverse is
$f^{-1}(x) = 3(x-6) = 3x - 18$

f) $f(x)$ is the operation of taking the multiplicative inverse of x. Since taking the multiplicative inverse twice returns to the original number, the inverse is taking the multiplicative inverse, i.e., $f^{-1}(x) = 1/x$

g) $f(x)$ is the composition of subtracting 9 from x, then taking the cube root of the result. Reversing the operations, the inverse is $f^{-1}(x) = x^3 + 9$.

h) $f(x)$ is the composition of cubing x, multiplying the result by -1, then adding 4. Reversing the operations, the inverse is $f^{-1}(x) = \sqrt[3]{-(x-4)} = \sqrt[3]{4-x}$.

i) $f(x)$ is the composition of adding 4 to x, taking the cube root of the result, then multiplying by 3. Reversing the operations, the inverse is $f^{-1}(x) = \left(\dfrac{x}{3}\right)^3 - 4$

j) $f(x)$ is the composition of adding 3 to x, taking the cube root of the result, then subtracting 9.
Reversing the operations, the inverse is $f^{-1}(x) = (x+9)^3 - 3$.

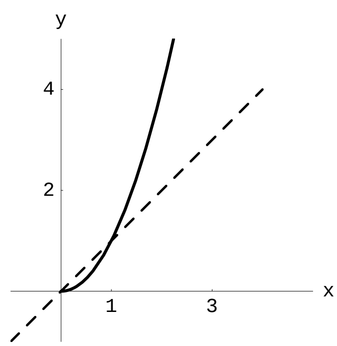
51. No, since they fail the Horizontal Line Test.

52. Yes, since they are symmetric about the line $y = x$.

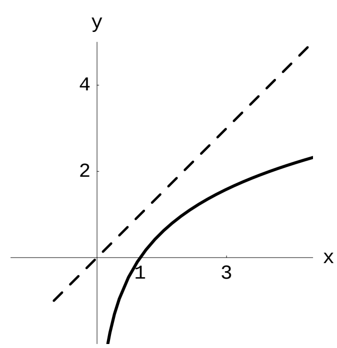
53. Yes, since the graphs are symmetric about the line $y = x$.

54. No since $f(x) = x^2$ fails the Horizontal Line Test.

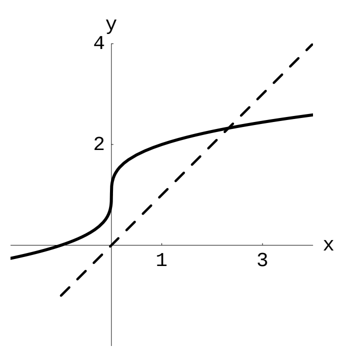
55. Graph of f^{-1}



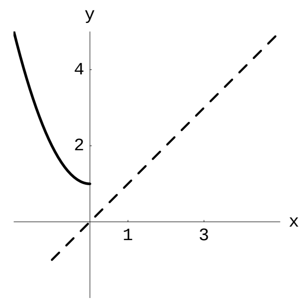
56. Graph of f^{-1}



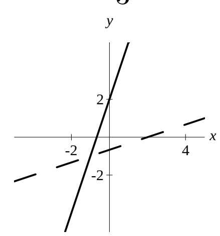
57. Graph of f^{-1}



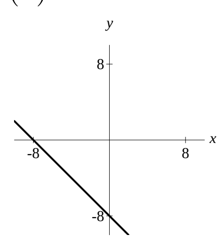
58. Graph of f^{-1}



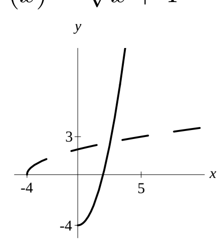
59. $f^{-1}(x) = \dfrac{x-2}{3}$



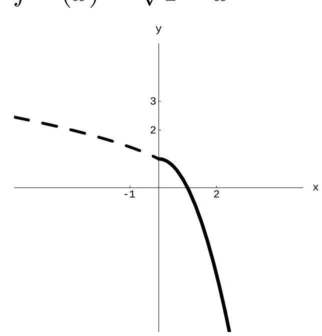
60. $f^{-1}(x) = -x - 8$



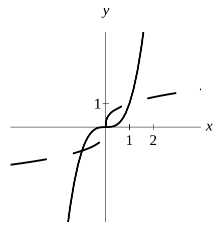
61. $f^{-1}(x) = \sqrt{x+4}$



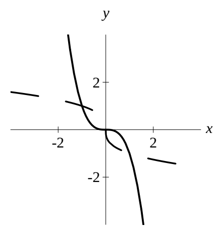
62. $f^{-1}(x) = \sqrt{1-x}$



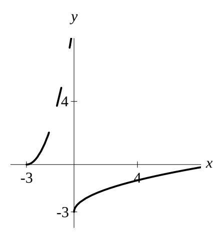
63. $f^{-1}(x) = \sqrt[3]{x}$



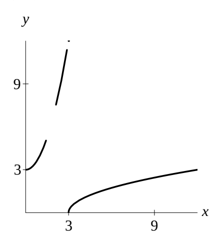
64. $f^{-1}(x) = -\sqrt[3]{x}$



65. $f^{-1}(x) = (x+3)^2$ for $x \geq -3$



66. $f^{-1}(x) = x^2 + 3$ for $x \geq 0$



67. Interchange x and y then solve for y.

$$\begin{aligned} x &= 3y - 7 \\ \frac{x+7}{3} &= y \\ \frac{x+7}{3} &= f^{-1}(x) \end{aligned}$$

68. Interchange x and y then solve for y.

$$\begin{aligned} x &= -2y + 5 \\ 2y &= 5 - x \\ f^{-1}(x) &= \frac{5-x}{2} \end{aligned}$$

69. Interchange x and y then solve for y.

$$\begin{aligned} x &= 2 + \sqrt{y-3} \quad \text{for } x \geq 2 \\ (x-2)^2 &= y - 3 \quad \text{for } x \geq 2 \\ f^{-1}(x) &= (x-2)^2 + 3 \quad \text{for } x \geq 2 \end{aligned}$$

70. Interchange x and y then solve for y.

$$\begin{aligned} x &= \sqrt{3y-1} \quad \text{for } x \geq 0 \\ x^2 + 1 &= 3y \quad \text{for } x \geq 0 \\ f^{-1}(x) &= \frac{x^2+1}{3} \quad \text{for } x \geq 0 \end{aligned}$$

71. Interchange x and y then solve for y.

$$\begin{aligned} x &= -y - 9 \\ y &= -x - 9 \\ f^{-1}(x) &= -x - 9 \end{aligned}$$

72. Interchange x and y then solve for y.

$$\begin{aligned} x &= -y + 3 \\ y &= -x + 3 \\ f^{-1}(x) &= -x + 3 \end{aligned}$$

73. Interchange x and y then solve for y.

$$\begin{aligned} x &= \frac{y+3}{y-5} \\ xy - 5x &= y + 3 \\ xy - y &= 5x + 3 \\ y(x-1) &= 5x + 3 \\ f^{-1}(x) &= \frac{5x+3}{x-1} \end{aligned}$$

74. Interchange x and y then solve for y.

$$\begin{aligned} x &= \frac{2y-1}{y-6} \\ xy - 6x &= 2y - 1 \\ xy - 2y &= 6x - 1 \\ y(x-2) &= 6x - 1 \\ f^{-1}(x) &= \frac{6x-1}{x-2} \end{aligned}$$

75. Interchange x and y then solve for y.

$$\begin{aligned} x &= -\frac{1}{y} \\ xy &= -1 \\ f^{-1}(x) &= -\frac{1}{x} \end{aligned}$$

76. Clearly $f^{-1}(x) = x$

77. Interchange x and y then solve for y.

$$\begin{aligned} x &= \sqrt[3]{y-9}+5 \\ x-5 &= \sqrt[3]{y-9} \\ (x-5)^3 &= y-9 \\ f^{-1}(x) &= (x-5)^3+9 \end{aligned}$$

78. Interchange x and y then solve for y.

$$\begin{aligned} x &= \sqrt[3]{\frac{y}{2}}+5 \\ x-5 &= \sqrt[3]{\frac{y}{2}} \\ (x-5)^3 &= \frac{y}{2} \\ f^{-1}(x) &= 2(x-5)^3 \end{aligned}$$

79. Interchange x and y then solve for y.

$$\begin{aligned} x &= (y-2)^2 \quad x \geq 0 \\ \sqrt{x} &= y-2 \\ f^{-1}(x) &= \sqrt{x}+2 \end{aligned}$$

80. Interchange x and y then solve for y.

$$\begin{aligned} x &= y^2 \quad x \geq 0 \\ -\sqrt{x} &= y \quad \text{since domain} \\ & \quad\ \text{of } f \text{ is } (-\infty, 0] \\ f^{-1}(x) &= -\sqrt{x} \quad x \geq 0 \end{aligned}$$

81. Note, $(g \circ f)(x) = 0.25(4x+4) - 1 = x$ and $(f \circ g)(x) = 4(0.25x - 1) + 4 = x$.

Yes, g and f are inverse functions of each other.

82. Note, $(g \circ f)(x) = -0.2(20 - 5x) + 4 = x$ and $(f \circ g)(x) = 20 - 5(-0.2x + 4) = x$.

Yes, g and f are inverse functions of each other.

83. Since $(f \circ g)(x) = \left(\sqrt{x-1}\right)^2 + 1 = x$ and and $(g \circ f)(x) = \sqrt{x^2+1-1} = \sqrt{x^2} = |x|$, g and f are not inverse functions of each other.

84. Since $(f \circ g)(x) = \sqrt[4]{x^4} = |x|$ and and $(g \circ f)(x) = (\sqrt[4]{x})^4 = x$, g and f are not inverse functions of each other.

85. We find

$$\begin{aligned} (f \circ g)(x) &= \frac{1}{1/(x-3)} + 3 \\ &= x - 3 + 3 \\ (f \circ g)(x) &= x \end{aligned}$$

and

$$\begin{aligned} (g \circ f)(x) &= \frac{1}{\left(\frac{1}{x}+3\right) - 3} \\ &= \frac{1}{1/x} \\ (g \circ f)(x) &= x. \end{aligned}$$

Then g and f are inverse functions of each other.

86. We obtain

$$\begin{aligned} (f \circ g)(x) &= 4 - \frac{1}{1/(4-x)} \\ &= 4 - (4-x) \\ (f \circ g)(x) &= x \end{aligned}$$

and

$$\begin{aligned} (g \circ f)(x) &= \frac{1}{4 - \left(4 - \frac{1}{x}\right)} \\ &= \frac{1}{1/x} \\ (g \circ f)(x) &= x \end{aligned}$$

Thus, g and f are inverse functions of each other.

87. We obtain

$$(f \circ g)(x) = \sqrt[3]{\frac{5x^3+2-2}{5}}$$

$$\begin{aligned} &= \sqrt[3]{\frac{5x^3}{5}} \\ &= \sqrt[3]{x^3} \\ (f \circ g)(x) &= x \end{aligned}$$

and

$$\begin{aligned} (g \circ f)(x) &= 5\left(\sqrt[3]{\frac{x-2}{5}}\right)^3 + 2 \\ &= 5\left(\frac{x-2}{5}\right) + 2 \\ &= (x-2) + 2 \\ (g \circ f)(x) &= x. \end{aligned}$$

Thus, g and f are inverse functions of each other.

88. We note

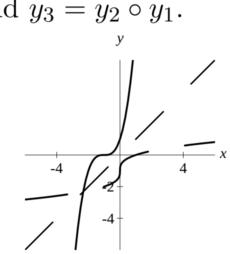
$$(f \circ g)(x) = \left(\sqrt[3]{x} + 3\right)^3 - 27 \neq x$$

and

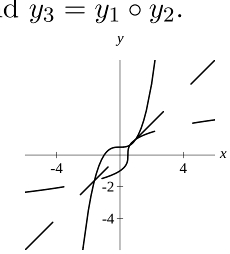
$$(g \circ f)(x) = \sqrt[3]{x^3 - 27} + 3 \neq x.$$

Then g and f are not inverse functions of each other.

89. y_1 and y_2 are inverse functions of each other and $y_3 = y_2 \circ y_1$.



90. y_1 and y_2 are inverse functions of each other and $y_3 = y_1 \circ y_2$.



91. $C = 1.08P$ expresses the total cost as a function of the purchase price; and $P = C/1.08$ is the purchase price as a function of the total cost.

92. $V(x) = x^3, S(x) = \sqrt[3]{x}$

93. The graph of t as a function of r satisfies the Horizontal Line Test and is invertible. Solving for r we find,

$$\begin{aligned} t - 7.89 &= -0.39r \\ r &= \frac{t - 7.89}{-0.39} \end{aligned}$$

and the inverse function is $r = \dfrac{7.89 - t}{0.39}$.

If $t = 5.55$ min., then $r = \dfrac{7.89 - 5.55}{0.39} = 6$ rowers.

94. Solving for F, we obtain

$$\begin{aligned} F - 32 &= \frac{9C}{5} \\ F &= \frac{9C}{5} + 32 \end{aligned}$$

and the inverse function is $F = \dfrac{9C}{5} + 32$; a formula that can convert Celsius temperature to Fahrenheit temperature.

95. Solving for w, we obtain

$$\begin{aligned} 1.496w &= V^2 \\ w &= \frac{V^2}{1.496} \end{aligned}$$

and the inverse function is $w = \dfrac{V^2}{1.496}$. If $V = 115$ ft./sec., then $w = \dfrac{115^2}{1.496} \approx 8,840$ lb.

96.

$r = \sqrt{5.625 \times 10^{-5} - \dfrac{V}{500}}$ where $0 \le V \le 0.028125$

97. **a)** Let $V = \$28,000$. The depreciation rate is

$$r = 1 - \left(\frac{28,000}{50,000}\right)^{1/5} \approx 0.109$$

or $r \approx 10.9\%$.

b) Writing V as a function of r we find

$$\begin{aligned} 1-r &= \left(\frac{V}{50,000}\right)^{1/5} \\ (1-r)^5 &= \frac{V}{50,000} \end{aligned}$$

and $V = 50,000(1-r)^5$.

98. Let $P = 80,558$. Then

$$r = \left(\frac{22,402}{10,000}\right)^{1/10} - 1 \approx 0.0839.$$

The average annual growth rate is $r \approx 8.4\%$.

Solving for P, we obtain

$$\begin{aligned} 1+r &= \left(\frac{P}{10,000}\right)^{1/10} \\ (1+r)^{10} &= \frac{P}{10,000} \end{aligned}$$

and $P = 10,000(1+r)^{10}$.

99. Since $g^{-1}(x) = \dfrac{x+5}{3}$ and $f^{-1}(x) = \dfrac{x-1}{2}$, we have

$$g^{-1} \circ f^{-1}(x) = \frac{\frac{x-1}{2}+5}{3} = \frac{x+9}{6}.$$

Likewise, since $(f \circ g)(x) = 6x - 9$, we get

$$(f \circ g)^{-1}(x) = \frac{x+9}{6}.$$

Hence, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

100. Since $\left(f \circ g \circ g^{-1} \circ f^{-1}\right)(x) =$ $\left(f \circ (g \circ g^{-1})\right)\left(f^{-1}(x)\right) = f(f^{-1}(x)) = x$ and the range of one function is the domain of the other function, we have $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

101. One can easily see that the slope of the line joining (a,b) to (b,a) is -1, and that their midpoint is $\left(\dfrac{a+b}{2}, \dfrac{a+b}{2}\right)$. This midpoint lies on the line $y = x$ whose slope is 1. Then $y = x$ is the perpendicular bisector of the line segment joining the points (a,b) and (b,a)

102. It is difficult to find the inverse mentally since the two steps $x-3$ and $x+2$ are done separately and simultaneously.

103. Dividing we get $\dfrac{x-3}{x+2} = 1 - \dfrac{5}{x+2}$

104. $f^{-1}(x) = \left(\dfrac{x-1}{-5}\right)^{-1} - 2$ or

$f^{-1}(x) = \dfrac{5}{1-x} - 2$

105. $(f \circ g)(2) = f(g(2)) = f(1) = \dfrac{2+3}{5} = 1$

$(f \cdot g)(2) = f(2)g(2)) = \dfrac{7}{5} \cdot 1 = \dfrac{7}{5}$

106. $y = -2\sqrt{x-5}$

107. Observe, the graph of $f(x) = -\sqrt{9-x^2}$ is a lower semicircle of radius 3 centered at $(0,0)$.

Then domain is $[-3,3]$, the range is $[-3,0]$, and increasing on $[0,3]$

108. No, two ordered pairs have the same first coordinates.

109.

$$\begin{aligned} 0.75 + 0.80 &= 0.225x - 0.125x \\ 1.55 &= 0.1x \\ 15.5 &= x \end{aligned}$$

The solution set is $\{15.5\}$.

110. The slope of the perpendicular line is $-1/2$.

Using $y = mx + b$ and the point $(2,4)$, we find

$$\begin{aligned} 4 &= -\frac{1}{2}(2) + b \\ 4 &= -1 + b \\ 5 &= b \end{aligned}$$

The perpendicular line is $y = -\dfrac{1}{2}x + 5$.

Thinking Outside the Box XXVII

Since $640,000 = 2^{10} \cdot 5^4$, we have either $x = 2^{10}$ and $y = 5^4$, or $x = 5^4$ and $y = 2^{10}$. In either case, $|x - y| = 399$.

2.5 Pop Quiz

1. No, since the second coordinate is repeated in $(1, 3)$ and $(2, 3)$.

2. 2, since the order pair $(5, 2)$ belong to f^{-1}.

3. Since $f^{-1}(x) = x/2$, $f^{-1}(8) = 8/2 = 4$.

4. No, since $f(1) = 1 = f(-1)$ or the second coordinate is repeated in $(1, 1)$ and $(1, -1)$.

5. $f^{-1}(x) = \dfrac{x+1}{2}$

6. Interchange x and y then solve for y.

$$\begin{aligned} x &= \sqrt[3]{y+1} - 4 \\ x + 4 &= \sqrt[3]{y+1} \\ (x+4)^3 &= y + 1 \\ (x+4)^3 - 1 &= y \end{aligned}$$

The inverse is $g^{-1}(x) = (x + 4)^3 - 1$.

7.

$$\begin{aligned} (h \circ j)(x) &= h(\sqrt[3]{x+5}) \\ &= \left(\sqrt[3]{x+5}\right)^3 - 5 \\ &= (x+5) - 5 \\ (h \circ j)(x) &= x \end{aligned}$$

2.5 Linking Concepts

a) $r = 365\left(\left(\dfrac{A}{P}\right)^{1/n} - 1\right)$.

b) For the first loan,

$$r = 365\left(\left(\frac{229}{200}\right)^{1/30} - 1\right) \approx 1.651$$

or $r \approx 165.1\%$ annually.

For the second loan,

$$r = 365\left(\left(\frac{339}{300}\right)^{1/30} - 1\right) \approx 1.49$$

or $r \approx 149\%$ annually.

For the third loan,

$$r = 365\left(\left(\frac{449}{400}\right)^{1/30} - 1\right) \approx 1.409$$

or $r \approx 140.9\%$ annually.

c) If one borrowed \$200 at an annual rate of $r = 165.1\%$ compounded daily, then after one year one will have to pay back

$$200\left(1 + \frac{1.651}{365}\right)^{365} = \$1,038.56$$

e) It charges high rates because of high risks.

For Thought

1. False

2. False, since cost varies directly with the number of pounds purchased.

3. True **4.** True

5. True, since the area of a circle varies directly with the square of its radius.

6. False, since $y = k/x$ is undefined when $x = 0$.

7. True **8.** True **9.** True

10. False, the surface area is not equal to

$$(\text{Surface Area }) = k \cdot \text{length} \cdot \text{width} \cdot \text{height}$$

for some constant k.

2.6 Exercises

1. varies directly

2. variation

3. varies inversely

4. varies jointlly

5. $G = kn$ **6.** $T = kP$ **7.** $V = k/P$

8. $m_1 = k/m_2$ **9.** $C = khr$ **10.** $V = khr^2$

11. $Y = \dfrac{kx}{\sqrt{z}}$ **12.** $W = krt/v$

13. A varies directly as the square of r

14. C varies directly as D

15. y varies inversely as x

16. m_1 varies inversely as m_2

17. Not a variation expression

18. Not a variation expression

19. a varies jointly as z and w

20. V varies jointly as L, W, and H

21. H varies directly as the square root of t and inversely as s

22. B varies directly as the square of y and inversely as the square root of x

23. D varies jointly as L and J and inversely as W

24. E varies jointly as m and the square of c.

25. Since $y = kx$ and $5 = k \cdot 9$, $k = 5/9$. Then $y = 5x/9$.

26. Since $h = kz$ and $210 = k \cdot 200$, $k = 21/20$. So $h = 21z/20$.

27. Since $T = k/y$ and $-30 = k/5$, $k = -150$. Thus, $T = -150/y$.

28. Since $H = k/n$ and $9 = k/(-6)$, $k = -54$. So $H = -54/n$.

29. Since $m = kt^2$ and $54 = k \cdot 18$, $k = 3$. Thus, $m = 3t^2$.

30. Since $p = k\sqrt[3]{w}$ and $\dfrac{\sqrt[3]{2}}{2} = k\sqrt[3]{4}$, we

get $k = \dfrac{\sqrt[3]{2}}{2\sqrt[3]{4}} = \dfrac{\sqrt[3]{4}}{4}$. Then $p = \dfrac{\sqrt[3]{4w}}{4}$.

31. Since $y = kx/\sqrt{z}$ and $2.192 = k(2.4)/\sqrt{2.25}$, we obtain $k = 1.37$. Hence, $y = 1.37x/\sqrt{z}$.

32. Since $n = kx\sqrt{b}$ and $-18.954 = k(-1.35)\sqrt{15.21}$, we find $k = 3.6$. So $n = 3.6x\sqrt{b}$

33. Since $y = kx$ and $9 = k(2)$, we obtain

$y = \dfrac{9}{2} \cdot (-3) = -27/2$.

34. Since $y = kz$ and $6 = k\sqrt{12}$, we obtain

$y = \dfrac{3}{\sqrt{3}} \cdot \sqrt{75} = 15$.

35. Since $P = k/w$ and $2/3 = \dfrac{k}{1/4}$, we find

$k = \dfrac{2}{3} \cdot \cdot \dfrac{1}{4} = \dfrac{1}{6}$. Thus, $P = \dfrac{1/6}{1/6} = 1$.

36. Since $H = k/q$ and $0.03 = \dfrac{k}{0.01}$, we get

$P = \dfrac{0.0003}{0.05} = 0.006$

37. Since $A = kLW$ and $30 = k(3)(5\sqrt{2})$,

we obtain $A = \sqrt{2}(2\sqrt{3})\dfrac{1}{2} = \sqrt{6}$.

38. Since $J = kGV$ and $\sqrt{3} = k\sqrt{2}\sqrt{8}$, we

find $J = \dfrac{\sqrt{3}}{4}\sqrt{6} \cdot 8 = 2\sqrt{18} = 6\sqrt{2}$.

39. Since $y = ku/v^2$ and $7 = k \cdot 9/36$,

we find $y = 28 \cdot 4/64 = 7/4$.

40. Since $q = k\sqrt{h}/j^3$ and $18 = k\sqrt{9}/8$,

we get $q = 48 \cdot \dfrac{4}{1/8} = 1536$.

41. Let L_i and L_f be the length in inches and feet, respectively. Then $L_i = 12L_f$ is a direct variation.

42. Let T_s and T_m be the time in seconds and minutes, respectively. Then $T_s = 60T_m$ is a direct variation.

43. Let P and n be the cost per person and the number of persons, respectively. Then
$P = 20/n$ is an inverse variation.

44. Let n and w be the number of rods and the weight of a rod, respectively.

Then $n = \dfrac{40,000}{w}$ is an inverse variation.

45. Let S_m and S_k be the speeds of the car in mph and kph, respectively. Then $S_m \approx S_k/1.6 \approx 0.6S_k$ is a direct variation.

46. Let W_p and W_k be the weight in pounds and kilograms, respectively. Then $W_p = 2.2W_k$ is a direct variation.

47. Not a variation **48.** Not a variation

49. Let A and W be the area and width, respectively. Then $A = 30W$ is a direct variation.

50. Let A and b be the area and base, respectively. Then $A = \frac{1}{2}10b = 5b$ is a direct variation.

51. Let n and p be the number of gallons and price per gallon, respectively. Since $np = 5$, we obtain that $n = \frac{5}{p}$ is an inverse variation.

52. Let L and W be the length and width, respectively. Then $L = 40/W$ is an inverse variation.

53. If p is the pressure at depth d, then $p = kd$. Since $4.34 = k(10)$, $k = 0.434$. At $d = 6000$ ft, the pressure is $p = 0.434(6000) = 2604$ lb per square inch.

54. Solving for the depth d in $2170 = 0.434d$, we find $d = \frac{2170}{0.434} = 5000$ feet.

55. If h is the number of hours, p is the number of pounds, and w is the number of workers then $h = kp/w$. Since $8 = k(3000)/6$, $k = 0.016$. Five workers can process 4000 pounds in $h = (0.016)(4000)/5 = 12.8$ hours.

56. Since $V = k\sqrt{A}$ and $154 = k\sqrt{16000}$, we find $k \approx 1.21747$. The view from 36000 feet is $V = (1.21747)\sqrt{36000} \approx 231$ miles.

57. Since $I = kPt$ and $20.80 = k(4000)(16)$, we find $k = 0.000325$. The interest from a deposit of \$6500 for 24 days is

$$I = (0.000325)(6500)(24) \approx \$50.70.$$

58. Since $d = kt^2$ and $16 = k(1^2)$, $k = 16$. Thus, after 2 seconds the cab falls $d = (16)4 = 64$ feet.

59. Since $C = kDL$ and $18.60 = k(6)(20)$, we obtain $k = 0.155$. The cost of a 16 ft pipe with a diameter of 8 inches is

$$C = 0.155(8)(16) = \$19.84.$$

60. Since $C = kDL$ and $36.60 = k(0.5)(20)$, we find $k = 3.66$. The cost of a 100 ft copper tubing with a $\frac{3}{4}$-inch diameter is

$$C = 3.66(0.75)(100) = \$274.50.$$

61. Since $w = khd^2$ and $14.5 = k(4)(6^2)$, we find $k = \frac{14.5}{144}$. Then a 5-inch high can with a diameter of 6 inches has weight $w = \frac{14.5}{144}(5)(6^2) = 18.125$ oz.

62. Since $V = kt/w$ and $10 = k(80)/600$, we get $k = 75$. At 90^oF and 800 pounds the volume is $V = 75(90)/(800) \approx 8.4375$ cubic inches.

63. Since $V = kh/l$ and $10 = k(50)/(200)$, we get $k = 40$. The velocity, if the head is 60 ft and the length is 300 ft, is $V = (40)(60)/(300) = 8$ ft/year.

64. Since $V = kiA$ and $3 = k(0.3)(10)$, $k = 1$. If the hydraulic gradient is 0.4 and the discharge is 5 gallons per minute then $5 = (1)(0.4)A$. The cross-sectional area is $A = 12.5$ ft^2.

65. No, it is not directly proportional otherwise the following ratios $\frac{42,506}{1.34} \approx 31,720$, $\frac{59,085}{0.295} \approx 200,288$, and $\frac{738,781}{0.958} \approx 771,170$ would be all the same but they are not.

66. Let $r = \frac{knw}{c}$. To find k, we solve

$$54 = \frac{k(50)(27)}{25}.$$

We obtain $k = 1$ and consequently $r = \frac{nw}{c}$. If $n = 40$, $c = 13$, and $w = 26$, then the gear ratio is $r = \frac{40(26)}{13} = 80$. If $n = 45$, $w = 27$, and $r = 67.5$, then by solving $67.5 = \frac{45(27)}{c}$ one obtains $c = 18$.

67. Since $g = ks/p$ and $76 = k(12)/(10)$, $k = \dfrac{190}{3}$. If Calvin studies for 9 hours and plays for 15 hours, then his score is

$$g = \frac{190}{3} \cdot \frac{s}{p} = \frac{190}{3} \cdot \frac{9}{15} = 38.$$

68. Since $c = klw$ and $263.40 = k(12)(9)$, we get $k = \dfrac{263.4}{108}$. A carpet which is 12 ft wide and that costs \$482.90 (i.e. $482.90 = \dfrac{263.4}{108} \cdot 12 \cdot l$) has length $l = 16.5$ ft.

69. Since $h = kv^2$ and $16 = k(32)^2$, we get $k = \dfrac{1}{64}$. To reach a height of $20'2.5''$, the velocity v must satisfy

$$20 + \frac{2.5}{12} = \frac{1}{64}v^2.$$

Solving for v, we find $v \approx 35.96$ ft/sec.

73. Interchange x and y then solve for y.

$$\begin{aligned} x &= \sqrt[3]{y-9} + 1 \\ (x-1)^3 &= y - 9 \\ (x-1)^3 + 9 &= y \\ f^{-1}(x) &= (x-1)^3 + 9 \end{aligned}$$

74. If s is the side of the square, then the diagonal is $d = \sqrt{2}s$ by the Pythagorean Theorem. If A is the area of the square, then $A = s^2$. Since $s = \sqrt{A}$, we obtain $d = \sqrt{2}s = \sqrt{2}\sqrt{A}$ or $d = \sqrt{2A}$

75. Let x be the average speed in the rain.Then

$$3x + 5(x+5) = 425$$

Solving for x, we find $x = 50$ mph.

76. Since $f(-x) = -f(x)$, the graph is symmetric about the origin.

77. The slope of the line is $1/2$. Using $y = mx + b$ and the point $(-4, 2)$, we find

$$\begin{aligned} 2 &= \frac{1}{2}(-4) + b \\ 2 &= -2 + b \\ 4 &= b \end{aligned}$$

The line is given by $y = \frac{1}{2}x + 4$, or $2y = x + 8$. A standard form is $x - 2y = -8$.

78. Since $-6 < 3x - 9 < 6$, we obtain

$$3 < 3x < 15.$$

The solution set is $(1, 5)$.

Thinking Outside the Box XXVIII

Let d be the distance Sharon walks. Since 2 minutes is the difference in the times of arrival at 4mph and 5 mph, we obtain

$$\frac{d}{4} - \frac{d}{5} = \frac{2}{60}.$$

The solution of the above equation is $d = 2/3$ mile. Since $d/4 = (2/3)/4 = 10/60$, Sharon must walk to school in 9 minutes to arrive on time. Thus, Sharon's speed in order to arrive on time is

$$r = \frac{d}{t} = \frac{2/3}{9/60} = \frac{40}{9} \text{ mph.}$$

2.6 Pop Quiz

1. Since $4 = k \cdot 20$, the constant of variation is $k = 4/20 = 1/5$.

2. $a = k/b = 10/2 = 5$

3. Since $C = kr^2$ and $108 = k \cdot 3^2$, we find $k = 12$. Then $C = 12 \cdot 4^2 = \$192$.

4. Since $C = kLW$ and $180 = k(6)(2)$, we obtain $k = 15$. Then $C = 15(5)(4) = \$300$.

2.6 Linking Concepts

a) $m = \dfrac{kd^3}{p^2}$ where m is the mass of the planet, d is the mean distance between the satellite and the planet, p is period of revolution of the satellite, and k is the proportion constant.

b) Solving $5.976 \times 10^{24} = \dfrac{k(384.4 \times 10^3)^3}{27.322^2}$ for k, we find $k \approx 7.8539 \times 10^{10} \approx 7.85 \times 10^{10}$.

c) The mass of Mars is

$$\frac{7.8539 \times 10^{10}(9330)^3}{(7.65/24)^2} \approx 6.278 \times 10^{23} \text{ kg.}$$

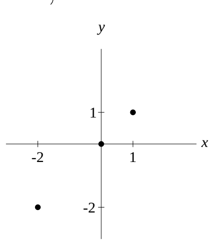
d) An approximate ratio of Earth's mass to Mars' mass is $\dfrac{5.976 \times 10^{24}}{6.278 \times 10^{23}} \approx 10$ or the ratio is approximately 10 to 1.

e) Solving for d, we obtain

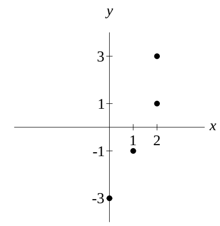
$$\begin{aligned}
\frac{(7.8539 \times 10^{10})d^3}{(42.47/24)^2} &= 1.921 \times 10^{27} \\
d^3 &= \frac{(1.921 \times 10^{27})\left(\frac{42.47}{24}\right)^2}{7.8539 \times 10^{10}} \\
d &= \sqrt[3]{\frac{(1.921 \times 10^{27})\left(\frac{42.47}{24}\right)^2}{7.8539 \times 10^{10}}} \\
d &\approx 424{,}678 \text{ km} \\
d &\approx 4.247 \times 10^5 \text{ km.}
\end{aligned}$$

Review Exercises

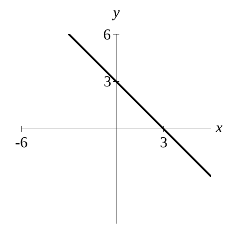
1. Function, domain and range are both $\{-2, 0, 1\}$



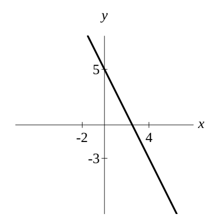
2. Not a function, domain is $\{0, 1, 2\}$, range is $\{\pm 1, \pm 3\}$



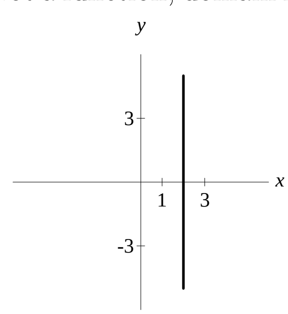
3. $y = 3 - x$ is a function, domain and range are both $(-\infty, \infty)$



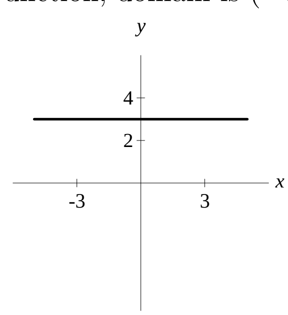
4. $2x + y = 5$ is a function, domain and range are both $(-\infty, \infty)$



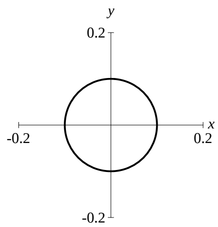
5. Not a function, domain is $\{2\}$, range is $(-\infty, \infty)$



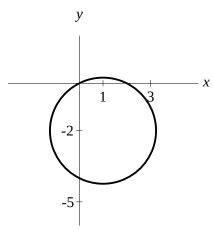
6. Function, domain is $(-\infty, \infty)$, range is $\{3\}$,



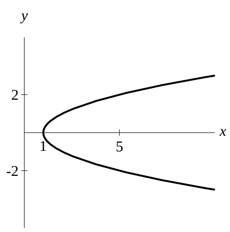
7. $x^2 + y^2 = 0.01$ is not a function, domain and range are both $[-0.1, 0.1]$



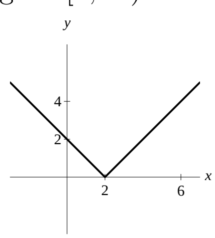
8. $(x-1)^2 + (y+2)^2 = 5$ is not a function, domain is $[1-\sqrt{5}, 1+\sqrt{5}]$, range is $[-2-\sqrt{5}, -2+\sqrt{5}]$



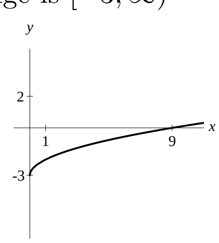
9. $x = y^2 + 1$ is not a function, domain is $[1, \infty)$, range is $(-\infty, \infty)$



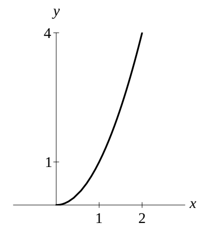
10. $y = |x - 2|$ is a function, domain is $(-\infty, \infty)$, range is $[0, \infty)$



11. $y = \sqrt{x} - 3$ is a function, domain is $[0, \infty)$, range is $[-3, \infty)$



12. $x = \sqrt{y}$ is a function, domain and range are both $[0, \infty)$



13. $9 + 3 = 12$ **14.** $6 - 7 = -1$

15. $24 - 7 = 17$ **16.** $1 + 3 = 4$

17. If $x^2 + 3 = 19$, then $x^2 = 16$ or $x = \pm 4$.

18. If $2x - 7 = 9$, then $2x = 16$ or $x = 8$.

19. $g(12) = 17$ **20.** $f(-1) = 4$

21. $7 + (-3) = 4$ **22.** $7 - (-11) = 18$

23. $(4)(-9) = -36$ **24.** 19

25. $f(-3) = 12$ **26.** $g(7) = 7$

27.

$$\begin{aligned} f(g(x)) &= f(2x-7) \\ &= (2x-7)^2 + 3 \\ &= 4x^2 - 28x + 52 \end{aligned}$$

28.

$$\begin{aligned} g(f(x)) &= g(x^2+3) \\ &= 2(x^2+3) - 7 \\ &= 2x^2 - 1 \end{aligned}$$

29. $(x^2+3)^2 + 3 = x^4 + 6x^2 + 12$

30. $2(2x-7) - 7 = 4x - 21$

31. $(a+1)^2 + 3 = a^2 + 2a + 4$

32. $2(a+2) - 7 = 2a - 3$

33.

$$\begin{aligned} \frac{f(3+h) - f(3)}{h} &= \frac{(9 + 6h + h^2) + 3 - 12}{h} \\ &= \frac{6h + h^2}{h} \\ &= 6 + h \end{aligned}$$

34.

$$\begin{aligned}\frac{g(5+h)-g(5)}{h} &= \frac{2(5+h)-7-3}{h}\\ &= \frac{2h}{h}\\ &= 2\end{aligned}$$

35.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &=\\ \frac{(x^2+2xh+h^2)+3-x^2-3}{h} &=\\ \frac{2xh+h^2}{h} &=\\ 2x+h &=\end{aligned}$$

36.

$$\begin{aligned}\frac{g(x+h)-g(x)}{h} &= \frac{2(x+h)-7-2x+7}{h}\\ &= \frac{2h}{h}\\ &= 2\end{aligned}$$

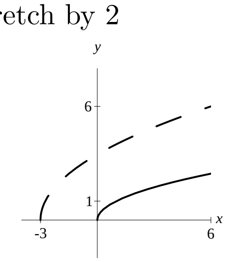
37. $g\left(\dfrac{x+7}{2}\right) = (x+7) - 7 = x$

38. $f\left(\sqrt{x-3}\right) = (x-3) + 3 = x$

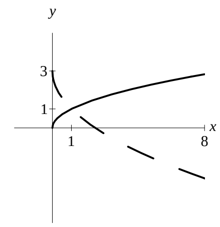
39. $g^{-1}(x) = \dfrac{x+7}{2}$

40. From number 39, $g^{-1}(-3) = \dfrac{-3+7}{2} = 2$

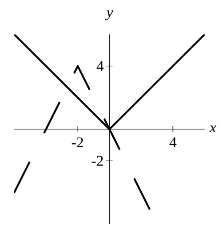
41. $f(x) = \sqrt{x}, g(x) = 2\sqrt{x+3}$; left by 3, stretch by 2



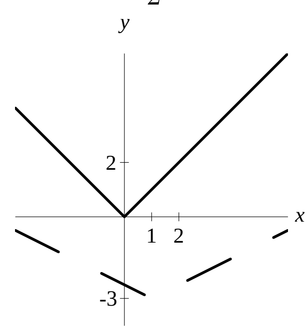
42. $f(x) = \sqrt{x}, g(x) = -2\sqrt{x} + 3$; stretch by 2, reflect about x-axis, up by 3



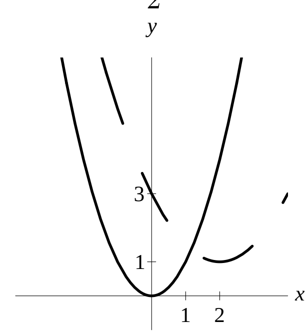
43. $f(x) = |x|, g(x) = -2|x+2| + 4$; left by 2, stretch by 2, reflect about x-axis, up by 4



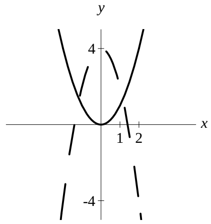
44. $f(x) = |x|, g(x) = \dfrac{1}{2}|x-1| - 3$; right by 1, stretch by $\dfrac{1}{2}$, down by 3



45. $f(x) = x^2, g(x) = \dfrac{1}{2}(x-2)^2 + 1$; right by 2, stretch by $\dfrac{1}{2}$, up by 1



46. $f(x) = x^2, g(x) = -2x^2 + 4$; stretch by 2, reflect about x-axis, up by 4



47. $(f \circ g)(x) = \sqrt{x} - 4$, domain $[0, \infty)$

48. $(f \circ g)(x) = \sqrt{x - 4}$, domain $[4, \infty)$

49. $(f \circ h)(x) = x^2 - 4$, domain $(-\infty, \infty)$

50. $(h \circ f)(x) = (x - 4)^2 = x^2 - 8x + 16$, domain $(-\infty, \infty)$

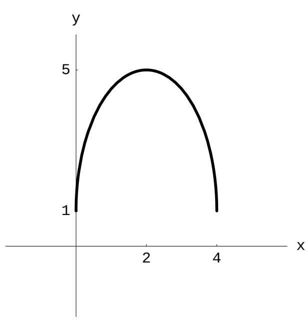
51. $(g \circ f \circ h)(x) = g(x^2 - 4) = \sqrt{x^2 - 4}$.

To find the domain, solve $x^2 - 4 \geq 0$. Then the domain is $[2, \infty) \cup (-\infty, -2]$

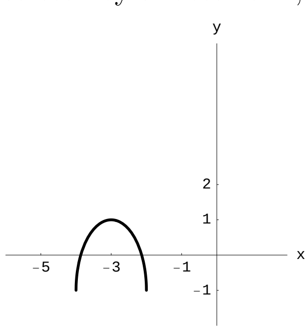
52. $(h \circ f \circ g)(x) = h(\sqrt{x} - 4) = (\sqrt{x} - 4)^2$.

Then $(h \circ f \circ g)(x) = x - 8\sqrt{x} + 16$. The domain is $[0, \infty)$.

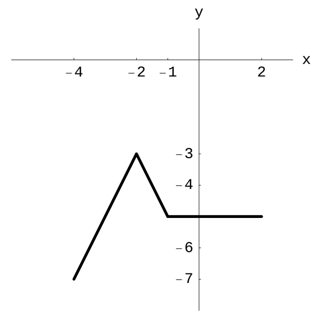
53. Translate the graph of f to the right by 2-units, stretch by a factor of 2, shift up by 1-unit.



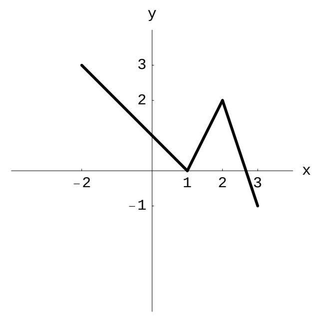
54. Translate the graph of f to the left by 3-units, stretch by a factor of 2, shift down by 1-unit.



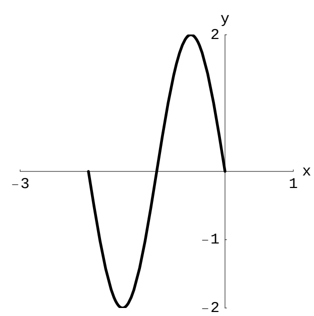
55. Translate the graph of f to the left by 1-unit, reflect about the x-axis, shift down by 3-units.



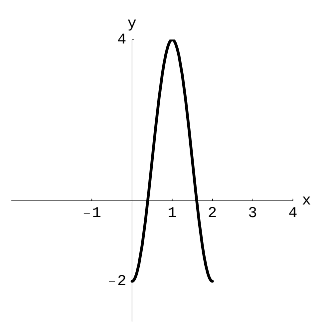
56. Translate the graph of f to the right by 1-unit, reflect about the x-axis, shift up by 2-units.



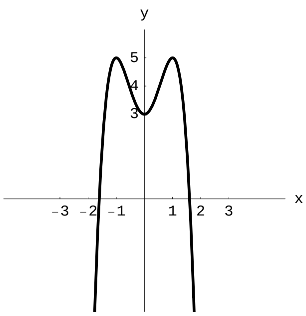
57. Translate the graph of f to the left by 2-units, stretch by a factor of 2, reflect about the x-axis.



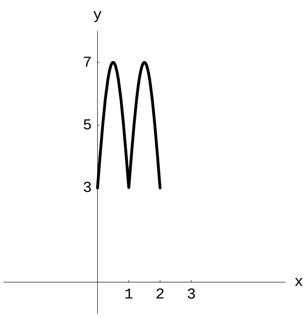
58. Stretch the graph of f by a factor of 3, reflect about the x-axis, shift up by 1-unit.



59. Stretch the graph of f by a factor of 2, reflect about the x-axis, shift up by 3-units.



60. Translate the graph of f to the right by 1-unit, stretch by a factor of 4, shift up by 3-units.



61. $F = f \circ g$ **62.** $G = g \circ f$

63. $H = f \circ h \circ g \circ j$ **64.** $M = j \circ h \circ g$

65. $N = h \circ f \circ j$ or $N = h \circ j \circ f$

66. $P = g \circ g \circ g \circ g \circ j$

67. $R = g \circ h \circ j$ **68.** $Q = j \circ g$

69.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{-5(x+h)+9+5x-9}{h} \\ &= \frac{-5h}{h} \\ &= -5 \end{aligned}$$

70.

$$\frac{f(x+h)-f(x)}{h} =$$

$$= \frac{\sqrt{x+h-7}-\sqrt{x-7}}{h} \cdot \frac{\sqrt{x+h-7}+\sqrt{x-7}}{\sqrt{x+h-7}+\sqrt{x-7}}$$

$$= \frac{(x+h-7)-(x-7)}{h\left(\sqrt{x+h-7}+\sqrt{x-7}\right)}$$

$$= \frac{1}{\sqrt{x+h-7}+\sqrt{x-7}}$$

71.

$$\frac{f(x+h)-f(x)}{h} =$$

$$= \frac{\frac{1}{2x+2h}-\frac{1}{2x}}{h} \cdot \frac{(2x+2h)(2x)}{(2x+2h)(2x)}$$

$$= \frac{(2x)-(2x+2h)}{h(2x+2h)(2x)}$$

$$= \frac{-2}{(2x+2h)(2x)}$$

$$= \frac{-1}{(x+h)(2x)}$$

72.

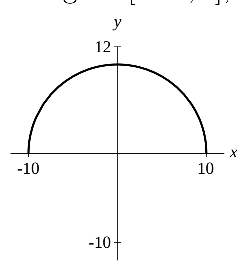
$$\frac{f(x+h)-f(x)}{h} =$$

$$= \frac{-5(x^2+2xh+h^2)+(x+h)+5x^2-x}{h}$$

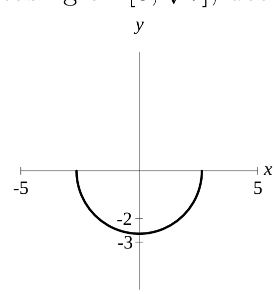
$$= \frac{-10xh-5h^2+h}{h}$$

$$= -10x-5h+1$$

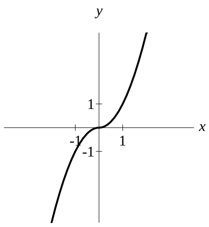
73. Domain is $[-10, 10]$, range is $[0, 10]$, increasing on $[-10, 0]$, decreasing on $[0, 10]$



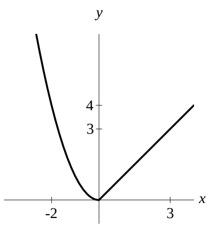
74. Domain is $[-\sqrt{7}, \sqrt{7}]$, range is $[-\sqrt{7}, 0]$, increasing on $[0, \sqrt{7}]$, decreasing on $[-\sqrt{7}, 0]$



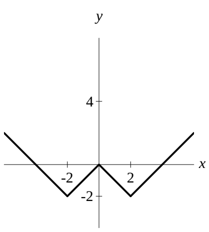
75. Domain and range are both $(-\infty, \infty)$, increasing on $(-\infty, \infty)$



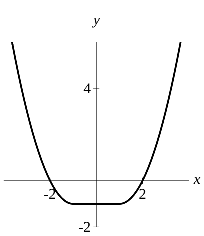
76. Domain $(-\infty, 4]$, range $[0, \infty)$, increasing on $[0, 4]$, decreasing on $(-\infty, 0]$



77. Domain is $(-\infty, \infty)$, range is $[-2, \infty)$, increasing on $[-2, 0]$ and $[2, \infty)$, decreasing on $(-\infty, -2]$ and $[0, 2]$



78. Domain is $(-\infty, \infty)$, range is $[-1, \infty)$, increasing on $[1, \infty)$, decreasing on $(-\infty, -1]$, constant on $[-1, 1]$



79. $y = |x| - 3$, domain is $(-\infty, \infty)$, range is $[-3, \infty)$

80. $y = |x - 3| - 1$, domain is $(-\infty, \infty)$, range is $[-1, \infty)$

81. $y = -2|x| + 4$, domain is $(-\infty, \infty)$, range is $(-\infty, 4]$

82. $y = -\frac{1}{2}|x| + 2$, domain is $(-\infty, \infty)$, range is $(-\infty, 2]$

83. $y = |x + 2| + 1$, domain is $(-\infty, \infty)$, range is $[1, \infty)$

84. $y = -|x + 1| - 2$, domain $(-\infty, \infty)$, range $(-\infty, -2]$

85. Symmetry: y-axis **86.** Symmetry: y-axis

87. Symmetric about the origin

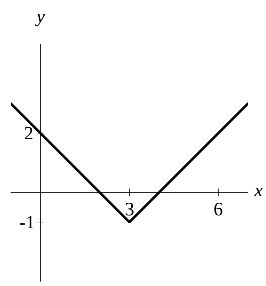
88. Symmetric about the origin

89. Neither symmetry **90.** Neither symmetry

91. Symmetric about the y-axis

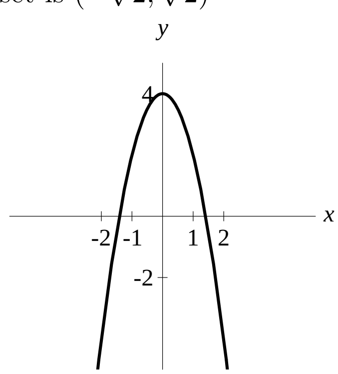
92. Symmetric about the y-axis

93. From the graph of $y = |x - 3| - 1$, the solution set is $(-\infty, 2] \cup [4, \infty)$

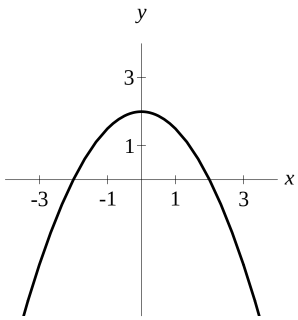


94. No solution since an absolute value is nonnegative

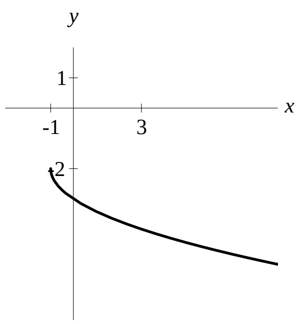
95. From the graph of $y = -2x^2 + 4$, the solution set is $(-\sqrt{2}, \sqrt{2})$



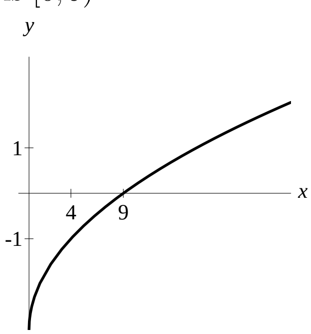
96. From the graph of $y = -\frac{1}{2}x^2 + 2$, the solution set is $(-\infty, -2] \cup [2, \infty)$



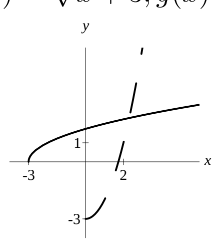
97. No solution since $-\sqrt{x+1} - 2 \leq -2$



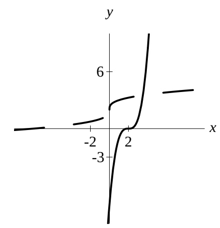
98. From the graph of $y = \sqrt{x} - 3$, the solution set is $[0, 9)$



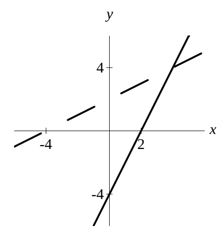
99. Inverse functions,
$f(x) = \sqrt{x+3}, g(x) = x^2 - 3$ for $x \geq 0$



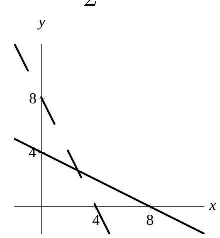
100. Inverse functions,
$f(x) = (x-2)^3, g(x) = \sqrt[3]{x} + 2$



101. Inverse functions,
$f(x) = 2x - 4, g(x) = \frac{1}{2}x + 2$



102. Inverse functions,
$f(x) = -\frac{1}{2}x + 4, g(x) = -2x + 8$



103. Not invertible, since there are two second components that are the same.

104. $f^{-1} = \{(1/3, -2), (1/4, -3), (1/5, -4)\}$ with domain $\{1/3, 1/4, 1/5\}$ and range $\{-2, -3, -4\}$

105. Inverse is $f^{-1}(x) = \frac{x+21}{3}$ with domain and range both $(-\infty, \infty)$

106. Not invertible **107.** Not invertible

108. Inverse is $f^{-1}(x) = 7 - x$ with domain and range both $(-\infty, \infty)$

109. Inverse is $f^{-1}(x) = x^2 + 9$ for $x \geq 0$ with domain $[0, \infty)$ and range $[9, \infty)$

110. Inverse is $f^{-1}(x) = (x+9)^2$ for $x \geq -9$ with domain $[-9, \infty)$ and range $[0, \infty)$

111. Inverse is $f^{-1}(x) = \frac{5x+7}{1-x}$ with domain $(-\infty, 1) \cup (1, \infty)$, and range $(-\infty, -5) \cup (-5, \infty)$

112. Inverse is $f^{-1}(x) = \frac{5x+3}{x+2}$ domain $(-\infty, -2) \cup (-2, \infty)$, and range $(-\infty, 5) \cup (5, \infty)$

113. Inverse is $f^{-1}(x) = -\sqrt{x-1}$ with domain $[1, \infty)$ and range $(-\infty, 0]$

114. Inverse is $f^{-1}(x) = \sqrt[4]{x} - 3$ for $x \geq 81$ with domain $[81, \infty)$, range $[0, \infty)$

115. Let x be the number of roses. The cost function is $C(x) = 1.20x + 40$, the revenue function is $R(x) = 2x$, and the profit function is $P(x) = R(x) - C(x)$ or $P(x) = 0.80x - 40$.

Since $P(50) = 0$, to make a profit she must sell at least 51 roses.

116. a) Since $V = \pi r^2 h$ and $V = 1$, we obtain $h = \frac{1}{\pi r^2}$.

b) Since $V = \pi r^2 h$ and $V = 1$, we get

$$\frac{1}{\pi h} = r^2 \text{ or } r = \frac{1}{\sqrt{\pi h}}.$$

c) From part a), we have $h = \frac{1}{\pi r^2}$.

Since $S = 2\pi r^2 + 2\pi r h$, we obtain

$$\begin{aligned} S &= 2\pi r^2 + 2\pi r \left(\frac{1}{\pi r^2}\right) \\ S &= 2\pi r^2 + \frac{2}{r}. \end{aligned}$$

117. Since $h(0) = 64$ and $h(2) = 0$, the range of $h = -16t^2 + 64$ is the interval $[0, 64]$. Then the domain of the inverse function is $[0, 64]$. Solving for t, we obtain

$$\begin{aligned} 16t^2 &= 64 - h \\ t^2 &= \frac{64-h}{16} \\ t &= \frac{\sqrt{64-h}}{4}. \end{aligned}$$

The inverse function is $t = \frac{\sqrt{64-h}}{4}$.

118. Solving for S in $T = 1.05S$, we obtain $S = \frac{T}{1.05}$.

119. Since $A = \pi \left(\frac{d}{2}\right)^2$, $d = 2\sqrt{\frac{A}{\pi}}$.

120. If A is the area of the square, then the length of one side is $\sqrt{A}$. Since one side of the square is twice the radius r then $\sqrt{A} = 2r$. Then $A = 4r^2$.

121. The average rate of change is $\frac{8-6}{4} = 0.5$ inch/lb.

122. The average rate of change is $\frac{130-40}{3} = 30$ mph/sec.

123. Since $D = kW$ and $9 = k \cdot 25$, we obtain $D = \frac{9}{25}100 = 36$.

124. Since $t = ku/v$ and $6 = k \cdot 8/2$, we find $k = 1.5$. Then $t = (1.5)(19)/3 = 19/2$.

125. Since $V = k\sqrt{h}$ and $45 = k\sqrt{1.5}$, the velocity of a Triceratops is $V = \frac{45}{\sqrt{1.5}} \cdot \sqrt{2.8} \approx 61$ kph.

126. Since $R = k/p$ and $21 = k/240$, we find $k = 5040$. If $p = 224$, then he needs $R = 5040/224 = 22.5$ rows.

127. Since $C = kd^2$ and $4.32 = k \cdot 36$, a 16-inch diameter globe costs $C = \frac{4.32}{36} \cdot 16^2 = \$30.72$.

128. $F = k\frac{m_1 m_2}{d^2}$ where m_1, m_2 are the masses and d is the distance between the centers of the objects.

130. No, it is not a function since there are two ordered pairs in a vertical line with the same first coordinate and different second coordinates.

Thinking Outside the Box

XXIX. The quadrilateral has vertices $A(0,0)$, $B(0,-9/2)$, $C(82/17,-15/17)$, and $D(7/2,0)$. The area of triangle ΔABC is

$$\frac{1}{2}\cdot\frac{9}{2}\cdot\frac{82}{17}=\frac{369}{34}$$

and the area of triangle ΔACD is

$$\frac{1}{2}\cdot\frac{7}{2}\cdot\frac{15}{17}=\frac{105}{68}.$$

The sum of areas of the two triangles is the area of the quadrilateral, i.e.,

$$\text{Area}=\frac{369}{34}+\frac{105}{68}=\frac{843}{68}\quad\text{square units.}$$

XXX. Note, the number of handshakes in a group with n people is

$$1+2+3+...+(n-1)=\frac{(n-1)n}{2}.$$

In the first delegation, the number of handshakes is $\frac{(n-1)n}{2}=190$ which implies that $n=20$, the number of members in the first delegation.

Since there were 480 handshakes between the first delegation and second delegation, the size of the second delegation is

$$\frac{480}{n}=\frac{480}{20}=24 \text{ delegates.}$$

Thus, the total number of delegates is $20+24$, or 44 delegates.

Chapter 2 Test

1. No, since $(0,5)$ and $(0,-5)$ are two ordered pairs with the same first coordinate and different second coordinates.

2. Yes, since to each x-coordinate there is exactly one y-coordinate, namely, $y=\frac{3x-20}{5}$.

3. No, since $(1,-1)$ and $(1,-3)$ are two ordered pairs with the same first coordinate and different second coordinates.

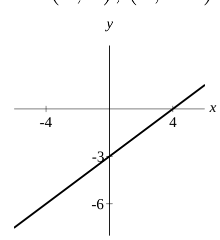
4. Yes, since to each x-coordinate there is exactly one y-coordinate, namely, $y=x^3-3x^2+2x-1$.

5. Domain is $\{2,5\}$, range is $\{-3,-4,7\}$

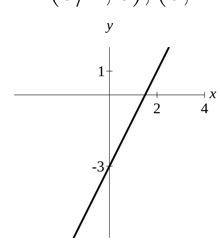
6. Domain is $[9,\infty)$, range is $[0,\infty)$

7. Domain is $[0,\infty)$, range is $(-\infty,\infty)$

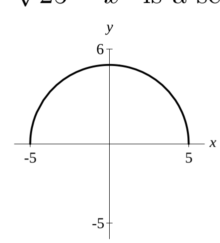
8. Graph of $3x-4y=12$ includes the points $(4,0),(0,-3)$



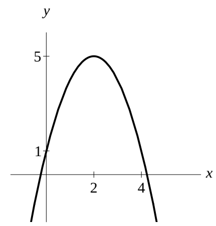
9. Graph of $y=2x-3$ includes the points $(3/2,0),(0,-3)$



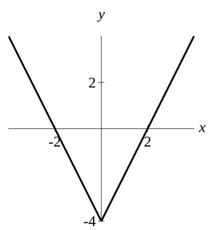
10. $y=\sqrt{25-x^2}$ is a semicircle with radius 5



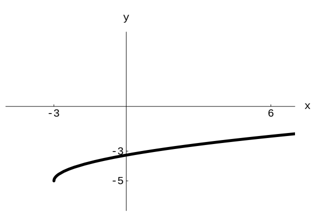
11. $y = -(x-2)^2 + 5$ is a parabola with vertex $(2, 5)$



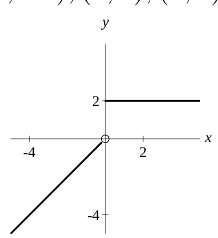
12. $y = 2|x| - 4$ includes the points $(0, -4), (\pm 3, 2)$



13. $y = \sqrt{x+3} - 5$ includes the points $(1, -3), (6, -2)$



14. Graph includes the points $(-2, -2), (0, 2), (3, 2)$



15. $\sqrt{9} = 3$ **16.** $f(5) = \sqrt{7}$

17. $f(3x-1) = \sqrt{(3x-1)+2} = \sqrt{3x+1}$.

18. $g^{-1}(x) = \dfrac{x+1}{3}$ **19.** $\sqrt{16} + 41 = 45$

20.

$$\begin{aligned}\frac{g(x+h) - g(x)}{h} &= \frac{3(x+h) - 1 - 3x + 1}{h} \\ &= \frac{3h}{h} \\ &= 3\end{aligned}$$

21. Increasing on $(3, \infty)$, decreasing on $(-\infty, 3)$

22. Symmetric about the y-axis

23. Add 1 to $-3 < x-1 < 3$ to obtain $-2 < x < 4$. Thus, the solution set $(-2, 4)$.

24. $f(x)$ is the composition of subtracting 2 from x, taking the cube root of the result, then adding 3. Reversing the operations, the inverse is $f^{-1}(x) = (x-3)^3 + 2$.

25. The range of $f(x) = \sqrt{x-5}$ is $[0, \infty)$. Then the domain of $f^{-1}(x)$ is $x \geq 0$ Note, $f(x)$ is the composition of subtracting 5 from x, then taking the square root of the result. Reversing the operations, the inverse is $f^{-1}(x) = x^2 + 5$ for $x \geq 0$.

26. $\dfrac{60-35}{200} = \$0.125$ per envelope

27. Since $I = k/d^2$ and $300 = k/4$, we get $k = 1200$. If $d = 10$, then $I = 1200/100 = 12$ candlepower.

28. Let s be the length of one side of the cube. By the Pythagorean Theorem we have $s^2 + s^2 = d^2$. Then $s = \dfrac{d}{\sqrt{2}}$ and the volume is $V = \left(\dfrac{d}{\sqrt{2}}\right)^3 = \dfrac{\sqrt{2}d^3}{4}$.

Tying It All Together

1. Add 3 to both sides of $2x - 3 = 0$ to obtain $2x = 3$. Thus, the solution set is $\left\{\dfrac{3}{2}\right\}$.

2. Add $2x$ to both sides of $-2x + 6 = 0$ to get $6 = 2x$. Then the solution set is $\{3\}$.

3. Note, $|x| = 100$ is equivalent to $x = \pm 100$. The solution set is $\{\pm 100\}$.

4. The equation is equivalent to $\dfrac{1}{2} = |x + 90|$. Then $x + 90 = \pm\dfrac{1}{2}$ and $x = \pm\dfrac{1}{2} - 90$. The solution set is $\{-90.5, -89.5\}$.

5. Note, $3 = 2\sqrt{x+30}$. If we divide by 2 and square both sides, we get $x + 30 = \dfrac{9}{4}$. Since $x = \dfrac{9}{4} - 30 = -27.75$, the solution set is $\{-27.75\}$.

6. Note, $\sqrt{x-3}$ is not a real number if $x < 3$. Since $\sqrt{x-3}$ is nonnegative for $x \geq 3$, it follows that $\sqrt{x-3} + 15$ is at least 15. In particular, $\sqrt{x-3} + 15 = 0$ has no real solution.

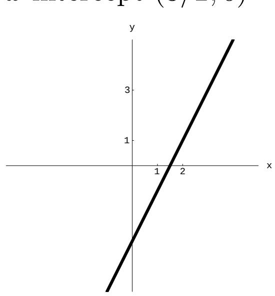
7. Rewriting, we obtain $(x-2)^2 = \dfrac{1}{2}$. By the square root property, $x - 2 = \pm\dfrac{\sqrt{2}}{2}$. Thus, $x = 2 \pm \dfrac{\sqrt{2}}{2}$. The solution set is $\left\{\dfrac{4 \pm \sqrt{2}}{2}\right\}$.

8. Rewriting, we get $(x+2)^2 = \dfrac{1}{4}$. By the square root property, $x + 2 = \pm\dfrac{1}{2}$. Thus, $x = -2 \pm \dfrac{1}{2}$. The solution set is $\left\{-\dfrac{3}{2}, -\dfrac{5}{2}\right\}$.

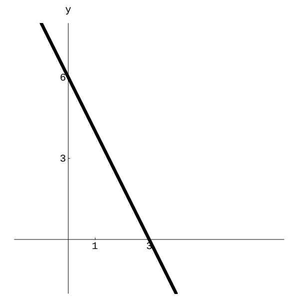
9. Squaring both sides of $\sqrt{9 - x^2} = 2$, we get $9 - x^2 = 4$. Then $5 = x^2$. The solution set is $\left\{\pm\sqrt{5}\right\}$.

10. Note, $\sqrt{49 - x^2}$ is not a real number if $49 - x^2 < 0$. Since $\sqrt{49 - x^2}$ is nonnegative for $49 - x^2 \geq 0$, we get that $\sqrt{49 - x^2} + 3$ is at least 3. In particular, $\sqrt{49 - x^2} + 3 = 0$ has no real solution.

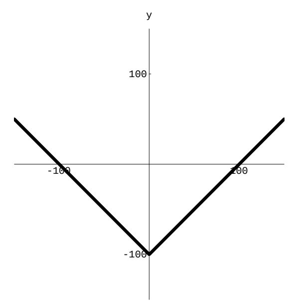
11. Domain is $(-\infty, \infty)$, range is $(-\infty, \infty)$, x-intercept $(3/2, 0)$



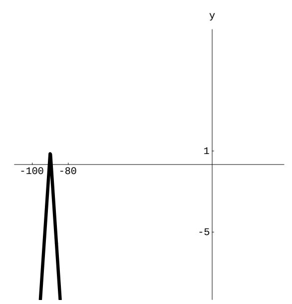
12. Domain is $(-\infty, \infty)$, range is $(-\infty, \infty)$, x-intercept $(3, 0)$



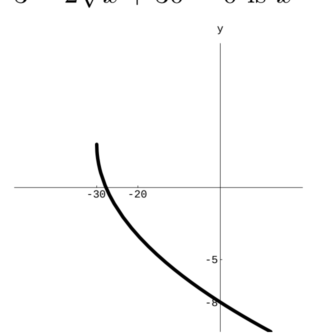
13. Domain is $(-\infty, \infty)$, range is $[-100, \infty)$, x-intercepts $(\pm 100, 0)$



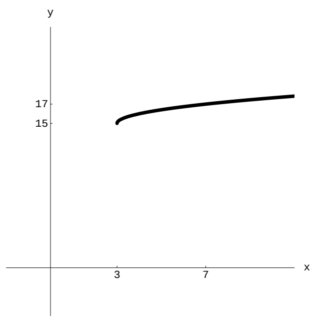
14. Domain is $(-\infty, \infty)$, range is $(-\infty, 1]$, x-intercepts $(-90.5, 0)$ and $(-89.5, 0)$.



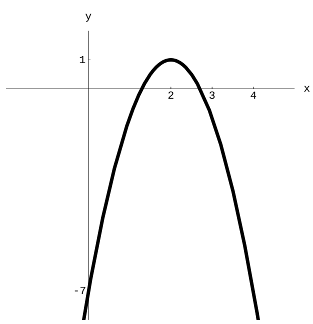
15. Domain is $[-30, \infty)$ since we need to require $x + 30 \geq 0$, range is $(-\infty, 3]$, x-intercept is $(-27.75, 0)$ since the solution to $3 - 2\sqrt{x+30} = 0$ is $x = -27.75$



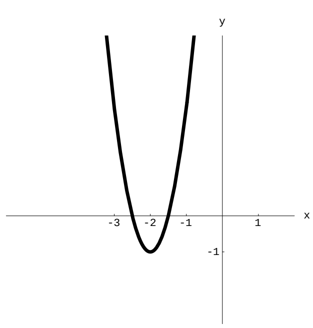
16. Domain is $[3, \infty)$ since we need to require $x - 3 \geq 0$, range is $[15, \infty)$, no x-intercept



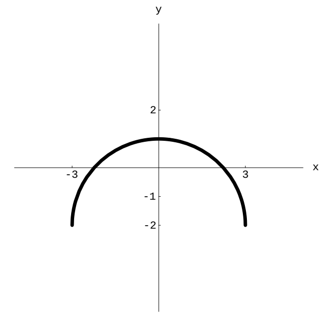
17. Domain is $(-\infty, \infty)$, range is $(-\infty, 1]$ since the vertex is $(2, 1)$, and the x-intercepts are $\left(\frac{4 \pm \sqrt{2}}{2}, 0\right)$ since the solutions of $0 = -2(x-2)^2 + 1$ are $x = \frac{4 \pm \sqrt{2}}{2}$.



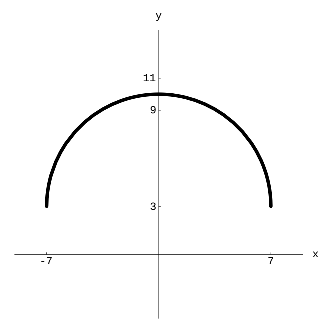
18. Domain is $(-\infty, \infty)$, range is $[-1, \infty)$ since the vertex is $(-2, -1)$, and the x-intercepts are $(-5/2, 0)$ and $(-3/2, 0)$ for the solutions to $0 = 4(x+2)^2 - 1$ are $x = -5/2, -3/2$.



19. Domain is $[-3, 3]$, range is $[-2, 1]$ since the graph is a semi-circle with center $(0, -2)$ and radius 3. The x-intercepts are $(\pm\sqrt{5}, 0)$ for the solutions to $0 = \sqrt{9 - x^2} - 2$ are $x = \pm\sqrt{5}$. The graph is shown in the next column.



20. Domain is $[-7, 7]$, range is $[3, 10]$ since the graph is a semi-circle with center $(0, 3)$ and radius 7, there are no x-intercepts as seen from the graph



21. Since $2x > 3$ or $x > 3/2$, the solution set is $(3/2, \infty)$.

22. Since $-2x \leq -6$ or $x \geq 3$, the solution set is $[3, \infty)$.

23. Based on the portion of the graph of $y = |x| - 100$ above the x-axis, and its x-intercepts, the solution set of $|x| - 100 \geq 0$ is $(-\infty, -100] \cup [100, \infty)$.

24. Based on the part of the graph of $y = 1 - 2|x + 90|$ above the x-axis, and its x-intercepts, the solution set of $1-2|x+90| > 0$ is $(-90.5, -89.5)$.

25. Based on the part of the graph of $y = 3 - 2\sqrt{x + 30}$ below the x-axis, and its x-intercepts, the solution set of $3 - 2\sqrt{x + 30} \leq 0$ is $[-27.75, \infty)$.

26. Solution set is $(-\infty, \infty)$ since the graph is entirely above the x-axis

27. Based on the portion of the graph of $y = -2(x-2)^2 + 1$ below the x-axis, and its x-intercepts, the solution set of $-2(x-2)^2+1<0$ is

$$\left(-\infty, \frac{4-\sqrt{2}}{2}\right) \cup \left(\frac{4+\sqrt{2}}{2}, \infty\right).$$

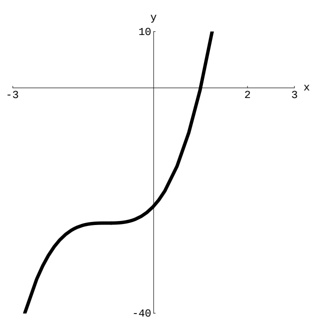
28. Based on the part of the graph of $y = 4(x+2)^2 - 1$ above the x-axis, and its x-intercepts, the solution set of $4(x+2)^2 - 1 < 0$ is $(-\infty, -5/2] \cup [-3/2, \infty)$.

29. Based on the part of the graph of $y = \sqrt{9-x^2} - 2$ above the x-axis, and its x-intercepts, the solution set of $\sqrt{9-x^2} - 2 \geq 0$ is $[-\sqrt{5}, \sqrt{5}]$.

30. Since the graph of $y = \sqrt{49-x^2} + 3$ is entirely above the line $y = 3$, the solution to $\sqrt{49-x^2} + 3 \leq 0$ is the empty set $\emptyset$.

31. $f(2) = 3(2+1)^3 - 24 = 3(27) - 24 = 57$

32. The graph of $f(x) = 3(x+1)^3 - 24$ is given.



33. $f = K \circ F \circ H \circ G$

34. Solving for x, we get

$$\begin{aligned} 3(x+1)^3 &= 24 \\ (x+1)^3 &= 8 \\ x+1 &= 2 \\ x &= 1. \end{aligned}$$

The solution set is $\{1\}$.

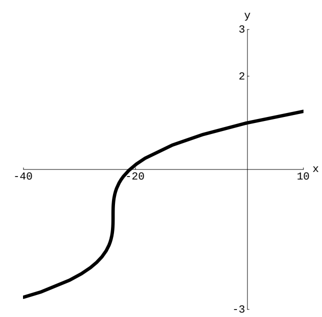
35. Based on the part of the graph of $y = 3(x+1)^3 - 24$ above the x-axis, and its x-intercept $(1, 0)$, the solution set of $3(x+1)^3 - 24 \geq 0$ is $[1, \infty)$.

36. Solving for x, we obtain

$$\begin{aligned} y + 24 &= 3(x+1)^3 \\ \frac{y+24}{3} &= (x+1)^3 \\ \sqrt[3]{\frac{y+24}{3}} &= x+1 \\ \sqrt[3]{\frac{y+24}{3}} - 1 &= x. \end{aligned}$$

37. Based on the answer from number 36, the inverse is $f^{-1}(x) = \sqrt[3]{\dfrac{x+24}{3}} - 1$

38. The graph of f^{-1} is given below.



39. Based on the part of the graph of f^{-1} above the x-axis, and its x-intercept $(-21, 0)$, the solution set of $f^{-1}(x) > 0$ is $(-21, \infty)$.

40. Since $f = K \circ F \circ H \circ G$, we get

$$f^{-1} = G^{-1} \circ H^{-1} \circ F^{-1} \circ K^{-1}.$$

Concepts of Calculus

1. a) Let $f(x) = x^2$.

$$\begin{aligned} \frac{f(2+h)-f(2)}{h} &= \frac{(2+h)^2 - 2^2}{h} \\ &= \frac{4+4h+h^2-4}{h} \\ &= \frac{4h+h^2}{h} \\ &= \frac{h(4+h)}{h} \\ \frac{f(2+h)-f(2)}{h} &= 4+h \end{aligned}$$

b) Note, $2+h$ gets closer and closer to 2 as h approaches 0. Using the results from part a), we conclude

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}(4+h) = 4.$$

2. a) Let $f(x)=x^2-2x$.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \\ \frac{\left((x+h)^2-2(x+h)\right)-\left(x^2-2x\right)}{h} &= \\ \frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h} &= \\ \frac{2xh+h^2-2h}{h} &= \\ 2x+h-2 &\end{aligned}$$

Thus, we obtain

$$\frac{f(x+h)-f(x)}{h}=2x+h-2.$$

b) Note, $2x+h-2$ gets closer and closer to $2x-2$ as h approaches 0. Then we obtain

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2x+h-2) = 2x-2.$$

Thus, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=2x-2$.

c) Based on part b), the instantaneous rate of change of $f(x)=x^2-2x$ is $2x-2$. And, when $x=5$ the instantaneous rate of change is $2(5)-2$ or 8.

3. a) Let $f(x)=\sqrt{x}$. Then we calculate the instantaneous rate of change of f.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \\ \frac{\sqrt{x+h}-\sqrt{x}}{h} &= \\ \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} &= \\ \frac{\left(\sqrt{x+h}\right)^2-\left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h}+\sqrt{x}\right)} &= \\ \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} &= \\ \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} &= \\ \frac{1}{\sqrt{x+h}+\sqrt{x}} &\end{aligned}$$

Thus, we obtain

$$\frac{f(x+h)-f(x)}{h}=\frac{1}{\sqrt{x+h}+\sqrt{x}}.$$

b) Note, as h approaches 0, we see that

$$\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

gets closer and closer to

$$\frac{1}{\sqrt{x+0}+\sqrt{x}} \quad \text{or} \quad \frac{1}{2\sqrt{x}}.$$

That is, by taking the limit, we have

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

c) Then the instantaneous rate of change of $f(x)=\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$. And, when $x=9$ the instantaneous rate of change

$$\frac{1}{2\sqrt{9}} \quad \text{or} \quad \frac{1}{6}.$$

4. a) Let $f(t)=-16t^2+128t$. Then

$$f(0)=-16(0)^2+128(0)=0$$

and

$$f(3)=-16(9)^2+128(3)=240.$$

b) The distance traveled in the first three seconds is

$$f(3)-f(0)=240-0=240 \text{ ft}.$$

c) The average rate of change of the height is

$$\frac{f(3)-f(0)}{3-0}=\frac{240 \text{ ft}}{3 \text{ sec}}=80 \text{ ft/sec}.$$

d) We calculate the difference quotient.

$$\begin{aligned}
\frac{f(t+h)-f(t)}{h} &= \\
\frac{\left(-16(t+h)^2+128(t+h)\right)-\left(-16t^2+128t\right)}{h} &= \\
\frac{\left(-16(t^2+2ht+h^2)+128t+128h\right)-\left(-16t^2+128t\right)}{h} &= \\
\frac{-32ht-16h^2-128h}{h} &= \\
-32t-16h-128 &=
\end{aligned}$$

Thus, we find

$$\frac{f(t+h)-f(t)}{h} = -32t-16h-128.$$

e) When h becomes closer and closer to 0, we obtain that

$$-32t-16h+128$$

gets closer and closer to

$$-32t+128.$$

Thus,

$$\lim_{h\to 0}\frac{f(t+h)-f(t)}{h} = -32t+128.$$

f) Note, based on the answer from part e), the instantaneous velocity of the ball at time t is

$$-32t+128 \quad \text{ft/sec.}$$

When $t=0$, the instantaneous velocity is

$$-32(0)+128 = 128 \quad \text{ft/sec.}$$

When $t=2$, the instantaneous velocity is

$$-32(2)+128 = 64 \quad \text{ft/sec.}$$

When $t=4$, the instantaneous velocity is

$$-32(4)+128 = 0 \quad \text{ft/sec.}$$

When $t=6$, the instantaneous velocity is

$$-32(6)+128 = -64 \quad \text{ft/sec.}$$

When $t=8$, the instantaneous velocity is

$$-32(8)+128 = -128 \quad \text{ft/sec.}$$

5. a) Let $f(x)=1/x$.

$$\begin{aligned}
\frac{f(x)-f(2)}{x-2} &= \frac{\frac{1}{x}-\frac{1}{2}}{x-2} \\
&= \frac{2-x}{2x(x-2)} \\
&= \frac{-1}{2x}
\end{aligned}$$

b) If x is close to 2, then

$$\begin{aligned}
\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} &= \lim_{x\to 2}\frac{-1}{2x} \\
&= \frac{-1}{2(2)}.
\end{aligned}$$

Thus, $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = -\frac{1}{4}$.

c) Let $h=x-c$. Note, h is close to 0 if x is near c, and conversely. Then

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c} = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}.$$

Thus, the instantaneous rate of change when $x=2$ is

$$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = -\frac{1}{4}$$

as shown in part b).

6. a) Let $f(x)=x^3$.

$$\begin{aligned}
\frac{f(x)-f(c)}{x-c} &= \frac{x^3-c^3}{x-c} \\
&= \frac{(x-c)(x^2+cx+c^2)}{x-c} \\
&= x^2+cx+c^2
\end{aligned}$$

b) If x is close to c, then

$$\begin{aligned}
\lim_{x\to c}\frac{f(x)-f(c)}{x-c} &= \lim_{x\to c}(x^2+cx+c^2) \\
&= c^2+c(c)+c^2.
\end{aligned}$$

Thus, $\lim_{x\to c}\frac{f(x)-f(c)}{x-c} = 3c^2$.

c) It is shown in 5c) that the instantaneous rate of change is a limit of an average rate of change as h approaches 0. Thus, the instantaneous rate of change when $x = c$ is

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = 3c^2$$

as shown in part b).