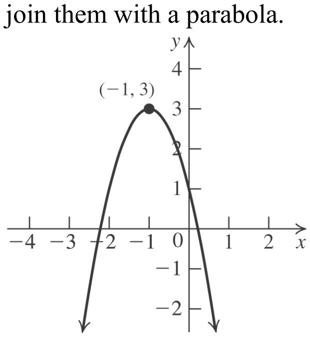
Chapter 2 Polynomial and Rational Functions

2.1 Quadratic Functions

2.1 Practice Problems

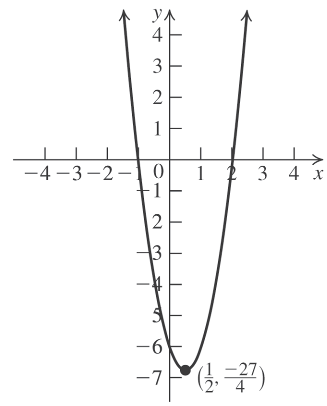
1. Substitute 1 for h, -5 for k, 3 for x, and 7 for y in the standard form for a quadratic equation to solve for a: $7 = a(3-1)^2 - 5 \Rightarrow 7 = 4a - 5 \Rightarrow$ $a = 3$. The equation is $y = 3(x-1)^2 - 5$. Since $a = 3 > 0$, f has a minimum value of -5 at $x = 1$.

2. The graph of $f(x) = -2(x+1)^2 + 3$ is a parabola with $a = -2$, $h = -1$ and $k = 3$. Thus, the vertex is $(-1, 3)$. The parabola opens down because $a < 0$. Now, find the x-intercepts:
$0 = -2(x+1)^2 + 3 \Rightarrow 2(x+1)^2 = 3 \Rightarrow$
$(x+1)^2 = \frac{3}{2} \Rightarrow x+1 = \pm\sqrt{\frac{3}{2}} \Rightarrow x = \pm\sqrt{\frac{3}{2}} - 1 \Rightarrow$
$x \approx 0.22$ or $x \approx -2.22$. Next, find the y-intercept: $f(0) = -2(0+1)^2 + 3 = 1$. Plot the vertex, the x-intercepts, and the y-intercept, and join them with a parabola.



3. The graph of $f(x) = 3x^2 - 3x - 6$ is a parabola with $a = 3$, $b = -3$ and $c = -6$. The parabola opens up because $a > 0$. Now, find the vertex:
$$h = -\frac{b}{2a} = -\frac{-3}{2(3)} = \frac{1}{2}$$
$$k = f(h) = f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) - 6 = -\frac{27}{4}$$
Thus, the vertex (h, k) is $\left(\frac{1}{2}, -\frac{27}{4}\right)$. Next, find the x-intercepts:
$3x^2 - 3x - 6 = 0 \Rightarrow 3(x^2 - x - 2) = 0 \Rightarrow$
$(x-2)(x+1) = 0 \Rightarrow x = 2$ or $x = -1$
Now, find the y-intercept:
$f(0) = 3(0)^2 - 3(0) - 6 = -6$.

Thus, the intercepts are $(-1, 0)$, $(2, 0)$ and $(0, -6)$. Use the fact that the parabola is symmetric with respect to its axis, $x = \frac{1}{2}$, to locate additional points. Plot the vertex, the x-intercepts, the y-intercept, and any additional points, and join them with a parabola.



4. The graph of $f(x) = 3x^2 - 6x - 1$ is a parabola with $a = 3$, $b = -6$ and $c = -1$. The parabola opens up because $a > 0$. Complete the square to write the equation in standard form:
$g(x) = 3x^2 - 6x - 1 \Rightarrow g(x) = 3(x^2 - 2x) - 1 \Rightarrow$
$g(x) = 3(x^2 - 2x + 1) - 1 - 3 = 3(x-1)^2 - 4$.
Thus, the vertex is $(-1, -4)$. The domain of f is $(-\infty, \infty)$ and the range is $[-4, \infty)$.
Next, find the x-intercepts:
$0 = 3(x-1)^2 - 4 \Rightarrow \frac{4}{3} = (x-1)^2 \Rightarrow$
$\pm\frac{2\sqrt{3}}{3} = x - 1 \Rightarrow 1 \pm \frac{2\sqrt{3}}{3} = x \Rightarrow x \approx 2.15$ or $x \approx -0.15$. Now, find the y-intercept: $f(0) = 3(0)^2 - 6(0) - 1 = -1$. Use the fact that the parabola is symmetric with respect to its axis, $x = -1$, to locate additional points. Plot the vertex, the x-intercepts, the y-intercept, and any additional points, and join them with a parabola.

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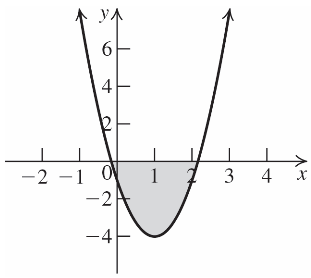


The graph of f is below the x-axis between the x-intercepts, so the solution set for

$g(x) = 3x^2 - 6x - 1 \le 0$ is $\left[1 - \frac{2\sqrt{3}}{3}, 1 + \frac{2\sqrt{3}}{3}\right]$

or $\left[\frac{3 - 2\sqrt{3}}{3}, \frac{3 + 2\sqrt{3}}{3}\right]$.

5. $h(t) = -\frac{g_E}{2}t^2 + v_0 t + h_0$

$g_E = 32 \text{ ft/s}^2$, $h_0 = 100$ ft, max height = 244 ft

a. Using the given values, we have

$$h(t) = -\frac{32}{2}t^2 + v_0 t + 100$$
$$= -16t^2 + v_0 t + 100$$

Using the formula for the vertex of a parabola gives $t = -\frac{v_0}{2(-16)} = \frac{v_0}{32}$. This is the time at which the maximum height $h(t) = 244$ ft is attained. Thus,

$$h(t) = 244 = h\left(\frac{v_0}{32}\right)$$
$$= -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) + 100$$
$$= -\frac{v_0^2}{64} + \frac{v_0^2}{32} + 100 = \frac{v_0^2}{64} + 100$$

Solving for v_0 yields

$$244 = \frac{v_0^2}{64} + 100 \Rightarrow 144 = \frac{v_0^2}{64} \Rightarrow$$
$$v_0^2 = 144 \cdot 64 \Rightarrow v_0 = \sqrt{144 \cdot 64} = 12 \cdot 8 = 96$$

Thus, $h(t) = -16t^2 + 96t + 100$.

b. Using the formula for the vertex of a parabola gives $t = -\frac{96}{2(-16)} = \frac{96}{32} = 3$.

The ball reached its highest point 3 seconds after it was released.

6. Let x = the length of the playground and y = the width of the playground.

Then $2(x + y) = 1000 \Rightarrow x + y = 500 \Rightarrow$
$y = 500 - x$.

The area of the playground is
$A(x) = xy = x(500 - x) = 500x - x^2$.

The vertex for the parabola is (h, k) where
$h = -\frac{500}{2(-1)} = 250$ and
$k = 500(250) - 250^2 = 62,500$.

Thus, the maximum area that can be enclosed is 62,500 ft². The playground is a square with side length 250 ft.

2.1 Exercises
Concepts and Vocabulary

1. The graph of $f(x) = ax^2 + bx + c,\ a \ne 0,$ is a <u>parabola</u>.

2. The vertex of the graph of $f(x) = a(x - h)^2 + k,\ a \ne 0,$ is <u>(h, k)</u>.

3. The graph of $f(x) = a(x - h)^2 + k,\ a \ne 0,$ is symmetric with respect to the vertical line <u>$x = h$</u>.

4. The x-coordinate of the vertex of $f(x) = ax^2 + bx + c,\ a \ne 0,$ is given by <u>$x = -\frac{b}{2a}$</u>.

5. True.

6. False. The graph of $f(x) = 2x^2$ is the graph of $g(x) = x^2$ compressed horizontally by a factor of 2.

7. False. If $a > 0$, then the parabola opens up and has a minimum.

8. True

Building Skills

9. f **10.** d **11.** a **12.** e

13. h **14.** b **15.** g **16.** c

17. Substitute −8 for y and 2 for x to solve for a: $-8 = a(2)^2 \Rightarrow -2 = a$. The equation is $y = -2x^2$.

18. Substitute 3 for y and –3 for x to solve for a: $3 = a(-3)^2 \Rightarrow \frac{1}{3} = a.$ The equation is $y = \frac{1}{3}x^2.$

19. Substitute 20 for y and 2 for x to solve for a: $20 = a(2)^2 \Rightarrow 5 = a.$ The equation is $y = 5x^2.$

20. Substitute –6 for y and –3 for x to solve for a: $-6 = a(-3)^2 \Rightarrow -\frac{2}{3} = a.$ The equation is $y = -\frac{2}{3}x^2.$

21. Substitute 0 for h, 0 for k, 8 for y, and –2 for x in the standard form for a quadratic equation to solve for a: $8 = a(-2-0)^2 + 0 \Rightarrow 8 = 4a \Rightarrow 2 = a.$ The equation is $y = 2x^2.$

22. Substitute 2 for h, 0 for k, 3 for y, and 1 for x in the standard form for a quadratic equation to solve for a: $3 = a(1-2)^2 + 0 \Rightarrow 3 = a.$ The equation is $y = 3(x-2)^2.$

23. Substitute –3 for h, 0 for k, –4 for y, and –5 for x in the standard form for a quadratic equation to solve for a: $-4 = a(-5-(-3))^2 + 0 \Rightarrow -1 = a.$ The equation is $y = -(x+3)^2.$

24. Substitute 0 for h, 1 for k, 0 for y, and –1 for x in the standard form for a quadratic equation to solve for a: $0 = a(-1-0)^2 + 1 \Rightarrow -1 = a.$ The equation is $y = -x^2 + 1.$

25. Substitute 2 for h, 5 for k, 7 for y, and 3 for x in the standard form for a quadratic equation to solve for a: $7 = a(3-2)^2 + 5 \Rightarrow 2 = a.$ The equation is $y = 2(x-2)^2 + 5.$

26. Substitute –3 for h, 4 for k, 0 for y, and 0 for x in the standard form for a quadratic equation to solve for a: $0 = a(0-(-3))^2 + 4 \Rightarrow -\frac{4}{9} = a.$ The equation is $y = -\frac{4}{9}(x+3)^2 + 4.$

27. Substitute 2 for h, –3 for k, 8 for y, and –5 for x in the standard form for a quadratic equation to solve for a: $8 = a(-5-2)^2 - 3 \Rightarrow \frac{11}{49} = a.$ The equation is $y = \frac{11}{49}(x-2)^2 - 3.$

28. Substitute –3 for h, –2 for k, –8 for y, and 0 for x in the standard form for a quadratic equation to solve for a: $-8 = a(0-(-3))^2 - 2 \Rightarrow -\frac{2}{3} = a.$ The equation is $y = -\frac{2}{3}(x+3)^2 - 2.$

29. Substitute $\frac{1}{2}$ for h, $\frac{1}{2}$ for k, $-\frac{1}{4}$ for y, and $\frac{3}{4}$ for x in the standard form for a quadratic equation to solve for a: $-\frac{1}{4} = a\left(\frac{3}{4} - \frac{1}{2}\right)^2 + \frac{1}{2} \Rightarrow -12 = a.$ The equation is $y = -12\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}.$

30. Substitute $-\frac{3}{2}$ for h, $-\frac{5}{2}$ for k, $\frac{55}{8}$ for y, and 1 for x in the standard form for a quadratic equation to solve for a: $\frac{55}{8} = a\left(1 - \left(-\frac{3}{2}\right)\right)^2 - \frac{5}{2} \Rightarrow \frac{3}{2} = a.$ The equation is $y = \frac{3}{2}\left(x + \frac{3}{2}\right)^2 - \frac{5}{2}.$

31. The vertex is (–2, 0), and the graph passes through (0, 3). Substitute −2 for h, 0 for k, 3 for y, and 0 for x in the standard form for a quadratic equation to solve for a: $3 = a(0-(-2))^2 + 0 \Rightarrow \frac{3}{4} = a.$ The equation is $y = \frac{3}{4}(x+2)^2.$

32. The vertex is (3, 0), and the graph passes through (0, 2). Substitute 3 for h, 0 for k, 2 for y, and 0 for x in the standard form for a quadratic equation to solve for a: $2 = a(0-3)^2 + 0 \Rightarrow \frac{2}{9} = a.$ The equation is $y = \frac{2}{9}(x-3)^2.$

33. The vertex is (3, –1), and the graph passes through (5, 2). Substitute 3 for h, –1 for k, 2 for y, and 5 for x in the standard form for a quadratic equation to solve for a: $2 = a(5-3)^2 - 1 \Rightarrow \frac{3}{4} = a.$ The equation is $y = \frac{3}{4}(x-3)^2 - 1.$

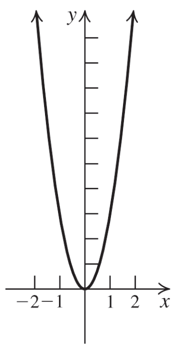
34. The vertex is (3, –1), and the graph passes through (0, 3). Substitute 3 for h, –1 for k, 3 for y, and 0 for x in the standard form for a quadratic equation to solve for a:

$$3 = a(0-3)^2 - 1 \Rightarrow 9a = 4 \Rightarrow a = \frac{4}{9}.$$

The equation is $y = \frac{4}{9}(x-3)^2 - 1.$

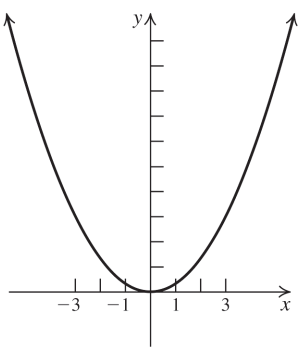
35. $f(x) = 3x^2$

Stretch the graph of $y = x^2$ vertically by a factor of 3.



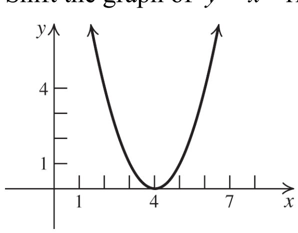
36. $f(x) = \frac{1}{3}x^2$

Compress the graph of $y = x^2$ vertically by a factor of $1/3$.



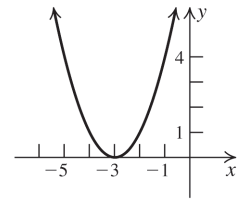
37. $g(x) = (x-4)^2$

Shift the graph of $y = x^2$ right four units.



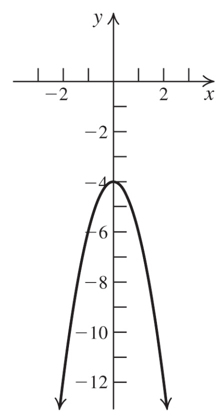
38. $g(x) = (x+3)^2$

Shift the graph of $y = x^2$ left three units.



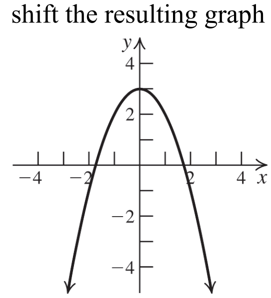
39. $f(x) = -2x^2 - 4$

Stretch the graph of $y = x^2$ vertically by a factor of 2, reflect the resulting graph in the x-axis, then shift the resulting graph down 4 units.



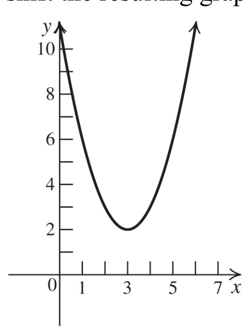
40. $f(x) = -x^2 + 3$

Reflect the graph of $y = x^2$ in the x-axis, then shift the resulting graph up 3 units.



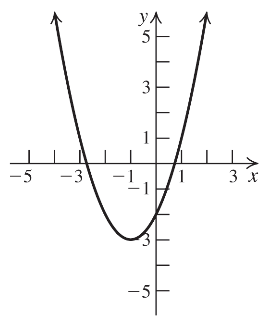
41. $g(x) = (x-3)^2 + 2$

Shift the graph of $y = x^2$ right three units, then shift the resulting graph up two units.



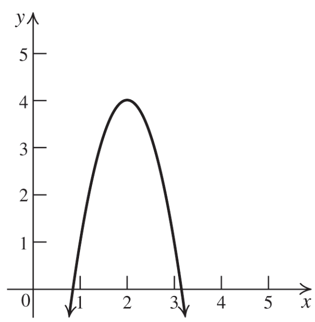
42. $g(x) = (x+1)^2 - 3$

Shift the graph of $y = x^2$ left one unit, then shift the resulting graph down three units.



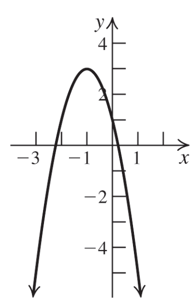
43. $f(x) = -3(x-2)^2 + 4$

Shift the graph of $y = x^2$ right two units, stretch the resulting graph vertically by a factor of 3, reflect the resulting graph about the x-axis, and then shift the resulting graph up four units.

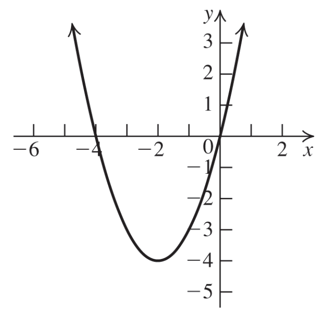


44. $f(x) = -2(x+1)^2 + 3$

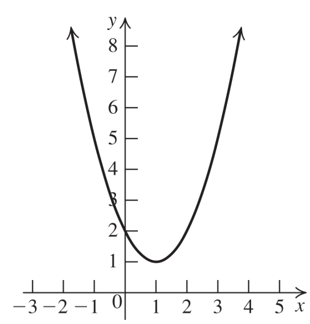
Shift the graph of $y = x^2$ left one unit, stretch the resulting graph vertically by a factor of 2, reflect the resulting graph about the x-axis, and then shift the resulting graph up three units.



45. Complete the square to write the equation in standard form: $y = x^2 + 4x \Rightarrow$ $y + 4 = x^2 + 4x + 4 \Rightarrow y = (x+2)^2 - 4.$ This is the graph of $y = x^2$ shifted two units left and four units down. The vertex is (–2, –4). The axis of symmetry is $x = -2.$ To find the x-intercepts, let $y = 0$ and solve $0 = (x+2)^2 - 4 \Rightarrow$ $(x+2)^2 = 4 \Rightarrow x + 2 = \pm 2 \Rightarrow x = -4$ or $x = 0.$ To find the y-intercept, let $x = 0$ and solve $y = (0+2)^2 - 4 \Rightarrow y = 0.$



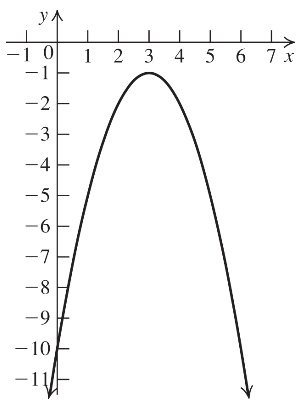
46. Complete the square to write the equation in standard form: $y = x^2 - 2x + 2 \Rightarrow$ $y + 1 = (x^2 - 2x + 1) + 2 \Rightarrow y = (x-1)^2 + 1.$ This is the graph of $y = x^2$ shifted one unit right and one unit up. The vertex is (1, 1). The axis of symmetry is $x = 1.$ To find the x-intercepts, let $y = 0$ and solve $0 = (x-1)^2 + 1 \Rightarrow$ $(x-1)^2 = -1 \Rightarrow$ there is no x-intercept. To find the y-intercept, let $x = 0$ and solve $y = (0-1)^2 + 1 \Rightarrow y = 2.$



47. Complete the square to write the equation in standard form:

$y = 6x - 10 - x^2 \Rightarrow y = -(x^2 - 6x + 10) \Rightarrow$
$y - 9 = -(x^2 - 6x + 9) - 10 \Rightarrow y = -(x-3)^2 - 1.$

This is the graph of $y = x^2$ shifted three units right, reflected about the x-axis, and then shifted one unit down. The vertex is (3, –1). The axis of symmetry is $x = 3$. To find the x-intercepts, let $y = 0$ and solve $0 = -(x-3)^2 - 1 \Rightarrow$ $-1 = (x-3)^2 \Rightarrow$ there is no x-intercept. To find the y-intercept, let $x = 0$ and solve $y = -(0-3)^2 - 1 \Rightarrow y = -10.$



48. Complete the square to write the equation in standard form:

$y = 8 + 3x - x^2 \Rightarrow y = -(x^2 - 3x - 8) \Rightarrow$

$y - \frac{9}{4} = -\left(x^2 - 3x + \frac{9}{4}\right) + 8 \Rightarrow$

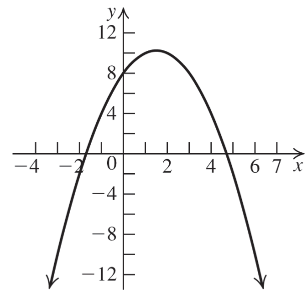
$y = -\left(x - \frac{3}{2}\right)^2 + \frac{41}{4}.$

This is the graph of $y = x^2$ shifted $3/2$ units right, reflected about the x-axis, and then shifted $\frac{41}{4}$ units up. The vertex is $\left(\frac{3}{2}, \frac{41}{4}\right)$. The axis of symmetry is $x = \frac{3}{2}$. To find the x-intercepts, let $y = 0$ and solve

$0 = -\left(x - \frac{3}{2}\right)^2 + \frac{41}{4} \Rightarrow \left(x - \frac{3}{2}\right)^2 = \frac{41}{4} \Rightarrow$

$x - \frac{3}{2} = \pm\frac{\sqrt{41}}{2} \Rightarrow x = \frac{3}{2} \pm \frac{1}{2}\sqrt{41}.$ To find the y-intercept, let $x = 0$ and solve

$y = -\left(0 - \frac{3}{2}\right)^2 + \frac{41}{4} \Rightarrow y = 8.$

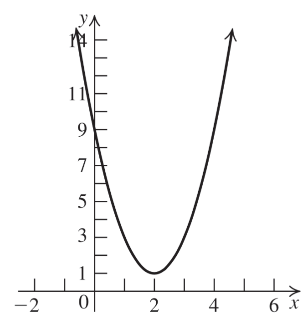


49. Complete the square to write the equation in standard form:

$y = 2x^2 - 8x + 9 \Rightarrow y = 2(x^2 - 4x) + 9 \Rightarrow$
$y + 8 = 2(x^2 - 4x + 4) + 9 \Rightarrow y = 2(x-2)^2 + 1.$

This is the graph of $y = x^2$ shifted 2 units right, stretched vertically by a factor of 2, and then shifted one unit up. The vertex is (2, 1). The axis of symmetry is $x = 2$. To find the x-intercepts, let $y = 0$ and solve

$0 = 2(x-2)^2 + 1 \Rightarrow -\frac{1}{2} = (x-2)^2 \Rightarrow$ there is no x-intercept. To find the y-intercept, let $x = 0$ and solve $y = 2(0-2)^2 + 1 \Rightarrow y = 9.$



50. Complete the square to write the equation in standard form:

$y = 3x^2 + 12x - 7 \Rightarrow y = 3(x^2 + 4x) - 7 \Rightarrow$
$y + 12 = 3(x^2 + 4x + 4) - 7 \Rightarrow y = 3(x+2)^2 - 19.$

This is the graph of $y = x^2$ shifted 2 units left, stretched vertically by a factor of 3, and then shifted 19 units down. The vertex is (–2, –19). The axis of symmetry is $x = -2$. To find the x-intercepts, let $y = 0$ and solve

$0 = 3(x+2)^2 - 19 \Rightarrow$

$\frac{19}{3} = (x+2)^2 \Rightarrow \pm\sqrt{\frac{19}{3}} = x + 2 \Rightarrow$

$\pm\frac{\sqrt{57}}{3} = x + 2 \Rightarrow -2 \pm \frac{1}{3}\sqrt{57} = x.$

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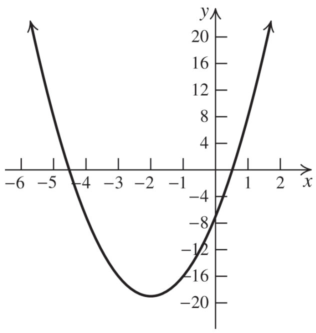
To find the y-intercept, let $x = 0$ and solve $y = 3(0+2)^2 - 19 \Rightarrow y = -7$.



51. Complete the square to write the equation in standard form:

$y = -3x^2 + 18x - 11 \Rightarrow y = -3(x^2 - 6x) - 11 \Rightarrow$

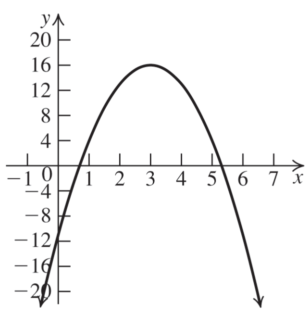
$y - 27 = -3(x^2 - 6x + 9) - 11 \Rightarrow$

$y = -3(x-3)^2 + 16$. This is the graph of $y = x^2$ shifted three units right, stretched vertically by a factor of three, reflected about the x-axis, and then shifted 16 units up. The vertex is (3, 16). The axis of symmetry is $x = 3$. To find the x-intercepts, let $y = 0$ and solve

$y = 0 = -3(x-3)^2 + 16 \Rightarrow$

$\frac{16}{3} = (x-3)^2 \Rightarrow \pm\frac{4\sqrt{3}}{3} = x - 3 \Rightarrow$

$3 \pm \frac{4}{3}\sqrt{3} = x$. To find the y-intercept, let $x = 0$ and solve $y = -3(0-3)^2 + 16 \Rightarrow y = -11$.



52. Complete the square to write the equation in standard form:

$y = -5x^2 - 20x + 13 \Rightarrow y = -5(x^2 + 4x) + 13 \Rightarrow$

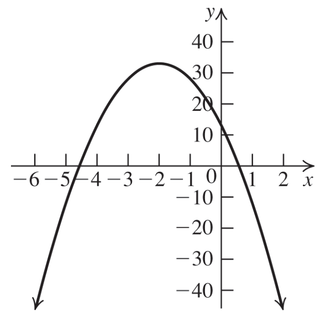
$y - 20 = -5(x^2 + 4x + 4) + 13 \Rightarrow$

$y = -5(x+2)^2 + 33$.

This is the graph of $y = x^2$ shifted two units left, stretched vertically by a factor of five, reflected about the x-axis, and then shifted 33 units up. The vertex is (–2, 33). The axis is $x = -2$. To find the x-intercepts, let $y = 0$ and solve $0 = -5(x+2)^2 + 33 \Rightarrow \frac{33}{5} = (x+2)^2 \Rightarrow$

$\pm\sqrt{\frac{33}{5}} = x + 2 \Rightarrow -2 \pm \frac{1}{5}\sqrt{165} = x$. To find the y-intercept, let $x = 0$ and solve

$y = -5(0+2)^2 + 33 \Rightarrow y = 13$.



53. **a.** $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\frac{-8}{2(1)}, f\left(-\frac{-8}{2(1)}\right)\right) = (4, -1)$.

c. The axis of symmetry is $x = 4$.

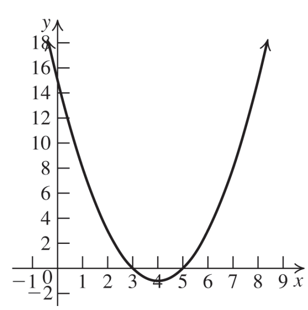
d. To find the x-intercepts, let $y = 0$ and solve

$0 = x^2 - 8x + 15 \Rightarrow 0 = (x-3)(x-5) \Rightarrow$

$x = 3$ or $x = 5$. To find the y-intercept, let $x = 0$ and solve

$y = 0^2 - 8(0) + 15 \Rightarrow y = 15$.

e.



54. **a.** $a = 1 > 0$, so the graph opens up.

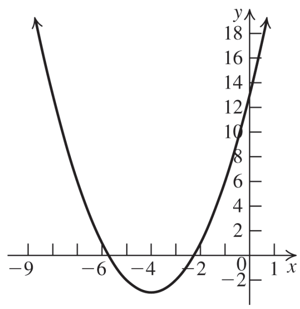
b. The vertex is

$\left(-\frac{8}{2(1)}, f\left(-\frac{8}{2(1)}\right)\right) = (-4, -3)$.

c. The axis of symmetry is $x = -4$.

d. To find the x-intercepts, let $y = 0$ and solve
$0 = x^2 + 8x + 13 \Rightarrow x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(13)}}{2(1)} \Rightarrow$
$x = \dfrac{-8 \pm \sqrt{12}}{2} = -4 \pm \sqrt{3}.$ To find the y-intercept, let $x = 0$ and solve
$y = 0^2 + 8(0) + 13 = 13.$

e.



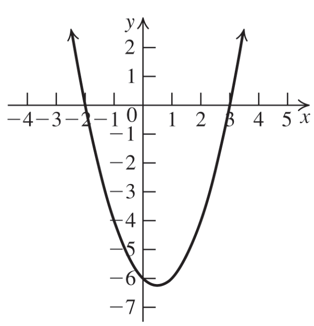
55. a. $a = 1 > 0$, so the graph opens up.

b. The vertex is
$\left(-\dfrac{-1}{2(1)}, f\left(-\dfrac{-1}{2(1)}\right)\right) = \left(\dfrac{1}{2}, -\dfrac{25}{4}\right).$

c. The axis of symmetry is $x = \dfrac{1}{2}.$

d. To find the x-intercepts, let $y = 0$ and solve
$0 = x^2 - x - 6 \Rightarrow 0 = (x-3)(x+2) \Rightarrow x = 3$ or $x = -2.$ To find the y-intercept, let $x = 0$ and solve $y = 0^2 - (0) - 6 \Rightarrow y = -6.$

e.



56. a. $a = 1 > 0$, so the graph opens up.

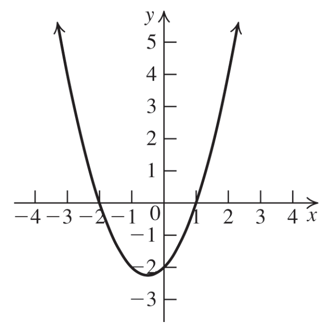
b. The vertex is
$\left(-\dfrac{1}{2(1)}, f\left(-\dfrac{1}{2(1)}\right)\right) = \left(-\dfrac{1}{2}, -\dfrac{9}{4}\right).$

c. The axis of symmetry is $x = -\dfrac{1}{2}.$

d. To find the x-intercepts, let $y = 0$ and solve
$0 = x^2 + x - 2 \Rightarrow 0 = (x-1)(x+2) \Rightarrow$
$x = 1$ or $x = -2.$

To find the y-intercept, let $x = 0$ and solve
$y = 0^2 + (0) - 2 \Rightarrow y = -2.$

e.



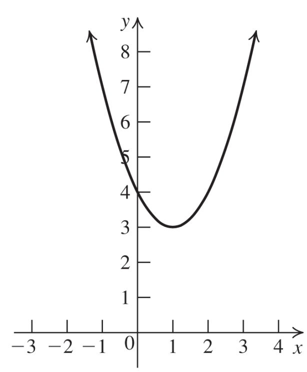
57. a. $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\dfrac{-2}{2(1)}, f\left(-\dfrac{-2}{2(1)}\right)\right) = (1, 3).$

c. The axis of symmetry is $x = 1.$

d. To find the x-intercepts, let $y = 0$ and solve
$0 = x^2 - 2x + 4 \Rightarrow$
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} \Rightarrow$
$x = \dfrac{2 \pm \sqrt{-12}}{2} \Rightarrow$ there are no x-intercepts.
To find the y-intercept, let $x = 0$ and solve
$y = 0^2 - 2(0) + 4 \Rightarrow y = 4.$

e.



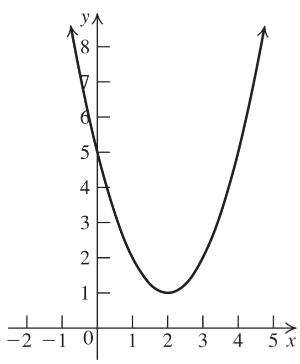
58. a. $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\dfrac{-4}{2(1)}, f\left(-\dfrac{-4}{2(1)}\right)\right) = (2, 1).$

c. The axis of symmetry is $x = 2.$

d. To find the x-intercepts, let $y = 0$ and solve
$0 = x^2 - 4x + 5 \Rightarrow$
$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \Rightarrow$
$x = \dfrac{4 \pm \sqrt{-4}}{2} \Rightarrow$ there are no x-intercepts. To find the y-intercept, let $x = 0$ and solve
$y = 0^2 - 4(0) + 5 \Rightarrow y = 5.$

e.



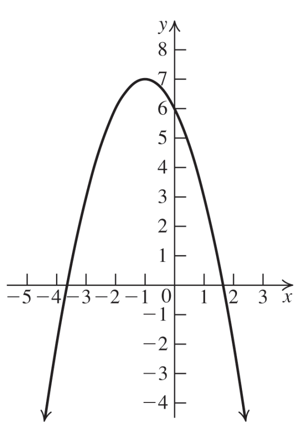
59. a. $a = -1 < 0$, so the graph opens down.

b. The vertex is
$\left(-\dfrac{-2}{2(-1)}, f\left(-\dfrac{-2}{2(-1)}\right)\right) = (-1, 7).$

c. The axis of symmetry is $x = -1$.

d. To find the x-intercepts, let $y = 0$ and solve
$0 = 6 - 2x - x^2 \Rightarrow$
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(6)}}{2(-1)} \Rightarrow x = \dfrac{2 \pm \sqrt{28}}{-2} \Rightarrow$
$x = -1 \pm \sqrt{7}.$
To find the y-intercept, let $x = 0$ and solve
$y = 6 - 2(0) - 0^2 \Rightarrow y = 6.$

e.



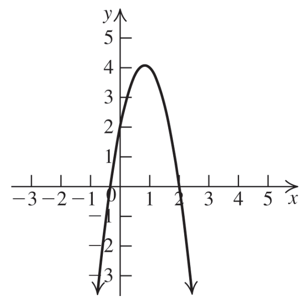
60. a. $a = -3 < 0$, so the graph opens down.

b. The vertex is
$\left(-\dfrac{5}{2(-3)}, f\left(-\dfrac{5}{2(-3)}\right)\right) = \left(\dfrac{5}{6}, \dfrac{49}{12}\right).$

c. The axis of symmetry is $x = \dfrac{5}{6}.$

d. To find the x-intercepts, let $y = 0$ and solve
$0 = 2 + 5x - 3x^2 \Rightarrow 0 = -(3x^2 - 5x - 2) \Rightarrow$
$0 = -(3x + 1)(x - 2) \Rightarrow x = -\dfrac{1}{3}$ or $x = 2.$ To find the y-intercept, let $x = 0$ and solve
$y = 2 + 5(0) - 3(0)^2 \Rightarrow y = 2.$

e.



61. a. $a = 1 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex:
$\left(-\dfrac{-4}{2(1)}, f\left(-\dfrac{-4}{2(1)}\right)\right) = (2, -1)$
The minimum value is –1.

b. The range of f is $[-1, \infty)$.

62. a. $a = -1 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex:
$\left(-\dfrac{6}{2(-1)}, f\left(-\dfrac{6}{2(-1)}\right)\right) = (3, 1)$
The maximum value is 1.

b. The range of f is $(-\infty, 1]$.

63. a. $a = -1 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex:
$\left(-\dfrac{4}{2(-1)}, f\left(-\dfrac{4}{2(-1)}\right)\right) = (2, 0)$
The maximum value is 0.

b. The range of f is $(-\infty, 0]$.

64. a. $a = 1 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex:
$$\left(-\frac{-6}{2(1)}, f\left(-\frac{-6}{2(1)}\right)\right) = (3, 0)$$
The minimum value is 0.

b. The range of f is $[0, \infty)$.

65. a. $a = 2 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex:
$$\left(-\frac{-8}{2(2)}, f\left(-\frac{-8}{2(2)}\right)\right) = (2, -5)$$
The minimum value is –5.

b. The range of f is $[-5, \infty)$.

66. a. $a = 3 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex:
$$\left(-\frac{12}{2(3)}, f\left(-\frac{12}{2(3)}\right)\right) = (-2, -17)$$
The minimum value is –17.

b. The range of f is $[-17, \infty)$.

67. a. $a = -4 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex:
$$\left(-\frac{12}{2(-4)}, f\left(-\frac{12}{2(-4)}\right)\right) = \left(\frac{3}{2}, 16\right)$$
The maximum value is 16.

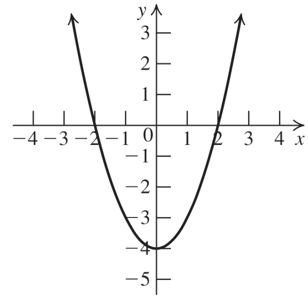
b. The range of f is $(-\infty, 16]$.

68. a. $a = -2 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex:
$$\left(-\frac{8}{2(-2)}, f\left(-\frac{8}{2(-2)}\right)\right) = (2, 3)$$
The maximum value is 3.

b. The range of f is $(-\infty, 3]$.

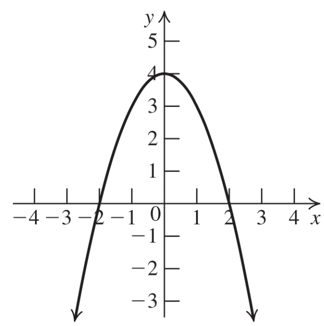
In exercises 69–82, the x-intercepts are the boundaries of the intervals.

69. $x^2 - 4 \le 0$
Vertex: (0, –4); x-intercepts: –2, 2



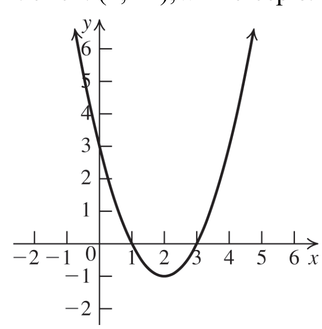
Solution: [–2, 2]

70. $4 - x^2 \ge 0$
Vertex: (0, 4); x-intercepts: –2, 2



Solution: [–2, 2]

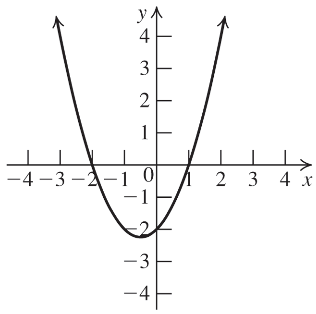
71. $x^2 - 4x + 3 < 0$
Vertex: (2, –1); x-intercepts: 1, 3



Solution: $(1, 3)$

72. $x^2 + x - 2 > 0$

Vertex: $\left(-\frac{1}{2}, -\frac{9}{4}\right)$, x-intercepts: –2, 1



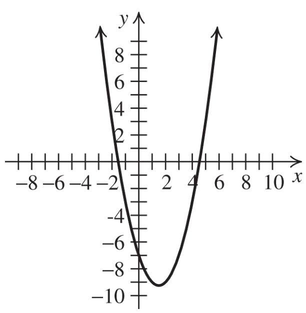
Solution: $(-\infty, -2) \cup (1, \infty)$

73. $x^2 - 3x - 7 \ge 0$

Vertex: $\left(\frac{3}{2}, -\frac{37}{4}\right)$

To find the x-intercepts, use the quadratic equation to solve $x^2 - 3x - 7 = 0$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-7)}}{2(1)} = \frac{3 \pm \sqrt{37}}{2}$$



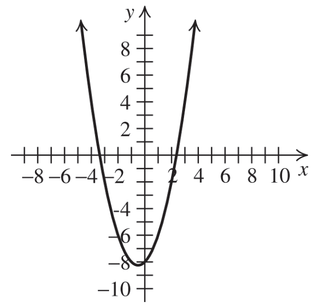
The solution of the inequality $x^2 - 3x - 7 \ge 0$ is $\left(-\infty, \frac{3}{2} - \frac{\sqrt{37}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{37}}{2}, \infty\right)$.

74. $x^2 + x - 8 \le 0$

Vertex: $\left(-\frac{1}{2}, -\frac{33}{4}\right)$

To find the x-intercepts, use the quadratic equation to solve $x^2 + x - 8 = 0$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)} = \frac{-1 \pm \sqrt{33}}{2}$$



The solution of the inequality $x^2 + x - 8 \le 0$ is $\left[-\frac{1}{2} - \frac{\sqrt{33}}{2}, -\frac{1}{2} + \frac{\sqrt{33}}{2}\right]$.

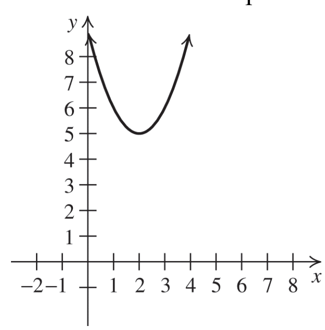
75. $x^2 - 4x + 9 \ge 0$

Vertex: (2, 5)

To find the x-intercepts, use the quadratic equation to solve $x^2 - 4x + 9 = 0$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(9)}}{2(1)} = \frac{4 \pm \sqrt{-20}}{2}$$

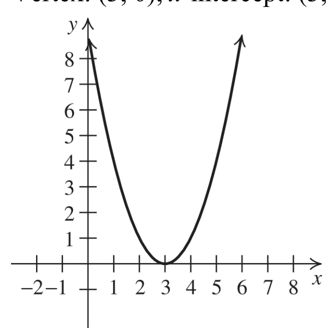
There are no x-intercepts.



Solution: $(-\infty, \infty)$

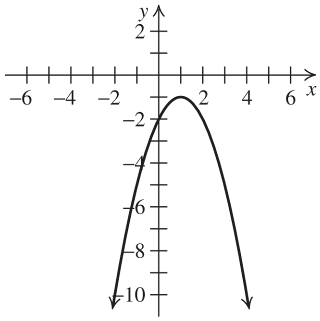
76. $x^2 - 6x + 9 \le 0$

Vertex: (3, 0); x-intercept: (3, 0)



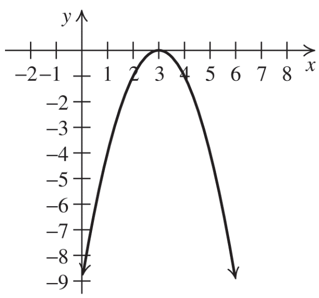
Solution: $\{3\}$

77. $-x^2+2x-2>0$
Vertex: (1,–1)



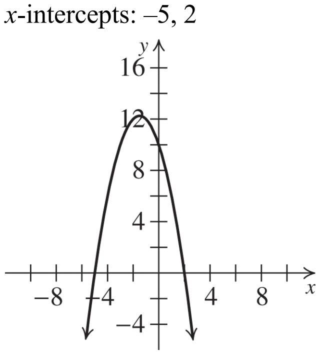
Solution: $\varnothing$

78. $-x^2+6x-9<0$
Vertex: (3, 0); x-intercept: (3, 0)



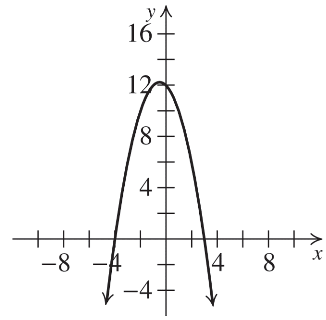
Solution: $(-\infty,-3)\cup(3,\infty)$

79. $-x^2-3x+10\geq 0$
Vertex: $\left(-\dfrac{3}{2},\dfrac{49}{4}\right)$
x-intercepts: –5, 2



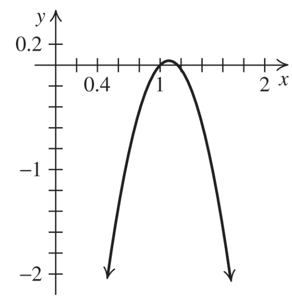
Solution: [–5, 2]

80. $-x^2-x+12\leq 0$
Vertex: $\left(-\dfrac{1}{2},\dfrac{49}{4}\right)$
x-intercepts: –4, 3



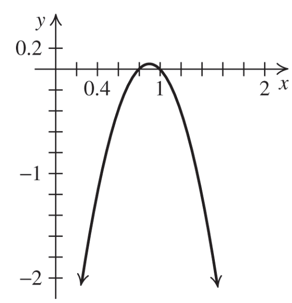
Solution: $(-\infty,-4]\cup[3,\infty)$

81. $-6x^2+13x-7<0$
Vertex: $\left(\dfrac{13}{12},\dfrac{1}{24}\right)$
x-intercepts: 1, $\dfrac{7}{6}$



Solution: $(-\infty,1)\cup\left(\dfrac{7}{6},\infty\right)$

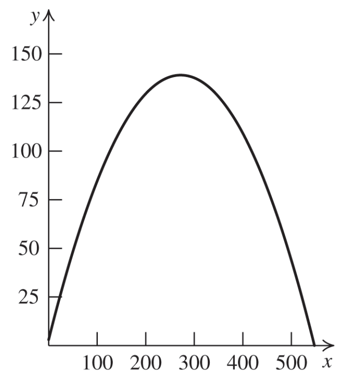
82. $-5x^2+9x-4>0$
Vertex: $\left(\dfrac{9}{10},\dfrac{1}{20}\right)$
x-intercepts: $\dfrac{4}{5}$, 1



Solution: $\left(\dfrac{4}{5},1\right)$

Applying the Concepts

83. $h(x) = -\dfrac{32}{132^2}x^2 + x + 3$



a. Using the graph, we see that the ball traveled approximately 550 ft horizontally.

b. Using the graph, we see that the ball went approximately 140 high.

c. $0 = -\dfrac{32}{132^2}x^2 + x + 3$

$$x = \frac{-1 \pm \sqrt{1^2 - 4\left(-\frac{32}{132^2}\right)(3)}}{2\left(-\frac{32}{132^2}\right)}$$
$$\approx -3 \text{ or } 547$$

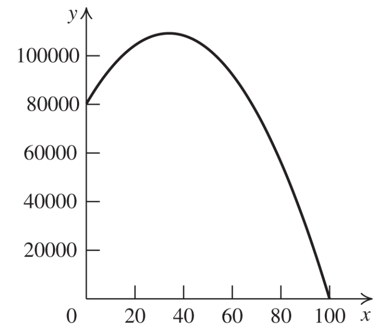
Thus, the ball traveled approximately 547 ft horizontally. The ball reached its maximum height at the vertex of the function, $\left(-\dfrac{b}{2a},\ h\left(-\dfrac{b}{2a}\right)\right)$.

$$-\frac{b}{2a} = -\frac{1}{2\left(-32/\left(132^2\right)\right)} \approx 272.25$$

$$h(272.25) = -\frac{32}{132^2}(272.25)^2 + 272.25 + 3$$
$$\approx 139$$

The ball reached approximately 139 ft

84. $R(x) = -25x^2 + 1700x + 80{,}000$



a. Using the graph, we see that the maximum revenue from the apartments is approximately \$110,000

b. Using the graph, we see that the maximum revenue is generated by about 35 \$25 increases.

c. The maximum revenue is at the vertex of the function, $\left(-\dfrac{b}{2a},\ R\left(-\dfrac{b}{2a}\right)\right)$.

$$-\frac{b}{2a} = -\frac{1700}{2(-25)} = 34$$

$$R(34) = -25(34)^2 + 1700(34) + 80{,}000$$
$$= 108{,}900$$

The maximum revenue of \$108,900 is reached at 34 \$25 increases.

85. The vertex of the function is $\left(-\dfrac{114}{2(-3)}, f\left(-\dfrac{114}{2(-3)}\right)\right) = (19, 1098)$.

The revenue is at its maximum when $x = 19$.

86. $p = 200 - 4x \Rightarrow R(x) = 200x - 4x^2$.

The vertex of the revenue function is $\left(-\dfrac{200}{2(-4)}, f\left(-\dfrac{200}{2(-4)}\right)\right) = (25, 2500)$.

The revenue is at its maximum when $x = 25$.

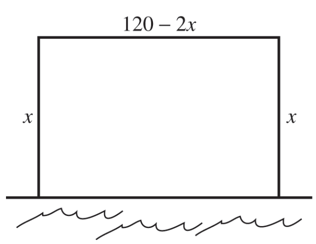
87. $\left(-\dfrac{-50}{2(1)}, f\left(-\dfrac{-50}{2(1)}\right)\right) = (25, -425)$.

The total cost is minimum when $x = 25$.

88. $p = 100 - x \Rightarrow R(x) = 100x - x^2$.
Profit = revenue – cost
$\Rightarrow P(x) = (100x - x^2) - (50 + 2x) \Rightarrow$
$P(x) = -x^2 + 98x - 50$. Now find the vertex of the profit function: $\left(-\frac{98}{2(-1)}, f\left(-\frac{98}{2(-1)}\right)\right) =$
$(49, 2351)$. So the maximum profit occurs when $x = 49$. The maximum profit is \$2351.

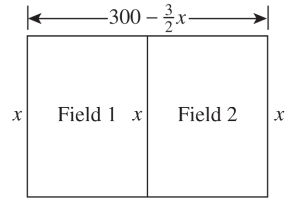
89. Let x = the length of the rectangle. Then $\frac{80 - 2x}{2} = 40 - x =$ the width of the rectangle.
The area of the rectangle = $x(40 - x)$
$= 40x - x^2$. Find the vertex to find the maximum value:
$\left(-\frac{40}{2(-1)}, f\left(-\frac{40}{2(-1)}\right)\right) = (20, 400)$.
The rectangle with the maximum area is a square with sides of length 20 units. Its area is 400 square units.

90. The fence encloses three sides of the region. Let x = the width of the region. Then $120 - 2x$ = the length of the region. The area of the region = $x(120 - 2x) = 120x - 2x^2$.



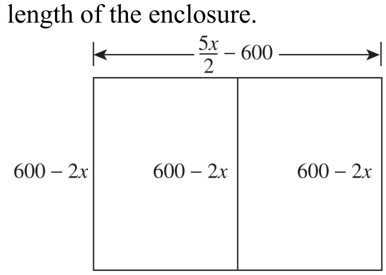
Find the vertex to find the maximum value:
$\left(-\frac{120}{2(-2)}, f\left(-\frac{120}{2(-2)}\right)\right) = (30, 1800)$.
The maximum area that can be enclosed is 1800 square meters.

91. Let x = the width of the fields. Then, $\frac{600 - 3x}{2} = 300 - \frac{3}{2}x =$ the length of the two fields together. (Note that there is fencing between the two fields, so there are three "widths.")



The total area = $x\left(300 - \frac{3}{2}x\right) = 300x - \frac{3}{2}x^2$.
Find the vertex to find the dimensions and maximum value:
$\left(-\frac{300}{2(-3/2)}, f\left(-\frac{300}{2(-3/2)}\right)\right) = (100, 15,000)$.
So the width of each field is 100 meters. The length of the two fields together is 300 – 1.5(100) = 150 meters, so the length of each field is 150/2 = 75 meters. The area of each field is 100(75) = 7500 square meters.

92. Let x = the amount of high fencing.
Then $8x$ = the cost of the high fencing. The cost of the low fencing = 2400 – $8x$, and
$\frac{2400 - 8x}{4} = 600 - 2x$ = the amount of low fencing. This is also the width of the enclosure.
So $\frac{x - 2(600 - 2x)}{2} = \frac{5x - 1200}{2} = \frac{5}{2}x - 600 =$ the length of the enclosure.



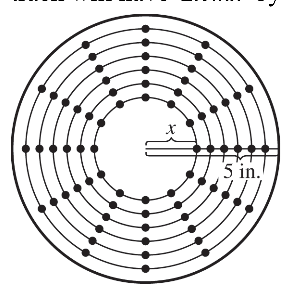
The area of the entire enclosure is
$(600 - 2x)\left(\frac{5}{2}x - 600\right) = -5x^2 + 2700x - 360,000$.
Use the vertex to find the dimensions and maximum area:
$\left(-\frac{2700}{2(-5)}, f\left(-\frac{2700}{2(-5)}\right)\right) = (270, 4500)$. So the area of the entire enclosure is 4500 square feet.
There are 270 feet of high fencing, so the dimensions of the enclosure are
$\frac{5(270)}{2} - 600 = 75$ feet by $600 - 2(270) = 60$ feet. The question asks for the dimensions and maximum area of each half of the enclosure, so each half has maximum area 2250 sq ft with dimensions 37.5 ft by 60 ft.

93. The yield per tree is modeled by the equation of a line passing through (26, 500) where the x-coordinate represents the number of trees planted, and the y-coordinate represents the number of apples per tree. The rate of change is –10; that is, for each tree planted the yield decreases by 10. So,
$y-500=-10(x-26)\Rightarrow y=-10x+760.$
Since there are x trees, the total yield = $x(-10x+760)=-10x^2+760x.$ Use the vertex to find the number of trees that will maximize the yield:
$$\left(-\frac{760}{2(-10)}, f\left(-\frac{760}{2(-10)}\right)\right)=(38,14,440).$$
So the maximum yield occurs when 38 trees are planted per acre.

94. Let x = the number of days. The original price is \$1.50 per pound and the price decreases \$0.02 per pound each day, so the price per pound is $(1.5 - 0.02x)$. The weight of the steer after x days is $300 + 8x$. So the selling price = number of pounds times price per pound $=(300+8x)(1.5-0.02x).$ The original cost of the steer is 1.5(300) = \$450, and the daily cost of the steer is x, so the total cost of the steer after x days is $x + 450$. The profit is selling price – cost
$$\begin{aligned}&=(300+8x)(1.5-0.02x)-(x+450)\\&=(-0.16x^2+6x+450)-(x+450)\\&=-0.16x^2+5x.\end{aligned}$$
The maximum profit occurs at the x-coordinate of the vertex: $\left(-\frac{5}{2(-0.16)}\right)=15.625.$
The maximum profit occurs after 16 days.

95. If 20 students or less go on the trip, the cost is \$72 per students. If more than 20 students go on the trip, the cost is reduced by \$2 per the number of students over 20. So the cost per student is a piecewise function based on the number of students, n, going on the trip:
$$f(n)=\begin{cases}72 & \text{if } n\le 20\\ 72-2(n-20)=112-2n & \text{if } n>20\end{cases}$$
The total revenue is
$$nf(n)=\begin{cases}72n & \text{if } n\le 20\\ n(112-2n)=112n-2n^2 & \text{if } n>20\end{cases}$$
The maximum revenue is either 1440 (the revenue if 20 students go on the trip) or the maximum of $112n-2n^2.$ Find this by using the vertex: $\left(-\frac{112}{(2)(-2)}, f\left(-\frac{112}{(2)(-2)}\right)\right)=(28,1568).$
The maximum revenue is \$1568 when 28 students go on the trip.

96. Assume that m bytes per inch can be put on any track. Let x = the radius of the innermost track. Then the maximum number of bytes that can be put on the innermost track is $2\pi mx.$ So, each track will have $2\pi mx$ bytes.



The total number of bytes on the disk is the number of bytes on each track times the number of tracks per inch times the radius (in inches):
$2\pi mx(p(5-x))=2\pi mp(5x-x^2)$
$=-2\pi mpx^2+10\pi mpx$. The maximum occurs at the x-coordinate of the vertex:
$$-\frac{b}{2a}=-\frac{10\pi mp}{2(-2\pi mp)}=\frac{10}{4}=2.5 \text{ inches.}$$

97. $h(t)=-\frac{g_M}{2}t^2+v_0t+h_0$
$g_M=1.6\,\text{m/s}^2$, $h_0=5$ m, max height = 25 m

a. Using the given values, we have
$$h(t)=-\frac{1.6}{2}t^2+v_0t+5=-0.8t^2+v_0t+5$$
Using the formula for the vertex of a parabola gives $t=-\frac{v_0}{2(-0.8)}=\frac{v_0}{1.6}.$ This is the time at which the height $h(t)$ = 25 m is attained. Thus,
$$\begin{aligned}h(t)=25&=h\left(\frac{v_0}{1.6}\right)\\&=-0.8\left(\frac{v_0}{1.6}\right)^2+v_0\left(\frac{v_0}{1.6}\right)+5\\&=-\frac{v_0{}^2}{3.2}+\frac{v_0{}^2}{1.6}+5=\frac{v_0{}^2}{3.2}+5\end{aligned}$$

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Solving for v_0 yields

$$25 = \frac{v_0^2}{3.2} + 5 \Rightarrow 20 = \frac{v_0^2}{3.2} \Rightarrow$$
$$v_0^2 = 64 \Rightarrow v_0 = 8$$

Thus, $h(t) = -0.8t^2 + 8t + 5.$

b. Using the formula for the vertex of a parabola gives $t = -\frac{8}{2(-0.8)} = \frac{8}{1.6} = 5.$

The ball reached its highest point 5 seconds after it was released.

98. $h(t) = -\frac{g_M}{2}t^2 + v_0 t + h_0$

$g_M = 1.6\,\text{m}/\text{s}^2$, $h_0 = 7$ m, max height = 52 m

a. Using the given values, we have

$$h(t) = -\frac{1.6}{2}t^2 + v_0 t + 7 = -0.8t^2 + v_0 t + 7$$

Using the formula for the vertex of a parabola gives $t = -\frac{v_0}{2(-0.8)} = \frac{v_0}{1.6}$. This is the time at which the height $h(t) = 52$ m is attained. Thus,

$$\begin{aligned} h(t) &= 52 = h\left(\frac{v_0}{1.6}\right) \\ &= -0.8\left(\frac{v_0}{1.6}\right)^2 + v_0\left(\frac{v_0}{1.6}\right) + 7 \\ &= -\frac{v_0^2}{3.2} + \frac{v_0^2}{1.6} + 7 = \frac{v_0^2}{3.2} + 7 \end{aligned}$$

Solving for v_0 yields

$$52 = \frac{v_0^2}{3.2} + 7 \Rightarrow 45 = \frac{v_0^2}{3.2} \Rightarrow$$
$$v_0^2 = 144 \Rightarrow v_0 = 12$$

Thus, $h(t) = -0.8t^2 + 12t + 7.$

b. Using the formula for the vertex of a parabola gives $t = -\frac{12}{2(-0.8)} = \frac{12}{1.6} = 7.5.$

The ball reached its highest point 7.5 seconds after it was released.

99. a. The maximum height occurs at the vertex: $\left(-\frac{64}{2(-16)}, f\left(-\frac{64}{2(-16)}\right)\right) = (2, 64),$ so the maximum height is 64 feet.

b. When the projectile hits the ground, $h = 0$, so solve

$$0 = -16t^2 + 64t \Rightarrow -16t(t-4) = 0 \Rightarrow$$
$t = 0$ or $t = 4$.

The projectile hits the ground at 4 seconds.

100. a. The maximum height occurs at the vertex:

$$\left(-\frac{64}{2(-16)}, f\left(-\frac{64}{2(-16)}\right)\right) = (2, 464).$$

The maximum height is 464 feet.

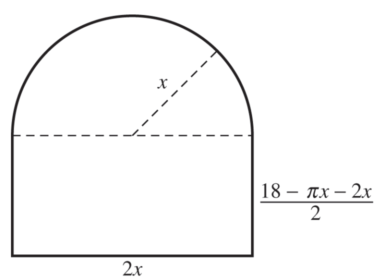
b. When the projectile hits the ground, $h = 0$, so solve $0 = -16t^2 + 64t + 400 \Rightarrow$

$$-16(t^2 - 4t - 25) = 0 \Rightarrow t^2 - 4t - 25 = 0 \Rightarrow$$
$$t = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-25)}}{2(1)} \Rightarrow t = \frac{4 \pm \sqrt{116}}{2} \Rightarrow$$

$t = 2 \pm \sqrt{29} \Rightarrow t \approx -3.39$ or $t \approx 7.39.$ Reject the negative solution (time cannot be negative). The projectile hits the ground at 7.39 seconds.

101. Let x = the radius of the semicircle. Then the length of the rectangle is $2x$. The circumference of the semicircle is πx, so the perimeter of the rectangular portion of the window is $18 - \pi x$.

The width of the rectangle = $\frac{18 - \pi x - 2x}{2}$.



The area of the semicircle is $\pi x^2/2$, and the area of the rectangle is $2x\left(\frac{18 - \pi x - 2x}{2}\right) =$ $18x - \pi x^2 - 2x^2$. So the total area is

$$18x - \pi x^2 - 2x^2 + \frac{\pi x^2}{2} = 18x - 2x^2 - \frac{\pi x^2}{2} \Rightarrow$$
$$18x - \left(2 + \frac{\pi}{2}\right)x^2.$$

The maximum area occurs at the x-coordinate of the vertex: $-\frac{18}{2\left(-2 - \frac{\pi}{2}\right)} = -\frac{18}{-4 - \pi} = \frac{18}{4 + \pi}.$

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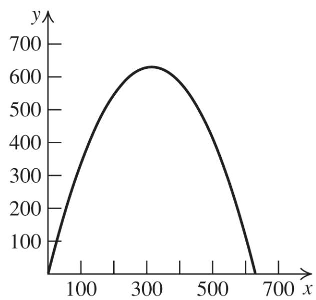
This is the radius of the semicircle. The length of the rectangle is $2\left(\frac{18}{4+\pi}\right)=\frac{36}{4+\pi}$ ft. The width of the rectangle is

$$\frac{18-\pi\left(\frac{18}{4+\pi}\right)-2\left(\frac{18}{4+\pi}\right)}{2}=\frac{18}{4+\pi}\text{ ft.}$$

102. a.



b. The width of the arch is the difference between the two x-intercepts:

$0=-0.00635x^2+4x\Rightarrow$

$0=4x(-0.0015875x+1)\Rightarrow x=0$ or

$x\approx 629.92.$ The arch is 629.92 feet wide.

c. The maximum height occurs at the vertex of the arch. Since we know that the arch is 629.92 feet wide, the vertex occurs at $x = 629.92/2 = 314.96.$

$f(314.96)=-0.00635(314.96)^2+4(314.96)$
$=629.92$ feet.

103. a. $V=\left(-\frac{1.18}{2(-0.01)},f\left(-\frac{1.18}{2(-0.01)}\right)\right)$
$=(59,36.81)$

b. $0=-0.01x^2+1.18x+2\Rightarrow$

$$x=\frac{-1.18\pm\sqrt{1.18^2-4(-0.01)(2)}}{2(-0.01)}$$
$$=\frac{-1.18\pm\sqrt{1.4724}}{-0.02}\approx\frac{-1.18\pm 1.21}{-0.02}$$

≈ -1.5 or 119.5. Reject the negative answer and round the positive answer to the nearest whole number. The ball hits the ground approximately 120 feet from the punter.

c. The maximum height occurs at the vertex. Since we know that the ball hits the ground at $x\approx 120,$ the vertex occurs at $x\approx 60.$

$f(60)=-0.01(60)^2+1.18(60)+2=36.8.$

The maximum height is approximately 37 ft.

d. The player is at $x = 6$ feet.

$f(6)=-0.01(6)^2+1.18(6)+2=8.72.$

The player must reach approximately 9 feet to block the ball.

e. $7=-0.01x^2+1.18x+2$

$0=-0.01x^2+1.18x-5$

$$x=\frac{-1.18\pm\sqrt{1.18^2-4(-0.01)(-5)}}{2(-0.01)}$$
$$=\frac{-1.18\pm\sqrt{1.1924}}{-0.02}\Rightarrow$$

$x\approx 4.4$ feet or $x\approx 113.6$ feet

104. The maximum height occurs at the vertex:

$$\left(-\frac{30}{2(-16)},f\left(-\frac{30}{2(-16)}\right)\right)=\left(\frac{15}{16},\frac{225}{16}\right).$$

The maximum height is approximately 14 feet, so it will never reach a height of 16 feet.

105. $f(x)=1.51-0.22x+0.029x^2$

a. We must complete the square to write the function in standard form.

$$0.029x^2-0.22x+1.51$$
$$=0.029\left(x^2-\frac{0.22}{0.029}x\right)+1.51$$
$$=0.029\left(x^2-\frac{0.22}{0.029}x+\frac{0.22^2}{4\cdot 0.029^2}\right)$$
$$+1.51-\left(\frac{0.22^2}{4\cdot 0.029^2}\right)(0.029)$$
$$=0.029\left(x-\frac{0.22}{2\cdot 0.029}\right)^2+1.09$$
$$=0.029(x-3.79)^2+1.09$$

b. Sales were at a minimum at $x\approx 3.79,$ or during year 4 (2010). This fits the original data.

c. The year 2011 is represented by $x = 5$.

$f(5)=1.51-0.22(5)+0.029(5)^2=1.135$

In 2011, there were about 1.14 million vehicles sold.

106. $f(x) = 0.822 + 0.896x - 0.11x^2$

a. We must complete the square to write the function in standard form.

$$\begin{aligned}&-0.11x^2 + 0.896x + 0.822\\&= -0.11\left(x^2 - \frac{0.896}{0.11}x\right) + 0.822\\&= -0.11\left(x^2 - \frac{0.896}{0.11}x + \frac{0.896^2}{4\cdot 0.11^2}\right)\\&\qquad + 0.822 - \left(\frac{0.896^2}{4\cdot 0.11^2}\right)(-0.11)\\&= -0.11\left(x - \frac{0.896}{2\cdot 0.11}\right)^2 + 2.65\\&= -0.11(x - 4.07)^2 + 2.65\end{aligned}$$

b. Foreclosure filings were at a maximum during year 4, 2010. This fits the original data.

c. The year 2013 is represented by $x = 7$.

$f(7) = 0.822 + 0.896(7) - 0.11(7)^2 \approx 1.7$

There were about 1.7 million foreclosure filings in 2013.

Beyond the Basics

107. $y = 3(x+2)^2 + 3 \Rightarrow y = 3(x^2 + 4x + 4) + 3 \Rightarrow$ $y = 3x^2 + 12x + 15$

108. Substitute the coordinates (1, 5) for x and y and (–3, –2) for h and k into the standard form $y = a(x-h)^2 + k$ to solve for a:

$5 = a(1+3)^2 - 2 \Rightarrow 7 = 16a \Rightarrow \frac{7}{16} = a$

The equation is $y = f(x) = \frac{7}{16}(x+3)^2 - 2 \Rightarrow$

$y = \frac{7}{16}x^2 + \frac{21}{8}x + \frac{31}{16}.$

109. The y-intercept is 4, so the graph passes through (0, 4). Substitute the coordinates (0, 4) for x and y and (1, –2) for h and k into the standard form $y = a(x-h)^2 + k$ to solve for a:

$4 = a(0-1)^2 - 2 \Rightarrow 6 = a.$ The x-coordinate of the vertex is $1 = -\frac{b}{2a} = -\frac{b}{2(6)} \Rightarrow b = -12.$

The y-intercept = c, so the equation is $f(x) = 6x^2 - 12x + 4.$

110. The x-coordinate of the vertex of the graph is halfway between the x-intercepts: $\frac{2+6}{2} = 4 = h.$ Substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find an expression relating a and k:

$0 = a(2-4)^2 + k \Rightarrow -4a = k.$

Now substitute the coordinates of the y-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y = a(x-h)^2 + k$ to solve for a:

$24 = a(0-4)^2 - 4a \Rightarrow 24 = 16a - 4a \Rightarrow 2 = a.$

Use this value to find k: $k = -4(2) = -8.$

The equation is

$y = 2(x-4)^2 - 8 \Rightarrow y = 2x^2 - 16x + 24.$

111. The x-coordinate of the vertex is 3. Substitute the coordinates of the x-intercept and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find an expression relating a and k: $0 = a(7-3)^2 + k \Rightarrow -16a = k.$ Now substitute the coordinates of the y-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y = a(x-h)^2 + k$ to solve for a: $14 = a(0-3)^2 - 16a \Rightarrow -2 = a.$ Use this value to find k: $k = -16(-2) = 32.$ The equation is

$y = -2(x-3)^2 + 32 \Rightarrow y = -2x^2 + 12x + 14.$

112. First, write $y = x^2 + 2x + 2$ in standard form by completing the square:

$y + 1 = (x^2 + 2x + 1) + 2 \Rightarrow y = (x+1)^2 + 1.$

Move the curve three units to the right by subtracting 3 from $x + 1$; move the curve two units down by subtracting 2 from 1:

$y = (x+1-3)^2 + (1-2) = (x-2)^2 - 1 \Rightarrow$

$y = x^2 - 4x + 3.$

113. The x-coordinate of the vertex is $\frac{-2+6}{2}=2.$ Substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y=a(x-h)^2+k$ to find an expression relating a and k:
$0=a(-2-2)^2+k \Rightarrow -16a=k.$
Now substitute the coordinates of the other x-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y=a(x-h)^2+k$ to solve for a:
$0=a(6-2)^2-16a \Rightarrow 1=a.$ Use this value to find k: $-16(1)=-16=k.$ So one equation is $y=(x-2)^2-16=x^2-4x-12.$ The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1: $y=-x^2+4x+12.$

114. The x-coordinate of the vertex is $\frac{-3+5}{2}=1.$ Substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y=a(x-h)^2+k$ to find an expression relating a and k:
$0=a(-3-1)^2+k \Rightarrow -16a=k.$ Now substitute the coordinates of the other x-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y=a(x-h)^2+k$ to solve for a: $0=a(5-1)^2-16a \Rightarrow 1=a.$ Use this value to find k: $-16(1)=-16=k.$ So one equation is $y=(x-1)^2-16=x^2-2x-15.$ The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1: $y=-x^2+2x+15.$

115. The x-coordinate of the vertex is $\frac{-7-1}{2}=-4.$ Substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y=a(x-h)^2+k$ to find an expression relating a and k:
$0=a(-7+4)^2+k \Rightarrow -9a=k.$
Now substitute the coordinates of the other x-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y=a(x-h)^2+k$ to solve for a:
$0=a(-1+4)^2-9a \Rightarrow 1=a.$ Use this value to find k: $-9(1)=-9=k.$ So one equation is $y=(x+4)^2-9=x^2+8x+7.$ The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1: $y=-x^2-8x-7.$

116. The x-coordinate of the vertex is $\frac{2+10}{2}=6.$ Substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y=a(x-h)^2+k$ to find an expression relating a and k:
$0=a(2-6)^2+k \Rightarrow -16a=k.$ Now substitute the coordinates of the other x-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y=a(x-h)^2+k$ to solve for a: $0=a(10-6)^2-16a \Rightarrow 1=a.$ Use this value to find k: $-16(1)=-16=k.$ So one equation is
$y=(x-6)^2-16=x^2-12x+20.$ The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1:
$y=-x^2+12x-20.$

117. First, complete the squares on x and y.

$$\begin{aligned}
&2x^2+y^2-4x+6y+15\\
&=2x^2-4x+y^2+6y+15\\
&=2\left(x^2-2x\right)+\left(y^2+6y\right)+15\\
&=2\left(x^2-2x+1\right)+\left(y^2+6y+9\right)+15-2(1)-9\\
&=2(x-1)^2+(y+3)^2+4
\end{aligned}$$

Now, consider the two functions $f=2(x-1)^2$ and $g=(y+3)^2+4.$
The minimum of f is 0 and the minimum of g is 4, so the minimum of $f+g=4.$

118. First, complete the squares on x and y.

$$\begin{aligned}&-3x^2 - y^2 - 12x + 2y - 11\\ &= -3x^2 - 12x - y^2 + 2y - 11\\ &= -3\left(x^2 + 4x\right) - \left(y^2 - 2y\right) - 11\\ &= -3\left(x^2 + 4x + 4\right) - \left(y^2 - 2y + 1\right) - 11 + 12 + 1\\ &= -3(x+2)^2 - (y-1)^2 + 2\end{aligned}$$

Now consider the two functions $f = -3(x+2)^2$ and $g = -(x-1)^2 + 2.$ The maximum of f is 0 and the maximum of g is 2, so the maximum of $f + g = 2.$

Critical Thinking/Discussion/Writing

119. $f(h+p) = a(h+p)^2 + b(h+p) + c$

$f(h-p) = a(h-p)^2 + b(h+p) + c$

$f(h+p) = f(h-p) \Leftrightarrow$

$$\begin{aligned}&a(h+p)^2 + b(h+p) + c\\ &\qquad = a(h-p)^2 + b(h-p) + c \Leftrightarrow\\ &ah^2 + 2ahp + ap^2 + bh + bp + c\\ &\qquad = ah^2 - 2ahp + ap^2 + bh - bp + c \Leftrightarrow\end{aligned}$$

$4ahp = -2bp \Leftrightarrow 2ah = -b$. (We can divide by p because $p \neq 0$.) Since $a \neq 0, 2ah = -b \Rightarrow$ $h = -\dfrac{b}{2a}.$

120. Write $y = 2x^2 - 8x + 9$ in standard form to find the axis of symmetry:

$y = 2x^2 - 8x + 9 \Rightarrow y + 8 = 2(x^2 - 4x + 4) + 9 \Rightarrow$

$y = 2(x-2)^2 + 1.$

The axis of symmetry is $x = 2.$

Using the results of exercise 119, we know that $f(2+p) = f(-1) = f(2-p).$

$2 + p = -1 \Rightarrow p = -3,$ so $2 - p = 2 - (-3) = 5.$

The point symmetric to the point (–1, 19) across the axis of symmetry is (5, 19).

121. a. $(f \circ g)(x)$

$$\begin{aligned}&= f(mx+b) = a\left[(mx+b) - h\right]^2 + k\\ &= a\left[(mx+b)^2 - 2h(mx+b) + h^2\right] + k\\ &= a\left[m^2x^2 + 2bmx + b^2 - 2hmx - 2hb + h^2\right] + k\\ &= am^2x^2 + 2abmx + ab^2 - 2ahmx\\ &\qquad - 2ahb + ah^2 + k\\ &= am^2x^2 + (2abm - 2ahm)x\\ &\qquad + \left(ab^2 - 2ahb + ah^2 + k\right)\end{aligned}$$

This is the equation of a parabola. The x-coordinate of the vertex is

$$-\frac{2abm - 2ahm}{2am^2} = -\frac{2am(b-h)}{2am^2} = -\frac{b-h}{m} \text{ or } \frac{h-b}{m}$$

The y-coordinate of the vertex is

$$\begin{aligned}&am^2\left(\frac{h-b}{m}\right)^2 + (2abm - 2ahm)\left(\frac{h-b}{m}\right)\\ &\qquad + \left(ab^2 - 2ahb + k\right)\\ &= a(h-b)^2 + 2a(b-h)(h-b)\\ &\qquad + \left(ab^2 - 2ahb + ah^2 + k\right)\\ &= ah^2 - 2ahb + ab^2 - 2ab^2 + 4abh - 2ah^2\\ &\qquad + ab^2 - 2ahb + ah^2 + k\\ &= k\end{aligned}$$

The vertex is $\left(\dfrac{h-b}{m}, k\right).$

b.
$$\begin{aligned}(g \circ f)(x) &= g\left[a(x-h)^2 + k\right]\\ &= m\left[a(x-h)^2 + k\right] + b\\ &= m\left(ax^2 - 2ahx + ah^2 + k\right) + b\\ &= max^2 - 2mahx + mah^2 + mk + b\end{aligned}$$

This is the equation of a parabola. The x-coordinate of the vertex is $-\dfrac{-2mah}{2ma} = h.$

The y-coordinate of the vertex is

$mah^2 - 2mah^2 + mah^2 + mk + b = mk + b$

The vertex is $(h, mk + b).$

122. If the discriminant equals zero, there is exactly one real solution. Thus, the vertex of $y = f(x)$ lies on the x-axis at $x = -\dfrac{b}{2a}.$

If the discriminant > 0, there are two unequal real solutions. This means that the graph of $y = f(x)$ crosses the x-axis in two places. If $a > 0$, then the vertex lies below the x-axis and the parabola crosses the x-axis; if $a < 0$, then the vertex lies above the x-axis and the parabola crosses the x-axis. If the discriminant < 0, there are two nonreal complex solutions. If $a > 0$, then the vertex lies above the x-axis and the parabola does not cross the x-axis (it opens upward); if $a < 0$, then the vertex lies below the x-axis and the parabola does not cross the x-axis (it opens downward).

123. Start by writing the function in standard form.

$$\begin{aligned} f(x) &= 2x^2 - 2ax + a^2 \\ &= 2\left(x^2 - ax\right) + a^2 \\ &= 2\left(x^2 - ax + \frac{a^2}{4}\right) + a^2 - \frac{a^2}{2} \\ &= 2\left(x - \frac{a}{2}\right)^2 + \frac{a^2}{2} \end{aligned}$$

The minimum value of this function is $\frac{a^2}{2}$.

Now, rewrite the function as

$$\begin{aligned} f(x) &= a^2 - 2ax + 2x^2 \\ &= \left(a^2 - 2ax\right) + 2x^2 \\ &= \left(a^2 - 2ax + x^2\right) + 2x^2 - x^2 \\ &= (a - x)^2 + x^2 \end{aligned}$$

This is the same function, so

$(a-x)^2 + x^2 \ge \frac{a^2}{2}$, or $x^2 + (a-x)^2 \ge \frac{a^2}{2}$.

124. Start by writing the function in standard form.

$$\begin{aligned} f(x) &= -x^2 + ax = -\left(x^2 - ax\right) \\ &= -\left(x^2 - ax + \frac{a^2}{4}\right) + \frac{a^2}{4} \\ &= -\left(x - \frac{a}{2}\right)^2 + \frac{a^2}{4} \end{aligned}$$

The maximum value of this function is $\frac{a^2}{4}$.

Now, rewrite the function as

$f(x) = ax - x^2 = x(a - x)$

This is the same function, so $x(a-x) \le \frac{a^2}{4}$.

Getting Ready for the Next Section

125. $-5^0 = -\left(5^0\right) = -1$

126. $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$

127. $\left(2x^2\right)\left(-3x^4\right) = (2)(-3)\left(x^2\right)\left(x^4\right) = -6x^{2+4} = -6x^6$

128. $\left(2x^2\right)(3x)\left(-x^3\right) = (2)(3)(-1)\left(x^2\right)(x)\left(x^3\right) = -6x^{2+1+3} = -6x^6$

129. $x^2\left(3 - \frac{3}{4}\right) = x^2\left(\frac{12}{4} - \frac{3}{4}\right) = \frac{9}{4}x^2$

130. $x^4\left(1 + \frac{3}{x^2} - \frac{5}{x^3}\right) = x^4(1) + x^4\left(\frac{3}{x^2}\right) + x^4\left(-\frac{5}{x^3}\right) = x^4 + 3x^2 - 5x$

131. $4x^2 - 9 = (2x+3)(2x-3)$

132. $x^2 + 6x + 9 = (x+3)^2$

133. $15x^2 + 11x - 12 = (3x+4)(5x-3)$

134. $14x^2 - 3x - 2 = (2x-1)(7x+2)$

135. $x^2(x-1) - 4(x-1) = \left(x^2 - 4\right)(x-1) = (x+2)(x-2)(x-1)$

136.
$$\begin{aligned} &9x^2(2x+7) - 25(2x+7) \\ &= \left(9x^2 - 25\right)(2x+7) \\ &= (3x+5)(3x-5)(2x+7) \end{aligned}$$

2.2 Polynomial Functions

2.2 Practice Problems

1. a. $f(x) = \frac{x^2+1}{x-1}$ is not a polynomial function because its domain is not $(-\infty, \infty)$.

b. $g(x) = 2x^7 + 5x^2 - 17$ is a polynomial function. Its degree is 7, the leading term is $2x^7$, and the leading coefficient is 2.

2.
$$\begin{aligned} P(x) &= 4x^3 + 2x^2 + 5x - 17 \\ &= x^3\left[4 + \frac{2}{x} + \frac{5}{x^2} - \frac{17}{x^3}\right] \end{aligned}$$

When $|x|$ is large, the terms $\frac{2}{x}$, $\frac{5}{x^2}$, and $-\frac{17}{x^3}$ are close to 0. Therefore, $P(x) = x^3(4 + 0 + 0 - 0) \approx 4x^3$.

3. Use the leading-term test to determine the end behavior of $y = f(x) = -2x^4 + 5x^2 + 3$. Here $n = 4$ and $a_n = -2 < 0$. Thus, Case 2 applies. The end behavior is described as $y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$.

4. First group the terms, then factor and solve $f(x)=0$:

$$\begin{aligned} f(x) &= 2x^3-3x^2+4x-6 \\ &= 2x^3+4x-3x^2-6 \\ &= 2x\left(x^2+2\right)-3\left(x^2+2\right) \\ &= (2x-3)\left(x^2+2\right) \end{aligned}$$

$0=(2x-3)\left(x^2+2\right)$

$0=2x-3 \Rightarrow x=\dfrac{3}{2}$ or

$0=x^2+2$ (no real solution)

The only real zero is $\dfrac{3}{2}$.

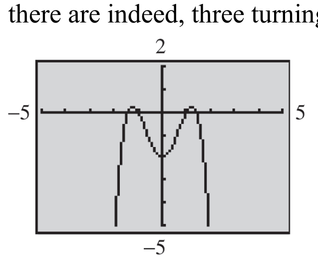
5. $f(x)=2x^3-3x-6$

$f(1)=-7$ and $f(2)=4$. Since $f(1)$ and $f(2)$ have opposite signs, by the Intermediate Value Theorem, *f* has a real zero between 1 and 2.

6. $f(x)=(x+1)^2(x-3)(x+5)=0 \Rightarrow$
$(x+1)^2=0$ or $x-3=0$ or $x+5=0 \Rightarrow$
$x=-1$ or $x=3$ or $x=-5$
$f(x)$ has three distinct zeros.

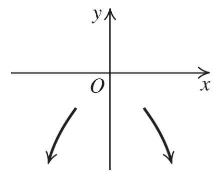
7. $f(x)=(x-1)^2(x+3)(x+5)=0 \Rightarrow$
$(x-1)^2=0$ or $x+3=0$ or $x+5=0 \Rightarrow$
$x=1$ (multiplicity 2) or $x=-3$ (multiplicity 1) or $x=-5$ (multiplicity 1)

8. $f(x)=-x^4+3x^2-2$ has at most three turning points. Using a graphing calculator, we see that there are indeed, three turning points.



9. $f(x)=-x^4+5x^2-4$

Since the degree, 4, is even and the leading coefficient is −1, the end behavior is as shown:



$y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$.

Now find the zeros of the function:

$0=-x^4+5x^2-4 \Rightarrow 0=-\left(x^4-5x^2+4\right)$
$=-\left(x^2-4\right)\left(x^2-1\right) \Rightarrow$
$0=x^2-4$ or $0=x^2-1 \Rightarrow$
$x=\pm 2$ or $x=\pm 1$

There are four zeros, each of multiplicity 1, so the graph crosses the *x*-axis at each zero.
Next, find the *y*-intercept:

$f(0)=-x^4+5x^2-4=-4$

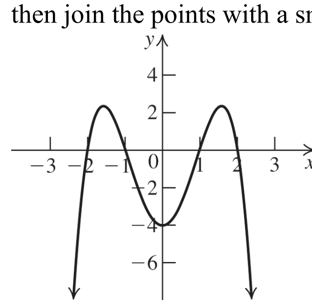
Determine symmetry:

$$\begin{aligned} f(-x) &= -(-x)^4+5(-x)^2-4 \\ &= -x^4+5x^2-4=f(x) \end{aligned}$$

So *f* is an even function and its graph is symmetric with respect to the *y*-axis.
Now find the intervals on which the graph lies above or below the *x*-axis. The four zeros divide the *x*-axis into five intervals, $(-\infty,-2)$, $(-2,-1),(-1,1),(1,2)$, and $(2,\infty)$. Determine the sign of a test value in each interval

Interval	Test point	Value of $f(x)$	Above/below *x*-axis
$(-\infty,-2)$	−3	−40	below
$(-2,-1)$	−1.5	2.1875	above
$(-1,1)$	0	−4	below
$(1,2)$	1.5	2.1875	above
$(2,\infty)$	3	−40	below

Plot the zeros, *y*-intercepts, and test points, and then join the points with a smooth curve.



10. $f(x) = x(x+2)^3(x-1)^2$

First, find the zeros of the function.

$x(x+2)^3(x-1)^2 = 0 \Rightarrow$

$x = 0 \;|\; x+2=0 \Rightarrow x=-2 \;|\; x-1=0 \Rightarrow x=1$

The function has three distinct zeros, –2, 1, and 0. The zero $x = -2$ has multiplicity 3, so the graph crosses the x-axis at –2, and near –2 the function looks like

$f(x) = -2(x+2)^3(-2-1)^2 = -18(x+2)^3$.

The zero $x = 0$ has multiplicity 1, so the graph crosses the x-axis at 0, and near 0 the function looks like $f(x) = x(0+2)^3(0-1)^2 = 8x$.

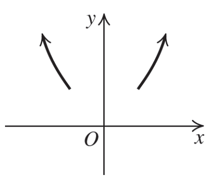
The zero $x = 1$ has multiplicity 2, so the graph touches the x-axis at 1, and near 1 the function looks like

$f(x) = 1(1+2)^3(x-1)^2 = 27(x-1)^2$.

We can determine the end behavior of the polynomial by finding its leading term as a product of the leading terms of each factor.

$f(x) = x(x+2)^3(x-1)^2 \approx x(x^3)(x^2) = x^6$

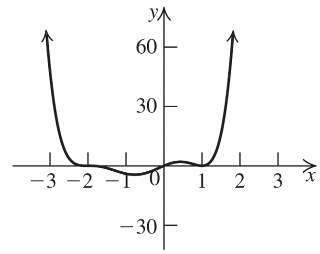
Therefore, the end behavior of $f(x)$ is Case 1:



Now find the intervals on which the graph lies above or below the x-axis. The three zeros divide the x-axis into four intervals, $(-\infty, -2)$, $(-2, 0)$, $(0, 1)$, and $(1, \infty)$. Determine the sign of a test value in each interval

Interval	Test point	Value of $f(x)$	Above/below x-axis
$(-\infty, -2)$	−3	48	above
$(-2, 0)$	−1	–4	below
$(0, 1)$	0.5	1.95	above
$(1, \infty)$	2	128	above

Plot the zeros, y-intercepts, and test points, and then join the points with a smooth curve.



11. $V = \frac{\pi}{3\sqrt{3}}x^3 = \frac{\pi}{3\sqrt{3}} \cdot 7^3 \approx 207.378 \text{ dm}^3$
≈ 207.378 L

2.2 Exercises
Concepts and Vocabulary

1. Consider the polynomial $2x^5 - 3x^4 + x - 6$. The degree of this polynomial is <u>5</u>, its leading term is <u>$2x^5$</u>, its leading coefficient is <u>2</u>, and its constant term is <u>−6</u>.
2. A number c for which $f(c) = 0$ is called a <u>zero</u> of the polynomial function f.
3. If c is a zero of even multiplicity for a polynomial function f, then the graph of f <u>touches</u> the x-axis at c.
4. If c is a zero of odd multiplicity for a polynomial function f, then the graph of f <u>crosses</u> the x-axis at c.
5. False. The graph of a polynomial function is a smooth curve. That means it has no corners or cusps.
6. True
7. False. The polynomial function of degree n has, at most, n zeros.
8. True

Building Skills

9. Polynomial function; degree: 5;
leading term: $2x^5$; leading coefficient: 2
10. Polynomial function; degree: 4;
leading term: $-7x^4$; leading coefficient: –7
11. Polynomial function; degree: 3;
leading term: $\frac{2}{3}x^3$; leading coefficient: $\frac{2}{3}$
12. Polynomial function; degree: 3; leading term: $\sqrt{2}x^3$; leading coefficient: $\sqrt{2}$

13. Polynomial function; degree: 4; leading term: πx^4; leading coefficient: π

14. Polynomial function; degree: 0; leading term: 5; leading coefficient: 5

15. Not a polynomial function (the graph has sharp corners; not a smooth curve; presence of $|x|$)

16. Not a polynomial function (the domain is restricted)

17. Not a polynomial function because the domain is not $(-\infty, \infty)$.

18. Not a polynomial function (the graph is not continuous)

19. Not a polynomial function (presence of $\sqrt{x}$)

20. Not a polynomial function (noninteger exponent)

21. Not a polynomial function because the domain is not $(-\infty, \infty)$.

22. Not a polynomial function (negative exponent)

23. Not a polynomial function (graph is not continuous)

24. Not a polynomial function (the domain is restricted)

25. Not a polynomial function (the graph is not continuous)

26. Not a polynomial function (the graph has sharp corners; not a smooth curve)

27. Not a polynomial function (not the graph of a function)

28. Not a polynomial function (the graph has sharp corners; not a smooth curve)

29. c **30.** f **31.** a

32. e **33.** d **34.** b

35. $f(x) = x - x^3$

The leading term is $-x^3$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

36. $f(x) = 2x^3 - 2x^2 + 1$

The leading term is $2x^3$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

37. $f(x) = 4x^4 + 2x^3 + 1$

The leading term is $4x^4$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

38. $f(x) = -x^4 + 3x^3 + x$

The leading term is $-x^4$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

39. $f(x) = (x+2)^2(2x-1)$

The leading term is $x^2(2x) = 2x^3$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

40. $f(x) = (x-2)^3(2x+1)$

The leading term is $x^3(2x) = 2x^4$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

41. $f(x) = (x+2)^2(4-x)$

The leading term is $x^2(-x) = -x^3$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

42. $f(x) = (x+3)^3(2-x)$

The leading term is $x^3(-x) = -x^4$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

43. $f(x) = 3(x-1)(x+2)(x-3)$
Zeros: –2, 1, 3
$x = -2$: multiplicity: 1, crosses the x-axis
$x = 1$: multiplicity: 1, crosses the x-axis
$x = 3$: multiplicity: 1, crosses the x-axis

44. $f(x) = -5(x+1)(x+2)(x-3)$
Zeros: –2, –1, 3
$x = -2$: multiplicity: 1, crosses the x-axis
$x = -1$: multiplicity: 1, crosses the x-axis
$x = 0$: multiplicity: 1, crosses the x-axis

45. $f(x) = (x+2)^2(2x-1)$

Zeros: –2, $\frac{1}{2}$

$x = -2$: multiplicity 2, touches but does not cross the x-axis

$x = \frac{1}{2}$: multiplicity 1, crosses the x-axis

46. $f(x) = (x-2)^3(2x+1)$

Zeros: $-\frac{1}{2}$, 2

$x = -\frac{1}{2}$: multiplicity 1, crosses the x-axis

$x = 2$: multiplicity 3, crosses the x-axis

47. $f(x) = x^2(x^2-9)(3x+2)^3$

Zeros: $-3, -\frac{2}{3}$, 0, 3

$x = -3$: multiplicity 1, crosses the x-axis

$x = -\frac{2}{3}$: multiplicity 3, crosses the x-axis

$x = 0$: multiplicity 2, touches but does not cross the x-axis

$x = 3$: multiplicity 1, crosses the x-axis

48. $f(x) = -x^3(x^2-4)(3x-2)^2$

Zeros: -2, 0, $\frac{2}{3}$, 2

$x = -2$: multiplicity 1, crosses the x-axis

$x = 0$: multiplicity 3, crosses the x-axis

$x = \frac{2}{3}$: multiplicity 2, touches but does not cross the x-axis

$x = 2$: multiplicity 1, crosses the x-axis

49. $f(x) = (x^2+1)(3x-2)^2$

Zero: $\frac{2}{3}$, multiplicity 2, touches but does not cross the x-axis

50. $f(x) = (x^2+1)(x+1)(x-2)$

Zeros: -1, 2

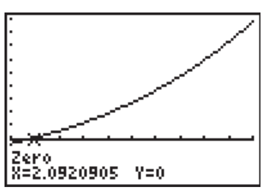
$x = -1$: multiplicity 1, crosses the x-axis

$x = 2$; multiplicity 1, crosses the x-axis

51. $f(2) = 2^4 - 2^3 - 10 = -2$;

$f(3) = 3^4 - 3^3 - 10 = 44$.

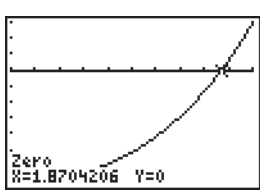
Because the sign changes, there is a real zero between 2 and 3. The zero is approximately 2.09.



52. $f(1) = 1^4 - 1^2 - 2(1) - 5 = -7$;

$f(2) = 2^4 - 2^2 - 2(2) - 5 = 3$.

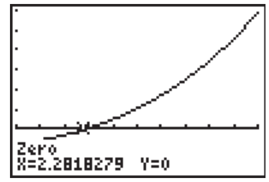
Because the sign changes, there is a real zero between 1 and 2. The zero is approximately 1.87.



53. $f(2) = 2^5 - 9(2)^2 - 15 = -19$;

$f(3) = 3^5 - 9(3)^2 - 15 = 147$.

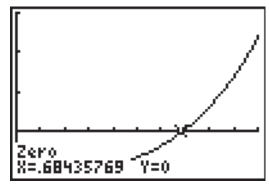
Because the sign changes, there is a real zero between 2 and 3. The zero is approximately 2.28.



54. $f(0) = 0^5 + 5(0^4) + 8(0^3) + 4(0^2) - 0 - 5 = -5$;

$f(1) = 1^5 + 5(1^4) + 8(1^3) + 4(1^2) - 1 - 5 = 12$.

Because the sign changes, there is a real zero between 0 and 1. The zero is approximately 0.68.



55. The least possible degree is 3. The zeros are –2, 1, and 3, each with multiplicity 1. The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+2)(x-1)(x-3)$.

56. The least possible degree is 3.The zeros are –1, 1, and 3, each with multiplicity 1. The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-(x+1)(x-1)(x-3)$.

57. The least possible degree is 3.The zeros are –3 (multiplicity 1) and 2 (multiplicity 2). The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+3)(x-2)^2$.

58. The least possible degree is 3.The zeros are –1 (multiplicity 2) and 2 (multiplicity 1). The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-(x+1)^2(x-2)$.

59. The least possible degree is 4. The zeros are –1 (multiplicity 2) and 2 (multiplicity 2). The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+1)^2(x-2)^2$.

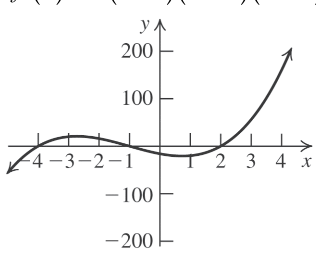
60. The least possible degree is 4. The zeros are –2 (multiplicity 2) and 2 (multiplicity 2). The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-(x+2)^2(x-2)^2$.

61. The zeros are –2 (multiplicity 2), 2 (multiplicity 1), and 3 (multiplicity 1), so the least possible degree is 4. The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+2)^2(x-2)(x-3)$.

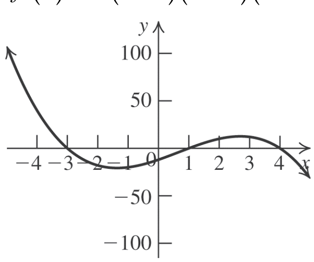
62. The zeros are –3 (multiplicity 1), 0 (multiplicity 2), and 2 (multiplicity 1), so the least possible degree is 4. The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-x^2(x+3)(x-2)$.

For exercises 63–74, use the procedure shown on pages 163–164 of the text to graph the function.

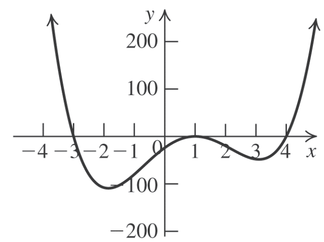
63. $f(x) = 2(x+1)(x-2)(x+4)$



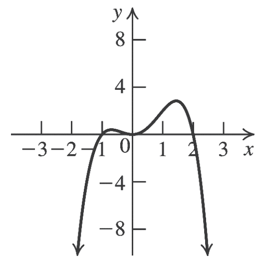
64. $f(x) = -(x-1)(x+3)(x-4)$



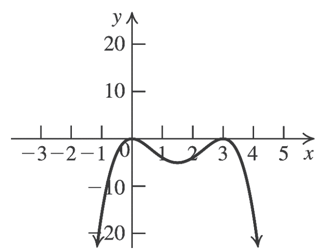
65. $f(x) = (x-1)^2(x+3)(x-4)$



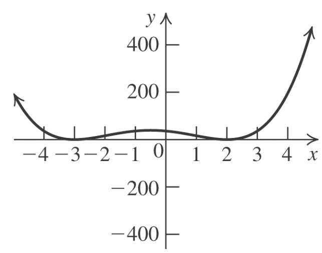
66. $f(x) = -x^2(x+1)(x-2)$



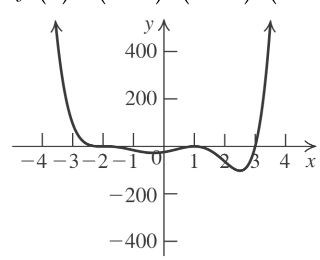
67. $f(x) = -x^2(x-3)^2$



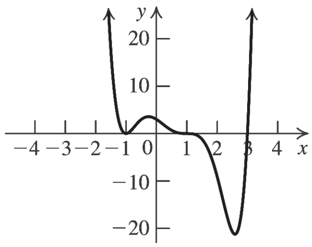
68. $f(x) = (x-2)^2(x+3)^2$



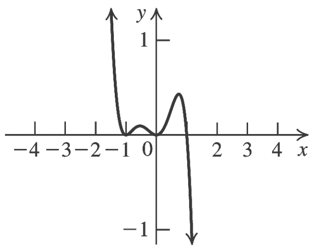
69. $f(x) = (x-1)^2(x+2)^3(x-3)$



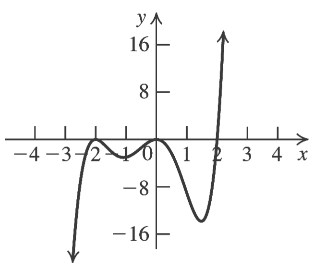
70. $f(x) = (x-1)^3(x+1)^2(x-3)$



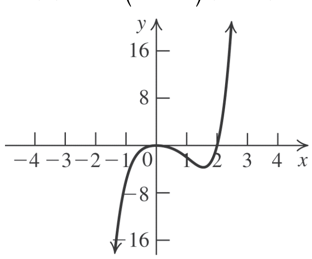
71. $f(x) = -x^2(x^2-1)(x+1)$



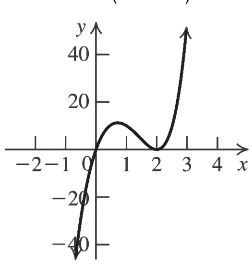
72. $f(x) = x^2(x^2-4)(x+2)$



73. $f(x) = x^2(x^2+1)(x-2)$



74. $f(x) = x(x^2+9)(x-2)^2$



Applying the Concepts

75. **a.** $P = \dfrac{0.04825}{746}(40)^3 \approx 4.14$ hp

b. $P = \dfrac{0.04825}{746}(65)^3 \approx 17.76$ hp

c. $P = \dfrac{0.04825}{746}(80)^3 \approx 33.12$ hp

d. $\dfrac{P(2v)}{P(v)} = \dfrac{\frac{0.04825}{746}(2v)^3}{\frac{0.04825}{746}v^3} = \dfrac{8v^3}{v^3} = 8$

76. **a.** $E = 12800(1.75)^4 = 120,050\text{ J}$

b. $E = 12800(0.5833)^4 \approx 1481.76\text{ J}$

$$E = 12800\left(\frac{L}{3}\right)^4 = 12800L^4\left(\frac{1}{3}\right)^4$$
$$= 12800L^4\left(\frac{1}{81}\right) \approx 158.02\text{ J}$$

It reduced by a factor of $\frac{1}{81}$.

c. $E = 12800(0.07)^4 \approx 0.31\text{ J}$

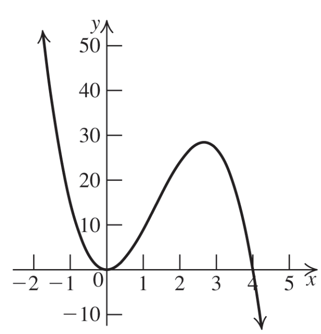
$$E = 12800\left(\frac{L}{25}\right)^4 = 12800L^4\left(\frac{1}{25}\right)^4$$
$$= 12800L^4\left(\frac{1}{390,625}\right) \approx 0.03\text{ J}$$

It reduced by a factor of $\frac{1}{390,625}$.

d. $\dfrac{E\left(\frac{L}{2}\right)}{E(L)} = \dfrac{12800\left(\frac{L}{2}\right)^4}{12800L^4} = \dfrac{\frac{L^4}{16}}{L^4} = \dfrac{1}{16}$

77. **a.** $3x^2(4-x) = 0 \Rightarrow x = 0$ or $x = 4$. $x = 0$, multiplicity 2; $x = 4$, multiplicity 1.

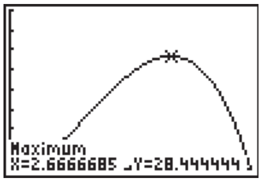
b.



c. There are 2 turning points.

d. Domain: [0, 4]. The portion between the x-intercepts is the graph of $R(x)$.

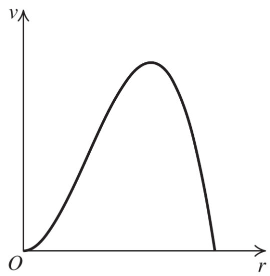
78.



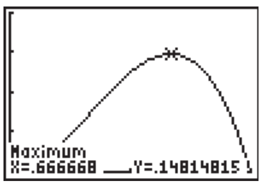
$R(x)$ is a maximum at $x \approx 2.67$.

79. a. Domain [0, 1]

b.



80.

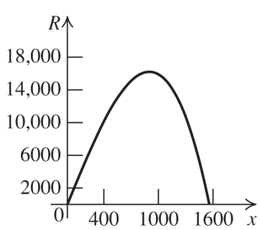


v is a maximum when $r \approx 0.67$.

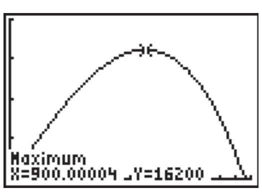
81. a. $R(x) = x\left(27 - \left(\dfrac{x}{300}\right)^2\right) = 27x - \dfrac{x^3}{90,000}$

b. The domain of $R(x)$ is the same as the domain of p. $p \geq 0$ when $p \leq 900\sqrt{3}$. The domain is $\left[0, 900\sqrt{3}\right]$.

c.



82. a.



About 900 pairs of slacks were sold.

b. \$16,200

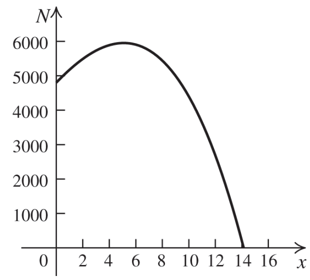
83. a. $N(x) = (x+12)\left(400 - 2x^2\right)$

b. The low end of the domain is 0. (There cannot be fewer than 0 workers.) The upper end of the domain is the value where productivity is 0, so solve $N(x) = 0$ to find the upper end of the domain.

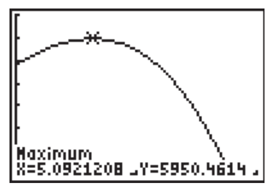
$(x+12)(400 - 2x^2) = 0 \Rightarrow x = -12$ (reject this) or $400 - 2x^2 = 0 \Rightarrow 200 = x^2 \Rightarrow$ $x = \pm 10\sqrt{2}$ (reject the negative solution).

The domain is $[0, 10\sqrt{2}]$.

c.



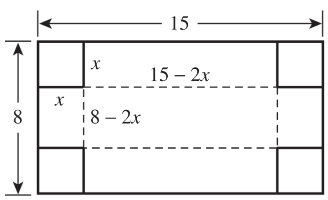
84. a.



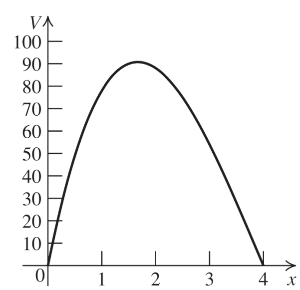
The maximum number of oranges that can be picked per hour is about 5950.

b. The number of employees = 12 + 5 = 17.

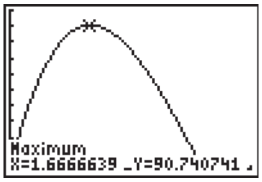
85. a. $V(x) = x(8 - 2x)(15 - 2x)$



b.



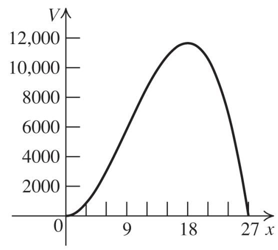
86.



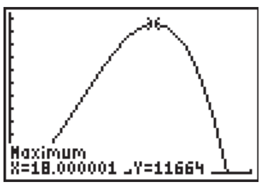
The largest possible value of the volume of the box is 90.74 cubic units.

87. a. $V(x) = x^2(108 - 4x)$

b.



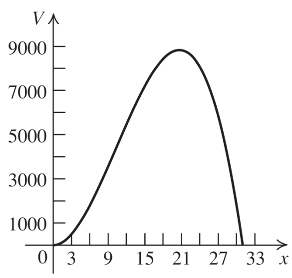
88.



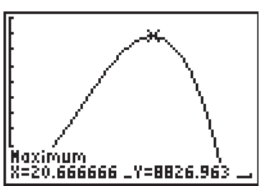
The largest possible volume of the box is 11,664 cubic inches.

89. a. $V(x) = x^2(62 - 2x)$

b.

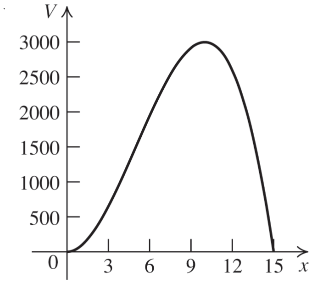


90.

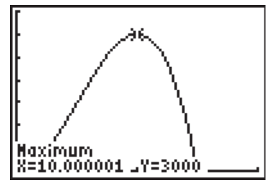


The volume is greatest when $x \approx 20.67$, so the dimensions of the suitcase with the largest volume are approximately 20.67 in. × 20.67 in. × 20.67 in.

91. a. $V(x) = 2x^2(45 - 3x)$

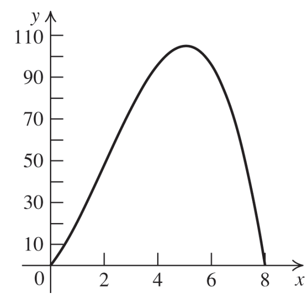


92.

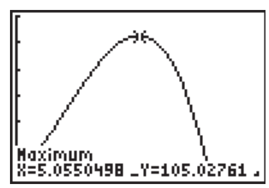


The volume is greatest when $x = 10$, so the width of the bag is 10 inches, the length is 2(10) = 20 inches, and the height is 45 – 20 – 10 = 15 inches.

93.



94.

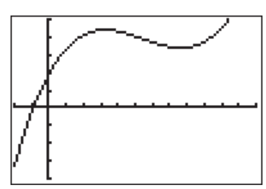


A worker is most efficient approximately five hours after 6:00 a.m. or 11:00 a.m.

95. $f(x) = 0.963 + 0.88x - 0.192x^2 + 0.0117x^3$

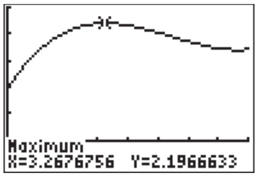
a. The year 2015 is represented by $x = 7$.
$f(7) \approx 1.73$ trillion

b.



[–2, 12] by [–2, 2.5]

It appears that there is one zero and two turning points.

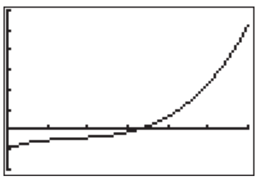
c. 

[0, 8] by [–0.5, 2.5]

The maximum occurs at $x \approx 3.27,$ which is during 2011. This fits the original data.

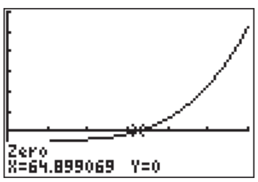
96. $f(x) = -0.23 + 0.0081x - 0.0002x^2 + 0.000002x^3$

a. The year 2025 is represented by $x = 120$.
$f(120) \approx 1.32$

b. 

[0, 120] by [–0.5, 1.5]

It appears that there is one zero and no turning points.

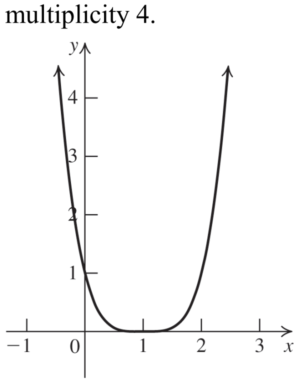
c. 

[0, 8] by [–0.5, 2.5]

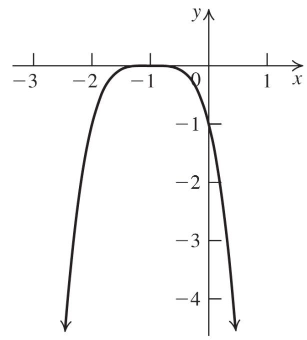
The zero occurs at $x \approx 65,$ which is the year 1970. This fits the original data.

Beyond the Basics

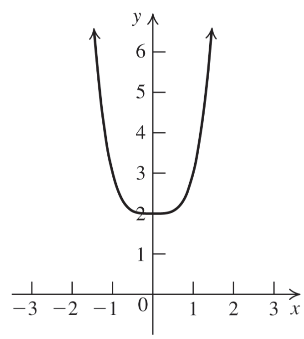
97. The graph of $f(x) = (x-1)^4$ is the graph of $y = x^4$ shifted one unit right.
$(x-1)^4 = 0 \Rightarrow x = 1$. The zero is $x = 1$ with multiplicity 4.



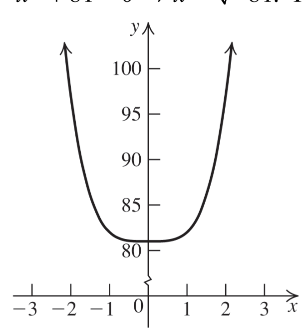
98. The graph of $f(x) = -(x+1)^4$ is the graph of $y = x^4$ shifted one unit left and then reflected across the x-axis. $-(x+1)^4 = 0 \Rightarrow x = -1$. The zero is $x = -1$ with multiplicity 4.



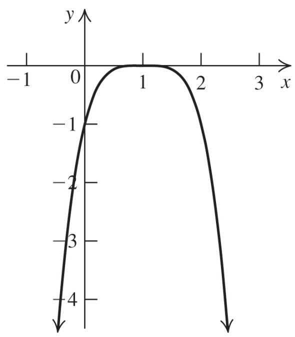
99. The graph of $f(x) = x^4 + 2$ is the graph of $y = x^4$ shifted two units up.
$x^4 + 2 = 0 \Rightarrow x = \sqrt[4]{-2}.$ There are no zeros.



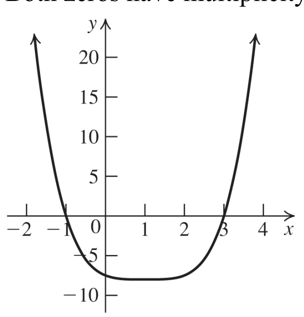
100. The graph of $f(x) = x^4 + 81$ is the graph of $y = x^4$ shifted 81 units up.
$x^4 + 81 = 0 \Rightarrow x = \sqrt[4]{-81}.$ There are no zeros.



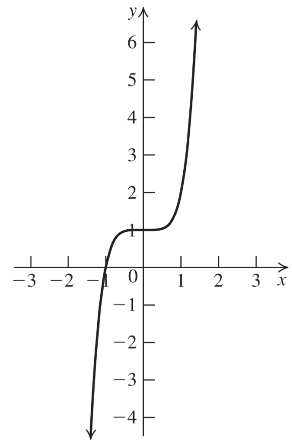
101. The graph of $f(x) = -(x-1)^4$ is the graph of $y = x^4$ shifted one unit right and then reflected across the x-axis. $-(x-1)^4 = 0 \Rightarrow x = 1$. The zero is $x = 1$ with multiplicity 4.



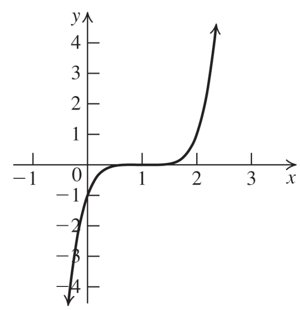
102. The graph of $f(x) = \frac{1}{2}(x-1)^4 - 8$ is the graph of $y = x^4$ shifted one unit right, compressed vertically by a factor of 2, and then shifted eight units down. $\frac{1}{2}(x-1)^4 - 8 = 0 \Rightarrow$ $(x-1)^4 = 16 \Rightarrow x - 1 = \pm 2 \Rightarrow x = 3$ or $x = -1$. Both zeros have multiplicity 1.



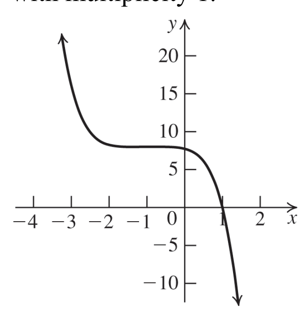
103. The graph of $f(x) = x^5 + 1$ is the graph of $y = x^5$ shifted one unit up. $x^5 + 1 = 0 \Rightarrow$ $x = \sqrt[5]{-1} \Rightarrow x = -1$. The zero is –1 with multiplicity 1.



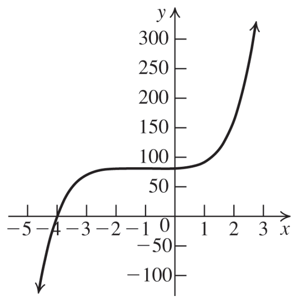
104. The graph of $f(x) = (x-1)^5$ is the graph of $y = x^5$ shifted one unit right. $(x-1)^5 = 0 \Rightarrow$ $x = 1$. The zero is 1 with multiplicity 5.



105. The graph of $f(x) = 8 - \frac{(x+1)^5}{4}$ is the graph of $y = x^5$ shifted one unit left, compressed vertically by one-fourth, reflected in the x-axis, and then shifted up eight units.
$8 - \frac{(x+1)^5}{4} = 0 \Rightarrow -\frac{(x+1)^5}{4} = -8 \Rightarrow$ $(x+1)^5 = 32 \Rightarrow x + 1 = 2 \Rightarrow x = 1$. The zero is 1 with multiplicity 1.



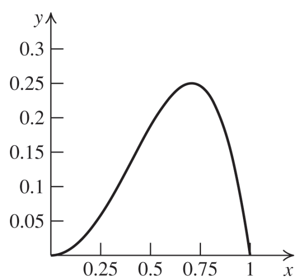
106. The graph of $f(x) = 81 + \frac{(x+1)^5}{3}$ is the graph of $y = x^5$ shifted one unit left, compressed vertically by one-third, and then shifted up 81 units. $81 + \frac{(x+1)^5}{3} = 0 \Rightarrow \frac{(x+1)^5}{3} = -81 \Rightarrow$ $(x+1)^5 = -243 \Rightarrow x+1 = -3 \Rightarrow x = -4.$ The zero is –4 with multiplicity 1.



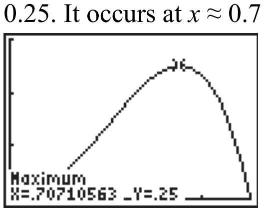
107. If $0 < x < 1$, then $x^2 < 1.$ Multiplying both sides of the inequality by x^2, we obtain $x^4 < x^2.$ Thus, $0 < x^4 < x^2.$

108. If $x > 1$, then $x^2 > 1.$ Multiplying both sides of the inequality by x^2, we obtain $x^4 > x^2.$ Thus, $x^4 > x^2 > 1.$

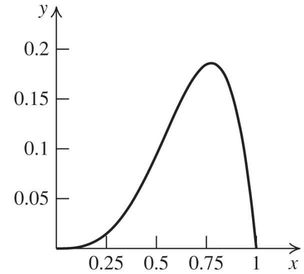
109.



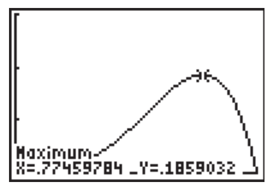
The graph of $y = x^2 - x^4$ represents the difference between the two functions $y = x^2$ and $y = x^4,$ so the maximum distance between the graphs occurs at the local maximum of $y = x^2 - x^4$. The maximum vertical distance is 0.25. It occurs at $x \approx 0.71.$



110.



The graph of $y = x^3 - x^5$ represents the difference between the two functions $y = x^3$ and $y = x^5,$ so the maximum distance between the graphs occurs at the local maximum of $y = x^3 - x^5$. The maximum vertical distance is 0.19. It occurs at $x \approx 0.77.$



Answers will vary in exercises 111–114. Sample answers are given.

111. $f(x) = x^2(x+1)$

112. $f(x) = -x(x+1)(x-1)$

113. $f(x) = 1 - x^4$

114. $f(x) = x^2(x+1)(x-1)$

115. The smallest possible degree is 5, because the graph has five *x*-intercepts and four turning points.

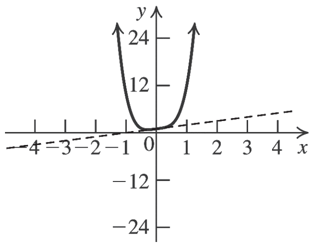
116. The smallest possible degree is 5, because the graph has four turns.

117. The smallest possible degree is 6, because the graph has five turning points.

118. The smallest possible degree is 6, because the graph has five turning points.

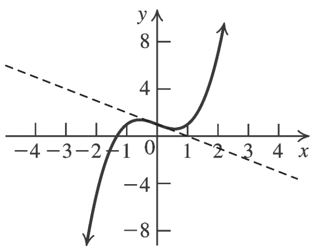
119. $f(x) = 10x^4 + x + 1$

For large values of x, $f(x) \approx 10x^4$. For small values of x, $f(x) \approx x + 1$.



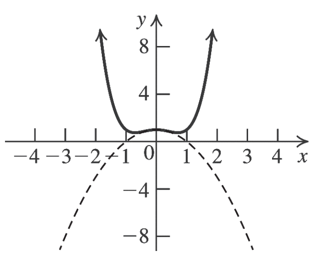
120. $f(x) = x^3 - x + 1$

For large values of x, $f(x) \approx x^3$. For small values of x, $f(x) \approx -x + 1$.



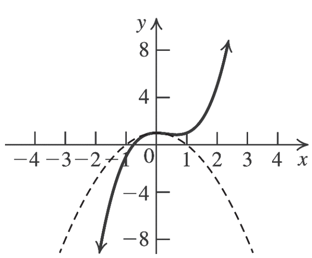
121. $f(x) = x^4 - x^2 + 1$

For large values of x, $f(x) \approx x^4$. For small values of x, $f(x) \approx -x^2 + 1$.



122. $f(x) = x^3 - x^2 + 1$

For large values of x, $f(x) \approx x^3$. For small values of x, $f(x) \approx -x^2 + 1$.



Critical Thinking/Discussion/Writing

123. If a human is scaled up by a factor of 20, then the total weight will increase $20^3 = 8000$ times but the bone strength will increase only $20^2 = 400$ times. The bones would probably break under the increased weight.

124. If a human is scaled down by a factor of 70, then the body surface area will decrease $70^2 = 4900$ times while the volume will decrease $70^3 = 343{,}000$ times. The body will rapidly lose heat and will have a very hard time maintaining body temperature unless its metabolic rate increases drastically.

125. It is not possible for a polynomial function to have no y-intercepts because the domain of any polynomial function is $(-\infty, \infty)$, which includes the point $x = 0$.

126. It is possible for a polynomial function to have no x-intercepts because the function can be shifted above the x-axis. An example is the function $y = x^2 + 1$.

127. It is not possible for the graph of a polynomial function of degree 3 to have exactly one local maximum and no local minimum because the graph of a function of degree 3 rises in one direction and falls in the other. This requires an even number of turning points. Since the degree is 3, there can be only zero or two turning points. Therefore, if there is a local maximum, there must also be another turning point, which will be a local minimum.

128. It is not possible for the graph of a polynomial function of degree 4 to have exactly one local maximum and exactly one local minimum because the graph rises in both directions or falls in both directions. This requires an odd number of turning points. Since the degree is 4, there can be only one or three turning points. If there were exactly one local maximum and exactly one local minimum, there would be two turning points.

129. If $P(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0$ and $Q(x) = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0$, then

$$(P \circ Q)(x) = a_m\left(b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0\right)^m + a_{m-1}\left(b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0\right)^{m-1} + \cdots + a_1\left(b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0\right) + a_0,$$

which is a polynomial of degree mn.

130. If $P(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0$ and $Q(x) = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0$, then

$$(P \circ Q)(x) = a_m\left(b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0\right)^m + a_{m-1}\left(b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0\right)^{m-1} + \cdots + a_1\left(b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0\right) + a_0,$$

and the leading coefficient is $a_m(b_n)^m$.

$$(Q \circ P)(x) = b_n\left(a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0\right)^n + b_{n-1}\left(a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0\right)^{n-1} + \cdots + b_1\left(a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0\right) + b_0,$$

and the leading coefficient is $b_n(a_m)^n$.

Getting Ready for the Next Section

131. $f(x) = x^3 - 3x^2 + 2x - 9$
$f(-2) = (-2)^3 - 3(-2)^2 + 2(-2) - 9 = -33$

132. $g(x) = x^4 + 2x^3 - 20x - 5$
$g(3) = (3)^4 + 2(3)^3 - 20(3) - 5 = 70$

133. $g(x) = \left(-7x^5 + 2x\right)(x - 13)$
$g(13) = \left[-7(13)^5 + 2(13)\right]\left[(13) - 13\right] = 0$

134. $f(x) = \left(x^8 - 15x^7 + 5x^3\right)(x + 12)$

$$\begin{aligned} &f(-12) \\ &= \left[(-12)^8 - 15(-12)^7 + 5(-12)^3\right]\left[(-12) + 12\right] \\ &= 0 \end{aligned}$$

135. $x^2 - 3x - 10 = (x - 5)(x + 2)$

136.
$$\begin{aligned} -2x^2 + x + 1 &= -\left(2x^2 - x - 1\right) \\ &= -(2x + 1)(x - 1) \\ &= (2x + 1)(1 - x) \end{aligned}$$

137. $(2x + 3)\left(x^2 + 6x - 7\right) = (2x + 3)(x + 7)(x - 1)$

138. $(x - 9)\left(6x^2 - 7x - 3\right) = (x - 9)(3x + 1)(2x - 3)$

2.3 Dividing Polynomials and the Rational Zeros Test

2.3 Practice Problems

1.
$$\begin{array}{r} 3x + 4 \\ x^2 + 0x + 1 \overline{\smash{\big)}\, 3x^3 + 4x^2 + x + 7} \\ (-)\underline{3x^3 + 0x^2 + 3x} \\ 4x^2 - 2x + 7 \\ (-)\ \underline{4x^2 + 0x + 4} \\ -2x + 3 \end{array}$$

Quotient: $3x + 4$; remainder: $-2x + 3$

2.
$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 5 \\ & & 6 & -3 & -9 \\ \hline & 2 & -1 & -3 & -4 \end{array}$$

The quotient is $2x^2 - x - 3$, remainder -4 or $2x^2 - x - 3 - \dfrac{4}{x - 3}$.

3.
$$\begin{array}{r|rrrr} 3 & 2 & 1 & -18 & -7 \\ & & 6 & 21 & 9 \\ \hline & 2 & 7 & 3 & 2 \end{array}$$

The quotient is $2x^2 + 7x + 3$, remainder 2.

4. $F(1) = 1^{110} - 2 \cdot 1^{57} + 5 = 1 - 2 + 5 = 4$, so the remainder when $F(x) = x^{110} - 2x^{57} + 5$ is divided by $x - 1$ is 4.

5.
$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 10 & 2 & -20 \\ & & -2 & 4 & -28 & 52 \\ \hline & 1 & -2 & 14 & -26 & 32 \end{array}$$

The remainder is 32, so $f(-2) = 32$.

6. Since −2 is a zero of the function $3x^3 - x^2 - 20x - 12$, $x + 2$ is a factor. Use synthetic division to find the depressed equation.

$$\begin{array}{r|rrrr} -2 & 3 & -1 & -20 & -12 \\ & & -6 & 14 & 12 \\ \hline & 3 & -7 & -6 & 0 \end{array}$$

Thus,
$3x^3 - x^2 - 20x - 12 = (x+2)(3x^2 - 7x - 6)$.

Now solve $3x^2 - 7x - 6 = 0$ to find the two remaining zeros:

$3x^2 - 7x - 6 = 0 \Rightarrow (x-3)(3x+2) = 0 \Rightarrow$
$x - 3 = 0 \text{ or } 3x + 2 = 0 \Rightarrow x = 3 \text{ or } x = -\frac{2}{3}$

The solution set is $\left\{-2, -\frac{2}{3}, 3\right\}$.

7. $C(x) = 0.23x^3 - 4.255x^2 + 0.345x + 41.05$
$C(3) = 10 \Rightarrow C(x) = (x-3)Q(x) + 10 \Rightarrow$
$C(x) - 10 = (x-3)Q(x)$

So, 3 is zero of $C(x) - 10 =$
$0.23x^3 - 4.255x^2 + 0.345x + 31.05$.
We need to find another positive zero of $C(x) - 10$. Use synthetic division to find the depressed equation.

$$\begin{array}{r|rrrr} 3 & 0.23 & -4.255 & 0.345 & 31.05 \\ & & 0.69 & -10.695 & -31.05 \\ \hline & 0.23 & -3.565 & -10.35 & 0 \end{array}$$

Solve the depressed equation $0.23x^2 - 3.565x - 10.35 = 0$ using the quadratic formula:

$$x = \frac{3.565 \pm \sqrt{(-3.565)^2 - 4(0.23)(-10.35)}}{2(0.23)}$$
$$\approx 18 \text{ or } -2.5$$

The positive zero is 18.
Check by verifying that $C(18) = 10$.

8. The factors of the constant term, −8, are $\{\pm 1, \pm 2, \pm 4, \pm 8\}$, and the factors of the leading coefficient, 2, are $\{\pm 1, \pm 2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8\right\}$. Use synthetic division to find one rational root:

$$\begin{array}{r|rrrr} -2 & 2 & 3 & -6 & -8 \\ & & -4 & 2 & 8 \\ \hline & 2 & -1 & -4 & 0 \end{array}$$

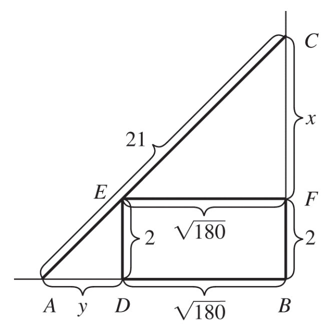
The remainder is 0, so −2 is a zero of the function.

$2x^3 + 3x^2 - 6x - 8 = (x+2)(2x^2 - x - 4)$

Now find the zeros of $2x^2 - x - 4$ using the quadratic formula:

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-4)}}{2(1)} = \frac{1 \pm \sqrt{33}}{2}$, which are not rational roots. The only rational zero is $\{-2\}$.

9.



From the similar right triangles $\triangle EFC$ and $\triangle ABC$, we have

$\frac{\sqrt{180}}{x} = \frac{y + \sqrt{180}}{x+2}$, or $y + \sqrt{180} = \sqrt{180} \cdot \frac{x+2}{x}$.

Using the Pythagorean theorem in right triangle *ABC* gives

$(x+2)^2 + \left(y + \sqrt{180}\right)^2 = 21^2$.

Now substitute.

$(x+2)^2 + \left(\sqrt{180} \cdot \frac{x+2}{x}\right)^2 = 21^2$.

Multiply both sides by x^2 to eliminate the fraction.

$x^2(x+2)^2 + \left(\sqrt{180}(x+2)\right)^2 = 21^2 x^2$

Now expand and simplify.

$$x^2(x^2 + 4x + 4) + 180(x^2 + 4x + 4) = 441x^2$$
$$x^4 + 4x^3 + 4x^2 + 180x^2 + 720x + 720 = 441x^2$$
$$x^4 + 4x^3 - 257x^2 + 720x + 720 = 0 = P(x)$$

(*continued on next page*)

(*continued*)

Use the factorization of $720 = 2^4 \cdot 3^2 \cdot 5$ to check for all possible rational zeros of the polynomial.
$\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm8, \pm9, \pm10,$
$\pm12, \pm15, \pm18, \pm20, \pm24, \pm30, \pm36,$
$\pm40, \pm48, \pm60, \pm72, \pm80, \pm90, \pm120,$
$\pm144, \pm180, \pm240, \pm360, \pm720$

We know that x must be a positive number less than 21. Using synthetic division, we see that $x = 4$ is a zero of $P(x)$.

$$\begin{array}{r|rrrrr} 4 & 1 & 4 & -257 & 720 & 720 \\ & & 4 & 32 & -900 & -720 \\ \hline & 1 & 8 & -225 & -180 & 0 \end{array}$$

So, the remainder is
$Q(x) = x^3 + 8x^2 - 225x - 180.$

Using synthetic division again, we see that $x = 12$ is another zero.

$$\begin{array}{r|rrrr} 12 & 1 & 8 & -225 & -180 \\ & & 12 & 240 & 180 \\ \hline & 1 & 20 & 15 & 0 \end{array}$$

There are no other real zeros.

$$\begin{aligned} P(x) &= x^4 + 4x^3 - 257x^2 + 720x + 720 \\ &= (x-4)(x-12)\left(x^2 + 20x + 15\right) \end{aligned}$$

Since the height = $x + 2$, the top of the ladder touches the wall at either 6 feet above the ground or 14 feet above the ground.

2.3 Exercises
Concepts and Vocabulary

1. Consider the equation $x^3 - 3x + 5 = \left(x^2 + 1\right)x - 4x + 5$. If both sides are divided by $x^2 + 1$, then the dividend is $\underline{x^3 - 3x + 5}$, the divisor is $\underline{x^2 + 1}$, the quotient is $\underline{x}$, and the remainder is $\underline{-4x + 5}$.

2. Consider the division $\frac{x^2+4x}{x-2} = x + 6 + \frac{12}{x-2}$. The dividend is $\underline{x^2 + 4x}$, the divisor is $\underline{x - 2}$, the quotient is $\underline{x + 6}$, and the remainder is $\underline{12}$.

3. The Remainder Theorem states that if a polynomial $F(x)$ is divided by $(x - a)$, then the remainder $R = \underline{F(a)}$.

4. The Factor Theorem states that $(x - a)$ is a factor of a polynomial $F(x)$ if and only if $\underline{F(a)} = 0$.

5. True

6. True

7. False. The possible rational zeros of $P(x) = 9x^3 - 9x^2 - x + 1$ are $\pm\frac{1}{9}, \pm\frac{1}{3}, \pm1.$

8. False. The remainder is a constant.

Building Skills

9.
$$\begin{array}{r} 3x-2 \\ 2x+1\overline{\smash{)}6x^2 - x - 2} \\ \underline{-(6x^2+3x)} \\ -4x-2 \\ \underline{-(-4x-2)} \\ 0 \end{array}$$

10.
$$\begin{array}{r} 2x^2+2x+\frac{7}{2} \\ 2x-3\overline{\smash{)}4x^3 - 2x^2 + x - 3} \\ \underline{-(4x^3-6x^2)} \\ 4x^2 + x \\ \underline{-(4x^2 - 6x)} \\ 7x - 3 \\ \underline{-\left(7x-\frac{21}{2}\right)} \\ \frac{15}{2} \end{array}$$

In exercises 9–14, insert zero coefficients for missing terms.

11.
$$\begin{array}{r} 3x^3-3x^2-3x+6 \\ x+1\overline{\smash{)}3x^4+0x^3-6x^2+3x-7} \\ \underline{-(3x^4+3x^3)} \\ -3x^3-6x^2 \\ \underline{-(-3x^3-3x^2)} \\ -3x^2+3x \\ \underline{-(-3x^2-3x)} \\ 6x-7 \\ \underline{-(6x+6)} \\ -13 \end{array}$$

12.
$$\begin{array}{r}
x^4-2x^2+5x+4 \\
x^2+0x+2\overline{\big)x^6+0x^5+0x^4+5x^3+0x^2+7x+3} \\
\underline{-(x^6+0x^5+2x^4)} \\
-2x^4+5x^3+0x^2 \\
\underline{-(-2x^4+0x^3-4x^2)} \\
5x^3+4x^2+7x \\
\underline{-(5x^3+0x^2+10x)} \\
4x^2-3x+3 \\
\underline{-(4x^2+0x+8)} \\
-3x-5
\end{array}$$

13.
$$\begin{array}{r}
2x-1 \\
2x^2-x-5\overline{\big)4x^3-4x^2-9x+5} \\
\underline{-(4x^3-2x^2-10x)} \\
-2x^2+x+5 \\
\underline{-(-2x^2+x+5)} \\
0
\end{array}$$

14.
$$\begin{array}{r}
y^3+y^2+y-5 \\
y^2+2y-3\overline{\big)y^5+3y^4+0y^3-6y^2+2y-7} \\
\underline{-(y^5+2y^4-3y^3)} \\
y^4+3y^3-6y^2 \\
\underline{-(y^4+2y^3-3y^2)} \\
y^3-3y^2+2y \\
\underline{-(y^3+2y^2-3y)} \\
-5y^2+5y-7 \\
\underline{-(-5y^2-10y+15)} \\
15y-22
\end{array}$$

15.
$$\begin{array}{r}
z^2+2z+1 \\
z^2-2z+1\overline{\big)z^4+0z^3-2z^2+0z+1} \\
\underline{-(z^4-2z^3+z^2)} \\
2z^3-3z^2+0z \\
\underline{-(2z^3-4z^2+2z)} \\
z^2-2z+1 \\
\underline{-(z^2-2z+1)} \\
0
\end{array}$$

16. First, arrange the terms in order of descending powers.
$$\begin{array}{r}
2x^2-3x+2 \\
3x^2-x-5\overline{\big)6x^4-11x^3-x^2+13x-10} \\
\underline{-(6x^4-2x^3-10x^2)} \\
-9x^3+9x^2+13x \\
\underline{-(-9x^3+3x^2+15x)} \\
6x^2-2x-10 \\
\underline{-(6x^2-2x-10)} \\
0
\end{array}$$

17.
$$\begin{array}{c|rrrr}
1 & 1 & -1 & -7 & 2 \\
 & & 1 & 0 & -7 \\ \hline
 & 1 & 0 & -7 & -5
\end{array}$$
The quotient is x^2-7 and the remainder is −5.

18.
$$\begin{array}{c|rrrr}
-2 & 2 & -3 & -1 & 2 \\
 & & -4 & 14 & -26 \\ \hline
 & 2 & -7 & 13 & -24
\end{array}$$
The quotient is $2x^2-7x+13$ and the remainder is −24.

19.
$$\begin{array}{c|rrrr}
-2 & 1 & 4 & -7 & -10 \\
 & & -2 & -4 & 22 \\ \hline
 & 1 & 2 & -11 & 12
\end{array}$$
The quotient is $x^2+2x-11$ and the remainder is 12.

20.
$$\begin{array}{c|rrrr}
3 & 1 & 1 & -13 & 2 \\
 & & 3 & 12 & -3 \\ \hline
 & 1 & 4 & -1 & -1
\end{array}$$
The quotient is x^2+4x-1 and the remainder is −1.

21.
$$\begin{array}{c|rrrrr}
2 & 1 & -3 & 2 & 4 & 5 \\
 & & 2 & -2 & 0 & 8 \\ \hline
 & 1 & -1 & 0 & 4 & 13
\end{array}$$
The quotient is x^3-x^2+4 and the remainder is 13.

22.
$$\begin{array}{c|rrrrr}
1 & 1 & -5 & -3 & 0 & 10 \\
 & & 1 & -4 & -7 & -7 \\ \hline
 & 1 & -4 & -7 & -7 & 3
\end{array}$$
The quotient is x^3-4x^2-7x-7 and the remainder is 3.

23.
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 4 & -3 & 1 \\ & & 1 & \frac{5}{2} & -\frac{1}{4} \\ \hline & 2 & 5 & -\frac{1}{2} & \frac{3}{4} \end{array}$$
The quotient is $2x^2 + 5x - \frac{1}{2}$ and the remainder is $\frac{3}{4}$.

24.
$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 8 & 1 & 1 \\ & & 1 & 3 & \frac{4}{3} \\ \hline & 3 & 9 & 4 & \frac{7}{3} \end{array}$$
The quotient is $3x^2 + 9x + 4$ and the remainder is $\frac{7}{3}$.

25.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -5 & 3 & 2 \\ & & -1 & 3 & -3 \\ \hline & 2 & -6 & 6 & -1 \end{array}$$
The quotient is $2x^2 - 6x + 6$ and the remainder is -1.

26.
$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -2 & 8 & 2 \\ & & -1 & 1 & -3 \\ \hline & 3 & -3 & 9 & -1 \end{array}$$
The quotient is $3x^2 - 3x + 9$ and the remainder is -1.

27. **a.**
$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & 1 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 5 \end{array}$$
The remainder is 5, so $f(1) = 5$.

b.
$$\begin{array}{r|rrrr} -1 & 1 & 3 & 0 & 1 \\ & & -1 & -2 & 2 \\ \hline & 1 & 2 & -2 & 3 \end{array}$$
The remainder is 3, so $f(-1) = 3$.

c.
$$\begin{array}{r|rrrr} \frac{1}{2} & 1 & 3 & 0 & 1 \\ & & \frac{1}{2} & \frac{7}{4} & \frac{7}{8} \\ \hline & 1 & \frac{7}{2} & \frac{7}{4} & \frac{15}{8} \end{array}$$
The remainder is $\frac{15}{8}$, so $f\left(\frac{1}{2}\right) = \frac{15}{8}$.

d.
$$\begin{array}{r|rrrr} 10 & 1 & 3 & 0 & 1 \\ & & 10 & 130 & 1300 \\ \hline & 1 & 13 & 130 & 1301 \end{array}$$
The remainder is 1301, so $f(10) = 1301$.

28. **a.**
$$\begin{array}{r|rrrr} -2 & 2 & -3 & 0 & 1 \\ & & -4 & 14 & -28 \\ \hline & 2 & -7 & 14 & -27 \end{array}$$
The remainder is –27, so $f(-2) = -27$.

b.
$$\begin{array}{r|rrrr} -1 & 2 & -3 & 0 & 1 \\ & & -2 & 5 & -5 \\ \hline & 2 & -5 & 5 & -4 \end{array}$$
The remainder is –4, so $f(-1) = -4$.

c.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -3 & 0 & 1 \\ & & -1 & 2 & -1 \\ \hline & 2 & -4 & 2 & 0 \end{array}$$
The remainder is 0, so $f\left(-\frac{1}{2}\right) = 0$.

d.
$$\begin{array}{r|rrrr} 7 & 2 & -3 & 0 & 1 \\ & & 14 & 77 & 539 \\ \hline & 2 & 11 & 77 & 540 \end{array}$$
The remainder is 540, so $f(7) = 540$.

29. **a.**
$$\begin{array}{r|rrrrr} 1 & 1 & 5 & -3 & 0 & -20 \\ & & 1 & 6 & 3 & 3 \\ \hline & 1 & 6 & 3 & 3 & -17 \end{array}$$
The remainder is –17, so $f(1) = -17$.

b.
$$\begin{array}{r|rrrrr} -1 & 1 & 5 & -3 & 0 & -20 \\ & & -1 & -4 & 7 & -7 \\ \hline & 1 & 4 & -7 & 7 & -27 \end{array}$$
The remainder is –27, so $f(-1) = -27$.

c.
$$\begin{array}{r|rrrrr} -2 & 1 & 5 & -3 & 0 & -20 \\ & & -2 & -6 & 18 & -36 \\ \hline & 1 & 3 & -9 & 18 & -56 \end{array}$$
The remainder is –56, so $f(-2) = -56$.

d.
$$\begin{array}{r|rrrrr} 2 & 1 & 5 & -3 & 0 & -20 \\ & & 2 & 14 & 22 & 44 \\ \hline & 1 & 7 & 11 & 22 & 24 \end{array}$$
The remainder is 24, so $f(2) = 24$.

30. **a.**
$$\begin{array}{r|rrrrr} 0.1 & 1 & 0.5 & -0.3 & 0 & -20 \\ & & 0.1 & 0.06 & -0.024 & -0.0024 \\ \hline & 1 & 0.6 & -0.24 & -0.024 & -20.0024 \end{array}$$
The remainder is –20.0024, so $f(0.1) = -20.0024$.

b.
$$\begin{array}{r|rrrrr} 0.5 & 1 & 0.5 & -0.3 & 0 & -20 \\ & & 0.5 & 0.5 & 0.1 & 0.05 \\ \hline & 1 & 1.0 & 0.2 & 0.1 & -19.95 \end{array}$$
The remainder is –19.95, so $f(0.5) = -19.95$.

c.
$$\begin{array}{r|rrrrr} 1.7 & 1 & 0.5 & -0.3 & 0 & -20 \\ & & 1.7 & 3.74 & 5.848 & 9.9416 \\ \hline & 1 & 2.2 & 3.44 & 5.848 & -10.0584 \end{array}$$
The remainder is -10.0584, so $f(1.7) = -10.0584$.

d.
$$\begin{array}{r|rrrrr} -2.3 & 1 & 0.5 & -0.3 & 0 & -20 \\ & & -2.3 & 4.14 & -8.832 & 20.3136 \\ \hline & 1 & -1.8 & 3.84 & -8.832 & 0.3136 \end{array}$$
The remainder is 0.3136, so $f(-2.3) = 0.3136$.

31. $f(1) = 2(1)^3 + 3(1)^2 - 6(1) + 1 = 0 \Rightarrow x - 1$ is a factor of $2x^3 + 3x^2 - 6x + 1$. Check as follows:
$$\begin{array}{r|rrrr} 1 & 2 & 3 & -6 & 1 \\ & & 2 & 5 & -1 \\ \hline & 2 & 5 & -1 & 0 \end{array}$$

32. $f(3) = 3(3)^3 - 9(3)^2 - 4(3) + 12 = 0 \Rightarrow x - 3$ is a factor of $3x^3 - 9x^2 - 4x + 12$. Check as follows:
$$\begin{array}{r|rrrr} 3 & 3 & -9 & -4 & 12 \\ & & 9 & 0 & -12 \\ \hline & 3 & 0 & -4 & 0 \end{array}$$

33. $f(-1) = 5(-1)^4 + 8(-1)^3 + (-1)^2 + 2(-1) + 4 = 0 \Rightarrow$

$x + 1$ is a factor of $5x^4 + 8x^3 + x^2 + 2x + 4$. Check as follows:
$$\begin{array}{r|rrrrr} -1 & 5 & 8 & 1 & 2 & 4 \\ & & -5 & -3 & 2 & -4 \\ \hline & 5 & 3 & -2 & 4 & 0 \end{array}$$

34. $f(-3) = 3(-3)^4 + 9(-3)^3 - 4(-3)^2 - 9(-3) + 9 = 0 \Rightarrow$

$x + 3$ is a factor of $3x^4 + 9x^3 - 4x^2 - 9x + 9$. Check as follows:
$$\begin{array}{r|rrrrr} -3 & 3 & 9 & -4 & -9 & 9 \\ & & -9 & 0 & 12 & -9 \\ \hline & 3 & 0 & -4 & 3 & 0 \end{array}$$

35. $f(2) = 2^4 + 2^3 - 2^2 - 2 - 18 = 0 \Rightarrow x - 2$ is a factor of $x^4 + x^3 - x^2 - x - 18$. Check as follows:
$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -1 & -1 & -18 \\ & & 2 & 6 & 10 & 18 \\ \hline & 1 & 3 & 5 & 9 & 0 \end{array}$$

36. $f(-3) = (-3)^5 + 3(-3)^4 + (-3)^2 + 8(-3) + 15 = 0 \Rightarrow$

$x + 3$ is a factor of $x^5 + 3x^4 + x^2 + 8x + 15$. Check as follows:
$$\begin{array}{r|rrrrrr} -3 & 1 & 3 & 0 & 1 & 8 & 15 \\ & & -3 & 0 & 0 & -3 & -15 \\ \hline & 1 & 0 & 0 & 1 & 5 & 0 \end{array}$$

37. $f(-2) = (-2)^6 - (-2)^5 - 7(-2)^4 + (-2)^3 + 8(-2)^2 + 5(-2) + 2 = 0 \Rightarrow$

$x + 2$ is a factor of $x^6 - x^5 - 7x^4 + x^3 + 8x^2 + 5x + 2$. Check as follows:
$$\begin{array}{r|rrrrrrr} -2 & 1 & -1 & -7 & 1 & 8 & 5 & 2 \\ & & -2 & 6 & 2 & -6 & -4 & -2 \\ \hline & 1 & -3 & -1 & 3 & 2 & 1 & 0 \end{array}$$

38. $f(2) = 2(2)^6 - 5(2)^5 + 4(2)^4 + (2)^3 - 7(2)^2 - 7(2) + 2 = 0 \Rightarrow$

$x - 2$ is a factor of $2x^6 - 5x^5 + 4x^4 + x^3 - 7x^2 - 7x + 2$. Check as follows:
$$\begin{array}{r|rrrrrrr} 2 & 2 & -5 & 4 & 1 & -7 & -7 & 2 \\ & & 4 & -2 & 4 & 10 & 6 & -2 \\ \hline & 2 & -1 & 2 & 5 & 3 & -1 & 0 \end{array}$$

39. $f(-1) = 0 = (-1)^3 + 3(-1)^2 + (-1) + k \Rightarrow$
$0 = 1 + k \Rightarrow k = -1$

40. $f(1) = 0 = -1^3 + 4(1)^2 + k(1) - 2 \Rightarrow 1 + k = 0 \Rightarrow$
$k = -1$

41. $f(2) = 0 = 2(2)^3 + (2^2)k - 2k - 2 \Rightarrow$
$14 + 2k = 0 \Rightarrow k = -7$

42. $f(1) = 0 = k^2 - 3(1^2)k - 2(1)k + 6 \Rightarrow$
$k^2 - 5k + 6 = 0 \Rightarrow (k-3)(k-2) = 0 \Rightarrow$
$k = 3$ or $k = 2$

In exercises 43–46, use synthetic division to find the remainder.

43.
$$\begin{array}{r|rrrr} 2 & -2 & 4 & -4 & 9 \\ & & -4 & 0 & -8 \\ \hline & -2 & 0 & -4 & 1 \end{array}$$
The remainder is 1, so $x - 2$ is not a factor of $-2x^3 + 4x^2 - 4x + 9$.

44. $\begin{array}{r|rrrr} -3 & -3 & -9 & 5 & 12 \\ & & 9 & 0 & -15 \\ \hline & -3 & 0 & 5 & -3 \end{array}$

The remainder is –3, so $x+3$ is not a factor of $-3x^3-9x^2+5x+12.$

45. $\begin{array}{r|rrrrr} -2 & 4 & 9 & 3 & 1 & 4 \\ & & -8 & -2 & -2 & 2 \\ \hline & 4 & 1 & 1 & -1 & 6 \end{array}$

The remainder is 6, so $x+2$ is not a factor of $4x^4+9x^3+3x^2+x+4.$

46. $\begin{array}{r|rrrrr} 3 & 3 & -8 & 5 & 7 & -3 \\ & & 9 & 3 & 24 & 93 \\ \hline & 3 & 1 & 8 & 31 & 90 \end{array}$

The remainder is 90, so $x-3$ is not a factor of $3x^4-8x^3+5x^2+7x-3.$

47. The factors of the constant term, 5, are $\{\pm1,\pm5\}$, and the factors of the leading coefficient, 3, are $\{\pm1,\pm3\}$. The possible rational zeros are $\left\{\pm\frac{1}{3},\pm1,\pm\frac{5}{3},\pm5\right\}.$

48. The factors of the constant term, 1, are $\{\pm1\}$, and the factors of the leading coefficient, 2, are $\{\pm1,\pm2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2},\pm1\right\}.$

49. The factors of the constant term, 6, are $\{\pm1,\pm2,\pm3,\pm6\}$, and the factors of the leading coefficient, 4, are $\{\pm1,\pm2,\pm4\}$.The possible rational zeros are $\left\{\pm\frac{1}{4},\pm\frac{1}{2},\pm\frac{3}{4},\pm1,\pm\frac{3}{2},\right.$ $\left.\pm2,\pm3,\pm6\right\}.$

50. The factors of the constant term, –35, are $\{\pm1,\pm5,\pm7\}$, and the factors of the leading coefficient, 6, are $\{\pm1,\pm2,\pm3,\pm6\}$. The possible rational zeros are

$\left\{\pm\frac{1}{6},\pm\frac{1}{2},\pm1,\pm\frac{5}{3},\pm\frac{5}{2},\pm5,\pm7,\pm\frac{35}{2},\pm\frac{1}{3},\pm\frac{5}{6},\right.$ $\left.\pm\frac{7}{6},\pm\frac{7}{3},\pm\frac{7}{2},\pm\frac{35}{6},\pm\frac{35}{3},\pm35\right\}.$

51. The factors of the constant term, 4, are $\{\pm1,\pm2,\pm4\}$, and the factors of the leading coefficient, 1, are $\{\pm1\}$. The possible rational zeros are $\{\pm1,\pm2,\pm4\}$. Use synthetic division to find one rational root:

$\begin{array}{r|rrrr} 1 & 1 & -1 & -4 & 4 \\ & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$

So, $x^3-x^2-4x+4=(x-1)(x^2-4)$
$=(x-1)(x-2)(x+2)\Rightarrow$
the rational zeros are {–2, 1, 2}.

52. The factors of the constant term, 2, are $\{\pm1,\pm2\}$, and the factors of the leading coefficient, 1, are $\{\pm1\}$. The possible rational zeros are $\{\pm1,\pm2\}$. Use synthetic division to find one rational root:

$\begin{array}{r|rrrr} -1 & 1 & 1 & 2 & 2 \\ & & -1 & 0 & -2 \\ \hline & 1 & 0 & 2 & 0 \end{array}$

So, $x^3+x^2+2x+2=(x+1)(x^2+2)\Rightarrow$ the rational zero is {–1}.

53. The factors of the constant term, 6, are $\{\pm1,\pm2,\pm3,\pm6\}$, and the factors of the leading coefficient, 1, are $\{\pm1\}$. The possible rational zeros are $\{\pm1,\pm2,\pm3,\pm6\}$. Use synthetic division to find one rational root:

$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$

So, $x^3-4x^2+x+6=(x+1)(x^2-5x+6)$
$=(x+1)(x-2)(x-3)\Rightarrow$
the rational zeros are {–1, 2, 3}.

54. The factors of the constant term, 6, are $\{\pm1,\pm2,\pm3,\pm6\}$, and the factors of the leading coefficient, 1, are $\{\pm1\}$. The possible rational zeros are $\{\pm1,\pm2,\pm3,\pm6\}$. Use synthetic division to find one rational root:

$\begin{array}{r|rrrr} -3 & 1 & 3 & 2 & 6 \\ & & -3 & 0 & -6 \\ \hline & 1 & 0 & 2 & 0 \end{array}$

So, $x^3+3x^2+2x+6=(x+3)(x^2+2)\Rightarrow$ the rational zero is {–3}.

55. The factors of the constant term, 6, are $\{\pm1, \pm2, \pm3, \pm6\}$, and the factors of the leading coefficient, 2, are $\{\pm1, \pm2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm2, \pm3, \pm6\right\}$.
Use synthetic division to find one rational root:

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$\begin{aligned} 2x^3 + x^2 - 13x + 6 &= (x-2)(2x^2 + 5x - 3) \\ &= (x-2)(x+3)(2x-1) \Rightarrow \end{aligned}$$

the rational zeros are $\left\{-3, \frac{1}{2}, 2\right\}$.

56. The factors of the constant term, –2, are $\{\pm1, \pm2\}$, and the factors of the leading coefficient, 3, are $\{\pm1, \pm3\}$. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm2\right\}$. Use synthetic division to find one rational root:

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 3 & -2 \\ & & 2 & 0 & 2 \\ \hline & 3 & 0 & 3 & 0 \end{array}$$

$3x^3 - 2x^2 + 3x - 2 = \left(x - \frac{2}{3}\right)(3x^2 + 3) \Rightarrow$ the rational zero is $\left\{\frac{2}{3}\right\}$.

57. The factors of the constant term, –2, are $\{\pm1, \pm2\}$, and the factors of the leading coefficient, 6, are $\{\pm1, \pm2, \pm3, \pm6\}$. The possible rational zeros are $\left\{\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{2}{3}, \pm1, \pm2\right\}$.
Use synthetic division to find one rational root:

$$\begin{array}{r|rrrr} -2 & 6 & 13 & 1 & -2 \\ & & -12 & -2 & 2 \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} 6x^3 + 13x^2 + x - 2 &= (x+2)(6x^2 + x - 1) \\ &= (x+2)(2x+1)(3x-1) \Rightarrow \end{aligned}$$

the rational zeros are $\left\{-2, -\frac{1}{2}, \frac{1}{3}\right\}$.

58. The factors of the constant term, 6, are $\{\pm1, \pm2, \pm3, \pm6\}$, and the factors of the leading coefficient, 2, are $\{\pm1, \pm2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm2, \pm3, \pm6\right\}$.
Use synthetic division to find one rational root:

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 3 & 4 & 6 \\ & & -3 & 0 & -6 \\ \hline & 2 & 0 & 4 & 0 \end{array}$$

$2x^3 + 3x^2 + 4x + 6 = \left(x + \frac{3}{2}\right)(2x^2 + 4) \Rightarrow$ the rational zero is $\left\{-\frac{3}{2}\right\}$.

59. The factors of the constant term, 2, are $\{\pm1, \pm2\}$, and the factors of the leading coefficient, 3, are $\{\pm1, \pm3\}$. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm2\right\}$.
Use synthetic division to find one rational root:

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 7 & 8 & 2 \\ & & -1 & -2 & -2 \\ \hline & 3 & 6 & 6 & 0 \end{array}$$

$3x^3 + 7x^2 + 8x + 2 = \left(x + \frac{1}{3}\right)(3x^2 + 6x + 6) \Rightarrow$ the rational zero is $\left\{-\frac{1}{3}\right\}$.

60. The factors of the constant term, 4, are $\{\pm1, \pm2, \pm4\}$, and the factors of the leading coefficient, 2, are $\{\pm1, \pm2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm1, \pm2, \pm4\right\}$. Use synthetic division to find one rational root.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & 8 & 4 \\ & & -1 & 0 & -4 \\ \hline & 2 & 0 & 8 & 0 \end{array}$$

$2x^3 + x^2 + 8x + 4 = \left(x + \frac{1}{2}\right)(2x^2 + 8) \Rightarrow$ the rational zero is $\left\{-\frac{1}{2}\right\}$.

61. The factors of the constant term, –2, are $\{\pm 1, \pm 2\}$, and the factors of the leading coefficient, 1, are $\{\pm 1\}$. The possible rational zeros are $\{\pm 1, \pm 2\}$. Use synthetic division to find one rational root:

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -1 & -1 & -2 \\ & & -1 & 2 & -1 & 2 \\ \hline & 1 & -2 & 1 & -2 & 0 \end{array}$$

So, –1 is a rational zero. Use synthetic division again to find another rational zero:.

$x^4 - x^3 - x^2 - x - 2 = (x-1)(x^3 - 2x^2 + x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 1 & -2 \\ & & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$x^4 - x^3 - x^2 - x - 2 = (x+1)(x-2)(x^2+1) \Rightarrow$ the rational zeros are {–1, 2}.

62. The factors of the constant term, –4, are $\{\pm 1, \pm 2, \pm 4\}$, and the factors of the leading coefficient, 2, are $\{\pm 1, \pm 2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4\right\}$. Use synthetic division to find one rational root:

$$\begin{array}{r|rrrrr} -1 & 2 & 3 & 8 & 3 & -4 \\ & & -2 & -1 & -7 & 4 \\ \hline & 2 & 1 & 7 & -4 & 0 \end{array}$$

So, –1 is a rational zero. Use synthetic division again to find another rational zero:

$$\begin{aligned} &2x^4 + 3x^3 + 8x^2 + 3x - 4 \\ &\qquad = (x+1)(2x^3 + x^2 + 7x - 4) \end{aligned}$$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 1 & 7 & -4 \\ & & 1 & 1 & 4 \\ \hline & 2 & 2 & 8 & 0 \end{array}$$

$$\begin{aligned} &2x^4 + 3x^3 + 8x^2 + 3x - 4 \\ &\qquad = (x+1)\left(x - \frac{1}{2}\right)(2x^2 + 2x + 8) \\ &\qquad = 2(x+1)\left(x - \frac{1}{2}\right)(x^2 + x + 4) \Rightarrow \end{aligned}$$

the rational zeros are $\left\{-1, \frac{1}{2}\right\}$.

63. The factors of the constant term, 12, are $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$, and the factors of the leading coefficient, 1, are $\{\pm 1\}$. The possible rational zeros are $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$. Use synthetic division to find one rational root:

$$\begin{array}{r|rrrrr} -3 & 1 & -1 & -13 & 1 & 12 \\ & & -3 & 12 & 3 & -12 \\ \hline & 1 & -4 & -1 & 4 & 0 \end{array}$$

So, –3 is a rational zero. Use synthetic division again to find another rational zero:

$$\begin{aligned} &x^4 - x^3 - 13x^2 + x + 12 \\ &\qquad = (x+3)(x^3 - 4x^2 - x + 4) \end{aligned}$$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & -1 & 4 \\ & & -1 & 5 & -4 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

So, –1 is also a rational zero.

$$\begin{aligned} &x^4 - x^3 - 13x^2 + x + 12 \\ &\qquad = (x+3)(x+1)(x^2 - 5x + 4) \\ &\qquad = (x+3)(x+1)(x-4)(x-1) \Rightarrow \end{aligned}$$

the rational zeros are {–3, –1, 1, 4}.

64. The factors of the constant term, –2, are $\{\pm 1, \pm 2\}$, and the factors of the leading coefficient, 3, are $\{\pm 1, \pm 3\}$. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm 2\right\}$. Use synthetic division to find one rational root:

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & 1 & 5 & -2 \\ & & -6 & 2 & -6 & 2 \\ \hline & 3 & -1 & 3 & -1 & 0 \end{array}$$

So, –2 is a rational zero. Use synthetic division again to find another rational zero:

$$\begin{aligned} &3x^4 + 5x^3 + x^2 + 5x - 2 \\ &\qquad = (x+2)(3x^3 - x^2 + 3x - 1) \end{aligned}$$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & 3 & -1 \\ & & 1 & 0 & 1 \\ \hline & 3 & 0 & 3 & 0 \end{array}$$

$$\begin{aligned} &3x^4 + 5x^3 + x^2 + 5x - 2 \\ &\qquad = (x+2)\left(x - \frac{1}{3}\right)(3x^2 + 3) \\ &\qquad = 3(x+2)\left(x - \frac{1}{3}\right)(x^2 + 1) \Rightarrow \end{aligned}$$

the rational zeros are $\left\{-2, \frac{1}{3}\right\}$.

65. The factors of the constant term, 1, are $\{\pm 1\}$, and the factors of the leading coefficient, 1, are $\{\pm 1\}$. The possible rational zeros are $\{\pm 1\}$. Use synthetic division to find one rational root:

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & 10 & -1 & 1 \\ & & 1 & -1 & 9 & 8 \\ \hline & 1 & -1 & 9 & 8 & 9 \end{array}$$

The remainder is 9 so, 1 is not a rational zero.
Try −1:

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & 10 & -1 & 1 \\ & & -1 & 3 & -13 & 14 \\ \hline & 1 & -3 & 13 & -14 & 15 \end{array}$$

The remainder is 15 so, −1 is not a rational zero. Therefore, there are no rational zeros.

66. The factors of the constant term, 2, are $\{\pm 1, \pm 2\}$, and the factors of the leading coefficient, 1, are $\{\pm 1\}$. The possible rational zeros are $\{\pm 1, \pm 2\}$. Testing each value $\big(f(-2), f(-1), f(1), f(2)\big)$ shows that none cause the value of the function to equal zero, so there are no rational zeros.

67. $f(x) = x^3 + 5x^2 - 8x + 2$

The possible rational zeros are $\{\pm 1, \pm 2\}$.

$$\begin{array}{r|rrrr} 1 & 1 & 5 & -8 & 2 \\ & & 1 & 6 & -2 \\ \hline & 1 & 6 & -2 & 0 \end{array}$$

So, 1 is a zero. Now solve the depressed equation $x^2 + 6x - 2 = 0$.

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(-2)}}{2(1)} = \frac{-6 \pm \sqrt{44}}{2}$$
$$= -3 \pm \sqrt{11}$$

The solution set is $\{1, -3 \pm \sqrt{11}\}$.

68. $f(x) = x^3 - 7x^2 - 5x + 3$

The possible rational zeros are $\{\pm 1, \pm 3\}$.

$$\begin{array}{r|rrrr} -1 & 1 & -7 & -5 & 3 \\ & & -1 & 8 & -3 \\ \hline & 1 & -8 & 3 & 0 \end{array}$$

So, −1 is a zero. Now solve the depressed equation $x^2 - 8x + 3 = 0$.

$$x = \frac{8 \pm \sqrt{64 - 4(1)(3)}}{2(1)} = \frac{8 \pm \sqrt{52}}{2}$$
$$= 4 \pm \sqrt{13}$$

The solution set is $\{-1, 4 \pm \sqrt{13}\}$.

69. $f(x) = x^4 - 3x^3 + 3x - 1$

The possible rational zeros are $\{\pm 1\}$.

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 0 & 3 & -1 \\ & & 1 & -2 & -2 & 1 \\ \hline & 1 & -2 & -2 & 1 & 0 \end{array}$$

So, 1 is a zero. Now find a zero of the depressed equation $g(x) = x^3 - 2x^2 - 2x + 1$.

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -2 & 1 \\ & & -1 & 3 & -1 \\ \hline & 1 & -3 & 1 & 0 \end{array}$$

So, −1 is a zero. Now solve the depressed equation $x^2 - 3x + 1 = 0$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{5}}{2}$$

The solution set is $\left\{\pm 1, \frac{3 \pm \sqrt{5}}{2}\right\}$.

70. $f(x) = x^4 - 6x^3 - 7x^2 + 54x - 18$

The possible rational zeros are $\{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18\}$.

$$\begin{array}{r|rrrrr} 3 & 1 & -6 & -7 & 54 & -18 \\ & & 3 & -9 & -48 & 18 \\ \hline & 1 & -3 & -16 & 6 & 0 \end{array}$$

So, 3 is a zero. Now find a zero of the depressed equation $g(x) = x^3 - 3x^2 - 16x + 6$.

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -16 & 6 \\ & & -3 & 18 & -6 \\ \hline & 1 & -6 & 2 & 0 \end{array}$$

So, −3 is a zero. Now solve the depressed equation $x^2 - 6x + 2 = 0$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(2)}}{2(1)} = \frac{6 \pm \sqrt{28}}{2} = 3 \pm \sqrt{7}$$

The solution set is $\{\pm 3, 3 \pm \sqrt{7}\}$.

71. $f(x) = 2x^3 - 9x^2 + 6x - 1$

The function has degree 3, so there are three zeros, one or three possible positive zeros; no possible negative zeros.

The possible rational zeros are $\left\{\pm 1, \pm\frac{1}{2}\right\}$.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -9 & 6 & -1 \\ & & 1 & -4 & 1 \\ \hline & 2 & -8 & 2 & 0 \end{array}$$

So, $1/2$ is a zero. Now solve the depressed equation $2x^2 - 8x + 2 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(2)}}{2(2)} = \frac{8 \pm \sqrt{48}}{4}$$
$$= 2 \pm \sqrt{3}$$

Solution set: $\left\{2 - \sqrt{3}, \frac{1}{2}, 2 + \sqrt{3}\right\}$.

72. $f(x) = 2x^3 - 3x^2 - 4x - 1$

The function has degree 3, so there are three zeros, one possible positive zero; zero or two possible negative zeros.

The possible rational zeros are $\left\{\pm 1, \pm\frac{1}{2}\right\}$.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -3 & -4 & -1 \\ & & -1 & 2 & 1 \\ \hline & 2 & -4 & -2 & 0 \end{array}$$

So, $-1/2$ is a zero. Now solve the depressed equation $2x^2 - 4x - 2 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-2)}}{2(2)} = \frac{4 \pm \sqrt{32}}{4}$$
$$= 1 \pm \sqrt{2}$$

Solution set: $\left\{1 - \sqrt{2}, -\frac{1}{2}, 1 + \sqrt{2}\right\}$.

73. $f(x) = x^4 + x^3 - 5x^2 - 3x + 6$

The function has degree 4, so there are four zeros, zero or two possible positive zeros; zero or two possible negative zeros.
The possible rational zeros are
$\{\pm 1, \pm 2, \pm 3, \pm 6\}$.

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -5 & -3 & 6 \\ & & 1 & 2 & -3 & -6 \\ \hline & 1 & 2 & -3 & -6 & 0 \end{array}$$

So, 1 is a zero. Now find a zero of the depressed equation $x^3 + 2x^2 - 3x - 6 = 0$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -3 & -6 \\ & & -2 & 0 & 6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

So, -2 is a zero. Now solve the depressed equation $x^2 - 3 = 0$.

$x^2 - 3 = 0 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

Solution set: $\{-2, -\sqrt{3}, 1, \sqrt{3}\}$

74. $f(x) = x^4 - 2x^3 - 5x^2 + 4x + 6$

The function has degree 4, so there are four zeros, zero or two possible positive real zeros; zero or two possible negative real zeros.
The possible rational zeros are
$\{\pm 1, \pm 2, \pm 3, \pm 6\}$.

$$\begin{array}{r|rrrrr} 3 & 1 & -2 & -5 & 4 & 6 \\ & & 3 & 3 & -6 & -6 \\ \hline & 1 & 1 & -2 & -2 & 0 \end{array}$$

So, 3 is a zero. Now find a zero of the depressed equation $x^3 + x^2 - 2x - 2 = 0$.

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -2 & -2 \\ & & -1 & 0 & 2 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

So, -1 is a zero. Now solve the depressed equation $x^2 - 2 = 0$.

$x^2 - 2 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

Solution set: $\{-\sqrt{2}, -1, \sqrt{2}, 3\}$

75. $f(x) = 2x^3 - 5x^2 + x + 2$

The possible rational zeros are $\pm\frac{1}{2}, \pm 1, \pm 2$.

Using synthetic division, we find that $x = 1$ is a zero.

$$\begin{array}{r|rrrr} 1 & 2 & -5 & 1 & 2 \\ & & 2 & -3 & -2 \\ \hline & 2 & -3 & -2 & 0 \end{array}$$

Now factor $Q(x) = 2x^2 - 3x - 2$.

$2x^2 - 3x - 2 = (2x + 1)(x - 2)$

So, the complete factorization of $P(x)$ is $(x - 1)(2x + 1)(x - 2)$.

76. $f(x)=2x^3+x^2-4x-3$

The possible rational zeros are
$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm1, \pm3.$

Using synthetic division, we find that $x=-1$ is a zero.

$$\begin{array}{r|rrrr} \underline{-1} & 2 & 1 & -4 & -3 \\ & & -2 & 1 & 3 \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

Now factor $Q(x)=2x^2-x-3.$

$2x^2-x-3=(2x-3)(x+1)$

So, the complete factorization of $P(x)$ is

$(x+1)^2(2x-3).$

77. $f(x)=2x^3+3x^2-17x-30$

The possible rational zeros are
$\pm1, \pm2, \pm3, \pm5, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}.$

Using synthetic division, we find that $x=-2$ is a zero.

$$\begin{array}{r|rrrr} \underline{-2} & 2 & 3 & -17 & -30 \\ & & -4 & 2 & 30 \\ \hline & 2 & -1 & -15 & 0 \end{array}$$

Now factor $Q(x)=2x^2-x-15.$

$2x^2-x-15=(2x+5)(x-3)$

So, the complete factorization of $P(x)$ is

$(x+2)(2x+5)(x-3).$

78. $f(x)=2x^3-3x^2-23x+12$

The possible rational zeros are
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{3}{2}.$

Using synthetic division, we find that $x=4$ is a zero.

$$\begin{array}{r|rrrr} \underline{4} & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

Now factor $Q(x)=2x^2+5x-3.$

$2x^2+5x-3=(2x-1)(x+3)$

So, the complete factorization of $P(x)$ is

$(x-4)(2x-1)(x+3).$

79. $f(x)=x^4-x^3-9x^2+11x+6$

The possible rational zeros are
$\pm1, \pm2, \pm3, \pm6.$

Using synthetic division, we find that $x=2$ is a zero.

$$\begin{array}{r|rrrrr} \underline{2} & 1 & -1 & -9 & 11 & 6 \\ & & 2 & 2 & -14 & -6 \\ \hline & 1 & 1 & -7 & -3 & 0 \end{array}$$

So, 2 is a zero. Now find a zero of the depressed function $Q(x)=x^3+x^2-7x-3.$

$$\begin{array}{r|rrrr} \underline{-3} & 1 & 1 & -7 & -3 \\ & & -3 & 6 & 3 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

So, −3 is a zero. Now find the zeros of the depressed function x^2-2x-1 using the quadratic formula.

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}=\frac{2\pm\sqrt{8}}{2}$$
$$=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$$

So, the complete factorization of $P(x)$ is

$(x-2)(x+3)\left(x-\left(1-\sqrt{2}\right)\right)\left(x-\left(1+\sqrt{2}\right)\right)$, or

$(x-2)(x+3)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right).$

80. $f(x)=x^4-6x^3+x^2+30x-8$

The possible rational zeros are
$\pm1, \pm2, \pm4, \pm8.$

Using synthetic division, we find that $x=-2$ is a zero.

$$\begin{array}{r|rrrrr} \underline{-2} & 1 & -6 & 1 & 30 & -8 \\ & & -2 & 16 & -34 & 8 \\ \hline & 1 & -8 & 17 & -4 & 0 \end{array}$$

So, –2 is a zero. Now find a zero of the depressed function $Q(x)=x^3-8x^2+17x-4.$

$$\begin{array}{r|rrrr} \underline{4} & 1 & -8 & 17 & -4 \\ & & 4 & -16 & 4 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

So, 4 is a zero. Now find the zeros of the depressed function x^2-4x+1 using the quadratic formula.

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{12}}{2}$$
$$=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$$

So, the complete factorization of $P(x)$ is

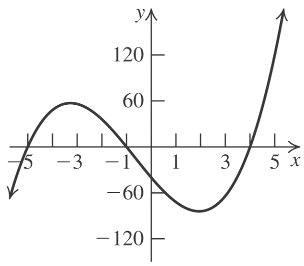
$(x+2)(x-4)\left(x-\left(2-\sqrt{3}\right)\right)\left(x-\left(2+\sqrt{3}\right)\right)$, or

$(x+2)(x-4)\left(x-2+\sqrt{3}\right)\left(x-2-\sqrt{3}\right).$

For exercises 81–88, use synthetic division to find the zeros of each function and then use the methods shown in Section 2.2 to complete the graph.

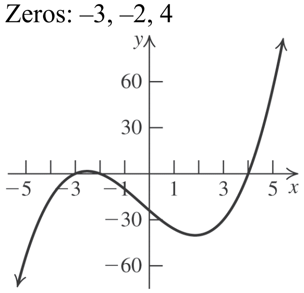
81. $f(x) = x^3 + 2x^2 - 19x - 20$

Zeros: –5, –1, 4



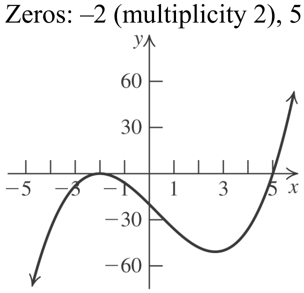
82. $f(x) = x^3 + x^2 - 14x - 24$

Zeros: –3, –2, 4



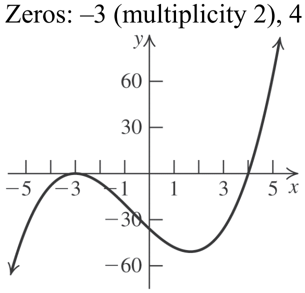
83. $f(x) = x^3 - x^2 - 16x - 20$

Zeros: –2 (multiplicity 2), 5



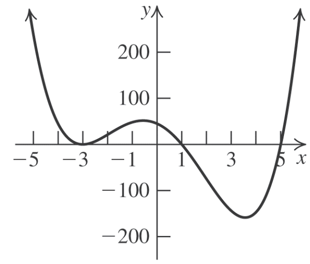
84. $f(x) = x^3 + 2x^2 - 15x - 36$

Zeros: –3 (multiplicity 2), 4



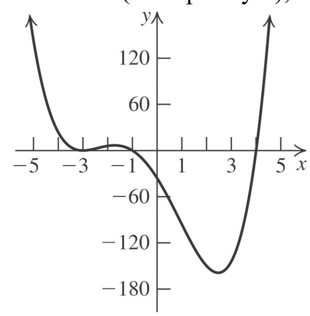
85. $f(x) = x^4 - 22x^2 - 24x + 45$

Zeros: –3 (multiplicity 2), 1, 5



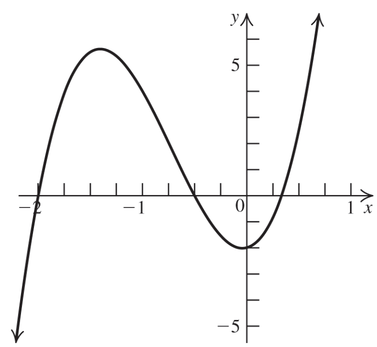
86. $f(x) = x^4 + 3x^3 - 13x^2 - 51x - 36$

Zeros: –3 (multiplicity 2), –1, 4



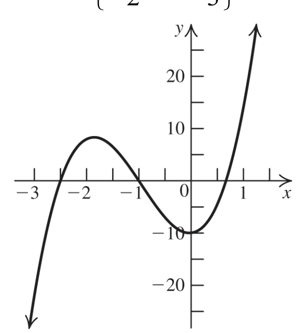
87. $f(x) = 6x^3 + 13x^2 + x - 2$

Zeros: $\left\{-2, -\frac{1}{2}, \frac{1}{3}\right\}$



88. $f(x) = 6x^3 + 17x^2 + x - 10$

Zeros: $\left\{-\frac{5}{2}, -1, \frac{2}{3}\right\}$



Applying the Concepts

89. $A = lw \Rightarrow l = \dfrac{A}{w} \Rightarrow$

$$\begin{array}{r}
2x^2 + 1 \\
x^2 - x + 2 \overline{\big) 2x^4 - 2x^3 + 5x^2 - x + 2} \\
-\,(\underline{2x^4 - 2x^3 + 4x^2}) \\
x^2 - x + 2 \\
-\,(\underline{x^2 - x + 2}) \\
0
\end{array}$$

The width is $2x^2 + 1$.

90. $V = lwh \Rightarrow h = \dfrac{V}{lw}$

$lw = (x+3)(x+1) = x^2 + 4x + 3$

$$\begin{array}{r}
x^2 - x + 1 \\
x^2 + 4x + 3 \overline{\big) x^4 + 3x^3 + 0x^2 + x + 3} \\
-\,(\underline{x^4 + 4x^3 + 3x^2}) \\
-x^3 - 3x^2 + x \\
-\,(\underline{-x^3 - 4x^2 - 3x}) \\
x^2 + 4x + 3 \\
-\,(\underline{x^2 + 4x + 3}) \\
0
\end{array}$$

The height is $x^2 - x + 1$.

91. **a.** $R(40) = 3000 \Rightarrow R(40) - 3000 = 0$ and $R(60) = 3000 \Rightarrow R(60) - 3000 = 0$. Thus, 40 and 60 are zeros of $R(x) - 3000$.

Therefore,

$$\begin{aligned} R(x) - 3000 &= a(x-40)(x-60) \\ &= a\left(x^2 - 100x + 2400\right) \end{aligned}$$

Since (30, 2400) lies on $R(x)$, we have

$2400 - 3000 = a(30-40)(30-60) \Rightarrow$

$-600 = a(-10)(-30) \Rightarrow a = -2$

Thus

$$\begin{aligned} R(x) - 3000 &= -2(x-40)(x-60) \\ &= -2x^2 + 200x - 4800 \end{aligned}$$

b. $R(x) - 3000 = -2x^2 + 200x - 4800 \Rightarrow$

$R(x) = -2x^2 + 200x - 1800$

c. The maximum weekly revenue occurs at the vertex of the function, $\left(-\dfrac{b}{2a}, R\left(-\dfrac{b}{2a}\right)\right)$.

$-\dfrac{b}{2a} = -\dfrac{200}{2(-2)} = 50$

$$\begin{aligned} R(50) &= -2(50)^2 + 200(50) - 1800 \\ &= 3200 \end{aligned}$$

The maximum revenue is \$3200 if the phone is priced at \$50.

92. **a.** $R(8) = 4725 \Rightarrow R(8) - 4725 = 0.$ and $R(12) = 4725 \Rightarrow R(12) - 4725 = 0.$ Thus, 8 and 12 are zeros of $R(x) - 4725$. Therefore,

$$\begin{aligned} R(x) - 4725 &= a(x-8)(x-12) \\ &= a\left(x^2 - 20x + 96\right) \end{aligned}$$

Since (6, 4125) lies on $R(x)$, we have

$4125 - 4725 = a(6-8)(6-12) \Rightarrow$

$-600 = a(-2)(-6) \Rightarrow a = -50$

Thus,

$$\begin{aligned} R(x) - 4725 &= -50(x-8)(x-12) \\ &= -50x^2 + 1000x - 4800 \end{aligned}$$

b. $R(x) - 4725 = -50x^2 + 1000x - 4800 \Rightarrow$

$R(x) = -50x^2 + 1000x - 75$

c. The maximum weekly revenue occurs at the vertex of the function, $\left(-\dfrac{b}{2a}, R\left(-\dfrac{b}{2a}\right)\right)$.

$-\dfrac{b}{2a} = -\dfrac{1000}{2(-50)} = 10$

$$\begin{aligned} R(0) &= -50(10)^2 + 1000(10) - 75 \\ &= 4925 \end{aligned}$$

The maximum revenue is \$4925 if tickets are priced at \$10 each.

93. Since $t = 11$ represents 2002, we have $C(11) = 97.6 \Rightarrow C(x) = (x-11)Q(x) + 97.6 \Rightarrow$ $C(t) - 97.6 = (t-11)Q(t)$. So

$$\begin{aligned}-0.0006t^3 - 0.0613t^2 + 2.0829t + 82.904 - 97.6 &= -0.0006t^3 - 0.0613t^2 + 2.0829t - 14.6960\\ &= (x-11)Q(x).\end{aligned}$$

Use synthetic division to find $Q(x)$:

$$\begin{array}{r|rrrr} 11 & -0.0006 & -0.0613 & 2.0829 & -14.6960\\ & & -0.0066 & -0.7469 & 14.6960\\ \hline & -0.0006 & -0.0679 & 1.3360 & 0\end{array}$$

Now solve the depressed equation to find another zero:

$$-0.0006^2 - 0.0679t + 1.3360 = 0 \Rightarrow t = \frac{0.0679 \pm \sqrt{(-0.0679)^2 - 4(-0.0006)(1.3360)}}{2(-0.0006)} \Rightarrow$$

$$t = \frac{0.0679 \pm \sqrt{0.00781681}}{-0.0012} = \frac{0.0679 \pm 0.0884}{-0.0012} \Rightarrow t \approx -130.26 \text{ or } t \approx 17.0939.$$

Since we must find t greater than 0, $t = 17$, and the year is 1991 + 17 = 2008.

94. Since $t = 1$ represents 1999, we have $s(1) = 737.7 \Rightarrow s(t) = (t-1)Q(t) + 737.7 \Rightarrow s(t) - 737.7 = (t-1)Q(t)$. So

$$\begin{aligned}-0.1779t^3 - 2.8292t^2 + 42.0240t + 698.6831 - 737.7 &= -0.1779t^3 - 2.8292t^2 + 42.0240t - 39.0169\\ &= (t-1)Q(t)\end{aligned}$$

Use synthetic division to find $Q(t)$:

$$\begin{array}{r|rrrr} 1 & -0.1779 & -2.8292 & 42.0240 & -39.0169\\ & & -0.1779 & -3.0071 & 39.0169\\ \hline & -0.1779 & -3.0071 & 39.0169 & 0\end{array}$$

Now solve the depressed equation to find another zero:

$$-0.1779t^2 - 3.0071t + 39.0169 = 0 \Rightarrow t = \frac{3.0071 \pm \sqrt{(-3.0071)^2 - 4(-0.1779)(39.0169)}}{2(-0.1779)} \Rightarrow$$

$$t = \frac{3.0071 \pm \sqrt{36.8071}}{-0.3558} \Rightarrow t \approx -25.5030 \text{ or } t \approx 8.5997$$

Since t must be positive, $t \approx 9$, and the year is 1998 + 9 = 2007.

95. $M(t) = -0.0027t^3 + 0.3681t^2 - 5.8645t + 195.2782$

Since $M(2) = 185.0$, $M(t) = (t-2)Q(t) + 195.2782 \Rightarrow M(t) - 195.2782 = (t-2)Q(t)$

We must find two other zeros of

$$\begin{aligned}F(t) = M(t) - 185 &= -0.0027t^3 + 0.3681t^2 - 5.8645t + 195.2782 - 185\\ &= -0.0027t^3 + 0.3681t^2 - 5.8645t + 10.2782\end{aligned}$$

Because 2 is a zero of $F(t)$, use synthetic division to find $Q(t)$.

$$\begin{array}{r|rrrr} 2 & -0.0027 & 0.3681 & -5.8645 & 10.2782\\ & & -0.0054 & 0.7254 & -10.2782\\ \hline & -0.0027 & 0.3627 & -5.1391 & 0\end{array}$$

Now use the quadratic formula to solve the depressed equation.

$$t = \frac{-0.3627 \pm \sqrt{0.3627^2 - 4(-0.0027)(-5.1391)}}{2(-0.0027)} = \frac{-0.3627 \pm \sqrt{0.0760}}{-0.0054} \Rightarrow t \approx 16.1 \text{ or } t \approx 118$$

Thus, the model shows that the Marine Corps had about 186,000 when $t \approx 16$, or in the year 1990 + 16 = 2006.

96. $U(t) = -0.0374t^3 + 0.5934t^2 - 2.0553t + 6.7478$

Since $U(6) = 7.7,\ U(t) = (t-6)Q(x) + 7.7 \Rightarrow U(t) - 7.7 = (t-6)Q(6)$

We must find two other zeros of

$F(t) = U(t) - 7.7 = -0.0374t^3 + 0.5934t^2 - 2.0553t + 6.7478 - 7.7$

$= -0.0374t^3 + 0.5934t^2 - 2.0553t - 0.9522$

Because 6 is a zero of $F(t)$, use synthetic division to find $Q(t)$.

$$\begin{array}{r|rrrr} 6 & -0.0374 & 0.5934 & -2.0553 & -0.9522 \\ & & -0.2244 & 2.2140 & 0.9522 \\ \hline & -0.0374 & 0.3690 & 0.1587 & 0 \end{array}$$

Now use the quadratic formula to solve the depressed equation.

$$t = \frac{-0.3690 \pm \sqrt{0.3690^2 - 4(-0.0374)(0.1587)}}{2(-0.0374)} = \frac{-0.3690 \pm \sqrt{0.1599}}{-0.0748} \Rightarrow t \approx -0.41 \text{ or } t \approx 10.3$$

Reject the negative solution.

According to the model, there was a 7.7% unemployment level in 2002 + 10 = 2012.

97. Use synthetic division to solve the equation $628 = 3x^3 - 6x^2 + 108x + 100 \Rightarrow 3x^3 - 6x^2 + 108x - 528 = 0$. The factors of the constant term, −528, are $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm11, \pm48, \pm66, \pm88, \pm132, \pm176, \pm264, \pm528\}$. The factors of the leading coefficient, 3, are $\{\pm1, \pm3\}$. Only the positive, whole number possibilities make sense for the problem, so the possible rational zeros are {1, 2, 3, 4, 6, 8, 11, 16, 22, 44, 48, 66, 88, 132, 176, 264, 528}.

$$\begin{array}{r|rrrr} 4 & 3 & -6 & 108 & -528 \\ & & 12 & 24 & 528 \\ \hline & 3 & 6 & 132 & 0 \end{array}$$

Thus, $x = 4$.

98. The demand function is $p(x) = 330 + 10x - x^2 \Rightarrow$ the revenue function is $330x + 10x^2 - x^3$.

Profit = revenue – cost = $910 = (330x + 10x^2 - x^3) - (3x^3 - 6x^2 + 108x + 100) \Rightarrow$

$910 = -4x^3 + 16x^2 + 222x - 100 \Rightarrow 0 = -4x^3 + 16x^2 + 222x - 1010$. The factors of the constant term, –1010, are $\{\pm1, \pm2, \pm5, \pm10, \pm101, \pm202, \pm505, \pm1010\}$. The factors of the leading coefficient, –4, are $\{\pm1, \pm2, \pm4\}$.

Find the zero using synthetic division:

$$\begin{array}{r|rrrr} 5 & -4 & 16 & 222 & -1010 \\ & & -20 & -20 & 1010 \\ \hline & -4 & -4 & 202 & 0 \end{array}$$

Thus, $x = 5$.

99. The cost function gives the result as a number of thousands, so set it equal to 125:

$x^3 - 15x^2 + 5x + 50 = 125 \Rightarrow x^3 - 15x^2 + 5x - 75 = 0$. The factors of the constant term, –75, are $\{\pm1, \pm3, \pm5, \pm15, \pm25, \pm75\}$. The factors of the leading coefficient, 1, are $\{\pm1\}$. Only the positive solutions make sense for the problem, so the possible rational zeros are {1, 3, 5, 15, 25, 75}. Use synthetic division to find the zero:

$$\begin{array}{r|rrrr} 15 & 1 & -15 & 5 & -75 \\ & & 15 & 0 & 75 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

The total monthly cost is $125,000 when 15 hundred (1500) units are produced.

100. The demand function is $p(x) = -3x + 3 + \dfrac{74}{x}$, so the revenue function is $xp(x) = -3x^2 + 3x + 74.$

The break-even point occurs when revenue – cost = 0:

$$\begin{aligned}(-3x^2 + 3x + 74) - (x^3 - 15x^2 + 5x + 50) &= 0\\ -x^3 + 12x^2 - 2x + 24 &= 0\\ (-x^3 - 2x) + (12x^2 + 24) &= 0\\ -x(x^2 + 2) + 12(x^2 + 2) &= 0\\ (12 - x)(x^2 + 2) &= 0\\ x &= 12\end{aligned}$$

So 1200 printers must be sold to break even.

101. Let x represent the edge of the taller box. Then $x - 3$ represents the edge of the shorter box. The volume of the taller box is x^3, and the volume of the shorter box is $(x-3)^3$. We must solve $x^3 + (x-3)^3 = 1843.$

$$\begin{aligned}x^3 + (x-3)^3 &= 1843\\ x^3 + x^3 - 9x^2 + 27x - 27 &= 1843\\ 2x^3 - 9x^2 + 27x - 1870 &= 0\end{aligned}$$

There are many possible rational zeros, so we start with an educated guess. The edge must be greater than 3 and less than 12. (Note that $12^3 = 1728$, which is close to the total volume of the two boxes.) The possible rational zeros between 3 and 12 are 5, 10, 11, and $\frac{11}{2}$. Using synthetic division, we find that 11 is a zero.

$$\begin{array}{r|rrrr} 11 & 2 & -9 & 27 & -1870\\ & & 22 & 143 & 1870\\ \hline & 2 & 13 & 170 & 0\end{array}$$

Using the quadratic formula to find the zeros of $2x^2 + 13x + 170$, we have

$$x = \frac{-13 \pm \sqrt{13^2 - 4(2)(170)}}{2(2)} = \frac{-13 \pm \sqrt{-1191}}{4},$$

which is not a real number. Thus, there are no other real zeros.

Therefore, the taller box has edge 11 inches and the shorter box has edge 11 – 3 = 8 inches.

102. Let x represent the width of the pool. Then the volume of the deep water section is x^3, and the volume of the shallow water section is $3x$. The total volume is 536 cubic meters, so we have $x^3 + 3x = 536 \Rightarrow x^3 + 3x - 536 = 0.$

There are many possible rational zeros, so we start with an educated guess. The edge must be greater than 1 and less than 9. (Note that $9^3 = 729$, which is more the total volume of the pool.) The possible rational zeros between 1 and 8 are 2, 4, 8. Using synthetic division, we find that 8 is a zero.

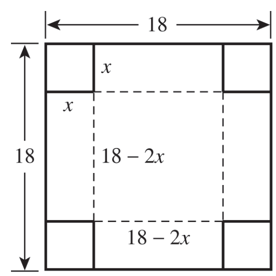
$$\begin{array}{r|rrrr} 8 & 1 & 0 & 3 & -536\\ & & 8 & 64 & 536\\ \hline & 1 & 8 & 67 & 0\end{array}$$

Using the quadratic formula to find the zeros of $x^2 + 8x + 67$, we have

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(67)}}{2(1)} = \frac{-8 \pm \sqrt{-204}}{2},$$ which is not a real number. Thus, there are no other real zeros.

Therefore, the dimensions of the deep water section are $8\text{ m} \times 8\text{ m} \times 8\text{ m}$.

103. The length and width of the box are $18 - 2x$, and the height is x, so the volume is $x(18 - 2x)^2$.



$432 = x(18 - 2x)^2 \Rightarrow$

$432 = 4x^3 - 72x^2 + 324x \Rightarrow$

$4x^3 - 72x^2 + 324x - 432 = 0.$

The factors of the constant term, 432, are $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm9, \pm12, \pm16, \pm18, \pm24, \pm27, \pm36, \pm48, \pm54, \pm72, \pm108, \pm144, \pm216, \pm432\}$.

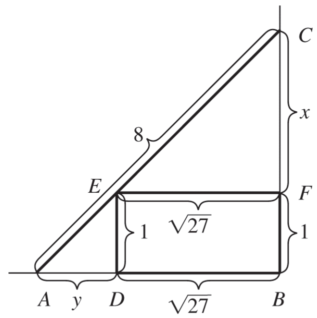
The factors of the leading coefficient, 4, are $\{\pm1, \pm2, \pm4\}$. There are many possible rational zeros, but the only ones that make sense for the problem are $\left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, 3, 4, 6, 8\right\}$.

Use synthetic division to find the zero:

$$\begin{array}{r|rrrr} 3 & 4 & -72 & 324 & -432\\ & & 12 & -180 & 432\\ \hline & 4 & -60 & 144 & 0\end{array}$$

The corners should be 3 inches by 3 inches.

104.



From the similar right triangles $\triangle EFC$ and $\triangle ABC$, we have

$$\frac{\sqrt{27}}{x} = \frac{y+\sqrt{27}}{x+1}, \text{ or } y+\sqrt{27} = \sqrt{27}\cdot\frac{x+1}{x}.$$

Using the Pythagorean theorem in right triangle ABC gives

$$(x+1)^2 + \left(y+\sqrt{27}\right)^2 = 8^2.$$

Now substitute.

$$(x+1)^2 + \left(\sqrt{27}\cdot\frac{x+1}{x}\right)^2 = 8^2.$$

Multiply both sides by x^2 to eliminate the fraction.

$$x^2(x+1)^2 + \left(\sqrt{27}(x+1)\right)^2 = 8^2x^2$$

Now expand and simplify.

$$x^2\left(x^2+2x+1\right) + 27\left(x^2+2x+1\right) = 64x^2$$
$$x^4+2x^3+x^2+27x^2+54x+27 = 64x^2$$
$$x^4+2x^3-36x^2+54x+27 = 0 = P(x)$$

The possible rational zeros of the polynomial are $\pm1, \pm3, \pm9, \pm27$.

We know that x must be a positive number less than 8. Using synthetic division, we see that $x = 3$ is a zero of $P(x)$.

3	1	2	−36	54	27
		3	15	−63	−27
	1	5	−21	−9	0

So, the remainder is $Q(x) = x^3+5x^2-21x-9$.

There are no other real zeros.

$$P(x) = x^4+2x^3-36x^2+54x+27$$
$$= (x-3)\left(x^3+5x^2-21x-9\right)$$

The possible rational zeros of $Q(x)$ are $\pm1, \pm3, \pm9$. Using synthetic division, we see that $x = 3$ is a zero of $Q(x)$.

3	1	5	−21	−9
		3	24	9
	1	8	3	0

Thus,

$$P(x) = x^4+2x^3-36x^2+54x+27$$
$$= (x-3)^2\left(x^2+8x+3\right)$$

Since the height = $x + 1$, the top of the ladder touches the wall at 3 + 1 = 4 feet above the ground.

Beyond the Basics

105. Divide $4x^3+8x^2-11x+3$ by $\left(x-\frac{1}{2}\right)$:

1/2	4	8	−11	3
		2	5	−3
	4	10	−6	0

Now divide $4x^2+10x-6$ by $\left(x-\frac{1}{2}\right)$:

1/2	4	10	−6
		2	6
	4	12	0

Since $\left(x-\frac{1}{2}\right)$ does not divide $4x+12$, $\left(x-\frac{1}{2}\right)$ is a root of multiplicity 2 of $4x^3+8x^2-11x+3$.

106. Divide $9x^3+3x^2-8x-4$ by $\left(x+\frac{2}{3}\right)$:

−2/3	9	3	−8	−4
		−6	2	4
	9	−3	−6	0

Now divide $9x^2-3x-6$ by $\left(x+\frac{2}{3}\right)$:

−2/3	9	−3	−6
		−6	6
	9	−9	0

Since $\left(x+\frac{2}{3}\right)$ does not divide $9x-9$, $\left(x+\frac{2}{3}\right)$ is a root of multiplicity 2 of $9x^3+3x^2-8x-4$.

107. Let x represent b. Then we can use synthetic division to divide x^3-a^3 by $x-a$.

a	1	0	0	$-a^3$
		a	a^2	a^3
	1	a	a^2	0

Then $x^3-a^3 = (x-a)\left(x^2+ax+a^2\right)$ or $b^3-a^3 = (b-a)\left(b^2+ab+a^2\right)$.

108. Let x represent b. Then we can use synthetic division to divide $x^4 - a^4$ by $x - a$.

$$\begin{array}{r|rrrrr} a & 1 & 0 & 0 & 0 & -a^4 \\ & & a & a^2 & a^3 & a^4 \\ \hline & 1 & a & a^2 & a^3 & 0 \end{array}$$

Then $x^4 - a^4 = (x-a)\left(x^3 + ax^2 + a^2x + a^3\right)$, or $b^3 - a^3 = (b-a)\left(b^3 + ab^2 + a^2b + a^3\right)$.

109. If $x^2 - 1$ is a factor of $x^{10} + ax + 4$, then so are $x - 1$ and $x + 1$. Use synthetic division to show that one or the other is not a factor.

$$\begin{array}{r|rrrrrrrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 4 \\ & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & a+1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & a+1 & 0 \end{array}$$

$4 + (a+1) = 0 \Rightarrow a = -5$

Now see if $x + 1$ is a factor of $x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x - 4$.

$$\begin{array}{r|rrrrrrrrrr} -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -4 \\ & & -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -5 \end{array}$$

$x + 1$ is not a factor, so there is no real number a such that $x^2 - 1$ is a factor of $x^{10} + ax + 4$.

110. If $b^2 - a^2$ divides $b^{2n} - a^{2n}$, then $b - a$ and $b + a$, must divide $b^{2n} - a^{2n}$. Let x represent b. Then we can use synthetic division to divide $x^{2n} - a^{2n}$ by $x - a$ and by $x + a$.

$$\begin{array}{r|rrrrrr} a & 1 & 0 & 0 & 0 & \cdots & -a^{2n} \\ & & a & a^2 & a^3 & \cdots & a^{2n} \\ \hline & 1 & a & a^2 & a^3 & \cdots & 0 \end{array}$$

$$\begin{array}{r|rrrrrrr} -a & 1 & 0 & 0 & 0 & 0 & \cdots & -a^{2n} \\ & & -a & a^2 & -a^3 & a^4 & \cdots & a^{2n} \\ \hline & 1 & -a & a^2 & -a^3 & a^4 & \cdots & 0 \end{array}$$

Thus, $b^2 - a^2$ divides $b^{2n} - a^{2n}$.

111. $f(x) = (x-3)Q_1(x) + 2$ and $g(x) = (x-3)Q_2(x) - 5$.

$$\begin{aligned} f(x)\cdot g(x) &= \left[(x-3)Q_1(x) + 2\right]\left[(x-3)Q_2(x) - 5\right] \\ &= (x-3)^2 Q_1(x)Q_2(x) - 5(x-3)Q_1(x) + 2(x-3)Q_2(x) - 10 \end{aligned}$$

$$\begin{aligned} \frac{f(x)\cdot g(x)}{x-3} &= \frac{(x-3)^2 Q_1(x)Q_2(x) - 5(x-3)Q_1(x) + 2(x-3)Q_2(x) - 10}{x-3} \\ &= (x-3)Q_1(x)Q_2(x) - 5Q_1(x) + 2Q_2(x) - \frac{10}{x-3} \end{aligned}$$

The remainder is –10.

112. $f(x) = (x+2)Q(x) - 2$

$$\begin{aligned} \left[f(x)\right]^3 + 1 &= \left[(x+2)Q(x) - 2\right]^3 + 1 = (x+2)^3\left[Q(x)\right]^3 - 6(x+2)^2\left[Q(x)\right]^2 + 12(x+2)Q(x) - 8 + 1 \\ &= (x+2)^3\left[Q(x)\right]^3 - 6(x+2)^2\left[Q(x)\right]^2 + 12(x+2)Q(x) - 7 \end{aligned}$$

$$\begin{aligned} \frac{\left[f(x)\right]^3 + 1}{x+2} &= \frac{(x+2)^3\left[Q(x)\right]^3 - 6(x+2)^2\left[Q(x)\right]^2 + 12(x+2)Q(x) - 7}{x+2} \\ &= (x+2)^2\left[Q(x)\right]^3 - 6(x+2)\left[Q(x)\right]^2 + 12Q(x) - \frac{7}{x+2} \end{aligned}$$

The remainder is –7.

113. $2x^4 + 2x^3 + \frac{1}{2}x^2 + 2x - \frac{3}{2} = 0 \Rightarrow$

$4x^4 + 4x^3 + x^2 + 4x - 3 = 0$. The factors of the constant term, –3, are $\{\pm 1, \pm 3\}$. The factors of the leading coefficient are $\{\pm 1, \pm 2, \pm 4\}$. So, the possible rational zeros are $\left\{\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{3}{2}, \pm 3\right\}$. Use synthetic division to find one zero:

$$\begin{array}{r|rrrrr} \frac{1}{2} & 4 & 4 & 1 & 4 & -3 \\ & & 2 & 3 & 2 & 3 \\ \hline & 4 & 6 & 4 & 6 & 0 \end{array}$$

So, $\frac{1}{2}$ is a rational zero and

$$\begin{aligned} &4x^4 + 4x^3 + x^2 + 4x - 3 \\ &= \left(x - \frac{1}{2}\right)(4x^3 + 6x^2 + 4x + 6) = \\ &= 2\left(x - \frac{1}{2}\right)(2x^3 + 3x^2 + 2x + 3) = \\ &= 2\left(x - \frac{1}{2}\right)\left(x^2(2x+3) + 1(2x+3)\right) = \\ &= 2\left(x - \frac{1}{2}\right)\left(x^2 + 1\right)(2x+3) = 0 \Rightarrow \end{aligned}$$

$x = -\frac{3}{2}$ is another rational zero.

114. i. Simplifying the fraction if necessary, we can assume that $\frac{p}{q}$ is in lowest terms. Since $\frac{p}{q}$ is a zero of F, we have $F\left(\frac{p}{q}\right) = 0$.

ii. Substitute $\frac{p}{q}$ for x in the equation $F(x) = 0$.

iii. Multiply the equation in (ii) by q^n.

iv. Subtract $a_0 q^n$ from both sides of the equation.

v. The left side of the equation in (iv) is
$a_n p^n + a_{n-1}p^{n-1}q + \cdots + a_1 pq^{n-1} =$
$p\left(a_n p^{n-1} + a_{n-1}p^{n-2}q + \cdots + a_1 q^{n-1}\right)$.
Therefore p is a factor.

vi. $a = b \Leftrightarrow \frac{a}{p} = \frac{b}{p}$.

vii. Since p and q have no common prime factors, p must be a factor of a_0.

viii. Rearrange the terms of the equation in (iii).

ix. The left side of the equation in (viii) is
$a_{n-1}p^{n-1}q + \cdots + a_1 pq^{n-1} + a_0 q^n =$
$q\left(a_{n-1}p^{n-1} + \cdots + a_1 pq^{n-2} + a_0 q^{n-1}\right)$.
Therefore q is a factor.

x. $a = b \Leftrightarrow \frac{a}{q} = \frac{b}{q}$.

xi. Since p and q have no common prime factors, q must be a factor of a_n.

Critical Thinking/Discussion/Writing

115. $f(x) = x^3 - cx + 2$

Possible rational zeros are $\pm 1, \pm 2$.

Use synthetic division to see which zeros lead to an integer value of c.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -c & 2 \\ & & -1 & 1 & c-1 \\ \hline & 1 & -1 & -c+1 & 0 \end{array}$$

$2 + (c-1) = 0 \Rightarrow c = -1$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -c & 2 \\ & & 1 & 1 & -c+1 \\ \hline & 1 & 1 & -c+1 & 0 \end{array}$$

$2 + (-c+1) = 0 \Rightarrow c = 3$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -c & 2 \\ & & 2 & 4 & -2c+8 \\ \hline & 1 & 2 & -c+4 & 0 \end{array}$$

$2 + (-2c+8) = 0 \Rightarrow c = 5$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -c & 2 \\ & & -2 & 4 & 2c-8 \\ \hline & 1 & -2 & -c+4 & 0 \end{array}$$

$2 + (2c-8) = 0 \Rightarrow c = 3$

The values of c for which $f(x) = x^3 - cx + 2$ has at least one rational zero are –1, 3, and 5.

116. $f(x) = x^3 - cx - 6$

Possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6$.

Use synthetic division to see which zeros lead to an integer value of c.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -c & -6 \\ & & -1 & 1 & c-1 \\ \hline & 1 & -1 & -c+1 & 0 \end{array}$$

$-6 + (c-1) = 0 \Rightarrow c = 7$

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$$\begin{array}{r|rrrr} 1 & 1 & 0 & -c & -6 \\ & & 1 & 1 & -c+1 \\ \hline & 1 & 1 & -c+1 & 0 \end{array}$$

$-6+(-c+1)=0 \Rightarrow c=-5$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -c & -6 \\ & & 2 & 4 & -2c+8 \\ \hline & 1 & 2 & -c+4 & 0 \end{array}$$

$-6+(-2c+8)=0 \Rightarrow c=1$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -c & -6 \\ & & -2 & 4 & 2c-8 \\ \hline & 1 & -2 & -c+4 & 0 \end{array}$$

$-6+(2c-8)=0 \Rightarrow c=7$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -c & -6 \\ & & 3 & 9 & -3c+27 \\ \hline & 1 & 3 & -c+9 & 0 \end{array}$$

$-6+(-3c+27)=0 \Rightarrow c=7$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -c & -6 \\ & & -3 & 9 & 3c-27 \\ \hline & 1 & -3 & -c+9 & 0 \end{array}$$

$-6+(3c-27)=0 \Rightarrow c=11$

$$\begin{array}{r|rrrr} 6 & 1 & 0 & -c & -6 \\ & & 6 & 36 & -6c+216 \\ \hline & 1 & 6 & -c+36 & 0 \end{array}$$

$-6+(-6c+216)=0 \Rightarrow c=35$

$$\begin{array}{r|rrrr} -6 & 1 & 0 & -c & -6 \\ & & -6 & 36 & 6c-216 \\ \hline & 1 & -6 & -c+36 & 0 \end{array}$$

$-6+(6c-216)=0 \Rightarrow c=37$

The values of c for which $f(x)=x^3-cx-6$ has at least one rational zero are –5, 1, 7, 11, 35, and 37.

117. Let $x=\sqrt{3} \Rightarrow x^2=3 \Rightarrow x^2-3=0$. The only possible rational zeros of this equation are ± 1 and ± 3. Because $\sqrt{3}$ is neither of these, it must be irrational.

118. $x=\sqrt[3]{4} \Rightarrow x^3=\left(\sqrt[3]{4}\right)^3 \Rightarrow x^3-4=0.$

The only possible rational zeros of this equation are $\{\pm 1, \pm 2, \pm 4\}$. Since $\sqrt[3]{4}$ is not in the set, it must be irrational.

119. $x=3-\sqrt{2} \Rightarrow 3-x=\sqrt{2} \Rightarrow 9-6x+x^2=2 \Rightarrow x^2-6x+7=0.$ The possible rational zeros of this equation are $\{\pm 1, \pm 7\}$. Since $3-\sqrt{2}$ is not in the set, it must be irrational.

120. $x=9^{2/3} \Rightarrow x^3=\left(9^{2/3}\right)^3 \Rightarrow x^3-81=0.$ The possible rational zeros of this equation are $\{\pm 1, \pm 3, \pm 9, \pm 27, \pm 81\}$. Since $9^{2/3}$ is not in the set, it must be irrational.

121. a. $x+a$ is a factor of x^n+a^n if n is an odd integer. The possible rational zeros of x^n+a^n are $\{\pm a, \pm a^2, \pm a^3, \ldots, \pm a^n\}$. Since $x+a$ is a factor means that $-a$ is a root, then $(-a)^n+a^n=0 \Rightarrow (-a)^n=-a^n$ only for odd values of n.

b. $x+a$ is a factor of x^n-a^n if n is an even integer. The possible rational zeros of x^n-a^n are $\{\pm a, \pm a^2, \pm a^3, \ldots, \pm a^n\}$. Since $x+a$ is a factor means that $-a$ is a root, then $(-a)^n-a^n=0 \Rightarrow (-a)^n=a^n$ only for even values of n.

c. There is no value of n for which $x-a$ is a factor of x^n+a^n. The possible rational zeros of x^n+a^n are $\{\pm a, \pm a^2, \pm a^3, \ldots, \pm a^n\}$. If $x-a$ is a factor, then a is a root, and $a^n+a^n=0 \Rightarrow a^n=-a^n$, which is not possible.

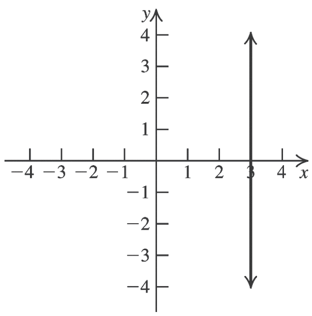
d. $x-a$ is a factor of x^n-a^n for all positive integers n. The possible rational zeros of x^n-a^n are $\{\pm a, \pm a^2, \pm a^3, \ldots, \pm a^n\}$. If $x-a$ is a factor, then a is a root, and $a^n-a^n=0$, which is true for all values of n. However, if n is negative, then $x^n-a^n=\dfrac{1}{x^{-n}}-\dfrac{1}{a^{-n}}$ and $x-a$ is not a factor.

122. a. According to exercise 121d, $x-a$ is a factor of x^n-a^n for any positive integer n. Using this result with $x=7$, $a=2$, and $n=11$, we find that $7-2=5$ is a factor of $7^{11}-2^{11}$. Therefore, $7^{11}-2^{11}$ is divisible by 5.

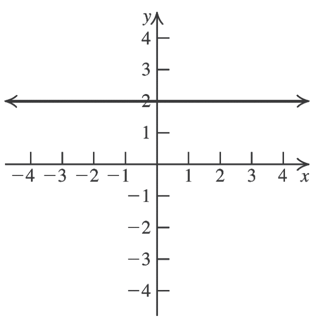
b. $19^{20} - 10^{20} = \left(19^{10} - 10^{10}\right)\left(19^{10} + 10^{10}\right).$
According to exercises 121b and 121d, $x + a$ is a factor of $x^n - a^n$ if n is an odd integer, and $x - a$ is a factor of $x^n - a^n$ for any positive integer n. Using these results with $x = 19$, $a = 10$, and $n = 10$, we find that $19 - 10 = 9$ and $19 + 10 = 29$ are both factors of $19^{10} - 10^{10}$, and therefore, $19^{20} - 10^{20}$ is divisible by 9 and 29. Since 9 and 29 have no common factors, $19^{20} - 10^{20}$ must be divisible by their product, $9(29) = 261$.

Getting Ready for the Next Section

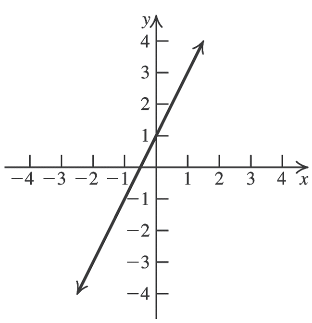
123.



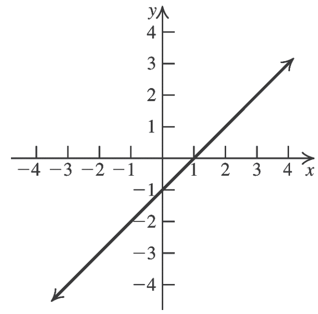
124.



125.



126.



127. $\dfrac{-3}{2(2)+1} = -\dfrac{3}{5}$

128. $\dfrac{7-(-1)}{2(-1)^2 + 3(-1)} = \dfrac{8}{-1} = -8$

129. $\dfrac{2(-3)+3}{5-2(-3)^2} = \dfrac{-3}{-13} = \dfrac{3}{13}$

130. $\dfrac{(2)^2 + 4(2) - 1}{9-(2)^3} = 11$

2.4 Rational Functions

2.4 Practice Problems

1. $f(x) = \dfrac{x-3}{x^2 - 4x - 5}$
The domain of f consists of all real numbers for which $x^2 - 4x - 5 \neq 0$.
$x^2 - 4x - 5 = 0 \Rightarrow (x-5)(x+1) = 0 \Rightarrow$
$x = 5$ or $x = -1$
Thus, the domain is $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$.

2. a. $g(x) = \dfrac{3}{x-2}$
Let $f(x) = \dfrac{1}{x}$. Then
$g(x) = \dfrac{3}{x-2} = 3\left(\dfrac{1}{x-2}\right) = 3f(x-2).$
The graph of $y = f(x-2)$ is the graph of $y = f(x)$ shifted two units to the right. This moves the vertical asymptote two units to the right. The graph of $y = 3f(x-2)$ is the graph of $y = f(x-2)$ stretched vertically three units. The domain of g is $(-\infty, 2) \cup (2, \infty)$. The range of g is $(-\infty, 0) \cup (0, \infty)$.

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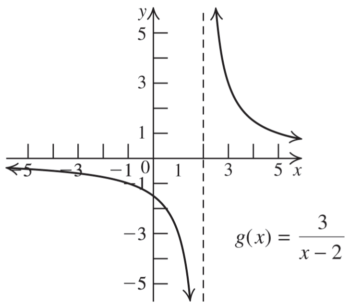
The vertical asymptote is $x = 2$. The horizontal asymptote is $y = 0$.



b. $h(x) = \dfrac{2x+5}{x+1}$

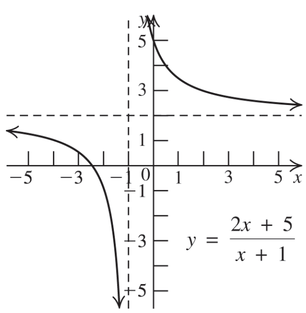
$$\begin{array}{r} 2 \\ x+1\overline{)2x+5} \\ \underline{2x+2} \\ 3 \end{array}$$

$h(x) = \dfrac{2x+5}{x+1} = 2 + \dfrac{3}{x+1}$

Let $f(x) = \dfrac{1}{x}$. Then

$h(x) = \dfrac{2x+5}{x+1} = 2 + \dfrac{3}{x+1} = 2 + 3f(x+1).$

The graph of $y = h(x)$ is the graph of $y = f(x)$ shifted one units to the left and then stretched vertically three units. The graph is then shifted two units up. This moves the vertical asymptote one unit to the left. The horizontal asymptote is shifted two units up. The domain of h is $(-\infty, -1) \cup (-1, \infty)$. The range of h is $(-\infty, 2) \cup (2, \infty)$. The vertical asymptote is $x = -1$. The horizontal asymptote is $y = 2$.



3. $f(x) = \dfrac{x+1}{x^2+3x-10}$

The vertical asymptotes are located at the zeros of the denominator.

$x^2 + 3x - 10 = 0 \Rightarrow (x+5)(x-2) = 0 \Rightarrow$
$x = -5$ or $x = 2$

The vertical asymptotes are $x = -5$ and $x = 2$.

4. $f(x) = \dfrac{3-x}{x^2-9} = \dfrac{-(x-3)}{(x-3)(x+3)} = -\dfrac{1}{x+3}$

$x + 3 = 0 \Rightarrow x = -3$

The vertical asymptote is $x = -3$.

5. a. $f(x) = \dfrac{2x-5}{3x+4}$

Since the numerator and denominator both have degree 1, the horizontal asymptote is $y = \dfrac{2}{3}$.

b. $g(x) = \dfrac{x^2+3}{x-1}$

The degree of the numerator is greater than the degree of the denominator, so there are no horizontal asymptotes.

6. $f(x) = \dfrac{2x}{x^2-1}$

First find the intercepts:

$\dfrac{2x}{x^2-1} = 0 \Rightarrow x = 0$

The graph passes through the origin.

Find the vertical asymptotes:

$x^2 - 1 = 0 \Rightarrow (x-1)(x+1) = 0 \Rightarrow$
$x = 1$ or $x = -1$

Find the horizontal asymptote: The degree of the numerator is less than the degree of the denominator, so the horizontal asymptote is the x-axis.

Test for symmetry:

$f(-x) = \dfrac{2(-x)}{(-x)^2-1} = -\dfrac{2x}{x^2-1} = -f(x)$

f is an odd function and is symmetric with respect to the origin.

Use test numbers to determine where the graph of f is above and below the x-axis.

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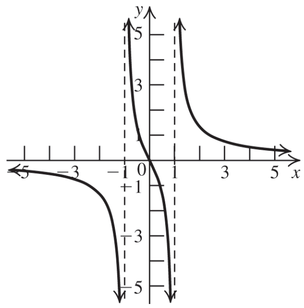
Interval	Test point	Value of $f(x)$	Above/below x-axis
$(-\infty,-1)$	-3	$-\frac{3}{4}$	below
$(-1,0)$	$-\frac{1}{2}$	$\frac{4}{3}$	above
$(0, 1)$	$\frac{1}{2}$	$-\frac{4}{3}$	below
$(1,\infty)$	2	$\frac{4}{3}$	above



7. $f(x)=\dfrac{2x^2-1}{2x^2+x-3}$

First find the intercepts:

$\dfrac{2x^2-1}{2x^2+x-3}=0 \Rightarrow x=\pm\dfrac{\sqrt{2}}{2}$; $f(0)=\dfrac{1}{3}$

Find the vertical asymptotes:

$2x^2+x-3=0 \Rightarrow (2x+3)(x-1)=0 \Rightarrow$

$x=-\dfrac{3}{2}$ or $x=1$

Find the horizontal asymptote: The degree of the numerator is the same as the degree of the denominator, so the horizontal asymptote is

$y=\dfrac{2}{2}=1$.

Test for symmetry:

$$f(-x)=\frac{2(-x)-1}{2(-x)^2+(-x)-3}=-\frac{-2x-1}{2x^2-x-3}$$
$$\neq f(x) \text{ and } \neq -f(x)$$

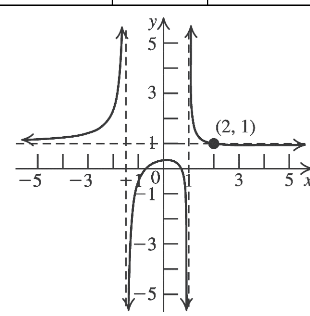
f is neither even nor odd, so there is no symmetry with respect to the y-axis or the origin.

$$f(x)=\frac{2x^2-1}{2x^2+x-3}=1+\frac{2-x}{2x^2+x-3}$$

The zero of $R(x)=2-x$ is 2 and the zeros of $D(x)=2x^2+x-3=(2x+3)(x-1)$ are $x=-\dfrac{3}{2}$ and $x=1$.

Use test numbers to determine where the graph of f is above and below the horizontal asymptote $y=1$.

Interval	Test point	Value of $f(x)$	Above/below x-axis
$\left(-\infty,-\frac{3}{2}\right)$	-2	$\frac{7}{3}$	above
$\left(-\frac{3}{2},1\right)$	0	$\frac{1}{3}$	below
$(1, 2)$	$\frac{3}{2}$	$\frac{7}{6}$	above
$(2,\infty)$	3	$\frac{17}{18}$	below



Notice that the graph crosses the horizontal asymptote at (2, 1).

8. $f(x)=\dfrac{x^2+1}{x^2+2}$

First find the intercepts:

$\dfrac{x^2+1}{x^2+2}=0 \Rightarrow x=\pm i \Rightarrow$ there is no x-intercept.

$f(0)=\dfrac{1}{2}$

Find the vertical asymptotes:

$x^2+2=0 \Rightarrow x=i\sqrt{2} \Rightarrow$ there is no vertical asymptote.

Find the horizontal asymptote: The degree of the numerator is the same as the degree of the denominator, so the horizontal asymptote is

$y=\dfrac{1}{1}=1$.

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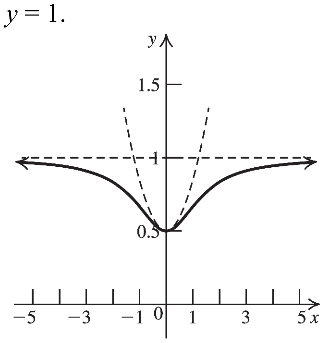
Test for symmetry:

$$f(-x) = \frac{(-x)^2 + 1}{(-x)^2 + 2} = \frac{x^2 + 1}{x^2 + 2} = f(x)$$

f is an even function and is symmetric with respect to the *y*-axis.

$$f(x) = \frac{x^2 + 1}{x^2 + 2} = 1 + \frac{-1}{x^2 + 2}$$

Neither $R(x) = -1$ nor $D(x) = x^2 + 2$ have real zeros. Since $\frac{-1}{x^2 + 2}$ is negative for all values of *x*, the graph of $f(x)$ is always below the line $y = 1$.



9. $f(x) = \frac{x^2 + 2}{x - 1}$

First find the intercepts:

$\frac{x^2 + 2}{x - 1} = 0 \Rightarrow x = \pm i\sqrt{2} \Rightarrow$ there is no *x*-intercept. $f(0) = -2$

Find the vertical asymptotes:

$x - 1 = 0 \Rightarrow x = 1$

Find the horizontal asymptote: The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote. However, there is an oblique asymptote:

$\frac{x^2 + 2}{x - 1} = x + 1 + \frac{3}{x - 1} \Rightarrow y = x + 1$ is the oblique asymptote.

The graph is above the line $y = x + 1$ on $(1, \infty)$ and below the line on $(-\infty, 1)$.

Test for symmetry:

$$f(-x) = \frac{(-x)^2 + 2}{-x - 1} = \frac{x^2 + 2}{-x - 1} \neq f(x)$$

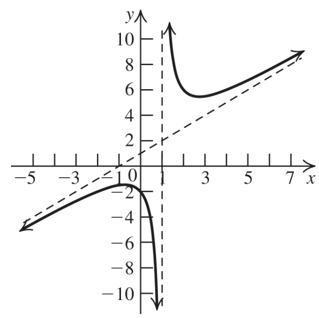
and $\neq -f(x)$

f is neither even nor odd, so there is no symmetry with respect to the *y*-axis or the origin.

The intervals determined by the zeros of the numerator and of the denominator of

$f(x) - (x + 1) = \frac{x^2 + 2}{x - 1} - (x + 1) = \frac{3}{x - 1}$ divide the *x*-axis into two intervals, $(-\infty, 1)$ and $(1, \infty)$. Use test numbers to determine where the graph of *f* is above and below the *x*-axis.

Interval	Test point	Value of $f(x)$	Above/below $y = x + 1$
$(-\infty, 1)$	−1	$-\frac{3}{2}$	below
$(1, \infty)$	2	6	above

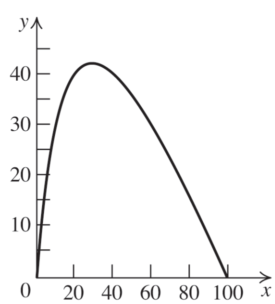


10. $R(x) = \frac{x(100 - x)}{x + 20}$

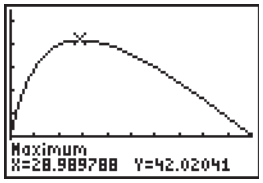
a. $R(10) = \frac{10(100 - 10)}{10 + 20} = 30$ billion dollars.

This means that if income is taxed at a rate of 10%, then the total revenue for the government will be 30 billion dollars. Similarly, $R(20) = \$40$ billion, $R(30) = \$42$ billion, $R(40) = \$40$ billion, $R(50) \approx \$35.7$ billion, $R(60) = \$30$ billion.

b.



c.



From the graphing calculator screen, we see that a tax rate of about 29% generates the maximum tax revenue of about $42.02 billion.

2.4 Exercises
Concept and Vocabulary

1. A function *f* is rational if it can be expressed in the form $\frac{N(x)}{D(x)}$ where $N(x)$ and $D(x)$ are polynomials.

2. A line $x = a$ is called a vertical asymptote of the rational function *f* if $|f(x)| \to \underline{\infty}$ as $x \to \underline{a^+}$ or as $x \to \underline{a^-}$.

3. A line $y = k$ is called a horizontal asymptote of the rational function *f* if $|f(x)| \to \underline{k}$ as $x \to \underline{\infty}$ or as $x \to \underline{-\infty}$.

4. If *a* is a zero of denominator $D(x)$ of the rational function $f(x) = \dfrac{N(x)}{D(x)}$ expressed in lowest terms, then $\underline{x = a}$ is a vertical asymptote of *f*.

5. True

6. False. The graph of a rational function can cross its horizontal asymptote.

7. True

8. False

Building Skills

9. $(-\infty, -4) \cup (-4, \infty)$

10. $(-\infty, 1) \cup (1, \infty)$

11. $(-\infty, \infty)$

12. $(-\infty, \infty)$

13. $x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = -2, 3$
The domain of the function is $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$.

14. $x^2 - 6x - 7 = 0 \Rightarrow (x-7)(x+1) = 0 \Rightarrow x = -1, 7$
The domain of the function is $(-\infty, -1) \cup (-1, 7) \cup (7, \infty)$.

15. $x^2 - 6x + 8 = 0 \Rightarrow (x-4)(x-2) = 0 \Rightarrow x = 2, 4$
The domain of the function is $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$.

16. $x^2 - 3x + 2 = 0 \Rightarrow (x-2)(x-1) = 0 \Rightarrow x = 1, 2$
The domain of the function is $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$.

17. As $x \to 1^+, f(x) \to \underline{\infty}$.

18. As $x \to 1^-, f(x) \to \underline{\infty}$.

19. As $x \to -2^+, f(x) \to \underline{\infty}$.

20. As $x \to -2^-, f(x) \to \underline{-\infty}$.

21. As $x \to \infty, f(x) \to \underline{1}$.

22. As $x \to -\infty, f(x) \to \underline{1}$.

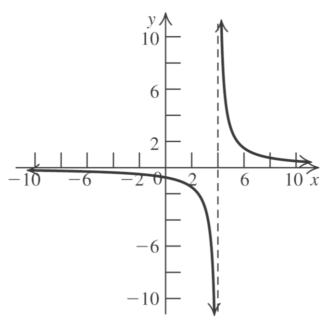
23. The domain of *f* is $\underline{(-\infty, -2) \cup (-2, 1) \cup (1, \infty)}$.

24. There are two vertical asymptotes.

25. The equations of the vertical asymptotes of the graph are $\underline{x = -2}$ and $\underline{x = 1}$.

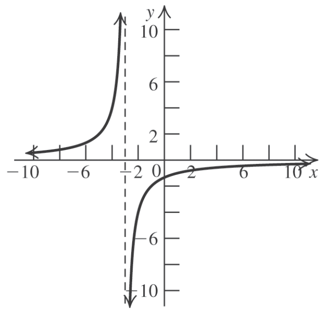
26. The equation of the horizontal asymptote of the graph is $\underline{y = 1}$.

27.



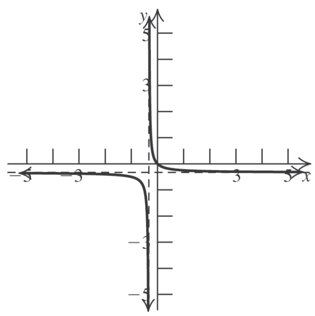
Domain: $(-\infty, -4) \cup (-4, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
Vertical asymptote: $x = 4$
Horizontal asymptote: $y = 0$

28.



Domain: $(-\infty,-3)\cup(-3,\infty)$
Range: $(-\infty, 0)\cup(0, \infty)$
Vertical asymptote: $x=-3$
Horizontal asymptote: $y=0$

29.



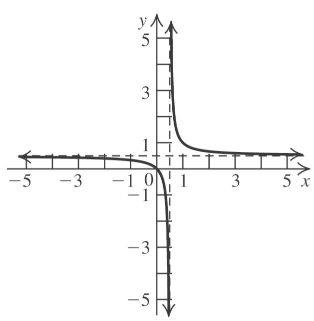
Domain: $\left(-\infty,-\frac{1}{3}\right)\cup\left(-\frac{1}{3},\infty\right)$

Range: $\left(-\infty,-\frac{1}{3}\right)\cup\left(-\frac{1}{3},\infty\right)$

Vertical asymptote: $x=-\frac{1}{3}$

Horizontal asymptote: $y=-\frac{1}{3}$

30.



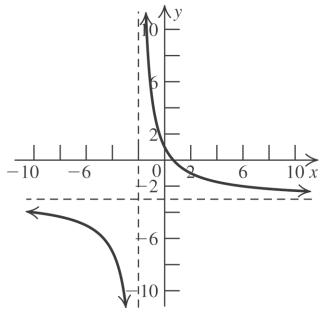
Domain: $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

Range: $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

Vertical asymptote: $x=\frac{1}{2}$

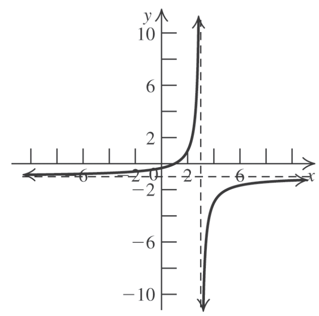
Horizontal asymptote: $y=\frac{1}{2}$

31.



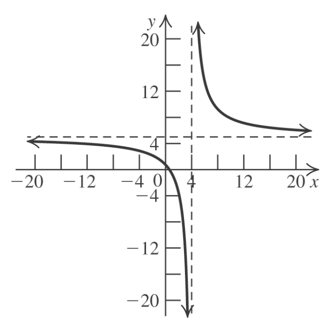
Domain: $(-\infty,-2)\cup(-2,\infty)$
Range: $(-\infty,-3)\cup(-3,\infty)$
Vertical asymptote: $x=-2$
Horizontal asymptote: $y=-3$

32.



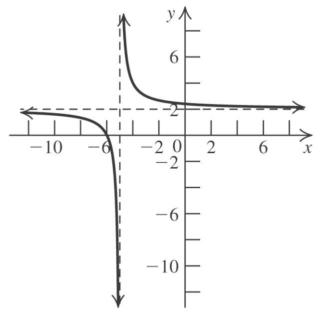
Domain: $(-\infty, 3)\cup(3, \infty)$
Range: $(-\infty, -1)\cup(-1, \infty)$
Vertical asymptote: $x=3$
Horizontal asymptote: $y=-1$

33.



Domain: $(-\infty, 4)\cup(4,\infty)$
Range: $(-\infty, 5)\cup(5, \infty)$
Vertical asymptote: $x=4$
Horizontal asymptote: $y=5$

34.



Domain: $(-\infty,-5)\cup(-5,\infty)$
Range: $(-\infty, 2)\cup(2, \infty)$
Vertical asymptote: $x = -5$
Horizontal asymptote: $y = 2$

In exercises 35−44, to find the vertical asymptotes, first eliminate any common factors in the numerator and denominator, and then set the denominator equal to zero and solve for x.

35. $x = 1$ **36.** $x = 2$

37. $x = -4, x = 3$ **38.** $x = -\dfrac{3}{2}, x = \dfrac{4}{3}$

39. $h(x) = \dfrac{x^2-1}{x^2+x-6} = \dfrac{(x-1)(x+1)}{(x-2)(x+3)}$.
The equations of the vertical asymptotes are $x = -3$ and $x = 2$.

40. $h(x) = \dfrac{x^2-4}{3x^2+x-4} = \dfrac{(x-2)(x+2)}{(3x+4)(x-1)}$.
The equations of the vertical asymptotes are $x = -\dfrac{4}{3}$ and $x = 1$.

41. $f(x) = \dfrac{x^2-6x+8}{x^2-x-12} = \dfrac{(x-4)(x-2)}{(x-4)(x+3)}$.
Disregard the common factor. The vertical asymptote is $x = -3$.

42. $f(x) = \dfrac{x^2-9}{x^3-4x} = \dfrac{(x-3)(x+3)}{x(x-2)(x+2)}$.
The equations of the vertical asymptotes are $x = 0, x = 2,$ and $x = -2$.

43. There is no vertical asymptote.

44. There is no vertical asymptote.

For exercises 45−52, locate the horizontal asymptote as follows:

- If the degree of the numerator of a rational function is less than the degree of the denominator, then the x-axis ($y = 0$) if the horizontal asymptote.
- If the degree of the numerator of a rational function equals the degree of the denominator, the horizontal asymptote is the line with the equation $y = \dfrac{a_n}{b_m}$, where a_n is the coefficient of the leading term of the numerator and b_m is the coefficient of the leading term of the denominator.
- If the degree of the numerator of a rational function is greater than the degree of the denominator, then there is no horizontal asymptote.

45. $y = 0$ **46.** $y = 0$

47. $y = \dfrac{2}{3}$ **48.** $y = -\dfrac{3}{4}$

49. There is no horizontal asymptote.

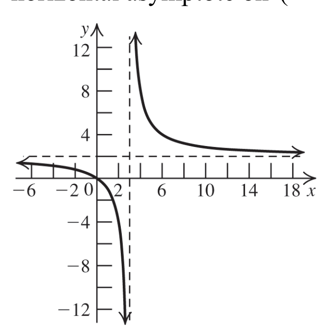
50. $y = 0$ **51.** $y = 0$

52. There is no horizontal asymptote.

53. d **54.** f **55.** e

56. b **57.** a **58.** c

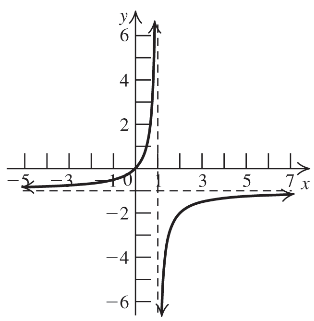
59. $0 = \dfrac{2x}{x-3} \Rightarrow x = 0$ is the x-intercept.
$\dfrac{2(0)}{0-3} = 0 \Rightarrow y = 0$ is the y-intercept. The vertical asymptote is $x = 3$. The horizontal asymptote is $y = 2$.
$f(-x) = \dfrac{2(-x)}{(-x)-3} = \dfrac{2x}{x+3} \neq f(x)$ and
$f(-x) \neq -f(x) \Rightarrow$ there are no symmetries.
The intervals to be tested are $(-\infty, 3)$ and $(3,\infty)$. The graph is above the horizontal asymptote on $(3,\infty)$ and below the horizontal asymptote on $(-\infty,3)$.



60. $0 = \dfrac{-x}{x-1} \Rightarrow x = 0$ is the x-intercept.

$\dfrac{0}{0-1} = 0 \Rightarrow y = 0$ is the y-intercept. The vertical asymptote is $x = 1$. The horizontal asymptote is $y = -1$. $f(-x) = \dfrac{-(-x)}{(-x)-3}$

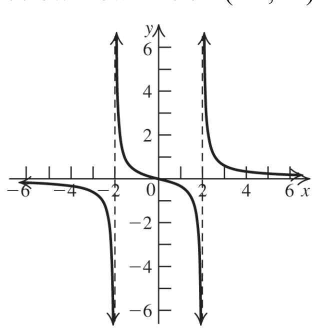
$= \dfrac{x}{-x-3} \neq f(x)$ and $\neq -f(x) \Rightarrow$ there are no symmetries. The intervals to be tested are $(-\infty, 1)$, and $(1, \infty)$. The graph is above the horizontal asymptote on $(-\infty, 1)$ and below the horizontal asymptote on $(1, \infty)$.



61. $0 = \dfrac{x}{x^2-4} \Rightarrow x = 0$ is the x-intercept.

$\dfrac{0}{0^2-4} = 0 \Rightarrow y = 0$ is the y-intercept. The vertical asymptotes are $x = -2$ and $x = 2$. The horizontal asymptote is the x-axis.

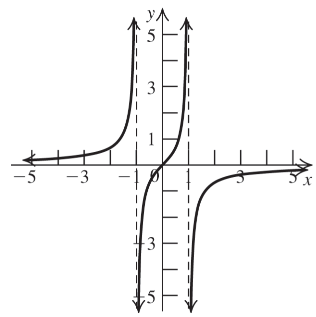
$f(-x) = \dfrac{-x}{(-x)^2-4} = -\dfrac{x}{x^2-4} = -f(x) \Rightarrow$ the function is odd, and the graph is symmetric with respect to the origin. The intervals to be tested are $(-\infty, -2), (-2, 0), (0, 2)$ and $(2, \infty)$. The graph is above the x-axis on $(-2, 0) \cup (2, \infty)$ and below the x-axis on $(-\infty, -2) \cup (0, 2)$.



62. $0 = \dfrac{x}{1-x^2} \Rightarrow x = 0$ is the x-intercept.

$\dfrac{0}{1-0^2} = 0 \Rightarrow y = 0$ is the y-intercept. The vertical asymptotes are $x = -1$ and $x = 1$. The horizontal asymptote is the x-axis.

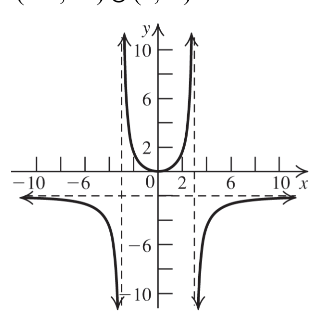
$f(-x) = \dfrac{-x}{1-(-x)^2} = -\dfrac{x}{1-x^2} = -f(x) \Rightarrow$ the function is odd, and the graph is symmetric with respect to the origin. The intervals to be tested are $(-\infty, -1), (-1, 0), (0, 1)$ and $(1, \infty)$. The graph is above the x-axis on $(-\infty, -1) \cup (0, 1)$ and below the x-axis on $(-1, 0) \cup (1, \infty)$.



63. $0 = \dfrac{-2x^2}{x^2-9} \Rightarrow x = 0$ is the x-intercept.

$\dfrac{0^2}{0^2-4} = 0 \Rightarrow y = 0$ is the y-intercept. The vertical asymptotes are $x = -3$ and $x = 3$. The horizontal asymptote is $y = -2$.

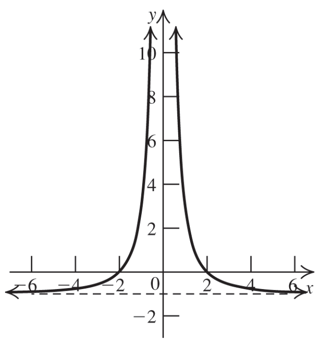
$f(-x) = \dfrac{(-x)^2}{(-x)^2-9} = \dfrac{x^2}{x^2-9} = f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, -3), (-3, 3)$, and $(3, \infty)$. The graph is above the horizontal asymptote on $(-3, 3)$ and below the horizontal asymptote on $(-\infty, -3) \cup (3, \infty)$.



64. $0 = \dfrac{4 - x^2}{x^2} \Rightarrow x = \pm 2$ is the x-intercept.

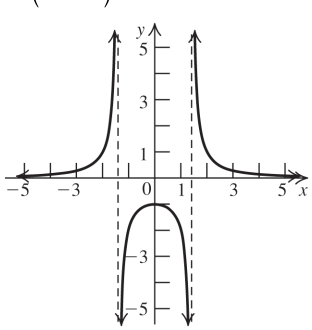
$\dfrac{4 - 0^2}{0^2} \Rightarrow$ there is no y-intercept. The vertical asymptote is the y-axis. The horizontal asymptote is $y = -1$.

$f(-x) = \dfrac{4 - (-x)^2}{(-x)^2} = \dfrac{4 - x^2}{x^2} = f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, 0)$, and $(0, \infty)$. The graph is above the horizontal asymptote on $(-\infty, 0) \cup (0, \infty)$.



65. $0 = \dfrac{2}{x^2 - 2} \Rightarrow$ there is no x-intercept.

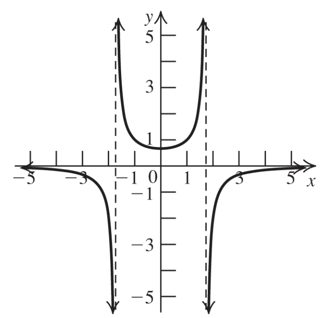
$\dfrac{2}{0^2 - 2} = 0 \Rightarrow y = -1$ is the y-intercept. The vertical asymptotes are $x = \pm\sqrt{2}$. The horizontal asymptote is the x-axis. $f(-x) = \dfrac{2}{(-x)^2 - 2} = \dfrac{2}{x^2 - 2} = f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, -\sqrt{2}), (-\sqrt{2}, \sqrt{2})$ and $(\sqrt{2}, \infty)$. The graph is above the x-axis on $\left(-\infty, -\sqrt{2}\right)$ $\cup\left(\sqrt{2}, \infty\right)$ and below the x-axis on $\left(-\sqrt{2}, \sqrt{2}\right)$.



66. $0 = \dfrac{-2}{x^2 - 3} \Rightarrow$ there is no x-intercept.

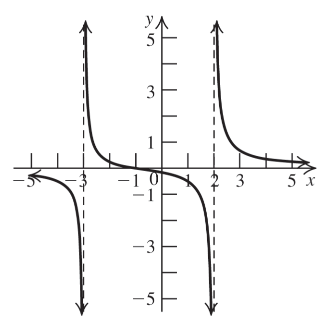
$\dfrac{-2}{0^2 - 3} = 0 \Rightarrow y = \dfrac{2}{3}$ is the y-intercept. The vertical asymptotes are $x = \pm\sqrt{3}$. The horizontal asymptote is the x-axis.

$f(-x) = \dfrac{-2}{(-x)^2 - 3} = \dfrac{-2}{x^2 - 2} = f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, -\sqrt{3}), (-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, \infty)$. The graph is above the x-axis on $\left(-\sqrt{3}, \sqrt{3}\right)$ and below the x-axis on $\left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$.



67. $0 = \dfrac{x + 1}{(x - 2)(x + 3)} \Rightarrow x = -1$ is the x-intercept.

$\dfrac{0 + 1}{(0 - 2)(0 + 3)} = -\dfrac{1}{6} \Rightarrow y = -\dfrac{1}{6}$ is the y-intercept. The vertical asymptotes are $x = -3$ and $x = 2$. The horizontal asymptote is the x-axis. $f(-x) = \dfrac{(-x) + 1}{((-x) - 2)((-x) + 3)} \neq f(x)$ and $\neq -f(x) \Rightarrow$ there are no symmetries. The intervals to be tested are $(-\infty, -3), (-3, -1),$ $(-1, 2),$ and $(2, \infty)$. The graph is above the x-axis on $(-3, -1) \cup (2, \infty)$ and below the x-axis on $(-\infty, -3) \cup (-1, 2)$.

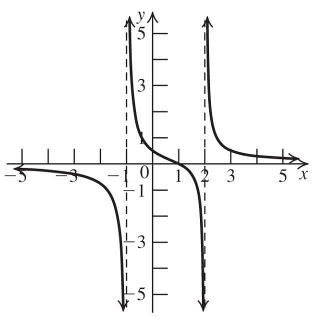


68. $0 = \dfrac{x-1}{(x+1)(x-2)} \Rightarrow x = 1$ is the x-intercept.

$\dfrac{0-1}{(0+1)(0-2)} = \dfrac{1}{2} \Rightarrow y = \dfrac{1}{2}$ is the y-intercept.

The vertical asymptotes are $x = -1$ and $x = 2$. The horizontal asymptote is the x-axis.

$f(-x) = \dfrac{(-x)-1}{((-x)+1)((-x)-2)} \neq f(x)$ and $\neq -f(x) \Rightarrow$ there are no symmetries. The intervals to be tested are $(-\infty,-1), (-1,1),$ $(1,2),$ and $(2,\infty)$. The graph is above the x-axis on $(-1,1) \cup (2,\infty)$ and below the x-axis on $(-\infty,-1) \cup (1,2)$.



69. $f(x) = \dfrac{2x}{x^2+3x+2}$

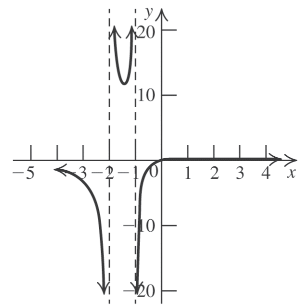
$\dfrac{2x}{x^2+3x+2} = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$ is the x-intercept.

$\dfrac{2(0)}{(0)^2+3(0)+2} = 0 \Rightarrow y = 0$ is the y-intercept.

$x^2+3x+2 = (x+1)(x+2) \Rightarrow x = -1,\ x = -2$ are the vertical asymptotes. The horizontal asymptote is the x-axis.

$$f(-x) = \frac{2(-x)}{(-x)^2+3(-x)+2} = \frac{-2x}{x^2-3x+2} \neq -f(x) \text{ or } f(x)$$

There are no symmetries. The intervals to be tested are $(-\infty,-2), (-2,-1),$ $(-1,0)$ and $(0,\infty)$. The graph is below the x-axis on $(-\infty,-2) \cup (-1,\ 0)$ and above the x-axis on $(-2,-1) \cup (0,\infty)$.



70. $f(x) = \dfrac{2x}{x^2-6x+8}$

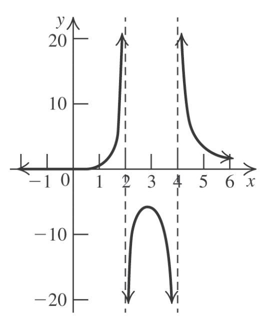
$\dfrac{2x}{x^2-6x+8} = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$ is the x-intercept.

$\dfrac{2(0)}{(0)^2-6(0)+8} = 0 \Rightarrow y = 0$ is the y-intercept.

$x^2-6x+8 = (x-2)(x-4) \Rightarrow x = 2,\ x = 4$ are the vertical asymptotes. The horizontal asymptote is the x-axis.

$$f(-x) = \frac{2(-x)}{(-x)^2-6(-x)+8} = \frac{-2x}{x^2+x+8} \neq -f(x) \text{ or } f(x)$$

There are no symmetries. The intervals to be tested are $(-\infty,0), (0,\ 2),$ $(2,\ 4)$ and $(4,\infty)$. The graph is below the x-axis on $(-\infty,0) \cup (2,\ 4)$ and above the x-axis on $(0,\ 2) \cup (4,\infty)$.



71. $f(x)=\dfrac{x-1}{x^2(x+1)}$

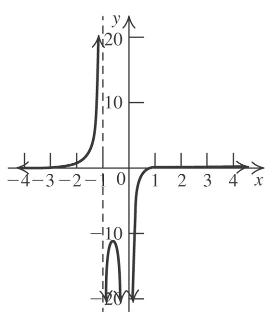
$\dfrac{x-1}{x^2(x+1)}=0 \Rightarrow x-1=0 \Rightarrow x=1$ is the x-intercept.

$\dfrac{0-1}{0^2(0+1)}$ is undefined, so there is no y-intercept.

$x^2(x+1)=0 \Rightarrow x=0,\ x=-1$ are the vertical asymptotes. The horizontal asymptote is the x-axis.

$$f(-x)=\frac{-x-1}{(-x)^2(-x+1)}=\frac{-x-1}{x^2(-x+1)}\neq -f(x) \text{ or } f(x)$$

There are no symmetries. The intervals to be tested are $(-\infty,-1),(-1,0),\ (0,1)$ and $(1,\infty)$. The graph is above the x-axis on $(-\infty,-1)\cup(1,\ \infty)$ and below the x-axis on $(-1,\ 0)\cup(0,\ 1)$.



72. $f(x)=\dfrac{x-1}{x(x+1)^2}$

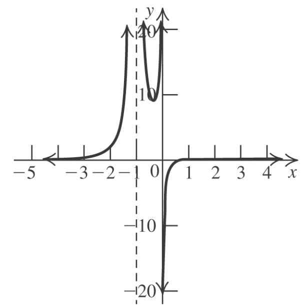
$\dfrac{x-1}{x(x+1)^2}=0 \Rightarrow x-1=0 \Rightarrow x=1$ is the x-intercept.

$\dfrac{0-1}{0(0+1)^2}$ is undefined, so there is no y-intercept.

$x(x+1)^2=0 \Rightarrow x=0,\ x=-1$ are the vertical asymptotes. The horizontal asymptote is the x-axis.

$$f(-x)=\frac{-x-1}{(-x)(-x+1)^2}\neq -f(x) \text{ or } f(x)$$

There are no symmetries. The intervals to be tested are $(-\infty,-1),(-1,0),\ (0,1)$ and $(1,\infty)$. The graph is below the x-axis on $(0,\ 1)$ and above the x-axis on $(-\infty,-1)\cup(-1,\ 0)\cup(1,\ \infty)$.



73. $f(x)=\dfrac{x-3}{x^2-7x+12}$

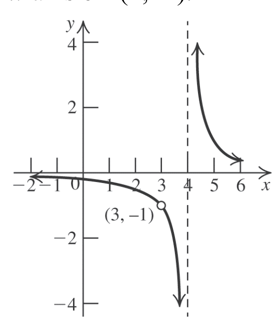
$\dfrac{x-3}{x^2-7x+12}=\dfrac{x-3}{(x-3)(x-4)}=\dfrac{1}{x-4}=0 \Rightarrow$ there is no x-intercept.

$\dfrac{0-3}{0^2-7(0)+12}=-\dfrac{1}{4}$ is the y-intercept.

There is a hole at $(3,-1)$ and $x=4$ is the vertical asymptote. The horizontal asymptote is the x-axis.

$$f(-x)=\frac{-x-3}{(-x)^2-7(-x)+12}=\frac{-x-3}{x^2+7x+12}\neq -f(x) \text{ or } f(x)$$

There are no symmetries.
The intervals to be tested are $(-\infty,3),(3,4),$ and $(4,\infty)$. The graph is below the x-axis on $(-\infty,-1)\cup(3,\ 4)$ and above the x-axis on $(4,\ \infty)$.



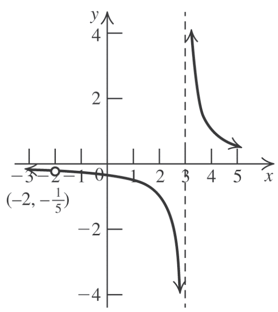
74. $f(x) = \dfrac{x+2}{x^2 - x - 6}$

$\dfrac{x+2}{x^2 - x - 6} = \dfrac{x+2}{(x+2)(x-3)} = \dfrac{1}{x-3} = 0 \Rightarrow$ there is no x-intercept. There is a hole at $\left(-2, -\frac{1}{5}\right)$.

$\dfrac{0+2}{0^2 - 0 - 6} = -\dfrac{1}{3}$ is the y-intercept.
$x = 3$ is the vertical asymptote. The horizontal asymptote is the x-axis.

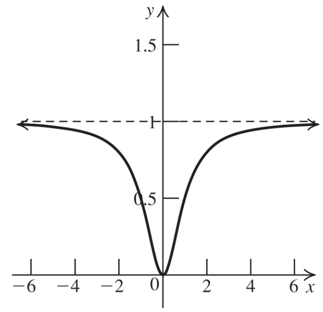
$$f(-x) = \frac{-x+2}{(-x)^2 - (-x) - 6} = \frac{-x+2}{x^2 + x - 6} \neq -f(x) \text{ or } f(x)$$

There are no symmetries.
The intervals to be tested are $(-\infty, -2), (-2, 3),$ and $(3, \infty)$. The graph is below the x-axis on $(-\infty, -2) \cup (-2, 3)$ and above the x-axis on $(3, \infty)$.



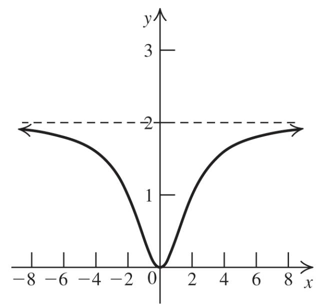
75. $0 = \dfrac{x^2}{x^2 + 1} \Rightarrow x = 0$ is the x-intercept.

$\dfrac{0^2}{0^2 + 1} = 0 \Rightarrow y = 0$ is the y-intercept. There is no vertical asymptote. The horizontal asymptote is $y = 1$. $f(-x) = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1} = f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, 0)$ and $(0, \infty)$. The graph is above the x-axis on $(-\infty, 0) \cup (0, \infty)$ and below the horizontal asymptote on $(-\infty, \infty)$.



76. $0 = \dfrac{2x^2}{x^2 + 4} \Rightarrow x = 0$ is the x-intercept.

$\dfrac{2(0^2)}{0^2 + 4} = 0 \Rightarrow y = 0$ is the y-intercept. There is no vertical asymptote. The horizontal asymptote is $y = 2$. $f(-x) = \dfrac{2(-x)^2}{(-x)^2 + 4} = \dfrac{x^2}{x^2 + 4} = f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, 0)$ and $(0, \infty)$. The graph is above the x-axis on $(-\infty, 0) \cup (0, \infty)$ and below the horizontal asymptote on $(-\infty, \infty)$.



77. $f(x) = \dfrac{x^3 - 4x}{x^3 - 9x} = \dfrac{x(x-2)(x+2)}{x(x-3)(x+3)} = \dfrac{(x-2)(x+2)}{(x-3)(x+3)}$

$f(x) = 0 \Rightarrow x = \pm 2$ are the x-intercepts.

$\dfrac{x^3 - 4x}{x^3 - 9x} = \dfrac{x(x^2 - 4)}{x(x^2 - 9)} = \dfrac{x^2 - 4}{x^2 - 9} \Rightarrow \dfrac{0^2 - 4}{0^2 - 9} = \dfrac{4}{9} \Rightarrow$

$\dfrac{4}{9}$ is the y-intercept. However, there is a hole at $\left(0, \dfrac{4}{9}\right)$ since $0^3 - 9(0) = 0$

$x^2 - 9 = 0 \Rightarrow x(x+3)(x-3) = 0 \Rightarrow x = -3$ and $x = 3$ are the vertical asymptotes.

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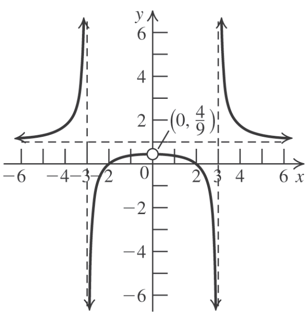
The degree of the numerator is the same as the degree of the denominator, so the horizontal asymptote is $y = 1$.

$$f(-x) = \frac{(-x)^3 - 4(-x)}{(-x)^3 - 9(-x)} = \frac{-x^3 + 4x}{-x^3 + 9x} = \frac{x^3 - 4x}{x^3 - 9x}$$

$= f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, -3), (-3, 0)$, $(0, 3)$, and $(3, \infty)$. The graph is above the horizontal asymptote on $(-\infty, -3) \cup (3, \infty)$ and below the horizontal asymptote on $(-3, 0) \cup (0, 3)$.



78. $f(x) = \dfrac{x^3 + 32x}{x^3 + 8x} = \dfrac{x(x^2 + 32)}{x(x^2 + 8)} = \dfrac{x^2 + 32}{x^2 + 8}$

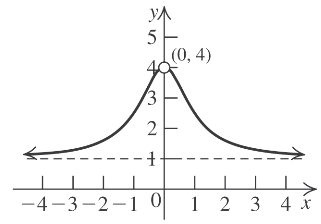
$0 = \dfrac{x^2 + 32}{x^2 + 8} \Rightarrow x^2 = -32 \Rightarrow$ there are no x-intercepts.

$\dfrac{0^2 + 32}{0^2 + 8} = \dfrac{32}{8} \Rightarrow y = 4$ is the y-intercept.

However, there is a hole at (0, 4) since $0^3 + 8(0) = 0$. Since there is no real solution for $x^2 + 8 = 0$, there are no vertical asymptotes. The horizontal asymptote is $y = 1$ since the degrees of the numerator and the denominator are equal and the leading coefficients are the same.

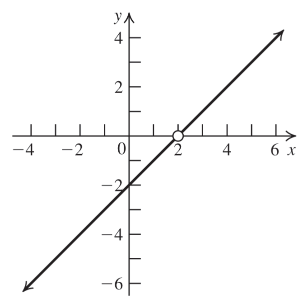
$$f(-x) = \frac{(-x)^3 + 32(-x)}{(-x)^3 + 8(-x)} = \frac{-x^3 - 32x}{-x^3 - 8x}$$

$= \dfrac{x^3 + 32x}{x^3 + 8x} = f(x) \Rightarrow$ the function is even, and the graph is symmetric with respect to the y-axis. The intervals to be tested are $(-\infty, 0)$ and $(0, \infty)$. The graph is above the horizontal asymptote on $(-\infty, 0) \cup (0, \infty)$.



79. $0 = \dfrac{(x-2)^2}{x-2} \Rightarrow$ there is no x-intercept. (There is a hole at $x = 2$.) $\dfrac{(0-2)^2}{0-2} = -2 \Rightarrow y = -2$ is the y-intercept. There are no vertical asymptotes. There are no horizontal asymptotes.

$f(-x) = \dfrac{(-x-2)^2}{-x-2} \neq f(x)$ and $\neq -f(x) \Rightarrow$ there are no symmetries. The intervals to be tested are $(-\infty, 2)$ and $(2, \infty)$. The graph is above the x-axis on $(2, \infty)$ and below the x-axis on $(-\infty, 2)$.



80. $0 = \dfrac{(x-1)^2}{x-1} \Rightarrow$ there is no x-intercept. (There is a hole at $x = 1$.) $\dfrac{(0-1)^2}{0-1} = -1 \Rightarrow y = -1$ is the y-intercept. There are no vertical asymptotes. There are no horizontal asymptotes.

$f(-x) = \dfrac{(-x-1)^2}{-x-1} \neq f(x)$ and $\neq -f(x) \Rightarrow$ there are no symmetries. The intervals to be tested are $(-\infty, 1)$ and $(1, \infty)$. The graph is above the x-axis on $(1, \infty)$ and below the x-axis on $(-\infty, 1)$.

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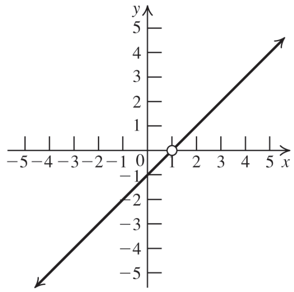


81. The x-intercept is 1 and the vertical asymptote is $x = 2$. The horizontal asymptote is $y = -2$, so the degree of the numerator equals the degree of the denominator, and the ratio of the leading terms of the numerator and the denominator is -2.

Thus, the equation is of the form $y = a\left(\frac{x-1}{x-2}\right)$.

The y-intercept is -1, so we have
$-1 = a\left(\frac{0-1}{0-2}\right) \Rightarrow a = -2.$

Thus, the equation is $f(x) = \frac{-2(x-1)}{x-2}$.

82. The x-intercept is -2 and the vertical asymptote is $x = 1$. The horizontal asymptote is $y = \frac{1}{2}$, so the degree of the numerator equals the degree of the denominator, and the ratio of the leading terms of the numerator and the denominator is $\frac{1}{2}$. Thus, the equation is of the form $y = a\left(\frac{x+2}{x-1}\right)$. The y-intercept is -1, so we have $-1 = a\left(\frac{0+2}{0-1}\right) \Rightarrow a = \frac{1}{2}$.

Thus, the equation is $f(x) = \frac{x+2}{2(x-1)}$.

83. The x-intercepts are 1 and 3, and the vertical asymptotes are $x = 0$ and $x = 2$. The horizontal asymptote is $y = 1$, so the degree of the numerator equals the degree of the denominator, and the ratio of the leading terms of the numerator and the denominator is 1. Thus, the equation is of the form $y = a\frac{(x-1)(x-3)}{x(x-2)}$.

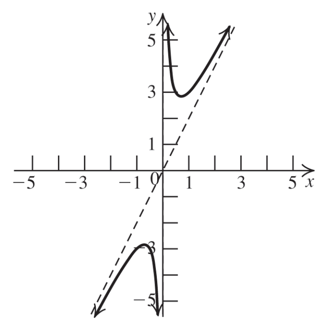
There is no y-intercept, so the equation is
$f(x) = \frac{(x-1)(x-3)}{x(x-2)}$.

84. The x-intercepts are -2 and 3. The vertical asymptotes are $x = -1$ and $x = 2$. The horizontal asymptote is $y = 1$, so the degree of the numerator equals the degree of the denominator, and the ratio of the leading terms of the numerator and the denominator is 1. Thus, the equation is of the form $y = a\frac{(x+2)(x-3)}{(x+1)(x-2)}$.

The y-intercept is 3, so we have
$3 = a \cdot \frac{(0+2)(0-3)}{(0+1)(0-2)} \Rightarrow a = 1.$ Thus, the equation is $f(x) = \frac{(x+2)(x-3)}{(x+1)(x-2)}$.

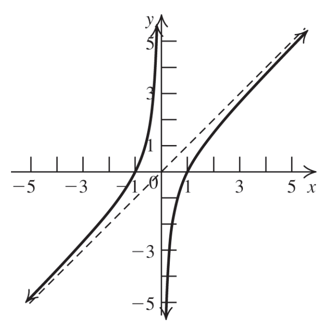
85. $\frac{2x^2+1}{x} = 2x + \frac{1}{x}$.

The oblique asymptote is $y = 2x$.



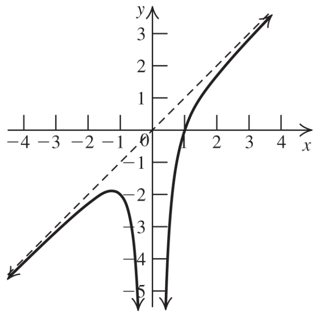
86. $\frac{x^2-1}{x} = x - \frac{1}{x}$.

The oblique asymptote is $y = x$.



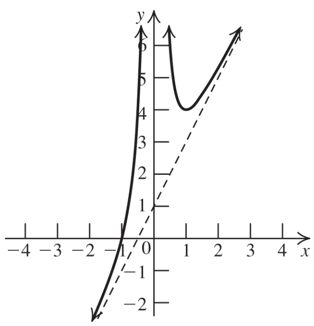
87. $\dfrac{x^3-1}{x^2} = x - \dfrac{1}{x^2}$.

The oblique asymptote is $y = x$.



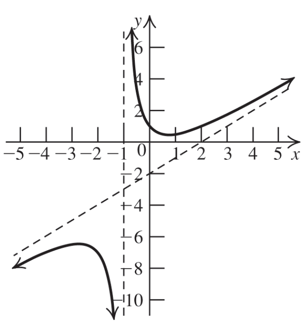
88. $\dfrac{2x^3+x^2+1}{x^2} = 2x+1+\dfrac{1}{x^2}$.

The oblique asymptote is $y = 2x+1$.



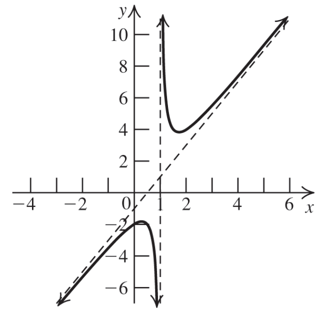
89.
$$\begin{array}{r|l} & x-2 \\ \hline x+1 & x^2 - x + 1 \\ & \underline{x^2 + x} \\ & \quad -2x+1 \\ & \quad \underline{-2x-2} \\ & \qquad\quad 3 \end{array}$$

The oblique asymptote is $y = x - 2$.



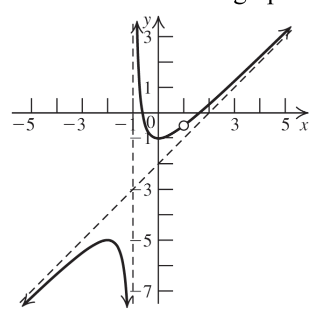
90.
$$\begin{array}{r|l} & 2x-1 \\ \hline x-1 & 2x^2 - 3x + 2 \\ & \underline{2x^2 - 2x} \\ & \qquad -x+2 \\ & \qquad \underline{-x+1} \\ & \qquad\quad 1 \end{array}$$

The oblique asymptote is $y = 2x - 1$.



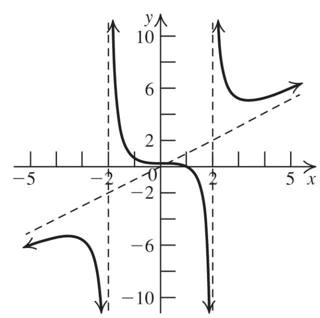
91.
$$\begin{array}{r|l} & x-2 \\ \hline x^2-1 & x^3 - 2x^2 + 0x + 1 \\ & \underline{x^3 \qquad\quad - x} \\ & \quad -2x^2 + x + 1 \\ & \quad \underline{-2x^2 \qquad + 2} \\ & \qquad\qquad x - 1 \end{array}$$

The oblique asymptote is $y = x - 2$. Note that there is a hole in the graph at $x = 1$.



92.
$$\begin{array}{r|l} & x \\ \hline x^2-4 & x^3 + 0x^2 + 0x - 1 \\ & \underline{x^3 \qquad\quad - 4x} \\ & \qquad\qquad 4x - 1 \end{array}$$

The oblique asymptote is $y = x$.



Applying the Concepts

93. a. $C(x) = 0.5x + 2000$

b. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{0.5x+2000}{x} = 0.5 + \dfrac{2000}{x}$

c. $\overline{C}(100) = 0.5 + \dfrac{2000}{100} = 20.5$

$\overline{C}(500) = 0.5 + \dfrac{2000}{500} = 4.5$

$\overline{C}(1000) = 0.5 + \dfrac{2000}{1000} = 2.5$

These show the average cost of producing 100, 500, and 1000 trinkets, respectively.

d. The horizontal asymptote of $\overline{C}(x)$ is $y = 0.5$. It means that the average cost approaches the fixed daily cost of producing each trinket as the number of trinkets approaches ∞.

94. a. $\overline{C}(x) = \dfrac{-0.002x^2 + 6x + 7000}{x}$
$= -0.002x + 6 + \dfrac{7000}{x}$

b. $\overline{C}(100) = -0.002(100) + 6 + \dfrac{7000}{100} = 75.8$

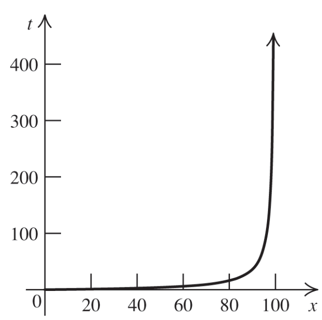
$\overline{C}(500) = -0.002(500) + 6 + \dfrac{7000}{500} = 19$

$\overline{C}(1000) = -0.002(1000) + 6 + \dfrac{7000}{1000} = 11$

These show the average cost of producing 100, 500, and 1000 CD players, respectively.

c. The oblique asymptote is $y = -0.002x + 6$. For large values of x, this is a good approximation of the average cost of producing x CD players.

95. a.



b. $f(50) = \dfrac{4(50)+1}{100-50} \approx 4$ min

$f(75) = \dfrac{4(75)+1}{100-75} \approx 12$ min

$f(95) = \dfrac{4(95)+1}{100-95} \approx 76$ min

$f(99) = \dfrac{4(99)+1}{100-99} \approx 397$ min

c. **(i)** As $x \to 100^-$, $f(x) \to \infty$.

(ii) The statement is not applicable because the domain is $x < 100$.

d. No, the bird doesn't ever collect all the seed from the field.

96. a. $C(50) = \dfrac{1000}{100-50} = 20$

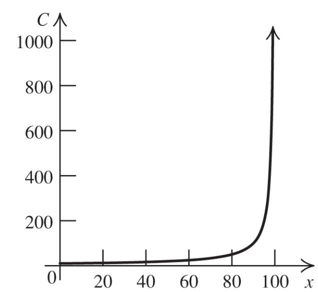
$C(75) = \dfrac{1000}{100-75} = 40$

$C(90) == \dfrac{1000}{100-90} = 100$

$C(99) = \dfrac{1000}{100-99} = 1000$

These show the estimated cost (in millions of dollars) of catching and convicting 50%, 75%, 90%, and 99% of the criminals, respectively.

b.

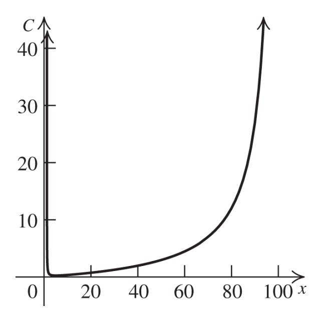


c. As $x \to 100^-$, $C(x) \to \infty$.

d. $30 = \dfrac{1000}{100-x} \Rightarrow 3000 - 30x = 1000 \Rightarrow$
$2000 = 30x \Rightarrow x = 66.67\%$

97. a. $C(50) = \dfrac{3(50^2)+50}{50(100-50)} = \$3.02$ billion

b.



c. $30 = \dfrac{3x^2+50}{x(100-x)} = \dfrac{3x^2+50}{100x-x^2} \Rightarrow$
$3000x - 30x^2 = 3x^2 + 50 \Rightarrow$
$-33x^2 + 3000x - 50 = 0 \Rightarrow$
$x = \dfrac{-3000 \pm \sqrt{3000^2 - 4(-33)(-50)}}{2(-33)}$
$= \dfrac{-3000 \pm \sqrt{8,993,400}}{-66} \approx \dfrac{-3000 \pm 2998.9}{-66}$
≈ 90.89 or -0.017
Reject the negative solution. Approximately 90.89% of the impurities can be removed at a cost of $30 billion.

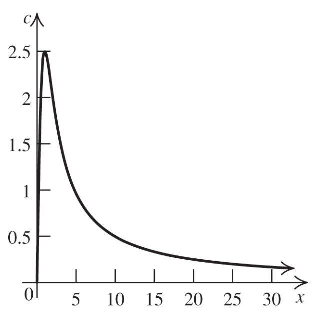
98. a. The horizontal asymptote is $y = a$. a is called the saturation level because as the concentration of the nutrient is increased, the growth rate is pushed close to a.

b. $g(k) = a\dfrac{k}{k+k} = a\dfrac{k}{2k} = \dfrac{a}{2}$

99. a. $P(0) = \dfrac{8(0)+16}{2(0)+1} = 16 \text{ thousand} = 16,000$

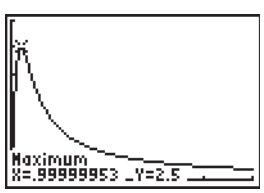
b. The horizontal asymptote is $y = 4$. This means that the population will stabilize at 4000.

100. a.



b. The horizontal asymptote is $y = 0$. This means that, as time passes, the concentration of the drug approaches 0.

c.



The concentration of the drug in the bloodstream is maximal at 1 hour after the injection.

d. $2 = \dfrac{5t}{t^2+1} \Rightarrow 2t^2 - 5t + 2 = 0 \Rightarrow$
$(2t-1)(x-2) = 0 \Rightarrow t = \dfrac{1}{2}$ or $t = 2$
The concentration level equal 2 ml/l at $\frac{1}{2}$ hr and 2 hr after the injection.

101. a. $f(x) = \dfrac{10x + 200,000}{x - 2500}$

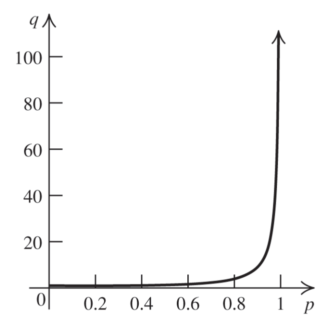
b. $C(10,000) = \dfrac{10(10,000) + 200,000}{10,000 - 2500} = \$40$

c. $20 > \dfrac{10x + 200,000}{x - 2500} \Rightarrow$
$20x - 50,000 > 10x + 200,000 \Rightarrow$
$10x > 250,000 \Rightarrow x > 25,000$
More than 25,000 books must be sold to bring the average cost under $20.

d. The vertical asymptote is $x = 2500$. This represents the number of free samples. The horizontal asymptote is $y = 10$. This represents the cost of printing and binding one book.

102. a. $(1+q)(1-p) = 1 \Rightarrow 1 - p + q - pq = 1 \Rightarrow$
$q(1-p) = p \Rightarrow q = \dfrac{p}{1-p}$

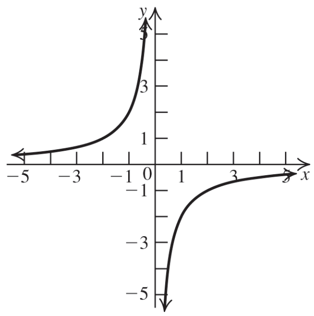
b.



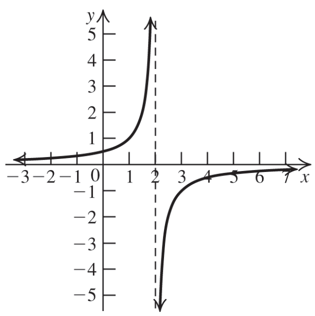
c. $q = \dfrac{0.25}{1-0.25} = 0.33 = 33.33\%$

Beyond the Basics

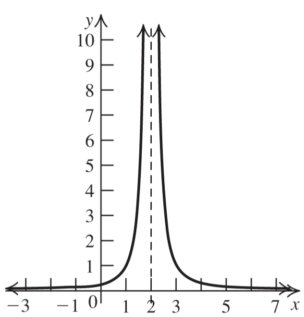
103. Stretch the graph of $y = \dfrac{1}{x}$ vertically by a factor of 2, and then reflect the graph about the *x*-axis.



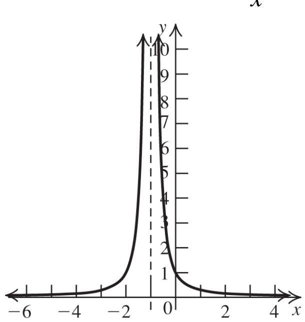
104. Shift the graph of $y = \dfrac{1}{x}$ two units right, and then reflect the graph about the *x*-axis.



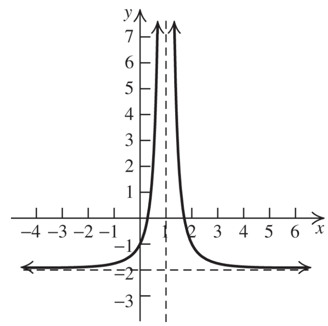
105. Shift the graph of $y = \dfrac{1}{x^2}$ two units right.



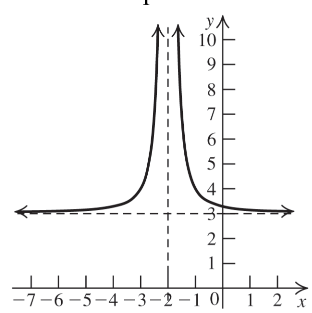
106. Shift the graph of $y = \dfrac{1}{x^2}$ one unit left.



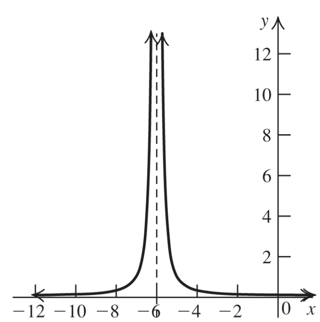
107. Shift the graph of $y = \dfrac{1}{x^2}$ one unit right and two units down.



108. Shift the graph of $y = \dfrac{1}{x^2}$ two units left and three units up.



109. Shift the graph of $y = \dfrac{1}{x^2}$ six units left.



110.
$$\begin{array}{r} 3 \\ x^2+6x+9 \overline{\smash{)}\,3x^2+18x+28} \Rightarrow \\ \underline{3x^2+18x+27} \\ 1 \end{array}$$

$$\frac{3x^2+18x+28}{x^2+6x+9} = \frac{3(x^2+6x+9)+1}{x^2+6x+9} = \frac{3(x^2+6x+9)}{x^2+6x+9} + \frac{1}{x^2+6x+9} = 3 + \frac{1}{(x+3)^2}$$

Shift the graph of $y = \dfrac{1}{x^2}$ three units left and three units up.

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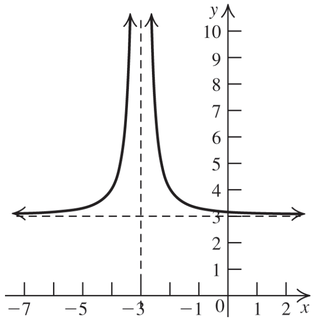
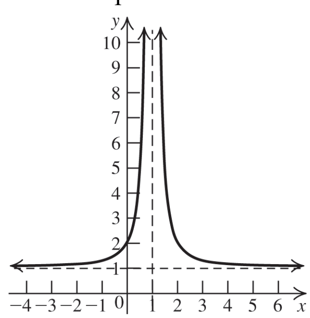


111. $x^2-2x+1\overline{\big)x^2-2x+2} \Rightarrow$ quotient 1

$$\frac{x^2-2x+1}{1}$$

$$\frac{x^2-2x+2}{x^2-2x+1}=\frac{(x^2-2x+1)+1}{x^2-2x+1}=$$
$$\frac{x^2-2x+1}{x^2-2x+1}+\frac{1}{x^2-2x+1}=1+\frac{1}{(x-1)^2}$$

Shift the graph of $y=\dfrac{1}{x^2}$ one unit right and one unit up.

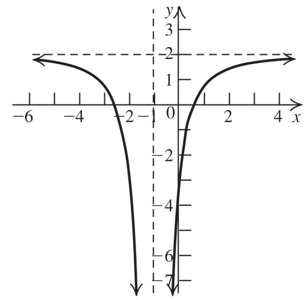


112. $x^2+2x+1\overline{\big)2x^2+4x-3} \Rightarrow$ quotient 2

$$\frac{2x^2+4x+2}{-5}$$

$$\frac{2x^2+4x-3}{x^2+2x+1}=\frac{2(x^2+2x+1)-5}{x^2+2x+1}=$$
$$\frac{2(x^2+2x+1)}{x^2+2x+1}-\frac{5}{x^2+2x+1}=2-\frac{5}{(x+1)^2}$$

Shift the graph of $y=\dfrac{1}{x^2}$ one unit left, stretch the graph vertically by a factor of 5, reflect it in the x-axis, and shift it two units up.



113. a. If c is a zero of $f(x)$, then $x=c$ is a vertical asymptote because c is not a factor of the numerator.

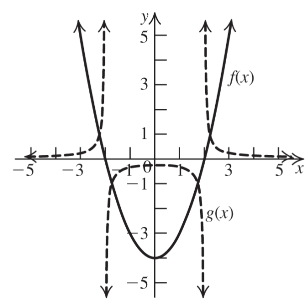
b. Because the numerators are positive, the signs of $f(x)$ and $g(x)$ are the same.

c. If the graphs intersect for some value x, then $f(x)=g(x)$.

$$f(x)=g(x)=\frac{1}{f(x)}\Rightarrow\left(f(x)\right)^2=1\Rightarrow$$
$$f(x)=\pm1.$$

d. If $f(x)$ increases (decreases, remains constant), then the denominator of $g(x)$ increases (decreases, remains constant). Therefore, $g(x)$ decreases (increases, remains constant).

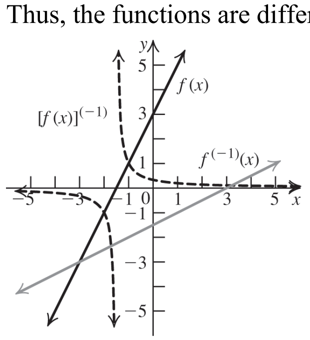
114.



115. $f(x)=2x+3\Rightarrow\left[f(x)\right]^{-1}=\dfrac{1}{2x+3}$ and $y=2x+3$ becomes $x=2y+3\Rightarrow$

$$\frac{x-3}{2}=\frac{x}{2}-\frac{3}{2}=y=f^{-1}(x).$$

Thus, the functions are different.



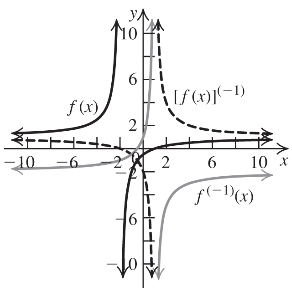
116. $f(x) = \dfrac{x-1}{x+2} \Rightarrow \left[f(x)\right]^{-1} = \dfrac{x+2}{x-1}$ and

$y = \dfrac{x-1}{x+2}$ becomes $x = \dfrac{y-1}{y+2} \Rightarrow$

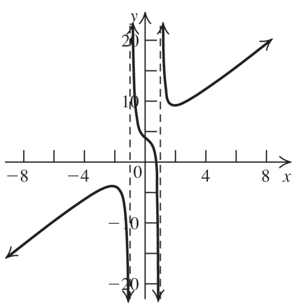
$xy + 2x = y - 1 \Rightarrow xy - y = -2x - 1 \Rightarrow$

$y(x-1) = -2x - 1 \Rightarrow$

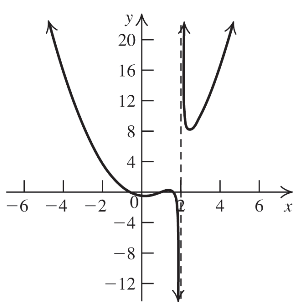
$y = \dfrac{-2x-1}{x-1} = \dfrac{2x+1}{1-x} = y = f^{-1}(x).$



117. $g(x)$ has the oblique asymptote $y = 2x + 3$.
$g(x) \to -\infty$ as $x \to -\infty$, and $g(x) \to \infty$ as $x \to \infty$.



118. $f(x)$ has no oblique asymptote. $f(x) \to \infty$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$. For large $|x|$, the graph behaves like the graph of $y = x^2$.

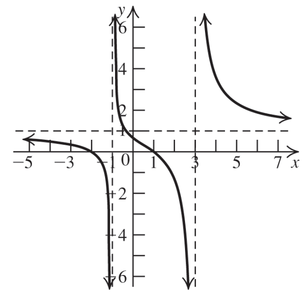


119. The horizontal asymptote is $y = 1$.

$\dfrac{x^2+x-2}{x^2-2x-3} = 1 \Rightarrow x^2 + x - 2 = x^2 - 2x - 3 \Rightarrow$

$3x = -1 \Rightarrow x = -\dfrac{1}{3}$. The point of intersection is

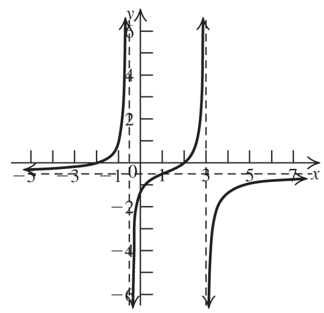
$\left(-\dfrac{1}{3}, 1\right)$.



120. The horizontal asymptote is $y = -\dfrac{1}{2}$.

$\dfrac{4-x^2}{2x^2-5x-3} = -\dfrac{1}{2} \Rightarrow -8 + 2x^2 = 2x^2 - 5x - 3 \Rightarrow$

$x = 1$. The point of intersection is $\left(1, -\dfrac{1}{2}\right)$.



121. Vertical asymptote at $x = 3 \Rightarrow$ the denominator is $x - 3$. Horizontal asymptote at $y = -1 \Rightarrow$ the ratio of the leading coefficients of the numerator and denominator $= -1$. The x–intercept $= 2 \Rightarrow$ the numerator is $x - 2$. So the equation is of the form $f(x) = -\dfrac{x-2}{x-3}$ or $\dfrac{2-x}{x-3}$.

122. Vertical asymptotes at $x = -1$ and $x = 1 \Rightarrow$ the denominator is $(x+1)(x-1)$. Horizontal asymptote at $y = 1 \Rightarrow$ the degrees of the numerator and denominator are the same. The x–intercept $= 0 \Rightarrow$ the numerator is x^2. So the equation is of the form

$f(x) = \dfrac{x^2}{(x-1)(x+1)} = \dfrac{x^2}{x^2-1}$.

123. Vertical asymptote at $x=0 \Rightarrow$ the denominator is x. The slant asymptote $y=-x \Rightarrow$ the degree of the numerator is one more than the degree of the denominator and the quotient of the numerator and the denominator is −. The x-intercepts −1 and 1 $\Rightarrow$ the numerator is $(x-1)(x+1)$. So the equation is of the form

$$f(x)=a\frac{(x-1)(x+1)}{x}=a\left(\frac{x^2-1}{x}\right).$$

$$\begin{array}{r} x \\ x\overline{)x^2-1} \\ \underline{x^2} \\ -1 \end{array}$$

However, the slant asymptote is $y=-x$, so $a=-1$ and the equation is

$$f(x)=-\frac{x^2-1}{x}=\frac{-x^2+1}{x}.$$

124. Vertical asymptote at $x=3 \Rightarrow$ the denominator is $x-3$. The slant asymptote $y=x+4 \Rightarrow$ the degree of the numerator is one more than the degree of the denominator, and the quotient of the numerator and the denominator is $x+4$. Thus, $(x-3)(x+4)+a$ is the numerator.

$$f(4)=14 \Rightarrow f(x)=\frac{(x-3)(x+4)+a}{x-3} \Rightarrow$$

$$14=\frac{4^2+4-12+a}{4-3} \Rightarrow a=6$$

Therefore, the equation is

$$f(x)=\frac{(x-3)(x+4)+6}{x-3}=\frac{x^2+x-6}{x-3}$$

125. Vertical asymptote at $x=2 \Rightarrow$ the denominator is $x-2$. Horizontal asymptote at $y=1 \Rightarrow$ the degrees of the numerator and denominator are the same and the leading coefficients of the numerator and denominator are the same. So the numerator is $x+a$, where a is chosen so that $f(0)=-2 \Rightarrow f(x)=\frac{x+4}{x-2}$.

126. Vertical asymptotes at $x=2$ and $x=-1 \Rightarrow$ the denominator is $(x-2)(x+1)$. Horizontal asymptote at $y=0 \Rightarrow$ the degree of the numerator < the degree of the denominator. So the numerator is $x+a$, where a is chosen so that $f(0)=2 \Rightarrow f(x)=\frac{x-4}{(x-2)(x+1)}$. Verify that the x-intercept is 4:

$$f(4)=\frac{4-4}{(4-2)(4+1)}=0.$$

127. $f(x)\to-\infty$ as $x\to1^-$ and $f(x)\to\infty$ as $x\to1^+ \Rightarrow$ the denominator is zero if $x=1$. Since $x\to1$ from both directions, the denominator is $(x-1)^2$. $f(x)\to4$ as $x\to\pm\infty \Rightarrow$ the horizontal asymptote is 4. So the leading coefficient of the numerator is 4 and the degree of the numerator is the same as the degree of the denominator. The numerator is $4x^2+a$, where a is chosen so that

$$f\left(\frac{1}{2}\right)=0 \Rightarrow \frac{4(1/2)^2+a}{(1/2-1)^2} \Rightarrow a=-1.\text{ So,}$$

$$f(x)=\frac{4x^2-1}{(x-1)^2}.$$

128. No vertical asymptotes $\Rightarrow$ the denominator has no real zeros. $f(x)$ symmetric about the y-axis $\Rightarrow f(x)$ is an even function. So the degree of the numerator and the degree of the denominator can equal 2. Then a possible denominator is x^2+1. $f(x)\to2$ as $x\to\pm\infty \Rightarrow$ the horizontal asymptote is 2. So the numerator is $2x^2+a$ where a is chosen so that $f(0)=0 \Rightarrow a=0$. $f(x)=\frac{2x^2}{x^2+1}$.

129. Vertical asymptote at $x=1 \Rightarrow$ the denominator could be $x-1$. Because the oblique asymptote is $y=3x+2$, the numerator is $(3x+2)(x-1)+a$, where a can be any number (no x-intercepts or function values are given). Let $a=1$. So.

$$f(x)=\frac{(3x+2)(x-1)+1}{x-1}=\frac{3x^2-x+1}{x-1}.$$

130. A rational function cannot have both a horizontal asymptote and an oblique asymptote because there would values of x which are mapped to two different function values. In that case, it wouldn't be a function.

Critical Thinking/Discussion/Writing

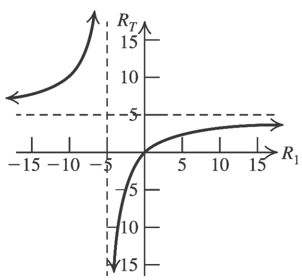
131. $\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2},\ R_2=5\Omega$

a. $\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{5} \Rightarrow \frac{1}{R_T}=\frac{5+R_1}{5R_1}=R_T=\frac{5R_1}{R_1+5}$

b. $\dfrac{5R_1}{R_1+5} \Rightarrow R_1 = 0$ is the x-intercept.

$\dfrac{5R_1}{R_1+5} = \dfrac{5(0)}{0+5} = 0 \Rightarrow y = 0$ is the y-intercept. The vertical asymptote is $x = -5$. The horizontal asymptote is $y = 5$.

$R_T(-R_1) = \dfrac{5(-R_1)}{(-R_1)+5} = -\dfrac{5(-R_1)}{-R_1+5} \neq f(x)$ and $f(-x) \neq -f(x) \Rightarrow$ there are no symmetries. The intervals to be tested are $(-\infty, -5)$, $(-5, 0)$ and $(0, \infty)$. The graph is above the horizontal asymptote on $(-\infty, -5)$ and below the horizontal asymptote on $(-5, 0) \cup (0, \infty)$. The graph is below the x-axis on $(-5, 0)$ and above the x-axis on $(-\infty, -5) \cup (0, \infty)$.



c. R_1 must be greater than 0, so $0 < R_T < 5$. The horizontal asymptote is $R_T = 5$, so as $R_1 \to \infty$, $R_T \to 5$.

132. $K = \dfrac{\alpha^2}{1-\alpha} \cdot c,\ c = 2$

a. $K = \dfrac{2\alpha^2}{1-\alpha} \Rightarrow \alpha = 0$ is the x-intercept.

$\dfrac{2(0)^2}{1-(0)} = 0 \Rightarrow K = 0$ is the y-intercept. The vertical asymptote is $\alpha = 1$. The degree of the numerator is greater than the degree of the denominator, so there is an oblique asymptote.

$\dfrac{2\alpha^2}{1-\alpha} = -2\alpha - 2 + \dfrac{2\alpha}{1-\alpha}$

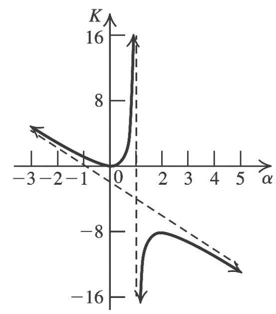
The oblique asymptote is $K = -2\alpha - 2$.

$\dfrac{2(-\alpha)^2}{1-(-\alpha)} = \dfrac{2\alpha^2}{1+\alpha} \neq f(x)$ and $f(-x) \neq -f(x) \Rightarrow$ there are no symmetries.

Use the intervals determined by the zeros of the numerator and of the denominator of

$K - (-2\alpha - 2) = \dfrac{2\alpha^2}{1-\alpha} - (-2\alpha - 2) = \dfrac{2}{1-\alpha}$

to create a sign graph. The numerator 2 has no zero and the denominator has one zero, 1. The intervals to be tested are $(-\infty, 1)$, and $(1, \infty)$. The graph is above the oblique asymptote on $(-\infty, 1)$ and below the oblique asymptote on $(1, \infty)$.



b. As $\alpha \to 0^+$ (meaning as α approaches 0 from the right), $K \to 0$. In other words, α is a positive number that is becoming smaller and smaller. As it approaches 0, $K \approx 2\alpha^2$.

c. As $\alpha \to 1^-$ (meaning as α approaches 1 from the left), $K \to \infty$. In other words, α is a positive number > 0 that is becoming closer and closer to 1. As it approaches 1, the denominator approaches 0, so Ostwald's law cannot hold for strong elctrolytes.

133. Answers may vary. Sample answers are given:

a. $f(x) = \dfrac{1}{x}$ **b.** $f(x) = \dfrac{x^2+2}{x^2+x-2}$

c. $f(x) = \dfrac{x^3+2x^2-7x-1}{x^3+x^2-6x+5}$

134. Answers may vary. Sample answers are given:

a. $f(x) = \dfrac{x^2+1}{x}$

b. $f(x) = \dfrac{x^2+2x-2}{x}$

c. $f(x) = \dfrac{x^5+x^4+x^2-4}{x^4}$

135. Since $y = ax + b$ is the oblique asymptote, we know that $R(x) = ax + b + \frac{r(x)}{D(x)}$. We are told that the graph of $R(x)$ intersects the asymptote at the points $(c_1, R(c_1)), (c_2, R(c_2)), \ldots (c_n, R(c_n))$, so $R(x) = ax + b$ when $\frac{r(x)}{D(x)} = 0$. Thus, $r(x) = (x - c_2)(x - c_2) \cdot \cdots \cdot (x - c_n)$. Recall that the numerator of a rational function must have degree greater than that of the denominator in order for there to be an oblique asymptote, so the denominator, $D(x)$, can be any polynomial whose degree is greater than or equal to $n + 2$. Thus, a possible rational function is

$$R(x) = (ax + b) + \frac{K(x - c_1)(x - c_2)\cdots(x - c_n)}{D(x)}$$
$$= \frac{(ax + b)D(x) + K(x - c_1)(x - c_2)\cdots(x - c_n)}{D(x)},$$

where $D(x)$ is a polynomial of degree $n + 1$ and none of its zeros are at $c_1, c_2, \ldots, c_n$.

136. $f(x) = x^2 + \frac{1}{x^2}$

a. $f(x) \approx \frac{1}{x^2}$ for small values of x. For large values of x, $f(x) \approx x^2$.

b. $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

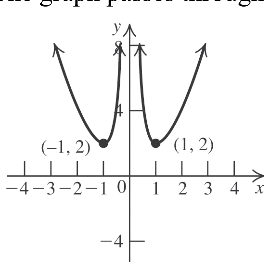
$\left(x - \frac{1}{x}\right)^2 \ge 0$, so $\left(x - \frac{1}{x}\right)^2 + 2 \ge 2$.

$\left(x - \frac{1}{x}\right)^2 + 2 = 2 \Rightarrow \left(\frac{x^2 - 1}{x}\right)^2 + 2 = 2 \Rightarrow$

$\left(\frac{x^2 - 1}{x}\right)^2 = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

The graph passes through (–1, 2) and (1, 2).

c.



Getting Ready for the Next Section

137. $\frac{x-1}{x+3} - 3 = \frac{x-1}{x+3} - \frac{3(x+3)}{x+3} = \frac{x - 1 - 3x - 9}{x+3}$
$= \frac{-2x - 10}{x+3} = -\frac{2(x+5)}{x+3}$

138. $\frac{2-x}{x+1} - 4 = \frac{2-x}{x+1} - \frac{4(x+1)}{x+1} = \frac{2 - x - 4x - 4}{x+1}$
$= \frac{-5x - 2}{x+1} = -\frac{5x+2}{x+1}$

139. $\frac{5}{x-2} - \frac{4}{x+1} = \frac{5(x+1) - 4(x-2)}{(x-2)(x+1)}$
$= \frac{5x + 5 - 4x + 8}{(x-2)(x+1)} = \frac{x+13}{(x-2)(x+1)}$

140. $\frac{2}{x+3} - \frac{3}{x-5} = \frac{2(x-5) - 3(x+3)}{(x+3)(x-5)}$
$= \frac{2x - 10 - 3x - 9}{(x+3)(x-5)}$
$= \frac{-x - 19}{(x+3)(x-5)} = -\frac{x+19}{(x+3)(x-5)}$

For exercises 141–144, instead of graphing the function to determine the solution set as shown in Section 2.1, we use test points.

141. $(x-1)(x+2) > 0$

The zeros occur at $x = -2$ and $x = 1$, so the intervals to be tested are $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$.

Interval	Test point	Value of $f(x)$	Result
$(-\infty, -2)$	–3	4	+
$(-2, 1)$	0	–2	–
$(1, \infty)$	2	4	+

$(x-1)(x+2) > 0$ for $(-\infty, -2) \cup (1, \infty)$.

142. $(x+2)(x+4)<0$

The zeros occur at $x = -2$ and $x = -4$, so the intervals to be tested are $(-\infty, -4)$, $(-4, -2)$, and $(-1, \infty)$.

Interval	Test point	Value of $f(x)$	Result
$(-\infty, -4)$	–5	3	+
$(-4, -2)$	–3	–1	–
$(-2, \infty)$	0	8	+

$(x+2)(x+4)<0$ for $(-4, -2)$.

143. $x^2+3x-10 \ge 0 \Rightarrow (x+5)(x-2) \ge 0$

The zeros occur at $x = -5$ and $x = 2$, so the intervals to be tested are $(-\infty, -5]$, $[-5, 2]$, and $[2, \infty)$.

Interval	Test point	Value of $f(x)$	Result
$(-\infty, -5]$	–10	60	+
$[-5, 2]$	0	–10	–
$[2, \infty)$	10	120	+

$x^2+3x-10 \ge 0$ for $(-\infty, -5] \cup [2, \infty)$.

144. $x^2-6x-7 \le 0 \Rightarrow (x-7)(x+1) \le 0$

The zeros occur at $x = -1$ and $x = 7$, so the intervals to be tested are $(-\infty, -1]$, $[-1, 7]$, and $[7, \infty)$.

Interval	Test point	Value of $f(x)$	Result
$(-\infty, -1]$	–10	153	+
$[-1, 7]$	0	–7	–
$[7, \infty)$	10	33	+

$x^2-6x-7 \le 0$ for $[-1, 7]$.

2.5 Polynomial and Rational Inequalities

2.5 Practice Problems

1. $x^3+2x^2-5x-21 \ge 4x-2x^2+15 \Rightarrow$
$x^3+4x^2-9x-36 \ge 0$

Now solve $x^3+4x^2-9x-36=0$.
The possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$.
Using synthetic division, we see that $x = 3$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -9 & -36 \\ & & 3 & 21 & 36 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$$x^3+4x^2-9x-36 = (x-3)(x^2+7x+12) = (x-3)(x+3)(x+4) = 0$$

So, the zeros are $x = -4$, $x = -3$, and $x = 3$, and the intervals to be tested are $(-\infty, -4]$, $[-4, -3]$, $[-3, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -4]$	–5	–16	–
$[-4, -3]$	–3.5	1.625	+
$[-3, 3]$	0	–36	–
$[3, \infty)$	5	144	+

Solution set: $[-4, -3] \cup [3, \infty)$.

2. $x(x+2)^2(x-1)>0$

The boundary points are $x = 0$, $x = -2$, and $x = 1$, so the intervals are $(-\infty, -2)$, $(-2, 0)$, $(0, 1)$, and $(1, \infty)$. The leading term of $P(x) = x(x+2)^2(x-1) \approx x(x)^2(x) = x^4$, so the end behavior of the graph is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$. Because 0 and 1 are zeros of odd multiplicity, the graph of $P(x)$ crosses the x-axis at these points.
Because –2 is a zeros of even multiplicity, the graph touches but does not cross the x-axis at $x = -2$.

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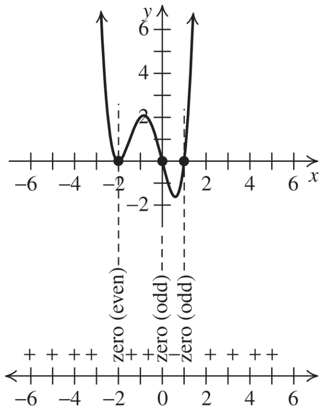
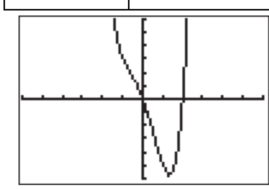


To solve the inequality, examine the graph to find the intervals on which $P(x)$ is positive. The solution set is $(-\infty, -2) \cup (-2, 0) \cup (1, \infty)$.

3. $x(x-2)(x^2+2x+2) \ge 0$

Note that the quadratic term x^2+2x+2 has no real zeros and is always positive. The real zeros of $x(x-2)(x^2+2x+2)$ are $x = 0$ and $x = 2$. So, the intervals are $(-\infty, 0]$, $[0, 2]$, and $[2, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 0]$	–2	16	+
$[0, 2]$	1	–5	–
$[2, \infty)$	3	51	+



[–6, 6] by [–6, 6]

We can verify the solution by examining the graph. Solution set: $(-\infty, 0] \cup [2, \infty)$.

4. $\frac{3}{x} \le \frac{1}{x-4} \Rightarrow \frac{3}{x} - \frac{1}{x-4} \le 0 \Rightarrow$

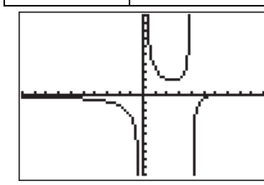
$R(x) = \frac{3(x-4)-x}{x(x-4)} = \frac{2x-12}{x(x-4)} \le 0$

numerator: $2x - 12 = 0 \Rightarrow x = 6$

denominator : $x(x-4) = 0 \Rightarrow x = 0,\ x = 4$

The boundary points are 0, 4, and 6. The lines $x = 0$ and $x = 4$ are the vertical asymptotes of $R(x)$. The intervals are $(-\infty, 0)$, $(0, 4)$, $(4, 6]$, and $[6, \infty)$. Note that 0 and 4 are not included in the intervals to be tested because they cause the denominator of $R(x)$ to equal 0.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, 0)$	–2	$-\frac{4}{3}$	–
$(0, 4)$	1	$\frac{10}{3}$	+
$(4, 6]$	5	$-\frac{2}{5}$	–
$[6, \infty)$	10	$\frac{2}{15}$	+



[–10, 10] by [–10, 10]

We can verify the solution by examining the graph. Solution set: $(-\infty, 0) \cup (4, 6]$.

5. $\frac{5(x+1)}{x+3} < 3 \Rightarrow \frac{5(x+1)}{x+3} - 3 < 0 \Rightarrow$

$R(x) = \frac{5x+5-3(x+3)}{x+3} = \frac{2x-4}{x+3} < 0$

numerator: $2x - 4 = 0 \Rightarrow x = 2$

denominator : $x + 3 = 0 \Rightarrow x = -3$

The boundary points are 2 and 3. The line $x = 3$ is the vertical asymptote of $R(x)$. The intervals are $(-\infty, -3)$, $(-3, 2)$, and $(2, \infty)$. Note that –3 is not included in the intervals to be tested because it causes the denominator of $R(x)$ to equal 0. To determine the sign of $R(x)$, form the corresponding polynomial $P(x) = (2x-4)(x+3)$. The signe rule says that $P(x)$ has the same sign as $R(x)$ on the intervals listed above. $P(x)$ is a quadratic polynomial with leading coefficient $a_2 = 2 > 0$.

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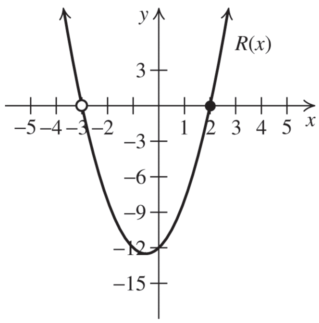
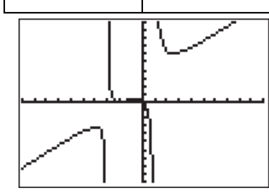


From the graph, it is clear that $R(x) < 0$ when $-3 < x < 2$. Thus, the solution set is $(-3, 2)$.

6. $R(x) = \dfrac{x(x+2)^2}{(x+3)(x-1)} \geq 0$

numerator: $x(x+2)^2 = 0 \Rightarrow x = 0,\ x = -2$
denominator: $(x+3)(x-1) = 0 \Rightarrow x = -3,\ x = 1$
The lines $x = -3$ and $x = 1$ are the vertical asymptotes of $R(x)$. The intervals to be tested are $(-\infty, -3)$, $(-3, -2]$, $[-2, 0]$, $[0, 1)$, and $(1, \infty)$. Note that –3 and 1 are not included in the intervals to be tested because they cause the denominator of $R(x)$ to equal 0.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -3)$	–4	$-\frac{16}{5}$	–
$(-3, -2]$	$-\frac{5}{2}$	$\frac{5}{14}$	+
$[-2, 0]$	–1	$\frac{1}{4}$	+
$[0, 1)$	$\frac{1}{2}$	$-\frac{25}{14}$	–
$(1, \infty)$	2	$\frac{32}{5}$	+



[–10, 10] by [–10, 10]
We can verify the solution by examining the graph. Solution set: $(-3, -2] \cup [-2, 0] \cup (1, \infty)$ or $(-3, 0] \cup (1, \infty)$.

2.5 Exercises
Concepts and Vocabulary

1. For the polynomial inequalities $P(x) \geq 0$ or $P(x) \leq 0$, the zeros of the polynomial function $P(x)$ are always <u>included</u> in the solution set.

2. For the polynomial inequalities $P(x) > 0$ or $P(x) < 0$, the zeros of the polynomial function $P(x)$ are always <u>excluded</u> from the solution set.

3. To solve a rational inequality, we begin by locating all <u>zeros</u> and <u>vertical</u> asymptotes (if they exist).

4. For the rational inequalities $R(x) \geq 0$, $R(x) \leq 0$, $R(x) > 0$, or $R(x) < 0$, the zeros of the denominator of the rational function $R(x)$ (vertical asymptotes or holes) are always <u>excluded</u> from the solution set.

5. True

6. False. A polynomial function does not change sign at its zero if its graph touches, but does not cross the *x*-axis. A polynomial function changes sign at its zero if its graph crosses the *x*-axis.

7. False

8. True

Building Skills

9. From the graph, $P(x) \geq 0$ on $[-2, 1] \cup [3, \infty)$.

10. From the graph $P(x) \leq 0$ on $[-2, 0] \cup [3, \infty)$.

11. From the graph, $P(x) > 0$ on $(-3, 2) \cup (2, \infty)$.

12. From the graph, $P(x) < 0$ on $(2, \infty)$.

13. From the graph, $P(x) > 0$ on $(-\infty, -2) \cup (-2, 2) \cup (3, \infty)$.

14. From the graph, $P(x) < 0$ on $(-2, 1) \cup (1, 3)$.

15. From the graph, $P(x) \geq 0$ on $[-3, 2]$.

16. From the graph $P(x) \leq 0$ on $(-\infty, -3] \cup [-1, \infty)$.

17. $(x-3)(x-4)>0$

The zeros are $x = 3$ and $x = 4$. So, the intervals to be tested are $(-\infty, 3)$, $(3, 4)$, and $(4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 3)$	0	12	+
$(3, 4)$	3.5	–0.25	–
$(4, \infty)$	5	2	+

Solution set: $(-\infty, 3)\cup(4, \infty)$

18. $(x+2)(x-5)<0$

The zeros are $x = -2$ and $x = 5$. So, the intervals to be tested are $(-\infty, -2)$, $(-2, 5)$, and $(5, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2)$	–5	30	+
$(-2, 5)$	0	–10	–
$(5, \infty)$	6	8	+

Solution set: $(-2, 5)$

19. $(x+1)(x-8)\ge 0$

The zeros are $x = -1$ and $x = 8$. So, the intervals to be tested are $(-\infty, -1]$, $[-1, 8]$, and $[8, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -1]$	–2	10	+
$[-1, 8]$	0	–8	–
$[8, \infty)$	10	22	+

Solution set: $(-\infty, -1]\cup[8, \infty)$

20. $(x+4)(x+6)\le 0$

The zeros are $x = -4$ and $x = -6$. So, the intervals to be tested are $(-\infty, -6]$, $[-6, -4]$, and $[-4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -6]$	–10	24	+
$[-6, -4]$	–5	–1	–
$[-4, \infty)$	0	24	+

Solution set: $[-6, -4]$

21. $2(x-1)(x+5)(x-3)<0$

The zeros are $x = 1$, $x = -5$, and $x = 3$. So, the intervals to be tested are $(-\infty, -5)$, $(-5, 1)$, $(1, 3)$, and $(3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -5)$	–6	–126	–
$(-5, 1)$	0	30	+
$(1, 3)$	2	–14	–
$(3, \infty)$	6	330	+

Solution set: $(-\infty, -5)\cup(1, 3)$

22. $3(x+1)(x-2)(x-4)>0$

The zeros are $x = -1$, $x = 2$, and $x = 4$. So, the intervals to be tested are $(-\infty, -1)$, $(-1, 2)$, $(2, 4)$, and $(4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -1)$	–2	–72	–
$(-1, 2)$	0	24	+
$(2, 4)$	3	–12	–
$(4, \infty)$	5	54	+

Solution set: $(-1, 2)\cup(4, \infty)$

23. $(1-x)(x-4)(x+7) \le 0$

The zeros are $x = 1$, $x = 4$, and $x = -7$. So, the intervals to be tested are $(-\infty, -7]$, $[-7, 1]$, $[1, 4]$, and $[4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -7]$	–10	462	+
$[-7, 1]$	0	–28	–
$[1, 4]$	2	18	+
$[4, \infty)$	5	–48	–

Solution set: $[-7, 1] \cup [4, \infty)$

24. $(x+2)(4-x)(x+5) \ge 0$

The zeros are $x = -2$, $x = 4$, and $x = -5$. So, the intervals to be tested are $(-\infty, -5]$, $[-5, -2]$, $[-2, 4]$, and $[4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -5]$	–6	40	+
$[-5, -2]$	–4	–16	–
$[-2, 4]$	0	40	+
$[4, \infty)$	5	–70	–

Solution set: $(-\infty, -5] \cup [-2, 4]$

25. $x^2(x+2)(x-1) < 0$

The zeros are $x = 0$, $x = -2$, and $x = 1$. So, the intervals to be tested are $(-\infty, -2)$, $(-2, 0)$, $(0, 1)$, and $(1, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2)$	–3	36	+
$(-2, 0)$	–1	–2	–
$(0, 1)$	0.5	–0.3125	–
$(1, \infty)$	2	16	+

Solution set: $(-2, 0) \cup (0, 1)$

26. $(x+2)(x-1)^2(x-6) > 0$

The zeros are $x = -2$, $x = 1$, and $x = 6$. So, the intervals to be tested are $(-\infty, -2)$, $(-2, 1)$, $(1, 6)$, and $(6, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2)$	–3	144	+
$(-2, 1)$	0	–12	–
$(1, 6)$	2	–16	–
$(6, \infty)$	7	324	+

Solution set: $(-\infty, -2) \cup (6, \infty)$

27. $x^2(x-1)(x-3) \ge 0$

The zeros are $x = 0$, $x = 1$, and $x = 3$. So, the intervals to be tested are $(-\infty, 0]$, $[0, 1]$, $[1, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 0]$	–1	8	+
$[0, 1]$	0.5	0.3125	+
$[1, 3]$	2	–4	–
$[3, \infty)$	5	200	+

Solution set:
$(-\infty, 0] \cup [0, 1] \cup [3, \infty) = (-\infty, 1] \cup [3, \infty)$

28. $(x+4)(x-3)^2(x-1) \le 0$

The zeros are $x = -4$, $x = 3$, and $x = 1$. So, the intervals to be tested are $(-\infty, -4]$, $[-4, 1]$, $[1, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -4]$	–5	384	+
$[-4, 1]$	0	–36	–
$[1, 3]$	2	6	+
$[3, \infty)$	5	144	+

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Note that $(x+4)(x-3)^2(x-1)=0$ when $x=3$, so include that value in the solution set.
Solution set: $[-4, 1]\cup\{3\}$

29. $2x(x-1)(x^2+1)>0$

The real zeros are $x = 0$ and $x = 1$. So, the intervals to be tested are $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 0)$	–1	8	+
$(0, 1)$	0.5	–0.625	–
$(1, \infty)$	2	20	+

Solution set: $(-\infty, 0)\cup(1, \infty)$

30. $-3x(x+2)(x^2+3)>0$

The real zeros are $x = 0$ and $x = –2$. So, the intervals to be tested are $(-\infty, -2)$, $(-2, 0)$, and $(0, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2)$	–3	–108	–
$(-2, 0)$	–1	12	+
$(0, \infty)$	1	–36	–

Solution set: $(-2, 0)$

31. $x^2-x>2 \Rightarrow x^2-x-2>0 \Rightarrow$
$(x-2)(x+1)>0$

The zeros are $x = 2$ and $x = –1$. So, the intervals to be tested are $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -1)$	–3	10	+
$(-1, 2)$	0	–2	–
$(2, \infty)$	3	4	+

Solution set: $(-\infty, -1)\cup(2, \infty)$

32. $20-x^2>x \Rightarrow -x^2-x+20>0 \Rightarrow$
$-(x^2+x-20)>0 \Rightarrow -(x+5)(x-4)>0$

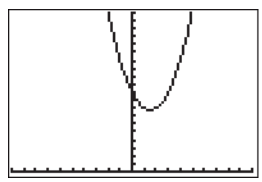
The zeros are $x = –5$ and $x = 4$. So, the intervals to be tested are $(-\infty, -5)$, $(-5, 4)$, and $(4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -5)$	–6	–10	–
$(-5, 4)$	0	20	+
$(4, \infty)$	5	–10	–

Solution set: $(-5, 4)$

33. $x^2+10\ge 3x \Rightarrow x^2-3x+10\ge 0$

$P(x)=x^2-3x+10$ cannot be factored, so there are no rational zeros. We must examine the graph. This is a parabola that opens upward and with vertex $\left(-\frac{b}{2a}, P\left(-\frac{b}{2a}\right)\right)=\left(\frac{3}{2}, \frac{31}{4}\right)$.

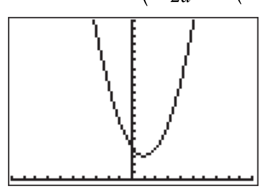


[–10, 10] by [0, 20]

The graph is always greater than 0, so the solution set is $(-\infty, \infty)$.

34. $x^2+4\le 2x \Rightarrow x^2-2x+4\le 0$

$P(x)=x^2-3x+4$ cannot be factored, so there are no rational zeros. We must examine the graph. This is a parabola that opens upward and with vertex $\left(-\frac{b}{2a}, P\left(-\frac{b}{2a}\right)\right)=(1, 1)$.



[–10, 10] by [0, 20]

The graph is never less than or equal to 0, so the solution set is $\varnothing$.

35. $x^3 + 4x^2 + x - 6 > 0$
The possible rational zeros are
$\pm 1, \pm 2, \pm 3, \pm 6.$
Using synthetic division, we find that $x = 1$ is a zero.

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

So,
$$x^3 + 4x^2 + x - 6 = (x-1)(x^2 + 5x + 6)$$
$$= (x-1)(x+2)(x+3)$$
The zeros are $x = -2$, $x = -3$, and $x = 1$. So, the intervals to be tested are $(-\infty, -3)$, $(-3, -2)$, $(-2, 1)$, and $(1, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -3)$	–4	–10	–
$(-3, -2)$	–2.5	0.875	+
$(-2, 1)$	0	–6	–
$(1, \infty)$	2	20	+

Solution set: $(-3, -2) \cup (1, \infty)$

36. $x^3 + x^2 - 10x + 8 < 0$
The possible rational zeros are
$\pm 1, \pm 2, \pm 4, \pm 8.$
Using synthetic division, we find that $x = 1$ is a zero.

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -10 & 8 \\ & & 1 & 2 & -8 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

So,
$$x^3 + x^2 - 10x + 8 = (x-1)(x^2 + 2x - 8)$$
$$= (x-1)(x-2)(x+4)$$
The zeros are $x = -4$, $x = 1$, and $x = 2$. So, the intervals to be tested are $(-\infty, -4)$, $(-4, 1)$, $(1, 2)$, and $(2, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -4)$	–5	–42	–
$(-4, 1)$	0	8	+
$(1, 2)$	1.5	–1.375	–
$(2, \infty)$	3	14	+

Solution set: $(-\infty, -4) \cup (1, 2)$

37. $x^3 - 12x \ge 16 \Rightarrow x^3 - 12x - 16 \ge 0$
The possible rational zeros are
$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16.$
Using synthetic division, we find that $x = -2$ is a zero.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -12 & -16 \\ & & -2 & 4 & 16 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

So,
$$x^3 - 12x - 16 = (x+2)(x^2 - 2x - 8)$$
$$= (x+2)(x+2)(x-4)$$
The zeros are $x = -2$ and $x = 4$. So, the intervals to be tested are $(-\infty, -2]$, $[-2, 4]$, and $[4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2]$	–3	–7	–
$[-2, 4]$	0	–16	–
$[4, \infty)$	5	49	+

Note that $x^3 - 12x \ge 16$ when $x = -2$, so include that value in the solution set.
Solution set: $\{-2\} \cup [4, \infty)$

38. $3x - x^3 \le 2 \Rightarrow -x^3 + 3x - 2 \le 0 \Rightarrow$
$-(x^3 - 3x + 2) \le 0$
The possible rational zeros are $\pm 1, \pm 2.$
Using synthetic division, we find that $x = -2$ is a zero.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

So,
$$-x^3 + 3x - 2 = -(x+2)(x^2 - 2x + 1)$$
$$= -(x+2)(x-1)^2$$
The zeros are $x = -2$ and $x = 1$. So, the intervals to be tested are $(-\infty, -2]$, $[-2, 1]$, and $[1, \infty)$.

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Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2]$	–3	16	+
$[-2, 1]$	0	–2	–
$[1, \infty)$	2	–4	–

Solution set: $[-2, 1] \cup [1, \infty) = [-2, \infty)$

39. $x^3 - 3x^2 + 3x + 5 \le 2x^2 + x - 3 \Rightarrow$
$x^3 - 5x^2 + 2x + 8 \le 0$
The possible rational zeros are
$\pm 1, \pm 2, \pm 4, \pm 8.$
Using synthetic division, we find that $x = 2$ is a zero.

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

So,
$$\begin{aligned} x^3 - 5x^2 + 2x + 8 &= (x-2)(x^2 - 3x - 4) \\ &= (x-2)(x-4)(x+1) \end{aligned}$$
The zeros are $x = -1$, $x = 2$, and $x = 4$. So, the intervals to be tested are $(-\infty, -1]$, $[-1, 2]$, $[2, 4]$, and $[4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -1]$	–2	–24	–
$[-1, 2]$	0	8	+
$[2, 4]$	3	–4	–
$[4, \infty)$	5	18	+

Solution set: $(-\infty, -1] \cup [2, 4]$

40. $x^3 - 4x^2 - 9x + 13 \ge -2x^2 + 2x + 1 \Rightarrow$
$x^3 - 2x^2 - 11x + 12 \ge 0$
The possible rational zeros are
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12.$
Using synthetic division, we find that $x = 1$ is a zero.

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -11 & 12 \\ & & 1 & -1 & -12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

So,
$$\begin{aligned} x^3 - 2x^2 - 11x + 12 &= (x-1)(x^2 - x - 12) \\ &= (x-1)(x-4)(x+3) \end{aligned}$$
The zeros are $x = -3$, $x = 1$, and $x = 4$. So, the intervals to be tested are $(-\infty, -3]$, $[-3, 1]$, $[1, 4]$, and $[4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -3]$	–4	–40	–
$[-3, 1]$	0	12	+
$[1, 4]$	3	–12	–
$[4, \infty)$	5	32	+

Solution set: $[-3, 1] \cup [4, \infty)$

41. $x^4 \ge 3x^2 \Rightarrow x^4 - 3x^2 \ge 0 \Rightarrow x^2(x^2 - 3) \ge 0 \Rightarrow$
$x^2(x + \sqrt{3})(x - \sqrt{3}) \ge 0$
The zeros are $\pm\sqrt{3}$, 0.
So, the intervals to be tested are $(-\infty, -\sqrt{3}]$, $[-\sqrt{3}, 0]$, $[0, \sqrt{3}]$, and $[\sqrt{3}, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -\sqrt{3}]$	–2	4	+
$[-\sqrt{3}, 0]$	–1	–2	–
$[0, \sqrt{3}]$	1	–2	–
$[\sqrt{3}, \infty)$	2	4	+

Note that $x^4 \ge 3x^2$ when $x = 0$, so include that value in the solution set.
Solution set: $(-\infty, -\sqrt{3}] \cup \{0\} \cup [\sqrt{3}, \infty)$

42. $x^4 \le 2x^2 \Rightarrow x^4 - 2x^2 \le 0 \Rightarrow x^2\left(x^2 - 2\right) \le 0 \Rightarrow$
$x^2\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) \le 0$
The zeros are $\pm\sqrt{2}$, 0.
So, the intervals to be tested are
$\left(-\infty, -\sqrt{2}\right]$, $\left[-\sqrt{2}, 0\right]$, $\left[0, \sqrt{2}\right]$, and
$\left[\sqrt{2}, \infty\right)$.

Interval	Test point	Value of $P(x)$	Result
$\left(-\infty, -\sqrt{2}\right]$	–2	8	+
$\left[-\sqrt{2}, 0\right]$	–1	–1	–
$\left[0, \sqrt{2}\right]$	1	–1	–
$\left[\sqrt{2}, \infty\right)$	2	8	+

Solution set: $\left[-\sqrt{2}, 0\right] \cup \left[0, \sqrt{2}\right] = \left[-\sqrt{2}, \sqrt{2}\right]$

43. From the graph, $R(x) \ge 0$ on $(-\infty, 0] \cup (1, \infty)$.

44. From the graph, $R(x) \le 0$ on $(-1, 0]$.

45. From the graph, $R(x) > 0$ on $(-1, 0) \cup (1, \infty)$.

46. From the graph, $R(x) < 0$ on $(-1, 0) \cup (0, 1)$.

47. From the graph, $R(x) \ge 0$ on $(-1, 0] \cup [2, \infty)$.

48. From the graph, $R(x) \le 0$ on $(-\infty, 0] \cup (1, 2]$.

49. From the graph, $R(x) > 0$ on
$(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$.

50. From the graph, $R(x) < 0$ on
$(-1, 0) \cup (0, 1) \cup (1, \infty)$.

For exercises 51–76, verify your answers by graphing the function.

51. $\dfrac{x-1}{x+2} > 0$
numerator: $x - 1 = 0 \Rightarrow x = 1$
denominator: $x + 2 = 0 \Rightarrow x = -2$
The line x = –2 is the vertical asymptote of $R(x)$, so –2 is not included in the intervals to be tested The boundary points are –2 and 1. The intervals are $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	–3	4	+
$(-2, 1)$	0	$-\frac{1}{2}$	–
$(1, \infty)$	2	$\frac{1}{4}$	+

Solution set: $(-\infty, -2) \cup (1, \infty)$

52. $\dfrac{x+3}{x-4} < 0$
numerator: $x + 3 = 0 \Rightarrow x = -3$
denominator: $x - 4 = 0 \Rightarrow x = 4$
The line x = 4 is the vertical asymptote of $R(x)$, so 4 is not included in the intervals to be tested. The boundary points are –3 and 4. The intervals are $(-\infty, -3)$, $(-3, 4)$, and $(4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -3)$	–4	$\frac{1}{8}$	+
$(-3, 4)$	0	$-\frac{3}{4}$	–
$(4, \infty)$	5	8	+

Solution set: $(-3, 4)$

53. $\dfrac{x+2}{x-8} \ge 0$
numerator: $x + 2 = 0 \Rightarrow x = -2$
denominator : $x - 8 = 0 \Rightarrow x = 8$
The line x = 8 is the vertical asymptote of $R(x)$, so 8 is not included in the intervals to be tested. The boundary points are –2 and 8. The intervals are $(-\infty, -2]$, $[-2, 8)$, and $(8, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2]$	–4	$\frac{1}{6}$	+
$[-2, 8)$	0	$-\frac{1}{4}$	–
$(8, \infty)$	10	6	+

Solution set: $(-\infty, -2] \cup (8, \infty)$

54. $\dfrac{x+5}{x-1} \le 0$

numerator: $x+5=0 \Rightarrow x=-5$
denominator : $x-1=0 \Rightarrow x=1$
The line $x = 1$ is the vertical asymptote of $R(x)$, so 1 is not included in the intervals to be tested. The boundary points are –5 and 1. The intervals are $(-\infty, -5]$, $[-5, 1)$, and $(1, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -5]$	–6	$\frac{1}{7}$	+
$[-5, 1)$	0	–5	–
$(1, \infty)$	2	7	+

Solution set: $[-5, 1)$

55. $\dfrac{3x-2}{4-3x} \ge 0$

numerator: $3x-2=0 \Rightarrow x=\frac{2}{3}$

denominator : $4-3x=0 \Rightarrow x=\frac{4}{3}$

The line $x=\frac{4}{3}$ is the vertical asymptote of $R(x)$, so $\frac{4}{3}$ is not included in the intervals to be tested. The boundary points are $\frac{2}{3}$ and $\frac{4}{3}$. The intervals are $\left(-\infty, \frac{2}{3}\right]$, $\left[\frac{2}{3}, \frac{4}{3}\right)$, and $\left(\frac{4}{3}, \infty\right)$.

Interval	Test point	Value of $R(x)$	Result
$\left(-\infty, \frac{2}{3}\right]$	0	$-\frac{1}{2}$	–
$\left[\frac{2}{3}, \frac{4}{3}\right)$	1	1	+
$\left(\frac{4}{3}, \infty\right)$	2	–2	–

Solution set: $\left[\frac{2}{3}, \frac{4}{3}\right)$

56. $\dfrac{1-2x}{3+2x} \ge 0$

numerator: $1-2x=0 \Rightarrow x=\frac{1}{2}$

denominator : $3+2x=0 \Rightarrow x=-\frac{3}{2}$

The line $x=-\frac{3}{2}$ is the vertical asymptote of $R(x)$, so $-\frac{3}{2}$ is not included in the intervals to be tested. The boundary points are $\frac{1}{2}$ and $-\frac{3}{2}$. The intervals are $\left(-\infty, -\frac{3}{2}\right)$, $\left(-\frac{3}{2}, \frac{1}{2}\right]$, and $\left[\frac{1}{2}, \infty\right)$.

Interval	Test point	Value of $R(x)$	Result
$\left(-\infty, -\frac{3}{2}\right)$	–2	–5	–
$\left(-\frac{3}{2}, \frac{1}{2}\right]$	0	$\frac{1}{3}$	+
$\left[\frac{1}{2}, \infty\right)$	1	$-\frac{1}{5}$	–

Solution set: $\left(-\frac{3}{2}, \frac{1}{2}\right]$

57. $\dfrac{(x-1)(x+2)}{x+1} > 0$

numerator: $(x-1)(x+2)=0 \Rightarrow x=-2,\ x=1$
denominator : $x+1=0 \Rightarrow x=-1$
The line $x = -1$ is the vertical asymptote of $R(x)$, so –1 is not included in the intervals to be tested. The boundary points are –2, –1, and 1. The intervals are $(-\infty, -2)$, $(-2, -1)$, $(-1, 1)$, and $(1, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	–3	–2	–
$(-2, -1)$	$-\frac{3}{2}$	$\frac{5}{2}$	+
$(-1, 1)$	0	–2	–
$(1, \infty)$	2	$\frac{4}{3}$	+

Solution set: $(-2, -1) \cup (1, \infty)$

58. $\frac{x(x-3)}{x+2} < 0$

numerator: $x(x-3) = 0 \Rightarrow x = 0,\ x = 3$

denominator : $x + 2 = 0 \Rightarrow x = -2$

The line $x = -2$ is the vertical asymptote of $R(x)$, so -2 is not included in the intervals to be tested. The boundary points are -2, 0, and 3. The intervals are $(-\infty, -2)$, $(-2, 0)$, $(0, 3)$, and $(3, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	-3	-18	$-$
$(-2, 0)$	-1	4	+
$(0, 3)$	2	$-\frac{1}{2}$	$-$
$(3, \infty)$	4	$\frac{2}{3}$	+

Solution set: $(-\infty, -2) \cup (0, 3)$

59. $\frac{(x+5)(x-2)}{x+3} \ge 0$

numerator: $(x+5)(x-2) = 0 \Rightarrow x = -5,\ x = 2$

denominator : $x + 3 = 0 \Rightarrow x = -3$

The line $x = -3$ is the vertical asymptote of $R(x)$, so -3 is not included in the intervals to be tested. The boundary points are -5, -3, and 2. The intervals are $(-\infty, -5]$, $[-5, -3)$, $(-3, 2]$ and $[2, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -5]$	-8	-6	$-$
$[-5, -3)$	-4	6	+
$(-3, 2]$	0	$-\frac{10}{3}$	$-$
$[2, \infty)$	3	$\frac{4}{3}$	+

Solution set: $[-5, -3) \cup [2, \infty)$

60. $\frac{x(x-1)}{x+4} \le 0$

numerator: $x(x-1) = 0 \Rightarrow x = 0,\ x = 1$

denominator : $x + 4 = 0 \Rightarrow x = -4$

The line $x = -4$ is the vertical asymptote of $R(x)$, so -4 is not included in the intervals to be tested. The boundary points are -4, 0, and 1. The intervals are $(-\infty, -4)$, $(-4, 0]$, $[0, 1]$ and $[1, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -4)$	-5	-30	$-$
$(-4, 0]$	-2	3	+
$[0, 1]$	$\frac{1}{2}$	$-\frac{1}{18}$	$-$
$[1, \infty)$	4	$\frac{3}{2}$	+

Solution set: $(-\infty, -4) \cup [0, 1]$

61. $\frac{(x+1)^2(x-1)}{(x+4)(x-3)} \ge 0$

numerator: $(x+1)^2(x-1) = 0 \Rightarrow x = -1,\ x = 1$

denominator :

$(x+4)(x-3) = 0 \Rightarrow x = -4,\ x = 3$

The lines $x = -4$ and $x = 3$ are the vertical asymptotes of $R(x)$, so -4 and 3 are not included in the intervals to be tested. The boundary points are -4, -1, 1, and 3. The intervals are $(-\infty, -4)$, $(-4, -1]$, $[-1, 1]$, $[1, 3)$, and $(3, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -4)$	-5	-12	$-$
$(-4, -1]$	-2	$\frac{3}{10}$	+
$[-1, 1]$	0	$\frac{1}{12}$	+
$[1, 3)$	2	$-\frac{3}{2}$	$-$
$(3, \infty)$	5	8	+

Solution set:

$(-4, -1] \cup [-1, 1] \cup (3, \infty) = (-4, 1] \cup (3, \infty)$

62. $\dfrac{(x+4)(x+2)}{(x-2)^2(x-1)} \geq 0$

numerator: $(x+4)(x+2) = 0 \Rightarrow x = -4,\ x = -2$

denominator :

$(x-2)^2(x-1) = 0 \Rightarrow x = 1,\ x = 2$

The lines $x = 1$ and $x = 2$ are the vertical asymptotes of $R(x)$, so 1 and 2 are not included in the intervals to be tested. The boundary points are –4, –2, 1, and 2. The intervals are $(-\infty, -4]$, $[-4, -2]$, $[-2, 1)$, $(1, 2)$, and $(2, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -4]$	–1	$-\frac{1}{98}$	–
$[-4, -2]$	–3	$\frac{1}{100}$	+
$[-2, 1)$	0	–2	–
$(1, 2)$	$\frac{3}{2}$	154	+
$(2, \infty)$	3	17.5	+

Solution set: $[-4, -2] \cup (1, 2) \cup (2, \infty)$

63. $\dfrac{(2-x)^3(x-3)}{x+1} \leq 0$

numerator: $(2-x)^3(x-3) = 0 \Rightarrow x = 2,\ x = 3$

denominator : $x+1 = 0 \Rightarrow x = -1$

The line $x = -1$ is the vertical asymptote of $R(x)$, so –1 is not included in the intervals to be tested. The boundary points are –1, 2, and 3. The intervals are $(-\infty, -1)$, $(-1, 2]$, $[2, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -1)$	–2	320	+
$(-1, 2]$	0	–24	–
$[2, 3]$	$\frac{5}{2}$	$\frac{1}{56}$	+
$[3, \infty)$	4	$-\frac{8}{5}$	–

Solution set: $(-1, 2] \cup [3, \infty)$

64. $\dfrac{(x-4)(1-x)^3}{x-2} \leq 0$

numerator: $(x-4)(1-x)^3 = 0 \Rightarrow x = 1,\ x = 4$

denominator : $x-2 = 0 \Rightarrow x = 2$

The line $x = 2$ is the vertical asymptote of $R(x)$, so 2 is not included in the intervals to be tested. The boundary points are 1, 2, and 4. The intervals are $(-\infty, 1]$, $[1, 2)$, $(2, 4]$, and $[4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, 1]$	0	2	+
$[1, 2)$	$\frac{3}{2}$	$-\frac{5}{8}$	–
$(2, 4]$	3	8	+
$[4, \infty)$	5	$-\frac{64}{3}$	–

Solution set: $[1, 2) \cup [4, \infty)$

65. $\dfrac{5x-9}{x-4} > 4 \Rightarrow \dfrac{5x-9}{x-4} - 4 > 0 \Rightarrow$

$\dfrac{5x-9-4(x-4)}{x-4} > 0 \Rightarrow \dfrac{x+7}{x-4} > 0$

numerator: $x+7 = 0 \Rightarrow x = -7$

denominator : $x-4 = 0 \Rightarrow x = 4$

The line $x = 4$ is the vertical asymptote of $R(x)$, so 4 is not included in the intervals to be tested. The boundary points are –7 and 4. The intervals are $(-\infty, -7)$, $(-7, 4)$ and $(4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -7)$	–8	$\frac{1}{12}$	+
$(-7, 4)$	0	$-\frac{7}{4}$	–
$(4, \infty)$	5	12	+

Solution set: $(-\infty, -7) \cup (4, \infty)$

66. $\dfrac{2x-17}{x-5} < 3 \Rightarrow \dfrac{2x-17}{x-5} - 3 < 0 \Rightarrow$
$\dfrac{2x-17-3(x-5)}{x-5} < 0 \Rightarrow \dfrac{-x-2}{x-5} < 0$
numerator: $-x-2=0 \Rightarrow x=-2$
denominator : $x-5=0 \Rightarrow x=5$
The line $x = 5$ is the vertical asymptote of $R(x)$, so 5 is not included in the intervals to be tested. The boundary points are –2 and 5. The intervals are $(-\infty, -2)$, $(-2, 5)$ and $(5, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	–3	$-\frac{1}{8}$	–
$(-2, 5)$	0	$\frac{2}{5}$	+
$(5, \infty)$	6	–8	–

Solution set: $(-\infty, -2) \cup (5, \infty)$

67. $\dfrac{x+8}{x+3} \ge 2 \Rightarrow \dfrac{x+8}{x+3} - 2 \ge 0 \Rightarrow$
$\dfrac{x+8-2(x+3)}{x+3} \ge 0 \Rightarrow \dfrac{-x+2}{x+3} \ge 0$
numerator: $-x+2=0 \Rightarrow x=2$
denominator : $x+3=0 \Rightarrow x=-3$
The line $x = -3$ is the vertical asymptote of $R(x)$, so –3 is not included in the intervals to be tested. The boundary points are –3 and 2. The intervals are $(-\infty, -3)$, $(-3, 2]$, and $[2, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -3)$	–4	–6	–
$(-3, 2]$	0	$\frac{2}{3}$	+
$[2, \infty)$	5	$-\frac{3}{8}$	–

Solution set: $(-3, 2]$

68. $\dfrac{4x-9}{x-3} \le 3 \Rightarrow \dfrac{4x-9}{x-3} - 3 \le 0 \Rightarrow$
$\dfrac{4x-9-3(x-3)}{x-3} \le 0 \Rightarrow \dfrac{x}{x-3} \le 0$
numerator: $x = 0$
denominator : $x-3=0 \Rightarrow x=3$
The line $x = 3$ is the vertical asymptote of $R(x)$, so 3 is not included in the intervals to be tested. The boundary points are 0 and 3. The intervals are $(-\infty, 0]$, $[0, 3)$ and $(3, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, 0]$	–3	$\frac{1}{2}$	+
$[0, 3)$	1	$-\frac{1}{2}$	–
$(3, \infty)$	4	4	+

Solution set: $[0, 3)$

69. $\dfrac{2}{x-3} > \dfrac{1}{x+2} \Rightarrow \dfrac{2}{x-3} - \dfrac{1}{x+2} > 0 \Rightarrow$
$\dfrac{2(x+2)-(x-3)}{(x-3)(x+2)} > 0 \Rightarrow \dfrac{x+7}{(x-3)(x+2)} > 0$
numerator: $x+7=0 \Rightarrow x=-7$
denominator :
$(x-3)(x+2)=0 \Rightarrow x=-2,\ x=3$
The lines $x = -2$ and $x = 3$ are the vertical asymptotes of $R(x)$, so –2 and 3 are not included in the intervals to be tested. The boundary points are –7, –2, and 3. The intervals are $(-\infty, -7)$, $(-7, -2)$, $(-2, 3)$, and $(3, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -7)$	–8	$-\frac{1}{66}$	–
$(-7, -2)$	–3	$\frac{2}{3}$	+
$(-2, 3)$	–1	$-\frac{3}{2}$	–
$(3, \infty)$	4	$\frac{11}{6}$	+

Solution set: $(-7, -2) \cup (3, \infty)$

70. $\frac{1}{x+2} < \frac{2}{x-5} \Rightarrow \frac{1}{x+2} - \frac{2}{x-5} < 0 \Rightarrow$
$\frac{(x-5)-2(x+2)}{(x-5)(x+2)} < 0 \Rightarrow \frac{-x-9}{(x-5)(x+2)} < 0$
numerator: $-x-9=0 \Rightarrow x=-9$
denominator :
$(x-5)(x+2)=0 \Rightarrow x=-2,\ x=5$
The lines $x = -2$ and $x = 5$ are the vertical asymptotes of $R(x)$, so –2 and 5 are not included in the intervals to be tested. The boundary points are –9, –2, and 5. The intervals are $(-\infty, -9)$, $(-9, -2)$, $(-2, 5)$, and $(5, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -9)$	–10	$\frac{1}{120}$	+
$(-9, -2)$	–3	$-\frac{3}{4}$	–
$(-2, 5)$	0	$\frac{9}{10}$	+
$(5, \infty)$	7	$-\frac{8}{9}$	–

Solution set: $(-9, -2) \cup (5, \infty)$

71. $\frac{x}{x+2} \ge \frac{x+1}{x-5} \Rightarrow \frac{x}{x+2} - \frac{x+1}{x-5} \ge 0 \Rightarrow$
$\frac{x(x-5)-(x+1)(x+2)}{(x+2)(x-5)} \ge 0 \Rightarrow$
$\frac{x^2-5x-x^2-3x-2}{(x+2)(x-5)} \ge 0 \Rightarrow \frac{-8x-2}{(x+2)(x-5)} \ge 0$
numerator: $-8x-2=0 \Rightarrow x=-\frac{1}{4}$
denominator : $(x+2)(x-5) \Rightarrow x=-2,\ x=5$
The lines $x = -2$ and $x = 5$ are the vertical asymptotes of $R(x)$, so –2 and 5 are not included in the intervals to be tested. The boundary points are –2, $-\frac{1}{4}$, and 5. The intervals are $(-\infty, -2)$, $\left(-2, -\frac{1}{4}\right]$, $\left[-\frac{1}{4}, 5\right)$, and $(5, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	–3	$\frac{11}{4}$	+
$\left(-2, -\frac{1}{4}\right]$	–1	–1	–
$\left[-\frac{1}{4}, 5\right)$	0	$\frac{1}{5}$	+
$(5, \infty)$	6	$-\frac{25}{4}$	–

Solution set: $(-\infty, -2) \cup \left[-\frac{1}{4}, 5\right)$

72. $\frac{x-1}{x+1} \le \frac{x-2}{x+3} \Rightarrow \frac{x-1}{x+1} - \frac{x-2}{x+3} \le 0 \Rightarrow$
$\frac{(x-1)(x+3)-(x-2)(x+1)}{(x+1)(x+3)} \le 0 \Rightarrow$
$\frac{x^2+2x-3-(x^2-x-2)}{(x+1)(x+3)} \le 0 \Rightarrow$
$\frac{3x-1}{(x+1)(x+3)} \le 0$
numerator: $3x-1=0 \Rightarrow x=\frac{1}{3}$
denominator : $(x+1)(x+3) \Rightarrow x=-3,\ x=-1$
The lines $x = -3$ and $x = -1$ are the vertical asymptotes of $R(x)$, so –3 and –1 are not included in the intervals to be tested. The boundary points are –3, –1, and $\frac{1}{3}$. The intervals are $(-\infty, -3)$, $(-3, -1)$, $\left(-1, \frac{1}{3}\right]$, and $\left[\frac{1}{3}, \infty\right)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -3)$	–5	–2	–
$(-3, -1)$	–2	7	+
$\left(-1, \frac{1}{3}\right]$	0	$-\frac{1}{3}$	–
$\left[\frac{1}{3}, \infty\right)$	1	$\frac{1}{4}$	+

Solution set: $(-\infty, -3) \cup \left(-1, \frac{1}{3}\right]$

73. $x+\frac{8}{x}\le 6 \Rightarrow x+\frac{8}{x}-6\le 0 \Rightarrow \frac{x^2-6x+8}{x}\le 0$

numerator:

$x^2-6x+8=0 \Rightarrow (x-2)(x-4)=0 \Rightarrow$
$x=2,\ x=4$

denominator : $x=0$

The line $x=0$ (the y-axis) is the vertical asymptote of $R(x)$, so 0 is not included in the intervals to be tested. The boundary points are 0, 2, and 4. The intervals are $(-\infty, 0), (0, 2], [2, 4],$ and $[4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, 0)$	–1	–15	–
$(0, 2]$	1	3	+
$[2, 4]$	3	$-\frac{1}{3}$	–
$[4, \infty)$	5	$\frac{3}{5}$	+

Solution set: $(-\infty, 0)\cup[2, 4]$

74. $x-\frac{12}{x}\ge 1 \Rightarrow x-\frac{12}{x}-1\ge 0 \Rightarrow \frac{x^2-x-12}{x}\ge 0$

numerator:

$x^2-x-12=0 \Rightarrow (x+3)(x-4)=0 \Rightarrow$
$x=-3,\ x=4$

denominator : $x=0$

The line $x=0$ (the y-axis) is the vertical asymptote of $R(x)$, so 0 is not included in the intervals to be tested. The boundary points are –3, 0, and 4. The intervals are $(-\infty, -3]$, $[-3, 0), (0, 4],$ and $[4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -3]$	–4	–2	–
$[-3, 0)$	–1	10	+
$(0, 4]$	1	–12	–
$[4, \infty)$	6	3	+

Solution set: $[-3, 0)\cup[4, \infty)$

75. $\frac{3x}{x^2-4}\ge 0$

numerator: $3x=0 \Rightarrow x=0$

denominator : $x^2-4=0 \Rightarrow x=\pm 2$

The lines $x=-2$ and $x=2$ are the vertical asymptotes of $R(x)$, so –2 and 2 are not included in the intervals to be tested. The boundary points are –2, 0, and 2. The intervals are $(-\infty, -2), (-2, 0], [0, 2),$ and $(2, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	–4	–1	–
$(-2, 0]$	–1	1	+
$[0, 2)$	1	–1	–
$(2, \infty)$	4	1	+

Solution set: $(-2, 0]\cup(2, \infty)$

76. $\frac{x^2+1}{x^2-4}\le 0$

numerator: $x^2+1=0$ has no real solutions.

denominator : $x^2-4=0 \Rightarrow x=\pm 2$

The lines $x=-2$ and $x=2$ are the vertical asymptotes of $R(x)$, so –2 and 2 are not included in the intervals to be tested. The boundary points are –2 and 2. The intervals are $(-\infty, -2), (-2, 2),$ and $(2, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	–3	2	+
$(-2, 2)$	0	$-\frac{1}{4}$	–
$(2, \infty)$	3	2	+

Solution set: $(-2, 2)$

Applying the Concepts

77. $d=0.05v^2+v$

$d=0.05(25)^2+25=56.25$

In a 25-mile-per-hour zone, the stopping distance is about 56.25 feet, so the driver was going over the speed limit because it took over 75 feet to stop.

78. $-16t^2 + 32t + 95 \geq 110$

$-16t^2 + 32t - 15 \geq 0$

$$t = \frac{-32 \pm \sqrt{32^2 - 4(-16)(-15)}}{2(-16)}$$
$$= \frac{-32 \pm \sqrt{64}}{-32} = \frac{-32 \pm 8}{-32}$$
$$= \frac{-40}{-32} = 1.25 \text{ or } \frac{-24}{-32} = 0.75$$

The height of the stone will be at least 110 feet between 0.75 second and 1.25 seconds.

79. $-Q^3 + 18Q^2 - 11Q - 102 > 0$

The possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 17, \pm 34, \pm 51, \pm 102$.

Using synthetic division, we find that 3 is a factor.

$$\begin{array}{r|rrrr} 3 & -1 & 18 & -11 & -102 \\ & & -3 & 45 & 102 \\ \hline & -1 & 15 & 34 & 0 \end{array}$$

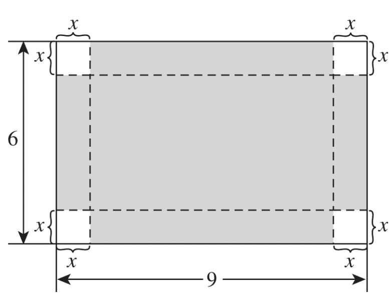
$$\begin{aligned} &-Q^3 + 18Q^2 - 11Q - 102 \\ &= (Q-3)\left(-Q^2 + 15Q + 34\right) \\ &= -(Q-3)\left(Q^2 - 15Q - 34\right) \\ &= -(Q-3)(Q+2)(Q-17) \end{aligned}$$

The zeros are $Q = -2$, $Q = 3$, and $Q = 17$. Because we are looking for an amount, it is not necessary to test any negative values. So, the intervals to be tested are $(0, 3)$, $(3, 17)$, and $(17, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(0, 3)$	1	–96	–
$(3, 17)$	4	78	+
$(17, \infty)$	18	–300	–

The company will make a profit if they sell between 3000 and 17,000 units of lip gloss.

80.



The length of the box is $9 - 2x$, the width is $6 - 2x$, and the height is x. Then, the volume is given by $V = x(9-2x)(6-2x)$.

$$x(9-2x)(6-2x) \geq 20$$
$$4x^3 - 30x^2 + 54x \geq 20$$
$$4x^3 - 30x^2 + 54x - 20 \geq 0$$

The possible rational zeros are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$.

Using synthetic division, we find that 2 is a factor.

$$\begin{array}{r|rrrr} 2 & 4 & -30 & 54 & -20 \\ & & 8 & -44 & 20 \\ \hline & 4 & -22 & 10 & 0 \end{array}$$

$$\begin{aligned} &4x^3 - 30x^2 + 54x - 20 \\ &= (x-2)\left(4x^2 - 22x + 10\right) \\ &= 2(x-2)\left(2x^2 - 11x + 5\right) \\ &= 2(x-2)(2x-1)(x-5) \end{aligned}$$

The zeros are $\frac{1}{2}$, 2, and 5. Because we are looking for a length, it is not necessary to test any negative values. The maximum value of x cannot be 3 or greater because that would eliminate the entire width. So, the intervals to be tested are $\left(0, \frac{1}{2}\right]$, $\left[\frac{1}{2}, 2\right]$, and $[2, 3)$.

Interval	Test point	Value of $P(x)$	Result
$\left(0, \frac{1}{2}\right]$	0.25	–8.3125	–
$\left[\frac{1}{2}, 2\right]$	1	8	+
$[2, 3)$	2.5	–10	–

If $0.5 \leq x \leq 2$, the volume of the resulting open-top box will be at least 20.

81. We want to find the value of Q that makes $TC_A < TC_B$.

$$Q^3 - 6Q^2 + 14Q + 10 < 5Q^2 - 20Q + 34$$
$$Q^3 - 11Q^2 + 34Q - 24 < 0$$

The possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$.

Using synthetic division, we find that 1 is a factor.

$$\begin{array}{r|rrrr} 1 & 1 & -11 & 34 & -24 \\ & & 1 & -10 & 24 \\ \hline & 1 & -10 & 24 & 0 \end{array}$$

(*continued on next page*)

(*continued*)

$$Q^3 - 11Q^2 + 34Q - 24$$
$$= (Q-1)(Q^2 - 10Q + 24)$$
$$= (Q-1)(Q-4)(Q-6)$$

The zeros are $Q = 1$, $Q = 4$, and $Q = 6$. Because we are looking for an amount, it is not necessary to test any negative values. So, the intervals to be tested are $(0, 1)$, $(1, 4)$, $(4, 6)$, and $(6, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(0, 1)$	0.5	–9.625	–
$(1, 4)$	2	8	+
$(4, 6)$	5	–4	–
$(6, \infty)$	7	18	–

It is more cost effective to produce headphones at company *A* when between 0 and 1000 headphones are produced and when between 4000 and 6000 headphones are produced.

82. $\dfrac{64-0.2x}{208-x} > 0.5 \Rightarrow \dfrac{64-0.2x}{208-x} - 0.5 > 0 \Rightarrow$

$\dfrac{64-0.2x-0.5(208-x)}{208-x} > 0 \Rightarrow$

$\dfrac{64-0.2x-104+0.5x}{208-x} > 0 \Rightarrow \dfrac{-40+0.3x}{208-x} > 0$

numerator: $-40 + 0.3x = 0 \Rightarrow x = \frac{400}{3}$

denominator : $208 - x = 0 \Rightarrow x = 208$

The line $x = 208$ is the vertical asymptote of $R(x)$, so 208 is not included in the intervals to be tested. The boundary points are $\frac{400}{3}$ and 208.

Because we are looking for an amount, it is not necessary to test any negative values. There are 208 cards in four decks, so it is not necessary to test values greater than 208. The intervals are $\left(0, \frac{400}{3}\right)$ and $\left(\frac{400}{3}, 208\right)$.

Interval	Test point	Value of $R(x)$	Result
$\left(0, \frac{400}{3}\right)$	100	–0.093	–
$\left(\frac{400}{3}, 208\right)$	150	0.086	+

Note that $\frac{400}{3} \approx 133.3$.

The likelihood that the next card dealt will be a jack, queen, king, or ace will be greater than 50% if between 134 and 208 cards are dealt.

83. The speed of the ferry against the current is $v - 2$, and the speed of the ferry with the current is $v + 2$. The time going against the current is $\frac{40}{v-2}$, and the time going with the current is $\frac{40}{v+2}$. So, we have

$$\frac{40}{v+2} + \frac{40}{v-2} \le 4.5$$
$$\frac{40(v-2)+40(v+2)}{(v+2)(v-2)} \le 4.5$$
$$\frac{80v}{(v+2)(v-2)} - 4.5 \le 0$$
$$\frac{80v - 4.5(v^2-4)}{(v+2)(v-2)} \le 0$$
$$\frac{-4.5v^2 + 80v + 18}{(v+2)(v-2)} \le 0$$

numerator:

$-4.5v^2 + 80v + 18 = 0$

$$v = \frac{-80 \pm \sqrt{80^2 - 4(-4.5)(18)}}{2(-4.5)} \approx -0.22, 18$$

denominator : $(v+2)(v-2) \Rightarrow v = -2, v = 2$

The lines $v = -2$ and $v = 2$ are the vertical asymptotes of $R(v)$, so –2 and 2 are not included in the intervals to be tested. Because we are looking for a speed, it is not necessary to test values less than or equal to 0. The boundary points are 0, 2, and 18. The intervals are $(0, 2)$, $(2, 18]$, and $[18, \infty)$.

Interval	Test point	Value of $R(v)$	Result
$(0, 2)$	1	–31.17	–
$(2, 18]$	5	14.55	+
$[18, \infty)$	20	–0.46	–

The ferry should travel at least 18 miles per hour to limit the round trip to at most 4.5 hours.

84. $\frac{40t}{t^2+16} \geq 3 \Rightarrow \frac{40t}{t^2+16} - 3 \geq 0 \Rightarrow$
$\frac{40t - 3(t^2+16)}{t^2+16} \geq 0 \Rightarrow \frac{-3t^2+40t-48}{t^2+16} \geq 0 \Rightarrow$
$\frac{-(t-12)(3t-4)}{t^2+16} \geq 0$

numerator: $-(t-12)(3t-4) = 0 \Rightarrow t = \frac{4}{3},\ t = 12$

denominator : $t^2+16 = 0$ has no real zeros. Because we are looking for an amount, it is not necessary to test any negative values. The boundary points are $\frac{4}{3}$ and 12. The intervals to be tested are $\left(0, \frac{4}{3}\right]$, $\left[\frac{4}{3}, 12\right]$, and $[12, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$\left(0, \frac{4}{3}\right]$	1	–0.65	–
$\left[\frac{4}{3}, 12\right]$	2	1	+
$[12, \infty)$	13	–0.19	–

The drug concentration is least 3 mg/dl for $\frac{4}{3}$ hr $\leq t \leq 12$ hr, or between 1 hr 20 min and 12 hr, inclusive.

85. We are looking for the values of I that make $P_A < P_B$.
$\frac{3I}{5+I} < \frac{2I}{2+I} \Rightarrow \frac{3I}{5+I} - \frac{2I}{2+I} < 0 \Rightarrow$
$\frac{3I(2+I) - 2I(5+I)}{(5+I)(2+I)} < 0 \Rightarrow$
$\frac{6I + 3I^2 - 10I - 2I^2}{(5+I)(2+I)} < 0 \Rightarrow \frac{I^2-4I}{(5+I)(2+I)} < 0$

numerator:
$I^2 - 4I = I(I-4) = 0 \Rightarrow I = 0,\ I = 4$

denominator :
$(5+I)(2+I) = 0 \Rightarrow I = -5,\ I = -2$

The lines $I = –5$ and $I = –2$ are the vertical asymptotes of $R(I)$, so –5 and –2 are not included in the intervals to be tested.
Because we are looking for an amount, it is not necessary to test any negative values. The boundary points are 0 and 4. The intervals to be tested are $(0, 4)$ and $(4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(0, 4)$	1	$-\frac{1}{6}$	–
$(4, \infty)$	5	$\frac{1}{14}$	+

$P_A < P_B$ for $0 < I < 4$.

86. $\frac{2x}{1+x^2} \geq 0.8 \Rightarrow \frac{2x}{1+x^2} - 0.8 \geq 0 \Rightarrow$
$\frac{2x - 0.8(1+x^2)}{1+x^2} \geq 0 \Rightarrow \frac{-0.8x^2+2x-0.8}{1+x^2} \geq 0 \Rightarrow$
$\frac{-0.4(2x^2-5x+2)}{1+x^2} \geq 0$
$\frac{-0.4(2x-1)(x-2)}{1+x^2} \geq 0$

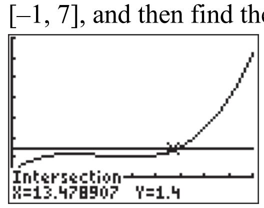
numerator:
$-0.4(2x-1)(x-2) = 0 \Rightarrow x = \frac{1}{2},\ x = 2$

denominator : $1+x^2$ has no real zeros
Because we are looking for an amount, it is not necessary to test any negative values. The boundary points are $\frac{1}{2}$ and 2. The intervals to be tested are $\left(0, \frac{1}{2}\right]$, $\left[\frac{1}{2}, 2\right]$, and $[2, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$\left(0, \frac{1}{2}\right]$	$\frac{1}{4}$	–0.33	–
$\left[\frac{1}{2}, 2\right]$	1	0.2	+
$[2, \infty)$	3	–0.2	–

The efficiency of this rocket is at least 80% for $\frac{1}{2} \leq x \leq 2$.

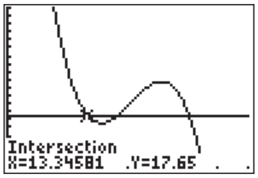
87. $f(x) = 0.29 + 0.436x - 0.066x^2 + 0.00295x^3$
We want to know when $f(x) > 1.4$. Graph $f(x)$ and $y = 1.4$ in the window [0, 20] by [–1, 7], and then find the intersection.

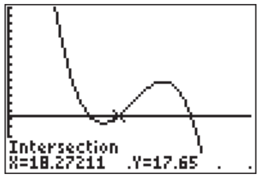


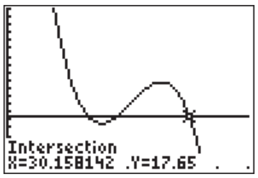
Vinyl sales exceeded 1.4 million between 13.5 years after 1993, or during 2007, and 2013.

88. $f(x) = 25.269 - 1.2405x + 0.064x^2 - 0.001036x^3$

We want to know when $f(x) < 17.65$. Graph $f(x)$ and $y = 17.65$. Use the window [0, 40] by [17, 19] to be able to see the intersections clearly.







The intersections occur at $x \approx 13.3$, which is during 1988, $x \approx 18.3$, which is during 1993, and at $x \approx 30.2$, which is during 2005. The price of a music album was less than \$17.65 between the years 1988 and 1993, and between 2005 and 2014.

Beyond the Basics

89. $f(x) = \sqrt{x^2 - 4x - 12}$

The domain consists of those values of x for which $x^2 - 4x - 12 \ge 0$.

$x^2 - 4x - 12 = 0 \Rightarrow (x-6)(x+2) = 0 \Rightarrow$
$x = 6,\ x = -2$

The intervals to be tested are $(-\infty, -2]$, $[-2, 6]$, and $[6, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2]$	–3	9	+
$[-2, 6]$	0	–12	–
$[6, \infty)$	7	9	+

The domain is $(-\infty, -2] \cup [6, \infty)$.

90. $f(x) = \sqrt{15 - 2x - x^2}$

The domain consists of those values of x for which $15 - 2x - x^2 \ge 0$.

$15 - 2x - x^2 = 0 \Rightarrow -(x+5)(x-3) = 0 \Rightarrow$
$x = -5,\ x = 3$

The intervals to be tested are $(-\infty, -5]$, $[-5, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -5]$	–6	–9	–
$[-5, 3]$	0	15	+
$[3, \infty)$	4	–9	–

The domain is $[-5, 3]$.

91. $f(x) = \sqrt{4x - x^3}$

The domain consists of those values of x for which $4x - x^3 \ge 0$.

$4x - x^3 = 0 \Rightarrow x(2-x)(2+x) = 0 \Rightarrow$
$x = 0,\ x = 2,\ x = -2$

The intervals to be tested are $(-\infty, -2]$, $[-2, 0]$, $[0, 2]$, and $[2, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2]$	–3	15	+
$[-2, 0]$	–1	–3	–
$[0, 2]$	1	3	+
$[2, \infty)$	3	–15	–

The domain is $(-\infty, -2] \cup [0, 2]$.

92. $f(x) = \sqrt{8x + x^4}$

The domain consists of those values of x for which $8x + x^4 \ge 0$.

$8x + x^4 = 0 \Rightarrow x\left(x^3 + 8\right) = 0 \Rightarrow$
$x(x+2)\left(x^2 - 2x + 4\right) = 0 \Rightarrow x = 0,\ x = -2$

The intervals to be tested are $(-\infty, -2]$, $[-2, 0]$, and $[0, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2]$	–3	57	+
$[-2, 0]$	–1	–7	–
$[0, \infty)$	1	9	+

The domain is $(-\infty, -2] \cup [0, \infty)$.

93. $f(x)=\sqrt{\dfrac{2-x}{x+5}}$

The domain consists of those values of x for which $\dfrac{2-x}{x+5}\geq 0$.

numerator: $2-x=0\Rightarrow x=2$

denominator : $x+5=0\Rightarrow x=-5$

The line $x=-5$ is the vertical asymptote of $R(x)$, so –5 is not included in the intervals to be tested. The boundary points are –5 and 2. The intervals to be tested are $(-\infty,-5)$, $(-5,2]$, and $[2,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty,-5)$	–6	–8	–
$(-5,2]$	0	$\frac{2}{5}$	+
$[2,\infty)$	3	$-\frac{1}{8}$	–

The domain is $(-5,2]$.

94. $f(x)=\sqrt{\dfrac{x+3}{x-2}}$

The domain consists of those values of x for which $\dfrac{x+3}{x-2}\geq 0$.

numerator: $x+3=0\Rightarrow x=-3$

denominator : $x-2=0\Rightarrow x=2$

The line $x=2$ is the vertical asymptote of $R(x)$, so 2 is not included in the intervals to be tested. The boundary points are –3 and 2. The intervals to be tested are $(-\infty,-3]$, $[-3,2)$, and $(2,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty,-3]$	–6	$\frac{3}{8}$	+
$[-3,2)$	0	$-\frac{3}{2}$	–
$(2,\infty)$	3	6	+

The domain is $(-\infty,-3]\cup(2,\infty)$.

95. $f(x)=\sqrt{1+\dfrac{1}{x-3}}=\sqrt{\dfrac{x-2}{x-3}}$

The domain consists of those values of x for which $1+\dfrac{1}{x-3}=\dfrac{x-2}{x-3}\geq 0$.

numerator: $x-2=0\Rightarrow x=2$

denominator : $x-3=0\Rightarrow x=3$

The line $x=3$ is the vertical asymptote of $R(x)$, so 3 is not included in the intervals to be tested. The boundary points are 2 and 3. The intervals to be tested are $(-\infty,2]$, $[2,3)$, and $(3,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty,2]$	0	$\frac{2}{3}$	+
$[2,3)$	2.5	–1	–
$(3,\infty)$	4	2	+

The domain is $(-\infty,2]\cup(3,\infty)$.

96. $f(x)=\sqrt{2-\dfrac{3x+11}{x+5}}=\sqrt{\dfrac{2(x+5)-(3x+11)}{x+5}}$
$=\sqrt{\dfrac{-x-1}{x+5}}$

The domain consists of those values of x for which $2-\dfrac{3x+11}{x+5}=\dfrac{-x-1}{x+5}\geq 0$.

numerator: $-x-1=0\Rightarrow x=-1$

denominator : $x+5=0\Rightarrow x=-5$

The line $x=-5$ is the vertical asymptote of $R(x)$, so –5 is not included in the intervals to be tested. The boundary points are –5 and –1. The intervals to be tested are $(-\infty,-5)$, $(-5,-1]$, and $[-1,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty,-5)$	–6	–5	–
$(-5,-1]$	–3	1	+
$[-1,\infty)$	0	$-\frac{1}{5}$	–

The domain is $(-5,-1]$.

97. $f(x)=\sqrt{1+\dfrac{3}{x^2-4}}=\sqrt{\dfrac{x^2-1}{x^2-4}}$

The domain consists of those values of x for which $1+\dfrac{3}{x^2-4}=\dfrac{x^2-1}{x^2-4}\ge 0.$

numerator: $x^2-1=0\Rightarrow x=\pm 1$

denominator : $x^2-4=0\Rightarrow x=\pm 2$

The lines $x = -2$ and $x = 2$ are the vertical asymptotes of $R(x)$, so –2 and 2 are not included in the intervals to be tested. The boundary points are –2, –1, 1 and 2. The intervals to be tested are $(-\infty,-2)$, $(-2,-1]$, $[-1, 1]$, $[1, 2)$, and $(2,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty,-2)$	–4	$\frac{5}{4}$	+
$(-2,-1]$	–1.5	$-\frac{5}{7}$	–
$[-1, 1]$	0	$\frac{1}{4}$	+
$[1, 2)$	1.5	$-\frac{5}{7}$	–
$(2,\infty)$	4	$\frac{5}{4}$	+

The domain is $(-\infty,-2)\cup[-1, 1]\cup(2,\infty)$.

98. $f(x)=\sqrt{x+\dfrac{1}{x}-\dfrac{5}{2}}=\sqrt{\dfrac{2x^2-5x+2}{2x}}$

The domain consists of those values of x for which $x+\dfrac{1}{x}-\dfrac{5}{2}=\dfrac{2x^2-5x+2}{2x}\ge 0.$

numerator:

$2x^2-5x+2=0\Rightarrow(2x-1)(x-2)=0\Rightarrow$

$x=\frac{1}{2},\ x=2$

denominator : $2x=0\Rightarrow x=0$

The line $x = 0$ (the y-axis) is the vertical asymptotes of $R(x)$, so 0 is not included in the intervals to be tested. The boundary points are 0, $\frac{1}{2}$, and 2. The intervals to be tested are $(-\infty, 0)$, $\left(0,\frac{1}{2}\right]$, $\left[\frac{1}{2}, 2\right]$, and $[2,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 0)$	–2	–5	–
$\left(0,\frac{1}{2}\right]$	$\frac{1}{4}$	$\frac{7}{4}$	+
$\left[\frac{1}{2}, 2\right]$	1	$-\frac{1}{2}$	–
$[2,\infty)$	4	$\frac{7}{4}$	+

The domain is $\left(0,\frac{1}{2}\right]\cup[2,\infty)$.

99. $P(x)=x^3-2x^2-11x+12$, range [–10, 18]

We must solve $-10\le P(x)$ and $P(x)\le 18$.

First inequality:

$x^3-2x^2-11x+12\ge -10\Rightarrow$

$x^3-2x^2-11x+22\ge 0$

Possible rational zeros: $\pm 1,\pm 2,\pm 11,\pm 22$

Using synthetic division, we find that 2 is a factor.

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -11 & 22 \\ & & 2 & 0 & -22 \\ \hline & 1 & 0 & -11 & 0 \end{array}$$

$$\begin{aligned} x^3-2x^2-11x+22&=(x-2)\left(x^2-11\right)\\ &=(x-2)\left(x-\sqrt{11}\right)\left(x+\sqrt{11}\right) \end{aligned}$$

The zeros are $x = 2$, $x=\sqrt{11}$, and $x=-\sqrt{11}$. The intervals to be tested are $\left(-\infty,-\sqrt{11}\right]$, $\left[-\sqrt{11}, 2\right]$, $\left[2,\sqrt{11}\right]$, and $\left[\sqrt{11},\infty\right)$. Note that $\sqrt{11}\approx 3.32$.

Interval	Test point	Value of $P(x)$	Result
$\left(-\infty,-\sqrt{11}\right]$	–4	–30	–
$\left[-\sqrt{11}, 2\right]$	–2	28	+
$\left[2,\sqrt{11}\right]$	3	–2	–
$\left[\sqrt{11},\infty\right)$	4	10	+

The solution set for the first inequality is $\left[-\sqrt{11}, 2\right]\cup\left[\sqrt{11},\infty\right)\approx[-3.32, 2]\cup[3.32,\infty)$.

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Second inequality:
$x^3 - 2x^2 - 11x + 12 \le 18 \Rightarrow$
$x^3 - 2x^2 - 11x - 6 \le 0$
Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$ Using synthetic division, we find that –2 is a factor.

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -11 & -6 \\ & & -2 & 8 & 6 \\ \hline & 1 & -4 & -3 & 0 \end{array}$$

$x^3 - 2x^2 - 11x - 6 = (x+2)(x^2 - 4x - 3)$

Use the quadratic formula to find the zeros of $x^2 - 4x - 3$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)} = \frac{4 \pm \sqrt{28}}{2}$$
$$= \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7} \approx -0.65, 4.65$$

The zeros are $x = -2$, $x = 2 - \sqrt{7}$, and $x = 2 + \sqrt{7}$. The intervals to be tested are $(-\infty, -2]$, $[-2, 2-\sqrt{7}]$, $[2-\sqrt{7}, 2+\sqrt{7}]$, and $[2+\sqrt{7}, \infty)$.

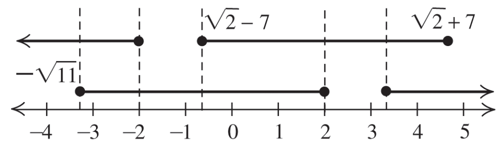
Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2]$	–3	–18	–
$[-2, 2-\sqrt{7}]$	–1	2	+
$[2-\sqrt{7}, 2+\sqrt{7}]$,	0	–6	–
$[2+\sqrt{7}, \infty)$	5	14	+

The solution set for the second inequality is $(-\infty, -2] \cup [2-\sqrt{7}, 2+\sqrt{7}]$,
$\approx (-\infty, -2] \cup [-0.65, 4.65]$.

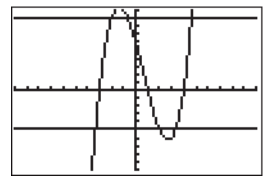
Now we must see where the two solution sets intersect. To make the work easier, use the decimal equivalents.

$$\{[-3.32, 2] \cup [3.32, \infty)\} \cap \{(-\infty, -2] \cup [-0.65, 4.65]\}$$
$$= [-3.32, -2] \cup [-0.65, 2] \cup [3.32, 4.65]$$
$$= [-\sqrt{11}, -2] \cup [2-\sqrt{7}, 2] \cup [\sqrt{11}, 2+\sqrt{7}]$$

The number line below shows the overlap.



Verify graphically by graphing $y_1 = P(x)$, $y_2 = -10$, and $y_3 = 18$, and then finding the intersections.



[–10, 10] by [–20, 20]

100. $P(x) = x^3 - 3x^2 - 4x + 20$, range [8, 14]

We must solve $8 \le P(x)$ and $P(x) \le 14$.

First inequality:
$x^3 - 3x^2 - 4x + 20 \ge 8 \Rightarrow$
$x^3 - 3x^2 - 4x + 12 \ge 0$
Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ Using synthetic division, we find that 2 is a factor.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & & 2 & -2 & 12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$x^3 - 3x^2 - 4x + 12 = (x-2)(x^2 - x - 6)$$
$$= (x-2)(x-3)(x+2)$$

The zeros are $x = -2$, $x = 2$ and $x = 3$ The intervals to be tested are $(-\infty, -2]$, $[-2, 2]$, $[2, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2]$	–3	–30	–
$[-2, 2]$	0	12	+
$[2, 3]$	2.5	–1.125	–
$[3, \infty)$	4	12	+

The solution set for the first inequality is $[-2, 2] \cup [3, \infty)$.

Second inequality:
$x^3 - 3x^2 - 4x + 20 \le 14 \Rightarrow$
$x^3 - 3x^2 - 4x + 6 \le 0$
Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

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(*continued*)

Using synthetic division, we find that 1 is a factor.

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -4 & 6 \\ & & 1 & -2 & -6 \\ \hline & 1 & -2 & -6 & 0 \end{array}$$

$x^3 - 3x^2 - 4x + 6 = (x-1)\left(x^2 - 2x - 6\right)$

Use the quadratic formula to find the zeros of $x^2 - 2x - 6$.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)} = \frac{2 \pm \sqrt{28}}{2}$$
$$= \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7} \approx -1.65,\ 3.65$$

The zeros are $x = 1 - \sqrt{7}$, $x = 1$, and $x = 1 + \sqrt{7}$. The intervals to be tested are $\left(-\infty, 1-\sqrt{7}\right]$, $\left[1-\sqrt{7}, 1\right]$, $\left[1, 1+\sqrt{7}\right]$, and $\left[1+\sqrt{7}, \infty\right)$.

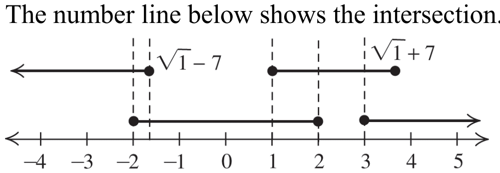
Interval	Test point	Value of $P(x)$	Result
$\left(-\infty, 1-\sqrt{7}\right]$	–2	–6	–
$\left[1-\sqrt{7}, 1\right]$	0	6	+
$\left[1, 1+\sqrt{7}\right]$	2	–6	–
$\left[1+\sqrt{7}, \infty\right)$	4	6	+

The solution set for the second inequality is $\left(-\infty, 1-\sqrt{7}\right] \cup \left[1, 1+\sqrt{7}\right]$, $\approx (-\infty, -1.65] \cup [1, 3.65]$.

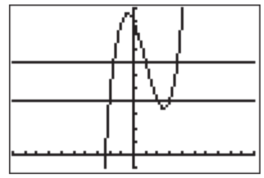
Now we must see where the two solution sets intersect. To make the work easier, use the decimal equivalents.

$$\{[-2, 2] \cup [3, \infty)\} \cap \{(-\infty, -1.65] \cup [1, 3.65]\}$$
$$= [-2, -1.65] \cup [1, 2] \cup [3, 3.65]$$
$$= \left[-2, 1-\sqrt{7}\right] \cup [1, 2] \cup \left[3, 1+\sqrt{7}\right]$$

The number line below shows the intersection.



Verify graphically by graphing $y_1 = P(x)$, $y_2 = -10$, and $y_3 = 18$, and then finding the intersections.



[–10, 10] by [–2, 22]

101. $R(x) = \dfrac{x+2}{x-3}$, range [2, 6]

We must solve $2 \le R(x)$ and $R(x) \le 6$.

First inequality:

$$\frac{x+2}{x-3} \ge 2 \Rightarrow \frac{x+2}{x-3} - 2 \ge 0 \Rightarrow$$
$$\frac{x+2-2(x-3)}{x-3} \ge 0 \Rightarrow \frac{-x+8}{x-3} \ge 0$$

numerator: $-x + 8 = 0 \Rightarrow x = 8$
denominator : $x - 3 = 0 \Rightarrow x = 3$

The line $x = 3$ is the vertical asymptote of $R(x)$, so 3 is not included in the intervals to be tested. The boundary points are 3 and 8. The intervals to be tested are $(-\infty, 3)$, $(3, 8]$, and $[8, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 3)$	2	–6	–
$(3, 8]$	4	4	+
$[8, \infty)$	11	$-\frac{3}{8}$	–

The solution set for the first inequality is $(3, 8]$.

Second inequality:

$$\frac{x+2}{x-3} \le 6 \Rightarrow \frac{x+2}{x-3} - 6 \le 0 \Rightarrow$$
$$\frac{x+2-6(x-3)}{x-3} \le 0 \Rightarrow \frac{-5x+20}{x-3} \le 0$$

numerator: $-5x + 20 = 0 \Rightarrow x = 4$
denominator : $x - 3 = 0 \Rightarrow x = 3$

The line $x = 3$ is the vertical asymptote of $R(x)$, so 3 is not included in the intervals to be tested. The boundary points are 3 and 4. The intervals to be tested are $(-\infty, 3)$, $(3, 4]$, and $[4, \infty)$.

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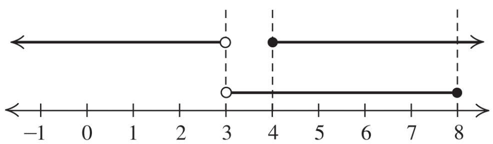
(*continued*)

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 3)$	2	–10	–
$(3, 4]$	3.5	5	+
$[4, \infty)$	8	–4	–

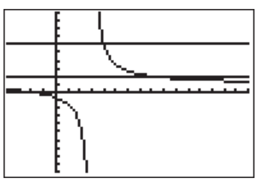
The solution set for the second inequality is $(-\infty, 3) \cup [4, \infty)$. Now we must see where the two solution sets intersect.

$(3, 8] \cap \{(-\infty, 3) \cup [4, \infty)\} = [4, 8]$

The number line below shows the intersection.



Verify graphically by graphing $y_1 = R(x)$, $y_2 = -10$, and $y_3 = 18$, and then finding the intersections.



[–4, 16] by [–10, 10]

102. $R(x) = \dfrac{3-x}{x-7}$, range [1, 3]

We must solve $1 \le R(x)$ and $R(x) \le 3$.

First inequality:

$\dfrac{3-x}{x-7} \ge 1 \Rightarrow \dfrac{3-x}{x-7} - 1 \ge 0 \Rightarrow$

$\dfrac{3-x-(x-7)}{x-7} \ge 0 \Rightarrow \dfrac{-2x+10}{x-7} \ge 0$

numerator: $-2x + 10 = 0 \Rightarrow x = 5$

denominator : $x - 7 = 0 \Rightarrow x = 7$

The line $x = 7$ is the vertical asymptote of $R(x)$, so 7 is not included in the intervals to be tested. The boundary points are 5 and 7. The intervals to be tested are $(-\infty, 5]$, $[5, 7)$, and $(7, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 5]$	–1	$-\frac{3}{2}$	–
$[5, 7)$	6	2	+
$(7, \infty)$	8	–6	–

The solution set for the first inequality is $[5, 7)$.

Second inequality:

$\dfrac{3-x}{x-7} \le 3 \Rightarrow \dfrac{3-x}{x-7} - 3 \le 0 \Rightarrow$

$\dfrac{3-x-3(x-7)}{x-7} \le 0 \Rightarrow \dfrac{-4x+24}{x-7} \le 0$

numerator: $-4x + 24 = 0 \Rightarrow x = 6$

denominator : $x - 7 = 0 \Rightarrow x = 7$

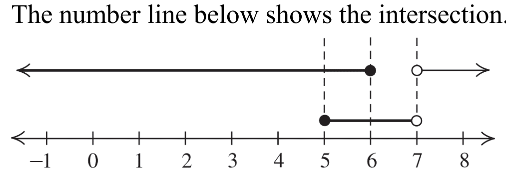
The line $x = 7$ is the vertical asymptote of $R(x)$, so 7 is not included in the intervals to be tested. The boundary points are 6 and 7. The intervals to be tested are $(-\infty, 6]$, $[6, 7)$, and $(7, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 6]$	3	–3	–
$[6, 7)$	6.5	4	+
$(7, \infty)$	8	–8	–

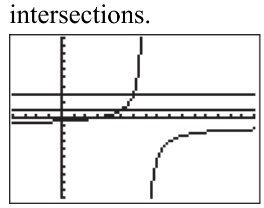
The solution set for the second inequality is $(-\infty, 6] \cup (7, \infty)$. Now we must see where the two solution sets intersect.

$[5, 7) \cap \{(-\infty, 6] \cup (7, \infty)\} = [5, 6]$

The number line below shows the intersection.



Verify graphically by graphing $y_1 = R(x)$, $y_2 = 1$, and $y_3 = 3$, and then finding the intersections.



[–4, 16] by [–10, 10]

103. $R(x)=\dfrac{5-x}{x^2-1}$, range [0, 1]

We must solve $0 \le R(x)$ and $R(x) \le 1$.

First inequality:

$\dfrac{5-x}{x^2-1} \ge 0$

numerator: $5-x=0 \Rightarrow x=5$

denominator : $x^2-1=0 \Rightarrow x=\pm 1$

The lines $x=-1$ and $x=1$ are the vertical asymptotes of $R(x)$, so –1 and 1 are not included in the intervals to be tested. The boundary points are –1, 1, and 5. The intervals to be tested are $(-\infty, -1)$, $(-1, 1)$, $(1, 5]$, and $[5, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -1)$	–3	1	+
$(-1, 1)$	0	–5	–
$(1, 5]$	3	$\frac{1}{4}$	+
$[5, \infty)$	9	$-\frac{1}{20}$	–

The solution set for the first inequality is $(-\infty, -1) \cup (1, 5]$.

Second inequality:

$\dfrac{5-x}{x^2-1} \le 1 \Rightarrow \dfrac{5-x}{x^2-1} - 1 \le 0 \Rightarrow$

$\dfrac{5-x-\left(x^2-1\right)}{x^2-1} \le 0 \Rightarrow \dfrac{-x^2-x+6}{x^2-1} \le 0$

numerator:

$-x^2-x+6=0 \Rightarrow -\left(x^2+x-6\right)=0 \Rightarrow$

$-(x+3)(x-2)=0 \Rightarrow x=-3,\ x=2$

denominator : $x^2-1=0 \Rightarrow x=\pm 1$

The lines $x=-1$ and $x=1$ are the vertical asymptotes of $R(x)$, so –1 and 1 are not included in the intervals to be tested.

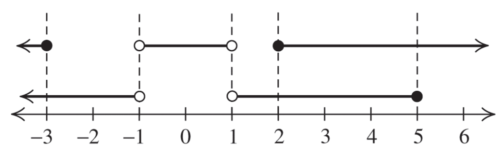
The boundary points are –3, –1, 1, and 2. The intervals to be tested are $(-\infty, -3]$, $[-3, -1)$, $(-1, 1)$, $(1, 2]$, and $[2, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -3]$	–4	$-\frac{2}{5}$	–
$[-3, -1)$	–2	$\frac{4}{3}$	+
$(-1, 1)$	0	–6	–
$(1, 2]$	1.5	$\frac{9}{5}$	+
$[2, \infty)$	3	$-\frac{3}{4}$	–

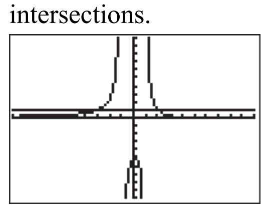
The solution set for the second inequality is $(-\infty, -3] \cup (-1, 1) \cup [2, \infty)$. Now we must see where the two solution sets intersect.

$$\{(-\infty, -1) \cup (1, 5]\} \cap \{(-\infty, -3] \cup (-1, 1) \cup [2, \infty)\} = (-\infty, -3] \cup [2, 5]$$

The number line below shows the intersection.



Verify graphically by graphing $y_1 = R(x)$, $y_2 = 1$, and $y_3 = 3$, and then finding the intersections.



[–10, 10] by [–10, 10]

104. $R(x)=\dfrac{2x-1}{x^2-4}$, range [0, 1]

We must solve $0 \le R(x)$ and $R(x) \le 1$.

First inequality:

$\dfrac{2x-1}{x^2-4} \ge 0$

numerator: $2x-1=0 \Rightarrow x=\frac{1}{2}$

denominator : $x^2-4=0 \Rightarrow x=\pm 2$

The lines $x=-2$ and $x=2$ are the vertical asymptotes of $R(x)$, so –2 and 2 are not included in the intervals to be tested.

(*continued on next page*)

(*continued*)

The boundary points are –2, $\frac{1}{2}$, and 2. The intervals to be tested are $(-\infty, -2)$, $\left(-2, \frac{1}{2}\right]$, $\left[\frac{1}{2}, 2\right)$, and $(2, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2)$	–4	$-\frac{3}{4}$	–
$\left(-2, \frac{1}{2}\right]$	0	$\frac{1}{4}$	+
$\left[\frac{1}{2}, 2\right)$	1	$-\frac{1}{3}$	–
$(2, \infty)$	3	1	+

The solution set for the first inequality is $\left(-2, \frac{1}{2}\right]\cup(2, \infty)$.

Second inequality:

$$\frac{2x-1}{x^2-4} \le 1 \Rightarrow \frac{2x-1}{x^2-4} - 1 \le 0 \Rightarrow$$

$$\frac{2x-1-\left(x^2-4\right)}{x^2-4} \le 0 \Rightarrow \frac{-x^2+2x+3}{x^2-4} \le 0$$

numerator:

$$-x^2+2x+3 \Rightarrow -\left(x^2-2x-3\right)=0 \Rightarrow$$

$$-(x+1)(x-3)=0 \Rightarrow x=-1,\ x=3$$

denominator : $x^2-4=0 \Rightarrow x=\pm 2$

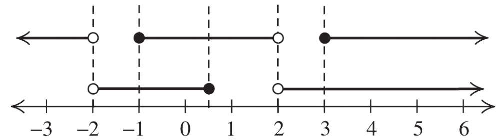
The lines $x = –2$ and $x = 2$ are the vertical asymptotes of $R(x)$, so –2 and 2 are not included in the intervals to be tested. The boundary points are –2, –1, 2, and 3. The intervals to be tested are $(-\infty, -2)$, $(-2, -1]$, $[-1, 2)$, $(2, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -2)$	–4	$-\frac{7}{4}$	–
$(-2, -1]$	–1.5	$\frac{9}{7}$	+
$[-1, 2)$	0	$-\frac{3}{4}$	–
$(2, 3]$	2.5	$\frac{7}{9}$	+
$[3, \infty)$	8	$-\frac{3}{4}$	–

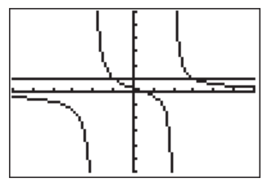
The solution set for the second inequality is $(-\infty, -2)\cup[-1, 2)\cup[3, \infty)$. Now we must see where the two solution sets intersect.

$$\left\{\left(-2, \tfrac{1}{2}\right]\cup(2, \infty)\right\}$$
$$\cap\left\{(-\infty, -2)\cup[-1, 2)\cup[3, \infty)\right\}$$
$$=\left[-1, \tfrac{1}{2}\right]\cup[3, \infty)$$

The number line below shows the intersection.



Verify graphically by graphing $y_1 = R(x)$, $y_2 = 1$, and $y_3 = 3$, and then finding the intersections.



[–6, 6] by [–6, 6]

105. $x+\frac{1}{x} \ge 2 \Rightarrow x+\frac{1}{x}-2 \ge 0 \Rightarrow \frac{x^2-2x+1}{x} \ge 0$

numerator:

$$x^2-2x+1=0 \Rightarrow (x-1)^2=0 \Rightarrow x=1$$

denominator : $x = 0$

The boundary points are 0 and 1, so the intervals to be tested are $(-\infty, 0)$, $(0, 1]$, and $[1, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 0)$	–1	–4	–
$(0, 1]$	0.5	$\frac{1}{2}$	+
$[1, \infty)$	2	$\frac{1}{2}$	+

Solution set: $(0, \infty)$

106. $x^4+(x-2)^4 \ge 2 \Rightarrow x^4+(x-2)^4-2 \ge 0 \Rightarrow$
$2x^4-8x^3+24x^2-32x+14 \ge 0$

From the hint, we know that $(x-1)^2$ is a factor. After using long division or synthetic division twice, we have

$2(x-1)^2(x^2-2x+7) \ge 0$

$(x-1)^2 = 0 \Rightarrow x = 1$

$x^2-2x+7=0 \Rightarrow$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(7)}}{2(1)}=\frac{2\pm\sqrt{-24}}{2}\Rightarrow$$

there are no real zeros.

The intervals to be tested are $(-\infty, 1]$ and $[1, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 1]$	0	14	+
$[1, \infty)$	2	14	+

The solution set is $(-\infty, 1] \cup [1, \infty) = (-\infty, \infty)$.

Therefore, the inequality is true for all real numbers.

107. $\frac{1}{x}+\frac{1}{a-x} \ge \frac{4}{a} \Rightarrow \frac{1}{x}+\frac{1}{a-x}-\frac{4}{a} \ge 0 \Rightarrow$

$$\frac{a(a-x)+ax-4x(a-x)}{ax(a-x)} \ge 0 \Rightarrow$$

$$\frac{a^2-4ax+4x^2}{ax(a-x)} \ge 0$$

numerator:

$a^2-4ax+4x^2 \Rightarrow (a-2x)^2=0 \Rightarrow x=\frac{a}{2}$

denominator : $ax(a-x)=0 \Rightarrow x=0,\ x=a$

The boundary points are 0, $\frac{a}{2}$, and a. The intervals to be tested are $(-\infty, 0)$, $\left(0, \frac{a}{2}\right]$, $\left[\frac{a}{2}, a\right)$, and $[a, \infty)$. Substitute the test values for x in $\frac{a^2-4ax+4x^2}{ax(a-x)}$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 0)$	–1	$-\frac{a^2+4a+4}{a(a+1)}$	–
$\left(0, \frac{a}{2}\right]$	$\frac{a}{4}$	$\frac{4}{3a}$	+
$\left[\frac{a}{2}, a\right)$	$\frac{3a}{4}$	$\frac{4}{3a}$	+
$[a, \infty)$	$a+1$	$-\frac{a^2+4a+4}{a(a+1)}$	–

The solution set is $\left(0, \frac{a}{2}\right] \cup \left[\frac{a}{2}, a\right) = (0, a)$.

Therefore, x is a positive number less than a.

108. $x^3+2a^3 \ge 3xa^2 \Rightarrow x^3-3xa^2+2a^3 \ge 0 \Rightarrow$
$(a-x)^2(2a+x) \ge 0$

The zeros of $P(x)$ are $x = a$ and $x = -2a$.

Because x is positive, it is not necessary to test negative values. Substitute the test values for x in $(a-x)^2(2a+x)$.

Interval	Test point	Value of $P(x)$	Result
$[0, a]$	$\frac{a}{2}$	$\frac{5a^3}{8}$	+
$[a, \infty)$	$a+1$	$3a+1$	+

The solution set is $[0, a] \cup [a, \infty) = [0, \infty)$.

Therefore, x is a positive number.

Critical Thinking/Discussion/Writing

For exercises 109–114, answers will vary. Verify your answer by graphing.

109. $(-4, 5)$
$(x+4)(x-5)<0$

110. $(-\infty, -2] \cup [4, \infty)$
$(x+2)(x-4) \ge 0$

111. $(-1, 2) \cup (3, \infty)$
$(x+1)(x-2)(x-3)>0$

112. $[-3, 1] \cup \{4\}$
$(x+3)(x-1)(x-4)^2 \le 0$

113. $(-1, 0) \cup (4, 5)$
$x(x+1)(x-4)(x-5)<0$

114. $(-\infty, -2] \cup [0, 1] \cup [4, \infty)$
$x(x+2)(x-1)(x-4) \geq 0$

For exercises 115–120, answers will vary. Verify your answer by graphing.

115. $[-2, 6)$
$\dfrac{x+2}{x-6} \leq 0$

116. $(-\infty, -1) \cup [3, \infty)$
$\dfrac{x-3}{x+1} \geq 0$

117. $[-1, 2) \cup [3, \infty)$
$\dfrac{(x+1)(x-3)}{x-2} \geq 0$

118. $(-1, 2] \cup (3, \infty)$
$\dfrac{x-2}{(x+1)(x-3)} \geq 0$

119. $(-1, 2) \cup [3, \infty)$
$\dfrac{x-3}{(x+1)(x-2)} \geq 0$

120. $(-\infty, -4) \cup [0, 1]$
$\dfrac{x(x-1)}{x+4} \leq 0$

121. If $P(x) \leq 0$, then $P(x)$ could be $(x+2)(x-1)(x-3)^2$.
Solving $(x+2)(x-1)(x-3)^2 \geq 0$ leads to the solution set
$(-\infty, -2] \cup [1, 3] \cup [3, \infty) = (-\infty, -2] \cup [1, \infty)$.
Thus, $P(x) = (x+2)(x-1)(x-3)^2$ satisfies both conditions.

122. Use the graphing transformation rules to find the solution sets.

a. $P(-x) = (-\infty, -6] \cup [-4, -2]$

b. $P(2x) = [1, 2] \cup [3, \infty)$

c. $P\left(\frac{1}{2}x\right) = [4, 8] \cup [12, \infty)$

123. $(x-a)(x-1) \geq 0$
For $-\infty < a \leq 1$, the solution set is $(-\infty, a] \cup [1, \infty)$. For $a = 1$, the solution set is $(-\infty, \infty)$. For $a > 1$, the solution set is $(-\infty, 1] \cup [a, \infty)$.

124. $\dfrac{x-a}{x-1} \geq 0$
For $-\infty < a \leq 1$, the solution set is $(-\infty, a] \cup (1, \infty)$. For $a = 1$, the solution set is $(-\infty, 1) \cup (1, \infty)$. For $a > 1$, the solution set is $(-\infty, 1) \cup [a, \infty)$.

125. $(x-a)(x+1)(x-1) \geq 0$
For $-\infty < a \leq -1$, the solution set is $[a, -1] \cup [1, \infty)$. For $a = -1$, the solution set is $\{-1\} \cup [1, \infty)$. For $-1 < a < 1$, the solution set is $[-1, a] \cup [1, \infty)$. For $a = 1$, the solution set is $[-1, \infty)$. For $a > 1$, the solution set is $[-1, 1] \cup [a, \infty)$.

126. $\dfrac{x-a}{(x-1)(x+1)} \geq 0$
For $a < -1$, the solution set is $[a, -1) \cup (1, \infty)$.
For $a = -1$, the solution set is $(1, \infty)$.
For $-1 < a < 1$, the solution set is $(-1, a] \cup (1, \infty)$. For $a = 1$, the solution set is $(-1, 1) \cup (1, \infty)$. For $a > 1$, the solution set is $(-1, 1) \cup [a, \infty)$.

127. P has to be a polynomial of odd degree, and its degree has to be less than or equal to $2k+1$. An example is $(-1, 1) \cup (2, 4) \cup (6, \infty)$ is the solution set of
$P(x) = (x+1)(x-1)(x-2)(x-4)(x-6) > 0$.
The solution set has $k = 2$ open intervals of the form (a, b) and one interval of the form (c, ∞). $P(x)$ has degree $5 = 2k + 1$.

128. P has to be a polynomial of even degree, and its degree has to be greater than or equal to $2k$. An example is $(-3, -2) \cup (-1, 1) \cup (2, 4)$ is the solution set of
$P(x) =$
$(x+3)(x+2)(x+1)(x-1)(x-2)(x-4) < 0$.
The solution set has $k = 3$ open intervals of the form (a, b). $P(x)$ has degree $6 = 2k$.

Getting Ready for the Next Section

129. $(x-i)(x+i) = x^2 - i^2 = x^2 - (-1) = x^2 + 1$

130. $(5x-2i)(5x+2i) = 25x^2 - 4i^2 = 25x^2 - 4(-1)$
$= 25x^2 + 4$

131. $$\begin{aligned}(x-1+2i)(x-1-2i) &= (x-1)^2-(2i)^2\\ &= x^2-2x+1-4i^2\\ &= x^2-2x+1-4(-1)\\ &= x^2-2x+5\end{aligned}$$

132. $$\begin{aligned}(x-2+3i)(x-2-3i) &= (x-2)^2-(3i)^2\\ &= x^2-4x+4-9i^2\\ &= x^2-4x+4-9(-1)\\ &= x^2-4x+13\end{aligned}$$

133. $$\begin{array}{r} x^2+3x-4 \\ x^2-2x+10\overline{)x^4+x^3+0x^2+38x-40} \\ \underline{x^4-2x^3+10x^2} \\ 3x^3-10x^2+38x \\ \underline{3x^3-6x^2+30x} \\ -4x^2+8x-40 \\ \underline{-4x^2+8x-40} \\ 0 \end{array}$$

$$\left(x^2-2x+10\right)\left(x^2+3x-4\right)= x^4+x^3+0x^2+38x-40$$

134. $$\begin{array}{r} x^2+x-6 \\ x^2-4x+20\overline{)x^4-3x^3+10x^2+44x-120} \\ \underline{x^4-4x^3+20x^2} \\ x^3-10x^2+44x \\ \underline{x^3-4x^2+20x} \\ -6x^2+24x-120 \\ \underline{-6x^2+24x-120} \\ 0 \end{array}$$

$$\left(x^2-4x+20\right)\left(x^2+x-6\right)= x^4-3x^3+10x^2+44x-120$$

135. $$\begin{array}{r} x^2-3x-4 \\ x^2-4x+1\overline{)x^4-7x^3+9x^2+13x-4} \\ \underline{x^4-4x^3+x^2} \\ -3x^3+8x^2+13x \\ \underline{-3x^3+12x^2-3x} \\ -4x^2+16x-4 \\ \underline{-4x^2+16x-4} \\ 0 \end{array}$$

$$\left(x^2-4x+1\right)\left(x^2-3x-4\right)= x^4-7x^3+9x^2+13x-4$$

136. $$\begin{array}{r} x^2+5x+6 \\ x^2-2x-4\overline{)x^4+3x^3-8x^2-32x-24} \\ \underline{x^4-2x^3-4x^2} \\ 5x^3-4x^2-32x \\ \underline{5x^3-10x^2-20x} \\ 6x^2-12x-24 \\ \underline{6x^2-12x-24} \\ 0 \end{array}$$

$$\left(x^2-2x-4\right)\left(x^2+5x+6\right)= x^4+3x^3-8x^2-32x-24$$

2.6 Zeros of a Polynomial Function

2.6 Practice Problems

1. $f(x)=2x^5+3x^2+5x-1$
There is one sign change in $f(x)$, so there is one positive zero.
$$\begin{aligned}f(-x) &= 2(-x)^5+3(-x)^2+5(-x)-1\\ &= -2x^5+3x^2-5x-1\end{aligned}$$
There are two sign changes in $f(-x)$, so there are either 2 or 0 negative zeros.

2. $f(x)=x^3-x^2-4x+4$
The possible rational zeros are $\{\pm1, \pm2, \pm4\}$.
There are two sign changes in $f(x)$, so there are either 2 or 0 positive zeros.
$$\begin{aligned}f(-x) &= (-x)^3-(-x)^2-4(-x)+4\\ &= -x^3-x^2+4x+4\end{aligned}$$
There is one sign change in $f(-x)$, so there is 1 negative zero. Testing each positive possibility shows that 2 is an upper bound. Testing the negative values shows that –2 is a lower bound.

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -4 & 4 \\ & & 2 & 2 & -4 \\ \hline & 1 & 1 & -2 & 0 \end{array} \qquad \begin{array}{r|rrrr} -2 & 1 & -1 & -4 & 4 \\ & & -2 & 6 & -4 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

3. a. $$\begin{aligned}P(x) &= 3(x+2)(x-1)[x-(1+i)][x-(1-i)]\\ &= 3(x+2)(x-1)(x-1-i)(x-1+i)\end{aligned}$$

b. $$\begin{aligned}P(x) &= 3(x+2)(x-1)[x-(1+i)][x-(1-i)]\\ &= 3(x+2)(x-1)\left(x^2-2x+2\right)\\ &= 3(x+2)\left(x^3-3x^2+4x-2\right)\\ &= 3\left(x^4-x^3-2x^2+6x-4\right)\\ &= 3x^4-3x^3-6x^2+18x-12\end{aligned}$$

4. Since $2 - 3i$ is a zero of multiplicity 2, so is $2 + 3i$. Since i is a zero, so is $-i$. The eight zeros are $-3, -3, 2 - 3i, 2 - 3i, 2 + 3i, 2 + 3i, i$, and $-i$.

5. The function has degree four, so there are four zeros. Since one zero is $2i$, another zero is $-2i$. So $(x-2i)(x+2i) = x^2+4$ is a factor of $P(x)$. Now divide to find the other factor:

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x^2+4 & x^4 - 3x^3 + 6x^2 - 12x + 8 \\ & \underline{x^4 \qquad + 4x^2} \\ & -3x^3 + 2x^2 - 12x \\ & \underline{-3x^3 \qquad - 12x} \\ & 2x^2 \qquad + 8 \\ & \underline{2x^2 \qquad + 8} \\ & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= x^4 - 3x^3 + 6x^2 - 12x + 8 \\ &= (x-2i)(x+2i)(x^2-3x+2) \\ &= (x-2i)(x+2i)(x-2)(x-1) \end{aligned}$$

The zeros of $P(x)$ are $1, 2, 2i$, and $-2i$.

6. $f(x) = x^4 - 8x^3 + 22x^2 - 28x + 16$

The function has degree 4, so there are four zeros. There are three sign changes, so there are either 3 or 1 positive zeros.

$$\begin{aligned} f(-x) &= (-x)^4 - 8(-x)^3 + 22(-x)^2 - 28(-x) + 16 \\ &= x^4 + 8x^3 + 22x + 28x + 16 \end{aligned}$$

There are no sign changes in $f(-x)$, so there are no negative zeros. The possible rational zeros are $\{\pm1, \pm2, \pm4, \pm8, \pm16\}$. Using synthetic division to test the positive values, we find that one zero is 2:

$$\begin{array}{r|rrrrr} 2 & 1 & -8 & 22 & -28 & 16 \\ & & 2 & -12 & 20 & -16 \\ \hline & 1 & -6 & 10 & -8 & 0 \end{array}$$

The zeros of the depressed function $x^3 - 6x^2 + 10x - 8$ are also zeros of P.

The possible rational zeros of the depressed function are $\{\pm1, \pm2, \pm4, \pm8\}$. Examine only the positive possibilities and find that 4 is a zero:

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 10 & -8 \\ & & 4 & -8 & 8 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

So, $x^4 + 8x^3 + 22x^2 - 28x + 16$
$= (x-2)(x-4)(x^2-2x+2)$.

Now find the zeros of $x^2 - 2x + 2$ using the quadratic formula:

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$$

The zeros are 2, 4, $1 + i$, and $1 - i$.

2.6 Exercises
Concepts and Vocabulary

1. If the nonzero terms of a polynomial are written in descending order, then a variation of sign occurs when the sign of two <u>consecutive</u> terms differ.

2. The number of positive zeros of a polynomial function $P(x)$, is either equal to the number <u>changes</u> of sign of $P(x)$ or less than that number by <u>an even integer</u>.

3. The Fundamental Theorem of Algebra states that a polynomial function of degree $n \geq 1$ has at least one <u>complex</u> zero.

4. If P is a polynomial function with real coefficients and if $z = a + bi$ is a zero of P, then $\underline{\bar{z} = a - bi}$ is also a zero of $P(x)$.

5. True.

6. False. If $2 + 3i$ is a zero of $P(x)$ with real coefficients, then so is $2 - 3i$.

7. True

8. False. $\sqrt{-16} = 4i$

Building Skills

9. $f(x) = 5x^3 - 2x^2 - 3x + 4$;

$$\begin{aligned} f(-x) &= 5(-x)^3 - 2(-x)^2 - 3(-x) + 4 \\ &= -5x^3 - 2x^2 + 3x + 4 \end{aligned}$$

There are two sign changes in $f(x)$, so there are either 2 or 0 positive zeros. There is one sign change in $f(-x)$, so there is one negative zero.

10. $g(x) = 3x^3 + x^2 - 9x - 3$;

$$\begin{aligned} g(-x) &= 3(-x)^3 + (-x)^2 - 9(-x) - 3 \\ &= -3x^3 + x^2 + 9x - 3 \end{aligned}$$

There is one sign change in $g(x)$, so there is one positive zero. There are two sign changes in $g(-x)$, so there are either 2 or 0 negative zeros.

11. $f(x) = 2x^3 + 5x^2 - x + 2;$
$f(-x) = 2(-x)^3 + 5(-x)^2 - (-x) + 2$
$= -2x^3 + 5x^2 + x + 2$
There are two sign changes in $f(x)$, so there are either 2 or 0 positive zeros. There is one sign change in $f(-x)$, so there is one negative zero.

12. $g(x) = 3x^4 + 8x^3 - 5x^2 + 2x - 3;$
$g(-x) = 3(-x)^4 + 8(-x)^3 - 5(-x)^2 + 2(-x) - 3$
$= 3x^4 - 8x^3 - 5x^2 - 2x - 3$
There are three sign changes in $g(x)$, so there are either 3 or 1 positive zeros. There is one sign change in $g(-x)$, so there is 1 negative zero.

13. $h(x) = 2x^5 - 5x^3 + 3x^2 + 2x - 1$
$h(-x) = 2(-x)^5 - 5(-x)^3 + 3(-x)^2 + 2(-x) - 1$
$= -2x^5 + 5x^3 + 3x^2 - 2x - 2$
There are three sign changes in $h(x)$, so there are either 1 or 3 positive zeros. There are two sign changes in $h(-x)$, so there are either 2 or 0 negative zeros.

14. $F(x) = 5x^6 - 7x^4 + 2x^3 - 1;$
$F(-x) = 5(-x)^6 - 7(-x)^4 + 2(-x)^3 - 1$
$= 5x^6 - 7x^4 - 2x^3 - 1$
There are three sign changes in $F(x)$, so there are either 3 or 1 positive zeros. There is one sign change in $F(-x)$, so there is 1 negative zero.

15. $G(x) = -3x^4 - 4x^3 + 5x^2 - 3x + 7;$
$G(-x) = -3(-x)^4 - 4(-x)^3 + 5(-x)^2$
$- 3(-x) + 7$
$= -3x^4 + 4x^3 + 5x^2 + 3x + 7$
There are three sign changes in $G(x)$, so there are either 3 or 1 positive zeros. There is one sign change in $G(-x)$, so there is 1 negative zero.

16. $H(x) = -5x^5 + 3x^3 - 2x^2 - 7x + 4;$
$H(-x) = -5(-x)^5 + 3(-x)^3 - 2(-x)^2$
$-7(-x) + 4$
$= 5x^5 - 3x^3 - 2x^2 + 7x + 4$
There are three sign changes in $H(x)$, so there are either 3 or 1 positive zeros. There are two sign changes in $H(-x)$, so there are 0 or 2 negative zeros.

17. $f(x) = x^4 + 2x^2 + 4$
$f(-x) = (-x)^4 + 2(-x)^2 + 4 = x^4 + 2x^2 + 4$
There are no sign changes in $f(x)$, nor are there sign changes in $f(-x)$. Therefore, there are no positive zeros and no negative zeros.

18. $f(x) = 3x^4 + 5x^2 + 6$
$f(-x) = 3(-x)^4 + 5(-x)^2 + 6 = 3x^4 + 5x^2 + 6$
There are no sign changes in $f(x)$, nor are there sign changes in $f(-x)$. Therefore, there are no positive zeros and no negative zeros.

19. $g(x) = 2x^5 + x^3 + 3x$
$g(-x) = 2(-x)^5 + (-x)^3 + 3(-x)$
$= -2x^5 - x^3 - 3x$
There are no sign changes in $g(x)$, nor are there sign changes in $g(-x)$. Therefore, there are no positive zeros and no negative zeros.

20. $g(x) = 2x^5 + 4x^3 + 5x$
$g(-x) = 2(-x)^5 + 4(-x)^3 + 5(-x)$
$= -2x^5 - 4x^3 - 5x$
There are no sign changes in $g(x)$, nor are there sign changes in $g(-x)$. Therefore, there are no positive zeros and no negative zeros.

21. $h(x) = -x^5 - 2x^3 + 4$
$h(-x) = -(-x)^5 - 2(-x)^3 + 4$
$= x^5 + 2x^3 + 4$
There is one sign change in $h(x)$, and no sign changes in $g(-x)$. Therefore, there is one positive zero and there are no negative zeros.

22. $h(x) = 2x^5 + 3x^3 + 1$
$h(-x) = 2(-x)^5 + 3(-x)^3 + 1$
$= -2x^5 - 3x^3 + 1$
There are no sign changes in $h(x)$, and one sign change in $g(-x)$. Therefore, there are no positive zeros and there is one negative zero.

23. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm 1, \pm 3\right\}$.

There are three sign changes in $f(x)$, so there are either 3 or 1 positive zeros. There are no sign changes in $f(-x)$, so there are no negative zeros. Testing each positive possibility shows that $1/3$ is an upper bound. Testing the negative values shows that $-1/3$ is a lower bound.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & 9 & -3 \\ & & 1 & 0 & 3 \\ \hline & 3 & 0 & 9 & 0 \end{array} \qquad \begin{array}{r|rrrr} -\frac{1}{3} & 3 & -1 & 9 & -3 \\ & & -1 & \frac{2}{3} & -\frac{29}{9} \\ \hline & 3 & -2 & \frac{29}{3} & -\frac{56}{9} \end{array}$$

24. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 3, \pm\frac{7}{2}, \pm 7, \pm\frac{21}{2} \pm 21\right\}$. There are two sign changes in $g(x)$, so there are either 2 or 0 positive zeros. There is one sign change in $g(-x)$, so there is one negative zero. Testing each positive possibility shows that $7/2$ is an upper bound. Testing the negative values shows that –3 is a lower bound.

$$\begin{array}{r|rrrr} \frac{7}{2} & 2 & -3 & -14 & 21 \\ & & 7 & 14 & 0 \\ \hline & 2 & 4 & 0 & 21 \end{array} \qquad \begin{array}{r|rrrr} -3 & 2 & -3 & -14 & 21 \\ & & -6 & 27 & -39 \\ \hline & 2 & -9 & 13 & -18 \end{array}$$

25. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm 1, \pm\frac{7}{3}, \pm 7\right\}$. There are no sign changes in $F(x)$, so there are no positive zeros. There are three sign changes in $F(-x)$, so there are either 3 or 1 negative zeros. Because there are no positive zeros, the smallest positive possible zeros, $1/3$, is an upper bound. Verify this with using synthetic division. Testing the negative values shows that $-7/3$ is a lower bound.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 2 & 5 & 7 \\ & & 1 & 1 & 2 \\ \hline & 3 & 3 & 6 & 9 \end{array} \qquad \begin{array}{r|rrrr} -\frac{7}{3} & 3 & 2 & 5 & 7 \\ & & -7 & \frac{35}{3} & -\frac{350}{9} \\ \hline & 3 & -5 & \frac{50}{3} & -\frac{287}{9} \end{array}$$

26. The possible rational zeros are $\{\pm 1, \pm 2, \pm 4\}$. There is one sign change in $G(x)$, so there is one positive zero. There are two sign changes in $G(-x)$, so there are either 2 or 0 negative zeros. Testing each positive possibility shows that 1 is an upper bound. Testing the negative values shows that –4 is a lower bound.

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 1 & -4 \\ & & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & 1 \end{array} \qquad \begin{array}{r|rrrr} -4 & 1 & 3 & 1 & -4 \\ & & -4 & 4 & -20 \\ \hline & 1 & -1 & 5 & -4 \end{array}$$

27. The possible rational zeros are $\{\pm 1, \pm 31\}$. There are two sign changes in $h(x)$, so there are either 2 or 0 positive zeros. There are two sign changes in $h(-x)$, so there are either 2 or 0 negative zeros. Testing each positive possibility shows that 31 is an upper bound. Testing the negative values shows that –31 is a lower bound.

$$\begin{array}{r|rrrrr} 31 & 1 & 3 & -15 & -9 & 31 \\ & & 31 & 1054 & 32,209 & 998,200 \\ \hline & 1 & 34 & 1039 & 32,200 & 998,231 \end{array}$$

$$\begin{array}{r|rrrrr} -31 & 1 & 3 & -15 & -9 & 31 \\ & & -31 & 868 & -26,443 & 820,012 \\ \hline & 1 & -28 & 853 & -26,452 & 820,043 \end{array}$$

28. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm 1, \pm\frac{13}{3}, \pm 13\right\}$. There are three sign changes in $H(x)$, so there are either 3 or 1 positive zeros. There is one sign change in $H(-x)$, so there is one negative zero. Testing each positive possibility shows that 13 is an upper bound. Testing the negative values shows that –1 is a lower bound.

$$\begin{array}{r|rrrrr} 13 & 3 & -20 & 28 & 19 & -13 \\ & & 39 & 247 & 3575 & 46,722 \\ \hline & 3 & 19 & 275 & 3594 & 46,709 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 3 & -20 & 28 & 19 & -13 \\ & & -3 & 23 & -51 & 32 \\ \hline & 3 & -23 & 51 & -32 & 19 \end{array}$$

29. The possible rational zeros are $\left\{\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm 1, \pm\frac{7}{6}, \pm\frac{7}{3}, \pm\frac{7}{2}, \pm 7\right\}$. There are two sign changes in $f(x)$, so there are either 2 or 0 positive zeros. There are two sign changes in $f(-x)$, so there are either 2 or 0 negative zeros. Testing each positive possibility shows that $7/2$ is an upper bound. Testing the negative values shows that $-7/2$ is a lower bound.

$$\begin{array}{r|rrrrr} \frac{7}{2} & 6 & 1 & -43 & -7 & 7 \\ & & 21 & 77 & 119 & 392 \\ \hline & 6 & 22 & 34 & 112 & 399 \end{array}$$

(*continued on next page*)

(*continued*)

$$\begin{array}{r|rrrrr} -\frac{7}{2} & 6 & 1 & -43 & -7 & 7 \\ & & -21 & 70 & -\frac{189}{2} & \frac{1421}{4} \\ \hline & 6 & -20 & 27 & -\frac{203}{2} & \frac{1449}{4} \end{array}$$

30. The possible rational zeros are $\left\{\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{5}{6}, \pm1, \pm\frac{5}{3}, \pm\frac{5}{2}, \pm5\right\}$. There is one sign change in $g(x)$, so there is one positive zero. There are three sign changes in $g(-x)$, so there are either 3 or 1 negative zeros. Testing each positive possibility shows that $1/2$ is an upper bound. Testing the negative values shows that -5 is a lower bound.

$$\begin{array}{r|rrrrr} \frac{1}{2} & 6 & 23 & 25 & -9 & -5 \\ & & 3 & 13 & 19 & 5 \\ \hline & 6 & 26 & 38 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -5 & 6 & 23 & 25 & -9 & -5 \\ & & -30 & 35 & -300 & 1545 \\ \hline & 6 & -7 & 60 & -309 & 1540 \end{array}$$

31. $x^2 + 25 = 0 \Rightarrow x^2 = -25 \Rightarrow x = \pm 5i$

32. $9x^2 + 16 = 0 \Rightarrow x^2 = -\frac{16}{9} \Rightarrow x = \pm\frac{4}{3}i$

33. $(x-2)^2 + 9 = 0 \Rightarrow (x-2)^2 = -9 \Rightarrow$
$x - 2 = \pm 3i \Rightarrow x = 2 \pm 3i$

34. $(x-1)^2 + 27 = 0 \Rightarrow (x-1)^2 = -27 \Rightarrow$
$x - 1 = \pm 3i\sqrt{3} \Rightarrow x = 1 \pm 3i\sqrt{3}$

35. $x^2 + 4x + 4 = -9 \Rightarrow (x+2)^2 = -9 \Rightarrow$
$x + 2 = \pm 3i \Rightarrow x = -2 \pm 3i$

36. $x^2 + 2x + 1 = -16 \Rightarrow (x+1)^2 = -16 \Rightarrow$
$x + 1 = \pm 4i \Rightarrow x = -1 \pm 4i$

37. $x^3 - 8 = 0 \Rightarrow (x-2)(x^2 + 2x + 4) = 0 \Rightarrow$

$$x = 2 \text{ or } x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm \sqrt{-12}}{2} = -1 \pm i\sqrt{3}$$

The solution set is $\left\{2, -1 \pm i\sqrt{3}\right\}$.

38. $x^4 - 1 = 0 \Rightarrow (x^2 - 1)(x^2 + 1) = 0 \Rightarrow$
$(x-1)(x+1)(x^2+1) = 0 \Rightarrow x = 1$ or $x = -1$ or $x = \pm i$.

39. $(x-2)(x-3i)(x+3i) = 0 \Rightarrow x = 2$ or $x = \pm 3i$

40. $(x-1)(x-2i)(2x-6i)(2x+6i) = 0 \Rightarrow$
$x = 1$ or $x = 2i$ or $x = \pm 3i$

41. The remaining zero is $3 - i$.

42. The remaining zero is $2 + i$.

43. The remaining zero is $2 + i$.

44. The remaining zeros are i and $1 + i$.

45. The remaining zeros are $-i$ and $-3i$.

46. The remaining zeros are $-2i, 4 - i,$ and $-1 - i$.

47.
$$\begin{aligned} P(x) &= 2(x-(5-i))(x-(5+i))(x-3i)(x+3i) \\ &= 2x^4 - 20x^3 + 70x^2 - 180x + 468 \end{aligned}$$

48.
$$\begin{aligned} P(x) &= -3(x-(2+3i))(x-(2-3i)) \\ &\quad \cdot (x-(1-4i))(x-(1+4i)) \\ &= -3x^4 + 18x^3 - 114x^2 + 282x - 663 \end{aligned}$$

49.
$$\begin{aligned} P(x) &= 7(x-5)^2(x-1)(x-(3-i))(x-(3+i) \\ &= 7x^5 - 119x^4 + 777x^3 - 2415x^2 \\ &\quad + 3500x - 1750 \end{aligned}$$

50.
$$\begin{aligned} P(x) &= 4(x-3)x^3(x-(2-3i))(x-(2+3i)) \\ &= 4x^6 - 28x^5 + 100x^4 - 156x^3 \end{aligned}$$

51. The function has degree four, so there are four zeros. Since one zero is $3i$, another zero is $-3i$. So $(x-3i)(x+3i) = x^2 + 9$ is a factor of $P(x)$. Now divide to find the other factor:

$$\begin{array}{r} x^2 + x \\ x^2+9 \overline{\smash{\big)} x^4 + x^3 + 9x^2 + 9x} \\ \underline{x^4 + 9x^2} \\ x^3 + 9x \\ \underline{x^3 + 9x} \\ 0 \end{array}$$

$$\begin{aligned} P(x) &= x^4 + x^3 + 9x^2 + 9x \\ &= (x-3i)(x+3i)(x^2+x) \\ &= (x-3i)(x+3i)(x+1)x \Rightarrow \text{ the zeros of} \end{aligned}$$
$P(x)$ are $-1, 0, -3i, 3i$.

52. The function has degree four, so there are four zeros. Since one zero is $1-i,$ another zero is $1+i.$ So $(x-(1-i))(x-(1+i)) = x^2-2x+2$ is a factor of $P(x)$. Now divide to find the other factor:

$$\begin{array}{r|l}
 & x^2-1 \\ \hline
x^2-2x+2 & x^4-2x^3+x^2+2x-2 \\
 & \underline{x^4-2x^3+2x^2} \\
 & \quad -x^2+2x-2 \\
 & \quad \underline{-x^2+2x-2} \\
 & \quad\quad 0
\end{array}$$

$$\begin{aligned}
P(x) &= x^4-2x^3+x^2+2x-2 \\
&= (x-(1-i))(x-(1+i))(x^2-1) \\
&= (x-(1-i))(x-(1+i))(x-1)(x+1) \Rightarrow
\end{aligned}$$

the zeros of $P(x)$ are $-1, 1, 1-i, 1+i.$

53. The function has degree five, so there are five zeros. Since one zero is $3-i,$ another zero is $3+i.$ So $(x-(3-i))(x-(3+i)) = x^2-6x+10$ is a factor of $P(x)$. Now divide to find the other factor:

$$\begin{array}{r|l}
 & x^3+x^2-2x \\ \hline
x^2-6x+10 & x^5-5x^4+2x^3+22x^2-20x \\
 & \underline{x^5-6x^4+10x^3} \\
 & \quad x^4-8x^3+22x^2 \\
 & \quad \underline{x^4-6x^3+10x^2} \\
 & \quad\quad -2x^3+12x^2-20x \\
 & \quad\quad \underline{-2x^3+12x^2-20x} \\
 & \quad\quad\quad 0
\end{array}$$

$$\begin{aligned}
P(x) &= x^5-5x^4+2x^3+22x^2-20x \\
&= (x-(3-i))(x-(3+i))(x^3+x^2-2x) \\
&= (x-(3-i))(x-(3+i))x(x^2+x-2) \\
&= (x-(3-i))(x-(3+i))x(x+2)(x-1) \Rightarrow
\end{aligned}$$

the zeros of $P(x)$ are $-2, 0, 1, 3-i, 3+i.$

54. The function has degree five, so there are five zeros. Since one zero is i, another zero is $-i$. So $(x-i)(x+i) = x^2+1$ is a factor of $P(x)$. Now divide to find the other factor:

$$\begin{array}{r|l}
 & 2x^3-11x^2+17x-6 \\ \hline
x^2+1 & 2x^5-11x^4+19x^3-17x^2+17x-6 \\
 & \underline{2x^5 \qquad\quad +2x^3} \\
 & \quad -11x^4+17x^3-17x^2 \\
 & \quad \underline{-11x^4 \qquad\quad -11x^2} \\
 & \quad\quad 17x^3-6x^2+17x \\
 & \quad\quad \underline{17x^3 \qquad\quad +17x} \\
 & \quad\quad\quad -6x^2 \qquad -6 \\
 & \quad\quad\quad \underline{-6x^2 \qquad -6} \\
 & \quad\quad\quad\quad 0
\end{array}$$

$$\begin{aligned}
P(x) &= 2x^5-11x^4+19x^3-17x^2+17x-6 \\
&= (x-i)(x+i)(2x^3-11x^2+17x-6)
\end{aligned}$$

Now find the zeros of $2x^3-11x^2+17x-6.$ There are either three or one positive, rational zeros. The possible zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 6\right\}$. Using synthetic division, we find that $x = 2$ is one of the zeros:

$$\begin{array}{r|rrrr}
2 & 2 & -11 & 17 & -6 \\
 & & 4 & -14 & 6 \\ \hline
 & 2 & -7 & 3 & 0
\end{array}$$

$$\begin{aligned}
P(x) &= 2x^5-11x^4+19x^3-17x^2+17x-6 \\
&= (x-i)(x+i)(2x^3-11x^2+17x-6) \\
&= (x-i)(x+i)(x-2)(2x^2-7x+3) \\
&= (x-i)(x+i)(x-2)(2x-1)(x-3) \Rightarrow
\end{aligned}$$

the zeros of $P(x)$ are $\frac{1}{2}, 2, 3, -i, i.$

55. The function has degree 3, so there are three zeros. There are three sign changes, so there are 1 or 3 positive zeros. The possible rational zeros are $\{\pm 1, \pm 17\}$. Using synthetic division we find that one zero is 1:

$$\begin{array}{r|rrrr}
1 & 1 & -9 & 25 & -17 \\
 & & 1 & -8 & 17 \\ \hline
 & 1 & -8 & 17 & 0
\end{array}$$

$x^3-9x^2+25x-17 = (x-1)(x^2-8x+17).$ Now solve the depressed equation $x^2-8x+17 = 0 \Rightarrow$

$$x = \frac{8 \pm \sqrt{(-8)^2-4(1)(17)}}{2(1)} \Rightarrow x = \frac{8 \pm \sqrt{-4}}{2} \Rightarrow$$

$x = 4 \pm i.$ The zeros are $1, 4 \pm i.$

56. The function has degree 3, so there are three zeros. There are two sign changes, so there are either 2 or 0 positive zeros. There is one sign change in $f(-x)$, so there is one negative zero. The possible rational zeros are $\{\pm 1, \pm 13\}$. Using synthetic division, we find that one zero is −1:

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 7 & 13 \\ & & -1 & 6 & -13 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$x^3 - 5x^2 + 7x + 13 = (x+1)(x^2 - 6x + 13).$ Now solve the depressed equation

$x^2 - 6x + 13 = 0 \Rightarrow$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} \Rightarrow x = \frac{6 \pm \sqrt{-16}}{2} \Rightarrow$$

$x = 3 \pm 2i.$ The zeros are $-1, 3 \pm 2i.$

57. The function has degree 3, so there are three zeros. There are two sign changes, so there are either 2 or 0 positive zeros. There is one sign change in $f(-x)$, so there is one negative zero. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \pm\frac{5}{3}, \pm 2, \pm\frac{8}{3}, \pm\frac{10}{3}, \pm 4, \pm 5, \pm\frac{20}{3}, \pm 8, \pm 10, \pm\frac{40}{3}, \pm 20, \pm 40\right\}$. Using synthetic division to test the negative values we find that one zero is $-\frac{4}{3}$:

$$\begin{array}{r|rrrr} -\frac{4}{3} & 3 & -2 & 22 & 40 \\ & & -4 & 8 & -40 \\ \hline & 3 & -6 & 30 & 0 \end{array}$$

$$3x^3 - 2x^2 + 22x + 40 = \left(x + \frac{4}{3}\right)(3x^2 - 6x + 30) = 3\left(x + \frac{4}{3}\right)(x^2 - 2x + 10).$$

Now solve the depressed equation

$x^2 - 2x + 10 = 0 \Rightarrow$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)} \Rightarrow x = \frac{2 \pm \sqrt{-36}}{2} \Rightarrow$$

$x = 1 \pm 3i.$ The zeros are $-\frac{4}{3}, 1 \pm 3i.$

58. The function has degree 3, so there are three zeros. There are three sign changes, so there are either 3 or 1 positive zeros. There are no sign changes in $f(-x)$, so there are no negative zeros. The possible rational zeros are $\left\{\pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \pm 2, \pm 4\right\}$. Using synthetic division to test the positive values we find that one zero is $\frac{1}{3}$:

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & 12 & -4 \\ & & 1 & 0 & 4 \\ \hline & 3 & 0 & 12 & 0 \end{array}$$

$$3x^3 - x^2 + 12x - 4 = \left(x - \frac{1}{3}\right)(3x^2 + 12)$$
$$= 3\left(x - \frac{1}{3}\right)(x^2 + 4) = 3\left(x - \frac{1}{3}\right)(x - 2i)(x + 2i).$$

The zeros are $\frac{1}{3}, \pm 2i.$

59. The function has degree 4, so there are four zeros. There are two sign changes, so there are 4, 2, or 0 positive zeros. There are no sign changes in $f(-x)$, so there are no negative zeros. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 3, \pm\frac{9}{2}, \pm 9\right\}$. Using synthetic division to test the positive values we find that one zero is 1:

$$\begin{array}{r|rrrrr} 1 & 2 & -10 & 23 & -24 & 9 \\ & & 2 & -8 & 15 & -9 \\ \hline & 2 & -8 & 15 & -9 & 0 \end{array}$$

The zeros of the depressed function $2x^3 - 8x^2 + 15x - 9$ are also zeros of P. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 3, \pm\frac{9}{2}, \pm 9\right\}$.

Using synthetic division to test the positive values we find that one zero is 1:

$$\begin{array}{r|rrrr} 1 & 2 & -8 & 15 & -9 \\ & & 2 & -6 & 9 \\ \hline & 2 & -6 & 9 & 0 \end{array}$$

Thus, $2x^4 - 10x^3 + 23x^2 - 24x + 9 = (x-1)^2(2x^2 - 6x + 9).$ Now solve the depressed equation $2x^2 - 6x + 9 = 0.$

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$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(9)}}{2(2)} \Rightarrow x = \frac{6 \pm \sqrt{-36}}{4} \Rightarrow$$

$x = \frac{6 \pm 6i}{4} = \frac{3}{2} \pm \frac{3i}{2}$. The zeros of P are $1, \frac{3}{2} \pm \frac{3i}{2}$.

60. The function has degree 4, so there are four zeros. There are two sign changes, so there are either 2 or 0 positive zeros. There are two sign change in $f(-x)$, so there are either 2 or 0 negative zeros. The possible rational zeros are $\left\{\pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{1}{3}, \pm\frac{4}{9}, \pm\frac{2}{3}, \pm\frac{8}{9}, \pm 1, \pm\frac{4}{3}, \pm 2, \pm\frac{8}{3}, \pm 4, \pm 8\right\}$. Using synthetic division to test the negative values we find that one zero is –2:

$$\begin{array}{r|rrrrr} \underline{-2} & 9 & 30 & 14 & -16 & 8 \\ & & -18 & -24 & 20 & -8 \\ \hline & 9 & 12 & -10 & 4 & 0 \end{array}$$

The zeros of the depressed function $9x^3 + 12x^2 - 10x + 4$ are also zeros of P. The possible rational zeros are $\left\{\pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{1}{3}, \pm\frac{4}{9}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \pm 2, \pm 4\right\}$. Using synthetic division to test the negative values we find that one zero is –2:

$$\begin{array}{r|rrrr} \underline{-2} & 9 & 12 & -10 & 4 \\ & & -18 & 12 & -4 \\ \hline & 9 & -6 & 2 & 0 \end{array}$$

Now solve the depressed equation $9x^2 - 6x + 2 = 0 \Rightarrow$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(9)(2)}}{2(9)} \Rightarrow x = \frac{6 \pm \sqrt{-36}}{18} \Rightarrow$$

$x = \frac{6 \pm 6i}{18} = \frac{1}{3} \pm \frac{1}{3}i$. The zeros are $-2, \frac{1}{3} \pm \frac{1}{3}i$.

61. The function has degree 4, so there are four zeros. There are three sign changes, so there are either 3 or 1 positive zeros. There is one sign change in $f(-x)$, so there is one negative zero. The possible rational zeros are $\{\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30\}$. Using synthetic division to test the negative values, we find that one zero is –3:

$$\begin{array}{r|rrrrr} \underline{-3} & 1 & -4 & -5 & 38 & -30 \\ & & -3 & 21 & -48 & 30 \\ \hline & 1 & -7 & 16 & -10 & 0 \end{array}$$

The zeros of the depressed function $x^3 - 7x^2 + 16x - 10$ are also zeros of P. The possible rational zeros of the depressed function are $\{\pm 1, \pm 2, \pm 5, \pm 10\}$. Since we have already found the negative zero, we examine only the positive possibilities and find that 1 is a zero:

$$\begin{array}{r|rrrr} \underline{1} & 1 & -7 & 16 & -10 \\ & & 1 & -6 & 10 \\ \hline & 1 & -6 & 10 & 0 \end{array}$$

$x^4 - 4x^3 - 5x^2 + 38x - 30$
$= (x+3)(x-1)(x^2 - 6x + 10)$. Now solve the depressed equation:

$$x^2 - 6x + 10 = 0 \Rightarrow x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)} \Rightarrow$$

$x = \frac{6 \pm \sqrt{-4}}{2} \Rightarrow x = 3 \pm i.$

The zeros are $-3, 1, 3 \pm i$.

62. The function has degree 4, so there are four zeros. There is one sign change, so there is one positive zero. There are three sign changes in $f(-x)$, so there are either 3 or 1 negative zeros. The possible rational zeros are $\{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18\}$. Using synthetic division to test the positive values we find that one zero is 1:

$$\begin{array}{r|rrrrr} \underline{1} & 1 & 1 & 7 & 9 & -18 \\ & & 1 & 2 & 9 & 18 \\ \hline & 1 & 2 & 9 & 18 & 0 \end{array}$$

The zeros of the depressed function $x^3 + 2x^2 + 9x + 18$ are also zeros of P. We can factor by grouping to find the next zero:

$x^3 + 2x^2 + 9x + 18 = x^2(x+2) + 9(x+2)$
$= (x+2)(x^2 + 9)$. So –2 is a zero, as are $\pm 3i$.

The zeros are $-2, 1, \pm 3i$.

63. The function has degree 5, so there are five zeros. There are five sign changes, so there are either 5, 3, or 1 positive zeros. There are no sign changes in $f(-x)$, so there are no negative zeros. The possible rational zeros are $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, 3, 6\right\}$. Using synthetic division to test the positive values, we find that one zero is $\frac{1}{2}$:

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$$\begin{array}{r|rrrrrr} \frac{1}{2} & 2 & -11 & 19 & -17 & 17 & -6 \\ & & 1 & -5 & 7 & -5 & 6 \\ \hline & 2 & -10 & 14 & -10 & 12 & 0 \end{array}$$

The zeros of the depressed function $2x^4 - 10x^3 + 14x^2 - 10x + 12$ are also zeros of P. Use synthetic division again to find the next zero, 2:

$$\begin{array}{r|rrrrr} 2 & 2 & -10 & 14 & -10 & 12 \\ & & 4 & -12 & 4 & -12 \\ \hline & 2 & -6 & 2 & -6 & 0 \end{array}$$

$$2x^5 - 11x^4 + 19x^3 - 17x^2 + 17x - 6$$
$$= \left(x - \frac{1}{2}\right)(x-2)(2x^3 - 6x^2 + 2x - 6)$$
$$= 2\left(x - \frac{1}{2}\right)(x-2)(x^3 - 3x^2 + x - 3).$$ Use factoring by grouping to factor $x^3 - 3x^2 + x - 3$:

$$x^3 - 3x^2 + x - 3 = x^2(x-3) + 1(x-3)$$
$= (x^2+1)(x-3)$. So the remaining zeros are 3 and $\pm i$. The zeros are $\frac{1}{2}, 2, 3, \pm i$.

64. The function has degree 5, so there are five zeros. There are four sign changes, so there are either 4, 2, or 0 positive zeros. There is one sign change in $f(-x)$, so there is one negative zero. The possible rational zeros are $\{\pm 1, \pm 2, \pm 4\}$. Using synthetic division to test the negative values, we find that one zero is –2:

$$\begin{array}{r|rrrrrr} -2 & 1 & -2 & -1 & 8 & -10 & 4 \\ & & -2 & 8 & -14 & 12 & -4 \\ \hline & 1 & -4 & 7 & -6 & 2 & 0 \end{array}$$

The zeros of the depressed function $x^4 - 4x^3 + 7x^2 - 6x + 2$ are also zeros of P. Use synthetic division again to find the next zero, 1:

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 7 & -6 & 2 \\ & & 1 & -3 & 4 & -2 \\ \hline & 1 & -3 & 4 & -2 & 0 \end{array}$$

$$x^5 - 2x^4 - x^3 + 8x^2 - 10x + 4$$
$= (x+2)(x-1)(x^3 - 3x^2 + 4x - 2)$.

The zeros of the depressed function $x^3 - 3x^2 + 4x - 2$ are also zeros of P. Use synthetic division again to find the next zero, 1:

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$$x^5 - 2x^4 - x^3 + 8x^2 - 10x + 4$$
$$= (x+2)(x-1)(x^3 - 3x^2 + 4x - 2)$$
$$= (x-2)(x-1)(x-1)(x^2 - 2x + 2).$$

Now solve the depressed equation:

$x^2 - 2x + 2 = 0 \Rightarrow$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \dfrac{2 \pm \sqrt{-4}}{2} = 1 \pm i.$ The zeros are $-2, 1$ (multiplicity 2)$, 1 \pm i$.

65. Since one zero is $3i$, $-3i$ is another zero. There are two zeros, so the degree of the equation is at least 2. Thus, the equation is of the form $f(x) = a(x-3i)(x+3i) = a(x^2+9)$. The graph passes through (0, 3), so we have $3 = a(0^2+9) \Rightarrow a = \frac{1}{3}$. Thus, the equation of the function is $f(x) = \frac{1}{3}(x^2+9)$.

66. Since one zero is $-i$, another zero is i. From the graph, we see that 3 is another zero. There are three zeros, so the degree of the equation is at least 3. Thus, the equation is of the form $f(x) = a(x-3)(x-i)(x+i) = a(x-3)(x^2+1)$
The graph passes through (0, −3), so we have $-3 = a(0-3)(0^2+1) \Rightarrow a = 1$. Thus, the equation of the function is
$f(x) = (x-3)(x^2+1)$.

67. Since i and $2i$ are zeros, so are $-i$, and $-2i$. From the graph, we see that 2 is also a zero. There are five zeros, so the degree of the equation is at least 5. Thus, the equation is of the form

$$f(x) = a(x-i)(x+i)(x-2i)(x+2i)(x-2)$$
$$= a(x^2+1)(x^2+4)(x-2)$$

The graph passes through (0, 4), so we have
$4 = a(0^2+1)(0^2+4)(0-2) \Rightarrow a = -\frac{1}{2}$.

Thus, the equation is
$f(x) = -\frac{1}{2}(x^2+1)(x^2+4)(x-2)$ or
$f(x) = \frac{1}{2}(x^2+1)(x^2+4)(2-x)$.

68. Since $-2i$ is a zero, so is $2i$. From the graph, we see that -1, 0, and 1 are also zeros. There are five zeros, so the degree of the equation is at least 5. Thus, the equation is of the form

$$f(x) = a(x+2i)(x-2i)(x+1)(x-0)(x-1) = ax\left(x^2+4\right)\left(x^2-1\right)$$

The y-intercept is (0, 0), so we have $0 = a(0)\left(0^2+4\right)\left(0^2-1\right)$ which is true for all values of a. Thus, the equation is $f(x) = ax\left(x^2+4\right)\left(x^2-1\right),\ a<0.$

Beyond the Basics

69. There are three cube roots. We know that one root is 1. Using synthetic division, we find $0 = x^3 - 1 = (x-1)(x^2+x+1)$:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

Solve the depressed equation

$$x^2+x+1=0 \Rightarrow x = \frac{-1\pm\sqrt{1^2-4(1)(1)}}{2(1)} = \frac{-1\pm\sqrt{-3}}{2} = -\frac{1}{2}\pm\frac{i\sqrt{3}}{2}.$$

So, the cube roots of 1 are 1 and $-\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$.

70. The solutions of the equation and the zeros of the polynomial are the same. There are n roots.

71. The polynomial has degree 6, so there are six zeros. There are no sign changes in $P(x)$, nor are there sign changes in $P(-x)$, so according to Descartes's Rule of Signs, there are neither positive nor negative zeros. Therefore, all six zeros must be complex.

72. The polynomial has degree 5, so there are five zeros. There are no sign changes in $P(x)$, while there is one sign change in $P(-x)$. According to Descartes's Rule of Signs, there is one negative zero, and the remaining four zeros must be complex.

73. The polynomial has degree 7, so there are seven zeros. There are no sign changes in $P(x)$, while there is one sign change in $P(-x)$. According to Descartes's Rule of Signs, there is one negative zero, and the remaining six zeros must be complex.

74. The polynomial has degree 3, so there are three zeros. There is one sign change in $P(x)$, while there are no sign changes in $P(-x)$. According to Descartes's Rule of Signs, there is one positive zero, and the two remaining zeros must be complex.

75. Since $1+2i$ is a zero, so is $1-2i$. The equation is of the form

$$f(x) = a(x-2)(x-(1+2i))(x-(1-2i)) = a(x-2)\left(x^2-2x+5\right)$$

The y-intercept is 40, so we have

$$40 = a(0-2)\left(0^2-2(0)+5\right) \Rightarrow a=-4$$

Thus, the equation is $f(x) = -4(x-2)\left(x^2-2x+5\right)$. Because $a<0$, $y\to\infty$ as $x\to-\infty$ and $y\to-\infty$ as $x\to\infty$.

76. Since $2-3i$ is a zero, so is $2+3i$. The equation is of the form

$$f(x) = a(x-1)(x-(2-3i))(x-(2+3i)) = a(x-1)\left(x^2-4x+13\right)$$

The y-intercept is -26, so we have

$$-26 = a(0-1)\left(0^2-4(0)+13\right) \Rightarrow a=2$$

Thus, the equation is $f(x) = 2(x-1)\left(x^2-4x+13\right)$. Because $a>0$, $y\to\infty$ as $x\to\infty$ and $y\to-\infty$ as $x\to-\infty$.

77. Since $3+i$ is a zero, so is $3-i$. The equation is of the form

$$f(x) = a(x-1)(x+1)(x-(3+i))(x+(3+i)) = a\left(x^2-1\right)\left(x^2-6x+10\right)$$

The y-intercept is 20, so we have

$$20 = a\left(0^2-1\right)\left(0^2-6(0)+10\right) \Rightarrow a=-2$$

Thus, the equation is $f(x) = -2\left(x^2-1\right)\left(x^2-6x+10\right)$. Because $a<0$, $y\to-\infty$ as $x\to-\infty$ and $y\to-\infty$ as $x\to\infty$.

78. Since $1-2i$ and $3-2i$ are zeros, so are $1+2i$ and $3+2i$. The equation is of the form

$$f(x) = a(x-(1-2i))(x-(1+2i))\cdot(x-(3-2i))(x+(3-2i)) = a\left(x^2-2x+5\right)\left(x^2-6x+13\right)$$

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The y-intercept is 130, so we have $130 = a\left(0^2 - 2(0) + 5\right)\left(0^2 - 6(0) + 13\right) \Rightarrow a = 2$

Thus, the equation is $f(x) = 2\left(x^2 - 2x + 5\right)\left(x^2 - 6x + 13\right)$. Because $a > 0$, $y \to \infty$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$.

79. $P(x) = x^2 + (i-2)x - 2i,\ x = -i$

$$\begin{array}{r|rrr} -i & 1 & i-2 & -2i \\ & & -i & 2i \\ \hline & 1 & -2 & 0 \end{array}$$

$x^2 + (i-2)x - 2i = (x+i)(x-2)$

80. $P(x) = x^2 + 3ix - 2,\ x = -2i$

$$\begin{array}{r|rrr} -2i & 1 & 3i & -2 \\ & & -2i & 2 \\ \hline & 1 & i & 0 \end{array}$$

$x^2 + 3ix - 2 = (x+2i)(x+i)$

81. $P(x) = x^3 - (3+i)x^2 - (4-3i)x + 4i,\ x = i$

$$\begin{array}{r|rrrr} i & 1 & -(3+i) & -(4-3i) & 4i \\ & & i & -3i & -4i \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$x^3 - (3+i)x^2 - (4-3i)x + 4i = \left(x^2 - 3x - 4\right)(x-i) = (x-4)(x+1)(x-i)$

82. $P(x) = x^3 - (4+2i)x^2 + (7+8i)x - 14i,\ x = 2i$

$$\begin{array}{r|rrrr} 2i & 1 & -(4+2i) & 7+8i & -14i \\ & & 2i & -8i & 14i \\ \hline & 1 & -4 & 7 & 0 \end{array}$$

$x^3 - (4+2i)x^2 + (7+8i)x - 14i = \left(x^2 - 4x + 7\right)(x-2i)$

Now solve $x^2 - 4x + 7 = 0$ to find linear factors.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)} = \frac{4 \pm \sqrt{-12}}{2} = \frac{4 \pm 2i\sqrt{3}}{2} = 2 \pm i\sqrt{3}$$

Thus,

$$\begin{aligned} x^3 - (4+2i)x^2 + (7+8i)x - 14i &= \left(x^2 - 4x + 7\right)(x-2i) = \left(x - \left(2+\sqrt{3}\right)\right)\left(x - \left(2-\sqrt{3}\right)\right)(x-2i) \\ &= \left(x - 2 - i\sqrt{3}\right)\left(x - 2 + i\sqrt{3}\right)(x-2i) \end{aligned}$$

Critical Thinking/Discussion/Writing

83. Factoring the polynomial, we have $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 = a_n(x - r_1)(x - r_2)\cdots(x - r_n)$. Expanding the right side, we find that the coefficient of x^{n-1} is $-a_n r_1 - a_n r_2 - \ldots - a_n r_n$ and the constant term is $(-1)^n a_n r_1 r_2 \cdots r_n$. Comparing the coefficients with those on the left side, we obtain $a_{n-1} = -a_n(r_1 + r_2 + \ldots r_n)$. Because $a_n \neq 0$, $-\dfrac{a_{n-1}}{a_n} = r_1 + r_2 + \ldots r_n$ and $-(1)^n \dfrac{a_0}{a_n} = r_1 r_2 \cdots r_n$.

84. Because r_1 and r_2 are roots of the equation, we have

$r_1 = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $r_2 = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$. Then

$$r_1^2 + r_2^2 = (r_1+r_2)^2 - 2r_1r_2 = \left(\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}\right)^2 - 2\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\cdot\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \left(\frac{-2b}{2a}\right)^2 - 2\left(\frac{(-b)^2-(b^2-4ac)}{4a^2}\right) = \frac{(-b)^2}{a^2} - \left(\frac{4ac}{2a^2}\right) = \frac{6^2-2ac}{a^2}$$

85. a. $x^3+6x=20 \Rightarrow x^3+6x-20=0$. There is one sign change, so there is one positive root. There are no sign changes in $f(-x)$, so there are no negative roots. Therefore, there is only one real solution.

b. Substituting $v-u$ for x and remembering that $v^3-u^3=20$ and $uv=2$, we have

$$\begin{aligned}(v-u)^3+6(v-u) &= v^3-3v^2u+3vu^2-u^3+6(v-u) = (v^3-u^3)-(3v^2u-3vu^2)+6(v-u)\\ &= (v^3-u^3)-3vu(v-u)+6(v-u) = (v^3-u^3)-(6-3uv)(v-u)\\ &= 20-(6-3(2))(v-u) = 20\end{aligned}$$

Therefore, $x=v-u$ is the solution.

c. To solve the system $v^3-u^3=20, vu=2$, solve the second equation for v, and substitute that value into the first equation, keeping in mind that u cannot be zero, so division by u is permitted:

$$v^3-u^3=20 \Rightarrow \left(\frac{2}{u}\right)^3 - u^3 = \frac{8}{u^3}-u^3=20 \Rightarrow$$

$$-u^3+\frac{8}{u^3}-20=0.$$

Let $-u^3=a$, so

$$a-\frac{8}{a}-20=0 \Rightarrow a^2-20a-8=0 \Rightarrow$$

$$a=\frac{20\pm\sqrt{400+32}}{2}=10\pm6\sqrt{3}=-u^3 \Rightarrow$$

$$\sqrt[3]{-10\pm6\sqrt{3}}=u \Rightarrow$$

$$\begin{aligned}v &= \frac{2}{\sqrt[3]{-10\pm6\sqrt{3}}}\\ &= \frac{2}{\sqrt[3]{-10\pm6\sqrt{3}}}\cdot\frac{\sqrt[3]{10\pm6\sqrt{3}}}{\sqrt[3]{10\pm6\sqrt{3}}}\\ &= \sqrt[3]{10\pm6\sqrt{3}}\end{aligned}$$

d. If q is positive, then, according to Descartes's Rule of Signs, the polynomial x^3+px-q has one positive zero and no negative zeros. If q is negative, then it has no positive zeros and one negative zero. Either way, there is exactly one real solution. From (b), we have $v^3-u^3=q$, $vu=\frac{p}{3}$, and $x=v-u$.

Substituting, we have

$$\begin{aligned}x^3+px &= (v-u)^3+p(v-u)\\ &= v^3-3v^2u+3vu^2-u^3+p(v-u)\\ &= (v^3-u^3)-(3v^2u-3vu^2)+p(v-u)\\ &= (v^3-u^3)-3vu(v-u)+p(v-u)\\ &= (v^3-u^3)+(p-3uv)(v-u)\\ &= q+\left(p-3\cdot\frac{p}{3}\right)x = q \Rightarrow\end{aligned}$$

$x=v-u$ is the solution. Solving the system $v^3-u^3=q$, $vu=\frac{p}{3}$, we obtain

$$v^3-u^3=q \Rightarrow \left(\frac{p}{3u}\right)^3-u^3=\frac{p}{3^3u^3}-u^3=q$$

$$\Rightarrow -u^3+\frac{p^3}{3^3u^3}-q=0.$$

Let $-u^3=a$, so we have

$$a-\frac{p}{3^3a}-q=0 \Rightarrow a^2-qa-\frac{p^3}{3^3}=0 \Rightarrow$$

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$$a=\frac{q\pm\sqrt{q^2+4\left(\frac{p^3}{3^3}\right)}}{2}=\frac{q}{2}\pm\sqrt{\frac{q^2+4\left(\frac{p^3}{3^3}\right)}{4}}$$

$$=\frac{q}{2}\pm\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}=-u^3\Rightarrow$$

$$u=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}\text{ and}$$

$$v=\sqrt[3]{\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}\text{ or}$$

$$u=\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}\text{ and}$$

$$v=\sqrt[3]{\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}.$$

The difference $v-u$ is the same in both cases, so

$$x=v-u$$

$$=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}-\sqrt[3]{\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}.$$

e. Substituting $x=y-\frac{a}{3}$, we have

$$x^3+ax^2+bx+c=0\Rightarrow$$

$$\left(y-\frac{a}{3}\right)^3+a\left(y-\frac{a}{3}\right)^2+b\left(y-\frac{a}{3}\right)+c=0\Rightarrow$$

$$\left(y^3-ay^2+\frac{a^2}{3}y-\frac{a^3}{27}\right)+\left(ay^2-\frac{2a^2y}{3}+\frac{a^3}{9}\right)+by-\frac{ab}{3}+c=0\Rightarrow$$

$y^3+py=q$, where $p=b-\dfrac{a^2}{3}$ and

$$q=-\frac{2a^3}{27}+\frac{ab}{3}-c$$

$$y^3-\frac{a^2y}{3}+by+\frac{2a^3}{27}-\frac{ab}{3}+c=0\Rightarrow$$

$$y^3+\left(b-\frac{a^2}{3}\right)y=-\frac{2a^3}{27}+\frac{ab}{3}-c$$

86. $x^3+6x^2+10x+8=0\Rightarrow a=6, b=10, c=8$.

Substituting $x=y-\frac{6}{3}=y-2$ as in (1e), we have

$$(y-2)^3+6(y-2)^2+10(y-2)+8=0\Rightarrow$$

$$(y^3-6y^2+12y-8)+(6y^2-24y+24)+10y-20+8=0\Rightarrow$$

$y^3-2y+4=0\Rightarrow y^3-2y=-4.$ Then, using the results of (1d), we have

$$y=\sqrt[3]{\frac{-4}{2}+\sqrt{\left(\frac{-4}{2}\right)^2+\left(\frac{-2}{3}\right)^3}}-\sqrt[3]{-\frac{-4}{2}+\sqrt{\left(\frac{-4}{2}\right)^2+\left(\frac{-2}{3}\right)^3}}.$$

Using a calculator, we find that $y=-2$. So $x=-2-2=-4$. Now use synthetic division to find the depressed equation:

-4	1	6	10	8
		-4	-8	-8
	1	2	2	0

$$x^3+6x^2+10x+8=(x+4)(x^2+2x+2)=0.$$

$$x^2+2x+2=0\Rightarrow x=\frac{-2\pm\sqrt{2^2-4(1)(2)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{-4}}{2}=\frac{-2\pm 2i}{2}=-1\pm i$$

The solution set is $\{-4,-1\pm i\}$.

Getting Ready for the Next Section

87. a. $y-3=-\frac{2}{3}(x-(-5))\Rightarrow y-3=-\frac{2}{3}(x+5)\Rightarrow$

$$y=-\frac{2}{3}x-\frac{2}{3}(5)+3\Rightarrow y=-\frac{2}{3}x-\frac{1}{3}$$

b. $y=-\frac{2}{3}(1)-\frac{1}{3}=-1$

88. a. The slope of the line $4x+5y=6\Rightarrow$ $5y=-4x+6\Rightarrow y=-\frac{4}{5}x+\frac{6}{5}$ is $-\frac{4}{5}$. The slope of the line perpendicular to this line is $\frac{5}{4}$. The equation we are seeking is

$$y-(-3)=\frac{5}{4}(x-(-2))\Rightarrow$$

$$y+3=\frac{5}{4}(x+2)\Rightarrow$$

$$y=\frac{5}{4}x+\frac{5}{4}(2)-3\Rightarrow y=\frac{5}{4}x-\frac{1}{2}.$$

b. $y = \frac{5}{4}(10) - \frac{1}{2} = 12$

89. $7 = (-2)^2 k + 1 \Rightarrow 7 = 4k + 1 \Rightarrow 6 = 4k \Rightarrow$
$k = \frac{3}{2}$
$y = \frac{3}{2}(2)^2 + 1 = 7$

90. $12 = \frac{k}{\left(\frac{1}{2}\right)^2} \Rightarrow k = 3$
$y = \frac{3}{2^2} \Rightarrow y = \frac{3}{4}$

2.7 Variation

2.7 Practice Problems

1. $y = kx \Rightarrow 6 = 30k \Rightarrow \frac{1}{5} = k$
$y = \left(\frac{1}{5}\right)120 = 24$

2. $I = kV \Rightarrow 60 = 220k \Rightarrow \frac{3}{11} = k$
$75 = \left(\frac{3}{11}\right)V \Rightarrow V = 275$
A battery of 275 volts is needed to produce 60 amperes of current.

3. $y = kx^2 \Rightarrow 48 = k(2)^2 \Rightarrow 12 = k$
$y = 12(5)^2 = 300$

4. $A = \frac{k}{B} \Rightarrow 12 = \frac{k}{5} \Rightarrow 60 = k$
$A = \frac{60}{3} = 20$

5. $y = \frac{k}{\sqrt{x}} \Rightarrow \frac{3}{4} = \frac{k}{\sqrt{16}} \Rightarrow k = 3$
$2 = \frac{3}{\sqrt{x}} \Rightarrow \sqrt{x} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$

6. $F = G \cdot \frac{m_1 m_2}{r^2} \Rightarrow m \cdot g = G \cdot \frac{mM_{\text{Mars}}}{R_{\text{Mars}}^2} \Rightarrow$
$g = G \cdot \frac{M_{\text{Mars}}}{R_{\text{Mars}}^2}$
Note that the radius of Mars is given in kilometers, which must be converted to meters.

$$g = \frac{\left(6.67 \times 10^{-11} \text{ m}^3/\text{kg}/\text{sec}^2\right)\left(6.42 \times 10^{23} \text{ kg}\right)}{(3397 \text{ km})^2}$$
$$= \frac{\left(6.67 \times 10^{-11} \text{ m}^3/\text{kg}/\text{sec}^2\right)\left(6.42 \times 10^{23} \text{ kg}\right)}{\left(3.397 \times 10^6 \text{ m}\right)^2}$$
$$\approx 3.7 \text{ m/sec}^2$$

2.7 Exercises Concepts and Vocabulary

1. y varies directly as x if $\underline{y = kx}$.
2. y varies inversely as x if $\underline{y = \frac{k}{x}}$.
3. y varies directly as the nth power of x if $\underline{y = kx^n}$.
4. z varies jointly as x and y if $\underline{z = kxy}$.
5. True
6. False. A varies directly as the second power of r.
7. False.
8. True

Building Skills

9. $x = ky; 15 = 30k \Rightarrow \frac{1}{2} = k; x = \frac{1}{2}(28) = 14$

10. $y = kx; 3 = 2k \Rightarrow \frac{3}{2} = k; y = \frac{3}{2}(7) = \frac{21}{2}$

11. $s = kt^2; 64 = 2^2 k \Rightarrow k = 16; s = 5^2(16) = 400$

12. $y = kx^3; 270 = 3^3 k \Rightarrow k = 10;$
$80 = 10x^3 \Rightarrow x = 2$

13. $r = \frac{k}{u}; 3 = \frac{k}{11} \Rightarrow k = 33; r = \frac{33}{1/3} = 99$

14. $y = \frac{k}{z}; 24 = \frac{k}{1/6} \Rightarrow k = 4; y = \frac{4}{1} = 4$

15. $B = \frac{k}{A^3}; 1 = \frac{k}{2^3} \Rightarrow k = 8; B = \frac{8}{4^3} = \frac{1}{8}$

16. $y = \frac{k}{\sqrt[3]{x}}; 10 = \frac{k}{\sqrt[3]{2}} \Rightarrow k = 10\sqrt[3]{2};$
$40 = \frac{10\sqrt[3]{2}}{\sqrt[3]{x}} \Rightarrow 40^3 = \frac{2000}{x} \Rightarrow x = \frac{1}{32}$

17. $z = kxy; 42 = (2)(3)k \Rightarrow k = 7;\ 56 = 2(7)y \Rightarrow y = 4$

18. $m = \frac{kq}{p}; \frac{1}{2} = \frac{13k}{26} \Rightarrow k = 1; m = \frac{7(1)}{14} = \frac{1}{2}$

19. $z = kx^2; 32 = 4^2k \Rightarrow k = 2; z = 2(5^2) = 50$

20. $u = \frac{k}{t^3}; 9 = \frac{k}{2^3} \Rightarrow k = 72; u = \frac{72}{6^3} = \frac{1}{3}$

21. $P = kTQ^2; 36 = (17)(6^2)k \Rightarrow k = \frac{1}{17};$
$P = \frac{1}{17}(4)(9^2) = \frac{324}{17}$

22. $a = kb\sqrt{c}; 9 = 13\sqrt{81}k \Rightarrow k = \frac{1}{13};$
$a = \frac{1}{13}(5)\sqrt{9} = \frac{15}{13}$

23. $z = \frac{k\sqrt{x}}{y^2}; 24 = \frac{\sqrt{16}k}{3^2} \Rightarrow k = 54$
$27 = \frac{54\sqrt{x}}{2^2} \Rightarrow 2 = \sqrt{x} \Rightarrow 4 = x$

24. $z = \frac{kuv^3}{w^2}; 9 = \frac{4(3^3)k}{2^2} \Rightarrow \frac{1}{3} = k;$
$8 = \frac{\frac{1}{3}(27)(2^3)}{w^2} \Rightarrow 8 = \frac{72}{w^2} \Rightarrow w = \pm 3$

25. $\frac{16}{12} = \frac{8}{y} \Rightarrow y = 6$

26. $\frac{17}{22} = \frac{z}{110} \Rightarrow z = 85$

27. $\frac{100}{x_0} = \frac{y}{2x_0} \Rightarrow y = 200$

28. $\frac{2}{\sqrt{9}} = \frac{3}{\sqrt{y}} \Rightarrow \sqrt{y} = \frac{9}{2} \Rightarrow y = \frac{81}{4}$

Applying the Concepts

29. $y = Hx$, where y is the speed of the galaxies and x is the distance between them.

30. $R = kP$, where k is a constant.

31. a. $y = 30.5x$, where y is the length in centimeters and x is the length in feet.

b. **(i)** $y = 30.5(8) = 244$ cm

(ii) $y = 30.5\left(5\frac{1}{3}\right) \approx 162.67$ cm

c. **(i)** $57 = 30.5x \Rightarrow x \approx 1.87$ ft

(ii) $124 = 30.5x \Rightarrow x \approx 4.07$ ft

32. a. $y = 2.2x$, where y is the weight in pounds and x is the weight in kilograms.

b. **(i)** $y = 2.2(0.125) = 0.275$ lb

(ii) $y = 2.2(4) = 8.8$ lb

(iii) $y = 2.2(2.4) = 5.28$ lb

c. **(i)** $27 = 2.2x \Rightarrow x \approx 12.27$ kg

(ii) $160 = 2.2x \Rightarrow x \approx 72.73$ kg

33. $P = kQ; 7 = 20k \Rightarrow \frac{7}{20} = k; P = \frac{7}{20}(100) = 35$ g

34. $W = kh; 600 = 40k \Rightarrow k = 15;$
$W = 15(25) = \$375.$
The constant of proportionality is the hourly wage.

35. $d = kt^2; 64 = 2^2k \Rightarrow k = 16; 9 = 16t^2 \Rightarrow$
$t = 0.75$ sec

36. $F = kx; 10 = 4k \Rightarrow \frac{5}{2} = k; F = \frac{5}{2}(6) = 15$ lb

37. $P = \frac{k}{V}; 20 = \frac{k}{300} \Rightarrow k = 6000;$
$P = \frac{6000}{100} = 60 \text{ lb/in.}^2$

38. $P = \frac{kT_K}{V}; 36 = \frac{260k}{13} \Rightarrow k = \frac{9}{5}; P = \frac{9T_K}{5V}$

a. $40 = \frac{9(300)}{5V} \Rightarrow V = 13.5 \text{ in.}^3$

b. $P = \frac{9(280)}{5(39)} \approx 12.92 \text{ lb/in.}^2$

39. a. The astronaut is 6000 + 3960 miles from the Earth's center.
$W = \frac{k}{d^2}; 120 = \frac{k}{3960^2} \Rightarrow k = 120(3960^2)$
$W = \frac{120(3960^2)}{(6000 + 3960)^2} \approx 18.97$ lb

b. $200 = \frac{k}{3960^2} \Rightarrow k = 200(3960^2)$
$W = \frac{200(3960^2)}{3950^2} \approx 201.01$ lb

40. You should buy at the higher altitude because an object weighs less the further it is from the Earth's center.

41. 1740 km = 1,740,000 m = 1.740×10^6 m

$$g = G\bullet\frac{M}{R^2} = 6.67\times10^{-11}\times\frac{7.4\times10^{22}}{\left(1.740\times10^6\right)^2}$$
$$= \frac{(6.67)(7.4)(10^{11})}{1.740^2\times10^{12}} \approx 1.63 \text{ m/sec}^2$$

42. 696,000 km = 696,000,000 m = 6.96×10^8 m

$$g = G\bullet\frac{M}{R^2} = 6.67\times10^{-11}\times\frac{2\times10^{30}}{\left(6.96\times10^8\right)^2}$$
$$= \frac{(6.67)(2)(10^{19})}{6.96^2\times10^{16}} \approx 2.75\times10^2 \text{ m/sec}^2$$

43. a. $I = \frac{k}{d^2}; 320 = \frac{k}{10^2} \Rightarrow k = 32,000$
$I = \frac{32,000}{5^2} = 1280$ candla

b. $400 = \frac{32,000}{d^2} \Rightarrow d^2 = 80 \Rightarrow d \approx 8.94$ ft from the source.

44. a. $s = k\sqrt{d}; 48 = k\sqrt{96} \Rightarrow k = 2\sqrt{6}$
$s = 2\sqrt{6d}$

b. **(i)** $s = 2\sqrt{6(60)} \approx 37.95$ mph

(ii) $s = 2\sqrt{6(150)} = 60$ mph

(iii) $s = 2\sqrt{6(200)} \approx 69.28$ mph

c. $70 = 2\sqrt{6d} \Rightarrow \sqrt{6d} = 35 \Rightarrow$
$6d = 35^2 = 1225 \Rightarrow d \approx 204.17$ ft

45. $p = k\sqrt{l} \Rightarrow k = \frac{p}{\sqrt{l}} = \frac{2p}{2\sqrt{l}} = \frac{2p}{\sqrt{4l}} \Rightarrow$ the length is quadrupled if the period is doubled.

46. $V = kA; 400 = 100k \Rightarrow 4 = k$
$V = 4(120) = 480 \text{ cm}^3$

47. a. $H = kR^2N$

b. $k(2R)^2N = 4kR^2N = 4H \Rightarrow$ the horsepower is multiplied by 4 if the radius is doubled.

c. $kR^2(2N) = 2kR^2N = 2H \Rightarrow$ the horsepower is doubled if the number of pistons is doubled.

d. $k\left(\frac{R}{2}\right)^2(2N) = \frac{2kR^2N}{4} = \frac{kR^2N}{2} = \frac{H}{2} \Rightarrow$ the horsepower is halved if the radius is halved and the number of pistons is doubled.

48. Convert the dimensions given in inches to feet.

$$s = \frac{kwd^2}{l}; 576 = \frac{k\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^2}{25} \Rightarrow k = 172,800$$
$$s = \frac{172,800\left(\frac{1}{2}\right)\left(\frac{5}{6}\right)^2}{20} = 3000 \text{ lb}$$

Beyond the Basics

49. a. $E = kl^2v^3$

b. $1920 = k(10^2)(8^3) \Rightarrow k = \frac{3}{80} = 0.0375$

c. $E = 0.0375(8^2)(25^3) = 37,500$ watts

d. $kl^2(2v)^3 = 8kl^2v^3 = 8E$

e. $k(2l)^2v^3 = 4kl^2v^3 = 4E$

f. $k(2l)^2(2v)^3 = 4(8)kl^2v^3 = 32E$

50. a. $I_d = \frac{kI}{d^2}; 2 = \frac{100k}{2^2} \Rightarrow k = \frac{2}{25} = 0.08$

b. The distance from the bulb to the point on the floor is $\sqrt{2^2+3^2} = \sqrt{13}$ m.
$I_d = \frac{0.08(200)}{\left(\sqrt{13}\right)^2} \approx 1.23 \text{ watts/m}^2$

c. If the bulb is raised by 1 meter, then the distance from the bulb to the point on the floor is $\sqrt{3^2+3^2} = 3\sqrt{2}$ m.
$I_d = \frac{0.08(200)}{\left(3\sqrt{2}\right)^2} \approx 0.89 \text{ watts/m}^2$

51. a. $m = kw^{3/4}; 75 = k(75^{3/4}) \Rightarrow$
$k = \sqrt[4]{3}\sqrt{5} \approx 2.94$

b. $m = 2.94(450^{3/4}) \approx 287.25$ watts

c. $k(4w)^{3/4} = 4^{3/4}kw^{3/4} \approx 2.83kw^{3/4} \approx 2.83\,m$

d. $250 \approx 2.94w^{3/4} \Rightarrow w^{3/4} \approx 85.034 \Rightarrow$
$w \approx 373.93$ kg

52. $r_{\text{sun-Earth}} = 400r_{\text{moon-Earth}}$

$$F_{\text{s-E}} = 6.67\times10^{-11}\times\frac{m_{\text{sun}}m_{\text{Earth}}}{r^2_{\text{s-E}}}$$
$$= 6.67\times10^{-11}\times\frac{2\times10^{30}\times6\times10^{24}}{r^2_{\text{s-E}}}$$
$$= \frac{8.004\times10^{44}}{r^2_{\text{s-E}}} = \frac{8.004\times10^{44}}{(400r_{\text{m-E}})^2}$$
$$= \frac{8.004\times10^{44}}{1.6\times10^5r^2_{\text{m-E}}} = \frac{8.004\times10^{39}}{1.6r^2_{\text{m-E}}}$$
$$F_{\text{m-E}} = 6.67\times10^{-11}\times\frac{m_{\text{moon}}m_{\text{Earth}}}{r^2_{\text{m-E}}}$$
$$= 6.67\times10^{-11}\times\frac{7.4\times10^{22}\times6\times10^{24}}{r^2_{\text{m-E}}}$$
$$= \frac{2.9615\times10^{37}}{r^2_{\text{m-E}}}$$
$$\frac{8.004\times10^{39}}{1.6r^2_{\text{m-E}}}\div\frac{2.9615\times10^{37}}{r^2_{\text{m-E}}}$$
$$= \frac{8.004\times10^{39}}{1.6r^2_{\text{m-E}}}\times\frac{r^2_{\text{m-E}}}{2.9615\times10^{37}} \approx 168.92$$

The gravitational attraction between the sun and the Earth is approximately 168.92 times as strong as the gravitational pull between the Earth and the moon.

53. a. $T^2 = \dfrac{4\pi^2r^3}{G(M_1+M_2)}$

b. Because gravity is in terms of cubic meters per kilogram per second squared, convert the distance from kilometers to meters:
$1.5\times10^8\text{ km} = 1.5\times10^{11}\text{ m}$.

$$(3.15\times10^7)^2 \approx \frac{4\pi^2(1.5\times10^{11})^3}{6.67\times10^{-11}M_{\text{sun}}} \Rightarrow$$
$$9.9225\times10^{14} \approx \frac{4\pi^2(3.375)\times10^{33}}{6.67\times10^{-11}M_{\text{sun}}} \Rightarrow$$
$$M_{\text{sun}} \approx \frac{133.24\times10^{24}}{6.67\times10^{-11}\times9.9225\times10^{14}}$$
$$\approx 2.01\times10^{30}\text{ kg}$$

54. Because gravity is in terms of cubic meters per kilogram per second squared, convert the distance from kilometers to meters:
$384,000\text{ km} = 3.84\times10^8\text{ m}$.
27.3 days $= 27.3(24)(60)(60)$ sec
$= 2,358,720$ sec

$$2,358,720^2 = \frac{4\pi^2(3.84\times10^8)^3}{6.67\times10^{-11}M_{\text{Earth}}} \Rightarrow$$
$$M_{\text{Earth}} = \frac{4\pi^2(3.84\times10^8)^3}{6.67\times10^{-11}\times2,358,720^2} \Rightarrow$$
$$M_{\text{Earth}} \approx 6.02\times10^{24}\text{ kg}$$

55. a. $R = kN(P-N)$, where k is the constant of proportionality.

b. $45 = 1000(9000)k \Rightarrow k = 5\times10^{-6}$

c. $R = 5\times10^{-6}(5000)(5000) = 125$ people per day.

d. $100 = 5\times10^{-6}N(10,000-N) \Rightarrow$
$20,000,000 = 10,000N - N^2 \Rightarrow$
$N^2 - 10,000N + 20,000,000 = 0 \Rightarrow$
$$N = \frac{10,000\pm\sqrt{10,000^2-4(1)(2\times10^7)}}{2(1)}$$
$$= \frac{10,000\pm\sqrt{20,000,000}}{2} \approx 2764 \text{ or } 7236$$

Critical Thinking/Discussion/Writing

56. $I = \dfrac{kV}{R} \Rightarrow IR = kV$
$1.3I = \dfrac{kV}{1.2R} \Rightarrow 1.56IR = kV \Rightarrow$ the voltage must increase by 56%.

57. a. $v = kw^2$. The diamond is cut into two pieces whose weights are $\dfrac{2w}{5}$ and $\dfrac{3w}{5}$. The value of the first piece is $\left(\dfrac{4}{25}\right)(1000) = \$160$, and the value of the second piece is $\left(\dfrac{9}{25}\right)(1000) = \$360$. The two pieces together are valued at \$520, a loss of \$480.

b. The stone is broken into three pieces whose weights are $\frac{5w}{25}=\frac{w}{5}, \frac{9w}{25},$ and $\frac{11w}{25}$. So, the values of the three pieces are

$\left(\frac{1}{5}\right)^2(25,000)=\$1000,$

$\left(\frac{9}{25}\right)^2(25,000)=\$3240,$ and

$\left(\frac{11}{25}\right)^2(25,000)=\$4840,$ respectively. The total value is \$9,080, a loss of \$15,920.

c. The weights of the pieces are $\frac{w}{15}, \frac{2w}{15}, \frac{3w}{15}=\frac{w}{5}, \frac{4w}{15},$ and $\frac{5w}{15}=\frac{w}{3},$ respectively. If the original value is x, then

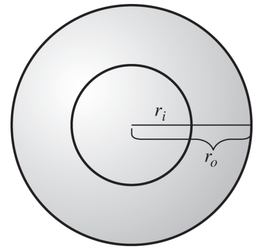
$$x-85,000=\left(\frac{1}{15}\right)^2 x+\left(\frac{2}{15}\right)^2 x+\left(\frac{1}{5}\right)^2 x +\left(\frac{4}{15}\right)^2 x+\left(\frac{1}{3}\right)^2 x \Rightarrow$$

$x-85,000=\frac{11}{45}x \Rightarrow x=\$112,500=$ the original value of the diamond. A diamond whose weight is twice that of the original diamond is worth 4 times the value of the original diamond = \$450,000.

58. $p=kd(n-f)$
$80=k(40)(30-f)$ and
$180=k(60)(35-f)$. Solving the first equation for k we have $k=-\frac{2}{f-30}$.
Substitute that value into the second equation and solve for f:

$180=\left(\frac{2}{30-f}\right)(60)(35-f) \Rightarrow$
$180=\frac{4200-120f}{30-f} \Rightarrow$
$5400-180f=4200-120f \Rightarrow$
$1200=60f \Rightarrow f=20$

59. $w_{\text{solid}}=kr_o{}^3; w_{\text{hollow}}=kr_o{}^3-kr_i{}^3=\frac{7}{8}kr_o{}^3$
$kr_o{}^3-kr_i{}^3=\frac{7}{8}kr_o{}^3 \Rightarrow r_o{}^3-r_i{}^3=\frac{7}{8}r_o{}^3 \Rightarrow$
$\frac{1}{8}r_o{}^3=r_i{}^3 \Rightarrow \frac{1}{8}=\frac{r_i{}^3}{r_o{}^3} \Rightarrow \frac{1}{2}=\frac{r_i}{r_o}$



60. $s=24-k\sqrt{w}; 20=24-k\sqrt{4} \Rightarrow k=2$
$0 \le 24-2\sqrt{w} \Rightarrow w \le 144$. The greatest number of wagons the engine can move is 144.

Getting Ready for the Next Section

61. $2^3=8$

62. $3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

63. $\left(\frac{1}{2}\right)^3=\frac{1^3}{2^3}=\frac{1}{8}$

64. $\left(\frac{1}{2}\right)^{-4}=2^4=16$

65. $2^{x-1}\cdot 2^{3-x}=2^{(x-1)+(3-x)}=2^2=4$

66. $5^{2x-3}\cdot 5^{3-x}=5^{(2x-3)+(3-x)}=5^x$

67. $\frac{2^{3x-2}}{2^{x-5}}=2^{(3x-2)-(x-5)}=2^{2x+3}$

68. $\left(2^{x-1}\right)^x=2^{(x-1)(x)}=2^{x^2-x}$

Chapter 2 Review Exercises

Building Skills

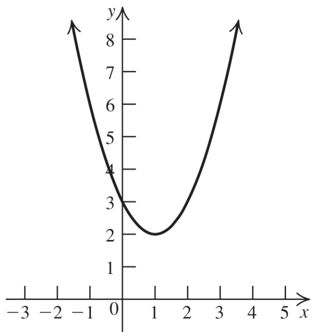
1.(i) opens up **(ii)** vertex: (1, 2)

(iii) axis: $x=1$

(iv) $0=(x-1)^2+2 \Rightarrow -2=(x-1)^2 \Rightarrow$ there are no x-intercepts.

(v) $y=(0-1)^2+2 \Rightarrow y=3$ is the y-intercept.

(vi) The function is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$.



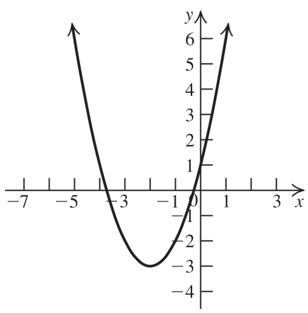
2.(i) opens up **(ii)** vertex: (–2, –3)

(iii) axis: $x = -2$

(iv) $0 = (x+2)^2 - 3 \Rightarrow 3 = (x+2)^2 \Rightarrow$
$x = -2 \pm \sqrt{3}$ are the x-intercepts.

(v) $y = (0+2)^2 - 3 \Rightarrow y = 1$ is the y-intercept.

(vi) The function is decreasing on $(-\infty, -2)$ and increasing on $(-2, \infty)$.



3.(i) opens down

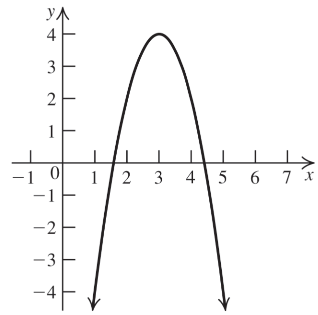
(ii) vertex: (3, 4)

(iii) axis: $x = 3$

(iv) $0 = -2(x-3)^2 + 4 \Rightarrow 2 = (x-3)^2 \Rightarrow$
$x = 3 \pm \sqrt{2}$ are the x-intercepts.

(v) $y = -2(0-3)^2 + 4 \Rightarrow y = -14$ is the y-intercept.

(vi) The function is increasing on $(-\infty, 3)$ and decreasing on $(3, \infty)$.



4.(i) opens down

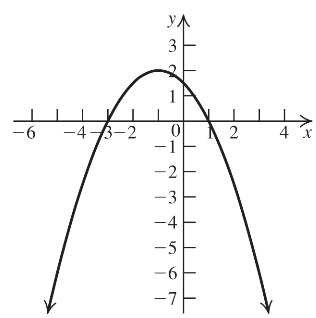
(ii) vertex: (–1, 2)

(iii) axis: $x = -1$

(iv) $0 = -\frac{1}{2}(x+1)^2 + 2 \Rightarrow 4 = (x+1)^2 \Rightarrow$
$x = -3$ and $x = 1$ are the x-intercepts.

(v) $y = -\frac{1}{2}(0+1)^2 + 2 \Rightarrow y = \frac{3}{2}$ is the y-intercept.

(vi) The function is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$.



5.(i) opens down

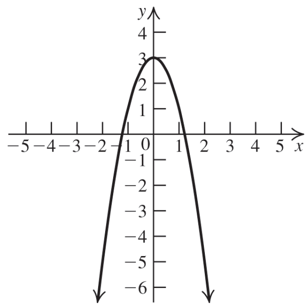
(ii) vertex: (0, 3)

(iii) axis: $x = 0$ (y-axis)

(iv) $0 = -2x^2 + 3 \Rightarrow \frac{3}{2} = x^2 \Rightarrow$
$x = \pm\frac{\sqrt{6}}{2}$ are the x-intercepts.

(v) $y = -2(0)^2 + 3 \Rightarrow y = 3$ is the y-intercept.

(vi) The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.



6.(i) opens up

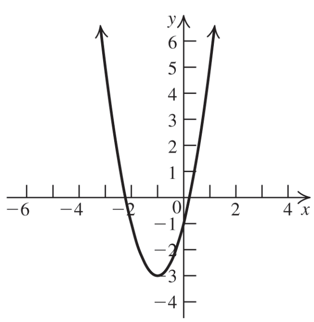
(ii) To find the vertex, write the equation in standard form by completing the square:
$y = 2x^2 + 4x - 1 \Rightarrow$
$y + 1 + 2 = 2(x^2 + 2x + 1) \Rightarrow$
$y = 2(x+1)^2 - 3 \Rightarrow$ the vertex is $(-1, -3)$.

(iii) axis: $x = -1$

(iv) $0 = 2x^2 + 4x - 1 \Rightarrow x = \dfrac{-4 \pm \sqrt{4^2 - 4(2)(-1)}}{2(2)} = \dfrac{-4 \pm \sqrt{24}}{4} \Rightarrow$
$x = -1 \pm \dfrac{\sqrt{6}}{2}$ are the x-intercepts.

(v) $y = 2(0)^2 + 4(0) - 1 \Rightarrow y = -1$ is the y-intercept.

(vi) The function is decreasing on $(-\infty, -1)$ and increasing on $(-1, \infty)$.



7.(i) opens up

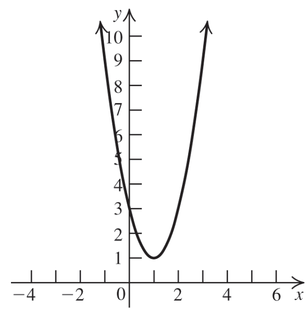
(ii) To find the vertex, write the equation in standard form by completing the square:
$y = 2x^2 - 4x + 3 \Rightarrow$
$y - 3 + 2 = 2(x^2 - 2x + 1) \Rightarrow$
$y = 2(x-1)^2 + 1 \Rightarrow$ the vertex is $(1, 1)$.

(iii) axis: $x = 1$

(iv) $0 = 2x^2 - 4x + 3 \Rightarrow$
$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(2)(3)}}{2(2)} = \dfrac{4 \pm \sqrt{-8}}{4} \Rightarrow$
there are no x-intercepts.

(v) $y = 2(0)^2 - 4(0) + 3 \Rightarrow y = 3$ is the y-intercept.

(vi) The function is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$.



8.(i) opens down

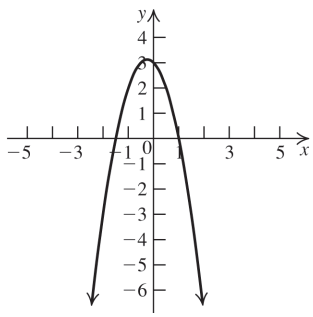
(ii) To find the vertex, write the equation in standard form by completing the square:
$y = -2x^2 - x + 3 \Rightarrow$
$y - 3 - \dfrac{1}{8} = -2\left(x^2 + \dfrac{1}{2}x + \dfrac{1}{16}\right) \Rightarrow$
$y = -2\left(x + \dfrac{1}{4}\right)^2 + \dfrac{25}{8} \Rightarrow$
the vertex is $\left(-\dfrac{1}{4}, \dfrac{25}{8}\right)$.

(iii) axis: $x = -\dfrac{1}{4}$

(iv) $0 = -2x^2 - x + 3 \Rightarrow$
$0 = -(2x+3)(x-1) \Rightarrow x = -\dfrac{3}{2}$ and $x = 1$ are the x-intercepts.

(v) $y = -2(0)^2 - 0 + 3 \Rightarrow y = 3$ is the y-intercept.

(vi) The function is increasing on $\left(-\infty, -\dfrac{1}{4}\right)$ and decreasing on $\left(-\dfrac{1}{4}, \infty\right)$.



9.(i) opens up

(ii) To find the vertex, write the equation in standard form by completing the square.

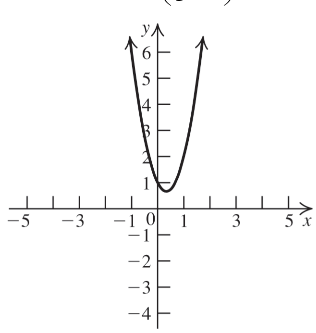
$$y = 3x^2 - 2x + 1 \Rightarrow$$
$$y - 1 + \frac{1}{3} = 3\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) \Rightarrow$$
$$y = 3\left(x - \frac{1}{3}\right)^2 + \frac{2}{3} \Rightarrow \text{ the vertex is } \left(\frac{1}{3}, \frac{2}{3}\right).$$

(iii) axis: $x = \dfrac{1}{3}$

(iv) $0 = 3x^2 - 2x + 1 \Rightarrow$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(1)}}{2(3)} = \frac{2 \pm \sqrt{-8}}{6} \Rightarrow$$
there are no x-intercepts.

(v) $y = 3(0)^2 - 3(0) + 1 \Rightarrow y = 1$ is the y-intercept.

(vi) The function is decreasing on $\left(-\infty, \dfrac{1}{3}\right)$ and increasing on $\left(\dfrac{1}{3}, \infty\right)$.



10.(i) opens up

(ii) To find the vertex, write the equation in standard form by completing the square:

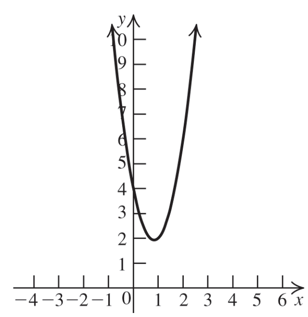
$$y = 3x^2 - 5x + 4 \Rightarrow$$
$$y - 4 + \frac{25}{12} = 3\left(x^2 - \frac{5}{3}x + \frac{25}{36}\right) \Rightarrow$$
$$y = 3\left(x - \frac{5}{6}\right)^2 + \frac{23}{12} \Rightarrow$$
the vertex is $\left(\dfrac{5}{6}, \dfrac{23}{12}\right)$.

(iii) axis: $x = \dfrac{5}{6}$

(iv) $0 = 3x^2 - 5x + 4 \Rightarrow$
$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(4)}}{2(3)} = \frac{5 \pm \sqrt{-23}}{6} \Rightarrow$$
there are no x-intercepts.

(v) $y = 3(0)^2 - 5(0) + 4 \Rightarrow y = 4$ is the y-intercept.

(vi) The function is decreasing on $\left(-\infty, \dfrac{5}{6}\right)$ and increasing on $\left(\dfrac{5}{6}, \infty\right)$.



In exercises 11–14, find the vertex using the formula $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

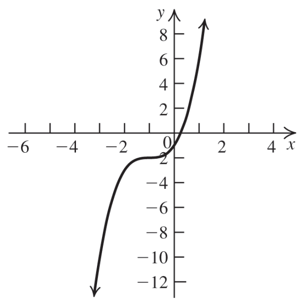
11. $a > 0 \Rightarrow$ the graph opens up, so f has a minimum value at its vertex. The vertex is
$$\left(-\frac{-4}{2(1)}, f\left(-\frac{-4}{2(1)}\right)\right) = (2, -1).$$

12. $a < 0 \Rightarrow$ the graph opens down, so f has a maximum value at its vertex. The vertex is
$$\left(-\frac{8}{2(-4)}, f\left(-\frac{8}{2(-4)}\right)\right) = (1, 1).$$

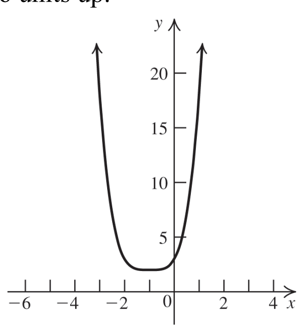
13. $a < 0 \Rightarrow$ the graph opens down, so *f* has a maximum value at its vertex. The vertex is $\left(-\dfrac{-3}{2(-2)}, f\left(-\dfrac{-3}{2(-2)}\right)\right) = \left(-\dfrac{3}{4}, \dfrac{25}{8}\right).$

14. $a > 0 \Rightarrow$ the graph opens up, so *f* has a minimum value at its vertex. The vertex is $\left(-\dfrac{-3/4}{2(1/2)}, f\left(-\dfrac{-3/4}{2(1/2)}\right)\right) = \left(\dfrac{3}{4}, \dfrac{55}{32}\right).$

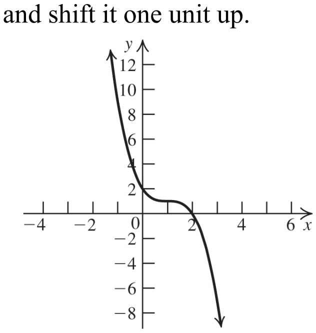
15. Shift the graph of $y = x^3$ one unit left and two units down.



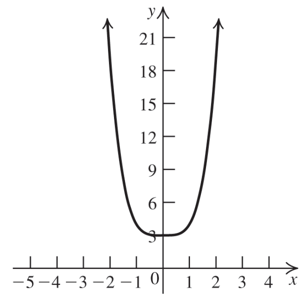
16. Shift the graph of $y = x^4$ one unit left and two units up.



17. Shift the graph of $y = x^3$ one unit right, reflect the resulting graph about the *y*-axis, and shift it one unit up.



18. Shift the graph of $y = x^4$ three units up.



19.(i) $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to \infty$ as $x \to \infty$

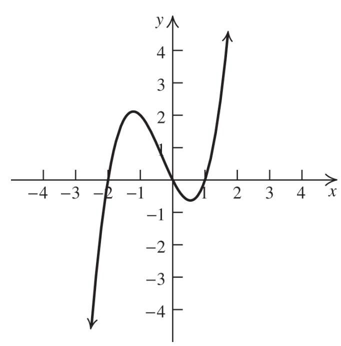
(ii) Zeros: $x = -2$, multiplicity 1, crosses the x-axis; $x = 0$, multiplicity 1, crosses the x-axis; $x = 1$, multiplicity 1, crosses the x-axis.

(iii) x-intercepts: –2, 0, 1;
$y = 0(0-1)(0+2) \Rightarrow y = 0$ is the y-intercept.

(iv) The intervals to be tested are $(-2, 0)$, $(0,1), (1,2),$ and $(2, \infty)$. The graph is above the x-axis on $(-2,0) \cup (1,\infty)$ and below the x-axis on $(-\infty,-2) \cup (0,1)$.

(v) $f(-x) = -x(-x-1)(-x+2) \neq f(x) \Rightarrow f$ is not even.
$-f(x) = -(x(x-1)(x+2)) \neq f(-x) \Rightarrow f$ is not odd. There are no symmetries.

(vi)



20.(i) $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to \infty$ as $x \to \infty$

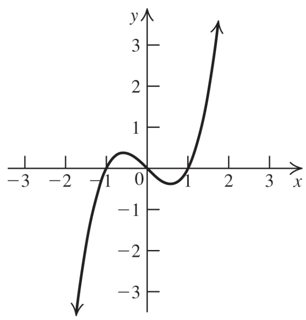
(ii) $f(x) = x^3 - x = x(x^2 - 1)$
$= x(x-1)(x+1).$
Zeros: $x = -1$, multiplicity 1, crosses the x-axis; $x = 0$, multiplicity 1, crosses the x-axis; $x = 1$, multiplicity 1, crosses the x-axis.

(iii) x-intercepts: –1, 0, 1;
$y = 0^3 - 0 \Rightarrow y = 0$ is the y-intercept.

(iv) The intervals to be tested are $(-\infty, -1)$, $(-1, 0), (0, 1)$, and $(1, \infty)$. The graph is above the x-axis on $(-1, 0) \cup (1, \infty)$ and below the x-axis on $(-\infty, -1) \cup (0, 1)$.

(v) $f(-x) = (-x)^3 - (-x) = -x^3 + x$
$= -f(x) \Rightarrow f$ is odd. f is symmetric with respect to the origin.

(vi)



21.(i) $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to \infty$

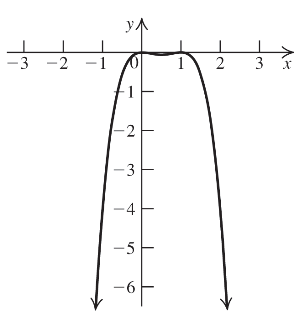
(ii) Zeros: $x = 0$, multiplicity 2, touches but does not cross the x- axis; $x = 1$, multiplicity 2, touches but does not cross the x- axis.

(iii) x-intercepts: 0, 1;
$y = -0^2(0-1)^2 \Rightarrow y = 0$ is the y-intercept.

(iv) The intervals to be tested are $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$. The graph is below the x-axis on $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

(v) $f(-x) = -(-x)^2(-x-1)$
$= -x^2(-x-1) \neq -f(x)$ or $f(x) \Rightarrow$
$= -f(x) \Rightarrow f$ is neither even nor odd.
There are no symmetries.

(vi)



22.(i) $f(x) \to \infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to \infty$

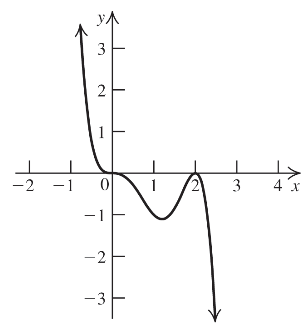
(ii) Zeros: $x = 0$, multiplicity 3, crosses the x-axis; $x = 2$, multiplicity 2, touches but does not cross the x-axis.

(iii) x-intercepts: 0, 2;
$y = -0^3(0-2)^2 \Rightarrow y = 0$ is the y-intercept.

(iv) The intervals to be tested are $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$. The graph is above the x-axis on $(-\infty, 0)$ and below the x-axis on $(0, 2) \cup (2, \infty)$.

(v) $f(-x) = -(-x)^3(-x-2)$
$= x^3(-x-2) \neq -f(x)$ or $f(x) \Rightarrow$
f is neither even nor odd. There are no symmetries.

(vi)



23.(i) $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to \infty$

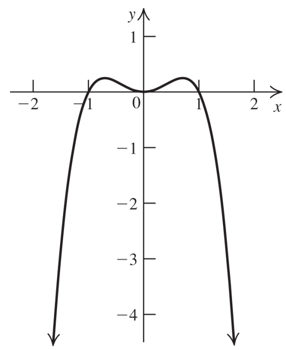
(ii) Zeros: $x = -1$, multiplicity 1, crosses the x-axis; $x = 0$, multiplicity 2, touches but does not cross; $x = 1$, multiplicity 1, crosses the x-axis.

(iii) x-intercepts: –1, 0, 1;
$y = -0^2(0^2 - 1) \Rightarrow y = 0$ is the y-intercept.

(iv) The intervals to be tested are $(-\infty, -1)$, $(-1, 0), (0, 1)$, and $(1, \infty)$. The graph is above the x-axis on $(-1, 0) \cup (0, 1)$ and below the x-axis on $(-\infty, -1) \cup (1, \infty)$.

(v) $f(-x) = -(-x)^2((-x)^2 - 1)$
$= -x^2(x^2 - 1) = f(x) \Rightarrow$
f is even, and f is symmetric with respect to the y-axis.

(vi)



24.(i) $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to \infty$

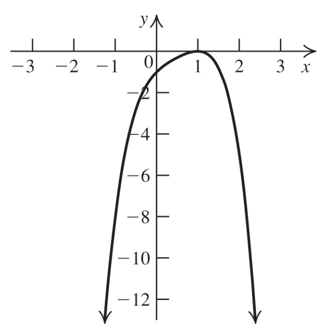
(ii) Zero: $x = 1$, multiplicity 2, touches but does not cross.

(iii) x-intercept: 1;
$y = -(0-1)^2(0^2+1) \Rightarrow y = -1$ is the y-intercept.

(iv) The intervals to be tested are $(-\infty, 1)$ and $(1, \infty)$. The graph is below the x-axis on $(-\infty, 1) \cup (1, \infty)$.

(v) $f(-x) = -(-x-1)^2((-x)^2+1) \neq f(x)$ or $-f(x) \Rightarrow f$ is neither even nor odd. There are no symmetries.

(vi)



25.
$$\begin{array}{r}
2x+3 \\
3x-2\,\overline{)\,6x^2+5x-13} \\
\underline{6x^2-4x} \\
9x-13 \\
\underline{9x-\;\;6} \\
-7
\end{array}$$

26.
$$\begin{array}{r}
4x-1 \\
2x-3\,\overline{)\,8x^2-14x+15} \\
\underline{8x^2-12x} \\
-2x+15 \\
\underline{-2x+\;\;3} \\
12
\end{array}$$

27.
$$\begin{array}{r}
8x^3-12x^2+14x-21 \\
x+1\,\overline{)\,8x^4-4x^3+\;\;2x^2-\;\;7x+165} \\
\underline{8x^4+\;\;8x^3} \\
-12x^3+\;\;2x^2 \\
\underline{-12x^3-12x^2} \\
14x^2-\;\;7x \\
\underline{14x^2+14x} \\
-21x+165 \\
\underline{-21x-\;\;21} \\
186
\end{array}$$

28.
$$\begin{array}{r}
x-1 \\
x^2-2x+6\,\overline{)\,x^3-3x^2+4x+7} \\
\underline{x^3-2x^2+6x} \\
-x^2-2x+7 \\
\underline{-x^2+2x-6} \\
-4x+13
\end{array}$$

29.
$$\begin{array}{r|rrrr}
3 & 1 & 0 & -12 & 3 \\
 & & 3 & 9 & -9 \\
\hline
 & 1 & 3 & -3 & -6
\end{array}$$

Quotient: x^2+3x-3 remainder –6.

30.
$$\begin{array}{r|rrrr}
6 & -4 & 3 & -5 & 0 \\
 & & -24 & -126 & -786 \\
\hline
 & -4 & -21 & -131 & -786
\end{array}$$

Quotient: $-4x^2-21x-131$ remainder –786.

31.
$$\begin{array}{r|rrrrr}
-1 & 2 & -3 & 5 & -7 & 165 \\
 & & -2 & 5 & -10 & 17 \\
\hline
 & 2 & -5 & 10 & -17 & 182
\end{array}$$

Quotient: $2x^3-5x^2+10x-17$ remainder 182.

32.
$$\begin{array}{r|rrrrrr}
-2 & 3 & -2 & 0 & 1 & -16 & -132 \\
 & & -6 & 16 & -32 & 62 & -92 \\
\hline
 & 3 & -8 & 16 & -31 & 46 & -224
\end{array}$$

Quotient: $3x^4-8x^3+16x^2-31x+46$ remainder –224.

33.(i) $f(2) = 2^3 - 3(2^2) + 11(2) - 29 = -11$

(ii)
$$\begin{array}{r|rrrr}
2 & 1 & -3 & 11 & -29 \\
 & & 2 & -2 & 18 \\
\hline
 & 1 & -1 & 9 & -11
\end{array}$$

The remainder is –11, so $f(2) = -11$.

34.(i) $f(-2)=2(-2)^3+(-2)^2-15(-2)-2=16$

(ii)
$$\begin{array}{r|rrrr} -2 & 2 & 1 & -15 & -2 \\ & & -4 & 6 & 18 \\ \hline & 2 & -3 & -9 & 16 \end{array}$$

The remainder is 16, so $f(-2) = 16$.

35.(i) $f(2)=(-3)^4-2(-3)^2-5(-3)+10=88$

(ii)
$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -2 & -5 & 10 \\ & & -3 & 9 & -21 & 78 \\ \hline & 1 & -3 & 7 & -26 & 88 \end{array}$$

The remainder is 88, so $f(-3) = 88$.

36.(i) $f(1)=1^5+2=3$

(ii)
$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & 2 \\ & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 3 \end{array}$$

The remainder is 3, so $f(1) = 3$.

37.
$$\begin{array}{r|rrrr} 2 & 1 & -7 & 14 & -8 \\ & & 2 & -10 & 8 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

The remainder is 0, so 2 is a zero. Now find the zeros of the depressed function

$x^2-5x+4=(x-4)(x-1) \Rightarrow$ 4 and 1 are also zeros. So the zeros of $x^3-7x^2+14x-8$ are 1, 2, and 4.

38.
$$\begin{array}{r|rrrr} -2 & 2 & -3 & -12 & 4 \\ & & -4 & 14 & -4 \\ \hline & 2 & -7 & 2 & 0 \end{array}$$

The remainder is 0, so –2 is a zero. Now find the zeros of the depressed function

$2x^2-7x+2$:

$$x=\frac{7\pm\sqrt{(-7)^2-4(2)(2)}}{2(2)}=\frac{7\pm\sqrt{33}}{4}.$$

So the zeros of $2x^3-3x^2-12x+4$ are –2 and $\frac{7\pm\sqrt{33}}{4}$.

39.
$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 14 & 13 & -6 \\ & & 1 & 5 & 6 \\ \hline & 3 & 15 & 18 & 0 \end{array}$$

The remainder is 0, so $\frac{1}{3}$ is a zero. Now find the zeros of the depressed function

$3x^2+15x+18=3(x+2)(x+3) \Rightarrow$ –2 and –3 are also zeros. So the zeros of $3x^3+14x^2+13x-6$ are –3, –2, and $\frac{1}{3}$.

40.
$$\begin{array}{r|rrrr} \frac{1}{4} & 4 & 19 & -13 & 2 \\ & & 1 & 5 & -2 \\ \hline & 4 & 20 & -8 & 0 \end{array}$$

The remainder is 0, so $1/4$ is a zero. Now find the zeros of the depressed function

$$4x^2+20x-8=\frac{-20\pm\sqrt{20^2-4(4)(-8)}}{2(4)}$$
$$=\frac{-20\pm4\sqrt{33}}{8}=\frac{-5\pm\sqrt{33}}{2}.$$

So the zeros of $4x^3+19x^2-13x+2$ are $\frac{-5\pm\sqrt{33}}{2}$ and $\frac{1}{4}$.

41. The factors of the constant term are $\{\pm1,\pm2,\pm3,\pm6\}$, and the factors of the leading coefficient are $\{\pm1\}$. The possible rational zeros are $\{\pm1,\pm2,\pm3,\pm6\}$.

42. The factors of the constant term are $\{\pm1,\pm2,\pm4,\pm8,\pm16\}$, and the factors of the leading coefficient are $\{\pm1,\pm3,\pm9\}$. The possible rational zeros are $\left\{\pm\frac{1}{9},\pm\frac{2}{9},\pm\frac{1}{3},\right.$

$\pm\frac{4}{9},\pm\frac{2}{3},\pm\frac{8}{9},\pm1,\pm\frac{4}{3},\pm\frac{16}{9},\pm2,\pm\frac{8}{3},\pm4,$

$\left.\pm\frac{16}{3},\pm8,\pm16\right\}.$

43. $f(x)=5x^3+11x^2+2x;$
$$f(-x)=5(-x)^3+11(-x)^2+2(-x)$$
$$=-5x^3+11x^2-2x$$

There are no sign changes in $f(x)$, so there are no positive zeros. There are two sign changes in $f(-x)$, so there are 2 or 0 negative zeros.

$$5x^3+11x^2+2x=x(5x^2+11x+2)$$
$$=x(5x+1)(2+x)\Rightarrow$$

the zeros are $-2,-\frac{1}{5},0.$

44. $f(x) = x^3 + 2x^2 - 5x - 6$
$f(-x) = (-x)^3 + 2(-x)^2 - 5(-x) - 6$
$= -x^3 + 2x^2 + 5x - 6$
There is one sign change in $f(x)$, so there is one positive zero. There are two sign changes in $f(-x)$, so there are 2 or 0 negative zeros. The possible rational zeros are $\{\pm1, \pm2, \pm3, \pm6\}$. Use synthetic division to find one zero:

-3	1	2	−5	−6
		−3	3	6
	1	−1	−2	0

The remainder is 0, so –3 is a zero. Now find the zeros of the depressed function.
$x^2 - x - 2 = (x-2)(x+1) \Rightarrow -1$ and 2 are also zeros. So the zeros of $x^3 + 2x^2 - 5x - 6$ are –3, –1, and 2.

45. $f(x) = x^3 + 3x^2 - 4x - 12$
$f(-x) = (-x)^3 + 3(-x)^2 - 4(-x) - 12$
$= -x^3 + 3x^2 + 4x - 12$
There is one sign change in $f(x)$, so there is one positive zero. There are two sign changes in $f(-x)$, so there are 2 or 0 negative zeros. The possible rational zeros are $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$. Use synthetic division to find one zero:

-3	1	3	−4	−12
		−3	0	12
	1	0	−4	0

The remainder is 0, so –3 is a zero. Now find the zeros of the depressed function
$x^2 - 4 = (x-2)(x+2) \Rightarrow -2$ and 2 are also zeros. So the zeros of $x^3 + 3x^2 - 4x - 12$ are –3, –2, and 2.

46. $f(x) = 2x^3 - 9x^2 + 12x - 5;$
$f(-x) = 2(-x)^3 - 9(-x)^2 + 12(-x) - 5$
$= -2x^3 + 9x^2 - 12x - 5$
There are three sign changes in $f(x)$, so there are 3 or 1 positive zeros. There are two sign changes in $f(-x)$, so there are 2 or 0 negative zeros. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm1, \pm\frac{5}{2}, \pm5\right\}$.

Use synthetic division to find one zero:

1	2	−9	12	−5
		2	−7	5
	2	−7	5	0

The remainder is 0, so 1 is a zero.
Now find the zeros of the depressed function $2x^2 - 7x + 5 = (2x-5)(x-1) \Rightarrow 5/2$ and 1 are also zeros. The zeros of $2x^3 - 9x^2 + 12x - 5$ are $5/2$ and 1 (multiplicity 2).

47. The function has degree three, so there are three zeros. Use synthetic division to find the depressed function:

2	1	0	−7	6
		2	4	−6
	1	2	−3	0

Now find the zeros of the depressed function $x^2 + 2x - 3 = (x+3)(x-1) \Rightarrow -3$ and 1 are zeros. The zeros of the original function are –3, 1, 2.

48. The function has degree four, so there are four zeros. Since 1 is a zero of multiplicity 2, $(x-1)^2$ is a factor of $x^4 + x^3 - 3x^2 - x + 2$. Use synthetic division twice to find the depressed function:

1	1	1	−3	−1	2	1	1	2	−1	−2
		1	2	−1	−2			1	3	2
	1	2	−1	−2	0		1	3	2	0

Alternatively, divide $x^4 + x^3 - 3x^2 - x + 2$ by $(x-1)^2 = x^2 - 2x + 1$. Now find the zeros of the depressed function
$x^2 + 3x + 2 = (x+2)(x+1) \Rightarrow -2$ and -1 are zeros. The zeros of the original function are –2, –1, and 1.

49. The function has degree four, so there are four zeros. Use synthetic division twice to find the depressed function:

-1	1	−2	6	−18	−27	3	1	−3	9	−27
		−1	3	−9	27			3	0	27
	1	−3	9	−27	0		1	0	9	0

Alternatively, divide $x^4 - 2x^3 + 6x^2 - 18x - 27$ by $(x+1)(x-3) = x^2 - 2x - 3.$ Now find the zeros of the depressed function $x^2 + 9 \Rightarrow \pm3i$ are zeros. The zeros of the original function are –1, 3, and $\pm3i$.

50. The function has degree three, so there are three zeros. Since one zero is 2 – i, another zero is 2 + i. Divide $4x^3 - 19x^2 + 32x - 15$ by $(x-(2-i))(x-(2+i)) = x^2 - 4x + 5$ to find the depressed function:

$$\begin{array}{r|l} & 4x - 3 \\ x^2 - 4x + 5 & 4x^3 - 19x^2 + 32x - 15 \\ & \underline{4x^3 - 16x^2 + 20x} \\ & -3x^2 + 12x - 15 \\ & \underline{-3x^2 + 12x - 15} \\ & 0 \end{array}$$

Now find the zeros of the depressed function: $4x - 3 = 0 \Rightarrow x = \frac{3}{4}$. Alternatively, find the possible rational zeros, and then use synthetic division to find the zero. The zeros of the original function are $\frac{3}{4}, 2 - i,$ and $2 + i$.

51. The function has degree four, so there are four zeros. Since one zero is –1 + 2i, another zero is –1 – 2i. Divide $x^4 + 2x^3 + 9x^2 + 8x + 20$ by $(x-(-1+2i))(x-(-1-2i)) = x^2 + 2x + 5$ to find the depressed function:

$$\begin{array}{r|l} & x^2 + 4 \\ x^2 + 2x + 5 & x^4 + 2x^3 + 9x^2 + 8x + 20 \\ & \underline{x^4 + 2x^3 + 5x^2} \\ & 4x^2 + 8x + 20 \\ & \underline{4x^2 + 8x + 20} \\ & 0 \end{array}$$

Now find the zeros of the depressed function: $x^2 + 4 = 0 \Rightarrow x = \pm 2i$. The zeros of the original function are $\pm 2i$ and $-1 \pm 2i$.

52. The function has degree five, so there are five zeros. Since two zeros are 2 + 2i, 2 – 2i are also zeros. Divide $x^5 - 7x^4 + 24x^3 - 32x^2 + 64$ by $\left((x-(2+2i))(x-(2-2i))\right)^2 = (x^2 - 4x + 8)^2$ $= x^4 - 8x^3 + 32x^2 - 64x + 64$ to find the depressed function. (Note it may be easier to divide by $x^2 - 4x + 8$ and then to divide the quotient by $x^2 - 4x + 8$.) The quotient is $x + 1$, so –1 is a zero. The zeros of the original function are -1 and $2 \pm 2i$.

53. The function has degree three, so there are three zeros. The possible rational zeros are $\{\pm 1, \pm 2, \pm 4\}$. Using synthetic division, we find that one zero is 1:

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -4 & 4 \\ & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

Now find the zeros of the depressed function: $x^2 - 4 = 0 \Rightarrow x = \pm 2$.
The solution set is {–2, 1, 2}.

54. The function has degree three, so there are three zeros. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 6\right\}$.
Using synthetic division, we find that one zero is $-1/2$:

$$\begin{array}{r|rrrr} -1/2 & 2 & 1 & -12 & -6 \\ & & -1 & 0 & 6 \\ \hline & 2 & 0 & -12 & 0 \end{array}$$

Now find the zeros of the depressed function: $2x^2 - 12 = 0 \Rightarrow x = \pm\sqrt{6}$.
The solution set is $\left\{-1/2, \pm\sqrt{6}\right\}$.

55. The function has degree three, so there are three zeros. The possible rational zeros are $\left\{\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{3}{2}, \pm 3\right\}$. Using synthetic division, we find that one zero is –1:

$$\begin{array}{r|rrrr} -1 & 4 & 0 & -7 & -3 \\ & & -4 & 4 & 3 \\ \hline & 4 & -4 & -3 & 0 \end{array}$$

Now find the zeros of the depressed function: $4x^2 - 4x - 3 = 0 \Rightarrow (2x-3)(2x+1) = 0 \Rightarrow$ $x = 3/2$ or $x = -1/2$.
The solution set is $\left\{-1/2, -1, 3/2\right\}$.

56. The function has degree three, so there are three zeros. The possible rational zeros are $\{\pm 1, \pm 2, \pm 3, \pm 6\}$. Using synthetic division, we find that one zero is –3:

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -8 & -6 \\ & & -3 & 6 & 6 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

Now find the zeros of the depressed function:
$x^2 - 2x - 2 = 0 \Rightarrow$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}.$$
The solution set is $\left\{-3, 1 \pm \sqrt{3}\right\}$.

57. The function has degree four, so there are four zeros. The possible rational zeros are $\{\pm1, \pm2\}$. Using synthetic division, we find that one zero is 2:

$$\begin{array}{r|rrrrr} \underline{2} & 1 & -1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 1 & 0 \end{array}$$

Now find the zeros of the depressed function: $x^3 + x^2 + x + 1 = 0.$ Again using synthetic division, we find that one zero is –1:

$$\begin{array}{r|rrrr} \underline{-1} & 1 & 1 & 1 & 1 \\ & & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

The zeros of the depressed function $x^2 + 1 = 0$ are $\pm i.$ The solution set is $\{-1, 2, \pm i\}$.

58. The function has degree four, so there are four zeros. The possible rational zeros are $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$. Using synthetic division, we find that one zero is 2:

$$\begin{array}{r|rrrrr} \underline{1} & 1 & -1 & -13 & 1 & 12 \\ & & 1 & 0 & -13 & -12 \\ \hline & 1 & 0 & -13 & -12 & 0 \end{array}$$

Now find the zeros of the depressed function: $x^3 - 13x - 12 = 0.$ Again using synthetic division, we find that one zero is –1:

$$\begin{array}{r|rrrr} \underline{-1} & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$x^2 - x - 12 = 0 \Rightarrow (x-4)(x+3) = 0 \Rightarrow x = 4$ or $x = -3$. The solution set is $\{-3, -1, 1, 4\}$.

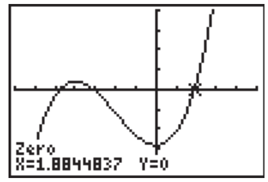
59. The only possible rational roots are $\{\pm1, \pm2\}$. None of these satisfies the equation.

60. The only possible rational roots are $\{\pm\frac{1}{3}, \pm1, \pm\frac{5}{3}, \pm5\}$. None of these satisfies the equation.

61. $f(1) = 1^3 + 6(1)^2 - 28 = -21$

$f(2) = 2^3 + 6(2)^2 - 28 = 4$

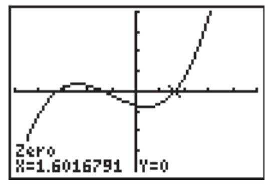
Because the sign changes, there is a real zero between 1 and 2. The zero is approximately 1.88.



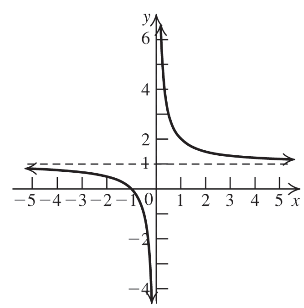
62. $f(1) = 1^3 + 3(1)^2 - 3(1) - 7 = -6$

$f(2) = 2^3 + 3(2)^2 - 3(2) - 7 = 7$

Because the sign changes, there is a real zero between 1 and 2. The zero is approximately 1.60.



63. $1 + \dfrac{1}{x} = 0 \Rightarrow x = -1$ is the x-intercept. There is no y-intercept. The vertical asymptote is the y-axis ($x = 0$). The horizontal asymptote is $y = 1.$ $f(-x) = 1 - \dfrac{1}{x} \neq -f(x)$ and $\neq f(x),$ so there are no symmetries. Testing the intervals $(-\infty, 0),$ and $(0, \infty),$ we find that the graph is below the horizontal asymptote on $(-\infty, 0)$ and above the horizontal asymptote on $(0, \infty)$.



64. $\dfrac{2-x}{x} = 0 \Rightarrow x = 2$ is the x-intercept. There is no y-intercept. The vertical asymptote is the y-axis ($x = 0$). The horizontal asymptote is $y = -1$.

$f(-x) = \dfrac{2+x}{-x} \neq -f(x)$ and $\neq f(x),$ so there are no symmetries. Testing the intervals $(-\infty, 0),$ and $(0, \infty),$ we find that the graph is above the horizontal asymptote on $(0, \infty)$ and below the horizontal asymptote on $(-\infty, 0)$.

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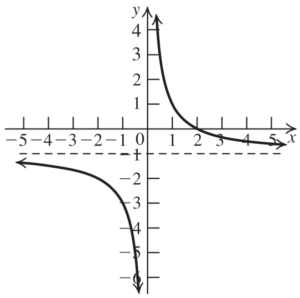
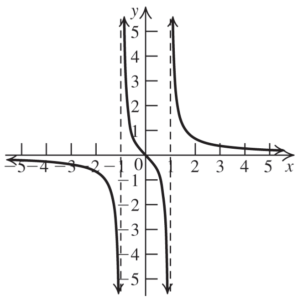


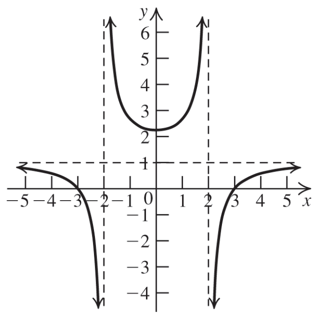
65. $\frac{x}{x^2-1}=0 \Rightarrow x=0$ is the x-intercept.

$\frac{0}{0^2-1}=0 \Rightarrow y=0$ is the y-intercept. The vertical asymptotes are $x = 1$ and $x = -1$. The horizontal asymptote is the x-axis.

$f(-x)=\frac{-x}{(-x)^2-1}=-f(x)$ and $\neq f(x)$, so there is symmetry with respect to the origin. Testing the intervals $(-\infty,-1),(-1,0),\ (0,1),$ and $(1,\infty)$, we find that the graph is above the x-axis on $(-\infty,-1)\cup(0,\infty)$ and below the x-axis on $(-1,0)\cup(0,1)$.



Exercise 65



Exercise 66

66. $\frac{x^2-9}{x^2-4}=0 \Rightarrow x=\pm 3$ are the x-intercepts.

$\frac{0^2-9}{0^2-4}=\frac{9}{4} \Rightarrow y=\frac{9}{4}$ is the y-intercept. The vertical asymptotes are $x = 2$ and $x = -2$. The horizontal asymptote is $y = 1$.

$f(-x)=\frac{(-x)^2-9}{(-x)^2-1}=f(x) \Rightarrow$ the function is even and the graph is symmetric with respect to the y-axis. Testing the intervals $(-\infty,-2),$ $(-2,2),$ and $(3,\infty),$ we find that the graph is above the horizontal asymptote on $(-2,2)$ and below the al asymptote on $(-\infty,-2)\cup(2,\infty)$.

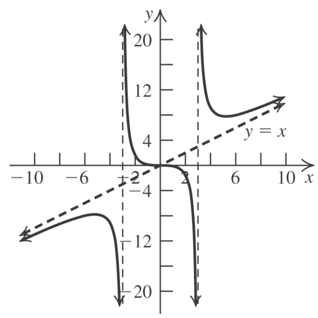
67. $\frac{x^3}{x^2-9}=0 \Rightarrow x=0$ is the x-intercept.

$\frac{0^3}{0^2-9}=0 \Rightarrow y=0$ is the y-intercept. The vertical asymptotes are $x = 3$ and $x = -3$. There is no horizontal asymptote.

$f(-x)=\frac{(-x)^3}{(-x)^2-9}\neq f(x)$ but $f(-x)=-f(x)$, so there is symmetry with respect to the origin.

$\frac{x^3}{x^2-9}=x+9x,$ so the oblique asymptote is $y = x$.

Testing the intervals $(-\infty,-3),(-3,0),(0,3),$ and $(3,\infty),$ we find that the graph is above the oblique asymptote on $(-3,0)\cup(3,\infty)$ and below the oblique asymptote on $(-\infty,-3)\cup(0,3)$.



68. $\frac{x+1}{x^2-2x-8}=0 \Rightarrow x=-1$ is the x-intercept.

$\frac{0+1}{0^2-2(0)-8}=-\frac{1}{8} \Rightarrow y=-\frac{1}{8}$ is the y-intercept. The vertical asymptotes are $x = 4$ and $x = -2$. The horizontal asymptote is the x-axis.

$f(-x)=\frac{(-x)+1}{(-x)^2-2(-x)-8}\neq f(x)$ and $\neq -f(x)$, so there are no symmetries.

Testing the intervals $(-\infty,-2),(-2,-1),(-1,4),$ and $(4,\infty),$ we find that the graph is above the x-axis on $(-2,-1)\cup(4,\ \infty))$ and below the x-axis on $(-\infty,-2)\cup(-1,4)$.

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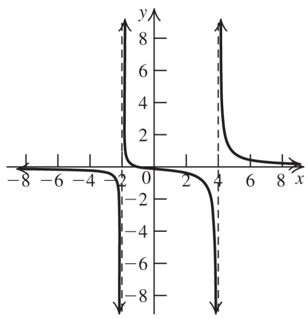
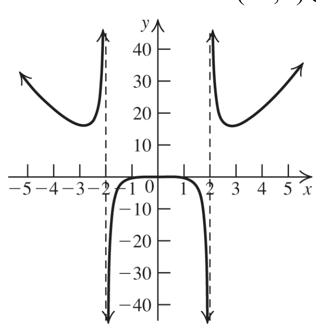


69. $\dfrac{x^4}{x^2-4}=0 \Rightarrow x=0$ is the x-intercept.

$\dfrac{0^4}{0^2-4}=0 \Rightarrow y=0$ is the y-intercept. The vertical asymptotes are $x = 2$ and $x = -2$. There is no horizontal asymptote.

$f(-x)=\dfrac{(-x)^4}{(-x)^2-4}=f(x)$, so the function is even, and the graph is symmetric with respect to the y-axis.

Testing the intervals $(-\infty,-2)$, $(-2,0),(0,2),$ and $(2,\infty),$ we find that the graph is above the x-axis on $(-\infty,-2)\cup(2,\infty)$ and below the x-axis on $(-2,0)\cup(0,2)$.



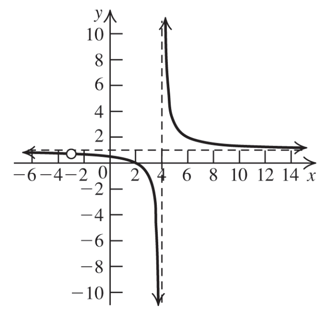
70. $\dfrac{x^2+x-6}{x^2-x-12}=0 \Rightarrow x=2$ is the x-intercept.

$\dfrac{0^2+0-6}{0^2-0-12}=\dfrac{1}{2} \Rightarrow y=\dfrac{1}{2}$ is the y-intercept.

The vertical asymptote is $x = 4$. The horizontal asymptote is $y = 1$.

$f(-x)=\dfrac{(-x)^2+(-x)-6}{(-x)^2-(-x)-12}\neq f(x)$ and $\neq -f(x),$ so there are no symmetries.

Testing the intervals $(-\infty,-3),(-3,4),$ and $(4,\infty),$ we find that the graph is above the horizontal asymptote on $(4,\infty)$ and below the horizontal asymptote on $(-\infty,-3)\cup(-3,4)$.



71. $(x-2)(x-3)(x+2)\le 0$

The real zeros are $x = -2$, $x = 2$, and $x = 3$. So, the intervals to be tested are $(-\infty,-2]$, $[-2,2]$, $[2,3]$, and $[3,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty,-2]$	–3	–30	–
$[-2,2]$	0	12	+
$[2,3]$	2.5	–1.125	–
$[3,\infty)$	4	12	+

Solution set: $(-\infty,-2]\cup[2,3]$

72. $(x-1)(x+1)^2(x+2)\ge 0$

The real zeros are $x = -2$, $x = -1$, and $x = 1$. So, the intervals to be tested are $(-\infty,-2]$, $[-2,-1]$, $[-1,1]$, and $[1,\infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty,-2]$	–3	16	+
$[-2,-1]$	–1.5	–0.3125	–
$[-1,1]$	0	–2	–
$[1,\infty)$	2	36	+

Solution set: $(-\infty,-2]\cup\{-1\}\cup[1,\infty)$

73. $x^3 - 4x^2 + 7x - 7 > x^2 - 4 \Rightarrow$
$x^3 - 5x^2 + 7x - 3 > 0$
The possible rational zeros are $\pm 1, \pm 3$
Using synthetic division, we find that $x = 3$ is a zero.

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 7 & -3 \\ & & 3 & -6 & 3 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$$\begin{aligned} x^3 - 5x^2 + 7x - 3 &= (x-3)\left(x^2 - 2x + 1\right) \\ &= (x-3)(x-1)^2 \end{aligned}$$

The zeros are $x = 1$ and $x = 3$. So, the intervals to be tested are $(-\infty, 1)$, $(1, 3)$, and $(3, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, 1)$	0	–3	–
$(1, 3)$	2	–1	–
$(3, \infty)$	4	9	+

Solution set: $(3, \infty)$

74. $x^3 - 2x^2 + 4x + 8 < 3x^2 + 2x \Rightarrow$
$x^3 - 5x^2 + 2x + 8 < 0$
The possible rational zeros are $\pm 1, \pm 2, \pm 4, \pm 8$.
Using synthetic division, we find that $x = 2$ is a zero.

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$$\begin{aligned} x^3 - 5x^2 + 2x + 8 &= (x-2)\left(x^2 - 3x - 4\right) \\ &= (x-2)(x-4)(x+1) \end{aligned}$$

The zeros are $x = -1$, $x = 2$ and $x = 4$. So, the intervals to be tested are $(-\infty, -1)$, $(-1, 2)$, $(2, 4)$ and $(4, \infty)$.

Interval	Test point	Value of $P(x)$	Result
$(-\infty, -1)$	–2	–24	–
$(-1, 2)$	0	8	+
$(2, 4)$	3	–4	–
$(4, \infty)$	5	18	+

Solution set: $(-\infty, -1) \cup (2, 4)$

75. $\dfrac{(x-1)(x+3)}{x+2} \ge 0$
numerator: $(x-1)(x+3) = 0 \Rightarrow x = 1,\ x = -3$
denominator : $x + 2 = 0 \Rightarrow x = -2$
The line $x = -2$ is the vertical asymptote of $R(x)$, so –2 is not included in the intervals to be tested. The boundary points are –3, –2, and 1. The intervals are $(-\infty, -3]$, $[-3, -2)$, $(-2, 1]$, and $[1, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -3]$	–5	–4	–
$[-3, -2)$	–2.5	$\frac{7}{2}$	+
$(-2, 1]$	0	$-\frac{3}{2}$	–
$[1, \infty)$	2	$\frac{5}{4}$	+

Solution set: $[-3, -2) \cup [1, \infty)$

76. $\dfrac{(x+1)^2(x-4)}{x-2} \le 0$
numerator:
$(x+1)^2(x-4) = 0 \Rightarrow x = -1,\ x = 4$
denominator : $x - 2 = 0 \Rightarrow x = 2$
The line $x = 2$ is the vertical asymptote of $R(x)$, so 2 is not included in the intervals to be tested. The boundary points are –1, 2, and 4. The intervals are $(-\infty, -1]$, $[-1, 2)$, $(2, 4]$, and $[4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -1]$	–2	$\frac{3}{2}$	+
$[-1, 2)$	0	2	+
$(2, 4]$	3	–16	–
$[4, \infty)$	5	12	+

Solution set: $\{-1\} \cup (2, 4]$ (Remember to include –1 in the solution set because it is a zero.)

77. $\dfrac{2x-3}{x+4} < 3 \Rightarrow \dfrac{2x-3}{x+4} - 3 < 0 \Rightarrow$
$\dfrac{2x-3-3(x+4)}{x+4} < 0 \Rightarrow \dfrac{-x-15}{x+4} < 0$
numerator: $-x-15 = 0 \Rightarrow x = -15$
denominator : $x+4=0 \Rightarrow x = -4$
The line $x = -4$ is the vertical asymptote of $R(x)$, so –4 is not included in the intervals to be tested. The boundary points are –15 and –4. The intervals are $(-\infty, -15)$, $(-15, -4)$ and $(-4, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -15)$	–16	$-\frac{1}{12}$	–
$(-15, -4)$	–12	$\frac{3}{8}$	+
$(-4, \infty)$	–3	–12	–

Solution set: $(-\infty, -15) \cup (-4, \infty)$

78. $\dfrac{2}{x+4} > \dfrac{3}{x} \Rightarrow \dfrac{2}{x+4} - \dfrac{3}{x} > 0 \Rightarrow$
$\dfrac{2x-3(x+4)}{x(x+4)} > 0 \Rightarrow \dfrac{-x-12}{x(x+4)} > 0$
numerator: $-x-12 = 0 \Rightarrow x = -12$
denominator : $x(x+4) = 0 \Rightarrow x = 0,\ x = -4$
The lines $x = -4$ and $x = 0$ are the vertical asymptotes of $R(x)$, so –4 and 0 are not included in the intervals to be tested. The boundary points are –12, –4, and 0. The intervals are $(-\infty, -12)$, $(-12, -4)$, $(-4, 0)$, and $(0, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -12)$	–13	$\frac{1}{117}$	+
$(-12, -4)$	–6	$-\frac{1}{2}$	–
$(-4, 0)$	–3	3	+
$(0, \infty)$	1	$-\frac{13}{5}$	–

Solution set: $(-\infty, -12) \cup (-4, 0)$

Applying the Concepts

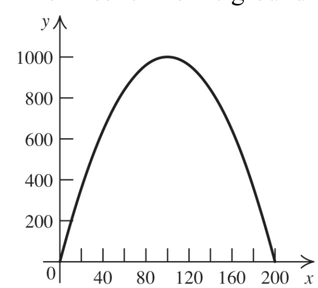
79. $y = kx; 12 = 4k \Rightarrow k = 3; y = 3(5) = 15$

80. $p = \dfrac{k}{q}; 4 = \dfrac{k}{3} \Rightarrow 12 = k; p = \dfrac{12}{4} = 3$

81. $s = kt^2; 20 = 2^2 k \Rightarrow 5 = k; s = 3^2(5) = 45$

82. $y = \dfrac{k}{x^2}; 3 = \dfrac{k}{8^2} \Rightarrow k = 192; 12 = \dfrac{192}{x^2} \Rightarrow x = \pm 4$

83. The maximum height occurs at the vertex,
$\left(-\dfrac{20}{2(-1/10)}, f\left(-\dfrac{20}{2(-1/10)}\right)\right) = (100, 1000).$
The maximum height is 1000. To find where the missile hits the ground, solve
$-\dfrac{1}{10}x^2 + 20x = 0 \Rightarrow x = 0$ or $x = 200.$
The missile hits the ground at $x = 200$.



84. Let x = the length of one piece, and the area of the square formed by that piece is x^2. Then $20 - x =$ the length of the other piece, $(20-x)^2 =$ the area of the square formed by that piece. The total area is $x^2 + (20-x)^2 = 2x^2 - 40x + 400.$ The minimum is at the x-coordinate of the vertex, $-\dfrac{-40}{2(2)} = 10.$ Each piece must be 10 cm long.

85. The area of each section is 400 square feet. Since the width is x, $y = \dfrac{400}{x}$. The total amount of fencing needed is $4x + \dfrac{2400}{x}$. Using a graphing calculator, we find that this is a minimum at $x \approx 24.5$ feet. So $y \approx \dfrac{400}{24.5} \approx 16.3$ feet. The dimensions of each pen should be approximately 24.5 ft by 16.3 ft.

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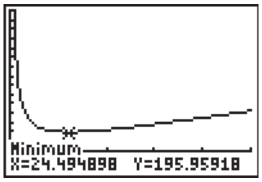


86. Since x = the width, $y = \dfrac{400}{x}$. Then

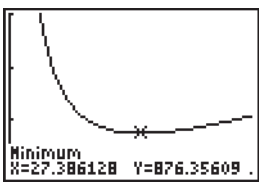
$6y = \dfrac{2400}{x} =$ the length of the heavy fencing.

The farmer needs $2x + \dfrac{2400}{x}$ feet of heavy fencing, so it will cost

$5\left(2x + \dfrac{2400}{x}\right) = 10x + \dfrac{12,000}{x}$.

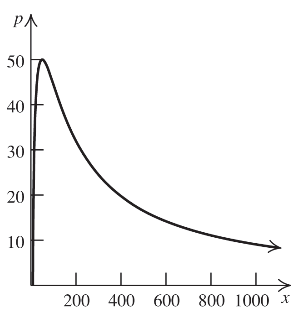
The low fencing will cost $3(2x) = 6x$. The total cost of the fencing is $\dfrac{12,000}{x} + 16x$.

Using a graphing calculator, we find this is a minimum when $x \approx 27.4$.



So $y \approx \dfrac{400}{27.4} \approx 14.6$. The cost will be minimized when the dimensions of each pen are 27.4 ft by 14.6 ft.

87. a.



b.

$$\frac{100^2 x}{(50+x)^2} > 42$$

$$\frac{10000x - 42(50+x)^2}{(50+x)^2} > 0$$

$$\frac{10000x - \left(42x^2 + 4200x + 105,000\right)}{(50+x)^2} > 0$$

$$\frac{-42x^2 + 5800x - 105,000}{(50+x)^2} > 0$$

$$\frac{-2(7x-150)(3x-350)}{(50+x)^2} > 0$$

numerator: $-2(7x-150)(3x-350) = 0 \Rightarrow$ $x = \frac{150}{7}$, $x = \frac{350}{3}$

denominator: $(50+x)^2 = 0 \Rightarrow x = -50$

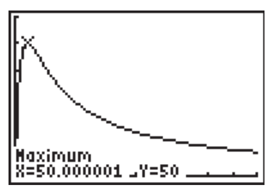
The line $x = -50$ is the vertical asymptote of $R(x)$, so -50 is not included in the intervals to be tested. In the context of this problem, $x > 0$, so it is not necessary to test negative values. The boundary points are 0, $\frac{150}{7}$, and $\frac{350}{3}$. The intervals are

$\left(0, \frac{150}{7}\right)$, $\left(\frac{150}{7}, \frac{350}{3}\right)$, and $\left(\frac{350}{3}, \infty\right)$.

Interval	Test point	Value of $R(x)$	Result
$\left(0, \frac{150}{7}\right)$	10	$-\frac{128}{9}$	–
$\left(\frac{150}{7}, \frac{350}{3}\right)$	50	8	+
$\left(\frac{350}{3}, \infty\right)$	200	–10	–

The power absorbed by the circuit is greater than 42 for $\frac{150}{7} < x < \frac{350}{3}$.

c.



The maximum occurs at $x = 50$.

88. a. The total area of the paper is 100 square inches. If the width of the paper is x, then the length is $100/x$. The width of the printed area is $x - 2(1) = x - 2$, and the length of the printed area is $\frac{100}{x} - 2\left(\frac{3}{2}\right) = \frac{100}{x} - 3$. The area of the printed region is $(x-2)\left(\frac{100}{x} - 3\right)$.

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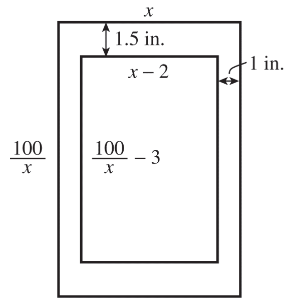


We want to solve $(x-2)\left(\frac{100}{x}-3\right) \geq 56.$

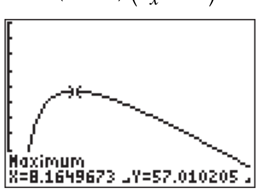
$$\begin{aligned}(x-2)\left(\frac{100}{x}-3\right) &\geq 56\\ (x-2)\left(\frac{100}{x}-3\right)-56 &\geq 0\\ -3x-\frac{200}{x}+106-56 &\geq 0\\ -3x-\frac{200}{x}+50 &\geq 0\\ \frac{-3x^2+50x-200}{x} &\geq 0\end{aligned}$$

numerator: $-3x^2+50x-200=0 \Rightarrow$
$-(x-10)(3x-20) \geq 0 \Rightarrow x=10, \frac{20}{3}$
denominator: $x=0$
Test intervals: $\left(0, \frac{20}{3}\right], \left[\frac{20}{3}, 10\right], [10, \infty)$

Interval	Test point	Value of $R(x)$	Result
$\left(0, \frac{20}{3}\right]$	5	–5	–
$\left[\frac{20}{3}, 10\right]$	8	1	+
$[10, \infty)$	12	$-\frac{8}{3}$	–

The width of the paper should be between $\frac{20}{3} \approx 6.67$ in. and 10 in.

b. The area of the printed region is given by $A=(x-2)\left(\frac{100}{x}-3\right).$



[0, 30] by [0, 100]

The maximum value of x, the width, is about 8.16 in., so the maximum length is $\frac{100}{8.16} \approx 12.25$ in. The maximum area of the printed region is about 57.01 in^2.

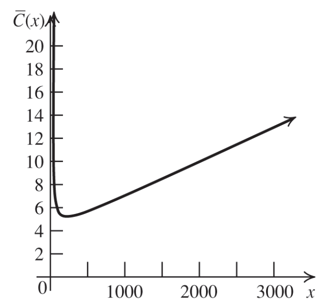
89. a. The revenue is $24x$. Profit = revenue – cost, so $P(x)=24x-\left(150+3.9x+\frac{3}{1000}x^2\right)$

$$=-\frac{3}{1000}x^2+20.1x-150.$$

b. The maximum occurs at the x-coordinate of the vertex: $-\frac{20.1}{2\left(-\frac{3}{1000}\right)}=3350$

c. $\bar{C}(x)=\frac{C(x)}{x}=\frac{\frac{3}{1000}x^2+3.9x+150}{x}$

$$=\frac{3}{1000}x+3.9+\frac{150}{x}$$



90. a. $19,340 \text{ feet}=\frac{19,340}{5280}=3.66 \text{ miles}$

$$p=\frac{69.1}{3.66+2.3} \approx 11.59 \text{ inches}$$

b. $0=\frac{69.1}{a+2.3} \Rightarrow 0=69.1,$ which is impossible. So there is no altitude at which the pressure is 0.

91. $280=40k \Rightarrow k=7; s=7(35)=\$245$

92. $d=ks^2; 25=30^2k \Rightarrow k=\frac{1}{36}$

$$s=\frac{1}{36}(66^2)=121 \text{ feet}$$

93. $I = \dfrac{ki}{d^2}$, where I = illumination, i = intensity, and d = distance from the source. The illumination 6 inches from the source is $I_6 = \dfrac{300k}{6^2}$, while the illumination x inches from the source is $I_x = \dfrac{300k}{x^2}$. $I_6 = 2I_x \Rightarrow$

$$\frac{300k}{36} = 2\left(\frac{300k}{x^2}\right) \Rightarrow x^2 = 72 \Rightarrow x = 6\sqrt{2} \text{ in.}$$

94. $V = kT; 1.2 = 295k \Rightarrow k = \dfrac{1.2}{295}$

$V = \dfrac{1.2}{295}(310) \approx 1.26$ cubic feet.

95. $I = \dfrac{k}{R}; 30 = \dfrac{k}{300} \Rightarrow k = 9000$

a. $I = \dfrac{9000}{250} = 36$ amp

b. $60 = \dfrac{9000}{R} \Rightarrow R = 150$ ohms

96. $L = \dfrac{k\sqrt{r}}{l^2}; 20 = \dfrac{k\sqrt{4}}{12^2} \Rightarrow 1440 = k$

$L = \dfrac{1440\sqrt{3}}{10^2} \approx 24.94$ tons

97. $R = ki(p - i)$

a. $255 = k(0.15)(20{,}000)(0.85)(20{,}000)$

$= \dfrac{1}{200{,}000} = 5 \times 10^{-6}$

b. $R = \dfrac{1}{200{,}000}(10{,}000)(10{,}000)$

$= 500$ people per day

c. $95 = \dfrac{1}{200{,}000}x(20{,}000 - x) \Rightarrow$

$x^2 - 20{,}000x + 19{,}000{,}000 = 0 \Rightarrow$

$(x - 1000)(x - 19{,}000) = 0 \Rightarrow x = 1000$ or

$x = 19{,}000$

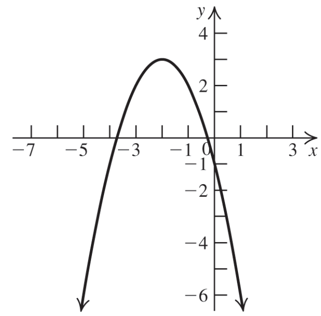
98. $F = (kq_1q_2)/d^2$. If the distance is quadrupled, then the force is $F_{\text{new}} = \dfrac{kq_1q_2}{(4d)^2} = \dfrac{kq_1q_2}{16d^2}$. So the original force is divided by 16: $\dfrac{96}{16} = 6$ units.

Chapter 2 Practice Test A

1. $x^2 - 6x + 2 = 0 \Rightarrow x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)} \Rightarrow$

$x = \dfrac{6 \pm \sqrt{28}}{2} = 3 \pm \sqrt{7}$ are the x-intercepts

2.



3. The vertex is at $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$

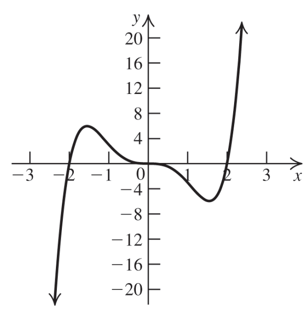
$= \left(-\dfrac{14}{2(-7)}, f\left(-\dfrac{14}{2(-7)}\right)\right) = (1, 10).$

4. The denominator is 0 when $x = -4$ or $x = 1$. The domain is $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$.

5. Using either long division or synthetic division, we find that the quotient is $x^2 - 4x + 3$ and the remainder is 0.

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -5 & 6 \\ & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

6.



7. The function has degree three, so there are three zeros. Since 2 is a zero, use synthetic division to find the depressed function:

$$\begin{array}{r|rrrr} 2 & 2 & -2 & -8 & 8 \\ & & 4 & 4 & -8 \\ \hline & 2 & 2 & -4 & 0 \end{array}$$

Now find the zeros of $2x^2 + 2x - 4$:

$2x^2 + 2x - 4 = 2(x^2 + x - 2) = 2(x + 2)(x - 1) \Rightarrow$ the zeros are $x = -2$ and $x = 1$. The zeros of the original function are –2, 1, 2.

8.
$$\begin{array}{r} -3x^2+5x+1 \\ 2x+3\overline{)-6x^3+\ x^2+17x+3} \\ \underline{-6x^3-9x^2} \\ 10x^2+17x \\ \underline{10x^2+15x} \\ 2x+3 \\ \underline{2x+3} \\ 0 \end{array}$$

9. Using synthetic division to find the remainder, we have $P(-2) = -53$.

$$\begin{array}{r|rrrrr} -2 & 1 & 5 & -7 & 9 & 17 \\ & & -2 & -6 & 26 & -70 \\ \hline & 1 & 3 & -13 & 35 & -53 \end{array}$$

10. The function has degree three, so there are three zeros. There are two sign changes in $f(x)$, so there are either 2 or 0 positive zeros. There is one sign change in $f(-x)$, so there is one negative zero. The possible rational zeros are $\{\pm1, \pm2, \pm4, \pm5, \pm10, \pm20\}$. Using synthetic division, we find that –2 is a zero:

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -4 & 20 \\ & & -2 & 14 & -20 \\ \hline & 1 & -7 & 10 & 0 \end{array}$$

Now find the zeros of the depressed function $x^2-7x+10: x^2-7x+10=(x-5)(x-2) \Rightarrow$ the zeros are $x = 2$ and $x = 5$. The zeros of the original function are –2, 2, 5.

11. The function has degree four, so there are four zeros. Factoring, we have $x^4+x^3-15x^2 = x^2(x^2+x-15) \Rightarrow 0$ is a zero of multiplicity 2. Now find the zeros of x^2+x-15:

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(-15)}}{2(1)} = \frac{-1 \pm \sqrt{61}}{2}.$$

The zeros of the original function are 0 and $\dfrac{-1 \pm \sqrt{61}}{2}$.

12. The factors of the constant term, 9, are $\{\pm1, \pm3, \pm9\}$, while the factors of the leading coefficient are $\{\pm1, \pm2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm3, \pm\frac{9}{2}, \pm9\right\}$.

13. $f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to \infty$ as $x \to \infty$.

14. $f(x) = (x^2-4)(x+2)^2$
$= (x-2)(x+2)(x+2)^2 \Rightarrow$
the zeros are –2 (multiplicity 3) and 2 (multiplicity 1).

15. There are two sign changes in $f(x)$, so there are either two or zero positive zeros. There is one sign change in $f(-x)$, so there is one negative zero.

16. The zeros of the denominator are $x = 5$ and $x = -4$, so those are the vertical asymptotes. The degree of the numerator equals the degree of the denominator, so the horizontal asymptote is $y = 2$.

17. $\dfrac{1}{x+4} \le \dfrac{3}{x-2} \Rightarrow \dfrac{1}{x+4} - \dfrac{3}{x-2} \le 0 \Rightarrow$
$\dfrac{x-2-3(x+4)}{(x+4)(x-2)} \le 0 \Rightarrow \dfrac{-2x-14}{(x+4)(x-2)} \le 0$
numerator: $-2x-14=0 \Rightarrow x=-7$
denominator:
$(x+4)(x-2)=0 \Rightarrow x=-4,\ x=2$
The intervals to be tested are $(-\infty, -7]$, $[-7, -4)$, $(-4, 2)$, and $(2, \infty)$.

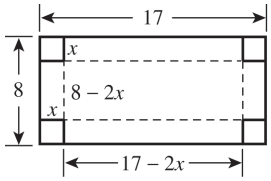
Interval	Test point	Value of $R(x)$	Result
$(-\infty, -7]$	–8	$\frac{1}{20}$	+
$[-7, -4)$	–6	$-\frac{1}{8}$	–
$(-4, 2)$	0	$\frac{7}{4}$	+
$(2, \infty)$	5	$-\frac{8}{9}$	–

Solution set: $[-7, -4) \cup (2, \infty)$

18. $y = \dfrac{kx}{t^2}$
$6 = \dfrac{8k}{2^2} \Rightarrow k = 3$
$y = \dfrac{3(12)}{3^2} = 4$

19. The minimum occurs at the x-coordinate of the vertex: $-\dfrac{-30}{2(1)} = 15$ thousand units.

20. $V = x(17-2x)(8-2x)$



Chapter 2 Practice Test B

1. B

$x^2 + 5x + 3 = 0 \Rightarrow$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)} = \frac{-5 \pm \sqrt{13}}{2}$$

2. D. The graph of $f(x) = 4-(x-2)^2$ is the graph of $f(x) = x^2$ shifted two units right, reflected across the x-axis, and then shifted 4 units up

3. A

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{12}{2(6)}, f\left(-\frac{12}{2(6)}\right)\right) = (-1, -11)$$

4. B. The denominator is 0 when $x = -3$ or $x = 2$.

5. D

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -8 & 6 \\ & & -3 & 9 & -3 \\ \hline & 1 & -3 & 1 & 3 \end{array}$$

6. C

$P(x) = x^4 + 2x^3 = x^3(x+2)$. So the zeros are 0 (multiplicity 3) and –2 (multiplicity 1).

7. C

$$\begin{array}{r|rrrr} 3 & 3 & -26 & 61 & -30 \\ & & 9 & -51 & 30 \\ \hline & 3 & -17 & 10 & 0 \end{array}$$

The zeros of the depressed function $3x^2 - 17x + 10$ are $x = \dfrac{2}{3}$ and $x = 5$.

8. B

$$\begin{array}{r} -5x^2 + 3x - 4 \\ 2x-3 \overline{\smash{\big)}\,-10x^3 + 21x^2 - 17x + 12} \\ \underline{-10x^3 + 15x^2} \qquad\qquad \\ 6x^2 - 17x \qquad \\ \underline{6x^2 - \ \ 9x} \qquad \\ -8x + 12 \\ \underline{-8x + 12} \\ 0 \end{array}$$

9. C

$$\begin{array}{r|rrrrr} -3 & 1 & 4 & 7 & 10 & 15 \\ & & -3 & -3 & -12 & 6 \\ \hline & 1 & 1 & 4 & -2 & 21 \end{array}$$

$P(-3) = 21$.

10. A. The polynomial has degree three, so there are three zeros. The possible rational zeros are $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$. Using synthetic division, we find that one zero is −3:

$$\begin{array}{r|rrrr} -3 & -1 & 1 & 8 & -12 \\ & & 3 & -12 & 12 \\ \hline & -1 & 4 & -4 & 0 \end{array}$$

The zeros of the depressed function $-x^2 + 4x - 4$ are $x = 2$ (multiplicity 2).

11. A. The polynomial has degree three, so there are three zeros. Factoring, we have $x^3 + x^2 - 30x = x(x^2 + x - 30) = x(x+6)(x-5)$

12. D. The factors of the constant term, 60, are $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 60\}$. Since the leading coefficient is 1, these are also the possible rational zeros.

13. C

14. B

$(x^2-1)(x+1)^2 = (x-1)(x+1)(x+1)^2$.

15. C. There are three sign changes in $P(x)$, so there are 3 or 1 positive zeros. There are two sign changes in $P(-x)$, so there are 2 or 0 negative zeros.

16. C. The zeros of the denominator are $x = 3$ and $x = -4$, so those are the vertical asymptotes. The degree of the numerator equals the degree of the denominator, so the horizontal asymptote is $y = 1$.

17. C

$$\frac{3}{x+2} \ge \frac{2}{x-1} \Rightarrow \frac{3}{x+2} - \frac{2}{x-1} \ge 0 \Rightarrow$$
$$\frac{3(x-1)-2(x+2)}{(x+2)(x-1)} \ge 0 \Rightarrow \frac{x-7}{(x+2)(x-1)} \ge 0$$

numerator: $x - 7 = 0 \Rightarrow x = 7$

denominator:

$(x+2)(x-1) = 0 \Rightarrow x = -2,\ x = 1$

The intervals to be tested are $(-\infty, -2)$, $(-2, 1)$, $(1, 7]$ and $[7, \infty)$.

Interval	Test point	Value of $R(x)$	Result
$(-\infty, -2)$	–3	$-\frac{5}{2}$	–
$(-2, 1)$	0	$\frac{7}{2}$	+
$(1, 7]$	2	$-\frac{5}{4}$	–
$[7, \infty)$	8	$\frac{1}{70}$	+

Solution set: $(-2, 1) \cup [7, \infty)$

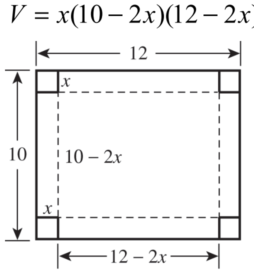
18. B

$$S = \frac{kt^2}{x^3}$$
$$27 = \frac{3^2 k}{1^3} \Rightarrow k = 3$$
$$S = \frac{3(6^2)}{3^3} = 4$$

19. B. The minimum occurs at the x-coordinate of the vertex: $-\frac{-24}{2(1)} = 12.$

20. D.

$V = x(10-2x)(12-2x)$



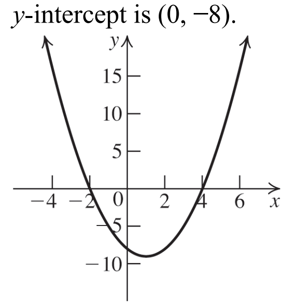
Cumulative Review Exercises Chapters 1–2

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$
$= \sqrt{(-1-2)^2 + (3-5)^2} = \sqrt{13}$

2. $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= \left(\frac{2+(-8)}{2}, \frac{-5+(-3)}{2}\right) = (-3, -4)$

3. $0 = x^2 - 2x - 8 \Rightarrow 0 = (x-4)(x+2) \Rightarrow$
$x - 4 = 0 \Rightarrow x = 4$ or $x + 2 = 0 \Rightarrow x = -2$
$f(0) = 0^2 - 2(0) - 8 = -8$

The x-intercepts are (−2, 0) and (4, 0), and the y-intercept is (0, −8).

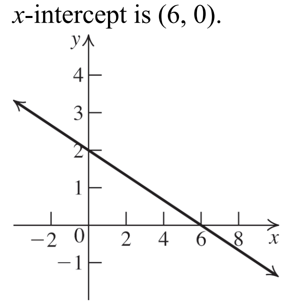


4. Write the equation in slope-intercept form:

$x + 3y - 6 = 0 \Rightarrow 3y = -x + 6 \Rightarrow y = -\frac{1}{3}x + 2$

The slope is $-\frac{1}{3}$, and the y-intercept is (0, 2).

$x + 3(0) - 6 = 0 \Rightarrow x - 6 = 0 \Rightarrow x = 6,$ so the x-intercept is (6, 0).



5. Write the equation in slope-intercept form:

$x = 2y - 6 \Rightarrow x + 6 = 2y \Rightarrow \frac{1}{2}x + 3 = y$

The slope is $\frac{1}{2}$, and the y-intercept is (0, 3).

The x-intercept is $x = 2(0) - 6 = -6$.

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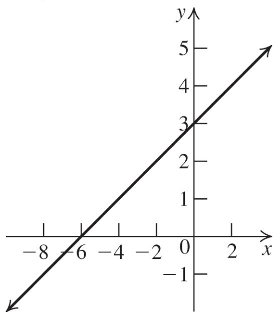


6. $(x-2)^2+(y+3)^2=16$

7. $x^2+y^2+2x-4y-4=0 \Rightarrow$
$(x^2+2x)+(y^2-4y)=4 \Rightarrow$
$(x^2+2x+1)+(y^2-4y+4)=4+1+4 \Rightarrow$
$(x+1)^2+(y-2)^2=9$
The center is (–1, 2). The radius is 3.

8. $y+2=3(x-1) \Rightarrow y=3x-5$

9. $2x+3y=5 \Rightarrow y=-\dfrac{2}{3}x+\dfrac{5}{3} \Rightarrow$ the slope is $-\dfrac{2}{3}$.

$y-3=-\dfrac{2}{3}(x-1) \Rightarrow y=-\dfrac{2}{3}x+\dfrac{11}{3}$.

10. $2x+3=0 \Rightarrow x=-\dfrac{3}{2}$.
The domain is $\left(-\infty,-\dfrac{3}{2}\right)\cup\left(-\dfrac{3}{2},\infty\right)$.

11. $\sqrt{4-2x}=0 \Rightarrow x=2$. The domain is $(-\infty, 2)$.

12. $f(-2)=(-2)^2-2(-2)+3=11;$
$f(3)=3^2-2(3)+3=6;$
$$\begin{aligned}f(x+h)&=(x+h)^2-2(x+h)+3\\&=x^2+2xh+h^2-2x-2h+3\\&=x^2+2(h-1)x+h^2-2h+3\end{aligned}$$
$$\begin{aligned}&\frac{f(x+h)-f(x)}{h}\\&=\frac{(x^2+2(h-1)x+h^2-2h+3)-(x^2-2x+3)}{h}\\&=\frac{2(h-1)x+2x+h^2-2h}{h}\\&=\frac{2hx-2x+2x+h^2-2h}{h}=\frac{h^2+2hx-2h}{h}\\&=h+2x-2\end{aligned}$$

13. a. $f(g(x))=\sqrt{x^2+1}$

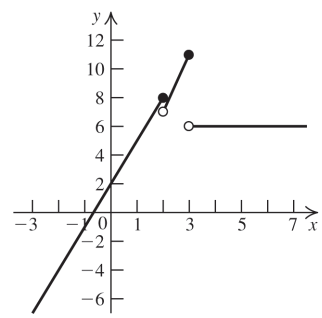
b. $g(f(x))=\left(\sqrt{x}\right)^2+1=x+1$

c. $f(f(x))=\sqrt{\sqrt{x}}=\sqrt[4]{x}$

d. $g(g(x))=\left(x^2+1\right)^2+1=x^4+2x^2+2$

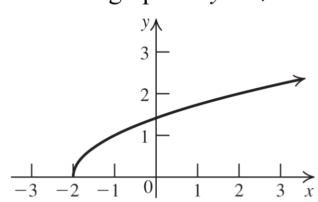
14. a. $f(1)=3(1)+2=5; f(3)=4(3)-1=11;$
$f(4)=6$

b.

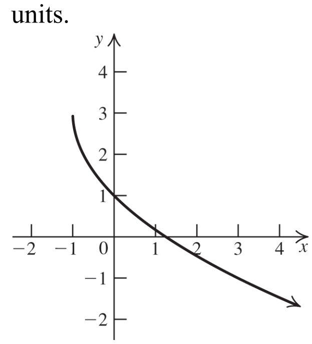


15. $y=2x-3$. Interchange x and y, and then solve for y.
$x=2y-3 \Rightarrow y=\dfrac{x+3}{2}=\dfrac{1}{2}x+\dfrac{3}{2}=f^{-1}(x)$.

16. a. Shift the graph of $y=\sqrt{x}$ two units left.

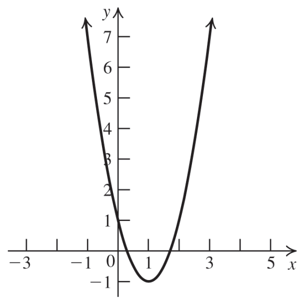


b. Shift the graph of $y=\sqrt{x}$ one unit left, stretch vertically by a factor of two, reflect about the x-axis, and then shift up three units.



17. The factors of the constant term, –6, are $\{\pm1, \pm2, \pm3, \pm6\}$, and the factors of the leading coefficient, 2, are $\{\pm1, \pm2\}$. The possible rational zeros are $\left\{\pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm2, \pm3, \pm6\right\}$.

18. a.



b. $2x^2 - 4x + 1 \ge 0$

Use the quadratic formula to find the zeros.

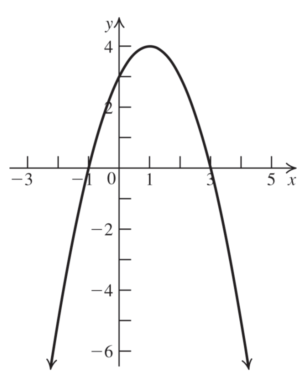
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} = \frac{4 \pm \sqrt{8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} = 1 \pm \frac{\sqrt{2}}{2}$$

The intervals to be tested are $\left(-\infty, 1 - \frac{\sqrt{2}}{2}\right]$, $\left[1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}\right]$, and $\left[1 + \frac{\sqrt{2}}{2}, \infty\right)$.

Interval	Test point	Value of $R(x)$	Result
$\left(-\infty, 1 - \frac{\sqrt{2}}{2}\right]$	0	1	+
$\left[1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}\right]$	1	–1	–
$\left[1 + \frac{\sqrt{2}}{2}, \infty\right)$	2	1	+

Solution set: $\left(-\infty, 1 - \frac{\sqrt{2}}{2}\right] \cup \left[1 + \frac{\sqrt{2}}{2}, \infty\right)$

19. a.

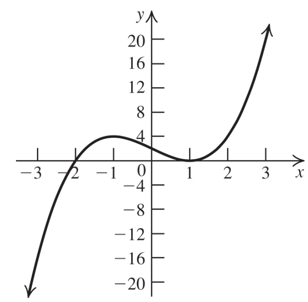


b. $-x^2 + 2x + 3 \ge 0$

$-x^2 + 2x + 3 = 0 \Rightarrow -\left(x^2 - 2x - 3\right) = 0 \Rightarrow$
$-(x+1)(x-3) = 0 \Rightarrow x = -1,\ x = 3$

The intervals to be tested are $(-\infty, -1]$, $[-1, 3]$, and $[3, \infty)$. From the graph in part (a), we see that the solution set is $[-1, 3]$.

20. a.

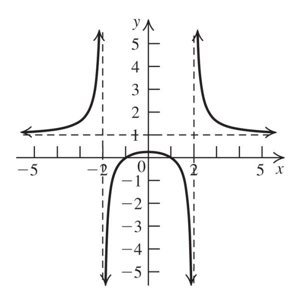


b. $(x-1)^2(x+2) \ge 0$

$(x-1)^2(x+2) = 0 \Rightarrow x = 1, -2$

The intervals to be tested are $(-\infty, -2]$, $[-2, 1]$, and $[1, \infty)$. From the graph in part (a), we see that the solution set is $[-2, 1] \cup [1, \infty) = [2, \infty)$.

21. a.



b. $\frac{x^2 - 1}{x^2 - 4} \ge 0$

numerator: $x^2 - 1 = 0 \Rightarrow x = \pm1$

denominator: $x^2 - 4 = 0 \Rightarrow x = \pm2$

The intervals to be tested are $(-\infty, -2)$, $(-2, -1]$, $[-1, 1]$, $[1, 2)$, and $(2, \infty)$. From the graph in part (a), we see that the solution set is $(-\infty, -2) \cup [-1, 1] \cup (2, \infty)$.

22. Since one zero is $1+i$, another zero is $1-i$. So $(x-(1-i))(x-(1+i)) = x^2-2x+2$ is a factor of $f(x)$. Now divide to find the other factor:

$$\begin{array}{r|l}
 & x^2 - x - 2 \\
x^2-2x+2 & x^4 - 3x^3 + 2x^2 + 2x - 4 \\
 & \underline{x^4 - 2x^3 + 2x^2} \\
 & -x^3 + 0x^2 + 2x \\
 & \underline{-x^3 + 2x^2 - 2x} \\
 & -2x^2 + 4x - 4 \\
 & \underline{-2x^2 + 4x - 4} \\
 & 0
\end{array}$$

$$\begin{aligned} f(x) &= x^4-3x^3+2x^2+2x-4 \\ &= (x-(1-i))(x-(1+i))(x^2-x-2) \\ &= (x-(1-i))(x-(1+i))(x-2)(x+1) \Rightarrow \end{aligned}$$

the zeros of $f(x)$ are $-1, 2, 1-i, 1+i$.

23. $y = k\sqrt{x}; 6 = k\sqrt{4} \Rightarrow k = 3; y = 3\sqrt{9} = 9$

24. Profit = revenue – cost

$$\begin{aligned} &150x - (0.02x^2 + 100x + 3000) \\ &\qquad = -0.02x^2 + 50x - 3000. \end{aligned}$$

The maximum occurs at the vertex:

$$\left(-\frac{50}{2(-0.02)}, f\left(-\frac{50}{2(-0.02)}\right)\right) = (1250,\ \$28,250).$$

25.

$$\begin{array}{r|rrrr}
10 & 0.02 & 48.8 & -2990 & 25,000 \\
 & & 0.2 & 490 & -25,000 \\ \hline
 & 0.02 & 49 & -2500 & 0
\end{array}$$

Now solve the depressed equation $0.02x^2 + 49x - 2500 = 0$.

$$x = \frac{-49 \pm \sqrt{49^2 - 4(0.02)(-2500)}}{2(0.02)}$$

$$= \frac{-49 \pm \sqrt{2601}}{0.04} = \frac{-49 \pm 51}{0.04} \Rightarrow x = 50 \text{ or}$$

$x = -2500$. There cannot be a negative amount of units sold, so another break-even point is 50.