Chapter 2
Polynomial and Rational Functions

Section 2.1

Check Point Exercises

1. a. $(5-2i)+(3+3i)$
$=5-2i+3+3i$
$=(5+3)+(-2+3)i$
$=8+i$

b. $(2+6i)-(12-i)$
$=2+6i-12+i$
$=(2-12)+(6+1)i$
$=-10+7i$

2. a. $7i(2-9i)=7i(2)-7i(9i)$
$=14i-63i^2$
$=14i-63(-1)$
$=63+14i$

b. $(5+4i)(6-7i)=30-35i+24i-28i^2$
$=30-35i+24i-28(-1)$
$=30+28-35i+24i$
$=58-11i$

3. $\dfrac{5+4i}{4-i}=\dfrac{5+4i}{4-i}\cdot\dfrac{4+i}{4+i}$
$=\dfrac{20+5i+16i+4i^2}{16+4i-4i-i^2}$
$=\dfrac{20+21i-4}{16+1}$
$=\dfrac{16+21i}{17}$
$=\dfrac{16}{17}+\dfrac{21}{17}i$

4. a. $\sqrt{-27}+\sqrt{-48}=i\sqrt{27}+i\sqrt{48}$
$=i\sqrt{9\cdot 3}+i\sqrt{16\cdot 3}$
$=3i\sqrt{3}+4i\sqrt{3}$
$=7i\sqrt{3}$

b. $(-2+\sqrt{-3})^2=(-2+i\sqrt{3})^2$
$=(-2)^2+2(-2)(i\sqrt{3})+(i\sqrt{3})^2$
$=4-4i\sqrt{3}+3i^2$
$=4-4i\sqrt{3}+3(-1)$
$=1-4i\sqrt{3}$

c. $\dfrac{-14+\sqrt{-12}}{2}=\dfrac{-14+i\sqrt{12}}{2}$
$=\dfrac{-14+2i\sqrt{3}}{2}$
$=\dfrac{-14}{2}+\dfrac{2i\sqrt{3}}{2}$
$=-7+i\sqrt{3}$

5. $x^2-2x+2=0$
$a=1, b=-2, c=2$
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$
$x=\dfrac{2\pm\sqrt{4-8}}{2}$
$x=\dfrac{2\pm\sqrt{-4}}{2}$
$x=\dfrac{2\pm 2i}{2}$
$x=1\pm i$
The solution set is $\{1+i, 1-i\}$.

Concept and Vocabulary Check 2.1

1. $\sqrt{-1}$; -1

2. complex; imaginary; real

3. $-6i$

4. $14i$

5. 18; $-15i$; $12i$; $-10i^2$; 10

6. $2+9i$

7. $2+5i$

8. i; i; $2i\sqrt{5}$

9. $-1\pm i\dfrac{\sqrt{6}}{2}$

Exercise Set 2.1

1. $(7+2i)+(1-4i) = 7+2i+1-4i$
$= 7+1+2i-4i$
$= 8-2i$

2. $(-2+6i)+(4-i)$
$= -2+6i+4-i$
$= -2+4+6i-i$
$= 2+5i$

3. $(3+2i)-(5-7i) = 3-5+2i+7i$
$= 3+2i-5+7i$
$= -2+9i$

4. $(-7+5i)-(-9-11i) = -7+5i+9+11i$
$= -7+9+5i+11i$
$= 2+16i$

5. $6-(-5+4i)-(-13-i) = 6+5-4i+13+i$
$= 24-3i$

6. $7-(-9+2i)-(-17-i) = 7+9-2i+17+i$
$= 33-i$

7. $8i-(14-9i) = 8i-14+9i$
$= -14+8i+9i$
$= -14+17i$

8. $15i-(12-11i) = 15i-12+11i$
$= -12+15i+11i$
$= -12+26i$

9. $-3i(7i-5) = -21i^2+15i$
$= -21(-1)+15i$
$= 21+15i$

10. $-8i(2i-7) = -16i^2+56i = -16(-1)+56i$
$= 9-25i^2 = 9+25 = 34 = 16+56i$

11. $(-5+4i)(3+i) = -15-5i+12i+4i^2$
$= -15+7i-4$
$= -19+7i$

12. $(-4-8i)(3+i) = -12-4i-24i-8i^2$
$= -12-28i+8$
$= -4-28i$

13. $(7-5i)(-2-3i) = -14-21i+10i+15i^2$
$= -14-15-11i$
$= -29-11i$

14. $(8-4i)(-3+9i) = -24+72i+12i-36i^2$
$= -24+36+84i$
$= 12+84i$

15. $(3+5i)(3-5i) = 9-15i+15i-25i^2$
$= 9+25$
$= 34$

16. $(2+7i)(2-7i) = 4-49i^2 = 4+49 = 53$

17. $(-5+i)(-5-i) = 25+5i-5i-i^2$
$= 25+1$
$= 26$

18. $(-7+i)(-7-i) = 49+7i-7i-i^2$
$= 49+1$
$= 50$

19. $(2+3i)^2 = 4+12i+9i^2$
$= 4+12i-9$
$= -5+12i$

20. $(5-2i)^2 = 25-20i+4i^2$
$= 25-20i-4$
$= 21-20i$

21. $\dfrac{2}{3-i} = \dfrac{2}{3-i}\cdot\dfrac{3+i}{3+i}$
$= \dfrac{2(3+i)}{9+1}$
$= \dfrac{2(3+i)}{10}$
$= \dfrac{3+i}{5}$
$= \dfrac{3}{5}+\dfrac{1}{5}i$

22. $\dfrac{3}{4+i} = \dfrac{3}{4+i}\cdot\dfrac{4-i}{4-i}$
$= \dfrac{3(4-i)}{16-i^2}$
$= \dfrac{3(4-i)}{17}$
$= \dfrac{12}{17}-\dfrac{3}{17}i$

23. $\frac{2i}{1+i} = \frac{2i}{1+i} \cdot \frac{1-i}{1-i} = \frac{2i-2i^2}{1+1} = \frac{2+2i}{2} = 1+i$

24. $\frac{5i}{2-i} = \frac{5i}{2-i} \cdot \frac{2+i}{2+i}$
$= \frac{10i+5i^2}{4+1}$
$= \frac{-5+10i}{5}$
$= -1+2i$

25. $\frac{8i}{4-3i} = \frac{8i}{4-3i} \cdot \frac{4+3i}{4+3i}$
$= \frac{32i+24i^2}{16+9}$
$= \frac{-24+32i}{25}$
$= -\frac{24}{25} + \frac{32}{25}i$

26. $\frac{-6i}{3+2i} = \frac{-6i}{3+2i} \cdot \frac{3-2i}{3-2i} = \frac{-18i+12i^2}{9+4}$
$= \frac{-12-18i}{13} = -\frac{12}{13} - \frac{18}{13}i$

27. $\frac{2+3i}{2+i} = \frac{2+3i}{2+i} \cdot \frac{2-i}{2-i}$
$= \frac{4+4i-3i^2}{4+1}$
$= \frac{7+4i}{5}$
$= \frac{7}{5} + \frac{4}{5}i$

28. $\frac{3-4i}{4+3i} = \frac{3-4i}{4+3i} \cdot \frac{4-3i}{4-3i}$
$= \frac{12-25i+12i^2}{16+9}$
$= \frac{-25i}{25}$
$= -i$

29. $\sqrt{-64} - \sqrt{-25} = i\sqrt{64} - i\sqrt{25}$
$= 8i - 5i = 3i$

30. $\sqrt{-81} - \sqrt{-144} = i\sqrt{81} - i\sqrt{144} = 9i - 12i$
$= -3i$

31. $5\sqrt{-16} + 3\sqrt{-81} = 5(4i) + 3(9i)$
$= 20i + 27i = 47i$

32. $5\sqrt{-8} + 3\sqrt{-18}$
$= 5i\sqrt{8} + 3i\sqrt{18} = 5i\sqrt{4 \cdot 2} + 3i\sqrt{9 \cdot 2}$
$= 10i\sqrt{2} + 9i\sqrt{2}$
$= 19i\sqrt{2}$

33. $\left(-2+\sqrt{-4}\right)^2 = \left(-2+2i\right)^2$
$= 4 - 8i + 4i^2$
$= 4 - 8i - 4$
$= -8i$

34. $\left(-5-\sqrt{-9}\right)^2 = (-5-i\sqrt{9})^2 = \left(-5-3i\right)^2$
$= 25 + 30i + 9i^2$
$= 25 + 30i - 9$
$= 16 + 30i$

35. $\left(-3-\sqrt{-7}\right)^2 = \left(-3-i\sqrt{7}\right)^2$
$= 9 + 6i\sqrt{7} + i^2\left(7\right)$
$= 9 - 7 + 6i\sqrt{7}$
$= 2 + 6i\sqrt{7}$

36. $\left(-2+\sqrt{-11}\right)^2 = \left(-2+i\sqrt{11}\right)^2$
$= 4 - 4i\sqrt{11} + i^2\left(11\right)$
$= 4 - 11 - 4i\sqrt{11}$
$= -7 - 4i\sqrt{11}$

37. $\frac{-8+\sqrt{-32}}{24} = \frac{-8+i\sqrt{32}}{24}$
$= \frac{-8+i\sqrt{16 \cdot 2}}{24}$
$= \frac{-8+4i\sqrt{2}}{24}$
$= -\frac{1}{3} + \frac{\sqrt{2}}{6}i$

38. $\frac{-12+\sqrt{-28}}{32} = \frac{-12+i\sqrt{28}}{32} = \frac{-12+i\sqrt{4 \cdot 7}}{32}$
$= \frac{-12+2i\sqrt{7}}{32} = -\frac{3}{8} + \frac{\sqrt{7}}{16}i$

39. $\frac{-6-\sqrt{-12}}{48} = \frac{-6-i\sqrt{12}}{48}$
$= \frac{-6-i\sqrt{4\cdot 3}}{48}$
$= \frac{-6-2i\sqrt{3}}{48}$
$= -\frac{1}{8}-\frac{\sqrt{3}}{24}i$

40. $\frac{-15-\sqrt{-18}}{33} = \frac{-15-i\sqrt{18}}{33} = \frac{-15-i\sqrt{9\cdot 2}}{33}$
$= \frac{-15-3i\sqrt{2}}{33} = -\frac{5}{11}-\frac{\sqrt{2}}{11}i$

41. $\sqrt{-8}\left(\sqrt{-3}-\sqrt{5}\right) = i\sqrt{8}(i\sqrt{3}-\sqrt{5})$
$= 2i\sqrt{2}\left(i\sqrt{3}-\sqrt{5}\right)$
$= -2\sqrt{6}-2i\sqrt{10}$

42. $\sqrt{-12}\left(\sqrt{-4}-\sqrt{2}\right) = i\sqrt{12}(i\sqrt{4}-\sqrt{2})$
$= 2i\sqrt{3}\left(2i-\sqrt{2}\right)$
$= 4i^2\sqrt{3}-2i\sqrt{6}$
$= -4\sqrt{3}-2i\sqrt{6}$

43. $\left(3\sqrt{-5}\right)\left(-4\sqrt{-12}\right) = \left(3i\sqrt{5}\right)\left(-8i\sqrt{3}\right)$
$= -24i^2\sqrt{15}$
$= 24\sqrt{15}$

44. $\left(3\sqrt{-7}\right)\left(2\sqrt{-8}\right)$
$= (3i\sqrt{7})(2i\sqrt{8}) = (3i\sqrt{7})(2i\sqrt{4\cdot 2})$
$= \left(3i\sqrt{7}\right)\left(4i\sqrt{2}\right) = 12i^2\sqrt{14} = -12\sqrt{14}$

45. $x^2-6x+10=0$
$x = \frac{6\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}$
$x = \frac{6\pm\sqrt{36-40}}{2}$
$x = \frac{6\pm\sqrt{-4}}{2}$
$x = \frac{6\pm 2i}{2}$
$x = 3\pm i$
The solution set is $\{3+i, 3-i\}$.

46. $x^2-2x+17=0$
$x = \frac{2\pm\sqrt{(-2)^2-4(1)(17)}}{2(1)}$
$x = \frac{2\pm\sqrt{4-68}}{2}$
$x = \frac{2\pm\sqrt{-64}}{2}$
$x = \frac{2\pm 8i}{2}$
$x = 1\pm 4i$
The solution set is $\{1+4i, 1-4i\}$.

47. $4x^2+8x+13=0$
$x = \frac{-8\pm\sqrt{8^2-4(4)(13)}}{2(4)}$
$= \frac{-8\pm\sqrt{64-208}}{8}$
$= \frac{-8\pm\sqrt{-144}}{8}$
$= \frac{-8\pm 12i}{8}$
$= \frac{4(-2\pm 3i)}{8}$
$= \frac{-2\pm 3i}{2}$
$= -1\pm\frac{3}{2}i$

The solution set is $\left\{-1+\frac{3}{2}i, -1-\frac{3}{2}i\right\}$.

48. $2x^2+2x+3=0$

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(2)(3)}}{2(2)}=\frac{-2\pm\sqrt{4-24}}{4}=\frac{-2\pm\sqrt{-20}}{4}=\frac{-2\pm 2i\sqrt{5}}{4}=\frac{2(-1\pm i\sqrt{5})}{4}=\frac{-1\pm i\sqrt{5}}{2}=-\frac{1}{2}\pm\frac{\sqrt{5}}{2}i$$

The solution set is $\left\{-\frac{1}{2}+\frac{\sqrt{5}}{2}i, -\frac{1}{2}-\frac{\sqrt{5}}{2}i\right\}$.

49. $3x^2-8x+7=0$

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(3)(7)}}{2(3)}=\frac{8\pm\sqrt{64-84}}{6}=\frac{8\pm\sqrt{-20}}{6}=\frac{8\pm 2i\sqrt{5}}{6}=\frac{2(4\pm i\sqrt{5})}{6}=\frac{4\pm i\sqrt{5}}{3}=\frac{4}{3}\pm\frac{\sqrt{5}}{3}i$$

The solution set is $\left\{\frac{4}{3}+\frac{\sqrt{5}}{3}i, \frac{4}{3}-\frac{\sqrt{5}}{3}i\right\}$.

50. $3x^2-4x+6=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(6)}}{2(3)}=\frac{4\pm\sqrt{16-72}}{6}=\frac{4\pm\sqrt{-56}}{6}=\frac{4\pm 2i\sqrt{14}}{6}=\frac{2(2\pm i\sqrt{14})}{6}=\frac{2\pm i\sqrt{14}}{3}=\frac{2}{3}\pm\frac{\sqrt{14}}{3}i$$

The solution set is $\left\{\frac{2}{3}+\frac{\sqrt{14}}{3}i, \frac{2}{3}-\frac{\sqrt{14}}{3}i\right\}$.

51. $(2-3i)(1-i)-(3-i)(3+i)$
$=(2-2i-3i+3i^2)-(3^2-i^2)$
$=2-5i+3i^2-9+i^2$
$=-7-5i+4i^2$
$=-7-5i+4(-1)$
$=-11-5i$

52. $(8+9i)(2-i)-(1-i)(1+i)$
$=(16-8i+18i-9i^2)-(1^2-i^2)$
$=16+10i-9i^2-1+i^2$
$=15+10i-8i^2$
$=15+10i-8(-1)$
$=23+10i$

53. $(2+i)^2-(3-i)^2$
$=(4+4i+i^2)-(9-6i+i^2)$
$=4+4i+i^2-9+6i-i^2$
$=-5+10i$

54. $(4-i)^2-(1+2i)^2$
$=(16-8i+i^2)-(1+4i+4i^2)$
$=16-8i+i^2-1-4i-4i^2$
$=15-12i-3i^2$
$=15-12i-3(-1)$
$=18-12i$

55. $5\sqrt{-16}+3\sqrt{-81}$
$=5\sqrt{16}\sqrt{-1}+3\sqrt{81}\sqrt{-1}$
$=5\cdot 4i+3\cdot 9i$
$=20i+27i$
$=47i$ or $0+47i$

56. $5\sqrt{-8}+3\sqrt{-18}$
$=5\sqrt{4}\sqrt{2}\sqrt{-1}+3\sqrt{9}\sqrt{2}\sqrt{-1}$
$=5\cdot 2\sqrt{2}\,i+3\cdot 3\sqrt{2}\,i$
$=10i\sqrt{2}+9i\sqrt{2}$
$=(10+9)i\sqrt{2}$
$=19i\sqrt{2}$ or $0+19i\sqrt{2}$

57. $f(x)=x^2-2x+2$
$f(1+i)=(1+i)^2-2(1+i)+2$
$=1+2i+i^2-2-2i+2$
$=1+i^2$
$=1-1$
$=0$

58. $f(x)=x^2-2x+5$
$f(1-2i)=(1-2i)^2-2(1-2i)+5$
$=1-4i+4i^2-2+4i+5$
$=4+4i^2$
$=4-4$
$=0$

59. $f(x)=\dfrac{x^2+19}{2-x}$
$f(3i)=\dfrac{(3i)^2+19}{2-3i}$
$=\dfrac{9i^2+19}{2-3i}$
$=\dfrac{-9+19}{2-3i}$
$=\dfrac{10}{2-3i}$
$=\dfrac{10}{2-3i}\cdot\dfrac{2+3i}{2+3i}$
$=\dfrac{20+30i}{4-9i^2}$
$=\dfrac{20+30i}{4+9}$
$=\dfrac{20+30i}{13}$
$=\dfrac{20}{13}+\dfrac{30}{13}i$

60. $f(x)=\dfrac{x^2+11}{3-x}$
$f(4i)=\dfrac{(4i)^2+11}{3-4i}=\dfrac{16i^2+11}{3-4i}$
$=\dfrac{-16+11}{3-4i}$
$=\dfrac{-5}{3-4i}$
$=\dfrac{-5}{3-4i}\cdot\dfrac{3+4i}{3+4i}$
$=\dfrac{-15-20i}{9-16i^2}$
$=\dfrac{-15-20i}{9+16}$
$=\dfrac{-15-20i}{25}$
$=\dfrac{-15}{25}-\dfrac{20}{25}i$
$=-\dfrac{3}{5}-\dfrac{4}{5}i$

61. $E = IR = (4-5i)(3+7i)$
$= 12 + 28i - 15i - 35i^2$
$= 12 + 13i - 35(-1)$
$= 12 + 35 + 13i = 47 + 13i$
The voltage of the circuit is
$(47 + 13i)$ volts.

62. $E = IR = (2-3i)(3+5i)$
$= 6 + 10i - 9i - 15i^2 = 6 + i - 15(-1)$
$= 6 + i + 15 = 21 + i$
The voltage of the circuit is $(21+i)$ volts.

63. Sum:
$\left(5 + i\sqrt{15}\right) + \left(5 - i\sqrt{15}\right)$
$= 5 + i\sqrt{15} + 5 - i\sqrt{15}$
$= 5 + 5$
$= 10$
Product:
$\left(5 + i\sqrt{15}\right)\left(5 - i\sqrt{15}\right)$
$= 25 - 5i\sqrt{15} + 5i\sqrt{15} - 15i^2$
$= 25 + 15$
$= 40$

64. – 72. Answers will vary.

73. makes sense

74. does not make sense; Explanations will vary. Sample explanation: Imaginary numbers are not undefined.

75. does not make sense; Explanations will vary. Sample explanation: $i = \sqrt{-1}$; It is not a variable in this context.

76. makes sense

77. false; Changes to make the statement true will vary. A sample change is: All irrational numbers are complex numbers.

78. false; Changes to make the statement true will vary. A sample change is: $(3 + 7i)(3 - 7i) = 9 + 49 = 58$ which is a real number.

79. false; Changes to make the statement true will vary. A sample change is:
$$\frac{7+3i}{5+3i} = \frac{7+3i}{5+3i} \cdot \frac{5-3i}{5-3i} = \frac{44-6i}{34} = \frac{22}{17} - \frac{3}{17}i$$

80. true

81.
$$\frac{4}{(2+i)(3-i)} = \frac{4}{6-2i+3i-i^2}$$
$$= \frac{4}{6+i+1}$$
$$= \frac{4}{7+i}$$
$$= \frac{4}{7+i} \cdot \frac{7-i}{7-i}$$
$$= \frac{28-4i}{49-i^2}$$
$$= \frac{28-4i}{49+1}$$
$$= \frac{28-4i}{50}$$
$$= \frac{28}{50} - \frac{4}{50}i$$
$$= \frac{14}{25} - \frac{2}{25}i$$

82.
$$\frac{1+i}{1+2i} + \frac{1-i}{1-2i}$$
$$= \frac{(1+i)(1-2i)}{(1+2i)(1-2i)} + \frac{(1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$= \frac{(1+i)(1-2i) + (1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$= \frac{1-2i+i-2i^2+1+2i-i-2i^2}{1-4i^2}$$
$$= \frac{1-2i+i+2+1+2i-i+2}{1+4}$$
$$= \frac{6}{5}$$
$$= \frac{6}{5} + 0i$$

83. $\dfrac{8}{1+\frac{2}{i}} = \dfrac{8}{\frac{i}{i}+\frac{2}{i}}$

$= \dfrac{8}{\frac{2+i}{i}}$

$= \dfrac{8i}{2+i}$

$= \dfrac{8i}{2+i}\cdot\dfrac{2-i}{2-i}$

$= \dfrac{16i-8i^2}{4-i^2}$

$= \dfrac{16i+8}{4+1}$

$= \dfrac{8+16i}{5}$

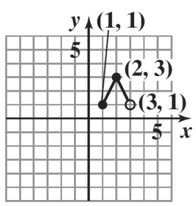
$= \dfrac{8}{5}+\dfrac{16}{5}i$

84. domain: $[0,2)$

range: $[0,2]$

85. $f(x)=1$ at $\frac{1}{2}$ and $\frac{3}{2}$.

86.



g(x) = f(x − 1) + 1

87. $0 = -2(x-3)^2+8$

$2(x-3)^2 = 8$

$(x-3)^2 = 4$

$x-3 = \pm\sqrt{4}$

$x = 3\pm 2$

$x = 1, 5$

88. $-x^2-2x+1=0$

$x^2+2x-1=0$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

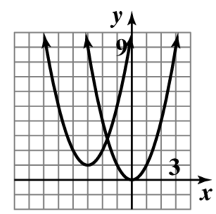
$= \dfrac{2\pm\sqrt{8}}{2}$

$= \dfrac{2\pm 2\sqrt{2}}{2}$

$= 1\pm\sqrt{2}$

The solution set is $\{1\pm\sqrt{2}\}$.

89. The graph of g is the graph of f shifted 1 unit up and 3 units to the left.



$f(x) = x^2$
$g(x) = (x+3)^2 + 1$

Section 2.2

Check Point Exercises

1. $f(x) = -(x-1)^2 + 4$

$f(x) = \underbrace{-}_{a=-1}\left(x - \underbrace{1}_{h=1}\right)^2 + \underbrace{4}_{k=4}$

Step 1: The parabola opens down because $a < 0$.
Step 2: find the vertex: (1, 4)
Step 3: find the x-intercepts:

$0 = -(x-1)^2 + 4$

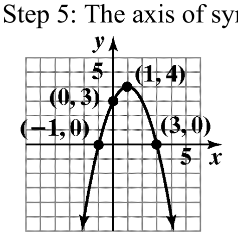
$(x-1)^2 = 4$
$x - 1 = \pm 2$
$x = 1 \pm 2$

$x = 3$ or $x = -1$

Step 4: find the y-intercept:

$f(0) = -(0-1)^2 + 4 = 3$

Step 5: The axis of symmetry is $x = 1$.



$f(x) = -(x-1)^2 + 4$

2. $f(x) = (x-2)^2 + 1$

Step 1: The parabola opens up because $a > 0$.
Step 2: find the vertex: (2, 1)
Step 3: find the x-intercepts:

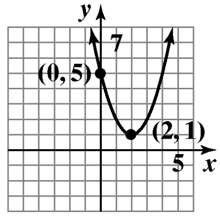
$0 = (x-2)^2 + 1$

$(x-2)^2 = -1$
$x - 2 = \sqrt{-1}$
$x = 2 \pm i$

The equation has no real roots, thus the parabola has no x-intercepts.
Step 4: find the y-intercept:

$f(0) = (0-2)^2 + 1 = 5$

Step 5: The axis of symmetry is $x = 2$.



$f(x) = (x-2)^2 + 1$

3. $f(x) = -x^2 + 4x + 1$

Step 1: The parabola opens down because $a < 0$.
Step 2: find the vertex:

$x = -\dfrac{b}{2a} = -\dfrac{4}{2(-1)} = 2$

$f(2) = -2^2 + 4(2) + 1 = 5$

The vertex is (2, 5).
Step 3: find the x-intercepts:

$0 = -x^2 + 4x + 1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(-1)(1)}}{2(-1)}$

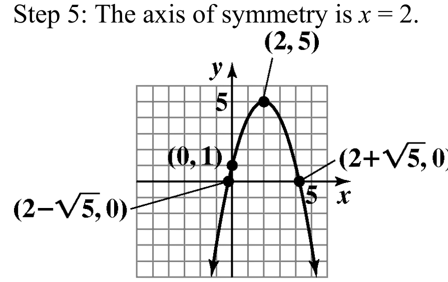
$x = \dfrac{-4 \pm \sqrt{20}}{-2}$

$x = 2 \pm \sqrt{5}$

The x-intercepts are $x \approx -0.2$ and $x \approx -4.2$.

Step 4: find the y-intercept: $f(0) = -0^2 + 4(0) + 1 = 1$

Step 5: The axis of symmetry is $x = 2$.



$f(x) = -x^2 + 4x + 1$

4. $f(x) = 4x^2 - 16x + 1000$

a. $a = 4$. The parabola opens upward and has a minimum value.

b. $x = \dfrac{-b}{2a} = \dfrac{16}{8} = 2$

$f(2) = 4(2)^2 - 16(2) + 1000 = 984$

The minimum point is 984 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[984, \infty)$

5. $f(x) = -0.005x^2 + 2x + 5$

a. The information needed is found at the vertex.
x-coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-2}{2(-0.005)} = 200$$

y-coordinate of vertex

$$y = -0.005(200)^2 + 2(200) + 5 = 205$$

The vertex is (200,205).
The maximum height of the arrow is 205 feet.
This occurs 200 feet from its release.

b. The arrow will hit the ground when the height reaches 0.

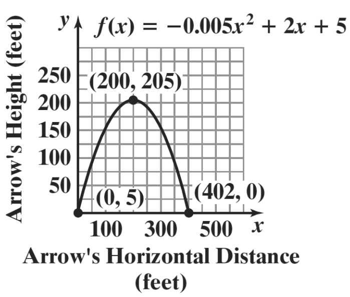
$$f(x) = -0.005x^2 + 2x + 5$$
$$0 = -0.005x^2 + 2x + 5$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(-0.005)(5)}}{2(-0.005)}$$
$$x \approx -2 \text{ or } x \approx 402$$

The arrow travels 402 feet before hitting the ground.

c. The starting point occurs when $x = 0$. Find the corresponding y-coordinate.

$$f(x) = -0.005(0)^2 + 2(0) + 5 = 5$$

Plot $(0,5)$, $(402,0)$, and $(200,205)$, and connect them with a smooth curve.



6. Let x = one of the numbers;
$x-8$ = the other number.
The product is $f(x) = x(x-8) = x^2 - 8x$
The x-coordinate of the minimum is

$$x = -\frac{b}{2a} = -\frac{-8}{2(1)} = -\frac{-8}{2} = 4.$$

$$f(4) = (4)^2 - 8(4)$$
$$= 16 - 32 = -16$$

The vertex is $(4,-16)$.
The minimum product is -16. This occurs when the two numbers are 4 and $4-8=-4$.

7. Maximize the area of a rectangle constructed with 120 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 2y = 120$$
$$2y = 120 - 2x$$
$$y = \frac{120 - 2x}{2} = 60 - x$$

We need to maximize $A = xy = x(60-x)$. Rewrite A as a function of x.

$$A(x) = x(60-x) = -x^2 + 60x$$

Since $a = -1$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{60}{2(-1)} = -\frac{60}{-2} = 30.$$

When the length x is 30, the width y is
$y = 60 - x = 60 - 30 = 30.$
The dimensions of the rectangular region with maximum area are 30 feet by 30 feet. This gives an area of $30 \cdot 30 = 900$ square feet.

Concept and Vocabulary Check 2.2

1. standard; parabola; (h,k); >0; <0

2. $-\frac{b}{2a}$; $f\left(-\frac{b}{2a}\right)$; $-\frac{b}{2a}$; $f\left(-\frac{b}{2a}\right)$

3. true

4. false

5. true

6. $x-8$; x^2-8x

7. $40-x$; $-x^2+40x$

Exercise Set 2.2

1. vertex: (1, 1)
$h(x) = (x-1)^2 + 1$

2. vertex: (–1, 1)
$g(x) = (x+1)^2 + 1$

3. vertex: (1, –1)
$j(x) = (x-1)^2 - 1$

4. vertex: (–1, –1)
$f(x) = (x+1)^2 - 1$

5. The graph is $f(x) = x^2$ translated down one.
$h(x) = x^2 - 1$

6. The point (–1, 0) is on the graph and $f(-1) = 0$. $f(x) = x^2 + 2x + 1$

7. The point (1, 0) is on the graph and $g(1) = 0$. $g(x) = x^2 - 2x + 1$

8. The graph is $f(x) = -x^2$ translated down one.
$j(x) = -x^2 - 1$

9. $f(x) = 2(x-3)^2 + 1$
$h = 3, k = 1$
The vertex is at (3, 1).

10. $f(x) = -3(x-2)^2 + 12$
$h = 2, k = 12$
The vertex is at (2, 12).

11. $f(x) = -2(x+1)^2 + 5$
$h = -1, k = 5$
The vertex is at (–1, 5).

12. $f(x) = -2(x+4)^2 - 8$
$h = -4, k = -8$
The vertex is at (–4, –8).

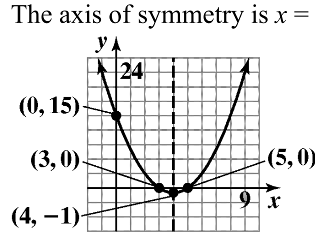
13. $f(x) = 2x^2 - 8x + 3$
$x = \frac{-b}{2a} = \frac{8}{4} = 2$
$f(2) = 2(2)^2 - 8(2) + 3$
$= 8 - 16 + 3 = -5$
The vertex is at (2, –5).

14. $f(x) = 3x^2 - 12x + 1$
$x = \frac{-b}{2a} = \frac{12}{6} = 2$
$f(2) = 3(2)^2 - 12(2) + 1$
$= 12 - 24 + 1 = -11$
The vertex is at (2, –11).

15. $f(x) = -x^2 - 2x + 8$
$x = \frac{-b}{2a} = \frac{2}{-2} = -1$
$f(-1) = -(-1)^2 - 2(-1) + 8$
$= -1 + 2 + 8 = 9$
The vertex is at (–1, 9).

16. $f(x) = -2x^2 + 8x - 1$
$x = \frac{-b}{2a} = \frac{-8}{-4} = 2$
$f(2) = -2(2)^2 + 8(2) - 1$
$= -8 + 16 - 1 = 7$
The vertex is at (2, 7).

17. $f(x) = (x-4)^2 - 1$
vertex: (4, –1)
x-intercepts:
$0 = (x-4)^2 - 1$
$1 = (x-4)^2$
$\pm 1 = x - 4$
$x = 3$ or $x = 5$
y-intercept:
$f(0) = (0-4)^2 - 1 = 15$
The axis of symmetry is $x = 4$.



$f(x) = (x - 4)^2 - 1$

domain: $(-\infty, \infty)$
range: $[-1, \infty)$

18. $f(x)=(x-1)^2-2$

vertex: (1, –2)
x-intercepts:

$0=(x-1)^2-2$

$(x-1)^2=2$

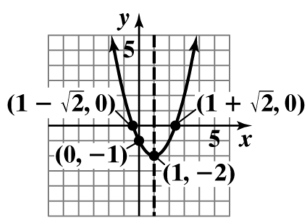
$x-1=\pm\sqrt{2}$

$x=1\pm\sqrt{2}$

y-intercept:

$f(0)=(0-1)^2-2=-1$

The axis of symmetry is $x = 1$.



$f(x) = (x - 1)^2 - 2$

domain: $(-\infty,\infty)$

range: $[-2,\infty)$

19. $f(x)=(x-1)^2+2$

vertex: (1, 2)
x-intercepts:

$0=(x-1)^2+2$

$(x-1)^2=-2$

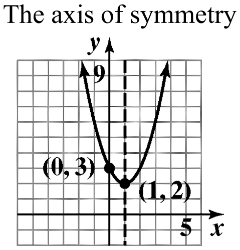
$x-1=\pm\sqrt{-2}$

$x=1\pm i\sqrt{2}$

No x-intercepts.
y-intercept:

$f(0)=(0-1)^2+2=3$

The axis of symmetry is $x = 1$.



$f(x) = (x - 1)^2 + 2$

domain: $(-\infty,\infty)$

range: $[2,\infty)$

20. $f(x)=(x-3)^2+2$

vertex: (3, 2)
x-intercepts:

$0=(x-3)^2+2$

$(x-3)^2=-2$

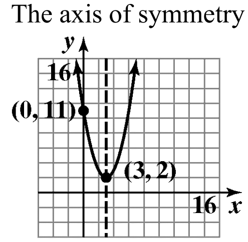
$x-3=\pm i\sqrt{2}$

$x=3\pm i\sqrt{2}$

No x-intercepts.
y-intercept:

$f(0)=(0-3)^2+2=11$

The axis of symmetry is $x = 3$.



$f(x) = (x - 3)^2 + 2$

domain: $(-\infty,\infty)$

range: $[2,\infty)$

21. $y-1=(x-3)^2$

$y=(x-3)^2+1$

vertex: (3, 1)
x-intercepts:

$0=(x-3)^2+1$

$(x-3)^2=-1$

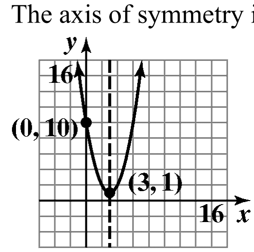
$x-3=\pm i$

$x=3\pm i$

No x-intercepts.
y-intercept: 10

$y=(0-3)^2+1=10$

The axis of symmetry is $x = 3$.



$y - 1 = (x - 3)^2$

domain: $(-\infty,\infty)$

range: $[1,\infty)$

22. $y-3=(x-1)^2$

$y=(x-1)^2+3$

vertex: (1, 3)

x-intercepts:

$0=(x-1)^2+3$

$(x-1)^2=-3$

$x-1=\pm i\sqrt{3}$

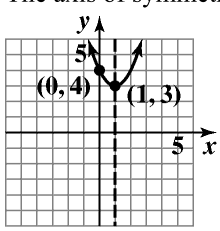
$x=1\pm i\sqrt{3}$

No x-intercepts

y-intercept:

$y=(0-1)^2+3=4$

The axis of symmetry is $x = 1$.



$y - 3 = (x - 1)^2$

domain: $(-\infty,\infty)$

range: $[3,\infty)$

23. $f(x)=2(x+2)^2-1$

vertex: (–2, –1)

x-intercepts:

$0=2(x+2)^2-1$

$2(x+2)^2=1$

$(x+2)^2=\frac{1}{2}$

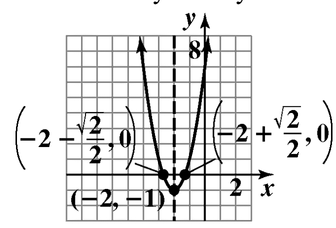
$x+2=\pm\frac{1}{\sqrt{2}}$

$x=-2\pm\frac{1}{\sqrt{2}}=-2\pm\frac{\sqrt{2}}{2}$

y-intercept:

$f(0)=2(0+2)^2-1=7$

The axis of symmetry is $x = -2$.



$f(x) = 2(x + 2)^2 - 1$

domain: $(-\infty,\infty)$

range: $[-1,\infty)$

24. $f(x)=\frac{5}{4}-\left(x-\frac{1}{2}\right)^2$

$f(x)=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

vertex: $\left(\frac{1}{2},\frac{5}{4}\right)$

x-intercepts:

$0=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

$\left(x-\frac{1}{2}\right)^2=\frac{5}{4}$

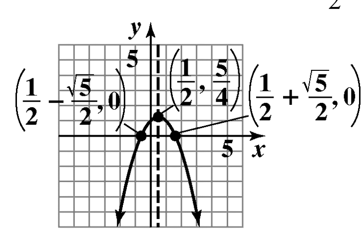
$x-\frac{1}{2}=\pm\frac{\sqrt{5}}{2}$

$x=\frac{1\pm\sqrt{5}}{2}$

y-intercept:

$f(0)=-\left(0-\frac{1}{2}\right)^2+\frac{5}{4}=1$

The axis of symmetry is $x=\frac{1}{2}$.



$f(x) = \frac{5}{4} - \left(x - \frac{1}{2}\right)^2$

domain: $(-\infty,\infty)$

range: $\left(-\infty,\frac{5}{4}\right]$

25. $f(x) = 4 - (x-1)^2$

$f(x) = -(x-1)^2 + 4$

vertex: (1, 4)

x-intercepts:

$0 = -(x-1)^2 + 4$

$(x-1)^2 = 4$

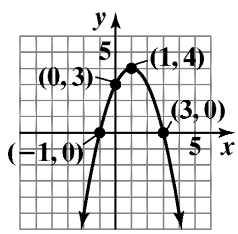
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept:

$f(x) = -(0-1)^2 + 4 = 3$

The axis of symmetry is $x = 1$.



$f(x) = 4 - (x-1)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 4]$

26. $f(x) = 1 - (x-3)^2$

$f(x) = -(x-3)^2 + 1$

vertex: (3, 1)

x-intercepts:

$0 = -(x-3)^2 + 1$

$(x-3)^2 = 1$

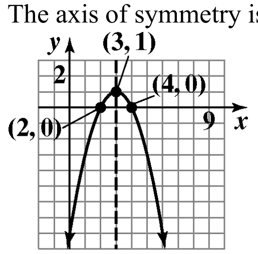
$x - 3 = \pm 1$

$x = 2$ or $x = 4$

y-intercept:

$f(0) = -(0-3)^2 + 1 = -8$

The axis of symmetry is $x = 3$.



$f(x) = 1 - (x-3)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 1]$

27. $f(x) = x^2 - 2x - 3$

$f(x) = \left(x^2 - 2x + 1\right) - 3 - 1$

$f(x) = (x-1)^2 - 4$

vertex: (1, –4)

x-intercepts:

$0 = (x-1)^2 - 4$

$(x-1)^2 = 4$

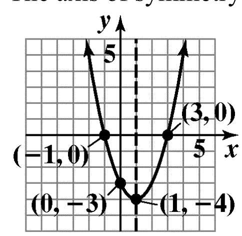
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept: –3

$f(0) = 0^2 - 2(0) - 3 = -3$

The axis of symmetry is $x = 1$.



$f(x) = x^2 + 3x - 10$

domain: $(-\infty, \infty)$

range: $[-4, \infty)$

28. $f(x) = x^2 - 2x - 15$

$f(x) = \left(x^2 - 2x + 1\right) - 15 - 1$

$f(x) = (x-1)^2 - 16$

vertex: (1, –16)

x-intercepts:

$0 = (x-1)^2 - 16$

$(x-1)^2 = 16$

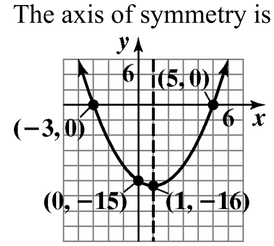
$x - 1 = \pm 4$

$x = -3$ or $x = 5$

y-intercept:

$f(0) = 0^2 - 2(0) - 15 = -15$

The axis of symmetry is $x = 1$.



$f(x) = x^2 - 2x - 15$

domain: $(-\infty, \infty)$

range: $[-16, \infty)$

29. $f(x) = x^2 + 3x - 10$

$f(x) = \left(x^2 + 3x + \frac{9}{4}\right) - 10 - \frac{9}{4}$

$f(x) = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

vertex: $\left(-\frac{3}{2}, -\frac{49}{4}\right)$

x-intercepts:

$0 = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{49}{4}$

$x + \frac{3}{2} = \pm\frac{7}{2}$

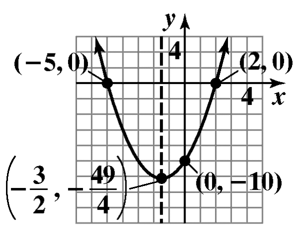
$x = -\frac{3}{2} \pm \frac{7}{2}$

$x = 2$ or $x = -5$

y-intercept:

$f(x) = 0^2 + 3(0) - 10 = -10$

The axis of symmetry is $x = -\frac{3}{2}$.



$f(x) = x^2 + 3x - 10$

domain: $(-\infty, \infty)$

range: $\left[-\frac{49}{4}, \infty\right)$

30. $f(x) = 2x^2 - 7x - 4$

$f(x) = 2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - 4 - \frac{49}{8}$

$f(x) = 2\left(x - \frac{7}{4}\right)^2 - \frac{81}{8}$

vertex: $\left(\frac{7}{4}, -\frac{81}{8}\right)$

x-intercepts:

$0 = 2\left(x - \frac{7}{4}\right)^2 - \frac{81}{8}$

$2\left(x - \frac{7}{4}\right)^2 = \frac{81}{8}$

$\left(x - \frac{7}{4}\right)^2 = \frac{81}{16}$

$x - \frac{7}{4} = \pm\frac{9}{4}$

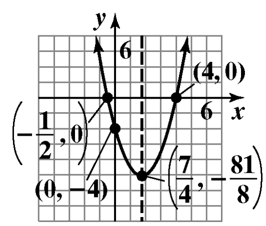
$x = \frac{7}{4} \pm \frac{9}{4}$

$x = -\frac{1}{2}$ or $x = 4$

y-intercept:

$f(0) = 2(0)^2 - 7(0) - 4 = -4$

The axis of symmetry is $x = \frac{7}{4}$.



$f(x) = 2x^2 - 7x - 4$

domain: $(-\infty, \infty)$

range: $\left[-\frac{81}{8}, \infty\right)$

31. $f(x) = 2x - x^2 + 3$

$f(x) = -x^2 + 2x + 3$

$f(x) = -\left(x^2 - 2x + 1\right) + 3 + 1$

$f(x) = -(x-1)^2 + 4$

vertex: (1, 4)

x-intercepts:

$0 = -(x-1)^2 + 4$

$(x-1)^2 = 4$

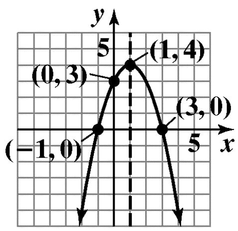
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept:

$f(0) = 2(0) - (0)^2 + 3 = 3$

The axis of symmetry is $x = 1$.



$f(x) = 2x - x^2 + 3$

domain: $(-\infty, \infty)$

range: $(-\infty, 4]$

32. $f(x) = 5 - 4x - x^2$

$f(x) = -x^2 - 4x + 5$

$f(x) = -\left(x^2 + 4x + 4\right) + 5 + 4$

$f(x) = -(x+2)^2 + 9$

vertex: (–2, 9)

x-intercepts:

$0 = -(x+2)^2 + 9$

$(x+2)^2 = 9$

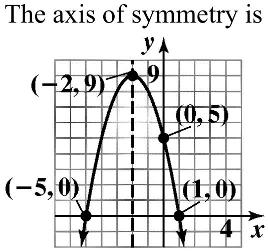
$x + 2 = \pm 3$

$x = -5, 1$

y-intercept:

$f(0) = 5 - 4(0) - (0)^2 = 5$

The axis of symmetry is $x = -2$.



$f(x) = 5 - 4x - x^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 9]$

33. $f(x) = x^2 + 6x + 3$

$f(x) = (x^2 + 6x + 9) + 3 - 9$

$f(x) = (x+3)^2 - 6$

vertex: $(-3, -6)$

x-intercepts:

$0 = (x+3)^2 - 6$

$(x+3)^2 = 6$

$x + 3 = \pm\sqrt{6}$

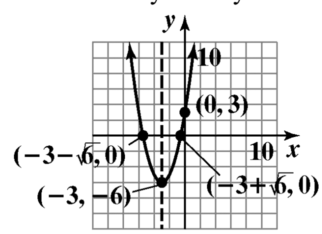
$x = -3 \pm \sqrt{6}$

y-intercept:

$f(0) = (0)^2 + 6(0) + 3$

$f(0) = 3$

The axis of symmetry is $x = -3$.



$f(x) = x^2 + 6x + 3$

domain: $(-\infty, \infty)$

range: $[-6, \infty)$

34. $f(x) = x^2 + 4x - 1$

$f(x) = (x^2 + 4x + 4) - 1 - 4$

$f(x) = (x+2)^2 - 5$

vertex: $(-2, -5)$

x-intercepts:

$0 = (x+2)^2 - 5$

$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

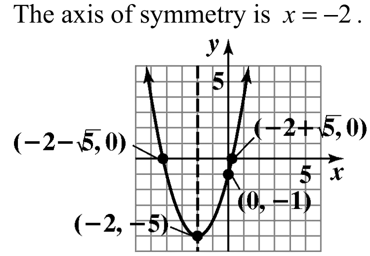
$x = -2 \pm \sqrt{5}$

y-intercept:

$f(0) = (0)^2 + 4(0) - 1$

$f(0) = -1$

The axis of symmetry is $x = -2$.



$f(x) = x^2 + 4x - 1$

domain: $(-\infty, \infty)$

range: $[-5, \infty)$

35. $f(x)=2x^2+4x-3$
$f(x)=2(x^2+2x\quad)-3$
$f(x)=2(x^2+2x+1)-3-2$
$f(x)=2(x+1)^2-5$

vertex: $(-1,-5)$

x-intercepts:

$0=2(x+1)^2-5$

$2(x+1)^2=5$

$(x+1)^2=\frac{5}{2}$

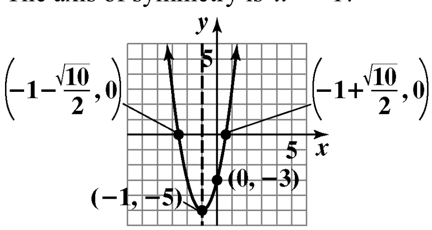
$x+1=\pm\sqrt{\frac{5}{2}}$

$x=-1\pm\frac{\sqrt{10}}{2}$

y-intercept:

$f(0)=2(0)^2+4(0)-3$
$f(0)=-3$

The axis of symmetry is $x=-1$.



$f(x) = 2x^2 + 4x - 3$

domain: $(-\infty,\infty)$

range: $[-5,\infty)$

36. $f(x)=3x^2-2x-4$

$f(x)=3\left(x^2-\frac{2}{3}x\quad\right)-4$

$f(x)=3\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)-4-\frac{1}{3}$

$f(x)=3\left(x-\frac{1}{3}\right)^2-\frac{13}{3}$

vertex: $\left(\frac{1}{3},-\frac{13}{3}\right)$

x-intercepts:

$0=3\left(x-\frac{1}{3}\right)^2-\frac{13}{3}$

$3\left(x-\frac{1}{3}\right)^2=\frac{13}{3}$

$\left(x-\frac{1}{3}\right)^2=\frac{13}{9}$

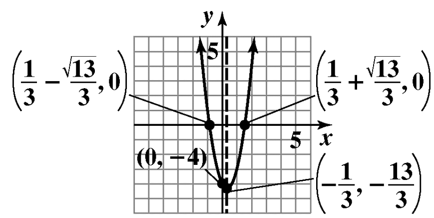
$x-\frac{1}{3}=\pm\sqrt{\frac{13}{9}}$

$x=\frac{1}{3}\pm\frac{\sqrt{13}}{3}$

y-intercept:

$f(0)=3(0)^2-2(0)-4$
$f(0)=-4$

The axis of symmetry is $x=\frac{1}{3}$.



$f(x) = 3x^2 - 2x - 4$

domain: $(-\infty,\infty)$

range: $\left[-\frac{13}{3},\infty\right)$

37. $f(x)=2x-x^2-2$
$f(x)=-x^2+2x-2$
$f(x)=-\left(x^2-2x+1\right)-2+1$
$f(x)=-(x-1)^2-1$

vertex: (1, –1)
x-intercepts:

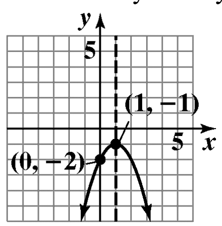
$0=-(x-1)^2-1$

$(x-1)^2=-1$

$x-1=\pm i$
$x=1\pm i$
No x-intercepts.
y-intercept:

$f(0)=2(0)-(0)^2-2=-2$

The axis of symmetry is $x = 1$.



$f(x) = 2x - x^2 - 2$

domain: $(-\infty, \infty)$

range: $(-\infty, -1]$

38. $f(x) = 6 - 4x + x^2$

$f(x) = x^2 - 4x + 6$

$f(x) = (x^2 - 4x + 4) + 6 - 4$

$f(x) = (x-2)^2 + 2$

vertex: (2, 2)

x-intercepts:

$0 = (x-2)^2 + 2$

$(x-2)^2 = -2$

$x - 2 = \pm i\sqrt{2}$

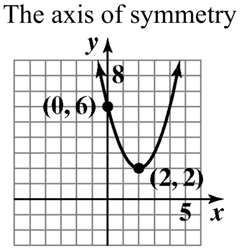
$x = 2 \pm i\sqrt{2}$

No x-intercepts

y-intercept:

$f(0) = 6 - 4(0) + (0)^2 = 6$

The axis of symmetry is $x = 2$.



$f(x) = 6 - 4x + x^2$

domain: $(-\infty, \infty)$

range: $[2, \infty)$

39. $f(x) = 3x^2 - 12x - 1$

a. $a = 3$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{12}{6} = 2$

$f(2) = 3(2)^2 - 12(2) - 1 = 12 - 24 - 1 = -13$

The minimum is -13 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-13, \infty)$

40. $f(x) = 2x^2 - 8x - 3$

a. $a = 2$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{8}{4} = 2$

$f(2) = 2(2)^2 - 8(2) - 3$

$= 8 - 16 - 3 = -11$

The minimum is -11 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-11, \infty)$

41. $f(x) = -4x^2 + 8x - 3$

a. $a = -4$. The parabola opens downward and has a maximum value.

b. $x = \frac{-b}{2a} = \frac{-8}{-8} = 1$

$f(1) = -4(1)^2 + 8(1) - 3$

$= -4 + 8 - 3 = 1$

The maximum is 1 at $x = 1$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 1]$

42. $f(x) = -2x^2 - 12x + 3$

a. $a = -2$. The parabola opens downward and has a maximum value.

b. $x = \frac{-b}{2a} = \frac{12}{-4} = -3$

$f(-3) = -2(-3)^2 - 12(-3) + 3$

$= -18 + 36 + 3 = 21$

The maximum is 21 at $x = -3$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 21]$

43. $f(x) = 5x^2 - 5x$

a. $a = 5$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{5}{10} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 5\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right)$

$= \frac{5}{4} - \frac{5}{2} = \frac{5}{4} - \frac{10}{4} = \frac{-5}{4}$

The minimum is $\frac{-5}{4}$ at $x = \frac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\frac{-5}{4}, \infty\right)$

44. $f(x) = 6x^2 - 6x$

a. $a = 6$. The parabola opens upward and has minimum value.

b. $x = \frac{-b}{2a} = \frac{6}{12} = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right)$$
$$= \frac{6}{4} - 3 = \frac{3}{2} - \frac{6}{2} = \frac{-3}{2}$$

The minimum is $\frac{-3}{2}$ at $x = \frac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\frac{-3}{2}, \infty\right)$

45. Since the parabola opens up, the vertex $(-1, -2)$ is a minimum point.
domain: $(-\infty, \infty)$. range: $[-2, \infty)$

46. Since the parabola opens down, the vertex $(-3, -4)$ is a maximum point.
domain: $(-\infty, \infty)$. range: $(-\infty, -4]$

47. Since the parabola has a maximum, it opens down from the vertex $(10, -6)$.
domain: $(-\infty, \infty)$. range: $(-\infty, -6]$

48. Since the parabola has a minimum, it opens up from the vertex $(-6, 18)$.
domain: $(-\infty, \infty)$. range: $[18, \infty)$

49. $(h, k) = (5, 3)$

$$f(x) = 2(x-h)^2 + k = 2(x-5)^2 + 3$$

50. $(h, k) = (7, 4)$

$$f(x) = 2(x-h)^2 + k = 2(x-7)^2 + 4$$

51. $(h, k) = (-10, -5)$

$$f(x) = 2(x-h)^2 + k$$
$$= 2[x-(-10)]^2 + (-5)$$
$$= 2(x+10)^2 - 5$$

52. $(h, k) = (-8, -6)$

$$f(x) = 2(x-h)^2 + k$$
$$= 2[x-(-8)]^2 + (-6)$$
$$= 2(x+8)^2 - 6$$

53. Since the vertex is a maximum, the parabola opens down and $a = -3$.
$(h, k) = (-2, 4)$

$$f(x) = -3(x-h)^2 + k$$
$$= -3[x-(-2)]^2 + 4$$
$$= -3(x+2)^2 + 4$$

54. Since the vertex is a maximum, the parabola opens down and $a = -3$.
$(h, k) = (5, -7)$

$$f(x) = -3(x-h)^2 + k$$
$$= -3(x-5)^2 + (-7)$$
$$= -3(x-5)^2 - 7$$

55. Since the vertex is a minimum, the parabola opens up and $a = 3$.
$(h, k) = (11, 0)$

$$f(x) = 3(x-h)^2 + k$$
$$= 3(x-11)^2 + 0$$
$$= 3(x-11)^2$$

56. Since the vertex is a minimum, the parabola opens up and $a = 3$.
$(h, k) = (9, 0)$

$$f(x) = 3(x-h)^2 + k$$
$$= 3(x-9)^2 + 0$$
$$= 3(x-9)^2$$

57. **a.** $y = -0.01x^2 + 0.7x + 6.1$
$a = -0.01,\ b = 0.7,\ c = 6.1$

x-coordinate of vertex

$$= \frac{-b}{2a} = \frac{-0.7}{2(-0.01)} = 35$$

y-coordinate of vertex

$$y = -0.01x^2 + 0.7x + 6.1$$
$$y = -0.01(35)^2 + 0.7(35) + 6.1 = 18.35$$

The maximum height of the shot is about 18.35 feet. This occurs 35 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$y = -0.01x^2 + 0.7x + 6.1$
$0 = -0.01x^2 + 0.7x + 6.1$
$a = -0.01,\ b = 0.7,\ c = 6.1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-0.7 \pm \sqrt{0.7^2 - 4(-0.01)(6.1)}}{2(-0.01)}$$
$x \approx 77.8$ or $x \approx -7.8$

The maximum horizontal distance is 77.8 feet.

c. The initial height can be found at $x = 0$.

$y = -0.01x^2 + 0.7x + 6.1$
$y = -0.01(0)^2 + 0.7(0) + 6.1 = 6.1$

The shot was released at a height of 6.1 feet.

58. **a.** $y = -0.04x^2 + 2.1x + 6.1$
$a = -0.04,\ b = 2.1,\ c = 6.1$

x-coordinate of vertex

$$= \frac{-b}{2a} = \frac{-2.1}{2(-0.04)} = 26.25$$

y-coordinate of vertex

$y = -0.04x^2 + 2.1x + 6.1$
$y = -0.04(26.25)^2 + 2.1(26.25) + 6.1 \approx 33.7$

The maximum height of the shot is about 33.7 feet. This occurs 26.25 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$y = -0.04x^2 + 2.1x + 6.1$
$0 = -0.04x^2 + 2.1x + 6.1$
$a = -0.04,\ b = 2.1,\ c = 6.1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2.1 \pm \sqrt{2.1^2 - 4(-0.04)(6.1)}}{2(-0.04)}$$
$x \approx 55.3$ or $x \approx -2.8$

The maximum horizontal distance is 55.3 feet.

c. The initial height can be found at $x = 0$.

$y = -0.04x^2 + 2.1x + 6.1$
$y = -0.04(0)^2 + 2.1(0) + 6.1 = 6.1$

The shot was released at a height of 6.1 feet.

59. $y = -0.8x^2 + 2.4x + 6$

a. The information needed is found at the vertex.
x-coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-2.4}{2(-0.8)} = 1.5$$

y-coordinate of vertex

$y = -0.8(1.5)^2 + 2.4(1.5) + 6 = 7.8$

The vertex is (1.5, 7.8).
The maximum height of the ball is 7.8 feet.
This occurs 1.5 feet from its release.

b. The ball will hit the ground when the height reaches 0.

$y = -0.8x^2 + 2.4x + 6$
$0 = -0.8x^2 + 2.4x + 6$

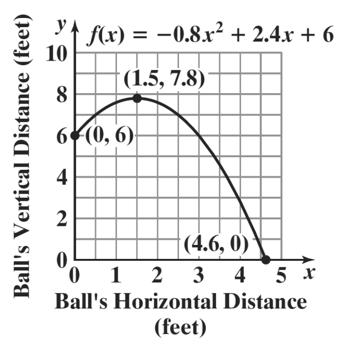
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2.4 \pm \sqrt{2.4^2 - 4(-0.8)(6)}}{2(-0.8)}$$
$x \approx -1.6$ or $x \approx 4.6$

The ball travels 4.6 feet before hitting the ground.

c. The starting point occurs when $x = 0$. Find the corresponding y-coordinate.

$y = -0.8(0)^2 + 2.4(0) + 6 = 6$

Plot $(0,6)$, $(1.5,7.8)$, and $(4.7,0)$, and connect them with a smooth curve.



60. $y = -0.8x^2 + 3.2x + 6$

a. The information needed is found at the vertex.
x-coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-3.2}{2(-0.8)} = 2$$

y-coordinate of vertex

$y = -0.8(2)^2 + 3.2(2) + 6 = 9.2$

The vertex is (2, 9.2).
The maximum height of the ball is 9.2 feet.
This occurs 2 feet from its release.

b. The ball will hit the ground when the height reaches 0.

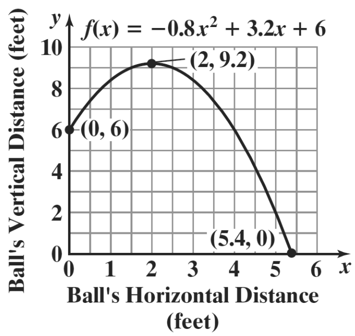
$$y = -0.8x^2 + 3.2x + 6$$
$$0 = -0.8x^2 + 3.2x + 6$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-3.2 \pm \sqrt{3.2^2 - 4(-0.8)(6)}}{2(-0.8)}$$
$$x \approx -1.4 \text{ or } x \approx 5.4$$

The ball travels 5.4 feet before hitting the ground.

c. The starting point occurs when $x = 0$. Find the corresponding y-coordinate.

$$y = -0.8(0)^2 + 3.2(0) + 6 = 6$$

Plot $(0,6)$, $(2,9.2)$, and $(5.4,0)$, and connect them with a smooth curve.



61. Let x = one of the numbers;
$16 - x$ = the other number.

The product is $f(x) = x(16 - x) = 16x - x^2 = -x^2 + 16x$

The x-coordinate of the maximum is

$$x = -\frac{b}{2a} = -\frac{16}{2(-1)} = -\frac{16}{-2} = 8.$$

$$f(8) = -8^2 + 16(8) = -64 + 128 = 64$$

The vertex is (8, 64). The maximum product is 64. This occurs when the two numbers are 8 and $16 - 8 = 8$.

62. Let x = one of the numbers
Let $20 - x$ = the other number

$$P(x) = x(20 - x) = 20x - x^2 = -x^2 + 20x$$

$$x = -\frac{b}{2a} = -\frac{20}{2(-1)} = -\frac{20}{-2} = 10$$

The other number is $20 - x = 20 - 10 = 10$.

The numbers which maximize the product are 10 and 10. The maximum product is $10 \cdot 10 = 100$.

63. Let x = one of the numbers;
$x - 16$ = the other number.

The product is $f(x) = x(x - 16) = x^2 - 16x$

The x-coordinate of the minimum is

$$x = -\frac{b}{2a} = -\frac{-16}{2(1)} = -\frac{-16}{2} = 8.$$

$$f(8) = (8)^2 - 16(8) = 64 - 128 = -64$$

The vertex is $(8, -64)$. The minimum product is -64.

This occurs when the two numbers are 8 and $8 - 16 = -8$.

64. Let x = the larger number. Then $x - 24$ is the smaller number. The product of these two numbers is given by

$$P(x) = x(x - 24) = x^2 - 24x$$

The product is minimized when

$$x = -\frac{b}{2a} = -\frac{(-24)}{2(1)} = 12$$

Since $12 - (-12) = 24$, the two numbers whose difference is 24 and whose product is minimized are 12 and -12.

The minimum product is $P(12) = 12(12 - 24) = -144$.

65. Maximize the area of a rectangle constructed along a river with 600 feet of fencing.
Let x = the width of the rectangle;
$600 - 2x$ = the length of the rectangle
We need to maximize.

$$A(x) = x(600 - 2x) = 600x - 2x^2 = -2x^2 + 600x$$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{600}{2(-2)} = -\frac{600}{-4} = 150.$$

When the width is $x = 150$ feet, the length is $600 - 2(150) = 600 - 300 = 300$ feet.

The dimensions of the rectangular plot with maximum area are 150 feet by 300 feet. This gives an area of $150 \cdot 300 = 45{,}000$ square feet.

66. From the diagram, we have that x is the width of the rectangular plot and $200-2x$ is the length. Thus, the area of the plot is given by

$$A = l \cdot w = (200-2x)(x) = -2x^2 + 200x$$

Since the graph of this equation is a parabola that opens down, the area is maximized at the vertex.

$$x = -\frac{b}{2a} = -\frac{200}{2(-2)} = 50$$

$$A = -2(50)^2 + 200(50) = -5000 + 10,000 = 5000$$

The maximum area is 5000 square feet when the length is 100 feet and the width is 50 feet.

67. Maximize the area of a rectangle constructed with 50 yards of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 2y = 50$$
$$2y = 50 - 2x$$
$$y = \frac{50-2x}{2} = 25 - x$$

We need to maximize $A = xy = x(25-x)$. Rewrite A as a function of x.

$$A(x) = x(25-x) = -x^2 + 25x$$

Since $a = -1$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{25}{2(-1)} = -\frac{25}{-2} = 12.5.$$

When the length x is 12.5, the width y is
$y = 25 - x = 25 - 12.5 = 12.5.$

The dimensions of the rectangular region with maximum area are 12.5 yards by 12.5 yards. This gives an area of $12.5 \cdot 12.5 = 156.25$ square yards.

68. Let x = the length of the rectangle
Let y = the width of the rectangle

$$2x + 2y = 80$$
$$2y = 80 - 2x$$
$$y = \frac{80-2x}{2}$$
$$y = 40 - x$$

$$A(x) = x(40-x) = -x^2 + 40x$$

$$x = -\frac{b}{2a} = -\frac{40}{2(-1)} = -\frac{40}{-2} = 20.$$

When the length x is 20, the width y is
$y = 40 - x = 40 - 20 = 20.$

The dimensions of the rectangular region with maximum area are 20 yards by 20 yards. This gives an area of $20 \cdot 20 = 400$ square yards.

69. Maximize the area of the playground with 600 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 3y = 600$$
$$3y = 600 - 2x$$
$$y = \frac{600-2x}{3}$$
$$y = 200 - \frac{2}{3}x$$

We need to maximize $A = xy = x\left(200 - \frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x) = x\left(200 - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + 200x$$

Since $a = -\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{200}{2\left(-\frac{2}{3}\right)} = -\frac{200}{-\frac{4}{3}} = 150.$$

When the length x is 150, the width y is

$$y = 200 - \frac{2}{3}x = 200 - \frac{2}{3}(150) = 100.$$

The dimensions of the rectangular playground with maximum area are 150 feet by 100 feet. This gives an area of $150 \cdot 100 = 15,000$ square feet.

70. Maximize the area of the playground with 400 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 3y = 400$$
$$3y = 400 - 2x$$
$$y = \frac{400-2x}{3}$$
$$y = \frac{400}{3} - \frac{2}{3}x$$

We need to maximize $A = xy = x\left(\frac{400}{3} - \frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x) = x\left(\frac{400}{3} - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + \frac{400}{3}x$$

Since $a=-\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{\frac{400}{3}}{2\left(-\frac{2}{3}\right)}=-\frac{\frac{400}{3}}{-\frac{4}{3}}=100.$$

When the length x is 100, the width y is

$$y=\frac{400}{3}-\frac{2}{3}x=\frac{400}{3}-\frac{2}{3}(100)=\frac{200}{3}=66\frac{2}{3}.$$

The dimensions of the rectangular playground with maximum area are 100 feet by $66\frac{2}{3}$ feet. This gives an area of $100\cdot 66\frac{2}{3}=6666\frac{2}{3}$ square feet.

71. Maximize the cross-sectional area of the gutter:

$$A(x)=x(20-2x)$$
$$=20x-2x^2=-2x^2+20x.$$

Since $a=-2$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{20}{2(-2)}=-\frac{20}{-4}=5.$$

When the height x is 5, the width is

$$20-2x=20-2(5)=20-10=10.$$

$$A(5)=-2(5)^2+20(5)$$
$$=-2(25)+100=-50+100=50$$

The maximum cross-sectional area is 50 square inches. This occurs when the gutter is 5 inches deep and 10 inches wide.

72.

$$A(x)=x(12-2x)=12x-2x^2$$
$$=-2x^2+12x$$
$$x=-\frac{b}{2a}=-\frac{12}{2(-2)}=-\frac{12}{-4}=3$$

When the height x is 3, the width is

$$12-2x=12-2(3)=12-6=6.$$

$$A(3)=-2(3)^2+12(3)=-2(9)+36$$
$$=-18+36=18$$

The maximum cross-sectional area is 18 square inches. This occurs when the gutter is 3 inches deep and 6 inches wide.

73. x = increase

$$A=(50+x)(8000-100x)$$
$$=400,000+3000x-100x^2$$

$$x=\frac{-b}{2a}=\frac{-3000}{2(-100)}=15$$

The maximum price is 50 + 15 = \$65.
The maximum revenue = 65(800 – 100·15) = \$422,500.

74. Maximize $A = (30 + x)(200 - 5x)$

$$= 6000 + 50x - 5x^2$$
$$x=\frac{-(50)}{2(-5)}=5$$

Maximum rental = 30 + 5 = \$35
Maximum revenue = 35(200 – 5·5) = \$6125

75. x = increase

$$A=(20+x)(60-2x)$$
$$=1200+20x-2x^2$$

$$x=\frac{-b}{2a}=\frac{-20}{2(-2)}=5$$

The maximum number of trees is 20 + 5 = 25 trees.
The maximum yield is 60 – 2·5=50 pounds per tree, 50 x 25 = 1250 pounds.

76. Maximize $A = (30 + x)(50 - x)$

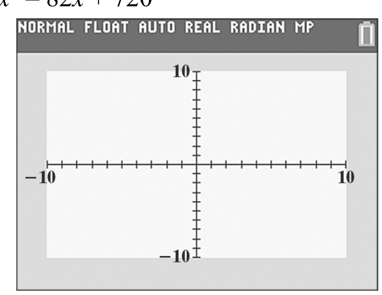
$$= 1500 + 20x - x^2$$
$$x=\frac{-20}{2(-1)}=10$$

Maximum number of trees = 30 + 10 = 40 trees
Maximum yield = (30 + 10)(50 – 10) = 1600 pounds

77. – 83. Answers will vary.

84. $y = 2x^2 - 82x + 720$

a.



You can only see a little of the parabola.

b. $a = 2$; $b = -82$

$$x = -\frac{b}{2a} = -\frac{-82}{4} = 20.5$$

$$\begin{aligned} y &= 2(20.5)^2 - 82(20.5) + 720 \\ &= 840.5 - 1681 + 720 \\ &= -120.5 \end{aligned}$$

vertex: (20.5, –120.5)

c. Ymax = 750

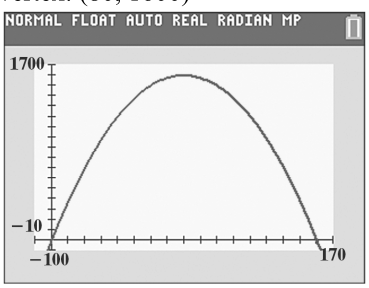
d. You can choose Xmin and Xmax so the x-value of the vertex is in the center of the graph. Choose Ymin to include the y-value of the vertex.

85. $y = -0.25x^2 + 40x$

$$x = \frac{-b}{2a} = \frac{-40}{-0.5} = 80$$

$$\begin{aligned} y &= -0.25(80)^2 + 40(80) \\ &= 1600 \end{aligned}$$

vertex: (80, 1600)

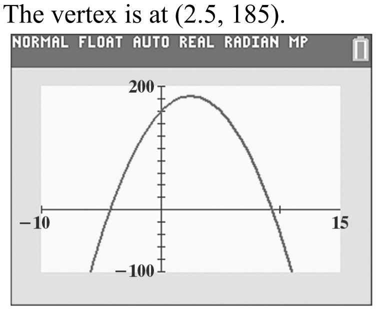


86. $y = -4x^2 + 20x + 160$

$$x = \frac{-b}{2a} = \frac{-20}{-8} = 2.5$$

$$\begin{aligned} y &= -4(2.5)^2 + 20(2.5) + 160 \\ &= -2.5 + 50 + 160 = 185 \end{aligned}$$

The vertex is at (2.5, 185).

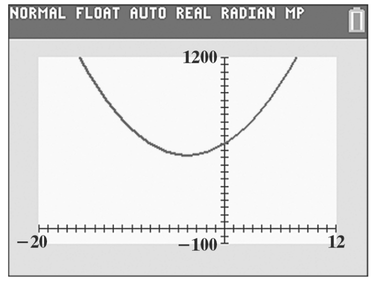


87. $y = 5x^2 + 40x + 600$

$$x = \frac{-b}{2a} = \frac{-40}{10} = -4$$

$$\begin{aligned} y &= 5(-4)^2 + 40(-4) + 600 \\ &= 80 - 160 + 600 = 520 \end{aligned}$$

vertex: (–4, 520)

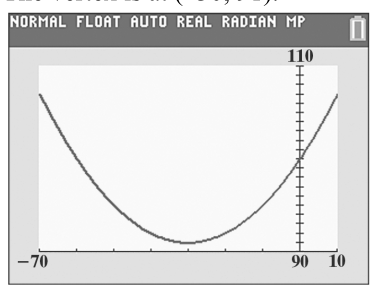


88. $y = 0.01x^2 + 0.6x + 100$

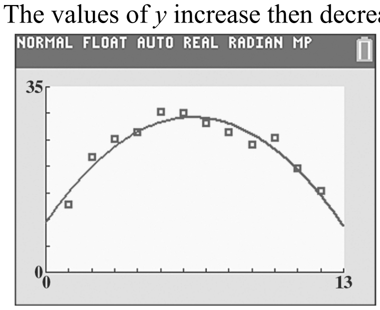
$$x = \frac{-b}{2a} = \frac{-0.6}{0.02} = -30$$

$$\begin{aligned} y &= 0.01(-30)^2 + 0.6(-30) + 100 \\ &= 9 - 18 + 100 = 91 \end{aligned}$$

The vertex is at (–30, 91).



89. a. The values of y increase then decrease.



b. $y = -0.48x^2 + 6.17x + 9.57$

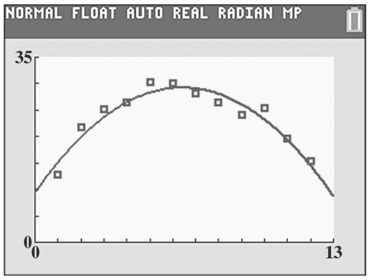
c. $x = \dfrac{-(6.17)}{2(-0.48)} \approx 6$

$$y = -0.48(6)^2 + 6.17(6) + 9.57 \approx 29.3$$

According to the model in part (b), *American Idol* had the greatest number of viewers, 29.3 million, in Season 6.

d. The greatest number of viewers actually occurred in Season 5, not Season 6, and the model underestimates the greatest number by 1.1 million.

e. Scatter plot and quadratic function of best fit:



90. does not make sense; Explanations will vary. Sample explanation: Some parabolas have the y-axis as the axis of symmetry.

91. makes sense

92. does not make sense; Explanations will vary. Sample explanation: If it is thrown vertically, its path will be a line segment.

93. does not make sense; Explanations will vary. Sample explanation: The football's path is better described by a quadratic model.

94. true

95. false; Changes to make the statement true will vary. A sample change is: The vertex is $(5,-1)$.

96. false; Changes to make the statement true will vary. A sample change is: The graph has no x–intercepts. To find x–intercepts, set $y = 0$ and solve for x.

$$0 = -2(x+4)^2 - 8$$
$$2(x+4)^2 = -8$$
$$(x+4)^2 = -4$$

Because the solutions to the equation are imaginary, we know that there are no x–intercepts.

97. false; Changes to make the statement true will vary. A sample change is: The x-coordinate of the maximum is $-\frac{b}{2a} = -\frac{1}{2(-1)} = -\frac{1}{-2} = \frac{1}{2}$ and the y–coordinate of the vertex of the parabola is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = \frac{5}{4}.$$

The maximum y–value is $\frac{5}{4}$.

98. $f(x) = 3(x+2)^2 - 5$; $(-1, -2)$
axis: $x = -2$
$(-1, -2)$ is one unit right of $(-2, -2)$. One unit left of $(-2, -2)$ is $(-3, -2)$.
point: $(-3, -2)$

99. Vertex $(3, 2)$ Axis: $x = 3$
second point $(0, 11)$

100. We start with the form $f(x) = a(x-h)^2 + k$.
Since we know the vertex is $(h,k) = (-3,-4)$, we have $f(x) = a(x+3)^2 - 4$. We also know that the graph passes through the point $(1,4)$, which allows us to solve for a.

$$4 = a(1+3)^2 - 4$$
$$8 = a(4)^2$$
$$8 = 16a$$
$$\frac{1}{2} = a$$

Therefore, the function is $f(x) = \frac{1}{2}(x+3)^2 - 4$.

101. We know $(h,k) = (-3,-4)$, so the equation is of the form

$$f(x) = a(x-h)^2 + k$$
$$= a[x-(-3)]^2 + (-1)$$
$$= a(x+3)^2 - 1$$

We use the point $(-2,-3)$ on the graph to determine the value of a:

$$f(x) = a(x+3)^2 - 1$$
$$-3 = a(-2+3)^2 - 1$$
$$-3 = a(1)^2 - 1$$
$$-3 = a - 1$$
$$-2 = a$$

Thus, the equation of the parabola is $f(x) = -2(x+3)^2 - 1$.

102. $2x + y - 2 = 0$
$y = 2 - 2x$

$d = \sqrt{x^2 + (2-2x)^2}$
$d = \sqrt{x^2 + 4 - 8x + 4x^2}$
$d = \sqrt{5x^2 - 8x + 4}$
Minimize $5x^2 - 8x + 4$
$x = \dfrac{-(-8)}{2(5)} = \dfrac{4}{5}$
$y = 2 - 2\left(\dfrac{4}{5}\right) = \dfrac{2}{5}$
$\left(\dfrac{4}{5}, \dfrac{2}{5}\right)$

103. $f(x) = (80 + x)(300 - 3x) - 10(300 - 3x)$
$= 24000 + 60x - 3x^2 - 3000 + 30x$
$= -3x^2 + 90x + 21000$

$x = \dfrac{-b}{2a} = \dfrac{-90}{2(-3)} = \dfrac{3}{2} = 15$
The maximum charge is 80 + 15 = \$95.00. the maximum profit is $-3(15)^2 + 9(15) + 21000 =$ \$21,675.

104. $440 = 2x + \pi y$
$440 - 2x = \pi y$
$\dfrac{440 - 2x}{\pi} = y$

Maximize $A = x\left(\dfrac{440 - 2x}{\pi}\right) = -\dfrac{2}{\pi}x^2 + \dfrac{440}{\pi}x$

$x = \dfrac{-\frac{440}{\pi}}{2\left(-\frac{2}{\pi}\right)} = \dfrac{\frac{-440}{\pi}}{-\frac{4}{\pi}} = \dfrac{440}{4} = 110$

$\dfrac{440 - 2(110)}{\pi} = \dfrac{220}{\pi}$

The dimensions are 110 yards by $\dfrac{220}{\pi}$ yards.

105. Answers will vary.

106. $3x + y^2 = 10$
$y^2 = 10 - 3x$
$y = \pm\sqrt{10 - 3x}$
Since there are values of x that give more than one value for y (for example, if $x = 0$, then $y = \pm\sqrt{10 - 0} = \pm\sqrt{10}$), the equation does not define y as a function of x.

107. a. domain: $\{x \mid -\infty < x < \infty\}$ or $(-\infty, \infty)$.

b. range: $\{y \mid -3 \le y < \infty\}$ or $[-3, \infty)$.

c. The x-intercepts are –2 and 4.

d. The y-intercept is –2.

e. $f(-4) = 2$

108. $f(x) = 4x^2 - 2x + 7$
$f(x+h) = 4(x+h)^2 - 2(x+h) + 7$
$= 4(x^2 + 2xh + h^2) - 2x - 2h + 7$
$= 4x^2 + 8xh + 4h^2 - 2x - 2h + 7$

$\dfrac{f(x+h) - f(x)}{h}$
$= \dfrac{4x^2 + 8xh + 4h^2 - 2x - 2h + 7 - \left(4x^2 - 2x + 7\right)}{h}$
$= \dfrac{4x^2 + 8xh + 4h^2 - 2x - 2h + 7 - 4x^2 + 2x - 7}{h}$
$= \dfrac{8xh + 4h^2 - 2h}{h}$
$= \dfrac{h\left(8x + 4h - 2\right)}{h}$
$= 8x + 4h - 2, \quad h \ne 0$

109. $x^3 + 3x^2 - x - 3 = x^2(x+3) - 1(x+3)$
$= (x+3)(x^2 - 1)$
$= (x+3)(x+1)(x-1)$

110. $f(x) = x^3 - 2x - 5$
$f(2) = (2)^3 - 2(2) - 5 = -1$
$f(3) = (3)^3 - 2(3) - 5 = 16$
The graph passes through (2, –1), which is below the x-axis, and (3, 16), which is above the x-axis. Since the graph of f is continuous, it must cross the x-axis somewhere between 2 and 3 to get from one of these points to the other.

111. $f(x) = x^4 - 2x^2 + 1$
$f(-x) = (-x)^4 - 2(-x)^2 + 1$
$= x^4 - 2x^2 + 1$
Since $f(-x) = f(x)$, the function is even.
Thus, the graph is symmetric with respect to the y-axis.

Section 2.3

Check Point Exercises

1. Since n is even and $a_n > 0$, the graph rises to the left and to the right.

2. It is not necessary to multiply out the polynomial to determine its degree. We can find the degree of the polynomial by adding the degrees of each of its factors. $f(x) = 2\underbrace{x^3}_{\text{degree 3}}\underbrace{(x-1)}_{\text{degree 1}}\underbrace{(x+5)}_{\text{degree 1}}$ has degree $3+1+1=5$.

$f(x) = 2x^3(x-1)(x+5)$ is of odd degree with a positive leading coefficient. Thus, the graph falls to the left and rises to the right.

3. Since n is odd and the leading coefficient is negative, the function falls to the right. Since the ratio cannot be negative, the model won't be appropriate.

4. The graph does not show the function's end behavior. Since $a_n > 0$ and n is odd, the graph should fall to the left but doesn't appear to do so.

5. $f(x) = x^3 + 2x^2 - 4x - 8$

$0 = x^2(x+2) - 4(x+2)$

$0 = (x+2)(x^2-4)$

$0 = (x+2)^2(x-2)$

$x = -2$ or $x = 2$

The zeros are –2 and 2.

6. $f(x) = x^4 - 4x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2-4) = 0$

$x^2(x+2)(x-2) = 0$

$x = 0$ or $x = -2$ or $x = 2$

The zeros are –2, 0, and 2.

7. $f(x) = -4\left(x+\frac{1}{2}\right)^2(x-5)^3$

$-4\left(x+\frac{1}{2}\right)^2(x-5)^3 = 0$

$x = -\frac{1}{2}$ or $x = 5$

The zeros are $-\frac{1}{2}$, with multiplicity 2, and 5, with multiplicity 3.

Because the multiplicity of $-\frac{1}{2}$ is even, the graph touches the x-axis and turns around at this zero. Because the multiplicity of 5 is odd, the graph crosses the x-axis at this zero.

8. $f(x) = 3x^3 - 10x + 9$

$f(-3) = 3(-3)^3 - 10(-3) + 9 = -42$

$f(-2) = 3(-2)^3 - 10(-2) + 9 = 5$

The sign change shows there is a zero between –3 and –2.

9. $f(x) = x^3 - 3x^2$

Since $a_n > 0$ and n is odd, the graph falls to the left and rises to the right.

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0$ or $x = 3$

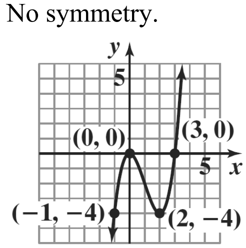
The x-intercepts are 0 and 3.

$f(0) = 0^3 - 3(0)^2 = 0$

The y-intercept is 0.

$f(-x) = (-x)^3 - 3(-x)^2 = -x^3 - 3x^2$

No symmetry.



$f(x) = x^3 - 3x^2$

10. $f(x) = 2(x+2)^2(x-3)$

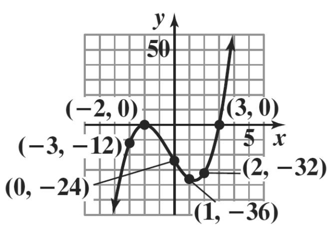
The leading term is $2 \cdot x^2 \cdot x$, or $2x^3$.
Since $a_n > 0$ and n is odd, the graph falls to the left and rises to the right.

$2(x+2)^2(x-3) = 0$
$x = -2$ or $x = 3$
The x-intercepts are –2 and 3.

$f(0) = 2(0+2)^2(0-3) = -12$
The y-intercept is –12.

$$f(-x) = 2((-x)+2)^2((-x)-3)$$
$$= 2(-x+2)^2(-x-3)$$

No symmetry.



$f(x) = 2(x + 2)^2 (x - 3)$

Concept and Vocabulary Check 2.3

1. 5; –2

2. false

3. end; leading

4. falls; rises

5. rises; falls

6. rises; rises

7. falls; falls

8. true

9. true

10. x-intercept

11. turns around; crosses

12. 0; Intermediate Value

13. $n-1$

Exercise Set 2.3

1. polynomial function;
degree: 3

2. polynomial function;
degree: 4

3. polynomial function;
degree: 5

4. polynomial function;
degree: 7

5. not a polynomial function

6. not a polynomial function

7. not a polynomial function

8. not a polynomial function

9. not a polynomial function

10. polynomial function;
degree: 2

11. polynomial function

12. Not a polynomial function because graph is not smooth.

13. Not a polynomial function because graph is not continuous.

14. polynomial function

15. (b)

16. (c)

17. (a)

18. (d)

19. $f(x) = 5x^3 + 7x^2 - x + 9$
Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.

20. $f(x) = 11x^3 - 6x^2 + x + 3$
Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.

21. $f(x) = 5x^4 + 7x^2 - x + 9$
Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

22. $f(x)=11x^4-6x^2+x+3$
Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

23. $f(x)=-5x^4+7x^2-x+9$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

24. $f(x)=-11x^4-6x^2+x+3$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

25. $f(x)=2(x-5)(x+4)^2$
$x = 5$ has multiplicity 1;
The graph crosses the x-axis.
$x = –4$ has multiplicity 2;
The graph touches the x-axis and turns around.

26. $f(x)=3(x+5)(x+2)^2$
$x = –5$ has multiplicity 1;
The graph crosses the x-axis.
$x = –2$ has multiplicity 2;
The graph touches the x-axis and turns around.

27. $f(x)=4(x-3)(x+6)^3$
x = 3 has multiplicity 1;
The graph crosses the x-axis.
$x = –6$ has multiplicity 3;
The graph crosses the x-axis.

28. $f(x)=-3\left(x+\frac{1}{2}\right)(x-4)^3$

$x=-\frac{1}{2}$ has multiplicity 1;
The graph crosses the x-axis.
$x = 4$ has multiplicity 3;
The graph crosses the x-axis.

29. $f(x)=x^3-2x^2+x$
$=x\left(x^2-2x+1\right)$
$=x(x-1)^2$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = 1$ has multiplicity 2;
The graph touches the x-axis and turns around.

30. $f(x)=x^3+4x^2+4x$
$=x\left(x^2+4x+4\right)$
$=x(x+2)^2$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = –2$ has multiplicity 2;
The graph touches the x-axis and turns around.

31. $f(x)=x^3+7x^2-4x-28$
$=x^2(x+7)-4(x+7)$
$=\left(x^2-4\right)(x+7)$
$=(x-2)(x+2)(x+7)$
$x = 2$, $x = –2$ and $x = –7$ have multiplicity 1;
The graph crosses the x-axis.

32. $f(x)=x^3+5x^2-9x-45$
$=x^2(x+5)-9(x+5)$
$=\left(x^2-9\right)(x+5)$
$=(x-3)(x+3)(x+5)$
$x = 3$, $x = –3$ and $x = –5$ have multiplicity 1;
The graph crosses the x-axis.

33. $f\left(x\right)=x^3-x-1$
$f(1) = –1$
$f(2) = 5$
The sign change shows there is a zero between the given values.

34. $f\left(x\right)=x^3-4x^2+2$
$f(0) = 2$
$f(1) = –1$
The sign change shows there is a zero between the given values.

35. $f\left(x\right)=2x^4-4x^2+1$
$f(–1) = –1$
$f(0) = 1$
The sign change shows there is a zero between the given values.

36. $f\left(x\right)=x^4+6x^3-18x^2$
$f(2) = –8$
$f(3) = 81$
The sign change shows there is a zero between the given values.

37. $f(x) = x^3 + x^2 - 2x + 1$

$f(-3) = -11$
$f(-2) = 1$
The sign change shows there is a zero between the given values.

38. $f(x) = x^5 - x^3 - 1$

$f(1) = -1$
$f(2) = 23$
The sign change shows there is a zero between the given values.

39. $f(x) = 3x^3 - 10x + 9$

$f(-3) = -42$
$f(-2) = 5$
The sign change shows there is a zero between the given values.

40. $f(x) = 3x^3 - 8x^2 + x + 2$

$f(2) = -4$
$f(3) = 14$
The sign change shows there is a zero between the given values.

41. $f(x) = x^3 + 2x^2 - x - 2$

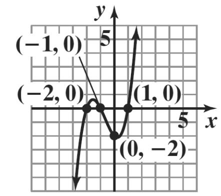
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b.
$$x^3 + 2x^2 - x - 2 = 0$$
$$x^2(x+2) - (x+2) = 0$$
$$(x+2)(x^2 - 1) = 0$$
$$(x+2)(x-1)(x+1) = 0$$
$$x = -2, x = 1, x = -1$$
The zeros at –2, –1, and 1 have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c.
$$f(0) = (0)^3 + 2(0)^2 - 0 - 2 = -2$$
The y-intercept is –2.

d.
$$f(-x) = (-x) + 2(-x)^2 - (-x) - 2 = -x^3 + 2x^2 + x - 2$$
$$-f(x) = -x^3 - 2x^2 + x + 2$$
The graph has neither origin symmetry nor y-axis symmetry.

e. The graph has 2 turning points and $2 \le 3 - 1$.



$f(x) = x^3 + 2x^2 - x - 2$

42. $f(x) = x^3 + x^2 - 4x - 4$

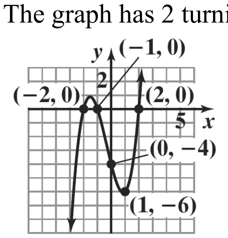
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b.
$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x+1)(x^2 - 4) = 0$$
$$(x+1)(x-2)(x+2) = 0$$
$x = -1$, or $x = 2$, or $x = -2$
The zeros at –2, –1 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The x-intercepts are –2, –1, and 2.

c. $f(0) = 0^3 + (0)^2 - 4(0) - 4 = -4$
The y-intercept is –4.

d.
$$f(-x) = -x^3 + x^2 + 4x - 4$$
$$-f(x) = -x^3 - x^2 + 4x + 4$$
neither symmetry

e. The graph has 2 turning points and $2 \le 3 - 1$.



43. $f(x) = x^4 - 9x^2$

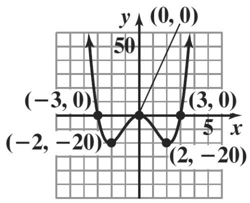
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b.
$$x^4 - 9x^2 = 0$$
$$x^2(x^2 - 9) = 0$$
$$x^2(x-3)(x+3) = 0$$
$$x = 0, x = 3, x = -3$$
The zeros at –3 and 3 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - 9x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 9x^2$

44. $f(x) = x^4 - x^2$

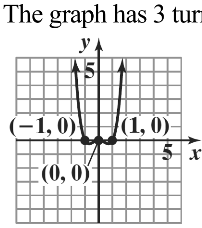
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - x^2 = 0$
$x^2(x^2 - 1) = 0$
$x^2(x-1)(x+1) = 0$
$x = 0, x = 1, x = -1$
f touches but does not cross the x-axis at 0.

c. $f(0) = (0)^4 - (0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - x^2$

45. $f(x) = -x^4 + 16x^2$

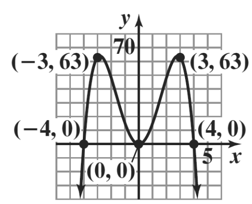
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-x^4 + 16x^2 = 0$
$x^2(-x^2 + 16) = 0$
$x^2(4 - x)(4 + x) = 0$
$x = 0, x = 4, x = -4$
The zeros at –4 and 4 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = -x^4 + 16x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^4 + 16x^2$

46. $f(x) = -x^4 + 4x^2$

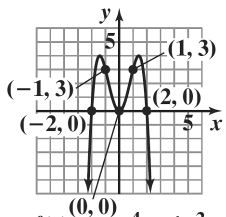
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-x^4 + 4x^2 = 0$
$x^2(4 - x^2) = 0$
$x^2(2 - x)(2 + x) = 0$
$x = 0, x = 2, x = -2$
The x-intercepts are –2,0, and 2. Since f has a double root at 0, it touches but does not cross the x-axis at 0.

c. $f(0) = -(0)^4 + 4(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = -x^4 + 4x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^4 + 4x^2$

47. $f(x) = x^4 - 2x^3 + x^2$

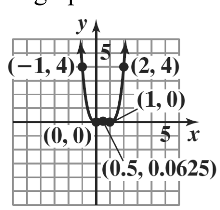
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - 2x^3 + x^2 = 0$
$x^2(x^2 - 2x + 1) = 0$
$x^2(x-1)(x-1) = 0$
$x = 0, x = 1$
The zeros at 1 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 0 and 1.

c. $f(0) = (0)^4 - 2(0)^3 + (0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 + 2x^3 + x^2$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 2x^3 + x^2$

48. $f(x) = x^4 - 6x^3 + 9x^2$

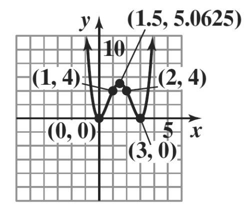
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - 6x^3 + 9x^2 = 0$
$x^2(x^2 - 6x + 9) = 0$
$x^2(x-3)^2 = 0$
$x = 0, x = 3$
The zeros at 3 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 3 and 0.

c. $f(0) = (0)^4 - 6(0)^3 + 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 + 6x^3 + 9x^2$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 6x^3 + 9x^2$

49. $f(x) = -2x^4 + 4x^3$

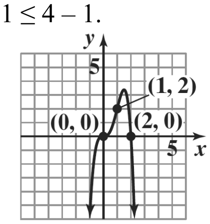
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4 + 4x^3 = 0$
$x^3(-2x + 4) = 0$
$x = 0, x = 2$
The zeros at 0 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -2(0)^4 + 4(0)^3 = 0$
The y-intercept is 0.

d. $f(-x) = -2x^4 - 4x^3$
The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -2x^4 + 4x^3$

50. $f(x) = -2x^4 + 2x^3$

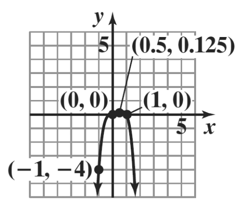
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4 + 2x^3 = 0$
$x^3(-2x+2) = 0$
$x = 0, x = 1$
The zeros at 0 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. The y-intercept is 0.

d. $f(-x) = -2x^4 - 2x^3$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 4 - 1$.



$f(x) = -2x^4 + 2x^3$

51. $f(x) = 6x^3 - 9x - x^5$

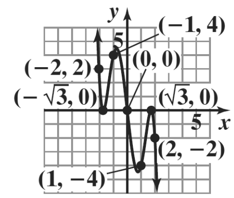
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5 + 6x^3 - 9x = 0$
$-x\left(x^4 - 6x^2 + 9\right) = 0$
$-x\left(x^2 - 3\right)\left(x^2 - 3\right) = 0$
$x = 0, \ x = \pm\sqrt{3}$
The root at 0 has odd multiplicity so $f(x)$ crosses the x-axis at (0, 0). The zeros at $-\sqrt{3}$ and $\sqrt{3}$ have even multiplicity so $f(x)$ touches the x-axis at $\sqrt{3}$ and $-\sqrt{3}$.

c. $f(0) = -(0)^5 + 6(0)^3 - 9(0) = 0$
The y-intercept is 0.

d. $f(-x) = x^5 - 6x^3 + 9x$
$f(-x) = -f(x)$
The graph has origin symmetry.

e. The graph has 4 turning points and $4 \le 5 - 1$.



$f(x) = 6x^3 - 9x - x^5$

52. $f(x) = 6x - x^3 - x^5$

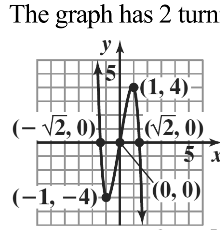
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5 - x^3 + 6x = 0$
$-x\left(x^4 + x^2 - 6\right) = 0$
$-x\left(x^2 + 3\right)\left(x^2 - 2\right) = 0$
$x = 0, x = \pm\sqrt{2}$
The zeros at $-\sqrt{2}$, 0, and $\sqrt{2}$ have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -(0)^5 - (0)^3 + 6(0) = 0$
The y-intercept is 0.

d. $f(-x) = x^5 + x^3 - 6x$
$f(-x) = -f(x)$
The graph has origin symmetry.

e. The graph has 2 turning points and $2 \le 5 - 1$.



$f(x) = 6x - x^3 - x^5$

53. $f(x) = 3x^2 - x^3$

a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^3 + 3x^2 = 0$
$-x^2(x-3) = 0$
$x = 0, x = 3$
The zero at 3 has odd multiplicity so $f(x)$ crosses the x-axis at that point. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

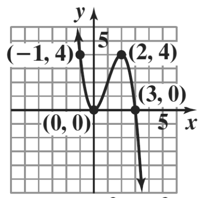
c. $f(0) = -(0)^3 + 3(0)^2 = 0$

The y-intercept is 0.

d. $f(-x) = x^3 + 3x^2$

The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 3 - 1$.



$f(x) = 3x^2 - x^3$

54. $f(x) = \frac{1}{2} - \frac{1}{2}x^4$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-\frac{1}{2}x^4 + \frac{1}{2} = 0$

$-\frac{1}{2}\left(x^4 - 1\right) = 0$

$-\frac{1}{2}\left(x^2 + 1\right)\left(x^2 - 1\right) = 0$

$-\frac{1}{2}\left(x^2 + 1\right)(x - 1)(x + 1) = 0$

$x = \pm 1$

The zeros at –1 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -\frac{1}{2}(0)^4 + \frac{1}{2} = \frac{1}{2}$

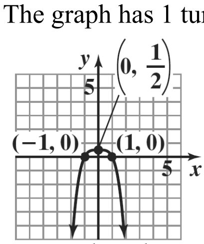
The y-intercept is $\frac{1}{2}$.

d. $f(-x) = \frac{1}{2} - \frac{1}{2}x^4$

$f(-x) = f(x)$

The graph has y-axis symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = \frac{1}{2} - \frac{1}{2}x^4$

55. $f(x) = -3(x - 1)^2\left(x^2 - 4\right)$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-3(x - 1)^2\left(x^2 - 4\right) = 0$

$x = 1, x = -2, x = 2$

The zeros at –2 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 1 has even multiplicity, so $f(x)$ touches the x-axis at (1, 0).

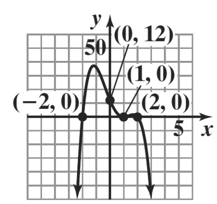
c. $f(0) = -3(0 - 1)^2(0^2 - 4)^3$
$= -3(1)(-4) = 12$

The y-intercept is 12.

d. $f(-x) = -3(-x - 1)^2\left(x^2 - 4\right)$

The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -3(x - 1)^2(x^2 - 4)$

56. $f(x) = -2(x - 4)^2\left(x^2 - 25\right)$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2(x - 4)^2\left(x^2 - 25\right) = 0$

$x = 4, x = -5, x = 5$

The zeros at –5 and 5 have odd multiplicity so $f(x)$ crosses the x-axis at these points. The root at 4 has even multiplicity so $f(x)$ touches the x-axis at (4, 0).

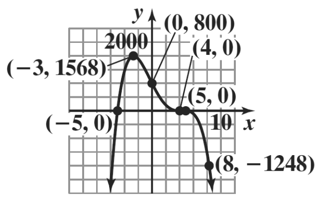
c. $f(0) = -2(0 - 4)^2(0^2 - 25)$
$= -2(16)(-25)$
$= 800$

The y-intercept is 800.

d. $f(-x) = -2(-x - 4)^2\left(x^2 - 2\right)$

The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4-1$.



$f(x) = -2(x-4)^2(x^2-25)$

57. $f(x) = x^2(x-1)^3(x+2)$

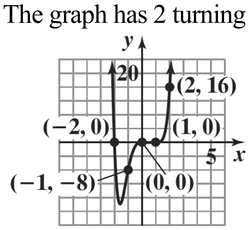
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = 1, x = -2$
The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = 0^2(0-1)^3(0+2) = 0$
The y-intercept is 0.

d. $f(-x) = x^2(-x-1)^3(-x+2)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6-1$.



$f(x) = x^2(x-1)^3(x+2)$

58. $f(x) = x^3(x+2)^2(x+1)$

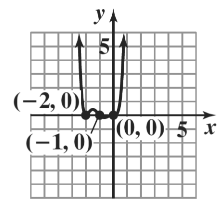
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = -2, x = -1$
The roots at 0 and –1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –2 has even multiplicity so $f(x)$ touches the axis at (–2, 0).

c. $f(0) = 0^3(0+2)^2(0+1) = 0$
The y-intercept is 0.

d. $f(-x) = -x^3(-x+2)^2(-x+1)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 6-1$.



$f(x) = x^3(x+2)^2(x+1)$

59. $f(x) = -x^2(x-1)(x+3)$

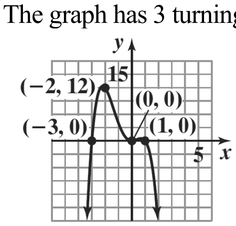
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -3$
The zeros at 1 and –3 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = -0^2(0-1)(0+3) = 0$
The y-intercept is 0.

d. $f(-x) = -x^2(-x-1)(-x+3)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4-1$.



$f(x) = -x^2(x-1)(x+3)$

60. $f(x) = -x^2(x+2)(x-2)$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 2, x = -2$
The zeros at 2 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

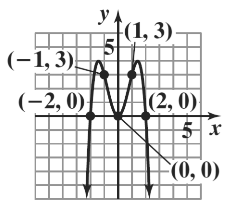
c. $f(0) = -0^2(0+2)(0-2) = 0$
The y-intercept is 0.

d. $f(-x) = -x^2(-x+2)(-x-2)$

$f(-x) = -x^2(-1)(x-2)(-1)(x+2)$
$f(-x) = -x^2(x+2)(x-2)$
$f(-x) = f(x)$

The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^2(x+2)(x-2)$

61. $f(x) = -2x^3(x-1)^2(x+5)$

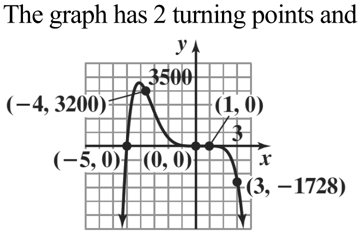
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -5$
The roots at 0 and –5 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 1 has even multiplicity so $f(x)$ touches the axis at (1, 0).

c. $f(0) = -2(0)^3(0-1)^2(0+5) = 0$
The y-intercept is 0.

d. $f(-x) = 2x^3(-x-1)^2(-x+5)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = -2x^3(x-1)^2(x+5)$

62. $f(x) = -3x^3(x-1)^2(x+3)$

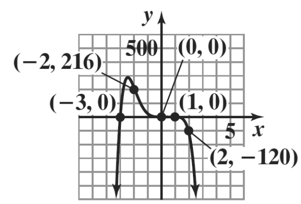
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -3$
The roots at 0 and –3 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 1 has even multiplicity so $f(x)$ touches the axis at (1, 0).

c. $f(0) = -3(0)^3(0-1)^2(0+3) = 0$
The y-intercept is 0.

d. $f(-x) = 3x^3(-x-1)^2(-x+3)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = -3x^3(x-1)^2(x+3)$

63. $f(x) = (x-2)^2(x+4)(x-1)$

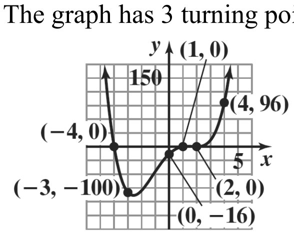
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and rises the right.

b. $x = 2, x = -4, x = 1$
The zeros at –4 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 2 has even multiplicity so $f(x)$ touches the axis at (2, 0).

c. $f(0) = (0-2)^2(0+4)(0-1) = -16$
The y-intercept is –16.

d. $f(-x) = (-x-2)^2(-x+4)(-x-1)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = (x-2)^2(x+4)(x-1)$

64. $f(x) = (x+3)(x+1)^3(x+4)$

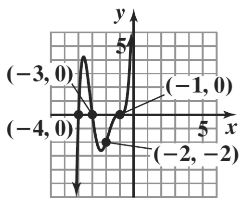
a. Since $a_n > 0$ and n is odd, $f(x)$ falls to the left and rises to the right.

b. $x = -3, x = -1, x = -4$
The zeros at all have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c. $f(0) = (0+3)(0+1)^3(0+4) = 12$
The y-intercept is 12.

d. $f(-x) = (-x+3)(-x+1)^3(-x+4)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points



$f(x) = (x+3)(x+1)^3(x+4)$

65. a. The x-intercepts of the graph are -2, 1, and 4, so they are the zeros. Since the graph actually crosses the x-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2, 1, and 4 are the zeros, $x+2$, $x-1$, and $x-4$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+2)(x-1)(x-4)$.

c. $f(0) = (0+2)(0-1)(0-4) = 8$

66. a. The x-intercepts of the graph are -3, 2, and 5, so they are the zeros. Since the graph actually crosses the x-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -3, 2, and 5 are the zeros, $x+3$, $x-2$, and $x-5$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+3)(x-2)(x-5)$.

c. $f(0) = (0+3)(0-2)(0-5) = 30$

67. a. The x-intercepts of the graph are -1 and 3, so they are the zeros. Since the graph crosses the x-axis at -1, it has odd multiplicity. Since the graph touches the x-axis and turns around at 3, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 3 are the zeros, $x+1$ and $x-3$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+1)(x-3)^2$.

c. $f(0) = (0+1)(0-3)^2 = 9$

68. a. The x-intercepts of the graph are -2 and 1, so they are the zeros. Since the graph crosses the x-axis at -2, it has odd multiplicity. Since the graph touches the x-axis and turns around at 1, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2 and 1 are the zeros, $x+2$ and $x-1$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+2)(x-1)^2$.

c. $f(0) = (0+2)(0-1)^2 = 2$

69. a. The x-intercepts of the graph are -3 and 2, so they are the zeros. Since the graph touches the x-axis and turns around at both -3 and 2, both have even multiplicity.

b. Since the graph has three turning points, the function must be at least of degree 4. Since -3 and 2 are the zeros, $x+3$ and $x-2$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x) = -(x+3)^2(x-2)^2$.

c. $f(0) = -(0+3)^2(0-2)^2 = -36$

70. a. The x-intercepts of the graph are -1 and 4, so they are the zeros. Since the graph touches the x-axis and turns around at both -1 and 4, both have even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 4 are the zeros, $x+1$ and $x-4$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x)=-(x+1)^2(x-4)^2$.

c. $f(0)=-(0+1)^2(0-4)^2=-16$

71. a. The x-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the x-axis at -1 and 1, they both have odd multiplicity. Since the graph touches the x-axis and turns around at -2, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is
$f(x)=(x+2)^2(x+1)(x-1)^3$.

c. $f(0)=(0+2)^2(0+1)(0-1)^3=-4$

72. a. The x-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the x-axis at -2 and 1, they both have odd multiplicity. Since the graph touches the x-axis and turns around at -1, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is
$f(x)=(x+2)(x+1)^2(x-1)^3$.

c. $f(0)=(0+2)(0+1)^2(0-1)^3=-2$

73. a. $f(x)=0.76x^3-30x^2-882x+37{,}807$
$f(40)=0.76(40)^3-30(40)^2-882(40)+37807$
$=3167$
The world tiger population in 2010 (40 years after 1970) was about 3167.
This is represented by the point $(40, 3167)$.

b. This underestimates the actual data shown in the bar graph by 33.

c. The leading coefficient is positive, thus the graph rises to the right.
No, if conservation efforts fail, the model will not be useful. The model indicates an increasing world tiger population that will actually decrease without conservation efforts.

74. a. $f(x)=0.76x^3-30x^2-882x+37{,}807$
$f(10)=0.76(10)^3-30(10)^2-882(10)+37{,}807$
$=26{,}747$
The world tiger population in 1980 (10 years after 1970) was about 26,747.
This is represented by the point (10, 26,747).

b. This underestimates the actual data shown in the bar graph by 1253.

c. The leading coefficient is positive, thus the graph rises to the right.
Yes, if conservation efforts succeed, the model will be useful. The model indicates an increasing world tiger population that might actually increase with conservation efforts.

b. Since the degree of g is odd and the leading coefficient is negative, the graph rises to the right. Based on the end behavior, the function will be a useful model over an extended period of time.

75. a. The woman's heart rate was increasing from 1 through 4 minutes and from 8 through 10 minutes.

b. The woman's heart rate was decreasing from 4 through 8 minutes and from 10 through 12 minutes.

c. There were 3 turning points during the 12 minutes.

d. Since there were 3 turning points, a polynomial of degree 4 would provide the best fit.

e. The leading coefficient should be negative. The graph falls to the left and to the right.

f. The woman's heart rate reached a maximum of about 116 ± 1 beats per minute. This occurred after 10 minutes.

g. The woman's heart rate reached a minimum of about 64 ± 1 beats per minute. This occurred after 8 minutes.

76. **a.** The average price per gallon in January was increasing from 2005 to 2006, 2007 to 2008, and 2009 to 2011.

b. The average price per gallon in January was decreasing from 2006 to 2007, and 2008 to 2009.

c. 4 turning points are shown in the graph.

d. Since there are 4 turning points, the degree of the polynomial function of best fit would be 5

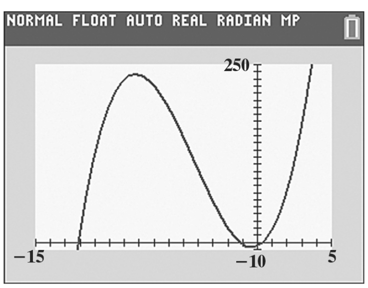
e. The leading coefficient would be positive because the graph falls to the left and rises to the right.

f. The maximum average January price per gallon was about \$3.15. This occurred in 2011.

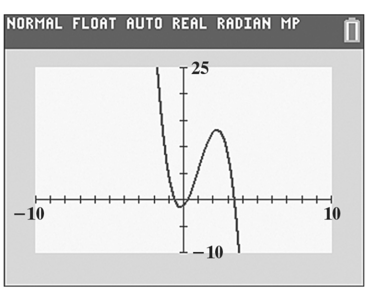
g. The minimum average January price per gallon was about \$1.85. This occurred in 2009.

77. – 93. Answers will vary.

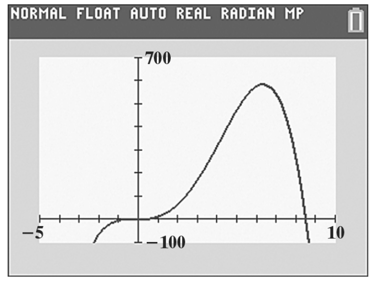
94.



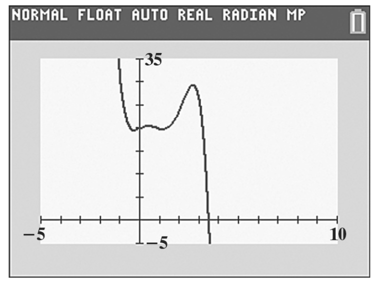
95.



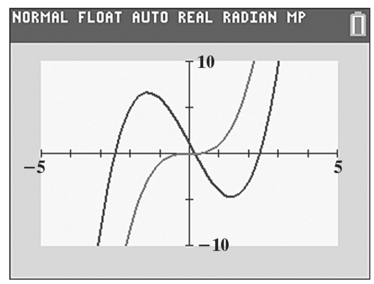
96.



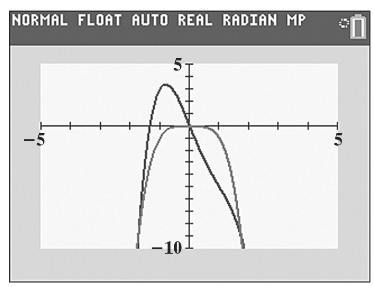
97.



98.



99.



100. makes sense

101. does not make sense; Explanations will vary. Sample explanation: Since $(x+2)$ is raised to an odd power, the graph crosses the x-axis at –2.

102. does not make sense; Explanations will vary. Sample explanation: A forth degree function has at most 3 turning points.

103. makes sense

104. false; Changes to make the statement true will vary. A sample change is: $f(x)$ falls to the left and rises to the right.

105. false; Changes to make the statement true will vary. A sample change is: Such a function falls to the right and will eventually have negative values.

106. true

107. false; Changes to make the statement true will vary. A sample change is: A function with origin symmetry either falls to the left and rises to the right, or rises to the left and falls to the right.

108. $f(x) = x^3 + x^2 - 12x$

109. $f(x) = x^3 - 2x^2$

110. Let x = the number of years after 1995.

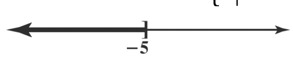
$$\begin{aligned} 315 - 13x &= 29 \\ -13x &= -286 \\ x &= \frac{-286}{-13} \\ x &= 22 \end{aligned}$$

Juries will render 29 death sentences 22 years after 1995, or 2017.

111.

$$\begin{aligned} \frac{2x-3}{4} &\ge \frac{3x}{4} + \frac{1}{2} \\ 4\left(\frac{2x-3}{4}\right) &\ge 4\left(\frac{3x}{4} + \frac{1}{2}\right) \\ 2x - 3 &\ge 3x + 2 \\ -5 &\ge x \\ x &\le -5 \end{aligned}$$

The solution set is $\{x | x \le -5\}$ or $(-\infty, -5]$.



112. $m = \frac{3-(-5)}{-10-(-2)} = \frac{8}{-8} = -1$,

so the slope is –1.

Using the point (–10, 3), we get the following point-slope equation:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= -1[x - (-10)] \\ y - 3 &= -(x + 10) \end{aligned}$$

Solve the equation for y to find the slope-intercept form:

$$\begin{aligned} y - 3 &= -(x + 10) \\ y - 3 &= -x - 10 \\ y &= -x - 7. \end{aligned}$$

113. $\frac{737}{21} = 35 + \frac{2}{21}$ or $35\frac{2}{21}$.

114. $6x^3 - x^2 - 5x + 4$

115. $2x^3 - x^2 - 11x + 6 = (x-3)(2x^2 + 3x - 2)$
$= (x-3)(2x-1)(x+2)$

Section 2.4

Check Point Exercises

1.

$$\begin{array}{r} x + 5 \\ x + 9\overline{)x^2 + 14x + 45} \\ \underline{x^2 + 9x} \\ 5x + 45 \\ \underline{5x + 45} \\ 0 \end{array}$$

The answer is $x + 5$.

2.

$$\begin{array}{r} 2x^2 + 3x - 2 \\ x - 3\overline{)2x^3 - 3x^2 - 11x + 7} \\ \underline{2x^3 - 6x^2} \\ 3x^2 - 11x \\ \underline{3x^2 - 9x} \\ -2x + 7 \\ \underline{-2x + 6} \\ 1 \end{array}$$

The answer is $2x^2 + 3x - 2 + \frac{1}{x-3}$.

3.

$$\begin{array}{r} 2x^2 + 7x + 14 \\ x^2 - 2x\overline{)2x^4 + 3x^3 + 0x^2 - 7x - 10} \\ \underline{2x^4 - 4x^3} \\ 7x^3 + 0x^2 \\ \underline{7x^3 - 14x^2} \\ 14x^2 - 7x \\ \underline{14x^2 - 28x} \\ 21x - 10 \end{array}$$

The answer is $2x^2 + 7x + 14 + \frac{21x - 10}{x^2 - 2x}$.

4.

–2	1	0	–7	–6
		–2	4	6
	1	–2	–3	0

The answer is $x^2 - 2x - 3$.

5.

$$\begin{array}{r|rrrr} -4 & 3 & 4 & -5 & 3 \\ & & -12 & 32 & -108 \\ \hline & 3 & -8 & 27 & -105 \end{array}$$

$f(-4) = -105$

6.

$$\begin{array}{r|rrrr} -1 & 15 & 14 & -3 & -2 \\ & & -15 & 1 & 2 \\ \hline & 15 & -1 & -2 & 0 \end{array}$$

$$15x^2 - x - 2 = 0$$
$$(3x+1)(5x-2) = 0$$
$$x = -\frac{1}{3} \quad \text{or} \quad x = \frac{2}{5}$$

The solution set is $\left\{-1, -\frac{1}{3}, \frac{2}{5}\right\}$.

Concept and Vocabulary Check 2.4

1. $2x^3 + 0x^2 + 6x - 4$

2. $6x^3$; $3x$; $2x^2$; $7x^2$

3. $2x^2$; $5x-2$; $10x^3 - 4x^2$; $10x^3 + 6x^2$

4. $6x^2 - 10x$; $6x^2 + 8x$; $18x$; -4; $18x - 4$

5. 9; $3x-5$; 9; $3x - 5 + \frac{9}{2x+1}$

6. divisor; quotient; remainder; dividend

7. 4; 1; 5; -7; 1

8. -5; 4; 0; -8; -2

9. true

10. $f(c)$

11. $x - c$

Exercise Set 2.4

1.

$$\begin{array}{r} x+3 \\ x+5\overline{)x^2+8x+15} \\ \underline{x^2+5x} \\ 3x+15 \\ \underline{3x+15} \\ 0 \end{array}$$

The answer is $x+3$.

2.

$$\begin{array}{r} x+5 \\ x-2\overline{)x^2+3x-10} \\ \underline{x^2-2x} \\ 5x-10 \\ \underline{5x-10} \\ 0 \end{array}$$

The answer is $x+5$.

3.

$$\begin{array}{r} x^2+3x+1 \\ x+2\overline{)x^3+5x^2+7x+2} \\ \underline{x^3+2x^2} \\ 3x^2+7x \\ \underline{3x^2+6x} \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$

The answer is x^2+3x+1.

4.

$$\begin{array}{r} x^2+x-2 \\ x-3\overline{)x^3-2x^2-5x+6} \\ \underline{x^3-3x^2} \\ x^2-5x \\ \underline{x^2-3x} \\ -2x+6 \\ \underline{-2x+6} \\ 0 \end{array}$$

The answer is x^2+x-2.

5.

$$\begin{array}{r} 2x^2+3x+5 \\ 3x-1\overline{)6x^3+7x^2+12x-5} \\ \underline{6x^3-2x^2} \\ 9x^2+12x \\ \underline{9x^2-3x} \\ 15x-5 \\ \underline{15x-5} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

6.

$$\begin{array}{r} 2x^2+3x+5 \\ 3x+4\overline{)6x^3+17x^2+27x+20} \\ \underline{6x^3+8x^2} \\ 9x^2+27x \\ \underline{9x^2+12x} \\ 15x+20 \\ \underline{15x+20} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

7.

$$\begin{array}{r} 4x+3+\frac{2}{3x-2} \\ 3x-2\overline{)12x^2+x-4} \\ \underline{12x^2-8x} \\ 9x-4 \\ \underline{9x-6} \\ 2 \end{array}$$

The answer is $4x+3+\frac{2}{3x-2}$.

8.

$$\begin{array}{r} 2x-3+\frac{3}{2x-1} \\ 2x-1\overline{)4x^2-8x+6} \\ \underline{4x^2-2x} \\ -6x+6 \\ \underline{-6x+6} \\ 3 \end{array}$$

The answer is $2x-3+\frac{3}{2x-1}$.

9.

$$\begin{array}{r} 2x^2+x+6-\frac{38}{x+3} \\ x+3\overline{)2x^3+7x^2+9x-20} \\ \underline{2x^3+6x^2} \\ x^2+9x \\ \underline{x^2+3x} \\ 6x-20 \\ \underline{6x+18} \\ -38 \end{array}$$

The answer is $2x^2+x+6-\frac{38}{x+3}$.

10.

$$\begin{array}{r} 3x+7+\frac{26}{x-3} \\ x-3\overline{)3x^2-2x+5} \\ \underline{3x^2-9x} \\ 7x+5 \\ \underline{7x-21} \\ 26 \end{array}$$

The answer is $3x+7+\frac{26}{x-3}$.

11.

$$\begin{array}{r} 4x^3+16x^2+60x+246+\frac{984}{x-4} \\ x-4\overline{)4x^4-4x^2+6x} \\ \underline{4x^4-16x^3} \\ 16x^3-4x^2 \\ \underline{16x^3-64x^2} \\ 60x^2+6x \\ \underline{60x^2-240x} \\ 246x \\ \underline{246x-984} \\ 984 \end{array}$$

The answer is

$$4x^3+16x^2+60x+246+\frac{984}{x-4}.$$

12.

$$\begin{array}{r} x^3+3x^2+9x+27 \\ x-3\overline{)x^4\qquad\qquad\qquad -81} \\ \underline{x^4-3x^3} \\ 3x^3 \\ \underline{3x^3-9x^2} \\ 9x^2 \\ \underline{9x^2-27x} \\ 27x-81 \\ \underline{27x-81} \\ 0 \end{array}$$

The answer is $x^3+3x^2+9x+27$.

13.

$$\begin{array}{r} 2x+5 \\ 3x^2-x-3\overline{)6x^3+13x^2-11x-15} \\ \underline{6x^3-2x^2-6x} \\ 15x^2-5x-15 \\ \underline{15x^2-5x-15} \\ 0 \end{array}$$

The answer is $2x+5$.

14.

$$\begin{array}{r} x^2+x-3 \\ x^2+x-2\overline{)x^4+2x^3-4x^2-5x-6} \\ \underline{x^4+x^3-2x^2} \\ x^3-2x^2-5x \\ \underline{x^3+x^2-2x} \\ -3x^2-3x-6 \\ \underline{-3x^2-3x+6} \\ -12 \end{array}$$

The answer is $x^2+x-3-\frac{12}{x^2+x-2}$.

15.

$$\begin{array}{r}
6x^2+3x-1 \\
3x^2+1\overline{\big)\,18x^4+9x^3+3x^2} \\
\underline{18x^4+6x^2} \\
9x^3-3x^2 \\
\underline{9x^3+3x} \\
-3x^2-3x \\
\underline{-3x^2-1} \\
-3x+1
\end{array}$$

The answer is $6x^2+3x-1-\dfrac{3x-1}{3x^2+1}$.

16.

$$\begin{array}{r}
x^2-4x+1 \\
2x^3+1\overline{\big)\,2x^5-8x^4+2x^3+x^2} \\
\underline{2x^5+x^2} \\
-8x^4+2x^3 \\
\underline{-8x^4-4x} \\
2x^3+4x \\
\underline{2x^3+1} \\
4x-1
\end{array}$$

The answer is $x^2-4x+1+\dfrac{4x-1}{2x^3+1}$.

17. $\left(2x^2+x-10\right)\div(x-2)$

2	2	1	–10
		4	10
	2	5	0

The answer is $2x+5$.

18. $\left(x^2+x-2\right)\div(x-1)$

1	1	1	–2
		1	2
	1	2	0

The answer is $x + 2$.

19. $\left(3x^2+7x-20\right)\div(x+5)$

–5	3	7	–20
		–15	40
	3	–8	20

The answer is $3x-8+\dfrac{20}{x+5}$.

20. $\left(5x^2-12x-8\right)\div(x+3)$

–3	5	12	–8
		–15	81
	5	–27	73

The answer is $5x-27+\dfrac{73}{x+3}$.

21. $\left(4x^3-3x^2+3x-1\right)\div(x-1)$

1	4	–3	3	–1
		4	1	4
	4	1	4	3

The answer is $4x^2+x+4+\dfrac{3}{x-1}$.

22. $\left(5x^3-6x^2+3x+11\right)\div(x-2)$

2	5	–6	3	11
		10	8	22
	5	4	11	33

The answer is $5x^2+4x+11+\dfrac{33}{x-2}$.

23. $\left(6x^5-2x^3+4x^2-3x+1\right)\div(x-2)$

2	6	0	–2	4	–3	1
		12	24	44	96	186
	6	12	22	48	93	187

The answer is

$6x^4+12x^3+22x^2+48x+93+\dfrac{187}{x-2}$.

24. $\left(x^5+4x^4-3x^2+2x+3\right)\div(x-3)$

3	1	4	0	–3	2	3
		3	21	63	180	546
	1	7	21	60	182	549

The answer is

$x^4+7x^3+21x^2+60x+182+\dfrac{549}{x-3}$.

25. $\left(x^2-5x-5x^3+x^4\right)\div(5+x)\Rightarrow$
$\left(x^4-5x^3+x^2-5x\right)\div(x+5)$

–5	1	–5	1	–5	0
		–5	50	–255	1300
	1	–10	51	–260	1300

The answer is
$x^3-10x^2+51x-260+\dfrac{1300}{x+5}$.

26. $\left(x^2-6x-6x^3+x^4\right)\div(6+x)\Rightarrow$
$\left(x^4-6x^3+x^2-6x\right)\div(x+6)$

–6	1	–6	1	–6	0
		–6	72	–438	2664
	1	–12	73	–444	2664

The answer is $x^3-12x^2+73x-444+\dfrac{2664}{x+6}$.

27. $\dfrac{x^5+x^3-2}{x-1}$

1	1	0	1	0	0	–2
		1	1	2	2	2
	1	1	2	2	2	0

The answer is $x^4+x^3+2x^2+2x+2$.

28. $\dfrac{x^7+x^5-10x^3+12}{x+2}$

–2	1	0	1	0	–10	0	0	12
		–2	4	–10	20	–20	40	–80
	1	–2	5	–10	10	–20	40	–68

The answer is $x^6-2x^5+5x^4-10x^3+10x^2$
$-20x+40-\dfrac{68}{x+2}$.

29. $\dfrac{x^4-256}{x-4}$

4	1	0	0	0	–256
		4	16	64	256
	1	4	16	64	0

The answer is $x^3+4x^2+16x+64$.

30. $\dfrac{x^7-128}{x-2}$

2	1	0	0	0	0	0	0	–128
		2	4	8	16	32	64	128
	1	2	4	8	16	32	64	0

The answer is
$x^6+2x^5+4x^4+8x^3+16x^2+32x+64$.

31. $\dfrac{2x^5-3x^4+x^3-x^2+2x-1}{x+2}$

–2	2	–3	1	–1	2	–1
		–4	14	–30	62	–128
	2	–7	15	–31	64	–129

The answer is
$2x^4-7x^3+15x^2-31x+64-\dfrac{129}{x+2}$.

32. $\dfrac{x^5-2x^4-x^3+3x^2-x+1}{x-2}$

2	1	–2	–1	3	–1	1
		2	0	–2	2	2
	1	0	–1	1	1	3

The answer is $x^4-x^2+x+1+\dfrac{3}{x-2}$.

33. $f(x)=2x^3-11x^2+7x-5$

4	2	–11	7	–5
		8	–12	–20
	2	–3	–5	–25

$f(4)=-25$

34.

3	1	–7	5	–6
		3	–12	–21
	1	–4	–7	–27

$f(3)=-27$

35. $f(x)=3x^3-7x^2-2x+5$

–3	3	–7	–2	5
		–9	48	–138
	3	–16	46	–133

$f(-3)=-133$

36.

$$\begin{array}{r|rrrr} -2 & 4 & 5 & -6 & -4 \\ & & -8 & 6 & 0 \\ \hline & 4 & -3 & 0 & -4 \end{array}$$

$f(-2) = -4$

37. $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

$$\begin{array}{r|rrrrr} 3 & 1 & 5 & 5 & -5 & -6 \\ & & 3 & 24 & 87 & 246 \\ \hline & 1 & 8 & 29 & 82 & 240 \end{array}$$

$f(3) = 240$

38.

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 5 & 5 & -6 \\ & & 2 & -6 & -2 & 6 \\ \hline & 1 & -3 & -1 & 3 & 0 \end{array}$$

$f(2) = 0$

39. $f(x) = 2x^4 - 5x^3 - x^2 + 3x + 2$

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -5 & -1 & 3 & 2 \\ & & -1 & 3 & -1 & -1 \\ \hline & 2 & -6 & 2 & 2 & 1 \end{array}$$

$f\left(-\frac{1}{2}\right) = 1$

40.

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 6 & 10 & 5 & 1 & 1 \\ & & -4 & -4 & -\frac{2}{3} & -\frac{2}{9} \\ \hline & 6 & 6 & 1 & \frac{1}{3} & \frac{7}{9} \end{array}$$

$f\left(-\frac{2}{3}\right) = \frac{7}{9}$

41. Dividend: $x^3 - 4x^2 + x + 6$
Divisor: $x + 1$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

The quotient is $x^2 - 5x + 6$.

$(x+1)(x^2 - 5x + 6) = 0$

$(x + 1)(x - 2)(x - 3) = 0$

$x = -1, x = 2, x = 3$

The solution set is $\{-1, 2, 3\}$.

42. Dividend: $x^3 - 2x^2 - x + 2$
Divisor: $x + 1$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -1 & 2 \\ & & -1 & 3 & -2 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

The quotient is $x^2 - 3x + 2$.

$(x+1)(x^2 - 3x + 2) = 0$

$(x + 1)(x - 2)(x - 1) = 0$

$x = -1, x = 2, x = 1$

The solution set is $\{-1, 2, 1\}$.

43. $2x^3 - 5x^2 + x + 2 = 0$

$$\begin{array}{r|rrrr} 2 & 2 & -5 & 1 & 2 \\ & & 4 & -2 & -2 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$(x-2)(2x^2 - x - 1) = 0$

$(x - 2)(2x + 1)(x - 1) = 0$

$x = 2,\ x = -\dfrac{1}{2},\ x = 1$

The solution set is $\left\{-\dfrac{1}{2}, 1, 2\right\}$.

44. $2x^3 - 3x^2 - 11x + 6 = 0$

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$(x + 2)(2x^2 - 7x + 3) = 0$

$(x + 2)(2x - 1)(x - 3) = 0$

$x = -2,\ x = \dfrac{1}{2},\ x = 3$

The solution set is $\left\{-2, \dfrac{1}{2}, 3\right\}$.

45. $12x^3+16x^2-5x-3=0$

$-\frac{3}{2}$	12	16	–5	–3
		–18	3	3
	12	–2	–2	0

$\left(x+\frac{3}{2}\right)(12x^2-2x-2)=0$

$\left(x+\frac{3}{2}\right)2\left(6x^2-x-1\right)=0$

$\left(x+\frac{3}{2}\right)2(3x+1)(2x-1)=0$

$x=-\frac{3}{2},\ x=-\frac{1}{3},\ x=\frac{1}{2}$

The solution set is $\left\{-\frac{3}{2}, -\frac{1}{3}, \frac{1}{2}\right\}$.

46. $3x^3+7x^2-22x-8=0$

$-\frac{1}{3}$	3	7	–22	–8
		–1	–2	8
	3	6	–24	0

$\left(x+\frac{1}{3}\right)3x^2+6x-24=0$

$\left(x+\frac{1}{3}\right)3(x+4)(x-2)=0$

$x=-4, x=2, x=-\frac{1}{3}$

The solution set is $\left\{-4, -\frac{1}{3}, 2\right\}$.

47. The graph indicates that 2 is a solution to the equation.

2	1	2	–5	–6
		2	8	6
	1	4	3	0

The remainder is 0, so 2 is a solution.

$x^3+2x^2-5x-6=0$

$(x-2)\left(x^2+4x+3\right)=0$

$(x-2)(x+3)(x+1)=0$

The solutions are 2, -3, and -1, or $\{-3,-1,2\}$.

48. The graph indicates that -3 is a solution to the equation.

–3	2	1	–13	6
		–6	15	–6
	2	–5	2	0

The remainder is 0, so -3 is a solution.

$2x^3+x^2-13x+6=0$

$(x+3)\left(2x^2-5x+2\right)=0$

$(x+3)(2x-1)(x-2)=0$

The solutions are -3, $\frac{1}{2}$, and 2, or $\left\{-3,\frac{1}{2},2\right\}$.

49. The table indicates that 1 is a solution to the equation.

1	6	–11	6	–1
		6	–5	1
	6	–5	1	0

The remainder is 0, so 1 is a solution.

$6x^3-11x^2+6x-1=0$

$(x-1)\left(6x^2-5x+1\right)=0$

$(x-1)(3x-1)(2x-1)=0$

The solutions are 1, $\frac{1}{3}$, and $\frac{1}{2}$, or $\left\{\frac{1}{3},\frac{1}{2},1\right\}$.

50. The table indicates that 1 is a solution to the equation.

1	2	11	–7	–6
		2	13	–6
	2	13	6	0

The remainder is 0, so 1 is a solution.

$2x^3+11x^2-7x-6=0$

$(x-1)\left(2x^2+13x+6\right)=0$

$(x-1)(2x+1)(x+6)=0$

The solutions are 1, $-\frac{1}{2}$, and -6, or $\left\{-6,-\frac{1}{2},1\right\}$.

51. a. $14x^3-17x^2-16x-177=0$

$$\begin{array}{r|rrrr} 3 & 14 & -17 & -16 & -177 \\ & & 42 & 75 & 177 \\ \hline & 14 & 25 & 59 & 0 \end{array}$$

The remainder is 0 so 3 is a solution.

$14x^3-17x^2-16x-177$
$=(x-3)\left(14x^2+25x+59\right)$

b. $f(x)=14x^3-17x^2-16x+34$

We need to find x when $f(x)=211$.

$$\begin{aligned} f(x)&=14x^3-17x^2-16x+34 \\ 211&=14x^3-17x^2-16x+34 \\ 0&=14x^3-17x^2-16x-177 \end{aligned}$$

This is the equation obtained in part **a.** One solution is 3. It can be used to find other solutions (if they exist).

$$14x^3-17x^2-16x-177=0$$
$$(x-3)\left(14x^2+25x+59\right)=0$$

The polynomial $14x^2+25x+59$ cannot be factored, so the only solution is $x = 3$. The female moth's abdominal width is 3 millimeters.

52. a.

$$\begin{array}{r|rrrr} 2 & 2 & 14 & 0 & -72 \\ & & 4 & 36 & 72 \\ \hline & 2 & 18 & 36 & 0 \end{array}$$

$$2h^3+14h^2-72=(h-2)\left(2h^2+18h+36\right)$$

b.

$$\begin{aligned} V&=lwh \\ 72&=(h+7)(2h)(h) \\ 72&=2h^3+14h^2 \\ 0&=2h^3+14h^2-72 \\ 0&=(h-2)\left(2h^2+18h+36\right) \\ 0&=(h-2)\left(2\left(h^2+9h+18\right)\right) \\ 0&=(h-2)\left(2(h+6)(h+3)\right) \\ 0&=2(h-2)(h+6)(h+3) \end{aligned}$$

$2(h-2)=0 \quad h+6=0 \quad h+3=0$
$h-2=0 \quad \cancel{h=-6} \quad \cancel{h=-3}$
$h=2$

The height is 2 inches, the width is $2\cdot 2=4$ inches and the length is $2+7=9$ inches. The dimensions are 2 inches by 4 inches by 9 inches.

53. $A=l\cdot w$ so

$$l=\frac{A}{w}=\frac{0.5x^3-0.3x^2+0.22x+0.06}{x+0.2}$$

$$\begin{array}{r|rrrr} -0.2 & 0.5 & -0.3 & 0.22 & 0.06 \\ & & -0.1 & 0.08 & -0.06 \\ \hline & 0.5 & -0.4 & 0.3 & 0 \end{array}$$

Therefore, the length of the rectangle is $0.5x^2-0.4x+0.3$ units.

54. $A=l\cdot w$ so,

$$l=\frac{A}{w}=\frac{8x^3-6x^2-5x+3}{x+\frac{3}{4}}$$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 8 & -6 & -5 & 3 \\ & & -6 & 9 & -3 \\ \hline & 8 & -12 & 4 & 0 \end{array}$$

Therefore, the length of the rectangle is $8x^2-12x+4$ units.

55. a.

$$f(30)=\frac{80(30)-8000}{30-110}=70$$

(30, 70) At a 30% tax rate, the government tax revenue will be $70 ten billion.

b.

$$\begin{array}{r|rr} 110 & 80 & -8000 \\ & & 8800 \\ \hline & 80 & 800 \end{array}$$

$$f(x)=80+\frac{800}{x-110}$$
$$f(30)=80+\frac{800}{80-110}=70$$

(30, 70) same answer as in **a.**

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

56. a.

$$f(40)=\frac{80(40)-8000}{40-110}=68.57$$

(40, 68.57) At a 40% tax rate, the government's revenue is $68.57 ten billion.

b.

$$\begin{array}{r|rr} 110 & 80 & -8000 \\ & & 8800 \\ \hline & 80 & 800 \end{array}$$

$$f(x) = 80 + \frac{800}{x-110}$$
$$f(40) = 80 + \frac{800}{40-110} = 68.57$$

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

57. – 65. Answers will vary.

66. does not make sense; Explanations will vary. Sample explanation: The division must account for the zero coefficients on the x^4, x^3, x^2 and x terms.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: The remainder theorem provides an alternative method for evaluating a function at a given value.

69. does not make sense; Explanations will vary. Sample explanation: The zeros of f are the same as the solutions of $f(x) = 0$.

70. false; Changes to make the statement true will vary. A sample change is: The degree of the quotient is 3, since $\frac{x^6}{x^3} = x^3$.

71. true

72. true

73. false; Changes to make the statement true will vary. A sample change is: The divisor is a factor of the divided only if the remainder is the whole number 0.

74.

$$\begin{array}{r} 5x^2 + 2x - 4 \\ 4x+3\overline{)20x^3 + 23x^2 - 10x + k} \\ \underline{20x^3 + 15x^2} \qquad\qquad \\ 8x^2 - 10 \qquad \\ \underline{8x^2 + 6x} \qquad \\ -16x + k \\ \underline{-16x - 12} \end{array}$$

To get a remainder of zero, k must equal –12.
$k = -12$

75. $f(x) = d(x) \cdot q(x) + r(x)$
$$2x^2 - 7x + 9 = d(x)(2x-3) + 3$$
$$2x^2 - 7x + 6 = d(x)(2x-3)$$
$$\frac{2x^2 - 7x + 6}{2x-3} = d(x)$$

$$\begin{array}{r} x - 2 \\ 2x-3\overline{)2x^2 - 7x + 6} \\ \underline{2x^2 - 3x} \qquad \\ -4x + 6 \\ \underline{-4x + 6} \end{array}$$

The polynomial is $x - 2$.

76.

$$\begin{array}{r} x^{2n} - x^n + 1 \\ x^n + 1\overline{)x^{3n} \qquad\qquad + 1} \\ \underline{x^{3n} + x^{2n}} \qquad\qquad \\ -x^{2n} \qquad\qquad \\ \underline{-x^{2n} - x^n} \qquad \\ x^n + 1 \\ \underline{x^n + 1} \\ 0 \end{array}$$

77. $2x - 4 = 2(x-2)$
Use synthetic division to divide by $x - 2$. Then divide the quotient by 2.

78. $x^4 - 4x^3 - 9x^2 + 16x + 20 = 0$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -9 & 16 & 20 \\ & & 5 & 5 & -20 & -20 \\ \hline & 1 & 1 & -4 & -4 & 0 \end{array}$$

The remainder is zero and 5 is a solution to the equation.
$$x^4 - 4x^3 - 9x^2 + 16x + 20 = (x-5)\left(x^3 + x^2 - 4x - 4\right)$$

To solve the equation, we set it equal to zero and factor.
$$(x-5)\left(x^3 + x^2 - 4x - 4\right) = 0$$
$$(x-5)\left(x^2(x+1) - 4(x+1)\right) = 0$$
$$(x-5)(x+1)\left(x^2 - 4\right) = 0$$
$$(x-5)(x+1)(x+2)(x-2) = 0$$

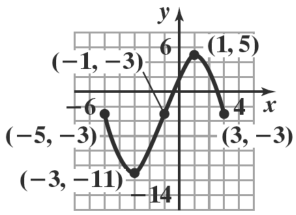
Apply the zero product principle.

$x - 5 = 0$, $x = 5$ $\qquad$ $x + 1 = 0$, $x = -1$

$x + 2 = 0$, $x = -2$ $\qquad$ $x - 2 = 0$, $x = 2$

The solutions are –2, –1, 2 and 5 and the solution set is $\{-2, -1, 2, 5\}$.

79. The graph of $y = f(x)$ is shifted 1 unit left, stretched by a factor of 2, reflected about the x-axis, then shifted down 3 units.



80. **a.** $(f \circ g)(x) = f(g(x))$
$= 2(2x^2 - x + 5) - 3$
$= 4x^2 - 2x + 10 - 3$
$= 4x^2 - 2x + 7$

b. $(g \circ f)(x) = g(f(x))$
$= 2(2x-3)^2 - (2x-3) + 5$
$= 2(4x^2 - 12x + 9) - 2x + 3 + 5$
$= 8x^2 - 24x + 18 - 2x + 3 + 5$
$= 8x^2 - 26x + 26$

c. $(g \circ f)(x) = 8x^2 - 26x + 26$
$(g \circ f)(1) = 8(1)^2 - 26(1) + 26$
$= 8 - 26 + 26$
$= 8$

81. $f(x) = \dfrac{x-10}{x+10}$

Replace $f(x)$ with *y*:

$y = \dfrac{x-10}{x+10}$

Interchange *x* and *y*:

$x = \dfrac{y-10}{y+10}$

Solve for *y*:

$x = \dfrac{y-10}{y+10}$
$x(y+10) = y - 10$
$xy + 10x = y - 10$
$xy - y = -10x - 10$
$y(x-1) = -10x - 10$
$y = \dfrac{-10x-10}{x-1}$

Replace *y* with $f^{-1}(x)$:

$f^{-1}(x) = \dfrac{-10x-10}{x-1}$

82. $x^2 + 4x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$
$x = \dfrac{-4 \pm \sqrt{20}}{2}$
$x = \dfrac{-4 \pm 2\sqrt{5}}{2}$
$x = -2 \pm \sqrt{5}$

The solution set is $\{-2 \pm \sqrt{5}\}$.

83. $x^2 + 4x + 6 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4(1)(6)}}{2(1)}$
$x = \dfrac{-4 \pm \sqrt{-8}}{2}$
$x = \dfrac{-4 \pm 2i\sqrt{2}}{2}$
$x = -2 \pm i\sqrt{2}$

The solution set is $\{-2 \pm i\sqrt{2}\}$.

84. $f(x) = a_n(x^4 - 3x^2 - 4)$

$f(3) = -150$
$a_n\left((3)^4 - 3(3)^2 - 4\right) = -150$
$a_n(81 - 27 - 4) = -150$
$a_n(50) = -150$
$a_n = -3$

Section 2.5

Check Point Exercises

1. $p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$
are the possible rational zeros.

2. $p: \pm 1, \pm 3$
$q: \pm 1, \pm 2, \pm 4$
$\frac{p}{q}: \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}$
are the possible rational zeros.

3. $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$ are possible rational zeros

1	1	8	11	–20
		1	9	20
	1	9	20	0

1 is a zero.
$x^2 + 9x + 20 = 0$
$(x+4)(x+5) = 0$
$x = -4$ or $x = -5$
The zeros are $-5, -4,$ and 1.

4. $\pm 1, \pm 2$ are possible rational zeros

2	1	1	-5	–2
		2	6	2
	1	3	1	0

2 is a zero.
$x^2 + 3x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{5}}{2}$$

The zeros are $2, \frac{-3-\sqrt{5}}{2},$ and $\frac{-3+\sqrt{5}}{2}$. .

5. $\pm 1, \pm 13$ are possible rational zeros.

1	1	–6	22	–30	13
		1	–5	17	–13
	1	–5	17	–13	0

1 is a zero.

1	1	5	17	–13
		1	–4	13
	1	–4	13	0

1 is a double root.
$x^2 - 4x + 13 = 0$

$$x = \frac{4 \pm \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2} = 2 + 3i$$

The solution set is $\{1, 2 - 3i, 2 + 3i\}$.

6. $(x+3)(x-i)(x+i) = (x+3)(x^2+1)$
$f(x) = a_n(x+3)(x^2+1)$
$f(1) = a_n(1+3)(1^2+1) = 8a_n = 8$
$a_n = 1$
$f(x) = (x+3)(x^2+1)$
$f(x) = x^3 + 3x^2 + x + 3$

7. $f(x) = x^4 - 14x^3 + 71x^2 - 154x + 120$
$f(-x) = x^4 + 14x^3 + 71x^2 + 154x + 120$
Since $f(x)$ has 4 changes of sign, there are 4, 2, or 0 positive real zeros.
Since $f(-x)$ has no changes of sign, there are no negative real zeros.

Concept and Vocabulary Check 2.5

1. a_0; a_n

2. true

3. false

4. n

5. $a - bi$

6. -6; $(x+6)(2x^2 - x - 1) = 0$

7. n; 1

8. false

9. true

10. true

Exercise Set 2.5

1. $f(x) = x^3 + x^2 - 4x - 4$

$p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

2. $f(x) = x^3 + 3x^2 - 6x - 8$

$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8$

3. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

p: ±1, ±2, ±3, ±6
q: ±1, ±3
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$

4. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

p: ±1, ±3, ±5, ±15
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

5. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

p: ±1, ±2, ±3, ±6
q: ±1, ±2, ±4
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}$

6. $f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

p: ±1, ±2, ±4, ±8
q: ±1, ±3
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

7. $f(x) = x^5 - x^4 - 7x^3 + 7x^2 - 12x - 12$

$p: \pm 1, \pm 2, \pm 3 \pm 4 \pm 6 \pm 12$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3 \pm 4 \pm 6 \pm 12$

8. $f(x) = 4x^5 - 8x^4 - x + 2$

$p: \pm 1, \pm 2$
$q: \pm 1, \pm 2, \pm 4$
$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{4}$

9. $f(x) = x^3 + x^2 - 4x - 4$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.

2	1	1	–4	–4
		2	6	4
	1	3	2	0

2 is a zero.
2, –2, –1 are rational zeros.

c. $x^3 + x^2 - 4x - 4 = 0$
$(x-2)(x^2 + 3x + 2) = 0$
$(x-2)(x+2)(x+1) = 0$

$x - 2 = 0 \quad x + 2 = 0 \quad x + 1 = 0$
$x = 2, \ x = -2, \ x = -1$

The solution set is {2, –2, –1}.

10. a. $f(x) = x^3 - 2x - 11x + 12$

p: ±1, ±2, ±3, ±4, ±6, ±12
q: ±1
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.

4	1	–2	–11	12
		4	8	–12
	1	2	–3	0

4 is a zero.
4, –3, 1 are rational zeros.

c. $x^3 - 2x^2 - 11x + 12 = 0$
$(x-4)\left(x^2 + 2x - 3\right) = 0$
$(x-4)\ (x+3)(x-1) = 0$
$x = 4, \ x = -3, \ x = 1$

The solution set is{4, –3, 1}.

11. $f(x) = 2x^3 - 3x^2 - 11x + 6$

a. $p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

b.

3	2	–3	–11	6
		6	9	–6
	2	3	–2	0

3 is a zero.

$3, \frac{1}{2}, -2$ are rational zeros.

c. $2x^3-3x^2-11x+6=0$

$(x-3)(2x^2+3x-2)=0$

$(x-3)(2x-1)(x+2)=0$

$x=3,\ x=\frac{1}{2},\ x=-2$

The solution set is $\left\{3, \frac{1}{2}, -2\right\}$.

12. a. $f(x)=2x^3-5x^2+x+2$

$p: \pm1, \pm2$

$q: \pm1, \pm2$

$\frac{p}{q}: \pm1, \pm2, \pm\frac{1}{2}$

b.

2	2	–5	1	2
		4	–2	–2
	2	–1	–1	0

2 is a zero.

$2, -\frac{1}{2}, 1$ are rational zeros.

c. $2x^3-5x^2+x+2=0$

$(x-2)\left(2x^2-x-1\right)=0$

$(x-2)\ (2x+1)(x-1)=0$

$x=2,\ x=-\frac{1}{2},\ x=1$

The solution set is $\left\{2, -\frac{1}{2}, 1\right\}$.

13. a. $f(x)=x^3+4x^2-3x-6$

$p: \pm1, \pm2, \pm3, \pm6$

$q: \pm1$

$\frac{p}{q}: \pm1, \pm2, \pm3, \pm6$

b.

–1	1	4	–3	–6
		–1	–3	6
	1	3	–6	0

–1 is a rational zero.

c. $x^2+3x-6=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-3\pm\sqrt{3^2-4(1)(-6)}}{2(1)}$

$=\frac{-3\pm\sqrt{33}}{2}$

The solution set is $\left\{-1, \frac{-3+\sqrt{33}}{2}, \frac{-3-\sqrt{33}}{2}\right\}$.

14. a. $f(x)=2x^3+x^2-3x+1$

$p: \pm1$

$q: \pm1, \pm2$

$\frac{p}{q}: \pm1, \pm\frac{1}{2}$

b.

$\frac{1}{2}$	2	1	–3	1
		1	1	–1
	2	2	–2	0

$\frac{1}{2}$ is a rational zero.

c. $2x^2+2x-2=0$

$x^2+x-1=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-1\pm\sqrt{1^2-4(1)(-1)}}{2(1)}$

$=\frac{-1\pm\sqrt{5}}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}\right\}$.

15. a. $f(x)=2x^3+6x^2+5x+2$

$p: \pm1, \pm2$

$q: \pm1, \pm2$

$\frac{p}{q}: \pm1, \pm2, \pm\frac{1}{2}$

b.

–2	2	6	5	2
		–4	–4	–2
	2	2	1	0

–2 is a rational zero.

c. $2x^2+2x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{2^2-4(2)(1)}}{2(2)}$$

$$=\frac{-2\pm\sqrt{-4}}{4}$$

$$=\frac{-2\pm 2i}{4}$$

$$=\frac{-1\pm i}{2}$$

The solution set is $\left\{-2,\frac{-1+i}{2},\frac{-1-i}{2}\right\}$.

16. a. $f(x)=x^3-4x^2+8x-5$

p: ±1, ±5

q: ±1

$\frac{p}{q}: \pm1, \pm5$

b.

1	1	–4	8	–5
		1	–3	5
	1	–3	5	0

1 is a rational zero.

c. $x^2-3x+5=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(5)}}{2(1)}$$

$$=\frac{3\pm\sqrt{-11}}{2}$$

$$=\frac{3\pm i\sqrt{11}}{2}$$

The solution set is $\left\{1,\frac{3+i\sqrt{11}}{2},\frac{3-i\sqrt{11}}{2}\right\}$.

17. $x^3-2x^2-11x+12=0$

a. $p: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$q: \pm1$

$\frac{p}{q}: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

b.

4	1	–2	–11	12
		4	8	–12
	1	2	–3	0

4 is a root.

–3, 1, 4 are rational roots.

c. $x^3-2x^2-11x+12$

$(x-4)(x^2+2x-3)=0$

$(x-4)(x+3)(x-1)=0$

$x-4=0\quad x+3=0\quad x-1=0$

$x=4\qquad x=-3\qquad x=1$

The solution set is {–3, 1, 4}.

18. a. $x^3-2x^2-7x-4=0$

p: ±1, ±2, ±4

q: ±1

$\frac{p}{q}: \pm1, \pm2, \pm4$

b.

4	1	–2	–7	–4
		4	8	4
	1	2	1	0

4 is a root.

–1, 4 are rational roots.

c. $x^3+2x^2-7x-4=0$

$(x-4)\left(x^2+2x+1\right)=0$

$(x-4)\quad (x+1)^2$

$x=4,\qquad x=-1$

The solution set is $\{4,-1\}$.

19. $x^3-10x-12=0$

a. $p: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$q: \pm1$

$\frac{p}{q}: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

b.

–2	1	0	–10	–12
		–2	4	12
	1	–2	–6	0

–2 is a rational root.

c. $x^3-10x-12=0$

$(x+2)(x^2-2x-6)=0$

$$x=\frac{2\pm\sqrt{4+24}}{2}=\frac{2\pm\sqrt{28}}{2}$$

$$=\frac{2\pm2\sqrt{7}}{2}=1\pm\sqrt{7}$$

The solution set is $\left\{-2, 1+\sqrt{7}, 1-\sqrt{7}\right\}$.

20. a. $x^3 - 5x^2 + 17x - 13 = 0$
p: ±1, ±13
q: ±1
$\frac{p}{q}: \pm 1, \pm 13$

b.

1	1	–5	17	–13
		1	–4	13
	1	–4	13	0

1 is a rational root.

c. $x^3 - 5x^2 + 17x - 13 = 0$
$(x-1)\left(x^2 - 4x + 13\right) = 0$
$x = \frac{4 + \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2}$
$= \frac{4 \pm 6i}{2} = 2 \pm 3i$
The solution set is $\{1,\ 2+3i,\ 2, -3i\}$.

21. $6x^3 + 25x^2 - 24x + 5 = 0$

a. $p: \pm 1, \pm 5$
$q: \pm 1, \pm 2, \pm 3, \pm 6$
$\frac{p}{q}: \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$

b.

–5	6	25	–24	5
		–30	25	–5
	6	–5	1	0

–5 is a root.
$-5, \frac{1}{2}, \frac{1}{3}$ are rational roots.

c. $6x^3 + 25x^2 - 24x + 5 = 0$
$(x+5)(6x^2 - 5x + 1) = 0$
$(x+5)(2x-1)(3x-1) = 0$
$x+5=0 \quad 2x-1=0 \quad 3x-1=0$
$x=-5, \quad x=\frac{1}{2}, \quad x=\frac{1}{3}$
The solution set is $\left\{-5, \frac{1}{2}, \frac{1}{3}\right\}$.

22. a. $2x^3 - 5x^2 - 6x + 4 = 0$
p: ±1, ±2, ±4
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm \frac{1}{2}$

b.

$\frac{1}{2}$	2	–5	–6	4
		1	–2	–4
	2	–4	–8	0

$\frac{1}{2}$ is a rational root.

c. $2x^3 - 5x^2 - 6x + 4 = 0$
$(x - \frac{1}{2})\left(2x^2 - 4x - 8\right) = 0$
$2\left(x - \frac{1}{2}\right)\left(x^2 - 2x - 4\right) = 0$
$x = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$
The solution set is $\left\{\frac{1}{2}, 1+\sqrt{5}, 1-\sqrt{5}\right\}$.

23. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.

2	1	–2	–5	8	4
		2	0	–10	–4
	1	0	–5	–2	0

2 is a root.
$-2, 2$ are rational roots.

c. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$
$(x-2)(x^3 - 5x - 2) = 0$

–2	1	0	–5	–2
		–2	4	2
	1	–2	–1	0

–2 is a zero of $x^3 - 5x - 2 = 0$.

$(x-2)(x+2)\left(x^2 - 2x - 1\right) = 0$
$x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$
$= 1 \pm \sqrt{2}$
The solution set is
$\left\{-2, 2, 1+\sqrt{2}, 1-\sqrt{2}\right\}$.

24. **a.** $x^4 - 2x^2 - 16x - 15 = 0$

p: ±1, ±3, ±5, ±15

q: ±1

$\frac{p}{q}: \pm 1, \pm 3 \pm 5 \pm 15$

b.

3	1	0	–2	–16	–15
		3	9	21	15
	1	3	7	5	0

3 is a root.

$-1, 3$ are rational roots.

c. $x^4 - 2x^2 - 16x - 15 = 0$

$(x-3)\left(x^3 + 3x^2 + 7x + 5\right) = 0$

–1	1	3	7	5
		–1	–2	–5
	1	2	5	0

–1 is a root of $x^3 + 3x^2 + 7x + 5$

$(x-3)\ (x+1)\ \left(x^2 + 2x + 5\right)$

$x = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$

$= \frac{-2 \pm 4i}{2} = -1 \pm 2i$

The solution set is $\{3, -1, -1+2i, -1-2i\}$.

25. $(x-1)\ (x+5i)\ (x-5i)$

$= (x-1)\ \left(x^2 + 25\right)$

$= x^3 + 25x - x^2 - 25$

$= x^3 - x^2 + 25x - 25$

$f(x) = a_n\left(x^3 - x^2 + 25x - 25\right)$

$f(-1) = a_n(-1-1-25-25)$

$-104 = a_n(-52)$

$a_n = 2$

$f(x) = 2\left(x^3 - x^2 + 25x - 25\right)$

$f(x) = 2x^3 - 2x^2 + 50x - 50$

26. $(x-4)\ (x+2i)\ (x-2i)$

$= (x-4)\ \left(x^2 + 4\right)$

$= x^3 - 4x^2 + 4x - 16$

$f(x) = a_n\left(x^3 - 4x^2 + 4x - 16\right)$

$f(-1) = a_n(-1-4-4-16)$

$-50 = a_n(-25)$

$a_n = 2$

$f(x) = 2\left(x^3 - 4x^2 + 4x - 16\right)$

$f(x) = 2x^3 - 8x^2 + 8x - 32$

27. $(x+5)(x-4-3i)(x-4+3i)$

$= (x+5)\left(x^2 - 4x + 3ix - 4x + 16 - 12i\right.$

$\left.-3ix + 12i - 9i^2\right)$

$= (x+5)\left(x^2 - 8x + 25\right)$

$= \left(x^3 - 8x^2 + 25x + 5x^2 - 40x + 125\right)$

$= x^3 - 3x^2 - 15x + 125$

$f(x) = a_n(x^3 - 3x^2 - 15x + 125)$

$f(2) = a_n\left(2^3 - 3(2)^2 - 15(2) + 125\right)$

$91 = a_n(91)$

$a_n = 1$

$f(x) = 1\left(x^3 - 3x^2 - 15x + 125\right)$

$f(x) = x^3 - 3x^2 - 15x + 125$

28. $(x-6)\ (x+5+2i)\ (x+5-2i)$

$= (x-6)\ \left(x^2 + 5x - 2ix + 5x + 25 - 10i + 2ix + 10i - 4i^2\right)$

$= (x-6)\ \left(x^2 + 10x + 29\right)$

$= x^3 + 10x^2 + 29x - 6x^2 - 60x - 174$

$= x^3 + 4x^2 - 31x - 174$

$f(x) = a_n\left(x^3 + 4x^2 - 31x - 174\right)$

$f(2) = a_n(8 + 16 - 62 - 174)$

$-636 = a_n(-212)$

$a_n = 3$

$f(x) = 3\left(x^3 + 4x^2 - 31x - 174\right)$

$f(x) = 3x^3 + 12x^2 - 93x - 522$

29. $(x-i)(x+i)(x-3i)(x+3i)$

$=\left(x^2-i^2\right)\left(x^2-9i^2\right)$

$=\left(x^2+1\right)\left(x^2+9\right)$

$=x^4+10x^2+9$

$f(x)=a_n(x^4+10x^2+9)$

$f(-1)=a_n((-1)^4+10(-1)^2+9)$

$20=a_n(20)$

$a_n=1$

$f(x)=x^4+10x^2+9$

30. $(x+2)\left(x+\frac{1}{2}\right)(x-i)(x+i)$

$=\left(x^2+\frac{5}{2}x+1\right)\left(x^2+1\right)$

$=x^4+x^2+\frac{5}{2}x^3+\frac{5}{2}x+x^2+1$

$=x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1$

$f(x)=a_n\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(1)=a_n\left[(1)^4+\frac{5}{2}(1)^3+2(1)^2+\frac{5}{2}(1)+1\right]$

$18=a_n(9)$

$a_n=2$

$f(x)=2\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(x)=2x^4+5x^3+4x^2+5x+2$

31. $(x+2)\ (x-5)\ (x-3+2i)\ (x-3-2i)$

$=\left(x^2-3x-10\right)\left(x^2-3x-2ix-3x+9+6i+2ix-6i-4i^2\right)$

$=\left(x^2-3x-10\right)\left(x^2-6x+13\right)$

$=x^4-6x+13x^2-3x^3+18x^2-39x-10x^2+60x-130$

$=x^4-9x^3+21x^2+21x-130$

$f(x)=a_n\left(x^4-9x^3+21x^2+21x-130\right)$

$f(1)=a_n(1-9+21+21-130)$

$-96=a_n(-96)$

$a_n=1$

$f(x)=x^4-9x^3+21x^2+21x-130$

32. $(x+4)\ (3x-1)\ (x-2+3i)\ (x-2-3i)$

$$
\begin{aligned}
&= \left(3x^2+11x-4\right)\left(x^2-2x-3ix-2x+4+6i+3ix-6i-9i^2\right)\\
&= \left(3x^2+11x-4\right)\left(x^2-4x+13\right)\\
&= 3x^4-12x^3+39x^2+11x^3-44x^2+143x-4x^2+16x-52\\
&= 3x^4-x^3-9x^2+159x-52
\end{aligned}
$$

$$f(x)=a_n\left(3x^4-x^3-9x^2+159x-52\right)$$
$$f(1)=a_n(3-1-9+159-52)$$
$$100=a_n(100)$$
$$a_n=1$$
$$f(x)=3x^4-x^3-9x^2+159x-52$$

33. $f(x)=x^3+2x^2+5x+4$

Since $f(x)$ has no sign variations,
no positive real roots exist.

$f(-x)=-x^3+2x^2-5x+4$

Since $f(-x)$ has 3 sign variations,
3 or 1 negative real roots exist.

34. $f(x)=x^3+7x^2+x+7$

Since $f(x)$ has no sign variations no positive real roots exist.

$f(-x)=-x^3+7x^2-x+7$

Since $f(-x)$ has 3 sign variations, 3 or 1 negative real roots exist.

35. $f(x)=5x^3-3x^2+3x-1$

Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.

$f(-x)=-5x^3-3x^2-3x-1$

Since $f(-x)$ has no sign variations, no negative real roots exist.

36. $f(x)=-2x^3+x^2-x+7$

Since $f(x)$ has 3 sign variations,
3 or 1 positive real roots exist.

$f(-x)=2x^3+x^2+x+7$

Since $f(-x)$ has no sign variations,
no negative real roots exist.

37. $f(x)=2x^4-5x^3-x^2-6x+4$

Since $f(x)$ has 2 sign variations, 2 or 0 positive real roots exist.

$f(-x)=2x^4+5x^3-x^2+6x+4$

Since $f(-x)$ has 2 sign variations, 2 or 0 negative real roots exist.

38. $f(x)=4x^4-x^3+5x^2-2x-6$

Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.

$f(-x)=4x^4+x^3+5x^2+2x-6$

Since $f(x)$ has 1 sign variations, 1 negative real roots exist.

39. $f(x) = x^3 - 4x^2 - 7x + 10$

$p: \pm 1, \pm 2, \pm 5, \pm 10$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 5, \pm 10$

Since $f(x)$ has 2 sign variations, 0 or 2 positive real zeros exist.

$f(-x) = -x^3 - 4x^2 + 7x + 10$

Since $f(-x)$ has 1 sign variation, exactly one negative real zeros exists.

–2	1	–4	–7	10
		–2	12	–10
	1	–6	5	0

–2 is a zero.

$$f(x) = (x+2)\left(x^2 - 6x + 5\right)$$
$$= (x+2)(x-5)(x-1)$$

$x = -2, x = 5, x = 1$

The solution set is $\{-2, 5, 1\}$.

40. $f(x) = x^3 + 12x^2 + 2x + 10$

p: ±1, ±2, ±5, ±10

q: ±1,

$\frac{p}{q}: \pm 1, \pm 2 \pm 5 \pm 10$

Since $f(x)$ has no sign variations, no positive zeros exist.

$f(-x) = -x^3 + 12x^2 - 21x + 10$

Since $f(-x)$ has 3 sign variations, 3 or 1 negative zeros exist.

–1	1	12	21	10
		–1	–11	–10
	1	11	10	0

–1 is a zero.

$$f(x) = (x+1)\left(x^2 + 11x + 10\right)$$
$$= (x+1)(x+10)(x+1)$$
$$x = -1, x = -10$$

The solution set is $\{-1, -10\}$.

41. $2x^3 - x^2 - 9x - 4 = 0$

$p: \pm 1, \pm 2, \pm 4$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4 \pm \frac{1}{2}$

1 positive real root exists.

$f(-x) = -2x^3 - x^2 + 9x - 4$ 2 or no negative real roots exist.

$-\frac{1}{2}$	2	–1	–9	–4
		–1	1	4
	2	–2	–8	0

$-\frac{1}{2}$ is a root.

$$\left(x + \frac{1}{2}\right)\left(2x^2 - 2x - 8\right) = 0$$
$$2\left(x + \frac{1}{2}\right)\left(x^2 - x - 4\right) = 0$$
$$x = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

The solution set is $\left\{-\frac{1}{2}, \frac{1+\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2}\right\}$.

42. $3x^3 - 8x^2 - 8x + 8 = 0$

p: ±1, ±2, ±4, ±8

q: ±1, ±3

$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

Since $f(x)$ has 2 sign variations, 2 or no positive real roots exist.

$f(-x) = -3x^3 - 8x^2 + 8x + 8$

Since $f(-x)$ has 1 sign changes, exactly 1 negative real zero exists.

$\frac{2}{3}$	3	–8	–8	8
		2	–4	–8
	3	–6	–12	0

$\frac{2}{3}$ is a zero.

$$f(x) = \left(x - \frac{2}{3}\right)\left(3x^2 - 6x - 12\right)$$
$$x = \frac{6 \pm \sqrt{36+144}}{6} = \frac{6 \pm 6\sqrt{5}}{6} = 1 \pm \sqrt{5}$$

The solution set is $\left\{\frac{2}{3}, 1+\sqrt{5}, 1-\sqrt{5}\right\}$.

43. $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$

$p: \pm 1, \ \pm 2, \ \pm 4, \ \pm 8$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \ \pm 2, \ \pm 4, \ \pm 8$

Since $f(x)$ has 2 sign changes, 0 or 2 positive roots exist.

$f(-x) = (-x)^4 - 2(-x)^3 + (-x)^2 - 12x + 8$
$= x^4 + 2x^3 + x^2 - 12x + 8$

Since $f(-x)$ has 2 sign changes, 0 or 2 negative roots exist.

–1	1	–2	1	12	8
		–1	4	–4	–8
	1	–3	4	8	0

–1	1	–3	4	8
		–1	4	–8
	1	–4	8	0

$0 = x^2 - 4x + 8$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16 - 32}}{2}$

$x = \frac{4 \pm \sqrt{-16}}{2}$

$x = \frac{4 \pm 4i}{2}$

$x = 2 \pm 2i$

The solution set is $\{-1, -1, 2 + 2i, 2 - 2i\}$.

44. $f(x) = x^4 - 4x^3 - x^2 + 14x + 10$

p: ±1, ±2, ±5, ±10
q: ±1
$\frac{p}{q}$: ±1, ±2, ±5, ±10

–1	1	-4	-1	14	10
		–1	5	-4	-10
	1	-5	4	10	0

–1	1	-5	4	10
		–1	6	–10
	1	-6	10	0

$f(x) = (x-1)(x-1)(x^2 - 6x + 10)$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)} \qquad x = 1$

$x = \frac{6 \pm \sqrt{36 - 40}}{2}$

$x = \frac{6 \pm \sqrt{-4}}{2}$

$x = \frac{6 \pm 2i}{2}$

$x = 3 \pm i$

The solution set is $\{-1, 3 - i, 3 + i\}$

45. $x^4 - 3x^3 - 20x^2 - 24x - 8 = 0$

$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4 \pm 8$

1 positive real root exists.
3 or 1 negative real roots exist.

–1	1	–3	–20	–24	–8
		–1	4	16	8
	1	–4	–16	–8	0

$(x+1)\left(x^3 - 4x^2 - 16x - 8\right) = 0$

–2	1	–4	–16	–8
		–2	12	8
	1	–6	–4	0

$(x+1)(x+2)\left(x^2 - 6x - 4\right) = 0$

$x = \frac{6 \pm \sqrt{36 + 16}}{2} = \frac{6 \pm \sqrt{52}}{2}$

$= \frac{6 \pm 2\sqrt{13}}{2} = \frac{3 \pm \sqrt{13}}{2}$

The solution set is
$\left\{-1, -2, 3 \pm \sqrt{13}, 3 - \sqrt{13}\right\}$.

46. $x^4 - x^3 + 2x^2 - 4x - 8 = 0$
p: ±1, ±2, ±4, ±8
q: ±1
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8$
1 negative real root exists.

–1	1	–1	2	–4	–8
		–1	2	–4	8
	1	–2	4	–8	0

$(x+1)\left(x^3 - 2x^2 + 4x - 8\right)$

2	1	–2	4	–8
		2	0	8
	1	0	4	0

$(x+1) \quad (x-2) \quad \left(x^2+4\right)$

$x+1=0 \quad x-2=0 \quad x^2+4=0$

$x=-1 \quad x=2 \quad x^2=-4$

$x = \pm 2i$

The solution set is $\{-1, 2, 2i, -2i\}$.

47. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$
$p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 3$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$
2 or no positive real zeros exists.
$f(-x) = 3x^4 + 11x^3 - x^2 - 19x + 6$
2 or no negative real zeros exist.

–1	3	–11	–1	19	6
		–3	14	–13	–6
	3	–14	13	6	0

$f(x) = (x+1)\left(3x^3 - 14x^2 + 13x + 6\right)$

2	3	–14	13	6
		6	–16	–6
	3	–8	–3	0

$f(x) = (x+1)(x-2)\left(3x^2 - 8x - 3\right)$
$= (x+1)(x-2)(3x+1)(x-3)$

$x=-1,\ x=2\ x=-\frac{1}{3},\ x=3$

The solution set is $\left\{-1, 2, -\frac{1}{3}, 3\right\}$.

48. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$
p: ±1, ±3, ±5, ±15
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$
2 or no positive real zeros exist.
$f(-x) = 2x^4 - 3x^3 - 11x^2 + 9x + 15$
2 or no negative real zeros exist.

1	2	3	–11	–9	15
		2	5	–6	–15
	2	5	–6	–15	0

$f(x) = (x-1)\left(2x^3 + 5x^2 - 6x - 15\right)$

$-\frac{5}{2}$	2	5	–6	–15
		–5	0	15
	2	0	–6	0

$f(x) = (x-1)\left(x+\frac{5}{2}\right)\left(2x^2 - 6\right)$
$= 2(x-1)\left(x+\frac{5}{2}\right)\left(x^2 - 3\right)$

$x^2 - 3 = 0$
$x^2 = 3$
$x = \pm\sqrt{3}$

$x=1,\ x=-\frac{5}{2},\ x=\sqrt{3},\ x=-\sqrt{3}$

The solution set is $\left\{1, -\frac{5}{2}, \sqrt{3}, -\sqrt{3}\right\}$.

49. $4x^4 - x^3 + 5x^2 - 2x - 6 = 0$
$p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 2, \pm 4$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$
3 or 1 positive real roots exists.
1 negative real root exists.

1	4	–1	5	–2	–6
		4	3	8	6
	4	3	8	6	0

$(x-1)(4x^3 + 3x^2 + 8x + 6) = 0$

$4x^3 + 3x^2 + 8x + 6 = 0$ has no positive real roots.

$-\frac{3}{4}$	4	3	8	6
		–3	0	–6
	4	0	8	0

$$(x-1)\left(x+\frac{3}{4}\right)\left(4x^2+8\right) = 0$$
$$4(x-1)\left(x+\frac{3}{4}\right)\left(x^2+2\right) = 0$$

$x^2 + 2 = 0$
$x^2 = -2$
$x = \pm i\sqrt{2}$

The solution set is $\left\{1, -\frac{3}{4}, i\sqrt{2}, -i\sqrt{2}\right\}$.

50. $3x^4 - 11x^3 - 3x^2 - 6x + 8 = 0$
p: ±1, ±2, ±4, ±8
q: ±1, ±3
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$
2 or no positive real roots exist.
$f(-x) = 3x^4 + 11x^3 - 3x^2 + 6x + 8$ 2 or no negative real roots exist.

4	3	–11	–3	–6	8
		12	4	4	–8
	3	1	1	–2	0

$$(x-4)\left(3x^3 + x^2 + x - 2\right) = 0$$

Another positive real root must exist.

$\frac{2}{3}$	3	1	1	–2
		2	2	2
	3	3	3	0

$$(x-4)\left(x-\frac{2}{3}\right)\left(3x^2+3x+3\right) = 0$$
$$3(x-4)\left(x-\frac{2}{3}\right)\left(x^2+x+1\right) = 0$$
$$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

The solution set is $\left\{4, \frac{2}{3}, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$.

51. $2x^5 + 7x^4 - 18x^2 - 8x + 8 = 0$
$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{2}$
2 or no positive real roots exists.
3 or 1 negative real root exist.

–2	2	7	0	–18	–8	8
		–4	–6	12	12	–8
	2	3	–6	–6	4	0

$(x+2)(2x^4 + 3x^3 - 6x^2 - 6x + 4) = 0$

$4x^3 + 3x^2 + 8x + 6 = 0$ has no positive real roots.

–2	2	3	–6	–6	4
		–4	2	8	–4
	2	–1	–4	2	0

$(x+2)^2(2x^3 - x^2 - 4x + 2)$

$\frac{1}{2}$	2	–1	–4	2
		1	0	2
	2	0	–4	0

$$(x+2)^2\left(x-\frac{1}{2}\right)\left(2x^2-4\right) = 0$$
$$2(x+2)^2\left(x-\frac{1}{2}\right)\left(x^2-2\right) = 0$$

$x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

The solution set is $\left\{-2, \frac{1}{2}, \sqrt{2}, -\sqrt{2}\right\}$.

52. $4x^5 + 12x^4 - 41x^3 - 99x^2 + 10x + 24 = 0$

p: ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24
q: ±1, ±2, ±4

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

2 or no positive real roots exist.

$f(-x) = -4x^5 + 12x^4 + 41x^3 - 99x^2 - 10x + 24$

3 or 1 negative real roots exist.

3	4	12	–41	–99	10	24
		12	72	93	–18	–24
	4	24	31	–6	–8	0

$(x-3)\left(4x^4 + 24x^3 + 31x^2 - 6x - 8\right) = 0$

–2	4	24	31	–6	–8
		–8	–32	2	8
	4	16	–1	–4	0

$(x-3)(x+2)\left(4x^3 + 16x^2 - x - 4\right) = 0$

–4	4	16	–1	4
		–16	0	4
	4	0	–1	0

$(x-3)(x+2)(x+4)\left(4x^2 - 1\right) = 0$

$4x^2 - 1 = 0$
$4x^2 = 1$
$x^2 = \frac{1}{4}$
$x = \pm\frac{1}{2}$

The solution set is $\left\{3, -2, -4, \frac{1}{2}, -\frac{1}{2}\right\}$.

53. $f(x) = -x^3 + x^2 + 16x - 16$

a. From the graph provided, we can see that -4 is an x-intercept and is thus a zero of the function. We verify this below:

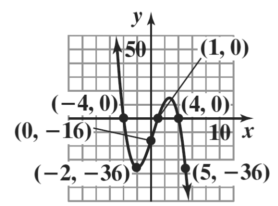
–4	–1	1	16	–16
		4	–20	16
	–1	5	–4	0

Thus,
$$-x^3 + x^2 + 16x - 16 = 0$$
$$(x+4)\left(-x^2 + 5x - 4\right) = 0$$
$$-(x+4)\left(x^2 - 5x + 4\right) = 0$$
$$-(x+4)(x-1)(x-4) = 0$$

$x + 4 = 0$ or $x - 1 = 0$ or $x - 4 = 0$
$x = -4$ $\quad x = 1$ $\quad x = 4$

The zeros are -4, 1, and 4.

b.



$f(x) = -x^3 + x^2 + 16x - 16$

54. $f(x) = -x^3 + 3x^2 - 4$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

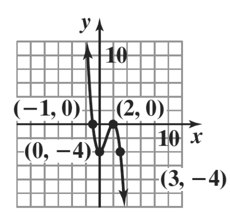
–1	–1	3	0	–4
		1	–4	4
	–1	4	–4	0

Thus,
$$-x^3 + 3x^2 - 4 = 0$$
$$(x+1)\left(-x^2 + 4x - 4\right) = 0$$
$$-(x+1)\left(x^2 - 4x + 4\right) = 0$$
$$-(x+1)(x-2)^2 = 0$$

$x + 1 = 0$ or $(x-2)^2 = 0$
$x = -1$ $\quad x - 2 = 0$
$x = 2$

The zeros are -1 and 2.

b.



$f(x) = -x^3 + 3x^2 - 4$

55. $f(x)=4x^3-8x^2-3x+9$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrr} -1 & 4 & -8 & -3 & 9 \\ & & -4 & 12 & -9 \\ \hline & 4 & -12 & 9 & 0 \end{array}$$

Thus,
$$4x^3-8x^2-3x+9=0$$
$$(x+1)(4x^2-12x+9)=0$$
$$(x+1)(2x-3)^2=0$$

$x+1=0$ or $(2x-3)^2=0$

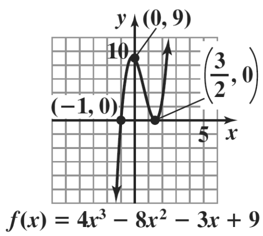
$x=-1$ $\quad 2x-3=0$

$2x=3$

$x=\frac{3}{2}$

The zeros are -1 and $\frac{3}{2}$.

b.



56. $f(x)=3x^3+2x^2+2x-1$

a. From the graph provided, we can see that $\frac{1}{3}$ is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 2 & 2 & -1 \\ & & 1 & 1 & 1 \\ \hline & 3 & 3 & 3 & 0 \end{array}$$

Thus,
$$3x^3+2x^2+2x-1=0$$
$$\left(x-\frac{1}{3}\right)(3x^2+3x+3)=0$$
$$3\left(x-\frac{1}{3}\right)(x^2+x+1)=0$$

Note that x^2+x+1 will not factor, so we use the quadratic formula:

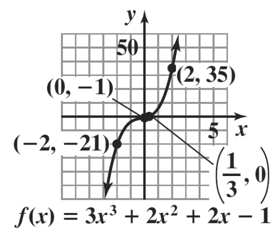
$x-\frac{1}{3}=0$ or $x^2+x+1=0$

$x=\frac{1}{3}$ $\quad a=1 \quad b=1 \quad c=1$

$$x=\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{-3}}{2}$$
$$=\frac{-1\pm\sqrt{3}i}{2}=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

The zeros are $\frac{1}{3}$ and $-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$.

b.



57. $f(x)=2x^4-3x^3-7x^2-8x+6$

a. From the graph provided, we can see that $\frac{1}{2}$ is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -3 & -7 & -8 & 6 \\ & & 1 & -1 & -4 & -6 \\ \hline & 2 & -2 & -8 & -12 & 0 \end{array}$$

Thus,
$$2x^4-3x^3-7x^2-8x+6=0$$
$$\left(x-\frac{1}{2}\right)(2x^3-2x^2-8x-12)=0$$
$$2\left(x-\frac{1}{2}\right)(x^3-x^2-4x-6)=0$$

To factor x^3-x^2-4x-6, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term -6:
$\pm1,\ \pm2,\ \pm3,\ \pm6$

Factors of the leading coefficient 1: ±1

The possible rational zeros are:

$$\frac{\text{Factors of } -6}{\text{Factors of } 1}=\frac{\pm1,\ \pm2,\ \pm3,\ \pm6}{\pm1}$$
$$=\pm1,\ \pm2,\ \pm3,\ \pm6$$

We test values from above until we find a zero. One possibility is shown next:

Test 3:

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -4 & -6 \\ & & 3 & 6 & 6 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

The remainder is 0, so 3 is a zero of *f*.

$$2x^4-3x^3-7x^2-8x+6=0$$
$$\left(x-\frac{1}{2}\right)\left(2x^3-2x^2-8x-12\right)=0$$
$$2\left(x-\frac{1}{2}\right)\left(x^3-x^2-4x-6\right)=0$$
$$2\left(x-\frac{1}{2}\right)(x-3)\left(x^2+2x+2\right)=0$$

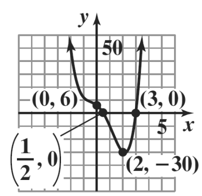
Note that x^2+x+1 will not factor, so we use the quadratic formula:

$a=1 \quad b=2 \quad c=2$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(2)}}{2(1)}$$
$$=\frac{-2\pm\sqrt{-4}}{2}=\frac{-2\pm 2i}{2}=-1\pm i$$

The zeros are $\frac{1}{2}$, 3, and $-1\pm i$.

b.



$f(x)=2x^4-3x^3-7x^2-8x+6$

58. $f(x)=2x^4+2x^3-22x^2-18x+36$

a. From the graph provided, we can see that 1 and 3 are *x*-intercepts and are thus zeros of the function. We verify this below:

1	2	2	−22	−18	36
		2	4	−18	−36
	2	4	−18	−36	0

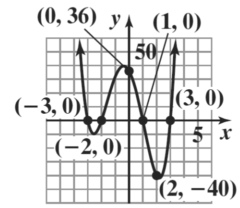
Thus, $2x^4+2x^3-22x^2-18x+36$
$=(x-1)\left(2x^3+4x^2-18x-36\right)$

3	2	4	−18	−36
		6	30	36
	2	10	12	0

Thus, $2x^4+2x^3-22x^2-18x+36=0$
$$(x-1)(x-3)\left(2x^2+10x+12\right)=0$$
$$2(x-1)(x-3)\left(x^2+5x+6\right)=0$$
$$2(x-1)(x-3)(x+3)(x+2)=0$$
$$x=1,\ x=3,\ x=-3,\ x=-2$$

The zeros are -3, -2, 1, and 3.

b.



$f(x)=2x^4+2x^3-22x^2-18x+36$

59. $f(x)=3x^5+2x^4-15x^3-10x^2+12x+8$

a. From the graph provided, we can see that 1 and 2 are *x*-intercepts and are thus zeros of the function. We verify this below:

1	3	2	−15	−10	12	8
		3	5	−10	−20	−8
	3	5	−10	−20	−8	0

Thus, $3x^5+2x^4-15x^3-10x^2+12x+8$
$=(x-1)\left(3x^4+5x^3-10x^2-20x-8\right)$

2	3	5	−10	−20	−8
		6	22	24	8
	3	11	12	4	0

Thus, $3x^5+2x^4-15x^3-10x^2+12x+8$
$=(x-1)\left(3x^4+5x^3-10x^2-20x-8\right)$
$=(x-1)(x-2)\left(3x^3+11x^2+12x+4\right)$

To factor $3x^3+11x^2+12x+4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1,\ \pm2,\ \pm4$
Factors of the leading coefficient 3: $\pm1,\ \pm3$

The possible rational zeros are:

$$\frac{\text{Factors of 4}}{\text{Factors of 3}}=\frac{\pm1,\ \pm2,\ \pm4}{\pm1,\ \pm3}$$
$$=\pm1,\ \pm2,\ \pm4,\ \pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm\frac{4}{3}$$

We test values from above until we find a zero. One possibility is shown next:

Test -1:

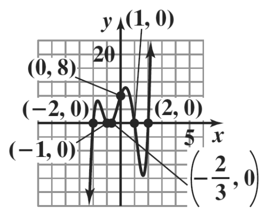
−1	3	11	12	4
		−3	−8	−4
	3	8	4	0

The remainder is 0, so -1 is a zero of *f*. We can now finish the factoring:

$$3x^5+2x^4-15x^3-10x^2+12x+8=0$$
$$(x-1)\left(3x^4+5x^3-10x^2-20x-8\right)=0$$
$$(x-1)(x-2)\left(3x^3+11x^2+12x+4\right)=0$$
$$(x-1)(x-2)(x+1)\left(3x^2+8x+4\right)=0$$
$$(x-1)(x-2)(x+1)(3x+2)(x+2)=0$$
$$x=1,\ x=2,\ x=-1,\ x=-\frac{2}{3},\ x=-2$$

The zeros are -2, -1, $-\frac{2}{3}$, 1 and 2.

b.



$f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

60. $f(x)=-5x^4+4x^3-19x^2+16x+4$

a. From the graph provided, we can see that 1 is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrrr} 1 & -5 & 4 & -19 & 16 & 4 \\ & & -5 & -1 & -20 & -4 \\ \hline & -5 & -1 & -20 & -4 & 0 \end{array}$$

Thus, $-5x^4+4x^3-19x^2+16x+4=0$
$$(x-1)\left(-5x^3-x^2-20x-4\right)=0$$
$$-(x-1)\left(5x^3+x^2+20x+4\right)=0$$

To factor $5x^3+x^2+20x+4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1,\ \pm2,\ \pm4$

Factors of the leading coefficient 5: $\pm1,\ \pm5$

The possible rational zeros are:

$$\frac{\text{Factors of 4}}{\text{Factors of 5}}=\frac{\pm1,\ \pm2,\ \pm4}{\pm1,\ \pm5}$$
$$=\pm1,\ \pm2,\ \pm4,\ \pm\frac{1}{5},\ \pm\frac{2}{5},\ \pm\frac{4}{5}$$

We test values from above until we find a zero. One possibility is shown next:

Test $-\frac{1}{5}$:

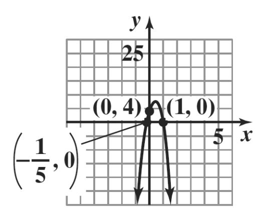
$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & 1 & 20 & 4 \\ & & -1 & 0 & -4 \\ \hline & 5 & 0 & 20 & 0 \end{array}$$

The remainder is 0, so $-\frac{1}{5}$ is a zero of *f*.

$$-5x^4+4x^3-19x^2+16x+4=0$$
$$(x-1)\left(-5x^3-x^2-20x-4\right)=0$$
$$-(x-1)\left(5x^3+x^2+20x+4\right)=0$$
$$-(x-1)\left(x+\frac{1}{5}\right)\left(5x^2+20\right)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)\left(x^2+4\right)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)(x+2i)(x-2i)=0$$
$$x=1,\ x=-\frac{1}{5},\ x=-2i,\ x=2i$$

The zeros are $-\frac{1}{5}$, 1, and $\pm2i$.

b.



$f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$

61. $V(x)=x(x+10)(30-2x)$

$$2000=x(x+10)(30-2x)$$
$$2000=-2x^3+10x^2+300x$$
$$2x^3-10x^2-300x+2000=0$$
$$x^3-5x^2-150x+1000=0$$

Find the roots.

$$\begin{array}{r|rrrr} 10 & 1 & -5 & -150 & 1000 \\ & & 10 & 50 & -1000 \\ \hline & 1 & 5 & -100 & 0 \end{array}$$

Use the remaining quadratic to find the other 2 roots.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(-100)}}{2(1)}$$
$$x\approx-12.8,\ 7.8$$

Since the depth must be positive, reject the negative value.

The depth can be 10 inches or 7.8 inches to obtain a volume of 2000 cubic inches.

62. $V(x) = x(x+10)(30-2x)$

$1500 = x(x+10)(30-2x)$

$1500 = -2x^3 + 10x^2 + 300x$

$2x^3 - 10x^2 - 300x + 1500 = 0$

$x^3 - 5x^2 - 150x + 750 = 0$

Find the roots.

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -150 & 750 \\ & & 5 & 0 & -750 \\ \hline & 1 & 0 & -150 & 0 \end{array}$$

Use the remaining quadratic to find the other 2 roots.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(0) \pm \sqrt{(0)^2 - 4(1)(-150)}}{2(1)}$

$x \approx -12.2, 12.2$

Since the depth must be positive, reject the negative value.

The depth can be 5 inches or 12.2 inches to obtain a volume of 1500 cubic inches.

63. **a.** The answers correspond to the points $(7.8, 2000)$ and $(10, 2000)$.

b. The range is $(0, 15)$.

64. **a.** The answers correspond to the points $(5, 1500)$ and $(12.2, 1500)$.

b. The range is $(0, 15)$.

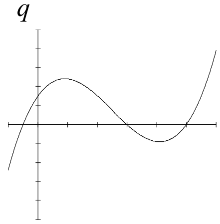
65. – 71. Answers will vary.

72. $2x^3 - 15x^2 + 22x + 15 = 0$

p: ±1, ±3, ±5, ±15

q: ±1, ±2

$\dfrac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{5}{2}, \pm\dfrac{15}{2}$



From the graph we see that the solutions are $-\dfrac{1}{2}$, 3 and 5.

73. $6x^3 - 19x^2 + 16x - 4 = 0$

p: ±1, ±2, ±4

q: ±1, ±2, ±3, ±6

$\dfrac{p}{q}: \pm 1, \pm 2, \pm 4, \pm\dfrac{1}{2}, \pm\dfrac{1}{3}, \pm\dfrac{2}{3}, \pm\dfrac{4}{3}, \pm\dfrac{1}{6}$

From the graph, we see that the solutions are $\dfrac{1}{2}, \dfrac{2}{3}$ and 2.

74. $2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$

p: ±1, ±2, ±3, ±6, ±9, ±18

q: ±1, ±2

$\dfrac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{9}{2}$

From the graph we see the solutions are

$-3, -\dfrac{3}{2}, -1, 2.$

75. $4x^4 + 4x^3 + 7x^2 - x - 2 = 0$

p: ±1, ±2

q: ±1, ±2, ±4

$\dfrac{p}{q}: \pm 1, \pm 2, \pm\dfrac{1}{2}, \pm\dfrac{1}{4}$

From the graph, we see that the solutions are

$-\dfrac{1}{2}$ and $\dfrac{1}{2}$.

76. $f(x) = 3x^4 + 5x^2 + 2$

Since $f(x)$ has no sign variations, it has no positive real roots.

$f(-x) = 3x^4 + 5x^2 + 2$

Since $f(-x)$ has no sign variations, no negative roots exist.

The polynomial's graph doesn't intersect the x-axis.

From the graph, we see that there are no real solutions.

77. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 8$

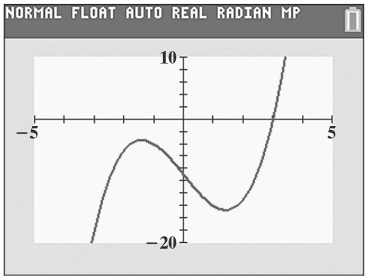
$f(x)$ has 5 sign variations, so either 5, 3, or 1 positive real roots exist.

$f(-x) = -x^5 - x^4 - x^3 - x^2 - x - 8$

$f(-x)$ has no sign variations, so no negative real roots exist.

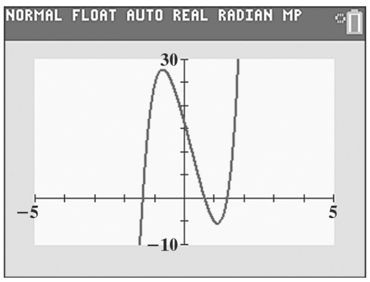
78. Odd functions must have at least one real zero. Even functions do not.

79. $f(x) = x^3 - 6x - 9$



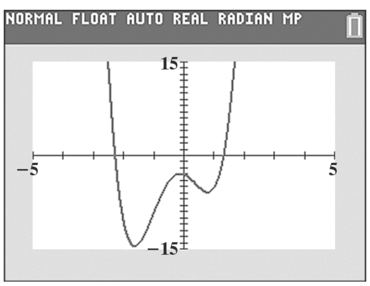
1 real zero
2 nonreal complex zeros

80. $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$

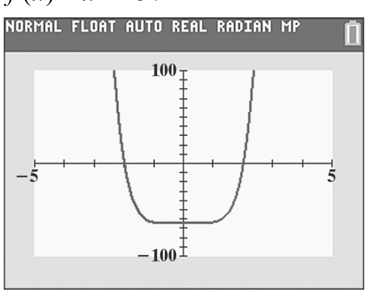


3 real zeros
2 nonreal complex zeros

81. $f(x) = 3x^4 + 4x^3 - 7x^2 - 2x - 3$



82. $f(x) = x^6 - 64$



2 real zeros
4 nonreal complex zeros

83. makes sense

84. does not make sense; Explanations will vary. Sample explanation: The quadratic formula is can be applied only of equations of degree 2.

85. makes sense

86. makes sense

87. false; Changes to make the statement true will vary. A sample change is: The equation has 0 sign variations, so no positive roots exist.

88. false; Changes to make the statement true will vary. A sample change is: Descartes' Rule gives the maximum possible number of real roots.

89. true

90. false; Changes to make the statement true will vary. A sample change is: Polynonials of degree *n* have at most *n* distinct solutions.

91. $(2x+1)(x+5)(x+2) - 3x(x+5) = 208$

$\left(2x^2 + 11x + 5\right)(x+2) - 3x^2 - 15x = 208$

$2x^3 + 4x^2 + 11x^2 + 22x + 5x$

$+10 - 3x^2 - 15x = 208$

$2x^3 + 15x^2 + 27x - 3x^2 - 15x - 198 = 0$

$2x^3 + 12x^2 + 12x - 198 = 0$

$2\left(x^3 + 6x^2 + 6x - 99\right) = 0$

3	1	6	6	–99
		3	27	99
	1	9	33	0

$x^2 + 9x + 33 = 0$

$b^2 - 4ac = -51$

$x = 3$ in.

92. Answers will vary

93. Because the polynomial has two obvious changes of direction; the smallest degree is 3.

94. Because the polynomial has no obvious changes of direction but the graph is obviously not linear, the smallest degree is 3.

95. Because the polynomial has two obvious changes of direction and two roots have multiplicity 2, the smallest degree is 5.

96. Two roots appear twice, the smallest degree is 5.

97. Answers will vary.

98. a. $(f \circ g)(x) = f(g(x))$
$= 4-(x+5)^2$
$= 4-x^2-10x-25$
$= -x^2-10x-21$

b. $\frac{f(x+h)-f(x)}{h}$
$= \frac{4-x^2-2xh-h^2-(4-x^2)}{h}$
$= \frac{4-x^2-2xh-h^2-4+x^2}{h}$
$= \frac{-2xh-h^2}{h}$
$= \frac{h(-2x-h)}{h}$
$= -2x-h, \quad h \neq 0$

99. Write the equation in slope-intercept form:
$x+5y-7=0$
$5y=-x+7$
$y=-\frac{1}{5}x+\frac{7}{5}$

The slope of this line is $-\frac{1}{5}$ thus the slope of any line perpendicular to this line is 5.

Use $m=5$ and the point (–5, 3) to write the equation.
$y-y_1=m(x-x_1)$
$y-3=5(x-(-5))$
$y-3=5(x+5)$
$y-3=5x+25$
$-5x+y-28=0$
$5x-y+28=0$ general form

100. $\frac{f(x_2)-f(x_1)}{x_2-x_1}=\frac{\sqrt{9}-\sqrt{4}}{9-4}=\frac{3-2}{5}=\frac{1}{5}$

101. The function is undefined at $x=1$ and $x=2$.

102. The equation of the vertical asymptote is $x=1$.

103. The equation of the horizontal asymptote is $y=0$.

Mid-Chapter 2 Check Point

1. $(6-2i)-(7-i)=6-2i-7+i=-1-i$

2. $3i(2+i)=6i+3i^2=-3+6i$

3. $(1+i)(4-3i)=4-3i+4i-3i^2$
$=4+i+3=7+i$

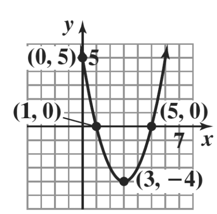
4. $\frac{1+i}{1-i}=\frac{1+i}{1-i}\cdot\frac{1+i}{1+i}=\frac{1+i+i+i^2}{1-i^2}$
$=\frac{1+2i-1}{1+1}$
$=\frac{2i}{2}$
$=i$

5. $\sqrt{-75}-\sqrt{-12}=5i\sqrt{3}-2i\sqrt{3}=3i\sqrt{3}$

6. $\left(2-\sqrt{-3}\right)^2=\left(2-i\sqrt{3}\right)^2$
$=4-4i\sqrt{3}+3i^2$
$=4-4i\sqrt{3}-3$
$=1-4i\sqrt{3}$

7. $x(2x-3)=-4$
$2x^2-3x=-4$
$2x^2-3x+4=0$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(4)}}{2(2)}$
$x=\frac{3\pm\sqrt{-23}}{4}$
$x=\frac{3}{4}\pm\frac{\sqrt{23}}{4}i$

8. $f(x)=(x-3)^2-4$
The parabola opens up because $a>0$.
The vertex is (3, –4).
x-intercepts:
$0=(x-3)^2-4$
$(x-3)^2=4$
$x-3=\pm\sqrt{4}$
$x=3\pm 2$
The equation has x-intercepts at $x=1$ and $x=5$.
y-intercept:
$f(0)=(0-3)^2-4=5$
domain: $(-\infty,\infty)$ range: $[-4,\infty)$



$f(x)=(x-3)^2-4$

9. $f(x) = 5 - (x+2)^2$

The parabola opens down because $a < 0$.
The vertex is (–2, 5).
x-intercepts:

$0 = 5 - (x+2)^2$

$(x+2)^2 = 5$

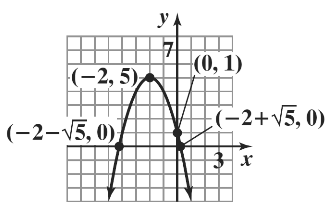
$x + 2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

y-intercept:

$f(0) = 5 - (0+2)^2 = 1$

domain: $(-\infty, \infty)$ range: $(-\infty, 5]$



$f(x) = 5 - (x+2)^2$

10. $f(x) = -x^2 - 4x + 5$

The parabola opens down because $a < 0$.

vertex: $x = -\dfrac{b}{2a} = -\dfrac{-4}{2(-1)} = -2$

$f(-2) = -(-2)^2 - 4(-2) + 5 = 9$

The vertex is (–2, 9).
x-intercepts:

$0 = -x^2 - 4x + 5$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(-1)(5)}}{2(-1)}$

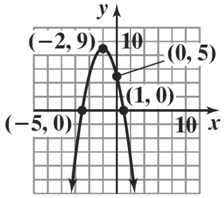
$x = \dfrac{4 \pm \sqrt{36}}{-2}$

$x = -2 \pm 3$

The x-intercepts are $x = 1$ and $x = -5$.
y-intercept:

$f(0) = -0^2 - 4(0) + 5 = 5$

domain: $(-\infty, \infty)$ range: $(-\infty, 9]$



$f(x) = -x^2 - 4x + 5$

11. $f(x) = 3x^2 - 6x + 1$

The parabola opens up because $a > 0$.

vertex: $x = -\dfrac{b}{2a} = -\dfrac{-6}{2(3)} = 1$

$f(1) = 3(1)^2 - 6(1) + 1 = -2$

The vertex is (1, –2).
x-intercepts:

$0 = 3x^2 - 6x + 1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$

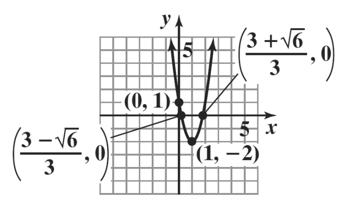
$x = \dfrac{6 \pm \sqrt{24}}{6}$

$x = \dfrac{3 \pm \sqrt{6}}{3}$

y-intercept:

$f(0) = 3(0)^2 - 6(0) + 1 = 1$

domain: $(-\infty, \infty)$ range: $[-2, \infty)$



$f(x) = 3x^2 - 6x + 1$

12. $f(x) = (x-2)^2 (x+1)^3$

$(x-2)^2 (x+1)^3 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^3 = 0$

$x - 2 = 0 \qquad x + 1 = 0$

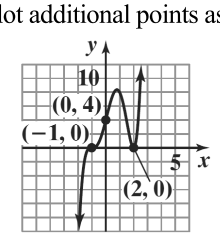
$x = 2 \qquad x = -1$

The zeros are -1 and 2.

The graph of *f* crosses the x-axis at -1, since the zero has multiplicity 3. The graph touches the x-axis and turns around at 2 since the zero has multiplicity 2.

Since *f* is an odd-degree polynomial, degree 5, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = (x-2)^2 (x+1)^3$

13. $f(x) = -(x-2)^2(x+1)^2$

$-(x-2)^2(x+1)^2 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^2 = 0$

$x-2 = 0$ $\quad$ $x+1 = 0$

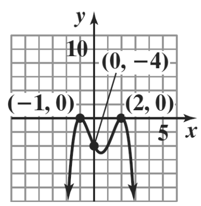
$x = 2$ $\quad$ $x = -1$

The zeros are -1 and 2.

The graph touches the *x*-axis and turns around both at -1 and 2 since both zeros have multiplicity 2.

Since *f* is an even-degree polynomial, degree 4, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x-2)^2(x+1)^2$

14. $f(x) = x^3 - x^2 - 4x + 4$

$x^3 - x^2 - 4x + 4 = 0$

$x^2(x-1) - 4(x-1) = 0$

$(x^2-4)(x-1) = 0$

$(x+2)(x-2)(x-1) = 0$

Apply the zero-product principle:

$x+2 = 0$ or $x-2 = 0$ or $x-1 = 0$

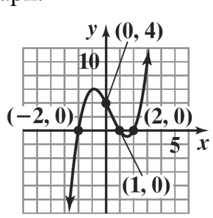
$x = -2$ $\quad$ $x = 2$ $\quad$ $x = 1$

The zeros are -2, 1, and 2.

The graph of *f* crosses the *x*-axis at all three zeros, -2, 1, and 2, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^3 - x^2 - 4x + 4$

15. $f(x) = x^4 - 5x^2 + 4$

$x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2-1) = 0$

$(x+2)(x-2)(x+1)(x-1) = 0$

Apply the zero-product principle,

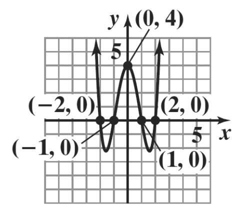
$x = -2,\ x = 2,\ x = -1,\ x = 1$

The zeros are -2, -1, 1, and 2.

The graph crosses the *x*-axis at all four zeros, -2, -1, 1, and 2., since all have multiplicity 1.

Since *f* is an even-degree polynomial, degree 4, and since the leading coefficient, 1, is positive, the graph rises to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^4 - 5x^2 + 4$

16. $f(x) = -(x+1)^6$

$-(x+1)^6 = 0$

$(x+1)^6 = 0$

$x+1 = 0$

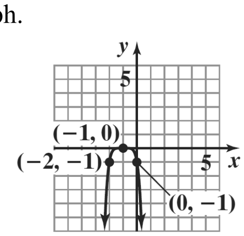
$x = -1$

The zero is are -1.

The graph touches the *x*-axis and turns around at -1 since the zero has multiplicity 6.

Since *f* is an even-degree polynomial, degree 6, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x+1)^6$

17. $f(x) = -6x^3 + 7x^2 - 1$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -1: ± 1
List all factors of the leading coefficient -6:
$\pm 1, \ \pm 2, \ \pm 3, \ \pm 6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } -6} = \frac{\pm 1}{\pm 1, \ \pm 2, \ \pm 3, \ \pm 6}$$
$$= \pm 1, \ \pm\frac{1}{2}, \ \pm\frac{1}{3}, \ \pm\frac{1}{6}$$

We test values from the above list until we find a zero. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & -6 & 7 & 0 & -1 \\ & & -6 & 1 & 1 \\ \hline & -6 & 1 & 1 & 0 \end{array}$$

The remainder is 0, so 1 is a zero. Thus,

$$\begin{aligned} -6x^3 + 7x^2 - 1 &= 0 \\ (x-1)\left(-6x^2 + x + 1\right) &= 0 \\ -(x-1)\left(6x^2 - x - 1\right) &= 0 \\ -(x-1)(3x+1)(2x-1) &= 0 \end{aligned}$$

Apply the zero-product property:

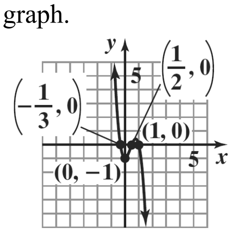
$x = 1, \quad x = -\frac{1}{3}, \quad x = \frac{1}{2}$

The zeros are $-\frac{1}{3}, \ \frac{1}{2}$, and 1.

The graph of *f* crosses the *x*-axis at all three zeros, $-\frac{1}{3}, \ \frac{1}{2}$, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -6, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -6x^3 + 7x^2 - 1$

18. $f(x) = 2x^3 - 2x$

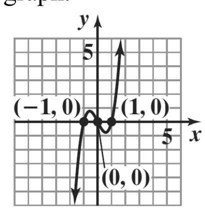
$$\begin{aligned} 2x^3 - 2x &= 0 \\ 2x\left(x^2 - 1\right) &= 0 \\ 2x(x+1)(x-1) &= 0 \end{aligned}$$

Apply the zero-product principle:
$x = 0, \qquad x = -1, \qquad x = 1$

The zeros are -1, 0, and 1.
The graph of *f* crosses the *x*-axis at all three zeros, -1, 0, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 2, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = 2x^3 - 2x$

19. $f(x) = x^3 - 2x^2 + 26x$

$$\begin{aligned} x^3 - 2x^2 + 26x &= 0 \\ x\left(x^2 - 2x + 26\right) &= 0 \end{aligned}$$

Note that $x^2 - 2x + 26$ does not factor, so we use the quadratic formula:

$x = 0 \quad$ or $\quad x^2 - 2x + 26 = 0$
$\qquad a = 1, \ \ b = -2, \ \ c = 26$

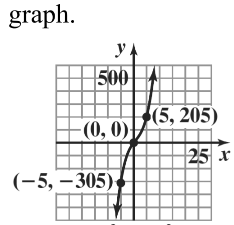
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(26)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{-100}}{2} = \frac{2 \pm 10i}{2} = 1 \pm 5i$$

The zeros are 0 and $1 \pm 5i$.

The graph of *f* crosses the *x*-axis at 0 (the only real zero), since it has multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^3 - 2x^2 + 26x$

20. $f(x) = -x^3 + 5x^2 - 5x - 3$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -3: $\pm 1, \pm 3$
List all factors of the leading coefficient -1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -3}{\text{Factors of } -1} = \frac{\pm 1, \pm 3}{\pm 1} = \pm 1, \pm 3$$

We test values from the previous list until we find a zero. One is shown next:

Test 3:

$$\begin{array}{r|rrrr} 3 & -1 & 5 & -5 & -3 \\ & & -3 & 6 & 3 \\ \hline & -1 & 2 & 1 & 0 \end{array}$$

The remainder is 0, so 3 is a zero. Thus,

$$-x^3 + 5x^2 - 5x - 3 = 0$$
$$(x-3)(-x^2 + 2x + 1) = 0$$
$$-(x-3)(x^2 - 2x - 1) = 0$$

Note that $x^2 - 2x - 1$ does not factor, so we use the quadratic formula:

$x - 3 = 0$ or $x^2 - 2x - 1 = 0$
$x = 3$ $\quad a = 1, \ b = -2, \ c = -1$

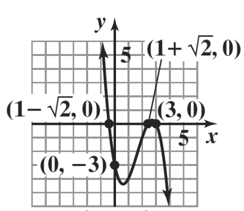
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

The zeros are 3 and $1 \pm \sqrt{2}$.

The graph of *f* crosses the *x*-axis at all three zeros, 3 and $1 \pm \sqrt{2}$, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -1, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



21. $x^3 - 3x + 2 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 2: $\pm 1, \pm 2$
Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } 2}{\text{Factors of } 1} = \frac{\pm 1, \pm 2}{\pm 1} = \pm 1, \pm 2$$

We test values from above until we find a root. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

The remainder is 0, so 1 is a root of the equation. Thus,

$$x^3 - 3x + 2 = 0$$
$$(x-1)(x^2 + x - 2) = 0$$
$$(x-1)(x+2)(x-1) = 0$$
$$(x-1)^2(x+2) = 0$$

Apply the zero-product property:

$(x-1)^2 = 0$ or $x + 2 = 0$
$x - 1 = 0$ $\quad x = -2$
$x = 1$

The solutions are -2 and 1, and the solution set is $\{-2, 1\}$.

22. $6x^3 - 11x^2 + 6x - 1 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -1: ± 1
Factors of the leading coefficient 6: $\pm 1, \pm 2, \pm 3, \pm 6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } 6} = \frac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$$

We test values from above until we find a root. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & 6 & -11 & 6 & -1 \\ & & 6 & -5 & 1 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

The remainder is 0, so 1 is a root of the equation. Thus,

$$6x^3 - 11x^2 + 6x - 1 = 0$$
$$(x-1)\left(6x^2 - 5x + 1\right) = 0$$
$$(x-1)(3x-1)(2x-1) = 0$$

Apply the zero-product property:

$x - 1 = 0$ or $3x - 1 = 0$ or $2x - 1 = 0$

$x = 1$ $\quad x = \frac{1}{3}$ $\quad x = \frac{1}{2}$

The solutions are $\frac{1}{3}$, $\frac{1}{2}$ and 1, and the solution set is $\left\{\frac{1}{3}, \frac{1}{2}, 1\right\}$.

23. $(2x+1)(3x-2)^3(2x-7) = 0$

Apply the zero-product property:

$2x + 1 = 0$ or $(3x-2)^3 = 0$ or $2x - 7 = 0$ T

$x = -\frac{1}{2}$ $\quad 3x - 2 = 0$ $\quad x = \frac{7}{2}$

$x = \frac{2}{3}$

he solutions are $-\frac{1}{2}$, $\frac{2}{3}$ and $\frac{7}{2}$, and the solution set is $\left\{-\frac{1}{2}, \frac{2}{3}, \frac{7}{2}\right\}$.

24. $2x^3 + 5x^2 - 200x - 500 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -500:
$\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm25,$
$\pm50, \pm100, \pm125, \pm250, \pm500$

Factors of the leading coefficient 2: $\pm1, \pm2$

The possible rational zeros are:

$$\frac{\text{Factors of } 500}{\text{Factors of } 2} = \pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm25, \pm50, \pm100, \pm125, \pm250, \pm500, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{25}{2}, \pm\frac{125}{2}$$

We test values from above until we find a root. One is shown next:

Test 10:

10	2	5	−200	−500
		20	250	500
	2	25	50	0

The remainder is 0, so 10 is a root of the equation. Thus,

$$2x^3 + 5x^2 - 200x - 500 = 0$$
$$(x-10)\left(2x^2 + 25x + 50\right) = 0$$
$$(x-10)(2x+5)(x+10) = 0$$

Apply the zero-product property:

$x - 10 = 0$ or $2x + 5 = 0$ or $x + 10 = 0$

$x = 10$ $\quad x = -\frac{5}{2}$ $\quad x = -10$

The solutions are -10, $-\frac{5}{2}$, and 10, and the solution set is $\left\{-10, -\frac{5}{2}, 10\right\}$.

25. $x^4 - x^3 - 11x^2 = x + 12$

$$x^4 - x^3 - 11x^2 - x - 12 = 0$$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -12:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Factors of the leading coefficient 1: ±1

The possible rational zeros are:

$$\frac{\text{Factors of } -12}{\text{Factors of } 1} = \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$$

We test values from this list we find a root. One possibility is shown next:

Test -3:

−3	1	−1	−11	−1	−12
		−3	12	−3	12
	1	−4	1	−4	0

The remainder is 0, so -3 is a root of the equation. Using the Factor Theorem, we know that $x - 1$ is a factor. Thus,

$$x^4 - x^3 - 11x^2 - x - 12 = 0$$
$$(x+3)\left(x^3 - 4x^2 + x - 4\right) = 0$$
$$(x+3)\left[x^2(x-4) + 1(x-4)\right] = 0$$
$$(x+3)(x-4)\left(x^2 + 1\right) = 0$$

As this point we know that -3 and 4 are roots of the equation. Note that $x^2 + 1$ does not factor, so we use the square-root principle: $x^2 + 1 = 0$

$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

The roots are -3, 4, and $\pm i$, and the solution set is $\{-3, 4, \pm i\}$.

26. $2x^4 + x^3 - 17x^2 - 4x + 6 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 6: $\pm 1, \pm 2, \pm 3, \pm 6$

Factors of the leading coefficient 4: $\pm 1, \pm 2$

The possible rational roots are:

$$\frac{\text{Factors of 6}}{\text{Factors of 2}} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$$
$$= \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$$

We test values from above until we find a root. One possibility is shown next:

Test -3:

$$\begin{array}{r|rrrrr} -3 & 2 & 1 & -17 & -4 & 6 \\ & & -6 & 15 & 6 & -6 \\ \hline & 2 & -5 & -2 & 2 & 0 \end{array}$$

The remainder is 0, so -3 is a root. Using the Factor Theorem, we know that $x+3$ is a factor of the polynomial. Thus,

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)\left(2x^3 - 5x^2 - 2x + 2\right) = 0$$

To solve the equation above, we need to factor $2x^3 - 5x^2 - 2x + 2$. We continue testing potential roots:

Test $\frac{1}{2}$:

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -5 & -2 & 2 \\ & & 1 & -2 & -2 \\ \hline & 2 & -4 & -4 & 0 \end{array}$$

The remainder is 0, so $\frac{1}{2}$ is a zero and $x - \frac{1}{2}$ is a factor.

Summarizing our findings so far, we have

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)\left(2x^3 - 5x^2 - 2x + 2\right) = 0$$
$$(x+3)\left(x - \frac{1}{2}\right)\left(2x^2 - 4x - 4\right) = 0$$
$$2(x+3)\left(x - \frac{1}{2}\right)\left(x^2 - 2x - 2\right) = 0$$

At this point, we know that -3 and $\frac{1}{2}$ are roots of the equation. Note that $x^2 - 2x - 2$ does not factor, so we use the quadratic formula:

$$x^2 - 2x - 2 = 0$$
$$a = 1, \quad b = -2, \quad c = -2$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

The solutions are -3, $\frac{1}{2}$, and $1 \pm \sqrt{3}$, and the solution set is $\left\{-3, \frac{1}{2}, 1 \pm \sqrt{3}\right\}$.

27. $P(x) = -x^2 + 150x - 4425$

Since $a = -1$ is negative, we know the function opens down and has a maximum at

$$x = -\frac{b}{2a} = -\frac{150}{2(-1)} = -\frac{150}{-2} = 75.$$

$$P(75) = -75^2 + 150(75) - 4425$$
$$= -5625 + 11,250 - 4425 = 1200$$

The company will maximize its profit by manufacturing and selling 75 cabinets per day. The maximum daily profit is \$1200.

28. Let x = one of the numbers;
$-18 - x$ = the other number

The product is $f(x) = x(-18 - x) = -x^2 - 18x$

The x-coordinate of the maximum is

$$x = -\frac{b}{2a} = -\frac{-18}{2(-1)} = -\frac{-18}{-2} = -9.$$

$$f(-9) = -9\left[-18 - (-9)\right]$$
$$= -9(-18 + 9) = -9(-9) = 81$$

The vertex is $(-9, 81)$. The maximum product is 81. This occurs when the two numbers are -9 and $-18 - (-9) = -9$.

29. Let x = height of triangle;
$40 - 2x$ = base of triangle

$$A = \frac{1}{2}bh = \frac{1}{2}x(40 - 2x)$$

$$A(x) = 20x - x^2$$

The height at which the triangle will have maximum area is $x = -\frac{b}{2a} = -\frac{20}{2(-1)} = 10.$

$$A(10) = 20(10) - (10)^2 = 100$$

The maximum area is 100 squares inches.

30.

$$\begin{array}{r} 2x^2 - x - 3 \\ 3x^2 - 1 \overline{)6x^4 - 3x^3 - 11x^2 + 2x + 4} \\ \underline{6x^4 \quad - 2x^2} \\ -3x^3 - 9x^2 + 2x \\ \underline{-3x^3 \quad + x} \\ -9x^2 + x + 4 \\ \underline{-9x^2 \quad + 3} \\ x + 1 \end{array}$$

$$2x^2 - x - 3 + \frac{x+1}{3x^2 - 1}$$

31. $\left(2x^4 - 13x^3 + 17x^2 + 18x - 24\right) \div (x - 4)$

4	2	–13	17	18	–24
		8	-20	–12	24
	2	–5	–3	6	0

The quotient is $2x^3 - 5x^2 - 3x + 6$.

32. $(x-1)(x-i)(x+i) = (x-1)(x^2+1)$

$f(x) = a_n(x-1)(x^2+1)$
$f(-1) = a_n(-1-1)\left((-1)^2+1\right) = -4a_n = 8$
$a_n = -2$
$f(x) = -2(x-1)(x^2+1)$ or $-2x^3 + 2x^2 - 2x + 2$

33. $(x-2)(x-2)(x-3i)(x+3i)$

$= (x-2)(x-2)(x^2+9)$

$f(x) = a_n(x-2)(x-2)(x^2+9)$
$f(0) = a_n(0-2)(0-2)(0^2+9)$
$36 = 36a_n$
$a_n = 1$
$f(x) = 1(x-2)(x-2)(x^2+9)$
$f(x) = x^4 - 4x^3 + 13x^2 - 36x + 36$

34. $f(x) = x^3 - x - 5$

$f(1) = 1^3 - 1 - 5 = -5$

$f(2) = 2^3 - 2 - 5 = 1$

Yes, the function must have a real zero between 1 and 2 because $f(1)$ and $f(2)$ have opposite signs.

Section 2.6

Check Point Exercises

1. Because division by 0 is undefined, we must exclude from the domain of each function values of x that cause the polynomial function in the denominator to be 0.

a. $x - 5 = 0$
$x = 5$

$\{x \mid x \neq 5\}$ or $(-\infty, 5) \cup (5, \infty)$.

b. $x^2 - 25 = 0$
$x^2 = 25$
$x = \pm 5$
$\{x \mid x \neq 5, x \neq -5\}$ or $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$.

c. The denominator cannot equal zero.
All real numbers or $(-\infty, \infty)$. .

2. **a.** $x^2 - 1 = 0$
$x^2 = 1$
$x = 1, x = -1$

b. $g(x) = \dfrac{x-1}{x^2-1} = \dfrac{x-1}{(x-1)(x+1)} = \dfrac{1}{x+1}$
$x = -1$

c. The denominator cannot equal zero.
No vertical asymptotes.

3. **a.** The degree of the numerator, 2, is equal to the degree of the denominator, 2. Thus, the leading coefficients of the numerator and denominator, 9 and 3, are used to obtain the equation of the horizontal asymptote.

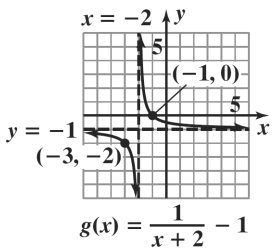
$y = \dfrac{9}{3} = 3$

$y = 3$ is a horizontal asymptote.

b. The degree of the numerator, 1, is less than the degree of the denominator, 2. Thus, the graph has the x-axis as a horizontal asymptote
$y = 0$ is a horizontal asymptote.

c. The degree of the numerator, 3, is greater than the degree of the denominator, 2. Thus, the graph has no horizontal asymptote.

4. Begin with the graph of $f(x) = \frac{1}{x}$.



Shift the graph 2 units to the left by subtracting 2 from each x-coordinate. Shift the graph 1 unit down by subtracting 1 from each y-coordinate.

5. $f(x) = \frac{3x-3}{x-2}$

$f(-x) = \frac{3(-x)-3}{-x-2} = \frac{-3x-3}{-x-2} = \frac{3x+3}{x+2}$

no symmetry

$f(0) = \frac{3(0)-3}{0-2} = \frac{3}{2}$

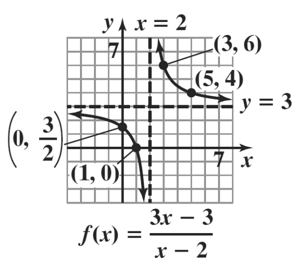
The y-intercept is $\frac{3}{2}$.

$3x - 3 = 0$
$3x = 3$
$x = 1$

The x-intercept is 1.
Vertical asymptote:
$x - 2 = 0$
$x = 2$

Horizontal asymptote:

$y = \frac{3}{1} = 3$



6. $f(x) = \frac{2x^2}{x^2-9}$

$f(-x) = \frac{2(-x)^2}{(-x)^2-9} = \frac{2x^2}{x^2-9} = f(x)$

The y-axis symmetry.

$f(0) = \frac{2(0)^2}{0^2-9} = 0$

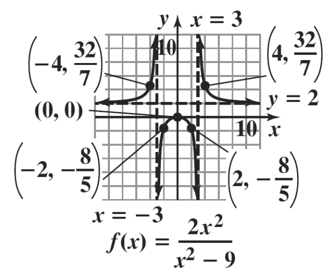
The y-intercept is 0.

$2x^2 = 0$
$x = 0$

The x-intercept is 0.
vertical asymptotes:
$x^2 - 9 = 0$
$x = 3, x = -3$

horizontal asymptote:

$y = \frac{2}{1} = 2$



7. $f(x) = \frac{x^4}{x^2+2}$

$f(-x) = \frac{(-x)^4}{(-x)^2+2} = \frac{x^4}{x^2+2} = f(x)$

y-axis symmetry

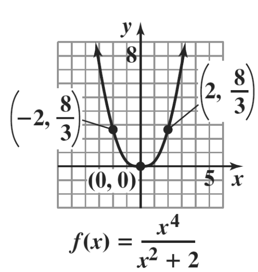
$f(0) = \frac{0^4}{0^2+2} = 0$

The y-intercept is 0.

$x^4 = 0$
$x = 0$

The x-intercept is 0.
vertical asymptotes:
$x^2 + 2 = 0$
$x^2 = -2$

no vertical asymptotes
horizontal asymptote:
Since $n > m$, there is no horizontal asymptote.



8.

2	2	–5	7
		4	–2
	2	–1	5

the equation of the slant asymptote is $y = 2x - 1$.

9. **a.** $C(x) = 500,000 + 400x$

b. $\bar{C}(x) = \dfrac{500,000 + 400x}{x}$

c.
$$\bar{C}(1000) = \frac{500,000 + 400(1000)}{1000} = 900$$
$$\bar{C}(10,000) = \frac{500,000 + 400(10,000)}{10,000} = 450$$
$$\bar{C}(100,000) = \frac{500,000 + 400(100,000)}{100,000} = 405$$
The average cost per wheelchair of producing 1000, 10,000, and 100,000 wheelchairs is \$900, \$450, and \$405, respectively.

d. $y = \dfrac{400}{1} = 400$
The cost per wheelchair approaches \$400 as more wheelchairs are produced.

10. $x - 10$ = the average velocity on the return trip. The function that expresses the total time required to complete the round trip is
$T(x) = \dfrac{20}{x} + \dfrac{20}{x-10}$.

Concept and Vocabulary Check 2.6

1. polynomial

2. false

3. true

4. vertical asymptote; $x = -5$

5. horizontal asymptote; $y = 0$; $y = \dfrac{1}{3}$

6. true

7. left; down

8. one more than

9. $y = 3x + 5$

10. $x - 20$; $\dfrac{30}{x-20}$

Exercise Set 2.6

1. $f(x) = \dfrac{5x}{x-4}$
$\{x \mid x \neq 4\}$

2. $f(x) = \dfrac{7x}{x-8}$
$\{x \mid x \neq 8\}$

3. $g(x) = \dfrac{3x^2}{(x-5)(x+4)}$
$\{x \mid x \neq 5, x \neq -4\}$

4. $g(x) = \dfrac{2x^2}{(x-2)(x+6)}$
$\{x \mid x \neq 2, x \neq -6\}$

5. $h(x) = \dfrac{x+7}{x^2 - 49}$
$x^2 - 49 = (x-7)(x+7)$
$\{x \mid x \neq 7, x \neq -7\}$

6. $h(x) = \dfrac{x+8}{x^2 - 64}$
$x^2 - 64 = (x-8)(x+8)$
$\{x \mid x \neq 8, x \neq -8\}$

7. $f(x) = \dfrac{x+7}{x^2 + 49}$
all real numbers

8. $f(x) = \dfrac{x+8}{x^2 + 64}$
all real numbers

9. $-\infty$

10. $+\infty$

11. $-\infty$

12. $+\infty$

13. 0

14. 0

15. $+\infty$

16. $-\infty$

17. $-\infty$

18. $+\infty$

19. 1

20. 1

21. $f(x)=\dfrac{x}{x+4}$

$x+4=0$
$x=-4$

vertical asymptote: $x=-4$
There are no holes.

22. $f(x)=\dfrac{x}{x-3}$

$x-3=0$
$x=3$

vertical asymptote: $x=3$
There are no holes.

23. $g(x)=\dfrac{x+3}{x(x+4)}$

$x(x+4)=0$
$x=0, x=-4$

vertical asymptotes: $x=0,\ x=-4$
There are no holes.

24. $g(x)=\dfrac{x+3}{x(x-3)}$

$x(x-3)=0$
$x=0, x=3$

vertical asymptotes: $x=0, x=3$
There are no holes.

25. $h(x)=\dfrac{x}{x(x+4)}=\dfrac{1}{x+4}$

$x+4=0$
$x=-4$

vertical asymptote: $x=-4$
There is a hole at $x=0$.

26. $h(x)=\dfrac{x}{x(x-3)}=\dfrac{1}{x-3}$

$x-3=0$
$x=3$

vertical asymptote: $x=3$
There is a hole at $x=0$.

27. $r(x)=\dfrac{x}{x^2+4}$

x^2+4 has no real zeros
There are no vertical asymptotes.
There are no holes.

28. $r(x)=\dfrac{x}{x^2+3}$

x^2+3 has no real zeros
There is no vertical asymptotes.
There are no holes.

29. $f(x)=\dfrac{x^2-9}{x-3}$

$=\dfrac{(x+3)(x-3)}{x-3}$

$=x+3$

There are no vertical asymptotes.
There is a hole at $x=3$.

30. $f(x)=\dfrac{x^2-25}{x-5}$

$=\dfrac{(x+5)(x-5)}{x-5}$

$=x+5$

There are no vertical asymptotes.
There is a hole at $x=5$.

31. $g(x)=\dfrac{x-3}{x^2-9}$

$=\dfrac{x-3}{(x+3)(x-3)}$

$=\dfrac{1}{x+3}$

vertical asymptote: $x=-3$
There is a hole at $x=3$.

32. $g(x)=\dfrac{x-5}{x^2-25}$

$=\dfrac{x-5}{(x+5)(x-5)}$

$=\dfrac{1}{x+5}$

vertical asymptote: $x=-5$
There is a hole at $x=5$.

33. $h(x) = \dfrac{x+7}{x^2+4x-21}$
$= \dfrac{x+7}{(x+7)(x-3)}$
$= \dfrac{1}{x-3}$
vertical asymptote: $x = 3$
There is a hole at $x = -7$.

34. $h(x) = \dfrac{x+6}{x^2+2x-24}$
$= \dfrac{x+6}{(x+6)(x-4)}$
$= \dfrac{1}{x-4}$
vertical asymptote: $x = 4$
There is a hole at $x = -6$.

35. $r(x) = \dfrac{x^2+4x-21}{x+7}$
$= \dfrac{(x+7)(x-3)}{x+7}$
$= x-3$
There are no vertical asymptotes.
There is a hole at $x = -7$.

36. $r(x) = \dfrac{x^2+2x-24}{x+6}$
$= \dfrac{(x+6)(x-4)}{x+6}$
$= x-4$
There are no vertical asymptotes.
There is a hole at $x = -6$.

37. $f(x) = \dfrac{12x}{3x^2+1}$
$n < m$
horizontal asymptote: $y = 0$

38. $f(x) = \dfrac{15x}{3x^2+1}$
$n < m$
horizontal asymptote: $y = 0$

39. $g(x) = \dfrac{12x^2}{3x^2+1}$
$n = m$,
horizontal asymptote: $y = \dfrac{12}{3} = 4$

40. $g(x) = \dfrac{15x^2}{3x^2+1}$
$n = m$
horizontal asymptote: $y = \dfrac{15}{3} = 5$

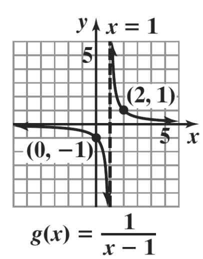
41. $h(x) = \dfrac{12x^3}{3x^2+1}$
$n > m$
no horizontal asymptote

42. $h(x) = \dfrac{15x^3}{3x^2+1}$
$n > m$
no horizontal asymptote

43. $f(x) = \dfrac{-2x+1}{3x+5}$
$n = m$
horizontal asymptote: $y = -\dfrac{2}{3}$

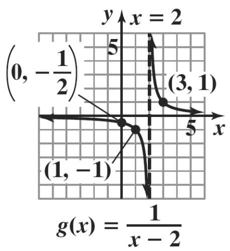
44. $f(x) = \dfrac{-3x+7}{5x-2}$
$n = m$ s
horizontal asymptote: $y = -\dfrac{3}{5}$

45. $g(x) = \dfrac{1}{x-1}$
Shift the graph of $f(x) = \dfrac{1}{x}$ 1 unit to the right.



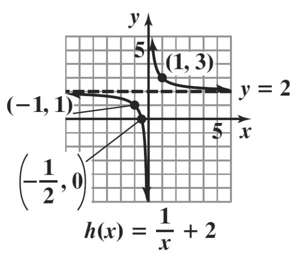
46. $g(x)=\dfrac{1}{x-2}$

Shift the graph of $f(x)=\dfrac{1}{x}$ 2 units to the right.



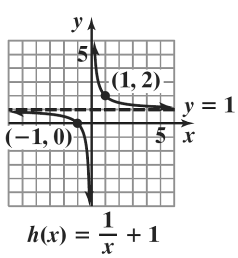
47. $h(x)=\dfrac{1}{x}+2$

Shift the graph of $f(x)=\dfrac{1}{x}$ 2 units up.



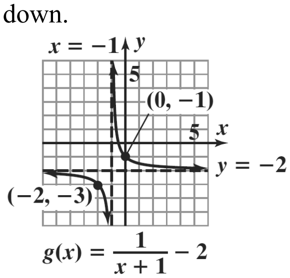
48. $h(x)=\dfrac{1}{x}+1$

Shift the graph of $f(x)=\dfrac{1}{x}$ 1 unit up.



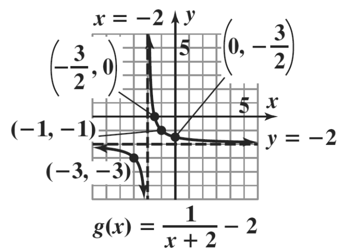
49. $g(x)=\dfrac{1}{x+1}-2$

Shift the graph of $f(x)=\dfrac{1}{x}$ 1 unit left and 2 units down.



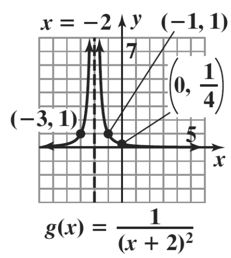
50. $g(x)=\dfrac{1}{x+2}-2$

Shift the graph of $f(x)=\dfrac{1}{x}$ 2 units left and 2 units down.



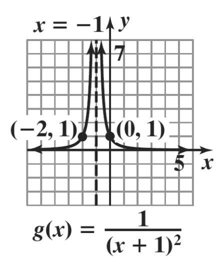
51. $g(x)=\dfrac{1}{(x+2)^2}$

Shift the graph of $f(x)=\dfrac{1}{x^2}$ 2 units left.



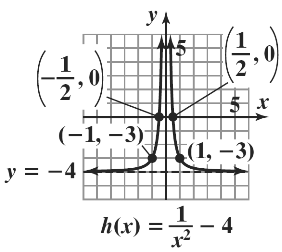
52. $g(x)=\dfrac{1}{(x+1)^2}$

Shift the graph of $f(x)=\dfrac{1}{x^2}$ 1 unit left.



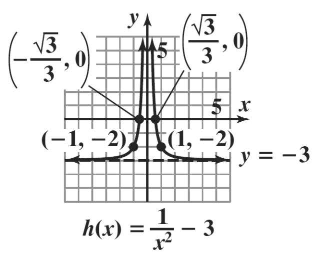
53. $h(x) = \frac{1}{x^2} - 4$

Shift the graph of $f(x) = \frac{1}{x^2}$ 4 units down.



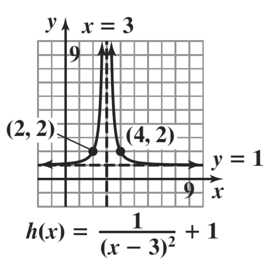
54. $h(x) = \frac{1}{x^2} - 3$

Shift the graph of $f(x) = \frac{1}{x^2}$ 3 units down.



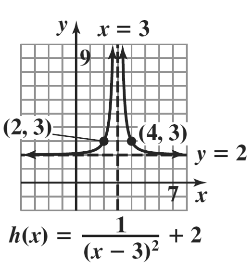
55. $h(x) = \frac{1}{(x-3)^2} + 1$

Shift the graph of $f(x) = \frac{1}{x^2}$ 3 units right and 1 unit up.



56. $h(x) = \frac{1}{(x-3)^2} + 2$

Shift the graph of $f(x) = \frac{1}{x^2}$ 3 units right and 2 units up.



57. $f(x) = \frac{4x}{x-2}$

$f(-x) = \frac{4(-x)}{(-x)-2} = \frac{4x}{x+2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \frac{4(0)}{0-2} = 0$

x-intercept: $4x = 0$

$x = 0$

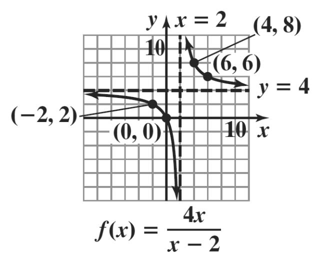
vertical asymptote:

$x - 2 = 0$

$x = 2$

horizontal asymptote:

$n = m$, so $y = \frac{4}{1} = 4$



58. $f(x) = \frac{3x}{x-1}$

$f(-x) = \frac{3(-x)}{(-x)-1} = \frac{3x}{x+1}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \frac{3(0)}{0-1} = 0$

x-intercept: $3x = 0$

$x = 0$

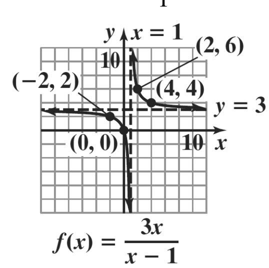
vertical asymptote:

$x - 1 = 0$

$x = 1$

horizontal asymptote:

$n = m$, so $y = \frac{3}{1} = 3$



59. $f(x)=\dfrac{2x}{x^2-4}$

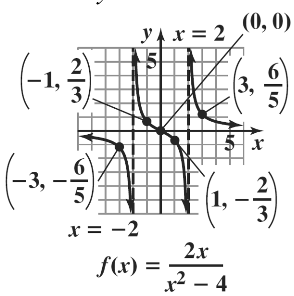
$f(-x)=\dfrac{2(-x)}{(-x)^2-4}=-\dfrac{2x}{x^2-4}=-f(x)$

Origin symmetry

y-intercept: $\dfrac{2(0)}{0^2-4}=\dfrac{0}{-4}=0$

x-intercept:
$2x=0$
$x=0$

vertical asymptotes:
$x^2-4=0$
$x=\pm 2$
horizontal asymptote:
$n<m$ so $y=0$



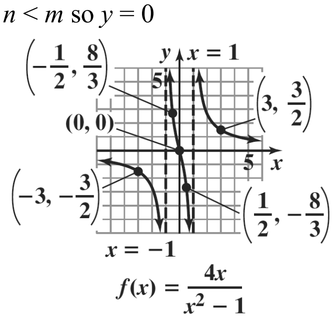
60. $f(x)=\dfrac{4x}{x^2-1}$

$f(-x)=\dfrac{4(-x)}{(-x)^2-1}=-\dfrac{4x}{x^2-1}=-f(x)$

Origin symmetry

y-intercept: $\dfrac{4(0)}{0^2-1}=0$

x-intercept:$4x=0$
$x=0$
vertical asymptotes:
$x^2-1=0$
$(x-1)(x+1)=0$
$x=\pm 1$
horizontal asymptote:
$n<m$ so $y=0$



61. $f(x)=\dfrac{2x^2}{x^2-1}$

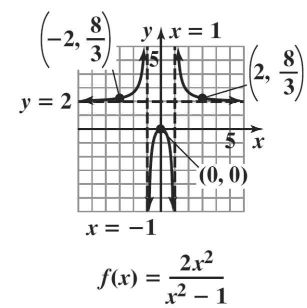
$f(-x)=\dfrac{2(-x)^2}{(-x)^2-1}=\dfrac{2x^2}{x^2-1}=f(x)$

y-axis symmetry

y-intercept: $y=\dfrac{2(0)^2}{0^2-1}=\dfrac{0}{1}=0$

x-intercept:
$2x^2=0$
$x=0$
vertical asymptote:
$x^2-1=0$
$x^2=1$
$x=\pm 1$

horizontal asymptote:
$n=m$, so $y=\dfrac{2}{1}=2$



62. $f(x)=\dfrac{4x^2}{x^2-9}$

$f(-x)=\dfrac{4(-x)^2}{(-x)^2-9}=\dfrac{4x^2}{x^2-9}=f(x)$

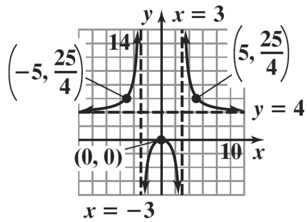
y-axis symmetry

y-intercept: $y=\dfrac{4(0)^2}{0^2-9}=0$

x-intercept:
$4x^2=0$
$x=0$
vertical asymptotes:
$x^2-9=0$
$(x-3)(x+3)=0$
$x=\pm 3$

horizontal asymptote:

$n = m$, so $y = \frac{4}{1} = 4$



$f(x) = \frac{4x^2}{x^2 - 9}$

63. $f(x) = \frac{-x}{x+1}$

$f(-x) = \frac{-(-x)}{(-x)+1} = \frac{x}{-x+1}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \frac{-(0)}{0+1} = \frac{0}{1} = 0$

x-intercept:

$-x = 0$

$x = 0$

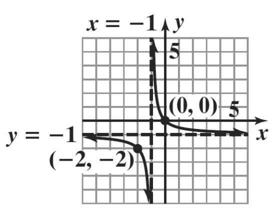
vertical asymptote:

$x + 1 = 0$

$x = -1$

horizontal asymptote:

$n = m$, so $y = \frac{-1}{1} = -1$



$f(x) = \frac{-x}{x+1}$

64. $f(x) = \frac{-3x}{x+2}$

$f(-x) = \frac{-3(-x)}{(-x)+2} = \frac{3x}{-x+2}$

$f-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept:

$y = \frac{-3(0)}{0+2} = 0$

x-intercept:

$-3x = 0$

$x = 0$

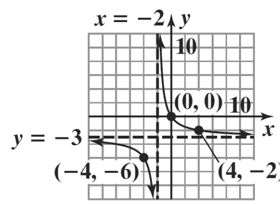
vertical asymptote:

$x + 2 = 0$

$x = -2$

horizontal asymptote:

$n = m$, so $y = \frac{-3}{1} = -3$



$f(x) = \frac{-3x}{x+2}$

65. $f(x) = -\frac{1}{x^2 - 4}$

$f(-x) = -\frac{1}{(-x)^2 - 4} = -\frac{1}{x^2 - 4} = f(x)$

y-axis symmetry

y-intercept: $y = -\frac{1}{0^2 - 4} = \frac{1}{4}$

x-intercept: $-1 \neq 0$

no x-intercept

vertical asymptotes:

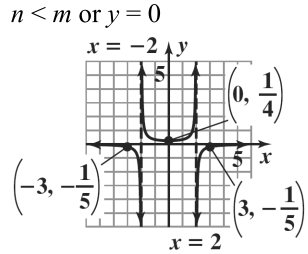
$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

horizontal asymptote:

$n < m$ or $y = 0$



$f(x) = -\frac{1}{x^2 - 4}$

66. $f(x) = -\dfrac{2}{x^2 - 1}$

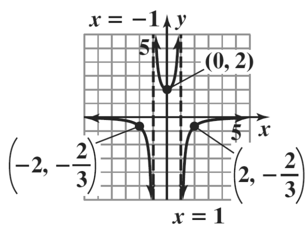
$f(-x) = -\dfrac{2}{(-x)^2 - 1} = -\dfrac{2}{x^2 - 1} = f(x)$

y-axis symmetry
y-intercept:

$y = -\dfrac{2}{0^2 - 1} = -\dfrac{2}{-1} = 2$

x-intercept: $-2 = 0$
no x-intercept
vertical asymptotes:

$x^2 - 1 = 0$
$(x - 1)(x + 1)$
$x = \pm 1$
horizontal asymptote:
$n < m$, so $y = 0$



$f(x) = -\dfrac{2}{x^2 - 1}$

67. $f(x) = \dfrac{2}{x^2 + x - 2}$

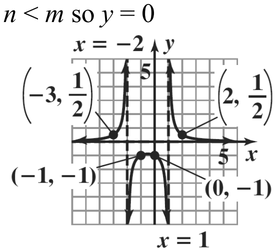
$f(-x) = -\dfrac{2}{(-x)^2 - x - 2} = \dfrac{2}{x^2 - x - 2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{2}{0^2 + 0 - 2} = \dfrac{2}{-2} = -1$

x-intercept: none
vertical asymptotes:

$x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$
$x = -2, x = 1$
horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{2}{x^2 + x - 2}$

68. $f(x) = \dfrac{-2}{x^2 - x - 2}$

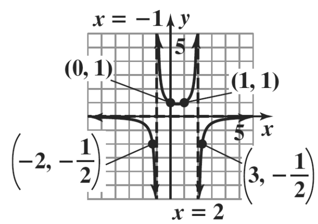
$f(-x) = \dfrac{-2}{(-x)^2 - (-x) - 2} = \dfrac{-2}{x^2 + x - 2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{-2}{0^2 - 0 - 2} = 1$

x-intercept: none
vertical asymptotes:

$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x = 2, x = -1$
horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{-2}{x^2 - x - 2}$

69. $f(x) = \dfrac{2x^2}{x^2 + 4}$

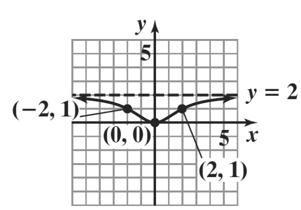
$f(-x) = \dfrac{2(-x)^2}{(-x)^2 + 4} = \dfrac{2x^2}{x^2 + 4} = f(x)$

y axis symmetry

y-intercept: $y = \dfrac{2(0)^2}{0^2 + 4} = 0$

x-intercept: $2x^2 = 0$
$x = 0$
vertical asymptote: none
horizontal asymptote:

$n = m$, so $y = \dfrac{2}{1} = 2$



$f(x) = \dfrac{2x^2}{x^2 + 4}$

70. $f(x)=\dfrac{4x^2}{x^2+1}$

$f(-x)=\dfrac{4(-x)^2}{(-x)^2+1}=\dfrac{4x^2}{x^2+1}=f(x)$

y axis symmetry

y-intercept: $y=\dfrac{4(0)^2}{0^2+1}=0$

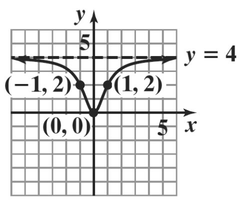
x-intercept: $4x^2=0$

$x=0$

vertical asymptote: none

horizontal asymptote:

$n=m$, so $y=\dfrac{4}{1}=4$



$f(x)=\dfrac{4x^2}{x^2+1}$

71. $f(x)=\dfrac{x+2}{x^2+x-6}$

$f(-x)=\dfrac{-x+2}{(-x)^2-(-x)-6}=\dfrac{-x+2}{x^2+x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0+2}{0^2+0-6}=-\dfrac{2}{6}=-\dfrac{1}{3}$

x-intercept:

$x+2=0$

$x=-2$

vertical asymptotes:

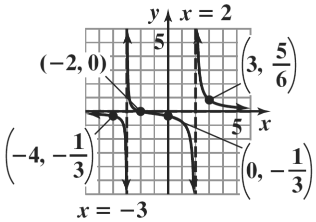
$x^2+x-6=0$

$(x+3)(x-2)$

$x=-3, x=2$

horizontal asymptote:

$n<m$, so $y=0$



$f(x)=\dfrac{x+2}{x^2+x-6}$

72. $f(x)=\dfrac{x-4}{x^2-x-6}$

$f(-x)=\dfrac{-x-4}{(-x)^2-(-x)-6}=-\dfrac{x+4}{x^2+x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0-4}{0^2-0-6}=\dfrac{2}{3}$

x-intercept:

$x-4=0, x=4$

vertical asymptotes:

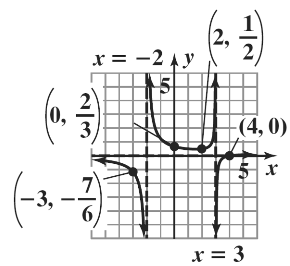
$x^2-x-6=0$

$(x-3)(x+2)$

$x=3, x=-2$

horizontal asymptote:

$n<m$, so $y=0$



$f(x)=\dfrac{x-4}{x^2-x-6}$

73. $f(x)=\dfrac{x-2}{x^2-4}$

$f(-x)=\dfrac{-x-2}{(-x)^2-4}=\dfrac{-x-2}{x^2-4}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0-2}{0^2-4}=\dfrac{-2}{-4}=\dfrac{1}{2}$

x-intercept:

$x-2=0, x=2$

vertical asymptotes:

$f(x)=\dfrac{x-2}{x^2-4}$

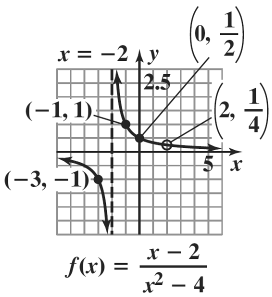
$=\dfrac{x-2}{(x-2)(x+2)}$

$=\dfrac{1}{x+2}$

$x=-2$ is a vertical asymptote.

Furthermore, the value 2 causes the original denominator to be zero, but the reduced form of the function's equation does not cause the denominator to be zero. Thus, there is a hole at $x=2$.

horizontal asymptote:
$n < m$, so $y = 0$



74. $f(x) = \dfrac{x-3}{x^2-9}$

$f(-x) = \dfrac{-x-3}{(-x)^2-9} = \dfrac{-x-3}{x^2-9}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

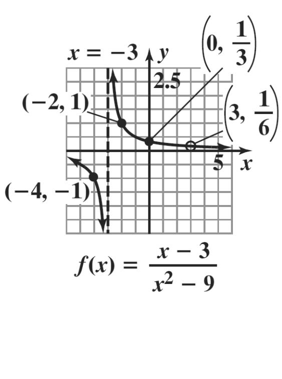
y-intercept: $y = \dfrac{0-3}{0^2-9} = \dfrac{-3}{-9} = \dfrac{1}{3}$

x-intercept:
$x-3=0, x=3$

vertical asymptotes:

$f(x) = \dfrac{x-3}{x^2-9}$
$= \dfrac{x-3}{(x-3)(x+3)}$
$= \dfrac{1}{x+3}$

$x=-3$ is a vertical asymptote.
Furthermore, the value 2 causes the original denominator to be zero, but the reduced form of the function's equation does not cause the denominator to be zero. Thus, there is a hole at $x = 3$.
horizontal asymptote:
$n < m$, so $y = 0$



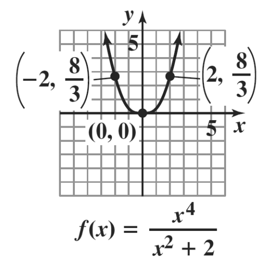
75. $f(x) = \dfrac{x^4}{x^2+2}$

$f(-x) = \dfrac{(-x)^4}{(-x)^2+2} = \dfrac{x^4}{x^2+2} = f(x)$

y-axis symmetry

y-intercept: $y = \dfrac{0^4}{0^2+2} = 0$

x-intercept: $x^4 = 0$
$x = 0$
vertical asymptote: none
horizontal asymptote:
$n > m$, so none



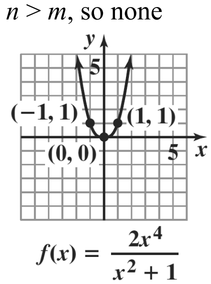
76. $f(x) = \dfrac{2x^4}{x^2+1}$

$f(-x) = \dfrac{2(-x)^4}{(-x)^2+1} = \dfrac{2x^4}{x^2+1} = f(x)$

y-axis symmetry

y-intercept: $y = \dfrac{2\left(0^4\right)}{0^2+2} = 0$

x-intercept: $2x^4 = 0$
$x = 0$
vertical asymptote: none
horizontal asymptote:
$n > m$, so none



77. $f(x)=\dfrac{x^2+x-12}{x^2-4}$

$f(-x)=\dfrac{(-x)^2-x-12}{(-x)^2-4}=\dfrac{x^2-x-12}{x^2-4}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^2+0-12}{0^2-4}=3$

x-intercept: $x^2+x-12=0$

$(x-3)(x+4)=0$

$x=3, x=-4$

vertical asymptotes:

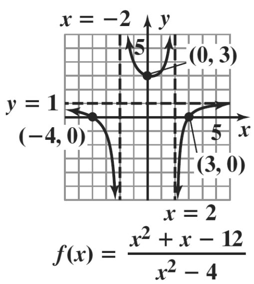
$x^2-4=0$

$(x-2)(x+2)=0$

$x=2, x=-2$

horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



78. $f(x)=\dfrac{x^2}{x^2+x-6}$

$f(-x)=\dfrac{(-x)^2}{(-x)^2-x-6}=\dfrac{x^2}{x^2-x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^2}{0^2+0-6}=0$

x-intercept: $x^2=0, x=0$

vertical asymptotes:

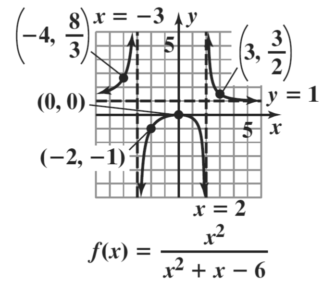
$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3, x=2$

horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



79. $f(x)=\dfrac{3x^2+x-4}{2x^2-5x}$

$f(-x)=\dfrac{3(-x)^2-x-4}{2(-x)^2+5x}=\dfrac{3x^2-x-4}{2x^2+5x}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{3(0)^2+0-4}{2(0)^2-5(0)}=\dfrac{-4}{0}$

no y-intercept

x-intercepts:

$3x^2+x-4=0$

$(3x+4)(x-1)=0$

$3x+4=0 \quad x-1=0$

$3x=-4$

$x=-\dfrac{4}{3}, x=1$

vertical asymptotes:

$2x^2-5x=0$

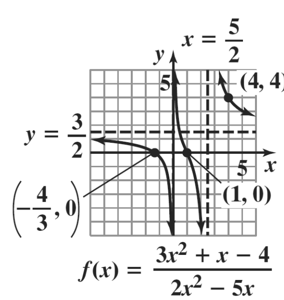
$x(2x-5)=0$

$x=0, 2x=5$

$x=\dfrac{5}{2}$

horizontal asymptote:

$n=m$, so $y=\dfrac{3}{2}$



80. $f(x) = \dfrac{x^2 - 4x + 3}{(x+1)^2}$

$f(-x) = \dfrac{(-x)^2 - 4(-x) + 3}{(-x+1)^2} = \dfrac{x^2 + 4x + 3}{(-x+1)^2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^2 - 4(0) + 3}{(0+1)^2} = \dfrac{3}{1} = 3$

x-intercept:

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$ and $x = 1$

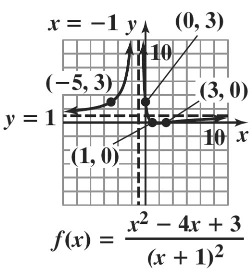
vertical asymptote:

$(x+1)^2 = 0$

$x = -1$

horizontal asymptote:

$n = m$, so $y = \dfrac{1}{1} = 1$



81. **a.** Slant asymptote:

$f(x) = x - \dfrac{1}{x}$

$y = x$

b. $f(x) = \dfrac{x^2 - 1}{x}$

$f(-x) = \dfrac{(-x)^2 - 1}{(-x)} = \dfrac{x^2 - 1}{-x} = -f(x)$

Origin symmetry

y-intercept: $y = \dfrac{0^2 - 1}{0} = \dfrac{-1}{0}$

no y-intercept

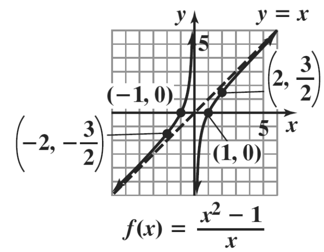
x-intercepts: $x^2 - 1 = 0$

$x = \pm 1$

vertical asymptote: $x = 0$

horizontal asymptote:

$n < m$, so none exist.



82. $f(x) = \dfrac{x^2 - 4}{x}$

a. slant asymptote:

$f(x) = x - \dfrac{4}{x}$

$y = x$

b. $f(x) = \dfrac{x^2 - 4}{x}$

$f(-x) = \dfrac{(-x)^2 - 4}{-x} = \dfrac{x^2 - 4}{-x} = -f(x)$

origin symmetry

y-intercept: $y = \dfrac{0^2 - 4}{0} = -\dfrac{4}{0}$

no y-intercept

x-intercept:

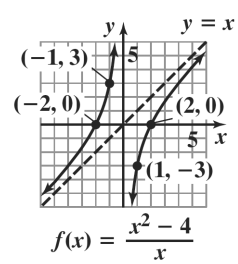
$x^2 - 4 = 0$

$x = \pm 2$

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



83. **a.** Slant asymptote:

$$f(x) = x + \frac{1}{x}$$

$y = x$

b. $$f(x) = \frac{x^2+1}{x}$$

$$f(-x) = \frac{(-x)^2+1}{-x} = \frac{x^2+1}{-x} = -f(x)$$

Origin symmetry

y-intercept: $y = \frac{0^2+1}{0} = \frac{1}{0}$

no y-intercept

x-intercept:

$x^2 + 1 = 0$

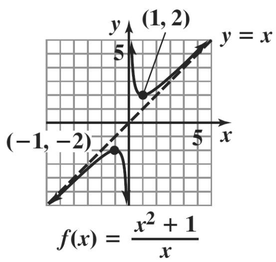
$x^2 = -1$

no x-intercept

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



84. $f(x) = \frac{x^2+4}{x}$

a. slant asymptote:

$$g(x) = x + \frac{4}{x}$$

$y = x$

b. $$f(x) = \frac{x^2+4}{x}$$

$$f(-x) = \frac{(-x)^2+4}{-x} = \frac{x^2+4}{-x} = -f(x)$$

origin symmetry

y-intercept: $y = \frac{0^2+4}{0} = \frac{4}{0}$

no y-intercept

$x^2 + 4 = 0$

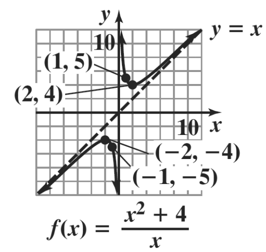
$x^2 = -4$

no x-intercept

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



85. **a.** Slant asymptote:

$$f(x) = x + 4 + \frac{6}{x-3}$$

$y = x + 4$

b. $$f(x) = \frac{x^2+x-6}{x-3}$$

$$f(-x) = \frac{(-x)^2+(-x)-6}{-x-3} = \frac{x^2-x-6}{-x-3}$$

$f(-x) \neq g(x)$, $g(-x) \neq -g(x)$

No symmetry

y-intercept: $y = \frac{0^2+0-6}{0-3} = \frac{-6}{-3} = 2$

x-intercept:

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3$ and $x = 2$

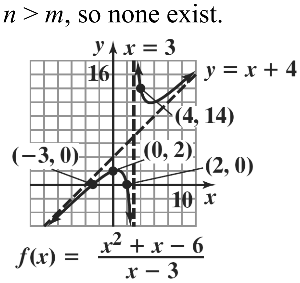
vertical asymptote:

$x - 3 = 0$

$x = 3$

horizontal asymptote:

$n > m$, so none exist.



86. $f(x) = \frac{x^2-x+1}{x-1}$

a. slant asymptote:

$$g(x) = x + \frac{1}{x-1}$$

$y = x$

b. $f(x) = \dfrac{x^2 - x - 1}{x - 1}$

$f(-x) = \dfrac{(-x)^2 - (-x) + 1}{-x - 1} = \dfrac{x^2 + x + 1}{-x - 1}$

no symmetry

$f(-x) \neq f(x), f(-x) \neq -g(x)$

y-intercept: $y = \dfrac{0^2 - 0 + 1}{0 - 1} = \dfrac{1}{-1} = -1$

x-intercept:

$x^2 - x + 1 = 0$

no x-intercept

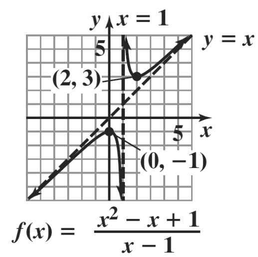
vertical asymptote:

$x - 1 = 0$

$x = 1$

horizontal asymptote:

$n > m$, so none



87. $f(x) = \dfrac{x^3 + 1}{x^2 + 2x}$

a. slant asymptote:

$$\begin{array}{r|l} & x - 2 \\ x^2 + 2x & x^3 \qquad + 1 \\ & \underline{x^3 + 2x^2} \\ & -2x^2 \\ & \underline{-2x^2 + 4x} \\ & -4x + 1 \end{array}$$

$y = x - 2$

b. $f(-x) = \dfrac{(-x)^3 + 1}{(-x)^2 + 2(-x)} = \dfrac{-x^3 + 1}{x^2 - 2x}$

$f(-x) \neq f(x),\ f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^3 + 1}{0^2 + 2(0)} = \dfrac{1}{0}$

no y-intercept

x-intercept: $x^3 + 1 = 0$

$x^3 = -1$

$x = -1$

vertical asymptotes:

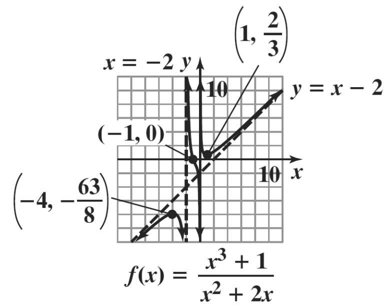
$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 0, x = -2$

horizontal asymptote:

$n > m$, so none



88. $f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

a. slant asymptote:

$$\begin{array}{r|l} & x + \dfrac{9x - 1}{x^2 - 9} \\ x^2 - 9 & x^3 \qquad - 1 \\ & \underline{x^3 - 9x} \\ & 9x - 1 \end{array}$$

$y = x$

b. $f(-x) = \dfrac{(-x)^3 - 1}{(-x)^2 - 9} = \dfrac{-x^3 - 1}{x^2 - 9}$

$f(-x) \neq f(x),\ f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^3 - 1}{0^2 - 9} = \dfrac{1}{9}$

x-intercept: $x^3 - 1 = 0$

$x^3 = 1$

$x = 1$

vertical asymptotes:

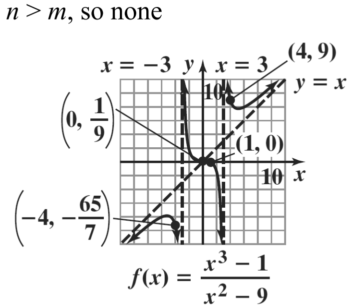
$x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x = 3, x = -3$

horizontal asymptote:

$n > m$, so none

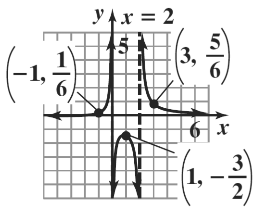


89. $\dfrac{5x^2}{x^2-4}\cdot\dfrac{x^2+4x+4}{10x^3}$

$=\dfrac{\cancel{5}\,\cancel{x^2}}{\cancel{(x+2)}(x-2)}\cdot\dfrac{(x+2)^{\cancel{2}}}{\underset{2}{\cancel{10}}\,x^{\cancel{3}1}}$

$=\dfrac{x+2}{2x(x-2)}$

So, $f(x)=\dfrac{x+2}{2x(x-2)}$



$f(x)=\dfrac{x+2}{2x(x-2)}$

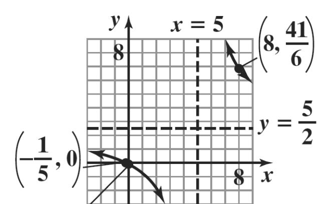
90. $\dfrac{x-5}{10x-2}\div\dfrac{x^2-10x+25}{25x^2-1}$

$=\dfrac{x-5}{10x-2}\cdot\dfrac{25x^2-1}{x^2-10x+25}$

$=\dfrac{\cancel{x-5}}{2\cancel{(5x-1)}}\cdot\dfrac{(5x+1)\cancel{(5x-1)}}{(x-5)^{\cancel{2}}}$

$=\dfrac{5x+1}{2(x-5)}$

So, $f(x)=\dfrac{5x+1}{2(x-5)}$



$f(x)=\dfrac{5x+1}{2(x-5)}$

91. $\dfrac{x}{2x+6}-\dfrac{9}{x^2-9}$

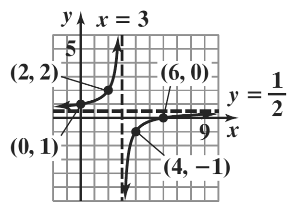
$\dfrac{x}{2x+6}-\dfrac{9}{x^2-9}=\dfrac{x}{2(x+3)}-\dfrac{9}{(x+3)(x-3)}$

$=\dfrac{x(x-3)-9(2)}{2(x+3)(x-3)}$

$=\dfrac{x^2-3x-18}{2(x+3)(x-3)}$

$=\dfrac{(x-6)\cancel{(x+3)}}{2\cancel{(x+3)}(x-3)}=\dfrac{x-6}{2(x-3)}$

So, $f(x)=\dfrac{x-6}{2(x-3)}$



$f(x)=\dfrac{x-6}{2(x-3)}$

92. $\dfrac{2}{x^2+3x+2}-\dfrac{4}{x^2+4x+3}$

$=\dfrac{2}{(x+2)(x+1)}-\dfrac{4}{(x+3)(x+1)}$

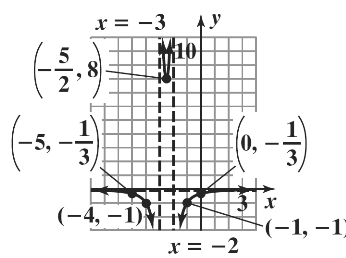
$=\dfrac{2(x+3)-4(x+2)}{(x+2)(x+1)(x+3)}$

$=\dfrac{2x+6-4x-8}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2x-2}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2\cancel{(x+1)}}{(x+2)\cancel{(x+1)}(x+3)}=\dfrac{-2}{(x+2)(x+3)}$

So, $f(x)=\dfrac{-2}{(x+2)(x+3)}$



$f(x)=\dfrac{-2}{(x+2)(x+3)}$

93. $\dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}} = \dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}} \cdot \dfrac{(x+2)(x-2)}{(x+2)(x-2)}$

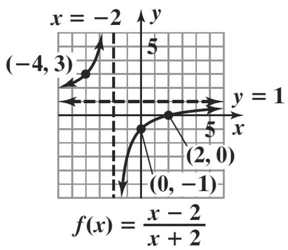
$= \dfrac{(x+2)(x-2)-3(x-2)}{(x+2)(x-2)+(x+2)}$

$= \dfrac{x^2-4-3x+6}{x^2-4+x+2}$

$= \dfrac{x^2-3x+2}{x^2+x-2}$

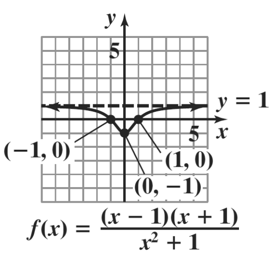
$= \dfrac{(x-2)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = \dfrac{x-2}{x+2}$

So, $f(x) = \dfrac{x-2}{x+2}$

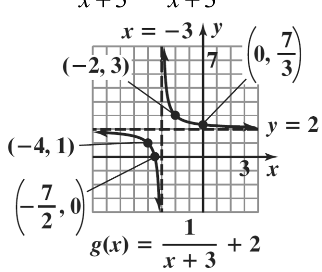


94. $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}} \cdot \dfrac{x}{x} = \dfrac{x^2-1}{x^2+1} = \dfrac{(x-1)(x+1)}{x^2+1}$

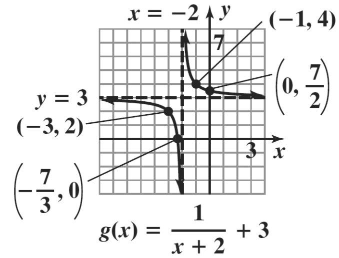
So, $f(x) = \dfrac{(x-1)(x+1)}{x^2+1}$



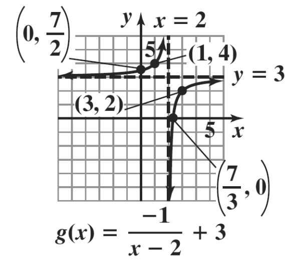
95. $g(x) = \dfrac{2x+7}{x+3} = \dfrac{1}{x+3} + 2$



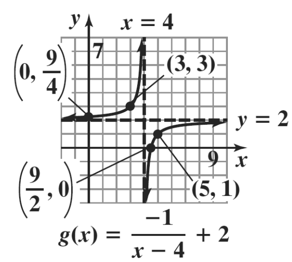
96. $g(x) = \dfrac{3x+7}{x+2} = \dfrac{1}{x+2} + 3$



97. $g(x) = \dfrac{3x-7}{x-2} = \dfrac{-1}{x-2} + 3$



98. $g(x) = \dfrac{2x-9}{x-4} = \dfrac{-1}{x-4} + 2$



99. **a.** $C(x) = 100x + 100{,}000$

b. $\bar{C}(x) = \dfrac{100x+100{,}000}{x}$

c. $\bar{C}(500) = \dfrac{100(500)+100{,}000}{500} = \$300$

When 500 bicycles are manufactured, it costs $300 to manufacture each.

$\bar{C}(1000) = \dfrac{100(1000)+100{,}000}{1000} = \$200$

When 1000 bicycles are manufactured, it costs $200 to manufacture each.

$\bar{C}(2000) = \dfrac{100(2000)+100{,}000}{2000} = \$150$

When 2000 bicycles are manufactured, it costs $150 to manufacture each.

$$\overline{C}(4000)=\frac{100(4000)+100,000}{4000}=\$125$$

When 4000 bicycles are manufactured, it costs \$125 to manufacture each.
The average cost decreases as the number of bicycles manufactured increases.

d. $n=m$, so $y=\frac{100}{1}=100$.
As greater numbers of bicycles are manufactured, the average cost approaches \$100.

100. a. $C(x) = 30x + 300,000$

b. $\overline{C}=\frac{300,000+30x}{x}$

c. $\overline{C}(1000)=\frac{300000+30(1000)}{1000}=330$

When 1000 shoes are manufactured, it costs \$330 to manufacture each.

$\overline{C}(10000)=\frac{300000+30(10000)}{10000}=60$

When 10,000 shoes are manufactured, it costs \$60 to manufacture each.

$\overline{C}(100,00)=\frac{300,000+30(100,000)}{100,000}=33$

When 100,000 shoes are manufactured, it costs \$33 to manufacture each.
The average cost decreases as the number of shoes manufactured increases.

d. $n=m$, so $y=\frac{30}{1}=30$.
As greater numbers of shoes are manufactured, the average cost approaches \$30.

101. a. From the graph the pH level of the human mouth 42 minutes after a person eats food containing sugar will be about 6.0.

b. From the graph, the pH level is lowest after about 6 minutes.

$$f(6)=\frac{6.5(6)^2-20.4(6)+234}{6^2+36}$$
$$=4.8$$

The pH level after 6 minutes (i.e. the lowest pH level) is 4.8.

c. From the graph, the pH level appears to approach 6.5 as time goes by. Therefore, the normal pH level must be 6.5.

d. $y=6.5$
Over time, the pH level rises back to the normal level.

e. During the first hour, the pH level drops quickly below normal, and then slowly begins to approach the normal level.

102. a. From the graph, the drug's concentration after three hours appears to be about 1.5 milligrams per liter.

$$C(3)=\frac{5(3)}{3^2+1}=\frac{15}{10}=1.5$$

This verifies that the drug's concentration after 3 hours will be 1.5 milligrams per liter.

b. The degree of the numerator, 1, is less than the degree of the denominator, 2, so the the horizontal asymptote is $y=0$.
Over time, the drug's concentration will approach 0 milligrams per liter.

103. $P(10)=\frac{100(10-1)}{10}=90 \quad (10, 90)$

For a disease that smokers are 10 times more likely to contact than non-smokers, 90% of the deaths are smoking related.

104. $P(9)=\frac{100(9-1)}{9}=89 \quad (9, 89)$

For a disease that smokers are 9 times more likely to have than non-smokers, 89% of the deaths are smoking related.

105. $y = 100$ As incidence of the diseases increases, the percent of death approaches, but never gets to be, 100%.

106. No, the percentage approaches 100%, but never reaches 100%.

107. a. $f(x)=\frac{p(x)}{q(x)}=\frac{1.75x^2-15.9x+160}{2.1x^2-3.5x+296}$

b. According to the graph, $\frac{2504.0}{3720.7}\approx 0.67$ or 67% of federal expenditures were spent on human resources in 2010.

c. According to the function,

$$f(x)=\frac{1.75(40)^2-15.9(40)+160}{2.1(40)^2-3.5(40)+296}\approx 0.66 \text{ or}$$

66% of federal expenditures were spent on human resources in 2010.

d. The degree of the numerator, 2, is equal to the degree of the denominator, 2. The leading coefficients of the numerator and denominator are 1.75 and 2.1, respectively. The equation of the horizontal asymptote is $y = \frac{1.75}{2.1}$ which is about 83%
Thus, about 83% of federal expenditures will be spent on human resources over time.

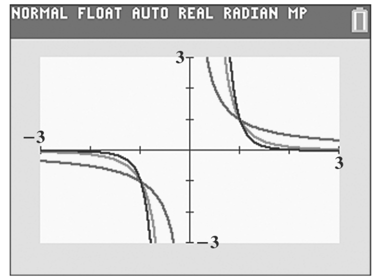
108. $x - 10$ = the average velocity on the return trip.
The function that expresses the total time required to complete the round trip is
$T(x) = \frac{600}{x} + \frac{600}{x-10}.$

109. $T(x) = \frac{90}{9x} + \frac{5}{x} = \frac{10}{x} + \frac{5}{x}$
The function that expresses the total time for driving and hiking is $T(x) = \frac{10}{x} + \frac{5}{x}.$

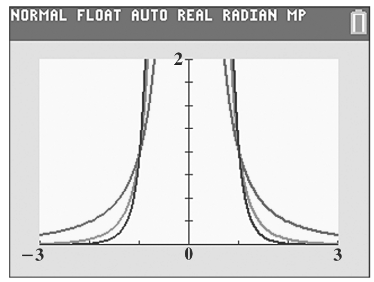
110. $A = xy = 2500$
$y = \frac{2500}{x}$
$P = 2x + 2y = 2x + 2\left(\frac{2500}{x}\right) = 2x + \frac{5000}{x}$
The perimeter of the floor, P, as a function of the width, x is $P(x) = 2x + \frac{5000}{x}.$

111. $A = lw$
$xy = 50$
$l = y + 2 = \frac{50}{x} + 2$
$w = x + 1$
$A = \left(\frac{50}{x} + 2\right)(x+1)$
$= 50 + \frac{50}{x} + 2x + 2$
$= 2x + \frac{50}{x} + 52$
The total area of the page is
$A(x) = 2x + \frac{50}{x} + 52.$

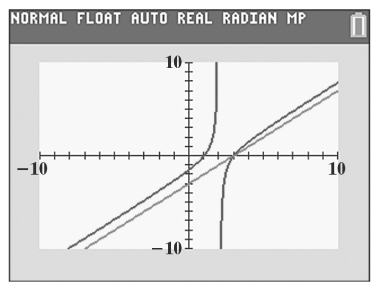
112. – 121. Answers will vary.

122.


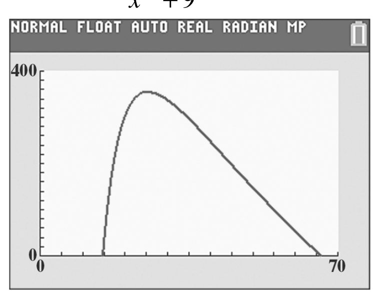
The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

123.


The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

124.


$g(x)$ is the graph of a line where $f(x)$ is the graph of a rational function with a slant asymptote.
In $g(x)$, $x - 2$ is a factor of $x^2 - 5x + 6$.

125. a. $f(x) = \frac{27725(x-14)}{x^2+9} - 5x$


b. The graph increases from late teens until about the age of 25, and then the number of arrests decreases.

c. At age 25 the highest number arrests occurs. There are about 356 arrests for every 100,000 drivers.

126. does not make sense; Explanations will vary. Sample explanation: A rational function can have at most one horizontal asymptote.

127. does not make sense; Explanations will vary. Sample explanation: The function has one vertical asymptote, $x = 2$.

128. makes sense

129. does not make sense; Explanations will vary. Sample explanation: As production level increases, the average cost for a company to produce each unit of its product decreases.

130. false; Changes to make the statement true will vary. A sample change is: The graph of a rational function may have both a vertical asymptote and a horizontal asymptote.

131. true

132. true

133. true

134. – 137. Answers will vary.

138. Let x = the number of miles driven in a week.

$$20 + 0.10x < 30 + 0.05x$$
$$0.05x < 10$$
$$x < 200$$

Driving less than 200 miles in a week makes Basic the better deal.

139. The graph (a) passes the vertical line test and is therefore is a <u>function</u>.
The graph (b) fails the vertical line test and is therefore <u>not a function</u>.
The graph (c) fails the vertical line test and is therefore <u>not a function</u>.
The graph (d) passes the vertical line test and is therefore is a <u>function</u>.

140. The graphs of (b) and (d) pass the horizontal line test and thus have an inverse.

141.

$$2x^2 + x = 15$$
$$2x^2 + x - 15 = 0$$
$$(2x-5)(x+3) = 0$$
$$2x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = \frac{5}{2} \qquad\qquad x = -3$$

The solution set is $\left\{-3, \frac{5}{2}\right\}$.

142.

$$x^3 + x^2 = 4x + 4$$
$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x+1)(x^2 - 4) = 0$$
$$(x+1)(x+2)(x-2) = 0$$

The solution set is $\{-2, -1, 2\}$.

143.

$$\frac{x+1}{x+3} - 2 = \frac{x+1}{x+3} - \frac{2(x+3)}{x+3}$$
$$= \frac{x+1}{x+3} - \frac{2x+6}{x+3}$$
$$= \frac{x+1-2x-6}{x+3}$$
$$= \frac{-x-5}{x+3} \text{ or } -\frac{x+5}{x+3}$$

Section 2.7

Check Point Exercises

1.
$$x^2 - x > 20$$
$$x^2 - x - 20 > 0$$
$$(x+4)(x-5) > 0$$

Solve the related quadratic equation.

$(x+4)(x-5)=0$

Apply the zero product principle.

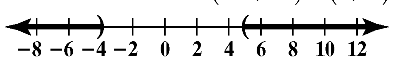
$x+4=0$ or $x-5=0$

$x=-4$ $\quad$ $x=5$

The boundary points are –2 and 4.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-4)$	–5	$(-5)^2-(-5)>20$ $30>20$, true	$(-\infty,-4)$ belongs to the solution set.
$(-4,5)$	0	$(0)^2-(0)>20$ $0>20$, false	$(-4,5)$ does not belong to the solution set.
$(5,\infty)$	10	$(10)^2-(10)>20$ $90>20$, true	$(5,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-4)\cup(5,\infty)$ or $\{x \mid x<-4 \text{ or } x>5\}$.



2.
$$2x^2 \le -6x-1$$
$$2x^2+6x+1 \le 0$$

Solve the related quadratic equation to find the boundary points.

$2x^2+6x+1=0$

$a=2 \quad b=6 \quad c=1$

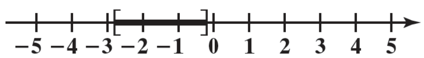
$$x=\frac{-(6)\pm\sqrt{(6)^2-4(2)(1)}}{2(2)}$$
$$=\frac{-6\pm\sqrt{36-8}}{4}$$
$$=\frac{-6\pm\sqrt{28}}{4}$$
$$=\frac{-6\pm2\sqrt{7}}{4}$$
$$=\frac{-3\pm\sqrt{7}}{2}$$

$x=\dfrac{-3+\sqrt{7}}{2}$ or $x=\dfrac{-3-\sqrt{7}}{2}$

$x\approx-0.2$ $\quad$ $x\approx-2.8$

Interval	Test Value	Test	Conclusion
$\left(-\infty, \frac{-3-\sqrt{7}}{2}\right)$	-10	$2(-10)^2 \le -6(-10)-1$ $200 \le 59$, false	$\left(-\infty, \frac{-3-\sqrt{7}}{2}\right)$ is not part of the solution set
$\left(\frac{-3-\sqrt{7}}{2}, \frac{-3+\sqrt{7}}{2}\right)$	-1	$2(-1)^2 \le -6(-1)-1$ $2 \le 5$, true	$\left(\frac{-3-\sqrt{7}}{2}, \frac{-3+\sqrt{7}}{2}\right)$ is part of the solution set
$\left(\frac{-3+\sqrt{7}}{2}, \infty\right)$	0	$2(0)^2 \le -6(0)-1$ $0 \le -1$, false	$\left(\frac{-3+\sqrt{7}}{2}, \infty\right)$ is not part of the solution set

The solution set is $\left(\frac{-3-\sqrt{7}}{2}, \frac{-3+\sqrt{7}}{2}\right)$.



3.

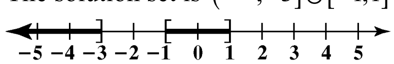
$$x^3 + 3x^2 \le x + 3$$
$$x^3 + 3x^2 - x - 3 \le 0$$
$$(x+1)(x-1)(x+3) \le 0$$
$$(x+1)(x-1)(x+3) = 0$$

$x+1=0$ or $x-1=0$ or $x+3=0$

$x=-1$ $\quad x=1$ $\quad x=-3$

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$(-4)^3+3(-4)^2 \le (-4)+3$ $-16 \le -1$ true	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1]$	–2	$(-2)^3+3(-2)^2 \le (-2)+3$ $4 \le 1$ false	$(-3,-1]$ does not belong to the solution set.
$[-1,1]$	0	$(0)^3+3(0)^2 \le (0)+3$ $0 \le 3$ true	$[-1,1]$ belongs to the solution set.
$[1,\infty)$	2	$(6+3)(6-5) > 0$ true	$[1,\infty)$ does not belong to the solution set.

The solution set is $(-\infty,-3] \cup [-1,1]$ or $\{x \mid x \le -3 \text{ or } -1 \le x \le 1\}$.



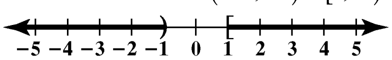
4.

$$\frac{2x}{x+1} \geq 1$$
$$\frac{2x}{x+1} - 1 \geq 0$$
$$\frac{x-1}{x+1} \geq 0$$

$x - 1 = 0$ or $x + 1 = 0$
$x = 1$ $\quad x = -1$

Test Interval	Test Number	Test	Conclusion
$(-\infty, -1)$	–2	$\frac{2(-2)}{-2+1} \geq 1$ $4 \geq 1$, true	$(-\infty, -1)$ belongs to the solution set.
$(-1, 1]$	0	$\frac{2(0)}{0+1} \geq 1$ $0 \geq 1$, false	$(-1, 1]$ does not belong to the solution set.
$[1, \infty)$	2	$\frac{2(2)}{2+1} \geq 1$ $\frac{4}{3} \geq 1$, true	$[1, \infty)$ belongs to the solution set.

The solution set is $(-\infty, -1) \cup [1, \infty)$ or $\{x | x < -1 \text{ or } x \geq 1\}$.



5.

$$-16t^2 + 80t > 64$$
$$-16t^2 + 80t - 64 > 0$$
$$-16(t-1)(t-4) > 0$$

$t - 1 = 0$ or $t - 4 = 0$
$t = 1$ $\quad t = 4$

Test Interval	Test Number	Test	Conclusion
$(-\infty, 1)$	0	$-16(0)^2 + 80(0) > 64$ $0 > 64$, false	$(-\infty, 1)$ does not belong to the solution set.
$(1, 4)$	2	$-16(2)^2 + 80(2) > 64$ $96 > 64$, true	$(1, 4)$ belongs to the solution set.
$(4, \infty)$	5	$-16(5)^2 + 80(5) > 64$ $0 > 64$, false	$(4, \infty)$ does not belong to the solution set.

The object will be more than 64 feet above the ground between 1 and 4 seconds.

Concept and Vocabulary Check 2.7

1. $x^2 + 8x + 15 = 0$; boundary

2. $(-\infty, -5)$; $(-5, -3)$; $(-3, \infty)$

3. true

4. true

5. $[-\infty, -2) \cup [1, \infty)$

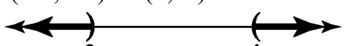
Exercise Set 2.7

1. $(x-4)(x+2)>0$
$x=4$ or $x=-2$

T	F	T

$-2 \quad 4$

Test -3: $(-3-4)(-3+2)>0$
$7>0$ True
Test 0: $(0-4)(0+2)>0$
$-8>0$ False
Test 5: $(5-4)(5+2)>0$
$7>0$ True
$(-\infty,-2)$ or $(4,\infty)$

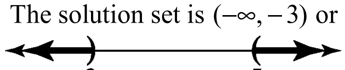


2. $(x+3)(x-5)>0$
$x=-3$ or $x=5$

T	F	T

$-3 \quad 5$

Test -4: $(-4+3)(-4-5)>0$
$9>0$ True
Test 0: $(0+3)(0-5)>0$
$-15>0$ False
Test 6: $(6+3)(6-5)>0$
$18>0$ True
The solution set is $(-\infty,-3)$ or $(5,\infty)$.

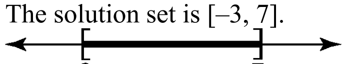


3. $(x-7)(x+3)\le 0$
$x=7$ or $x=-3$

F	T	F

$-3 \quad 7$

Test -4: $(-4-7)(-4+3)\le 0$
$11\le 0$ False
Test 0: $(0-7)(0+3)\le 0$
$-21\le 0$ True
Test 8: $(8-7)(8+3)\le 0$
$11\le 0$ False
The solution set is $[-3, 7]$.

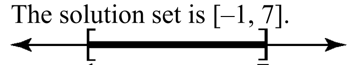


4. $(x+1)(x-7)\le 0$
$x=-1$ or $x=7$

F	T	F

$-1 \quad 7$

Test -2: $(-2+1)(-2-7)\le 0$
$9\le 0$ False
Test 0: $(0+1)(0-7)\le 0$
$-7\le 0$ True
Test 8: $(8+1)(8-7)\le 0$
$9\le 0$ False
The solution set is $[-1, 7]$.

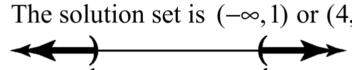


5. $x^2-5x+4>0$
$(x-4)(x-1)>0$
$x=4$ or $x=1$

T	F	T

$1 \quad 4$

Test 0: $0^2-5(0)+4>0$
$4>0$ True
Test 2: $2^2-5(2)+4>0$
$-2>0$ False
Test 5: $5^2-5(5)+4>0$
$4>0$ True
The solution set is $(-\infty,1)$ or $(4,\infty)$.

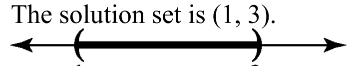


6. $x^2-4x+3<0$
$(x-1)(x-3)<0$
$x=1$ or $x=3$

F	T	F

$1 \quad 3$

Test 0: $0^2-4(0)+3<0$
$3<0$ False
Test 2: $2^2-4(2)+3<0$
$-1<0$ True
Test 4: $4^2-4(4)+3<0$
$3<0$ False
The solution set is $(1, 3)$.



7. $x^2+5x+4>0$

$(x+1)(x+4)>0$

$x=-1$ or $x=-4$

T	F	T

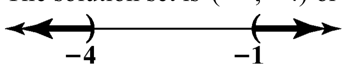
-4 -1

Test –5: $(-5)^2+5(-5)+4>0$
$4>0$ True

Test –3: $(-3)^2+5(-3)+4>0$
$-2>0$ False

Test 0: $0^2+5(0)+4>0$
$4>0$ True

The solution set is $(-\infty,-4)$ or $(-1,\infty)$.



8. $x^2+x-6>0$

$(x+3)(x-2)>0$

$x=-3$ or $x=2$

T	F	T

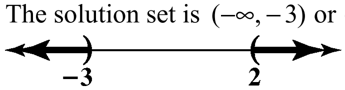
-3 2

Test –4: $(-4)^2-4-6>0$
$6>0$ True

Test 0: $(0)^2+0-6>0$
$-6>0$ False

Test 3: $3^2+3-6>0$
$6>0$ True

The solution set is $(-\infty,-3)$ or $(2,\infty)$.



9. $x^2-6x+9<0$

$(x-3)(x-3)<0$

$x=3$

F	F

3

Test 0: $0^2-6(0)+9<0$
$9<0$ False

Test 4: $4^2-6(4)+9<0$
$1<0$ False

The solution set is the empty set, $\varnothing$.

10. $x^2-2x+1>0$

$(x-1)(x-1)>0$

$x=1$

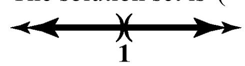
T	T

1

Test 0: $0^2-2(0)+1>0$
$1>0$ True

Test 2: $2^2-2(2)+1>0$
$1>0$ True

The solution set is $(-\infty,1)$ or $(1,\infty)$.



11. $3x^2+10x-8\le 0$

$(3x-2)(x+4)\le 0$

$x=\frac{2}{3}$ or $x=-4$

F	T	F

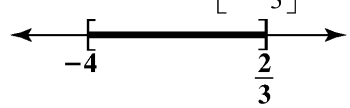
-4 $\frac{2}{3}$

Test –5: $3(-5)^2+10(-5)-8\le 0$
$17\le 0$ False

Test 0: $3(0)^2+10(0)-8\le 0$
$8\le 0$ True

Test 1: $3(1)^2+10(1)-8\le 0$
$5\le 0$ False

The solution set is $\left[-4,\frac{2}{3}\right]$.



12. $9x^2+3x-2\ge 0$

$(3x-1)(3x+2)\ge 0$

$3x=1 \quad 3x=-2$

$x=\frac{1}{3} \quad x=\frac{-2}{3}$

T	F	T

$\frac{-2}{3}$ $\quad$ $\frac{1}{3}$

Test –1: $9(-1)^2+3(-1)-2\ge 0$

$4\ge 0$ True

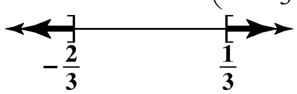
Test 0: $9(0)^2+3(0)-2\ge 0$

$-2\ge 0$ False

Test 1: $9(1)^2+3(1)-2\le 0$

$10\ge 0$ True

The solution set is $\left(-\infty,\frac{-2}{3}\right]$ or $\left[\frac{1}{3},\infty\right)$.



13. $2x^2+x<15$

$2x^2+x-15<0$

$(2x-5)(x+3)<0$

$2x-5=0$ or x+3=0

$2x=5$

$x=\frac{5}{2}$ or $x=-3$

F	T	F

-3 $\quad$ $\frac{5}{2}$

Test –4: $2(-4)^2+(-4)<15$

$28<15$ False

Test 0: $2(0)^2+0<15$

$0<15$ True

Test 3: $2(3)^2+3<15$

$21<15$ False

The solution set is $\left(-3,\frac{5}{2}\right)$.

14. $6x^2+x>1$

$6x^2+x-1>0$

$(2x+1)(3x-1)>0$

$2x+1=0$ or $3x-1=0$

$2x=-1 \qquad 3x=1$

$x=-\frac{1}{2} \qquad x=\frac{1}{3}$

T	F	T

$-\frac{1}{2}$ $\quad$ $\frac{1}{3}$

Test –1: $6(-1)^2+(-1)>1$

$5>1$ True

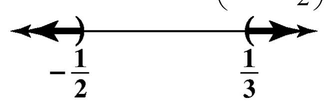
Test 0: $6(0)^2+0>1$

$0>1$ False

Test 1: $6(1)^2+1>1$

$7>1$ True

The solution set is $\left(-\infty,-\frac{1}{2}\right)$ or $\left(\frac{1}{3},\infty\right)$.



15. $4x^2+7x<-3$

$4x^2+7x+3<0$

$(4x+3)(x+1)<0$

$4x+3=0$ or x+1=0

4x-3=0

$x=-\frac{3}{4}$ or $x=-1$

F	T	F

-1 $\quad$ $-\frac{3}{4}$

Test –2: $4(-2)^2+7(-2)<-3$

$2<-3$ False

Test $-\frac{7}{8}$: $4\left(-\frac{7}{8}\right)^2+7\left(-\frac{7}{8}\right)<-3$

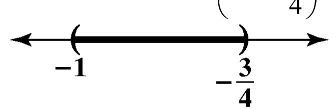
$\frac{49}{16}-\frac{49}{8}<-3$

$-\frac{49}{16}<-3$ True

Test 0: $4(0)^2+7(0)<-3$

$0<-3$ False

The solution set is $\left(-1,-\frac{3}{4}\right)$.



16. $3x^2+16x<-5$

$3x^2+16x+5<0$

$(3x+1)(x+5)<0$

$3x+1=0$ or $x+5=0$

$3x=-1$

$x=-\frac{1}{3}$ $\quad x=-5$

F	T	F
	-5	$-\frac{1}{3}$

Test –6: $3(-6)^2+16(-6)<-5$

$12<-5$ False

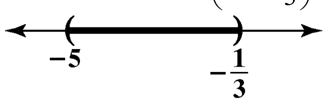
Test –2: $3(-2)^2+16(-2)<-5$

$-20<-5$ True

Test 0: $3(0)^2+16(0)<-5$

$0<-5$ False

The solution set is $\left(-5,-\frac{1}{3}\right)$.



17. $5x\le 2-3x^2$

$3x^2+5x-2\le 0$

$(3x-1)(x+2)\le 0$

$3x-1=0$ or $x+2=0$

$3x=1$

$3x-1=0$ or $x+2=0$

$3x=1$

$x=\frac{1}{3}$ or $x=-2$

F	T	F
	-2	$\frac{1}{3}$

Test –3: $5(-3)\le 2-3(-3)^2$

$-15\le -25$ False

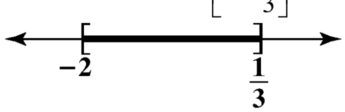
Test 0: $5(0)\le 2-3(0)^2$

$0\le 2$ True

Test 1: $5(1)\le 2-3(1)^2$

$5\le -1$ False

The solution set is $\left[-2,\frac{1}{3}\right]$.



18. $4x^2+1\ge 4x$

$4x^2-4x+1\ge 0$

$(2x-1)(2x-1)\ge 0$

$2x-1=0$

$x=\frac{1}{2}$

T	T
$\frac{1}{2}$	

Test 0: $4(0)^2+1\ge 4(0)$

$1\ge 0$ True

Test 1: $4(1)^2+1\ge 4(1)$

$5\ge 4$ True

The solution set is $(-\infty,\infty)$.

19. $x^2-4x\ge 0$

$x(x-4)\ge 0$

$x=0$ or $x-4=0$

$x=4$

T	F	T
	0	4

Test –1: $(-1)^2-4(-1)\ge 0$

$5\ge 0$ True

Test 1: $(1)^2-4(1)\ge 0$

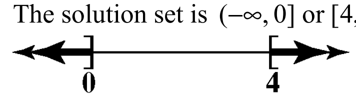
$-3\ge 0$ False

$0\le 2$ True

Test 5: $5^2-4(5)\ge 0$

$5\ge 0$ True

The solution set is $(-\infty,0]$ or $[4,\infty)$.



20. $x^2 + 2x < 0$
$x(x+2) < 0$
$x = 0$ or $x = -2$

F	T	F

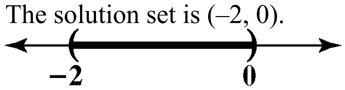
$-2 \quad 0$

Test –3: $(-3)^2 + 2(-3) < 0$
$3 < 0$ False

Test –1: $(-1)^2 + 2(-1) < 0$
$-1 < 0$ True

Test 1: $(1)^2 + 2(1) < 0$
$3 < 0$ False

The solution set is $(-2, 0)$.



21. $2x^2 + 3x > 0$
$x(2x+3) > 0$
$x = 0$ or $x = -\frac{3}{2}$

T	F	T

$-\frac{3}{2} \quad 0$

Test –2: $2(-2)^2 + 3(-2) > 0$
$2 > 0$ True

Test –1: $2(-1)^2 + 3(-1) > 0$
$-1 > 0$ False

Test 1: $2(1)^2 + 3(1) > 0$
$5 > 0$ True

The solution set is $\left(-\infty, -\frac{3}{2}\right)$ or $(0, \infty)$.

22. $3x^2 - 5x \le 0$
$x(3x-5) \le 0$
$x = 0$ or $x = \frac{5}{3}$

F	T	F

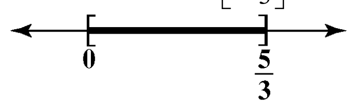
$0 \quad \frac{5}{3}$

Test –1: $3(-1)^2 - 5(-1) \le 0$
$8 \le 0$ False

Test 1: $3(1)^2 - 5(1) \le 0$
$-2 \le 0$ True

Test 2: $3(2)^2 - 5(2) \le 0$
$2 \le 0$ False

The solution set is $\left[0, \frac{5}{3}\right]$.



23. $-x^2 + x \ge 0$
$x^2 - x \le 0$
$x(x-1) \le 0$
$x = 0$ or $x = 1$

F	T	F

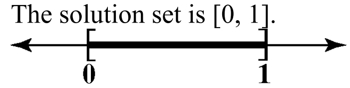
$0 \quad 1$

Test –1: $-(-1)^2 + (-1) \ge 0$
$-2 \ge 0$ False

Test $\frac{1}{2}$: $-\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) \ge 0$
$\frac{1}{4} \ge 0$ True

Test 2: $-(2)^2 + 2 \ge 0$
$-2 \ge 0$ False

The solution set is $[0, 1]$.



24. $-x^2 + 2x \ge 0$

$x(-x+2) \ge 0$

$x = 0$ or $x = 2$

F	T	F
	0	2

Test -1: $-(-1)^2 + 2(-1) \ge 0$

$-3 \ge 0$ False

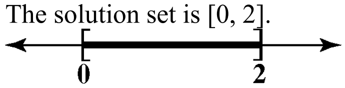
Test 1: $-(1)^2 + 2(1) \ge 0$

$1 \ge 0$ True

Test 3: $-(3)^2 + 2(3) \ge 0$

$-3 \ge 0$ False

The solution set is [0, 2].



25. $x^2 \le 4x - 2$

$x^2 - 4x + 2 \le 0$

Solve $x^2 - 4x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

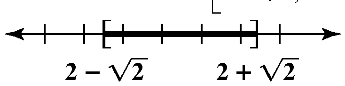
$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= 2 \pm \sqrt{2}$$

$x \approx 0.59$ or $x \approx 3.41$

F	T	F
	0.59	3.41

The solution set is $\left[2-\sqrt{2}, 2+\sqrt{2}\right]$ or $[0.59, 3.41]$.



26. $x^2 \le 2x + 2$

$x^2 - 2x - 2 \le 0$

Solve $x^2 - 2x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

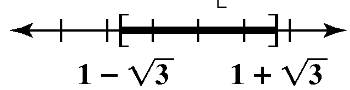
$$= \frac{2 \pm \sqrt{12}}{2}$$

$$= 1 \pm \sqrt{3}$$

$x \approx -0.73$ or $x \approx 2.73$

F	T	F
	–0.73	2.73

The solution set is $\left[1-\sqrt{3}, 1+\sqrt{3}\right]$ or $[-0.73, 2.73]$.



27. $x^2 - 6x + 9 < 0$

Solve $x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$(x-3)^2 = 0$

$x = 3$

F	F
	3

The solution set is the empty set, $\varnothing$.

28. $4x^2 - 4x + 1 \ge 0$

Solve $4x^2 - 4x + 1 = 0$

$(2x-1)(2x-1) = 0$

$(2x-1)^2 = 0$

$$x = \frac{1}{2}$$

T	T
	$\frac{1}{2}$

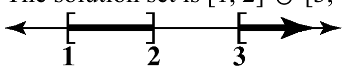
The solution set is $(-\infty, \infty)$.

29. $(x-1)(x-2)(x-3) \ge 0$

Boundary points: 1, 2, and 3
Test one value in each interval.

F	T	F	T
	1	2	3

The solution set is [1, 2] ∪ [3, ∞).

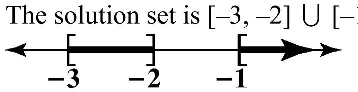


30. $(x+1)(x+2)(x+3) \ge 0$

Boundary points: –1, –2, and –3
Test one value in each interval.

F	T	F	T
	–3	–2	–1

The solution set is [–3, –2] ∪ [–1, ∞).

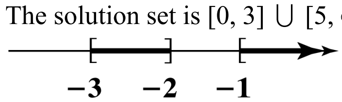


31. $x(3-x)(x-5) \le 0$

Boundary points: 0, 3, and 5
Test one value in each interval.

F	T	F	T
	0	3	5

The solution set is [0, 3] ∪ [5, ∞).

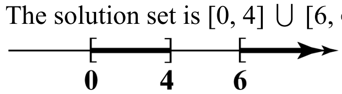


32. $x(4-x)(x-6) \le 0$

Boundary points: 0, 3, and 5
Test one value in each interval.

F	T	F	T
	0	4	6

The solution set is [0, 4] ∪ [6, ∞).



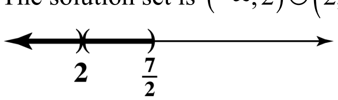
33. $(2-x)^2\left(x-\frac{7}{2}\right) < 0$

Boundary points: 2, and $\frac{7}{2}$

Test one value in each interval.

T	T	F
	2	$\frac{7}{2}$

The solution set is $(-\infty, 2) \cup \left(2, \frac{7}{2}\right)$.



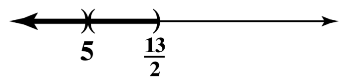
34. $(5-x)^2\left(x-\frac{13}{2}\right) < 0$

Boundary points: 5, and $\frac{13}{2}$

Test one value in each interval.

T	T	F
	5	$\frac{13}{2}$

The solution set is $(-\infty, 5) \cup \left(5, \frac{13}{2}\right)$.



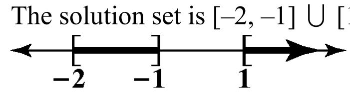
35. $x^3 + 2x^2 - x - 2 \ge 0$

$x^2(x+2) - 1(x+2) \ge 0$
$(x+2)(x^2-1) \ge 0$
$(x+2)(x-1)(x+1) \ge 0$

Boundary points: –2, –1, and 2
Test one value in each interval.

F	T	F	T
	–2	–1	2

The solution set is [–2, –1] ∪ [1, ∞).



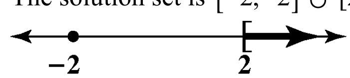
36. $x^3 + 2x^2 - 4x - 8 \ge 0$

$x^2(x+2) - 4(x+1) \ge 0$
$(x+2)(x^2-4) \ge 0$
$(x+2)(x+2)(x-2) \ge 0$

Boundary points: –2, and 2
Test one value in each interval.

F	F	T
	–2	2

The solution set is $[-2, -2]$ ∪ [2, ∞).



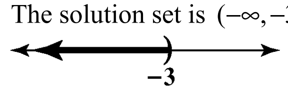
37. $x^3 + 2x^2 - x - 2 \ge 0$

$x^2(x-3) - 9(x-3) \ge 0$
$(x-3)(x^2-9) \ge 0$
$(x-3)(x+3)(x-3) \ge 0$

Boundary points: –3 and 3
Test one value in each interval.

T	F	F
	–3	3

The solution set is $(-\infty, -3]$.



38. $x^3+7x^2-x-7<0$

$x^2(x+7)-(x+7)<0$

$(x+7)(x^2-1)<0$

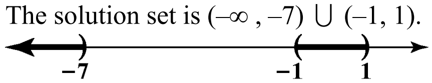
$(x+7)(x+1)(x-1)<0$

Boundary points: –7, –1 and 1

Test one value in each interval.

T	F	T	F
	–7	–1	1

The solution set is $(-\infty, -7) \cup (-1, 1)$.



39. $x^3+x^2+4x+4>0$

$x^2(x+1)+4(x+1)\ge 0$

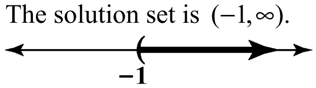
$(x+1)(x^2+4)\ge 0$

Boundary point: –1

Test one value in each interval.

F	T
–1	

The solution set is $(-1,\infty)$.



40. $x^3-x^2+9x-9>0$

$x^2(x-1)+9(x-1)\ge 0$

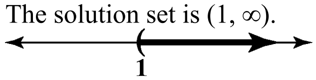
$(x-1)(x^2+9)\ge 0$

Boundary point: 1.

Test one value in each interval.

F	T
1	

The solution set is $(1, \infty)$.



41. $x^3-9x^2\ge 0$

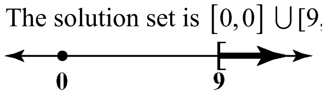
$x^2(x-9)\ge 0$

Boundary points: 0 and 9

Test one value in each interval.

F	F	T
	0	9

The solution set is $[0,0]\cup[9, \infty)$.



42. $x^3-4x^2\le 0$

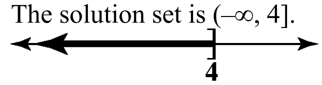
$x^2(x-4)\le 0$

Boundary points: 0 and 4.

Test one value in each interval.

T	T	F
	0	4

The solution set is $(-\infty, 4]$.



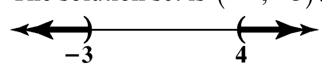
43. $\dfrac{x-4}{x+3}>0$

$x-4=0 \quad x+3=0$

$x=4 \quad x=-3$

T	F	T
	–3	4

The solution set is $(-\infty,-3)\cup(4,\infty)$.

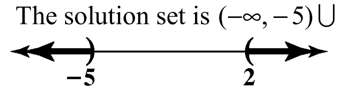


44. $\dfrac{x+5}{x-2}>0$

$x=-5$ or $x=2$

T	F	T
	–5	2

The solution set is $(-\infty,-5)\cup(2,\infty)$.

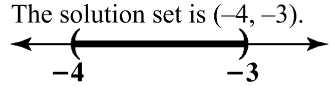


45. $\dfrac{x+3}{x+4}<0$

$x=-3$ or $x=-4$

F	T	F
	–4	–3

The solution set is $(-4, -3)$.

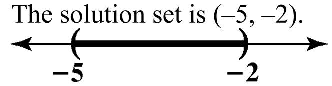


46. $\dfrac{x+5}{x+2}<0$

$x=-5$ or $x=-2$

F	T	F
	–5	–2

The solution set is $(-5, -2)$.



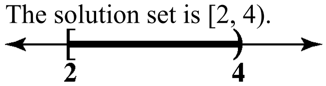
47. $\frac{-x+2}{x-4} \ge 0$

$x = 2$ or $x = 4$

F	T	F

$2 \quad 4$

The solution set is [2, 4).



48. $\frac{-x-3}{x+2} \le 0$

$x = -3$ or $x = -2$

T	F	T

$-3 \quad -2$

The solution set is $(-\infty, -3] \cup (-2, \infty)$.

49. $\frac{4-2x}{3x+4} \le 0$

$x = 2$ or $x = -\frac{4}{3}$

T	F	T

$-\frac{4}{3} \quad 2$

The solution set is $\left(-\infty, \frac{-4}{3}\right) \cup [2, \infty)$.

50. $\frac{3x+5}{6-2x} \ge 0$

$x = -\frac{5}{3}$ or $x = 3$

F	T	F

$-\frac{5}{3} \quad 3$

The solution set is $\left[-\frac{5}{3}, 3\right)$.

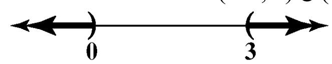
51. $\frac{x}{x-3} > 0$

$x = 0$ or $x = 3$

T	F	T

$0 \quad 3$

The solution set is $(-\infty, 0) \cup (3, \infty)$.



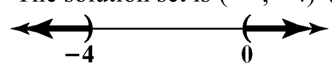
52. $\frac{x+4}{x} > 0$

$x = -4$ or $x = 0$

T	F	T

$-4 \quad 0$

The solution set is $(-\infty, -4) \cup (0, \infty)$.



53. $\frac{(x+4)(x-1)}{x+2} \le 0$

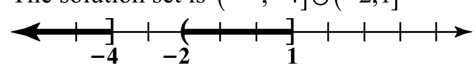
$x = -4$ or $x = -2$ or $x = 1$.

T	F	T	F

$-4 \quad -2 \quad 1$

Values of $x = -4$ or $x = 1$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -4] \cup (-2, 1]$



54. $\frac{(x+3)(x-2)}{x+1} \le 0$

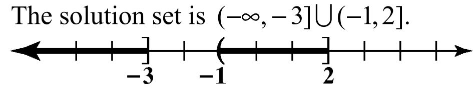
$x = -3$ or $x = -1$ or $x = 2$.

T	F	T	F

$-3 \quad -1 \quad 2$

Values of $x = -3$ or $x = 2$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -3] \cup (-1, 2]$.



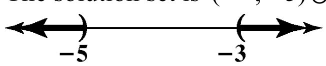
55.
$$\frac{x+1}{x+3}<2$$
$$\frac{x+1}{x+3}-2<0$$
$$\frac{x+1-2(x+3)}{x+3}<0$$
$$\frac{x+1-2x-6}{x+3}<0$$
$$\frac{-x-5}{x+3}<0$$

$x=$ or $x=-3$

T	F	T

$-5 \qquad -3$

The solution set is $(-\infty,-5)\cup(-3,\infty)$.



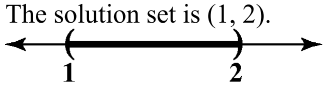
56.
$$\frac{x}{x-1}>2$$
$$\frac{x}{x-1}-2>0$$
$$\frac{x}{x-1}-\frac{2(x-1)}{x-1}>0$$
$$\frac{x-2x+2}{x-1}>0$$
$$\frac{-x+2}{x-1}>0$$

$x = 2$ or $x = 1$

F	T	F

$1 \qquad 2$

The solution set is (1, 2).

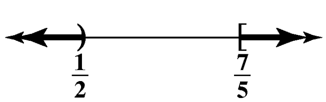


57.
$$\frac{x+4}{2x-1}\le 3$$
$$\frac{x+4}{2x-1}-3\le 0$$
$$\frac{x+4-3(2x-1)}{2x-1}\le 0$$
$$\frac{x+4-6x+3}{2x-1}\le 0$$
$$\frac{-5x+7}{2x-1}\le 0$$

T	F	T

$\frac{1}{2} \qquad \frac{7}{5}$

$x=\frac{7}{5}$ or $x=\frac{1}{2}$



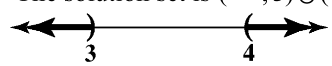
58.
$$\frac{1}{x-3}<1$$
$$\frac{1}{x-3}-1<0$$
$$\frac{1}{x-3}-\frac{x-3}{x-3}<0$$
$$\frac{1-x+3}{x-3}<0$$
$$\frac{-x+4}{x-3}<0$$

$x = 4$ or $x = 3$

T	F	T

$3 \qquad 4$

The solution set is $(-\infty,3)\cup(4,\infty)$.



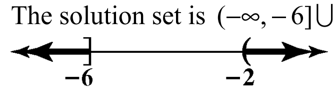
59.
$$\frac{x-2}{x+2}\le 2$$
$$\frac{x-2}{x+2}-2\le 0$$
$$\frac{x-2-2(x+2)}{x+2}\le 0$$
$$\frac{x-2-2x-4}{x+2}\le 0$$
$$\frac{-x-6}{x+2}\le 0$$

$x=-6$ or $x=-2$

T	F	T

$-6 \qquad -2$

The solution set is $(-\infty,-6]\cup(-2,\infty)$.



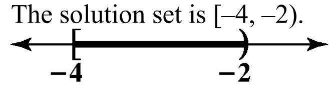
60.
$$\frac{x}{x+2}\ge 2$$
$$\frac{x}{x+2}-2\ge 0$$
$$\frac{x}{x+2}-\frac{2(x+2)}{x+2}\ge 0$$
$$\frac{x-2x-4}{x+2}\ge 0$$
$$\frac{-x-4}{x+2}\ge 0$$

$x = -4$ or $x = -2$

F	T	F

$-4 \qquad -2$

The solution set is $[-4, -2)$.



61. $f(x)=\sqrt{2x^2-5x+2}$

The domain of this function requires that $2x^2-5x+2\ge 0$

Solve $2x^2-5x+2=0$

$(x-2)(2x-1)=0$

$x=\frac{1}{2}$ or $x=2$

T	F	T

Boundary points: $\frac{1}{2}$, 2

The domain is $\left(-\infty,\frac{1}{2}\right]\cup[2,\infty)$.

62. $f(x)=\dfrac{1}{\sqrt{4x^2-9x+2}}$

The domain of this function requires that $4x^2-9x+2>0$

Solve $4x^2-9x+2=0$

$(x-2)(4x-1)=0$

$x=\frac{1}{4}$ or $x=2$

T	F	T

Boundary points: $\frac{1}{4}$, 2

The domain is $\left(-\infty,\frac{1}{4}\right)\cup(2,\infty)$.

63. $f(x)=\sqrt{\dfrac{2x}{x+1}-1}$

The domain of this function requires that $\dfrac{2x}{x+1}-1\ge 0$ or $\dfrac{x-1}{x+1}\ge 0$

$x=-1$ or $x=1$

T	F	T

Boundary points: -1, 1

The value $x=1$ results in 0 and, thus, it must be included in the domain.

The domain is $(-\infty,-1)\cup[1,\infty)$.

64. $f(x)=\sqrt{\dfrac{x}{2x-1}-1}$

The domain of this function requires that $\dfrac{x}{2x-1}-1\ge 0$ or $\dfrac{-x+1}{2x-1}\ge 0$

$x=\dfrac{1}{2}$ or $x=1$

F	T	F
$\frac{1}{2}$	1	

The value $x=1$ results in 0 and, thus, it must be included in the domain.
The domain is $(-\infty,-1)\cup[1,\infty)$.

65. $\left|x^2+2x-36\right|>12$

Express the inequality without the absolute value symbol:

$x^2+2x-36<-12$ or $x^2+2x-36>12$
$x^2+2x-24<0$ $\quad$ $x^2+2x-48>0$

Solve the related quadratic equations.

$x^2+2x-24=0$ or $x^2+2x-48=0$
$(x+6)(x-4)=0$ $\quad$ $(x+8)(x-6)=0$

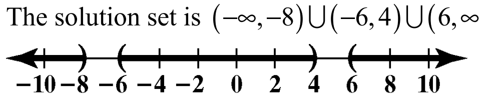
Apply the zero product principle.

$x+6=0$ or $x-4=0$ or $x+8=0$ or $x-6=0$
$x=-6$ $\quad$ $x=4$ $\quad$ $x=-8$ $\quad$ $x=6$

The boundary points are $-8,\ -6,\ 4$ and 6 .

Test Interval	Test Number	Test	Conclusion
$(-\infty,-8)$	-9	$\left\|(-9)^2+2(-9)-36\right\|>12$ $27>12$, True	$(-\infty,-8)$ belongs to the solution set.
$(-8,-6)$	-7	$\left\|(-7)^2+2(-7)-36\right\|>12$ $1>12$, False	$(-8,-6)$ does not belong to the solution set.
$(-6,4)$	0	$\left\|0^2+2(0)-36\right\|>12$ $36>12$, True	$(-6,4)$ belongs to the solution set.
$(4,6)$	5	$\left\|5^2+2(5)-36\right\|>12$ $1>12$, False	$(4,6)$ does not belong to the solution set.
$(6,\infty)$	7	$\left\|7^2+2(7)-36\right\|>12$ $27>12$, True	$(6,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-8)\cup(-6,4)\cup(6,\infty)$ or $\{x\,|\,x<-8 \text{ or } -6<x<4 \text{ or } x>6\}$.



66. $\left|x^2+6x+1\right|>8$

Express the inequality without the absolute value symbol:

$x^2+6x+1<-8$ or $x^2+6x+1>8$

$x^2+6x+9<0$ $\quad$ $x^2+6x-7>0$

Solve the related quadratic equations.

$x^2+6x+9=0$ or $x^2+6x-7=0$

$(x+3)^2=0$ $\quad$ $(x+7)(x-1)=0$

$x+3=\pm\sqrt{0}$ or $x+7=0$ or $x-1=0$

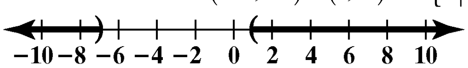
$x+3=0$ $\quad$ $x=-7$ $\quad$ $x=1$

$x=-3$

The boundary points are $-7,-3,$ and 1.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-7)$	-8	$\left\|(-8)^2+6(-8)+1\right\|>8$ $17\ge 8$, True	$(-\infty,-7)$ belongs to the solution set.
$(-7,-3)$	-5	$\left\|(-5)^2+6(-5)+1\right\|>8$ $4\ge 8$, False	$(-7,-3)$ does not belong to the solution set.
$(-3,1)$	0	$\left\|0^2+6(0)+1\right\|>8$ $1\ge 8$, False	$(-3,1)$ does not belong to the solution set.
$(1,\infty)$	2	$\left\|2^2+6(2)+1\right\|>8$ $17\ge 8$, True	$(1,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-7)\cup(1,\infty)$ or $\{x\,|\,x<-7 \text{ or } x>1\}$.



67. $\dfrac{3}{x+3}>\dfrac{3}{x-2}$

Express the inequality so that one side is zero.

$$\frac{3}{x+3}-\frac{3}{x-2}>0$$

$$\frac{3(x-2)}{(x+3)(x-2)}-\frac{3(x+3)}{(x+3)(x-2)}>0$$

$$\frac{3x-6-3x-9}{(x+3)(x-2)}<0$$

$$\frac{-15}{(x+3)(x-2)}<0$$

Find the values of x that make the denominator zero.

$x+3=0$ $\quad$ $x-2=0$

$x=-3$ $\quad$ $x=2$

The boundary points are -3 and 2.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\frac{3}{-4+3} > \frac{3}{-4-2}$ $-3 > \frac{1}{2}$, False	$(-\infty,-3)$ does not belong to the solution set.
$(-3,2)$	0	$\frac{3}{0+3} > \frac{3}{0-2}$ $1 > -\frac{3}{2}$, True	$(-3,2)$ belongs to the solution set.
$(2,\infty)$	3	$\frac{3}{3+3} > \frac{3}{3-2}$ $\frac{1}{2} > 3$, False	$(2,\infty)$ does not belong to the solution set.

The solution set is $(-3,2)$ or $\{x \mid -3 < x < 2\}$.

68. $\frac{1}{x+1} > \frac{2}{x-1}$

Express the inequality so that one side is zero.

$$\frac{1}{x+1} - \frac{2}{x-1} > 0$$

$$\frac{x-1}{(x+1)(x-1)} - \frac{2(x+1)}{(x+1)(x-1)} > 0$$

$$\frac{x-1-2x-2}{(x+1)(x-1)} < 0$$

$$\frac{-x-3}{(x+1)(x-1)} < 0$$

Find the values of x that make the numerator and denominator zero.

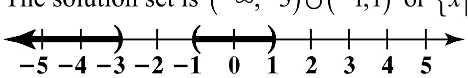
$-x-3=0$ $\quad$ $x+1=0$ $\quad$ $x-1=0$

$-3=x$ $\quad$ $x=-1$ $\quad$ $x=1$

The boundary points are -3, -1, and 1.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\frac{1}{-4+1} > \frac{2}{-3-1}$ $-\frac{1}{3} > -\frac{1}{2}$, True	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1)$	-2	$\frac{1}{-2+1} > \frac{2}{-2-1}$ $-1 > -\frac{2}{3}$, False	$(-3,-1)$ does not belong to the solution set.
$(-1,1)$	0	$\frac{1}{0+1} > \frac{2}{0-1}$ $1 > -2$, True	$(-3,1)$ belongs to the solution set.
$(1,\infty)$	2	$\frac{1}{2+1} > \frac{2}{2-1}$ $\frac{1}{3} > 1$, False	$(1,\infty)$ does not belong to the solution set.

The solution set is $(-\infty,-3)\cup(-1,1)$ or $\{x \mid x < -3 \text{ or } -1 < x < 1\}$.



69. $\dfrac{x^2-x-2}{x^2-4x+3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-x-2=0 \qquad x^2-4x+3=0$

$(x-2)(x+1)=0 \qquad (x-3)(x-1)=0$

Apply the zero product principle.

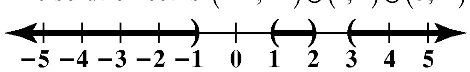
$x-2=0$ or $x+1=0 \qquad x-3=0$ or $x-1=0$

$x=2 \qquad x=-1 \qquad x=3 \qquad x=1$

The boundary points are -1, 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2	$\dfrac{(-2)^2-(-2)-2}{(-2)^2-4(-2)+3}>0$ $\dfrac{4}{15}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\dfrac{0^2-0-2}{0^2-4(0)+3}>0$ $-\dfrac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\dfrac{1.5^2-1.5-2}{1.5^2-4(1.5)+3}>0$ $\dfrac{5}{3}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\dfrac{2.5^2-2.5-2}{2.5^2-4(2.5)+3}>0$ $-\dfrac{7}{3}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\dfrac{4^2-4-2}{4^2-4(4)+3}>0$ $\dfrac{10}{3}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x\,|\,x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.



70. $\frac{x^2-3x+2}{x^2-2x-3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-3x+2=0 \qquad x^2-2x-3=0$

$(x-2)(x-1)=0 \qquad (x-3)(x+1)=0$

Apply the zero product principle.

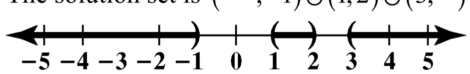
$x-2=0$ or $x-1=0 \qquad x-3=0$ or $x+1=0$

$x=2 \qquad x=1 \qquad x=3 \qquad x=-1$

The boundary points are -1, 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2 $\frac{x^2-3x+2}{x^2-2x-3}>0$	$\frac{(-2)^2-3(-2)+2}{(-2)^2-2(-2)-3}>0$ $\frac{12}{5}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\frac{0^2-3(0)+2}{0^2-2(0)-3}>0$ $-\frac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\frac{1.5^2-3(1.5)+2}{1.5^2-2(1.5)-3}>0$ $\frac{1}{15}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\frac{2.5^2-3(2.5)+2}{2.5^2-2(2.5)-3}>0$ $-\frac{3}{7}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\frac{4^2-3(4)+2}{4^2-2(4)-3}>0$ $\frac{6}{5}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x \mid x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.



71.

$$2x^3+11x^2 \ge 7x+6$$
$$2x^3+11x^2-7x-6 \ge 0$$

The graph of $f(x)=2x^3+11x^2-7x-6$ appears to cross the *x*-axis at -6, $-\frac{1}{2}$, and 1. We verify this numerically by substituting these values into the function:

$$f(-6)=2(-6)^3+11(-6)^2-7(-6)-6=2(-216)+11(36)-(-42)-6=-432+396+42-6=0$$

$$f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+11\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)-6=2\left(-\frac{1}{8}\right)+11\left(\frac{1}{4}\right)-\left(-\frac{7}{2}\right)-6=-\frac{1}{4}+\frac{11}{4}+\frac{7}{2}-6=0$$

$$f(1)=2(1)^3+11(1)^2-7(1)-6=2(1)+11(1)-7-6=2+11-7-6=0$$

Thus, the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x)\ge 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. Now, the curve is above the *x*-axis when $-6<x<-\frac{1}{2}$ and when $x>1$. Thus, the solution set is $\left\{x \,\middle|\, -6\le x\le -\frac{1}{2} \text{ or } x\ge 1\right\}$ or $\left[-6,-\frac{1}{2}\right]\cup[1,\infty)$.

72.

$$2x^3+11x^2<7x+6$$
$$2x^3+11x^2-7x-6<0$$

In Problem 63, we verified that the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x)<0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x<-6$ and when $-\frac{1}{2}<x<1$. Thus, the solution set is $\left\{x \,\middle|\, x<-6 \text{ or } -\frac{1}{2}<x<1\right\}$ or $(-\infty,-6)\cup\left(-\frac{1}{2},1\right)$.

73.

$$\frac{1}{4(x+2)} \le -\frac{3}{4(x-2)}$$
$$\frac{1}{4(x+2)}+\frac{3}{4(x-2)} \le 0$$

Simplify the left side of the inequality:

$$\frac{x-2}{4(x+2)}+\frac{3(x+2)}{4(x-2)}=\frac{x-2+3x+6}{4(x+2)(x-2)}=\frac{4x+4}{4(x+2)(x-2)}=\frac{4(x+1)}{4(x+2)(x-2)}=\frac{x+1}{x^2-4}.$$

The graph of $f(x)=\frac{x+1}{x^2-4}$ crosses the *x*-axis at -1, and has vertical asymptotes at $x=-2$ and $x=2$. Thus, the boundaries are -2, -1, and 1. We need to find the intervals on which $f(x)\le 0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x<-2$ and when $-1<x<2$. Thus, the solution set is $\{x \mid x<-2 \text{ or } -1\le x<2\}$ or $(-\infty,-2)\cup[-1,2)$.

74.
$$\frac{1}{4(x+2)} > -\frac{3}{4(x-2)}$$
$$\frac{1}{4(x+2)} + \frac{3}{4(x-2)} > 0$$
$$\frac{x+1}{(x+2)(x-2)} > 0$$

The boundaries are -2, -1, and 2. We need to find the intervals on which $f(x) > 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. The curve is above the *x*-axis when $-2 < x < -1$ and when $x > 2$. Thus, the solution set is $\{x \mid -2 < x < -1 \text{ or } x > 2\}$ or $(-2,-1) \cup (2,\infty)$.

75. $s(t) = -16t^2 + 8t + 87$

The diver's height will exceed that of the cliff when $s(t) > 87$

$$-16t^2 + 8t + 87 > 87$$
$$-16t^2 + 8t > 0$$
$$-8t(2t-1) > 0$$

The boundaries are 0 and $\frac{1}{2}$. Testing each interval shows that the diver will be higher than the cliff for the first half second after beginning the jump. The interval is $\left(0, \frac{1}{2}\right)$.

76. $s(t) = -16t^2 + 48t + 160$

The ball's height will exceed that of the rooftop when $s(t) > 160$

$$-16t^2 + 48t + 160 > 160$$
$$-16t^2 + 48t > 0$$
$$-16t(t-3) > 0$$

The boundaries are 0 and 3. Testing each interval shows that the ball will be higher than the rooftop for the first three seconds after the throw. The interval is $(0,3)$.

77. $f(x) = 0.0875x^2 - 0.4x + 66.6$

$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(35) = 0.0875(35)^2 - 0.4(35) + 66.6 \approx 160$ feet

$g(35) = 0.0875(35)^2 + 1.9(35) + 11.6 \approx 185$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well. The function values and the data are identical.

d. $0.0875x^2 - 0.4x + 66.6 > 540$

$0.0875x^2 - 0.4x + 473.4 > 0$

Solve the related quadratic equation.

$0.0875x^2 - 0.4x + 473.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.4) \pm \sqrt{(-0.4)^2 - 4(0.0875)(473.4)}}{2(0.0875)}$$

$x \approx -71$ or 76

Since the function's domain is $x \geq 30$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30, 76)$	50	$0.0875(50)^2 - 0.4(50) + 66.6 > 540$ $265.35 > 540$, False	$(30, 76)$ does not belong to the solution set.
$(76, \infty)$	100	$0.0875(100)^2 - 0.4(100) + 66.6 > 540$ $901.6 > 540$, True	$(76, \infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 540 feet for speeds exceeding 76 miles per hour. This is represented on graph (b) to the right of point (76, 540).

78. $f(x) = 0.0875x^2 - 0.4x + 66.6$

$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(55) = 0.0875(55)^2 - 0.4(55) + 66.6 \approx 309$ feet

$g(55) = 0.0875(55)^2 + 1.9(55) + 11.6 \approx 381$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well.

d. $0.0875x^2 + 1.9x + 11.6 > 540$

$0.0875x^2 + 1.9x + 528.4 > 0$

Solve the related quadratic equation.

$0.0875x^2 + 1.9x + 528.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1.9) \pm \sqrt{(1.9)^2 - 4(0.0875)(528.4)}}{2(0.0875)}$$

$x \approx -89$ or 68

Since the function's domain is $x \geq 30$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30, 68)$	50	$0.0875(50)^2 + 1.9(50) + 11.6 > 540$ $325.35 > 540$, False	$(30, 68)$ does not belong to the solution set.
$(68, \infty)$	100	$0.0875(100)^2 + 1.9(100) + 11.6 > 540$ $1076.6 > 540$, True	$(68, \infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 540 feet for speeds exceeding 68 miles per hour. This is represented on graph (a) to the right of point (68, 540).

79. Let x = the length of the rectangle.
Since $\text{Perimeter} = 2(\text{length}) + 2(\text{width})$, we know

$$\begin{aligned} 50 &= 2x + 2(\text{width}) \\ 50 - 2x &= 2(\text{width}) \\ \text{width} &= \frac{50-2x}{2} = 25 - x \end{aligned}$$

Now, $A = (\text{length})(\text{width})$, so we have that

$$\begin{aligned} A(x) &\le 114 \\ x(25-x) &\le 114 \\ 25x - x^2 &\le 114 \end{aligned}$$

Solve the related equation

$$\begin{aligned} 25x - x^2 &= 114 \\ 0 &= x^2 - 25x + 114 \\ 0 &= (x-19)(x-6) \end{aligned}$$

Apply the zero product principle:

$x - 19 = 0$ or $x - 6 = 0$
$x = 19$ $\quad x = 6$

The boundary points are 6 and 19.

Test Interval	Test Number	Test	Conclusion
$(-\infty, 6)$	0	$25(0) - 0^2 \le 114$ $0 \le 114$, True	$(-\infty, 6)$ belongs to the solution set.
$(6, 19)$	10	$25(10) - 10^2 \le 114$ $150 \le 114$, False	$(6, 19)$ does not belong to the solution set.
$(19, \infty)$	20	$25(20) - 20^2 \le 114$ $100 \le 114$, True	$(19, \infty)$ belongs to the solution set.

If the length is 6 feet, then the width is 19 feet. If the length is less than 6 feet, then the width is greater than 19 feet. Thus, if the area of the rectangle is not to exceed 114 square feet, the length of the shorter side must be 6 feet or less.

80. $2l + 2w = P$
$2l + 2w = 180$
$2l = 180 - 2w$
$l = 90 - w$

We want to restrict the area to 800 square feet. That is,

$$A \le 800$$
$$l \cdot w \le 800$$
$$(90 - w)w \le 800$$
$$90w - w^2 \le 800$$
$$-w^2 + 90w - 800 \le 0$$
$$w^2 - 90w + 800 \ge 0$$

$$w^2 - 90w + 800 = 0$$
$$(w - 80)(w - 10) = 0$$

$w - 80 = 0$ or $w - 10 = 0$
$w = 80$ $\quad$ $w = 10$

Assuming the width is the shorter side, we ignore the larger solution.

Test Interval	Test Number	Test	Conclusion
$(0,10)$	5	$90(5) - (5)^2 \le 800$ true	$(0,10)$ is part of the solution set
$(10,45)$	20	$90(20) - (20)^2 \le 800$ false	$(10,45)$ is not part of the solution set

The solution set is $\{w \mid 0 < w \le 10\}$ or $(0,10]$.

The length of the shorter side cannot exceed 10 feet.

81. – 85. Answers will vary.

86. The solution set is $(-\infty, -5) \cup (2, \infty)$.

87. The solution set is $\left\{x \middle| -3 \le x \le \frac{1}{2}\right\}$ or $\left[-3, \frac{1}{2}\right]$.

88. The solution set is $(-2, -1)$ or $(2, \infty)$.

89. The solution set is $(1, 4]$.

90. Graph $y_1 = \frac{x+2}{x-3}$ and $y_2 = 2$

y_1 less than or equal to y_2 for $x < 3$ or $x \ge 8$.

The solution set is $(-\infty, 3) \cup [8, \infty)$

91. Graph $y_1 = \frac{1}{x+1}$ and $y_2 = \frac{2}{x+4}$

y_1 less than or equal to y_2 for $-4 < x < -1$ or $x \ge 2$.

The solution set is $(-4, -1) \cup [2, \infty)$

92. **a.** $f(x) = 0.1125x^2 - 0.1x + 55.9$

b. $0.1125x^2 - 0.1x + 55.9 > 455$
$0.1125x^2 - 0.1x + 399.1 > 0$
Solve the related quadratic equation.
$0.1125x^2 - 0.1x + 399.1 = 0$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-0.1) \pm \sqrt{(-0.1)^2 - 4(0.1125)(399.1)}}{2(0.1125)}$$
$x \approx -59$ or 60
Since the function's domain must be $x \geq 0$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(0, 60)$	50	$0.1125(50)^2 - 0.1(50) + 55.9 > 455$ $332.15 > 455$, False	$(0, 60)$ does not belong to the solution set.
$(60, \infty)$	100	$0.1125(100)^2 - 0.1(100) + 55.9 > 455$ $1170.9 > 455$, True	$(60, \infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 455 feet for speeds exceeding 60 miles per hour.

93. **a.** $f(x) = 0.1375x^2 + 0.7x + 37.8$

b. $0.1375x^2 + 0.7x + 37.8 > 446$
$0.1375x^2 + 0.7x + 408.2 > 0$
Solve the related quadratic equation.
$0.1375x^2 + 0.7x + 408.2 = 0$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(0.7) \pm \sqrt{(0.7)^2 - 4(0.1375)(408.2)}}{2(0.1375)}$$
$x \approx -57$ or 52
Since the function's domain must be $x \geq 0$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(0, 52)$	10	$0.1375(10)^2 + 0.7(10) + 37.8 > 446$ $58.55 > 446$, False	$(0, 52)$ does not belong to the solution set.
$(52, \infty)$	100	$0.1375(100)^2 + 0.7(100) + 37.8 > 446$ $1482.8 > 446$, True	$(52, \infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 446 feet for speeds exceeding 52 miles per hour.

94. makes sense

95. does not make sense; Explanations will vary. Sample explanation: Polynomials are defined for all values.

96. makes sense

97. does not make sense; Explanations will vary. Sample explanation: To solve this inequality you must first subtract 2 from both sides.

98. false; Changes to make the statement true will vary. A sample change is: The solution set is $\{x | x < -5 \text{ or } x > 5\}$ or $(-\infty, -5) \cup (5, \infty)$.

99. false; Changes to make the statement true will vary. A sample change is: The inequality cannot be solved by multiplying both sides by $x + 3$. We do not know if $x + 3$ is positive or negative. Thus, we would not know whether or not to reverse the order of the inequality.

100. false; Changes to make the statement true will vary. A sample change is: The inequalities have different solution sets. The value, 1, is included in the domain of the first inequality, but not included in the domain of the second inequality.

101. true

102. One possible solution: $x^2 - 2x - 15 \le 0$

103. One possible solution: $\dfrac{x-3}{x+4} \ge 0$

104. Because any non-zero number squared is positive, the solution is all real numbers except 2.

105. Because any number squared other than zero is positive, the solution includes only 2.

106. Because any number squared is positive, the solution is the empty set, $\varnothing$.

107. Because any number squared other than zero is positive, and the reciprocal of zero is undefined, the solution is all real numbers except 2.

108. **a.** The solution set is all real numbers.

b. The solution set is the empty set, $\varnothing$.

c. $4x^2 - 8x + 7 > 0$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(7)}}{2(4)}$$
$$x = \frac{8 \pm \sqrt{64 - 112}}{8}$$
$$x = \frac{8 \pm \sqrt{-48}}{8} \Rightarrow \text{imaginary}$$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 > 0$

$7 > 0$ True

The inequality is true for all numbers.

$4x^2 - 8x + 7 < 0$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 = 7 < 0$ False

The solution set is the empty set.

109. $\sqrt{27 - 3x^2} \ge 0$

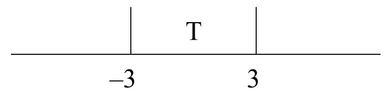
$27 - 3x^2 \ge 0$

$9 - x^2 \ge 0$

$(3 - x)(3 + x) \ge 0$

$3 - x = 0 \qquad 3 + x = 0$

$x = 3$ or $\qquad x = -3$



Test –4: $\sqrt{27 - 3(-4)^2} \ge 0$

$\sqrt{27 - 48} \ge 0$

$\sqrt{-21} \ge 0$

no graph- imaginary

Test 0: $\sqrt{27 - 3(0)^2} \ge 0$

$\sqrt{27} \ge 0$ True

Test 4: $\sqrt{27 - 3(4)^2} \ge 0$

$\sqrt{27 - 48} \ge 0$

$\sqrt{-21} \ge 0$

no graph -imaginary

The solution set is [–3, 3].

110. The slope of the line $y = -\frac{1}{4}x + \frac{1}{3}$ is $-\frac{1}{4}$. Thus the slope of the line perpendicular to this line is 4.

$y - y_1 = m(x - x_1)$

$y - 5 = 4(x - (-2))$

$y - 5 = 4(x + 2)$ point-slope

$y - 5 = 4x + 8$

$y = 4x + 13$ slope-intercept

$4x - y + 13 = 0$ general

111. Since $h(x) = \sqrt{36 - 2x}$ contains an even root; the quantity under the radical must be greater than or equal to 0.

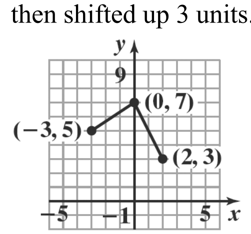
$36 - 2x \ge 0$

$-2x \ge -36$

$x \le 18$

Thus, the domain of h is $\{x \mid x \le 18\}$, or the interval $(-\infty, 18]$.

112. The graph of $y = f(x)$ is reflected about the y-axis, then shifted up 3 units.



113. **a.** $y = kx^2$
$64 = k \cdot 2^2$
$64 = 4k$
$16 = k$

b. $y = kx^2$
$y = 16x^2$

c. $y = kx^2$
$y = 16x^2$
$y = 16 \cdot 5^2$
$y = 400$

114. **a.** $y = \frac{k}{x}$
$12 = \frac{k}{8}$
$96 = k$

b. $y = \frac{k}{x}$
$y = \frac{96}{x}$

c. $y = \frac{96}{x}$
$y = \frac{96}{3}$
$y = 32$

115. $S = \frac{kA}{P}$
$12,000 = \frac{k \cdot 60,000}{40}$
$\frac{12,000 \cdot 40}{60,000} = k$
$8 = k$

Section 2.8

Check Point Exercises

1. y varies directly as x is expressed as $y = kx.$
The volume of water, W, varies directly as the time, t can be expressed as $W = kt$.
Use the given values to find k.
$W = kt$
$30 = k(5)$
$6 = k$
Substitute the value of k into the equation.
$W = kt$
$W = 6t$
Use the equation to find W when $t = 11$.
$W = 6t$
$= 6(11)$
$= 66$
A shower lasting 11 minutes will use 66 gallons of water.

2. y varies directly as the cube of x is expressed as $y = kx^3$.
The weight, w, varies directly as the cube of the length, l can be expressed as $w = kl^2$.
Use the given values to find k.
$w = kl^3$
$2025 = k(15)^3$
$0.6 = k$
Substitute the value of k into the equation.
$w = kl^3$
$w = 0.6l^3$
Use the equation to find w when $l = 25$.
$w = 0.6l^3$
$= 0.6(25)^3$
$= 9375$
The 25-foot long shark was 9375 pounds.

3. y varies inversely as x is expressed as $y = \frac{k}{x}$.
The length, L, varies inversely as the frequency, f can be expressed as $L = \frac{k}{f}$.
Use the given values to find k.
$L = \frac{k}{f}$
$8 = \frac{k}{640}$
$5120 = k$
Substitute the value of k into the equation.
$L = \frac{k}{f}$
$L = \frac{5120}{f}$
Use the equation to find f when $L = 10$.
$L = \frac{5120}{f}$
$10 = \frac{5120}{f}$
$10f = 5120$
$f = 512$
A 10 inch violin string will have a frequency of 512 cycles per second.

4. let M represent the number of minutes
let Q represent the number of problems
let P represent the number of people
M varies directly as Q and inversely as P is expressed as $M = \frac{kQ}{P}$.
Use the given values to find k.

$$M = \frac{kQ}{P}$$
$$32 = \frac{k(16)}{4}$$
$$8 = k$$

Substitute the value of k into the equation.

$$M = \frac{kQ}{P}$$
$$M = \frac{8Q}{P}$$

Use the equation to find M when $P = 8$ and $Q = 24$.

$$M = \frac{8Q}{P}$$
$$M = \frac{8(24)}{8}$$
$$M = 24$$

It will take 24 minutes for 8 people to solve 24 problems.

5. V varies jointly with h and r^2 and can be modeled as $V = khr^2$.

Use the given values to find k.

$$V = khr^2$$
$$120\pi = k(10)(6)^2$$
$$\frac{\pi}{3} = k$$

Therefore, the volume equation is $V = \frac{1}{3}hr^2$.

$$V = \frac{\pi}{3}(2)(12)^2 = 96\pi \text{ cubic feet}$$

Concept and Vocabulary Check 2.8

1. $y - kx$; constant of variation

2. $y = kx^n$

3. $y = \frac{k}{x}$

4. $y = \frac{kx}{z}$

5. $y = kxz$

6. directly; inversely

7. jointly; inversely

Exercise Set 2.8

1. Use the given values to find k.

$$y = kx$$
$$65 = k \cdot 5$$
$$\frac{65}{5} = \frac{k \cdot 5}{5}$$
$$13 = k$$

The equation becomes $y = 13x$.
When $x = 12$, $y = 13x = 13 \cdot 12 = 156$.

2.
$$y = kx$$
$$45 = k \cdot 5$$
$$9 = k$$

$$y = 9x = 9 \cdot 13 = 117$$

3. Since y varies inversely with x, we have $y = \frac{k}{x}$.
Use the given values to find k.

$$y = \frac{k}{x}$$
$$12 = \frac{k}{5}$$
$$5 \cdot 12 = 5 \cdot \frac{k}{5}$$
$$60 = k$$

The equation becomes $y = \frac{60}{x}$.

When $x = 2$, $y = \frac{60}{2} = 30$.

4.
$$y = \frac{k}{x}$$
$$6 = \frac{k}{3}$$
$$18 = k$$

$$y = \frac{18}{9} = 2$$

5. Since y varies inversely as x and inversely as the square of z, we have $y = \frac{kx}{z^2}$.

Use the given values to find k.

$$y = \frac{kx}{z^2}$$
$$20 = \frac{k(50)}{5^2}$$
$$20 = \frac{k(50)}{25}$$
$$20 = 2k$$
$$10 = k$$

The equation becomes $y = \frac{10x}{z^2}$.

When $x = 3$ and $z = 6$,

$$y = \frac{10x}{z^2} = \frac{10(3)}{6^2} = \frac{10(3)}{36} = \frac{30}{36} = \frac{5}{6}.$$

6.

$$a = \frac{kb}{c^2}$$
$$7 = \frac{k(9)}{(6)^2}$$
$$7 = \frac{k(9)}{36}$$
$$7 = \frac{k}{4}$$
$$28 = k$$

$$a = \frac{28(4)}{(8)^2} = \frac{28(4)}{64} = \frac{7}{4}$$

7. Since y varies jointly as x and z, we have $y = kxz$.

Use the given values to find k.

$$y = kxz$$
$$25 = k(2)(5)$$
$$25 = k(10)$$
$$\frac{25}{10} = \frac{k(10)}{10}$$
$$\frac{5}{2} = k$$

The equation becomes $y = \frac{5}{2}xz$.

When $x = 8$ and $z = 12$, $y = \frac{5}{2}(8)(12) = 240$.

8.

$$C = kAT$$
$$175 = k(2100)(4)$$
$$175 = k(8400)$$
$$\frac{1}{48} = k$$

$$C = \frac{1}{48}(2400)(6) = \frac{14400}{48} = 300$$

9. Since y varies jointly as a and b and inversely as the square root of c, we have $y = \frac{kab}{\sqrt{c}}$.

Use the given values to find k.

$$y = \frac{kab}{\sqrt{c}}$$
$$12 = \frac{k(3)(2)}{\sqrt{25}}$$
$$12 = \frac{k(6)}{5}$$
$$12(5) = \frac{k(6)}{5}(5)$$
$$60 = 6k$$
$$\frac{60}{6} = \frac{6k}{6}$$
$$10 = k$$

The equation becomes $y = \frac{10ab}{\sqrt{c}}$.

When $a = 5$, $b = 3$, $c = 9$,

$$y = \frac{10ab}{\sqrt{c}} = \frac{10(5)(3)}{\sqrt{9}} = \frac{150}{3} = 50.$$

10.

$$y = \frac{kmn^2}{p}$$
$$15 = \frac{k(2)(1)^2}{6}$$
$$15 = \frac{2k}{6}$$
$$15(6) = \frac{2k}{6}(6)$$
$$90 = 2k$$
$$k = 45$$

$$y = \frac{45mn^2}{p} = \frac{45(3)(4)^2}{10} = \frac{2160}{10} = 216$$

11. $x = kyz$;

Solving for y:

$$x = kyz$$
$$\frac{x}{kz} = \frac{kyz}{yz}.$$
$$y = \frac{x}{kz}$$

12. $x = kyz^2$;

Solving for y:

$$x = kyz^2$$
$$\frac{x}{kz^2} = \frac{kyz^2}{kz^2}$$
$$y = \frac{x}{kz^2}$$

13. $x = \frac{kz^3}{y}$;

Solving for y

$$x = \frac{kz^3}{y}$$
$$xy = y \cdot \frac{kz^3}{y}$$
$$xy = kz^3$$
$$\frac{xy}{x} = \frac{kz^3}{x}$$
$$y = \frac{kz^3}{x}$$

14.

$$x = \frac{k\sqrt[3]{z}}{y}$$
$$yx = y \cdot \frac{k\sqrt[3]{z}}{y}$$
$$yx = k\sqrt[3]{z}$$
$$\frac{yx}{x} = \frac{k\sqrt[3]{z}}{x}$$
$$y = \frac{k\sqrt[3]{z}}{x}$$

15. $x = \frac{kyz}{\sqrt{w}}$;

Solving for y:

$$x = \frac{kyz}{\sqrt{w}}$$
$$x\left(\sqrt{w}\right) = \left(\sqrt{w}\right)\frac{kyz}{\sqrt{w}}$$
$$x\sqrt{w} = kyz$$
$$\frac{x\sqrt{w}}{kz} = \frac{kyz}{kz}$$
$$y = \frac{x\sqrt{w}}{kz}$$

16.

$$x = \frac{kyz}{w^2}$$
$$\left(\frac{w^2}{kz}\right)x = \frac{w^2}{kz}\frac{kyz}{w^2}$$
$$y = \frac{xw^2}{kz}$$

17. $x = kz(y + w)$;

Solving for y:

$$x = kz(y + w)$$
$$x = kzy + kzw$$
$$x - kzw = kzy$$
$$\frac{x - kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x - kzw}{kz}$$

18.

$$x = kz(y - w)$$
$$x = kzy - kzw$$
$$x + kzw = kzy$$
$$\frac{x + kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x + kzw}{kz}$$

19. $x = \dfrac{kz}{y-w}$;

Solving for *y:*

$$\begin{aligned} x &= \frac{kz}{y-w} \\ (y-w)x &= (y-w)\frac{kz}{y-w} \\ xy - wx &= kz \\ xy &= kz + wx \\ \frac{xy}{x} &= \frac{kz+wx}{x} \\ y &= \frac{xw+kz}{x} \end{aligned}$$

20.

$$\begin{aligned} x &= \frac{kz}{y+w} \\ (y+w)x &= (y+w)\frac{kz}{y+w} \\ yx + xw &= kz \\ yx &= kz - xw \\ \frac{yx}{x} &= \frac{kz-xw}{x} \\ y &= \frac{kz-xw}{x} \end{aligned}$$

21. Since *T* varies directly as *B*, we have $T = kB$.
Use the given values to find *k*.

$$\begin{aligned} T &= kB \\ 3.6 &= k(4) \\ \frac{3.6}{4} &= \frac{k(4)}{4} \\ 0.9 &= k \end{aligned}$$

The equation becomes $T = 0.9B$.
When $B = 6$, $T = 0.9(6) = 5.4$.
The tail length is 5.4 feet.

22.

$$\begin{aligned} M &= kE \\ 60 &= k(360) \\ \frac{60}{360} &= \frac{k(360)}{360} \\ \frac{1}{6} &= k \end{aligned}$$

$$M = \frac{1}{6}(186) = 31$$

A person who weighs 186 pounds on Earth will weigh 31 pounds on the moon.

23. Since *B* varies directly as *D*, we have $B = kD$.
Use the given values to find *k*.

$$\begin{aligned} B &= kD \\ 8.4 &= k(12) \\ \frac{8.4}{12} &= \frac{k(12)}{12} \\ k &= \frac{8.4}{12} = 0.7 \end{aligned}$$

The equation becomes $B = 0.7D$.
When $B = 56$,

$$\begin{aligned} 56 &= 0.7D \\ \frac{56}{0.7} &= \frac{0.7D}{0.7} \\ D &= \frac{56}{0.7} = 80 \end{aligned}$$

It was dropped from 80 inches.

24.

$$\begin{aligned} d &= kf \\ 9 &= k(12) \\ \frac{9}{12} &= \frac{k(12)}{12} \\ 0.75 &= k \\ d &= 0.75f \\ 15 &= 0.75f \\ \frac{15}{0.75} &= \frac{0.75f}{0.75} \\ 20 &= f \end{aligned}$$

A force of 20 pounds is needed.

25. Since a man's weight varies directly as the cube of his height, we have $w = kh^3$.
Use the given values to find *k*.

$$\begin{aligned} w &= kh^3 \\ 170 &= k(70)^3 \\ 170 &= k(343,000) \\ \frac{170}{343,000} &= \frac{k(343,000)}{343,000} \\ 0.000496 &= k \end{aligned}$$

The equation becomes $w = 0.000496h^3$.
When *h* = 107,

$$\begin{aligned} w &= 0.000496(107)^3 \\ &= 0.000496(1,225,043) \approx 607. \end{aligned}$$

Robert Wadlow's weight was approximately 607 pounds.

26.

$$h = kd^2$$
$$50 = k \cdot 10^2$$
$$0.5 = k$$

$$h = 0.5d^2$$

a. $h = 0.5d^2$
$h = 0.5(30)^2$
$h = 450$
A water pipe with a 30 centimeter diameter can serve 450 houses.

b. $h = 0.5d^2$
$1250 = 0.5d^2$
$d^2 = 625$
$d = \sqrt{625}$
$d = 25$
A water pipe with a 25 centimeter diameter can serve 1250 houses.

27. Since the banking angle varies inversely as the turning radius, we have $B = \frac{k}{r}$.

Use the given values to find *k*.

$$B = \frac{k}{r}$$
$$28 = \frac{k}{4}$$
$$28(4) = 28\left(\frac{k}{4}\right)$$
$$112 = k$$

The equation becomes $B = \frac{112}{r}$.

When $r = 3.5$, $B = \frac{112}{r} = \frac{112}{3.5} = 32$.

The banking angle is 32° when the turning radius is 3.5 feet.

28.

$$t = \frac{k}{d}$$
$$4.4 = \frac{k}{1000}$$
$$(1000)4.4 = (1000)\frac{k}{1000}$$
$$4400 = k$$

$$t = \frac{4400}{d} = \frac{4400}{5000} = 0.88$$

The water temperature is 0.88° Celsius at a depth of 5000 meters.

29. Since intensity varies inversely as the square of the distance, we have pressure, we have $I = \frac{k}{d}$.

Use the given values to find *k*.

$$I = \frac{k}{d^2}.$$
$$62.5 = \frac{k}{3^2}$$
$$62.5 = \frac{k}{9}$$
$$9(62.5) = 9\left(\frac{k}{9}\right)$$
$$562.5 = k$$

The equation becomes $I = \frac{562.5}{d^2}$.

When $d = 2.5$, $I = \frac{562.5}{2.5^2} = \frac{562.5}{6.25} = 90$

The intensity is 90 milliroentgens per hour.

30.

$$i = \frac{k}{d^2}$$
$$3.75 = \frac{k}{40^2}$$
$$3.75 = \frac{k}{1600}$$
$$(1600)3.75 = (1600)\frac{k}{1600}$$
$$6000 = k$$

$$i = \frac{6000}{d^2} = \frac{6000}{50^2} = \frac{6000}{2500} = 2.4$$

The illumination is 2.4 foot-candles at a distance of 50 feet.

31. Since index varies directly as weight and inversely as the square of one's height, we have $I = \frac{kw}{h^2}$.

Use the given values to find *k*.

$$I = \frac{kw}{h^2}$$
$$35.15 = \frac{k(180)}{60^2}$$
$$35.15 = \frac{k(180)}{3600}$$
$$(3600)35.15 = \frac{k(180)}{3600}$$
$$126540 = k(180)$$
$$k = \frac{126540}{180} = 703$$

The equation becomes $I = \frac{703w}{h^2}$.

When $w = 170$ and $h = 70$,

$I = \frac{703(170)}{(70)^2} \approx 24.4.$

This person has a BMI of 24.4 and is not overweight.

32.
$$i = \frac{km}{c}$$
$$125 = \frac{k(25)}{20}$$
$$20(125) = (20)\frac{k(25)}{20}$$
$$2500 = 25k$$
$$\frac{2500}{25} = \frac{25k}{25}$$
$$100 = k$$

$$i = \frac{100m}{c}$$
$$80 = \frac{100(40)}{c}$$
$$80 = \frac{4000}{c}$$
$$80c = c \cdot \frac{4000}{c}$$
$$80c = 4000$$
$$\frac{80c}{80} = \frac{4000}{80}$$
$$c = 50$$

The chronological age is 50.

33. Since heat loss varies jointly as the area and temperature difference, we have $L = kAD$. Use the given values to find *k*.

$$L = kAD$$
$$1200 = k(3 \cdot 6)(20)$$
$$1200 = 360k$$
$$\frac{1200}{360} = \frac{360k}{360}$$
$$k = \frac{10}{3}$$

The equation becomes $L = \frac{10}{3}AD$

When $A = 6 \cdot 9 = 54$, $D = 10$,

$L = \frac{10}{3}(9 \cdot 6)(10) = 1800$.

The heat loss is 1800 Btu .

34.
$$e = kmv^2$$
$$36 = k(8)(3)^2$$
$$36 = k(8)(9)$$
$$36 = 72k$$
$$\frac{36}{72} = \frac{72k}{72}$$
$$k = 0.5$$

$$e = 0.5mv^2 = 0.5(4)(6)^2 = 0.5(4)(36) = 72$$

A mass of 4 grams and velocity of 6 centimeters per second has a kinetic energy of 72 ergs.

35. Since intensity varies inversely as the square of the distance from the sound source, we have $I = \frac{k}{d^2}$. If you move to a seat twice as far, then $d = 2d$. So we have $I = \frac{k}{(2d)^2} = \frac{k}{4d^2} = \frac{1}{4} \cdot \frac{k}{d^2}$. The intensity will be multiplied by a factor of $\frac{1}{4}$. So the sound intensity is $\frac{1}{4}$ of what it was originally.

36.
$$t = \frac{k}{a}$$
$$t = \frac{k}{3a} = \frac{1}{3} \cdot \frac{k}{a}$$

A year will seem to be $\frac{1}{3}$ of a year.

37. **a.** Since the average number of phone calls varies jointly as the product of the populations and inversely as the square of the distance, we have $C = \frac{kP_1P_2}{d^2}$.

b. Use the given values to find *k*.

$$C = \frac{kP_1P_2}{d^2}$$
$$326,000 = \frac{k(777,000)(3,695,000)}{(420)^2}$$
$$326,000 = \frac{k(2.87\times10^{12})}{176,400}$$
$$326,000 = 16269841.27k$$
$$0.02 \approx k$$

The equation becomes $C = \frac{0.02P_1P_2}{d^2}$.

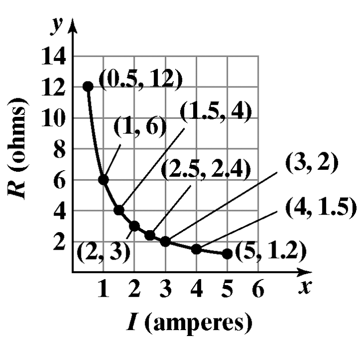
c. $C = \frac{0.02(650,000)(490,000)}{(400)^2}$
$\approx 39,813$

There are approximately 39,813 daily phone calls.

38.
$$f = kas^2$$
$$150 = k(4\cdot5)(30)^2$$
$$150 = k(20)(900)$$
$$150 = 18000k$$
$$\frac{150}{18000} = \frac{18000k}{150}$$
$$\frac{1}{120} = k$$
$$f = \frac{1}{120}as^2 = \frac{1}{120}(3\cdot4)(60)^2$$
$$= \frac{1}{120}(12)(3600)$$
$$= 360$$

Yes, the wind will exert a force of 360 pounds on the window.

39. **a.**



b. Current varies inversely as resistance. Answers will vary.

c. Since the current varies inversely as resistance we have $R = \frac{k}{I}$. Using one of the given ordered pairs to find k.

$$12 = \frac{k}{0.5}$$
$$12(0.5) = \frac{k}{0.5}(0.5)$$
$$k = 6$$

The equation becomes $R = \frac{6}{I}$.

40. – 48. Answers will vary.

49. does not make sense; Explanations will vary. Sample explanation: For an inverse variation, the independent variable can not be zero.

50. does not make sense; Explanations will vary. Sample explanation: A direct variation with a positive constant of variation will have both variables increase simultaneously.

51. makes sense

52. makes sense

53. Pressure, *P*, varies directly as the square of wind velocity, *v*, can be modeled as $P = kv^2$.

If $v = x$ then $P = k(x)^2 = kx^2$

If $v = 2x$ then $P = k(2x)^2 = 4kx^2$

If the wind speed doubles the pressure is 4 times more destructive.

54. Illumination, *I*, varies inversely as the square of the distance, *d*, can be modeled as $I = \frac{k}{d^2}$.

If $d = 15$ then $I = \frac{k}{15^2} = \frac{k}{225}$

If $d = 30$ then $I = \frac{k}{30^2} = \frac{k}{900}$

Note that $\frac{225}{900} = \frac{1}{4}$

If the distance doubles the illumination will be $\frac{1}{4}$ as intense.

55. The Heat, *H*, varies directly as the square of the voltage, *v*, and inversely as the resistance, *r*.

$$H = \frac{kv^2}{r}$$

If the voltage remains constant, to triple the heat the resistant must be reduced by a multiple of 3.

56. Illumination, I, varies inversely as the square of the distance, d, can be modeled as $I = \frac{k}{d^2}$.

If $I = x$ then $x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{k}{x}}$.

If $I = \frac{1}{50}x$ then $\frac{1}{50}x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{50k}{x}} = \sqrt{50}\sqrt{\frac{k}{x}}$.

Since $\sqrt{50} \approx 7$, the Hubble telescope is able to see about 7 times farther than a ground-based telescope.

57. Answers will vary.

58. Let x = the amount invested at 7%.
Let $20{,}000 - x$ = the amount invested at 9%.

$$0.07x + 0.09(20{,}000 - x) = 1550$$
$$0.07x + 1800 - 0.09x = 1550$$
$$-0.02x + 1800 = 1550$$
$$-0.02x = -250$$
$$x = 12{,}500$$
$$20{,}000 - x = 7500$$

\$12,500 was invested at 7% and \$7500 was invested at 9%.

59.
$$\sqrt{x+7} - 1 = x$$
$$\sqrt{x+7} = x + 1$$
$$x + 7 = x^2 + 2x + 1$$
$$x^2 + x - 6 = 0$$
$$(x-2)(x+3) = 0$$

$x - 2 = 0$ $\quad x + 3 = 0$
$x = 2$ $\quad x = -3$

The check indicates that 2 is a solution.
The solution set is $\{2\}$.

60. $f(x) = x^3 + 2$

Replace $f(x)$ with y:

$y = x^3 + 2$

Interchange x and y:

$x = y^3 + 2$

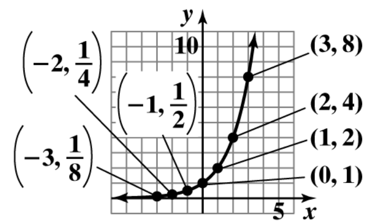
Solve for y:

$$x = y^3 + 2$$
$$x - 2 = y^3$$
$$\sqrt[3]{x-2} = y$$

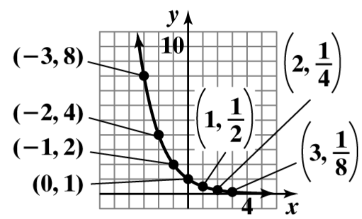
Replace y with $f^{-1}(x)$:

$f^{-1}(x) = \sqrt[3]{x-2}$

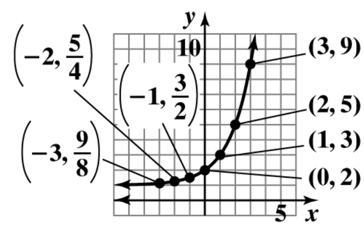
61.



62.



63.



Chapter 2 Review Exercises

1. $(8 - 3i) - (17 - 7i) = 8 - 3i - 17 + 7i$
$= -9 + 4i$

2. $4i(3i - 2) = (4i)(3i) + (4i)(-2)$
$= 12i^2 - 8i$
$= -12 - 8i$

3. $(7 - i)(2 + 3i)$
$= 7 \cdot 2 + 7(3i) + (-i)(2) + (-i)(3i)$
$= 14 + 21i - 2i + 3$
$= 17 + 19i$

4. $(3 - 4i)^2 = 3^2 + 2 \cdot 3(-4i) + (-4i)^2$
$= 9 - 24i - 16$
$= -7 - 24i$

5. $(7 + 8i)(7 - 8i) = 7^2 + 8^2 = 49 + 64 = 113$

6. $\frac{6}{5+i}=\frac{6}{5+i}\cdot\frac{5-i}{5-i}$
$=\frac{30-6i}{25+1}$
$=\frac{30-6i}{26}$
$=\frac{15-3i}{13}$
$=\frac{15}{13}-\frac{3}{13}i$

7. $\frac{3+4i}{4-2i}=\frac{3+4i}{4-2i}\cdot\frac{4+2i}{4+2i}$
$=\frac{12+6i+16i+8i^2}{16-4i^2}$
$=\frac{12+22i-8}{16+4}$
$=\frac{4+22i}{20}$
$=\frac{1}{5}+\frac{11}{10}i$

8. $\sqrt{-32}-\sqrt{-18}=i\sqrt{32}-i\sqrt{18}$
$=i\sqrt{16\cdot 2}-i\sqrt{9\cdot 2}$
$=4i\sqrt{2}-3i\sqrt{2}$
$=(4i-3i)\sqrt{2}$
$=i\sqrt{2}$

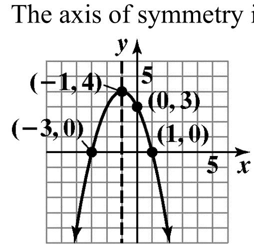
9. $(-2+\sqrt{-100})^2=(-2+i\sqrt{100})^2$
$=(-2+10i)^2$
$=4-40i+(10i)^2$
$=4-40i-100$
$=-96-40i$

10. $\frac{4+\sqrt{-8}}{2}=\frac{4+i\sqrt{8}}{2}=\frac{4+2i\sqrt{2}}{2}=2+i\sqrt{2}$

11. $x^2-2x+4=0$
$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$
$x=\frac{2\pm\sqrt{4-16}}{2}$
$x=\frac{2\pm\sqrt{-12}}{2}$
$x=\frac{2\pm 2i\sqrt{3}}{2}$
$x=1\pm i\sqrt{3}$
The solution set is $\{1-i\sqrt{3},\ 1+i\sqrt{3}\}$.

12. $2x^2-6x+5=0$
$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(5)}}{2(2)}$
$x=\frac{6\pm\sqrt{36-40}}{4}$
$x=\frac{6\pm\sqrt{-4}}{4}$
$x=\frac{6\pm 2i}{4}$
$x=\frac{6}{4}\pm\frac{2i}{4}$
$=\frac{3}{2}\pm\frac{1}{2}i$
The solution set is $\left\{\frac{3}{2}-\frac{1}{2}i,\ \frac{3}{2}+\frac{1}{2}i\right\}$.

13. $f(x)=-(x+1)^2+4$
vertex: (–1, 4)
x-intercepts:
$0=-(x+1)^2+4$
$(x+1)^2=4$
$x+1=\pm 2$
$x=-1\pm 2$
$x=-3$ or $x=1$
y-intercept:
$f(0)=-(0+1)^2+4=3$
The axis of symmetry is $x=-1$.



$f(x) = -(x+1)^2 + 4$

domain: $(-\infty,\infty)$ range: $(-\infty,4]$

14. $f(x) = (x+4)^2 - 2$

vertex: (–4, –2)

x-intercepts:

$0 = (x+4)^2 - 2$

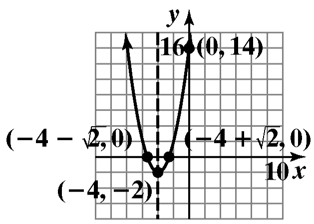
$(x+4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

y-intercept:

$f(0) = (0+4)^2 - 2 = 14 \;\; = -1$



***f*(*x*) = (*x* + 4)² − 2**

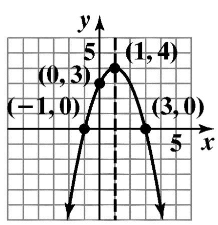
The axis of symmetry is $x = -4$.

domain: $(-\infty, \infty)$ range: $[-2, \infty)$

15. $f(x) = -x^2 + 2x + 3$

$= -(x^2 - 2x + 1) + 3 + 1$

$f(x) = -(x-1)^2 + 4$



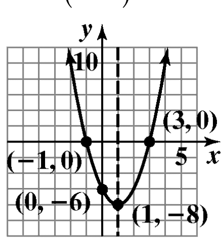
***f*(*x*) = − *x*² + 2*x* + 3**

domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

16. $f(x) = 2x^2 - 4x - 6$

$f(x) = 2(x^2 - 2x + 1) - 6 - 2$

$2(x-1)^2 - 8$



***f*(*x*) = 2*x*² − 4*x* − 6**

axis of symmetry: $x = 1$

domain: $(-\infty, \infty)$ range: $[-8, \infty)$

17. $f(x) = -x^2 + 14x - 106$

a. Since $a < 0$ the parabola opens down with the maximum value occurring at

$$x = -\frac{b}{2a} = -\frac{14}{2(-1)} = 7.$$

The maximum value is $f(7)$.

$f(7) = -(7)^2 + 14(7) - 106 = -57$

b. domain: $(-\infty, \infty)$ range: $(-\infty, -57]$

18. $f(x) = 2x^2 + 12x + 703$

a. Since $a > 0$ the parabola opens up with the minimum value occurring at

$$x = -\frac{b}{2a} = -\frac{12}{2(2)} = -3.$$

The minimum value is $f(-3)$.

$f(-3) = 2(-3)^2 + 12(-3) + 703 = 685$

b. domain: $(-\infty, \infty)$ range: $[685, \infty)$

19. **a.** The maximum height will occur at the vertex.

$f(x) = -0.025x^2 + x + 6$

$$x = -\frac{b}{2a} = -\frac{1}{2(-0.025)} = 20$$

$f(20) = -0.025(20)^2 + (20) + 6 = 16$

The maximum height of 16 feet occurs when the ball is 20 yards downfield.

b. $f(x) = -0.025x^2 + x + 6$

$f(0) = -0.025(0)^2 + (0) + 6 = 6$

The ball was tossed at a height of 6 feet.

c. The ball is at a height of 0 when it hits the ground.

$f(x) = -0.025x^2 + x + 6$

$0 = -0.025x^2 + x + 6$

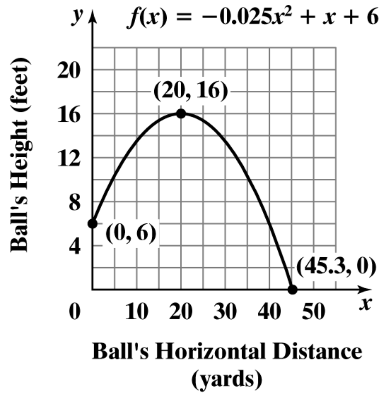
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(-0.025)(6)}}{2(-0.025)}$$

$x \approx 45.3,\ -5.3$ (reject)

The ball will hit the ground 45.3 yards downfield.

d. The football's path:



20. Maximize the area using $A = lw$.

$A(x) = x(1000 - 2x)$

$A(x) = -2x^2 + 1000x$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$x = -\frac{b}{2a} = -\frac{1000}{2(-2)} = -\frac{1000}{-4} = 250.$

The maximum area is achieved when the width is 250 yards. The maximum area is

$$\begin{aligned} A(250) &= 250(1000 - 2(250)) \\ &= 250(1000 - 500) \\ &= 250(500) = 125,000. \end{aligned}$$

The area is maximized at 125,000 square yards when the width is 250 yards and the length is $1000 - 2 \cdot 250 = 500$ yards.

21. Let x = one of the numbers
Let $14 + x$ = the other number

We need to minimize the function

$$\begin{aligned} P(x) &= x(14 + x) \\ &= 14x + x^2 \\ &= x^2 + 14x. \end{aligned}$$

The minimum is at

$x = -\frac{b}{2a} = -\frac{14}{2(1)} = -\frac{14}{2} = -7.$

The other number is $14 + x = 14 + (-7) = 7.$

The numbers which minimize the product are 7 and -7. The minimum product is $-7 \cdot 7 = -49$.

22. $3x + 4y = 1000$

$4y = 1000 - 3x$

$y = \frac{1000 - 3x}{4}$

$$\begin{aligned} A &= x\left(\frac{1000 - 3x}{4}\right) \\ &= -\frac{3}{4}x^2 + 250x \end{aligned}$$

$x = \frac{-b}{2a} = \frac{-250}{2\left(-\frac{3}{4}\right)} = 125$

$y = \frac{1000 - 3(125)}{4} = 166\frac{2}{3}$

125 feet by $166\frac{2}{3}$ feet will maximize the area.

23. $y = (35 + x)(150 - 4x)$

$y = 5250 + 10x - 4x^2$

$x = \frac{-b}{2a} = \frac{-10}{2(-4)} = \frac{5}{4} = 1.25$ or 1 tree

The maximum number of trees should be 35 + 1 = 36 trees.

$y = 36(150 - 4x) = 36(150 - 4 \cdot 1) = 5256$

The maximum yield will be 5256 pounds.

24. $f(x) = -x^3 + 12x^2 - x$

The graph rises to the left and falls to the right and goes through the origin, so graph (c) is the best match.

25. $g(x) = x^6 - 6x^4 + 9x^2$

The graph rises to the left and rises to the right, so graph (b) is the best match.

26. $h(x) = x^5 - 5x^3 + 4x$

The graph falls to the left and rises to the right and crosses the y-axis at zero, so graph (a) is the best match.

27. $f(x) = -x^4 + 1$

$f(x)$ falls to the left and to the right so graph (d) is the best match.

28. **a.** Since n is odd and $a_n > 0$, the graph rises to the right.

b. No, the model will not be useful. The model indicates increasing deforestation despite a declining rate in which the forest is being cut down.

c. The graph of function g falls to the right.

d. No, the model will not be useful. The model indicates the amount of forest cleared, in square kilometers, will eventually be negative, which is not possible.

29. In the polynomial, $f(x) = -x^4 + 21x^2 + 100$, the leading coefficient is –1 and the degree is 4. Applying the Leading Coefficient Test, we know that even-degree polynomials with negative leading coefficient will fall to the left and to the right. Since the graph falls to the right, we know that the elk population will die out over time.

30. $f(x) = -2(x-1)(x+2)^2(x+5)^3$
$x = 1$, multiplicity 1, the graph crosses the x-axis
$x = -2$, multiplicity 2, the graph touches the x-axis
$x = -5$, multiplicity 5, the graph crosses the x-axis

31. $f(x) = x^3 - 5x^2 - 25x + 125$
$= x^2(x-5) - 25(x-5)$
$= (x^2 - 25)(x-5)$
$= (x+5)(x-5)^2$
$x = -5$, multiplicity 1, the graph crosses the x-axis
$x = 5$, multiplicity 2, the graph touches the x-axis

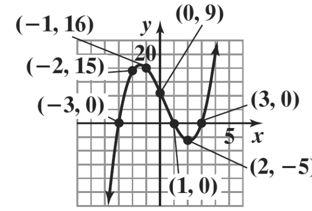
32. $f(x) = x^3 - 2x - 1$
$f(1) = (1)^3 - 2(1) - 1 = -2$
$f(2) = (2)^3 - 2(2) - 1 = 3$
The sign change shows there is a zero between the given values.

33. $f(x) = x^3 - x^2 - 9x + 9$

a. Since n is odd and $a_n > 0$, the graph falls to the left and rises to the right.

b. $f(-x) = (-x)^3 - (-x)^2 - 9(-x) + 9$
$= -x^3 - x^2 + 9x + 9$
$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

c. $f(x) = (x-3)(x+3)(x-1)$
zeros: 3, −3, 1

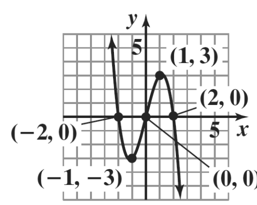


34. $f(x) = 4x - x^3$

a. Since n is odd and $a_n < 0$, the graph rises to the left and falls to the right.

b. $f(-x) = -4x + x^3$
$f(-x) = -f(x)$
origin symmetry

c. $f(x) = x(x^2 - 4) = x(x-2)(x+2)$
zeros: $x = 0, 2, -2$

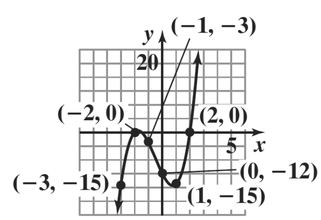


35. $f(x) = 2x^3 + 3x^2 - 8x - 12$

a. Since h is odd and $a_n > 0$, the graph falls to the left and rises to the right.

b. $f(-x) = -2x^3 + 3x^2 + 8x - 12$
$f(-x) \neq f(x),\ f(-x) = -f(x)$
no symmetry

c. $f(x) = (x-2)(x+2)(2x+3)$
zeros: $x = 2, -2, -\frac{3}{2}$



36. $g(x) = -x^4 + 25x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 25(-x)^2$
$\quad = -x^4 + 25x^2 = f(x)$

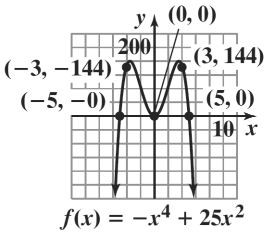
y-axis symmetry

c. $-x^4 + 25x^2 = 0$

$-x^2\left(x^2 - 25\right) = 0$

$-x^2(x-5)(x+5) = 0$

zeros: $x = -5, 0, 5$



37. $f(x) = -x^4 + 6x^3 - 9x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 6(-x)^3 - 9(-x)$
$\quad = -x^4 - 6x^3 - 9x^2 \, f(-x) \neq f(x)$

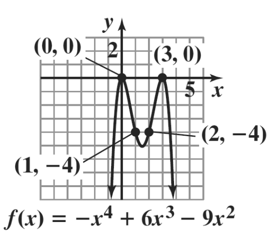
$f(-x) \neq -f(x)$

no symmetry

c. $= -x^2\left(x^2 - 6x + 9\right) = 0$

$-x^2(x-3)(x-3) = 0$

zeros: $x = 0, 3$



38. $f(x) = 3x^4 - 15x^3$

a. The graph rises to the left and to the right.

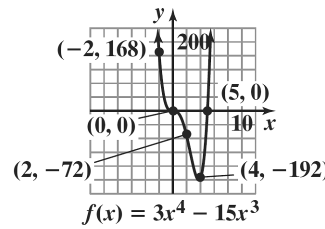
b. $f(-x) = 3(-x)^4 - 15(-x)^2 = 3x^4 + 15x^3$
$f(-x) \neq f(x), \ f(-x) \neq -f(x)$

no symmetry

c $3x^4 - 15x^3 = 0$

$3x^3(x-5) = 0$

zeros: $x = 0, 5$



39. $f(x) = 2x^2(x-1)^3(x+2)$

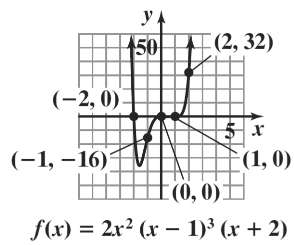
Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

$x = 0, x = 1, x = -2$

The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0)

$f(0) = 2(0)^2(0-1)^3(0+2) = 0$

The y-intercept is 0.



40. $f(x) = -x^3(x+4)^2(x-1)$

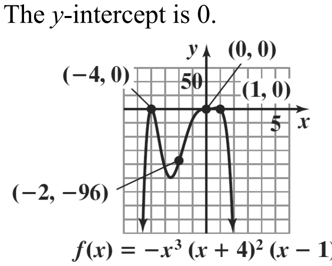
Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

$x = 0, x = -4, x = 1$

The roots at 0 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –4 has even multiplicity so $f(x)$ touches the axis at (–4, 0)

$f(0) = -(0)^3(0+4)^2(0-1) = 0$

The y-intercept is 0.



41.

$$\begin{array}{r} 4x^2-7x+5 \\ x+1\overline{)4x^3-3x^2-2x+1} \\ \underline{4x^3+4x^2} \\ -7x^2-2x \\ \underline{-7x^2-7x} \\ 5x+1 \\ \underline{5x+5} \\ -4 \end{array}$$

Quotient: $4x^2-7x+5-\dfrac{4}{x+1}$

42.

$$\begin{array}{r} 2x^2-4x+1 \\ 5x-3\overline{)10x^3-26x^2+17x-13} \\ \underline{10x^3+6x^2} \\ -20x^2+17x \\ \underline{-20x^2+12x} \\ 5x-13 \\ \underline{5x-\ 3} \\ -10 \end{array}$$

Quotient: $2x^2-4x+1-\dfrac{10}{5x-3}$

43.

$$\begin{array}{r} 2x^2+3x-1 \\ 2x^2+1\overline{)4x^4+6x^3+3x-1} \\ \underline{4x^2+2x^2} \\ 6x^3-2x^2+3x \\ \underline{6x^2+3x} \\ -2x^2-1 \\ \underline{-2x^2-1} \\ 0 \end{array}$$

44. $(3x^4+11x^3-20x^3+7x+35)\div(x+5)$

–5	3	11	–20	7	35
		–15	20	0	–35
	3	–4	0	7	0

Quotient: $3x^3-4x^2+7$

45. $(3x^4-2x^2-10x)\div(x-2)$

2	3	0	–2	–10	0
		6	12	20	20
	3	6	10	10	20

Quotient: $3x^3+6x^2+10x+10+\dfrac{20}{x-2}$

46. $f(x)=2x^3-7x^2+9x-3$

–13	2	–7	9	–3
		–26	429	–5694
	2	–33	438	–5697

Quotient: $f(-13)=-5697$

47. $f(x)=2x^3+x^2-13x+6$

2	2	1	–13	6
		4	10	–6
	2	5	–3	0

$f(x)=(x-2)(2x^2+5x-3)$
$\quad=(x-2)(2x-1)(x+3)$

Zeros: $x=2,\dfrac{1}{2},-3$

48. $x^3-17x+4=0$

4	1	0	–17	4
		4	16	–4
	1	4	–1	0

$(x-4)\left(x^2+4x-1\right)=0$

$x=\dfrac{-4\pm\sqrt{16+4}}{2}=\dfrac{-4\pm2\sqrt{5}}{2}=-2\pm\sqrt{5}$

The solution set is $\left\{4,-2+\sqrt{5},-2-\sqrt{5}\right\}$.

49. $f(x)=x^4-6x^3+14x^2-14x+5$

$p:\pm1,\pm5$
$q:\pm1$
$\dfrac{p}{q}:\pm1,\pm5$

50. $f(x)=3x^5-2x^4-15x^3+10x^2+12x-8$

$p:\pm1,\pm2,\pm4,\pm8$
$q:\pm1,\pm3$
$\dfrac{p}{q}:\pm1,\pm2,\pm4,\pm8,\pm\dfrac{8}{3},\pm\dfrac{4}{3},\pm\dfrac{2}{3},\pm\dfrac{1}{3}$

51. $f(x)=3x^4-2x^3-8x+5$

$f(x)$ has 2 sign variations, so $f(x)=0$ has 2 or 0 positive solutions.

$f(-x)=3x^4+2x^3+x+5$

$f(-x)$ has no sign variations, so $f(x)=0$ has no negative solutions.

52. $f(x) = 2x^5 - 3x^3 - 5x^2 + 3x - 1$

$f(x)$ has 3 sign variations, so $f(x) = 0$ has 3 or 1 positive real roots.

$f(-x) = -2x^5 + 3x^3 - 5x^2 - 3x - 1$

$f(-x)$ has 2 sign variations, so $f(x) = 0$ has 2 or 0 negative solutions.

53. $f(x) = f(-x) = 2x^4 + 6x^2 + 8$

No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

54. $f(x) = x^3 + 3x^2 - 4$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b. 1 sign variation $\Rightarrow$ 1 positive real zero
$f(-x) = -x^3 + 3x^2 - 4$
2 sign variations $\Rightarrow$ 2 or no negative real zeros

c.

1	1	3	0	–4
		1	4	–4
	1	4	4	0

1 is a zero.
1, –2 are rational zeros.

d. $(x-1)(x^2 + 4x + 4) = 0$
$(x-1)(x+2)^2 = 0$
$x = 1$ or $x = -2$
The solution set is $\{1, -2\}$.

55. $f(x) = 6x^3 + x^2 - 4x + 1$

a. $p: \pm 1$
$q: \pm 1, \pm 2, \pm 3, \pm 6$
$\frac{p}{q}: \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$

b. $f(x) = 6x^3 + x^2 - 4x + 1$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = -6x^3 + x^2 + 4x + 1$
1 sign variation; 1 negative real zero.

c.

–1	6	1	–4	1
		–6	5	–1
	6	–5	1	0

–1 is a zero.
$-1, \frac{1}{3}, \frac{1}{2}$ are rational zeros.

d.
$$6x^3 + x^2 - 4x + 1 = 0$$
$$(x+1)(6x^2 - 5x + 1) = 0$$
$$(x+1)(3x-1)(2x-1) = 0$$
$$x = -1 \text{ or } x = \frac{1}{3} \text{ or } x = \frac{1}{2}$$
The solution set is $\left\{-1, \frac{1}{3}, \frac{1}{2}\right\}$.

56. $f(x) = 8x^3 - 36x^2 + 46x - 15$

a. $p: \pm 1, \pm 3, \pm 5, \pm 15$
$q: \pm 1, \pm 2, \pm 4, \pm 8$
$$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{3}{8}, \pm \frac{5}{2}, \pm \frac{5}{4}, \pm \frac{5}{8}, \pm \frac{15}{2}, \pm \frac{15}{4}, \pm \frac{15}{8}$$

b. $f(x) = 8x^3 - 36x^2 + 46x - 15$
3 sign variations; 3 or 1 positive real solutions.
$f(-x) = -8x^3 - 36x^2 - 46x - 15$
0 sign variations; no negative real solutions.

c.

$\frac{1}{2}$	8	–36	46	–15
		4	–16	15
	8	–32	30	0

$\frac{1}{2}$ is a zero.
$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ are rational zeros.

d.

$$8x^3 - 36x^2 + 46x - 15 = 0$$
$$\left(x - \frac{1}{2}\right)(8x^2 - 32x + 30) = 0$$
$$2\left(x - \frac{1}{2}\right)(4x - 16x + 15) = 0$$
$$2\left(x - \frac{1}{2}\right)(2x - 5)(2x - 3) = 0$$

$x = \frac{1}{2}$ or $x = \frac{5}{2}$ or $x = \frac{3}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$.

57. $2x^3 + 9x^2 - 7x + 1 = 0$

a. p: ±1
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm \frac{1}{2}$

b. $f(x) = 2x^3 + 9x^2 - 7x + 1$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = -2x^3 + 9x^2 + 7x + 1$
1 sign variation; 1 negative real zero.

c.

$\frac{1}{2}$	2	9	–7	1
		1	5	–1
	2	10	–2	0

$\frac{1}{2}$ is a rational zero.

d.

$$2x^3 + 9x^2 - 7x + 1 = 0$$
$$\left(x - \frac{1}{2}\right)(2x^2 + 10x - 2) = 0$$
$$2\left(x - \frac{1}{2}\right)(x^2 + 5x - 1) = 0$$

Solving $x^2 + 5x - 1 = 0$ using the quadratic formula gives $x = \frac{-5 \pm \sqrt{29}}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{-5+\sqrt{29}}{2}, \frac{-5-\sqrt{29}}{2}\right\}$.

58. $x^4 - x^3 - 7x^2 + x + 6 = 0$

a. $p = \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

b. $f(x) = x^4 - x^3 - 7x^2 + x + 6$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = x^4 + x^3 - 7x^2 - x + 6$
2 sign variations; 2 or 0 negative real zeros.

c.

1	1	–1	–7	1	6
		1	0	–7	–6
	1	0	–7	–6	0

–1	1	0	–7	–6
		–1	1	6
	1	–1	–6	0

$-2, -1, 1, 3$ are rational zeros.

d.

$$x^4 - x^3 - 7x^2 + x + 6 = 0$$
$$(x-1)(x+1)(x^2 - x + 6) = 0$$
$$(x-1)(x+1)(x-3)(x+2) = 0$$

The solution set is $\{-2, -1, 1, 3\}$.

59. $4x^4 + 7x^2 - 2 = 0$

a. p: ±1, ±2
q: ±1, ±2, ±4
$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$

b. $f(x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 positive real zero.
$f(-x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 negative real zero.

c.

$\frac{1}{2}$	4	0	7	0	–2
		2	1	4	2
	4	2	8	4	0

$-\frac{1}{2}$	4	2	8	4
		–2	0	–4
	4	0	8	0

$-\frac{1}{2}, \frac{1}{2}$ are rational zeros.

d. $4x^4+7x^2-2=0$

$\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(4x^2+8)=0$

$4\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(x^2+2)=0$

Solving $x^2+2=0$ using the quadratic formula gives $x=\pm 2i$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}, 2i, -2i\right\}$.

60. $f(x)=2x^4+x^3-9x^2-4x+4$

a. p: ±1, ±2, ±4
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$

b. $f(x)=2x^4+x^3-9x^2-4x+4$
2 sign variations; 2 or 0 positive real zeros.
$f(-x)=2x^4-x^3-9x^2+4x+4$
2 sign variations; 2 or 0 negative real zeros.

c.

2	2	1	–9	–4	4
		4	10	2	–4
	2	5	1	–2	0

–1	2	5	1	–2
		–2	–3	2
	2	3	–2	0

$-2, -1, \frac{1}{2}, 2$ are rational zeros.

d. $2x^2+3x-2=0$
$(2x-1)(x+2)=0$

$x=-2$ or $x=\frac{1}{2}$

The solution set is $\left\{-2, -1, \frac{1}{2}, 2\right\}$.

61. $f(x)=a_n(x-2)(x-2+3i)(x-2-3i)$

$f(x)=a_n(x-2)(x^2-4x+13)$

$f(1)=a_n(1-2)\left[1^2-4(1)+13\right]$

$-10=-10a_n$

$a_n=1$

$f(x)=1(x-2)(x^2-4x+13)$

$f(x)=x^3-4x^2+13x-2x^2+8x-26$

$f(x)=x^3-6x^2+21x-26$

62. $f(x)=a_n(x-i)(x+i)(x+3)^2$

$f(x)=a_n(x^2+1)(x^2+6x+9)$

$f(-1)=a_n\left[(-1)^2+1\right]\left[(-1)^2+6(-1)+9\right]$

$16=8a_n$

$a_n=2$

$f(x)=2(x^2+1)(x^2+6x+9)$

$f(x)=2(x^4+6x^3+9x^2+x^2+6x+9)$

$f(x)=2x^4+12x^3+20x^2+12x+18$

63. $f(x)=2x^4+3x^3+3x-2$

p: ±1, ±2
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}$

–2	2	3	0	3	–2
		–4	2	–4	2
	2	–1	2	–1	0

$2x^4+3x^3+3x-2=0$

$(x+2)(2x^3-x^2+2x-1)=0$

$(x+2)[x^2(2x-1)+(2x-1)]=0$

$(x+2)(2x-1)(x^2+1)=0$

$x=-2$, $x=\frac{1}{2}$ or $x=\pm i$

The zeros are –2, $\frac{1}{2}$, $\pm i$.

$f(x)=(x-i)(x+i)(x+2)(2x-1)$

64. $g(x) = x^4 - 6x^3 + x^2 + 24x + 16$

p: ±1, ±2, ±4, ±8, ±16

q: ±1

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

–1	1	–6	1	24	16
		–1	7	–8	–16
	1	–7	8	16	0

$x^4 - 6x^3 + x^2 + 24x + 16 = 0$

$(x+1)(x^3 - 7x^2 + 8x + 16) = 0$

–1	1	–7	8	16
		–1	8	–16
	1	–8	16	0

$(x+1)^2(x^2 - 8x + 16) = 0$

$(x+1)^2(x-4)^2 = 0$

$x = -1$ or $x = 4$

$g(x) = (x+1)^2(x-4)^2$

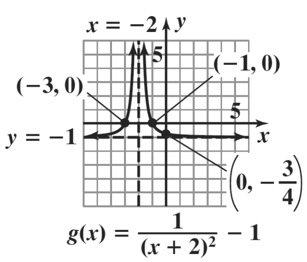
65. 4 real zeros, one with multiplicity two

66. 3 real zeros; 2 nonreal complex zeros

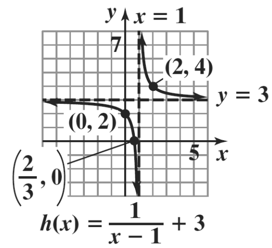
67. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros

68. 1 real zero; 4 nonreal complex zeros

69. $g(x) = \frac{1}{(x+2)^2} - 1$



70. $h(x) = \frac{1}{x-1} + 3$



71. $f(x) = \frac{2x}{x^2 - 9}$

Symmetry: $f(-x) = -\frac{2x}{x^2 - 9} = -f(x)$

origin symmetry

x-intercept:

$0 = \frac{2x}{x^2 - 9}$

$2x = 0$

$x = 0$

y-intercept: $y = \frac{2(0)}{0^2 - 9} = 0$

Vertical asymptote:

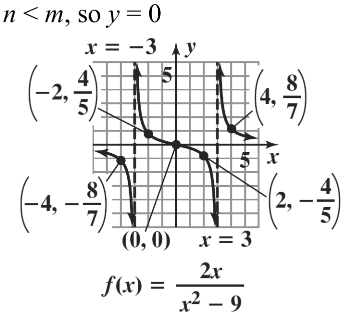
$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = 3$ and $x = -3$

Horizontal asymptote:

$n < m$, so $y = 0$



72. $g(x) = \frac{2x-4}{x+3}$

Symmetry: $g(-x) = \frac{-2x-4}{x+3}$

$g(-x) \neq g(x)$, $g(-x) \neq -g(x)$

No symmetry

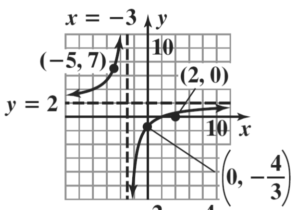
x-intercept:

$2x - 4 = 0$

$x = 2$

y-intercept: $y = \frac{2(0)-4}{(0)+3} = -\frac{4}{3}$

Vertical asymptote:
$x + 3 = 0$
$x = -3$
Horizontal asymptote:
$n = m$, so $y = \frac{2}{1} = 2$

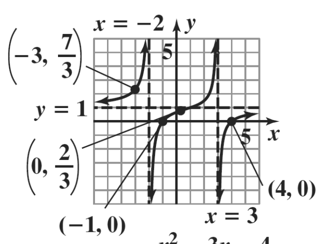


$f(x) = \frac{2x - 4}{x + 3}$

73. $h(x) = \frac{x^2 - 3x - 4}{x^2 - x - 6}$

Symmetry: $h(-x) = \frac{x^2 + 3x - 4}{x^2 + x - 6}$
$h(-x) \neq h(x)$, $h(-x) \neq -h(x)$
No symmetry
x-intercepts:
$x^2 - 3x - 4 = 0$
$(x - 4)(x + 1)$
$x = 4 \quad x = -1$
y-intercept: $y = \frac{0^2 - 3(0) - 4}{0^2 - 0 - 6} = \frac{2}{3}$
Vertical asymptotes:
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3, -2$

Horizontal asymptote:
$n = m$, so $y = \frac{1}{1} = 1$

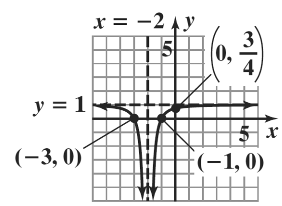


$h(x) = \frac{x^2 - 3x - 4}{x^2 - x - 6}$

74. $r(x) = \frac{x^2 + 4x + 3}{(x + 2)^2}$

Symmetry: $r(-x) = \frac{x^2 - 4x + 3}{(-x + 2)^2}$
$r(-x) \neq r(x)$, $r(-x) \neq -r(x)$
No symmetry
x-intercepts:
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3, -1$
y-intercept: $y = \frac{0^2 + 4(0) + 3}{(0 + 2)^2} = \frac{3}{4}$
Vertical asymptote:
$x + 2 = 0$
$x = -2$

Horizontal asymptote:
$n = m$, so $y = \frac{1}{1} = 1$



$r(x) = \frac{x^2 + 4x + 3}{(x + 2)^2}$

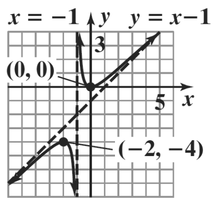
75. $y = \frac{x^2}{x + 1}$

Symmetry: $f(-x) = \frac{x^2}{-x + 1}$
$f(-x) \neq f(x)$, $f(-x) \neq -f(x)$
No symmetry
x-intercept:
$x^2 = 0$
$x = 0$
y-intercept: $y = \frac{0^2}{0 + 1} = 0$
Vertical asymptote:
$x + 1 = 0$
$x = -1$
$n > m$, no horizontal asymptote.

Slant asymptote:

$y = x - 1 + \dfrac{1}{x+1}$

$y = x - 1$



$y = \dfrac{x^2}{x+1}$

76. $y = \dfrac{x^2 + 2x - 3}{x - 3}$

Symmetry: $f(-x) = \dfrac{x^2 - 2x - 3}{-x - 3}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercepts:

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3, 1$

y-intercept: $y = \dfrac{0^2 + 2(0) - 3}{0 - 3} = \dfrac{-3}{-3} = 1$

Vertical asymptote:

$x - 3 = 0$

$x = 3$

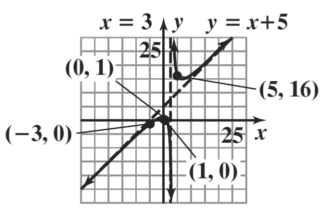
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$y = x + 5 + \dfrac{12}{x-3}$

$y = x + 5$



$y = \dfrac{x^2 + 2x - 3}{x - 3}$

77. $f(x) = \dfrac{-2x^3}{x^2 + 1}$

Symmetry: $f(-x) = \dfrac{2}{x^2 + 1} = -f(x)$

Origin symmetry

x-intercept:

$-2x^3 = 0$

$x = 0$

y-intercept: $y = \dfrac{-2(0)^3}{0^2 + 1} = \dfrac{0}{1} = 0$

Vertical asymptote:

$x^2 + 1 = 0$

$x^2 = -1$

No vertical asymptote.

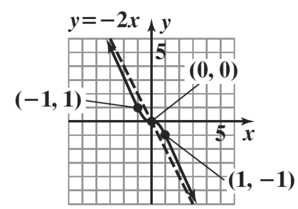
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$f(x) = -2x + \dfrac{2x}{x^2 + 1}$

$y = -2x$



$f(x) = \dfrac{-2x^3}{x^2 + 1}$

78. $g(x) = \dfrac{4x^2 - 16x + 16}{2x - 3}$

Symmetry: $g(-x) = \dfrac{4x^2 + 16x + 16}{-2x - 3}$

$g(-x) \neq g(x), g(-x) \neq -g(x)$

No symmetry

x-intercept:

$4x^2 - 16x + 16 = 0$

$4(x - 2)^2 = 0$

$x = 2$

y-intercept:

$y = \dfrac{4(0)^2 - 16(0) + 16}{2(0) - 3} = -\dfrac{16}{3}$

Vertical asymptote:

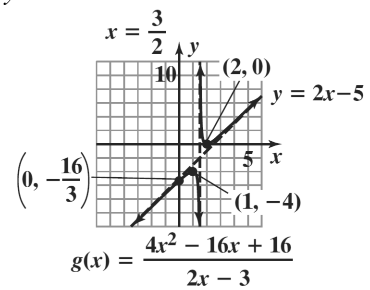
$2x - 3 = 0$

$x = \dfrac{3}{2}$

Horizontal asymptote:
$n > m$, so no horizontal asymptote.
Slant asymptote:

$$g(x) = 2x - 5 + \frac{1}{2x-3}$$

$y = 2x - 5$



79. **a.** $C(x) = 50{,}000 + 25x$

b. $\overline{C}(x) = \frac{25x + 50{,}000}{x}$

c. $\overline{C}(50) = \frac{25(50) + 50{,}000}{50} = 1025$

When 50 calculators are manufactured, it costs $1025 to manufacture each.

$\overline{C}(100) = \frac{25(100) + 50{,}000}{100} = 525$

When 100 calculators are manufactured, it costs $525 to manufacture each.

$\overline{C}(1000) = \frac{25(1000) + 50{,}000}{1000} = 75$

When 1,000 calculators are manufactured, it costs $75 to manufacture each.

$\overline{C}(100{,}000) = \frac{25(100{,}000) + 50{,}000}{100{,}000} = 25.5$ Wh
en 100,000 calculators are manufactured, it costs $25.50 to manufacture each.

d. $n = m$, so $y = \frac{25}{1} = 25$ is the horizontal asymptote. Minimum costs will approach $25.

80. $f(x) = \frac{150x + 120}{0.05x + 1}$

$n = m$, so $y = \frac{150}{0.05} = 3000$

The number of fish available in the pond approaches 3000.

81. $P(x) = \frac{72{,}900}{100x^2 + 729}$

$n < m$ so $y = 0$

As the number of years of education increases the percentage rate of unemployment approaches zero.

82. **a.** $P(x) = M(x) + F(x)$
$= 1.48x + 115.1 + 1.44x + 120.9$
$= 2.92x + 236$

b. $R(x) = \frac{M(x)}{P(x)} = \frac{1.48x + 115.1}{2.92x + 236}$

c. $y = \frac{1.48}{2.92} \approx 0.51$

Over time, the percentage of men in the U.S. population will approach 51%.

83. $T(x) = \frac{4}{x+3} + \frac{2}{x}$

84. $1000 = lw$

$\frac{1000}{w} = l$

$P = 2x + 2\left(\frac{1000}{x}\right)$

$P = 2x + \frac{2000}{x}$

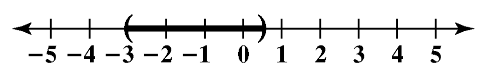
85. $2x^2 + 5x - 3 < 0$

Solve the related quadratic equation.

$2x^2 + 5x - 3 = 0$
$(2x-1)(x+3) = 0$

The boundary points are -3 and $\frac{1}{2}$.

Testing each interval gives a solution set of $\left(-3, \frac{1}{2}\right)$



86. $2x^2 + 9x + 4 \geq 0$

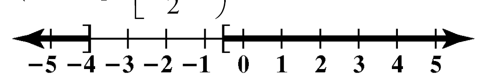
Solve the related quadratic equation.

$2x^2 + 9x + 4 = 0$
$(2x+1)(x+4) = 0$

The boundary points are -4 and $-\frac{1}{2}$.

Testing each interval gives a solution set of

$(-\infty, -4] \cup \left[-\frac{1}{2}, \infty\right)$



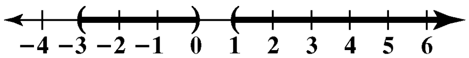
87. $x^3 + 2x^2 > 3x$

Solve the related equation.

$$x^3 + 2x^2 = 3x$$
$$x^3 + 2x^2 - 3x = 0$$
$$x\left(x^2 + 2x - 3\right) = 0$$
$$x(x+3)(x-1) = 0$$

The boundary points are -3, 0, and 1.

Testing each interval gives a solution set of $(-3, 0) \cup (1, \infty)$

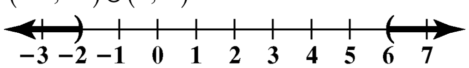


88. $\dfrac{x-6}{x+2} > 0$

Find the values of x that make the numerator and denominator zero.

The boundary points are –2 and 6.

Testing each interval gives a solution set of $(-\infty, -2) \cup (6, \infty)$.

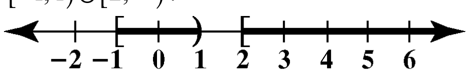


89. $\dfrac{(x+1)(x-2)}{x-1} \ge 0$

Find the values of x that make the numerator and denominator zero.

The boundary points are –1, 1 and 2. We exclude 1 from the solution set, since this would make the denominator zero.

Testing each interval gives a solution set of $[-1, 1) \cup [2, \infty)$.



90. $\dfrac{x+3}{x-4} \le 5$

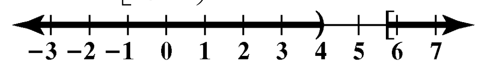
Express the inequality so that one side is zero.

$$\frac{x+3}{x-4} - 5 \le 0$$
$$\frac{x+3}{x-4} - \frac{5(x-4)}{x-4} \le 0$$
$$\frac{-4x+23}{x-4} \le 0$$

Find the values of x that make the numerator and denominator zero.

The boundary points are 4 and $\dfrac{23}{4}$. We exclude 4 from the solution set, since this would make the denominator zero.

Testing each interval gives a solution set of $(-\infty, 4) \cup \left[\dfrac{23}{4}, \infty\right)$.



91. **a.** $g(x) = 0.125x^2 + 2.3x + 27$

$g(35) = 0.125(35)^2 + 2.3(35) + 27 \approx 261$

The stopping distance on wet pavement for a motorcycle traveling 35 miles per hour is about 261 feet. This overestimates the distance shown in the graph by 1 foot.

b. $f(x) = 0.125x^2 - 0.8x + 99$

$$0.125x^2 - 0.8x + 99 > 267$$
$$0.125x^2 - 0.8x - 168 > 0$$

Solve the related quadratic equation.

$$0.125x^2 - 0.8x - 168 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.125)(-168)}}{2(0.125)}$$
$$x = -33.6,\ 40$$

Testing each interval gives a solution set of $(-\infty, -33.6) \cup (40, \infty)$.

Thus, speeds exceeding 40 miles per hour on dry pavement will require over 267 feet of stopping distance.

92. $s = -16t^2 + v_0 t + s_0$

$$32 < -16t^2 + 48t + 0$$
$$0 < -16t^2 + 48t - 32$$
$$0 < -16\left(t^2 - 3t + 2\right)$$
$$0 < -16(t-2)(t-1)$$

F	T	F
	1	2

The projectile's height exceeds 32 feet during the time period from 1 to 2 seconds.

93.

$$w = ks$$
$$28 = k \cdot 250$$
$$0.112 = k$$

Thus, $w = 0.112s$.

$w = 0.112(1200) = 134.4$

1200 cubic centimeters of melting snow will produce 134.4 cubic centimeters of water.

94. $d = kt^2$

$144 = k(3)^2$
$k = 16$
$d = 16t^2$
$d = 16(10)^2 = 1,600$ ft

95. $p = \frac{k}{w}$

$660 = \frac{k}{1.6}$
$1056 = k$

Thus, $p = \frac{1056}{w}$.

$p = \frac{1056}{2.4} = 440$

The pitch is 440 vibrations per second.

96. $l = \frac{k}{d^2}$

$28 = \frac{k}{8^2}$
$k = 1792$
$l = \frac{1792}{d^2}$
$l = \frac{1792}{4^2} = 112$ decibels

97. $t = \frac{kc}{w}$

$10 = \frac{k \cdot 30}{6}$
$10 = 5h$
$h = 2$
$t = \frac{2c}{w}$
$t = \frac{2(40)}{5} = 16$ hours

98. $V = khB$

$175 = k \cdot 15 \cdot 35$
$k = \frac{1}{3}$
$V = \frac{1}{3}hB$
$V = \frac{1}{3} \cdot 20 \cdot 120 = 800 \text{ ft}^3$

99. **a.** Use $L = \frac{k}{R}$ to find k.

$L = \frac{k}{R}$
$30 = \frac{k}{63}$
$63 \cdot 30 = 63 \cdot \frac{k}{63}$
$1890 = k$

Thus, $L = \frac{1890}{R}$.

b. This is an approximate model.

c. $L = \frac{1890}{R}$

$L = \frac{1890}{27} = 70$

The average life span of an elephant is 70 years.

Chapter 2 Test

1. $(6-7i)(2+5i) = 12 + 30i - 14i - 35i^2$
$= 12 + 16i + 35$
$= 47 + 16i$

2. $\frac{5}{2-i} = \frac{5}{2-i} \cdot \frac{2+i}{2+i}$
$= \frac{5(2+i)}{4+1}$
$= \frac{5(2+i)}{5}$
$= 2 + i$

3. $2\sqrt{-49} + 3\sqrt{-64} = 2(7i) + 3(8i)$
$= 14i + 24i$
$= 38i$

4. $x^2 = 4x - 8$

$x^2 - 4x + 8 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$
$x = \frac{4 \pm \sqrt{-16}}{2}$
$x = \frac{4 \pm 4i}{2}$
$x = 2 \pm 2i$

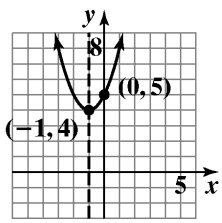
5. $f(x) = (x+1)^2 + 4$

vertex: (–1, 4)
axis of symmetry: $x = -1$
x-intercepts:

$(x+1)^2 + 4 = 0$
$x^2 + 2x + 5 = 0$
$x = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$

no x-intercepts
y-intercept:

$f(0) = (0+1)^2 + 4 = 5$



***f*(*x*) = (*x* + 1)² + 4**

domain: $(-\infty, \infty)$; range: $[4, \infty)$

6. $f(x) = x^2 - 2x - 3$

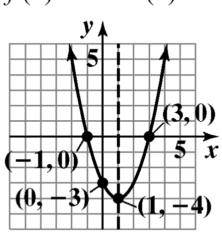
$x = \frac{-b}{2a} = \frac{2}{2} = 1$
$f(1) = 1^2 - 2(1) - 3 = -4$

vertex: (1, –4)
axis of symmetry $x = 1$
x-intercepts:

$x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$x = 3$ or $x = -1$

y-intercept:

$f(0) = 0^2 - 2(0) - 3 = -3$



***f*(*x*) = *x*² − 2*x* − 3**

domain: $(-\infty, \infty)$; range: $[-4, \infty)$

7. $f(x) = -2x^2 + 12x - 16$

Since the coefficient of x^2 is negative, the graph of $f(x)$ opens down and $f(x)$ has a maximum point.

$x = \frac{-12}{2(-2)} = 3$

$f(3) = -2(3)^2 + 12(3) - 16$
$= -18 + 36 - 16$
$= 2$

Maximum point: (3, 2)
domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

8. $f(x) = -x^2 + 46x - 360$

$x = -\frac{b}{2a} = \frac{-46}{-2} = 23$

23 computers will maximize profit.

$f(23) = -(23)^2 + 46(23) - 360 = 169$

Maximum daily profit = $16,900.

9. Let x = one of the numbers;
$14 - x$ = the other number.
The product is $f(x) = x(14-x)$

$f(x) = x(14-x) = -x^2 + 14x$

The x-coordinate of the maximum is

$x = -\frac{b}{2a} = -\frac{14}{2(-1)} = -\frac{14}{-2} = 7.$

$f(7) = -7^2 + 14(7) = 49$

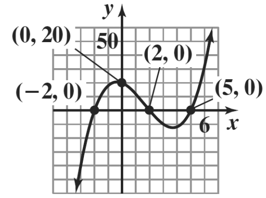
The vertex is (7, 49). The maximum product is 49. This occurs when the two numbers are 7 and $14 - 7 = 7$.

10. a. $f(x) = x^3 - 5x^2 - 4x + 20$

$x^3 - 5x^2 - 4x + 20 = 0$
$x^2(x-5) - 4(x-5) = 0$
$(x-5)(x-2)(x+2) = 0$
$x = 5, 2, -2$

The solution set is {5, 2, –2}.

b. The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



***f*(*x*) = *x*³ − 5*x*² − 4*x* + 20**

11. $f(x) = x^5 - x$

Since the degree of the polynomial is odd and the leading coefficient is positive, the graph of *f* should fall to the left and rise to the right. The *x*-intercepts should be –1 and 1.

12. **a.** The integral root is 2.

b.

2	6	–19	16	–4
		12	–14	4
	6	–7	2	0

$6x^2 - 7x + 2 = 0$

$(3x - 2)(2x - 1) = 0$

$x = \frac{2}{3}$ or $x = \frac{1}{2}$

The other two roots are $\frac{1}{2}$ and $\frac{2}{3}$.

13. $2x^3 + 11x^2 - 7x - 6 = 0$

$p: \pm 1, \pm 2, \pm 3, \pm 6$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

14. $f(x) = 3x^5 - 2x^4 - 2x^2 + x - 1$

$f(x)$ has 3 sign variations.

$f(-x) = -3x^5 - 2x^4 - 2x^2 - x - 1$

$f(-x)$ has no sign variations.

There are 3 or 1 positive real solutions and no negative real solutions.

15. $x^3 + 9x^2 + 16x - 6 = 0$

Since the leading coefficient is 1, the possible rational zeros are the factors of 6

$p = \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

–3	1	9	16	–6
		–3	–18	6
	1	6	–2	0

Thus $x = 3$ is a root.

Solve the quotient $x^2 + 6x - 2 = 0$ using the quadratic formula to find the remaining roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-2)}}{2(1)} = \frac{-6 \pm \sqrt{44}}{2} = -3 \pm \sqrt{11}$$

The zeros are -3, $-3 + \sqrt{11}$, and $-3 - \sqrt{11}$.

16. $f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15$

a. Possible rational zeros are:

$p: \pm 1, \pm 3, \pm 5, \pm 15$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

b. Verify that -1 and $\frac{3}{2}$ are zeros as it appears in the graph:

−1	2	−1	−13	5	15
		−2	3	10	−15
	2	−3	−10	15	0

$\frac{3}{2}$	2	−3	−10	15
		3	0	−15
	2	0	−10	0

Thus, -1 and $\frac{3}{2}$ are zeros, and the polynomial factors as follows:

$$2x^4 - x^3 - 13x^2 + 5x + 15 = 0$$
$$(x+1)\left(2x^3 - 3x^2 - 10x + 15\right) = 0$$
$$(x+1)\left(x - \frac{3}{2}\right)\left(2x^2 - 10\right) = 0$$

Find the remaining zeros by solving:

$2x^2 - 10 = 0$

$2x^2 = 10$

$x^2 = 5$

$x = \pm\sqrt{5}$

The zeros are -1, $\frac{3}{2}$, and $\pm\sqrt{5}$.

17. $f(x)$ has zeros at –2 and 1. The zero at –2 has multiplicity of 2.

$x^3 + 3x^2 - 4 = (x - 1)(x + 2)^2$

18. $f(x) = a_0(x+1)(x-1)(x+i)(x-i)$
$= a_0(x^2-1)(x^2+1)$
$= a_0(x^4-1)$

Since $f(3) = 160$, then

$a_0(3^4-1) = 160$
$a_0(80) = 160$
$a_0 = \dfrac{160}{80}$
$a_0 = 2$

$f(x) = 2(x^4-1) = 2x^4-2$

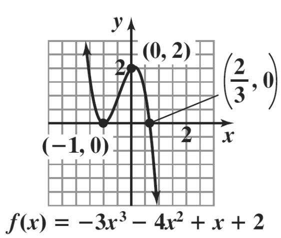
19. $f(x) = -3x^3-4x^2+x+2$
The graph shows a root at $x = -1$.
Use synthetic division to verify this root.

–1	–3	–4	1	2
		3	1	4
	–3	–1	2	0

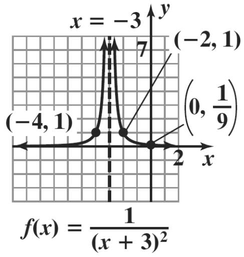
Factor the quotient to find the remaining zeros.
$-3x^2-x+2=0$
$-(3x-2)(x+1)=0$

The zeros (x-intercepts) are –1 and $\frac{2}{3}$.

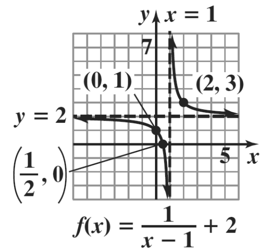
The y-intercept is $f(0) = 2$



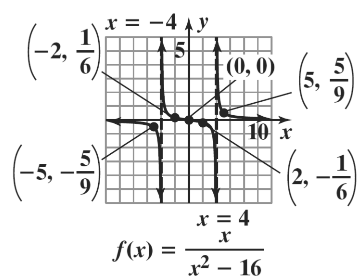
20. $f(x) = \dfrac{1}{(x+3)^2}$
domain: $\{x \mid x \neq -3\}$ or $(-\infty,-3)\cup(-3,\infty)$



21. $f(x) = \dfrac{1}{x-1}+2$
domain: $\{x \mid x \neq 1\}$ or $(-\infty,1)\cup(1,\infty)$



22. $f(x) = \dfrac{x}{x^2-16}$
domain: $\{x \mid x \neq 4, x \neq -4\}$
Symmetry: $f(-x) = \dfrac{-x}{x^2-16} = -f(x)$
y-axis symmetry
x-intercept: $x = 0$
y-intercept: $y = \dfrac{0}{0^2-16} = 0$
Vertical asymptotes:
$x^2-16=0$
$(x-4)(x+4)=0$
$x = 4, -4$
Horizontal asymptote:
$n < m$, so $y = 0$ is the horizontal asymptote.



23. $f(x) = \dfrac{x^2-9}{x-2}$
domain: $\{x \mid x \neq 2\}$
Symmetry: $f(-x) = \dfrac{x^2-9}{-x-2}$
$f(-x) \neq f(x), f(-x) \neq -f(x)$
No symmetry

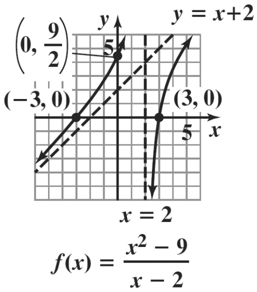
x-intercepts:
$x^2-9=0$
$(x-3)(x+3)=0$
$x = 3, -3$

y-intercept: $y=\dfrac{0^2-9}{0-2}=\dfrac{9}{2}$

Vertical asymptote:
$x-2=0$
$x=2$
Horizontal asymptote:
$n>m$, so no horizontal asymptote exists.

Slant asymptote: $f(x)=x+2-\dfrac{5}{x-2}$

$y=x+2$



24. $f(x)=\dfrac{x+1}{x^2+2x-3}$

$x^2+2x-3=(x+3)(x-1)$
domain: $\{x \mid x\neq -3, x\neq 1\}$

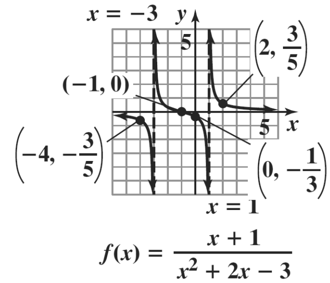
Symmetry: $f(-x)=\dfrac{-x+1}{x^2-2x-3}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$
No symmetry
x-intercept:
$x+1=0$
$x=-1$

y-intercept: $y=\dfrac{0+1}{0^2+2(0)-3}=-\dfrac{1}{3}$

Vertical asymptotes:
$x^2+2x-3=0$
$(x+3)(x-1)=0$
x –3, 1

Horizontal asymptote:
$n<m$, so $y=0$ is the horizontal asymptote.



25. $f(x)=\dfrac{4x^2}{x^2+3}$

domain: all real numbers

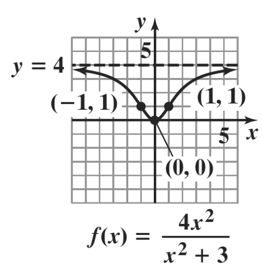
Symmetry: $f(-x)=\dfrac{4x^2}{x^2+3}=f(x)$

y-axis symmetry
x-intercept:
$4x^2=0$
$x=0$

y-intercept: $y=\dfrac{4(0)^2}{0^2+3}=0$

Vertical asymptote:
$x^2+3=0$
$x^2=-3$
No vertical asymptote.
Horizontal asymptote:

$n=m$, so $y=\dfrac{4}{1}=4$ is the horizontal asymptote.



26. **a.** $\overline{C}(x)=\dfrac{300{,}000+10x}{x}$

b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $x=\dfrac{10}{1}=10$.
This represents the fact that as the number of satellite radio players produced increases, the production cost approaches \$10 per radio.

27. $x^2 < x+12$

$x^2 - x - 12 < 0$

$(x+3)(x-4) < 0$

Boundary values: –3 and 4

Solution set: $(-3, 4)$

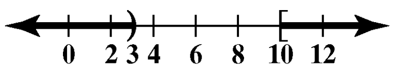
28. $\dfrac{2x+1}{x-3} \le 3$

$\dfrac{2x+1}{x-3} - 3 \le 0$

$\dfrac{10-x}{x-3} \le 0$

Boundary values: 3 and 10

Solution set: $(-\infty, 3) \cup [10, \infty)$



29. $i = \dfrac{k}{d^2}$

$20 = \dfrac{k}{15^2}$

$4500 = k$

$i = \dfrac{4500}{d^2} = \dfrac{4500}{10^2} = 45$ foot-candles

Cumulative Review Exercises (Chapters P–2)

1. domain: $(-2, 2)$ range: $[0, \infty)$

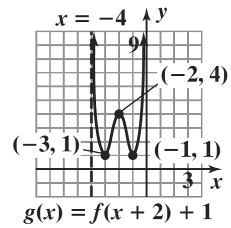
2. The zero at –1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.
The zero at 1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.

3. There is a relative maximum at the point (0, 3).

4. $(f \circ f)(-1) = f\left(f(-1)\right) = f(0) = 3$

5. $f(x) \to \infty$ as $\underline{x \to -2^+}$ or as $\underline{x \to 2^-}$

6.



7. $|2x-1| = 3$

$2x - 1 = 3$

$2x = 4$

$x = 2$

$2x - 1 = -3$

$2x = -2$

$x = -1$

The solution set is $\{-1, 2\}$.

8. $3x^2 - 5x + 1 = 0$

$x = \dfrac{5 \pm \sqrt{25-12}}{6} = \dfrac{5 \pm \sqrt{13}}{6}$

The solution set is $\left\{\dfrac{5+\sqrt{13}}{6}, \dfrac{5-\sqrt{13}}{6}\right\}$.

9. $9 + \dfrac{3}{x} = \dfrac{2}{x^2}$

$9x^2 + 3x = 2$

$9x^2 + 3x - 2 = 0$

$(3x-1)(3x+2) = 0$

$3x - 1 = 0 \qquad 3x + 2 = 0$

$x = \dfrac{1}{3}$ or $x = -\dfrac{2}{3}$

The solution set is $\left\{-\dfrac{2}{3}, \dfrac{1}{3}\right\}$.

10. $x^3 + 2x^2 - 5x - 6 = 0$

p: ±1, ±2, ±3, ±6

q: ±1

$\dfrac{p}{q}$: ±1, ±2, ±3, ±6

–3	1	2	–5	–6
		–3	3	6
	1	–1	–2	0

$x^3 + 2x^2 - 5x - 6 = 0$

$(x+3)(x^2 - x - 2) = 0$

$(x+3)(x+1)(x-2) = 0$

$x = -3$ or $x = -1$ or $x = 2$

The solution set is $\{-3, -1, 2\}$.

11. $|2x-5|>3$

$2x-5>3$
$2x>8$
$x>4$
$2x-5<-3$
$2x<2$
$x<1$

$(-\infty,1)$ or $(4,\infty)$

12. $3x^2>2x+5$

$3x^2-2x-5>0$
$3x^2-2x-5=0$
$(3x-5)(x+1)=0$

$x=\frac{5}{3}$ or $x=-1$

Test intervals are $(-\infty,-1)$, $\left(-1,\frac{5}{3}\right)$, $\left(\frac{5}{3},\infty\right)$.

Testing points, the solution is $(-\infty,-1)$ or $\left(\frac{5}{3},\infty\right)$.

13. $f(x)=x^3-4x^2-x+4$

x-intercepts:

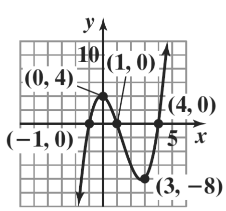
$x^3-4x^2-x+4=0$
$x^2(x-4)-1(x-4)=0$
$(x-4)(x^2-1)=0$
$(x-4)(x+1)(x-1)=0$

$x=-1,1,4$

x-intercepts:

$f(0)=0^3-4(0)^2-0+4=4$

The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



$f(x)=x^3-4x^2-x+4$

14. $f(x)=x^2+2x-8$

$x=\frac{-b}{2a}=\frac{-2}{2}=-1$

$f(-1)=(-1)^2+2(-1)-8$
$=1-2-8=-9$

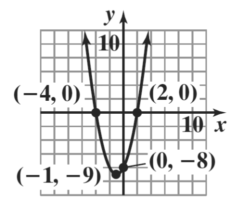
vertex: (–1, –9)

x-intercepts:

$x^2+2x-8=0$
$(x+4)(x-2)=0$

$x=-4$ or $x=2$

y-intercept: $f(0)=-8$



$f(x)=x^2+2x-8$

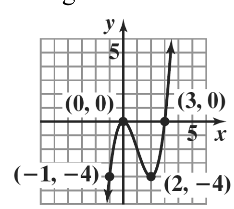
15. $f(x)=x^2(x-3)$

zeros: $x=0$ (multiplicity 2) and $x=3$

y-intercept: $y=0$

$f(x)=x^3-3x^2$

$n=3, a_n=0$ so the graph falls to the left and rises to the right.



$f(x)=x^2(x-3)$

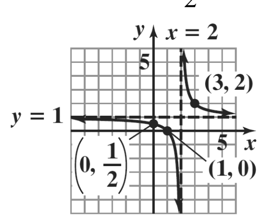
16. $f(x)=\frac{x-1}{x-2}$

vertical asymptote: $x=2$

horizontal asymptote: $y=1$

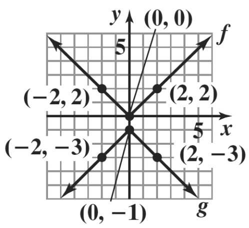
x-intercept: $x=1$

y-intercept: $y=\frac{1}{2}$

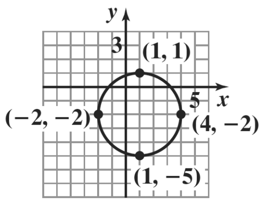


$f(x)=\frac{x-1}{x-2}$

17.



18.



19. $(f \circ g)(x) = f\left(g(x)\right)$

$$= 2(4x-1)^2 - (4x-1) - 1$$
$$= 32x^2 - 20x + 2$$

20. $\dfrac{f(x+h)-f(x)}{h}$

$$= \frac{\left[2(x+h)^2 - (x+h) - 1\right] - \left[2x^2 - x - 1\right]}{h}$$
$$= \frac{2x^2 + 4hx - x + 2h^2 - h - 1 - 2x^2 + x + 1}{h}$$
$$= \frac{4hx + 2h^2 - h}{h}$$
$$= 4x + 2h - 1$$

