INSTRUCTOR'S SOLUTIONS MANUAL

TO ACCOMPANY

POWER SYSTEM
ANALYSIS AND DESIGN

FIFTH EDITION

J. DUNCAN GLOVER

MULUKUTLA S. SARMA

THOMAS J. OVERBYE

Contents

Chapter 2	1
Chapter 3	27
Chapter 4	71
Chapter 5	95
Chapter 6	137
Chapter 7	175
Chapter 8	195
Chapter 9	231
Chapter 10	303
Chapter 11	323
Chapter 12	339
Chapter 13	353
Chapter 14	379

Chapter 2
Fundamentals

ANSWERS TO MULTIPLE-CHOICE TYPE QUESTIONS

2.1 b
2.2 a
2.3 c
2.4 a
2.5 b
2.6 c
2.7 a
2.8 c
2.9 a
2.10 c
2.11 a
2.12 b
2.13 b
2.14 c
2.15 a
2.16 b
2.17 A. a
B. b
C. a
2.18 c
2.19 a
2.20 A. c
B. a
C. b
2.21 a
2.22 a
2.23 b
2.24 a
2.25 a
2.26 b
2.27 a
2.28 b
2.29 a
2.30 (i) c
(ii) b
(iii) a
(iv) d
2.31 a
2.32 a

2.1 (a) $\bar{A}_1 = 5\angle 30° = 5[\cos 30° + j\sin 30°] = 4.33 + j\,2.5$

(b) $\bar{A}_2 = -3 + j4 = \sqrt{9+16}\ \angle \tan^{-1}\dfrac{4}{-3} = 5\angle 126.87° = 5e^{j126.87°}$

(c) $\bar{A}_3 = (4.33 + j2.5) + (-3 + j4) = 1.33 + j\,6.5 = 6.635\angle 78.44°$

(d) $\bar{A}_4 = (5\angle 30°)(5\angle 126.87°) = 25\angle 156.87° = -22.99 + j9.821$

(e) $\bar{A}_5 = (5\angle 30°)/(5\angle -126.87°) = 1\angle 156.87° = 1e^{j156.87°}$

2.2 (a) $\bar{I} = 400\angle -30° = 346.4 - j200$

(b) $i(t) = 5\sin(\omega t + 15°) = 5\cos(\omega t + 15° - 90°) = 5\cos(\omega t - 75°)$

$\bar{I} = \left(5/\sqrt{2}\right)\angle -75° = 3.536\angle -75° = 0.9151 - j3.415$

(c) $\bar{I} = \left(4/\sqrt{2}\right)\angle -30° + 5\angle -75° = (2.449 - j1.414) + (1.294 - j4.83)$

$= 3.743 - j\,6.244 = 7.28\angle -59.06°$

2.3 (a) $V_{\max} = 359.3\,\text{V};\ I_{\max} = 100\,\text{A}$

(b) $V = 359.3/\sqrt{2} = 254.1\text{V};\ I = 100/\sqrt{2} = 70.71\text{A}$

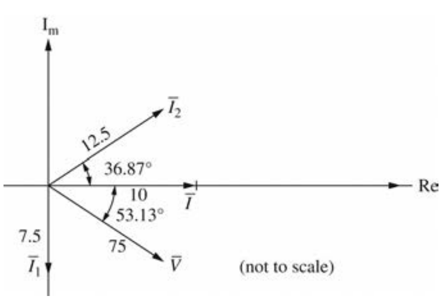
(c) $\bar{V} = 254.1\angle 15°\,\text{V};\ \bar{I} = 70.71\angle -85°\,\text{A}$

2.4 (a) $\bar{I}_1 = 10\angle 0° \dfrac{-j6}{8 + j6 - j6} = 10\dfrac{6\angle -90°}{8} = 7.5\angle -90°\,\text{A}$

$\bar{I}_2 = \bar{I} - \bar{I}_1 = 10\angle 0° - 7.3\angle -90° = 10 + j7.5 = 12.5\angle 36.87°\,\text{A}$

$\bar{V} = \bar{I}_2(-j6) = (12.5\angle 36.87°)(6\angle -90°) = 75\angle -53.13°\,\text{V}$

(b)



2.5 (a) $v(t) = 277\sqrt{2}\cos(\omega t + 30°) = 391.7\cos(\omega t + 30°)\,\text{V}$

(b) $\bar{I} = \bar{V}/20 = 13.85\angle 30°\,\text{A}$

$i(t) = 19.58\cos(\omega t + 30°)\,\text{A}$

(c) $\bar{Z} = j\omega L = j(2\pi 60)(10\times10^{-3}) = 3.771\angle 90°\ \Omega$

$\bar{I} = \bar{V}/\bar{Z} = (277\angle 30°)/(3.771\angle 90°) = 73.46\angle -60°$ A

$i(t) = 73.46\sqrt{2}\cos(\omega t - 60°) = 103.9\cos(\omega t - 60°)$ A

(d) $\bar{Z} = -j\,25\ \Omega$

$\bar{I} = \bar{V}/\bar{Z} = (277\angle 30°)/(25\angle -90°) = 11.08\angle 120°$ A

$i(t) = 11.08\sqrt{2}\cos(\omega t + 120°) = 15.67\cos(\omega t + 120°)$ A

2.6 (a) $\bar{V} = (100/\sqrt{2})\angle -30° = 70.7\angle -30°$; ω does not appear in the answer.

(b) $\upsilon(t) = 100\sqrt{2}\cos(\omega t + 20°)$; with $\omega = 377$,

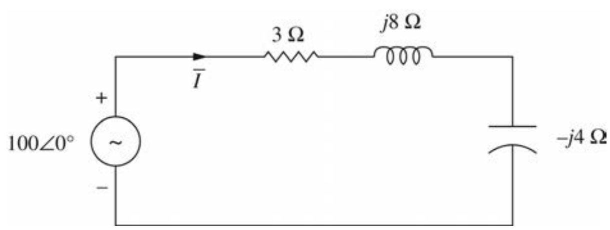
$\upsilon(t) = 141.4\cos(377t + 20°)$

(c) $\bar{A} = A\angle\alpha$; $\bar{B} = B\angle\beta$; $\bar{C} = \bar{A} + \bar{B}$

$c(t) = a(t) + b(t) = \sqrt{2}\,\mathrm{Re}\left[\bar{C}e^{j\omega t}\right]$

The resultant has the same frequency ω.

2.7 (a) The circuit diagram is shown below:



(b) $\bar{Z} = 3 + j8 - j4 = 3 + j4 = 5\angle 53.1°\ \Omega$

(c) $\bar{I} = (100\angle 0°)/(5\angle 53.1°) = 20\angle -53.1°$ A

The current lags the source voltage by 53.1°

Power Factor $= \cos 53.1° = 0.6$ Lagging

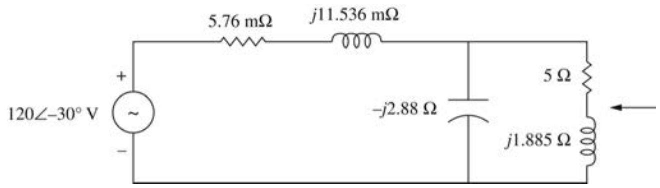
2.8 $\bar{Z}_{LT} = j(377)(30.6\times10^{-6}) = j11.536\ \mathrm{m}\Omega$

$\bar{Z}_{LL} = j(377)(5\times10^{-3}) = j1.885\ \Omega$

$$\bar{Z}_C = -j\frac{1}{(377)(921\times10^{-6})} = -j2.88\ \Omega$$

$$\bar{V} = \frac{120\sqrt{2}}{\sqrt{2}}\angle -30°\ \mathrm{V}$$

The circuit transformed to phasor domain is shown below:



2.9 $KVL: \ 120\angle 0° = (60\angle 0°)(0.1 + j0.5) + \bar{V}_{LOAD}$

$$\therefore \ \bar{V}_{LOAD} = 120\angle 0° - (60\angle 0°)(0.1 + j0.5)$$
$$= 114.1 - j30.0 = 117.9\angle -14.7° \text{ V} \ \leftarrow$$

2.10 (a) $p(t) = \upsilon(t)i(t) = [359.3\cos(\omega t + 15°)][100\cos(\omega t - 85°)]$

$$= \frac{1}{2}(359.3)(100)[\cos 100° + \cos(2\omega t - 70°)]$$
$$= -3120 + 1.797\times 10^4 \cos(2\omega t - 70°) \text{ W}$$

(b) $P = VI\cos(\delta - \beta) = (254.1)(70.71)\cos(15° + 85°)$

$= -3120$ W Absorbed

$= +3120$ W Delivered

(c) $Q = VI\sin(\delta - \beta) = (254.1)(70.71)\sin 100°$

$= 17.69$ kVAR Absorbed

(d) The phasor current $(-\bar{I}) = 70.71\angle -85° + 180° = 70.71\angle 95°$ A leaves the positive terminal of the generator.

The generator power factor is then $\cos(15° - 95°) = 0.1736$ leading

2.11 (a) $p(t) = \upsilon(t)i(t) = 391.7\times 19.58\cos^2(\omega t + 30°)$

$$= 0.7669\times 10^4 \left(\frac{1}{2}\right)[1 + \cos(2\omega t + 60°)]$$
$$= 3.834\times 10^3 + 3.834\times 10^3 \cos(2\omega t + 60°) \text{ W}$$

$P = VI\cos(\delta - \beta) = 277\times 13.85\cos 0° = 3.836 \text{ kW}$

$Q = VI\sin(\delta - \beta) = 0 \text{ VAR}$

Source Power Factor $= \cos(\delta - \beta) = \cos(30° - 30°) = 1.0$

(b) $p(t) = \upsilon(t)i(t) = 391.7\times 103.9\cos(\omega t + 30°)\cos(\omega t - 60°)$

$$= 4.07\times 10^4 \left(\frac{1}{2}\right)[\cos 90° + \cos(2\omega t - 30°)]$$
$$= 2.035\times 10^4 \cos(2\omega t - 30°) \text{ W}$$

$P = VI\cos(\delta - \beta) = 277\times 73.46\cos(30° + 60°) = 0 \text{ W}$

$Q = VI\sin(\delta-\beta) = 277\times 73.46\sin 90° = 20.35\,\text{kVAR}$

$pf = \cos(\delta-\beta) = 0$ Lagging

(c) $p(t) = \upsilon(t)i(t) = 391.7\times 15.67\cos(\omega t+30°)\cos(\omega t+120°)$

$$= 6.138\times 10^3\left(\frac{1}{2}\right)\left[\cos(-90°)+\cos(2\omega t+150°)\right] = 3.069\times 10^3\cos(2\omega t+150°)\,\text{W}$$

$P = VI\cos(\delta-\beta) = 277\times 11.08\cos(30°-120°) = 0\,\text{W}$

$Q = VI\sin(\delta-\beta) = 277\times 11.08\sin(-90°)$

$= -3.069\,\text{kVAR}$ Absorbed $= +3.069\,\text{kVAR}$ Delivered

$pf = \cos(\delta-\beta) = \cos(-90°) = 0$ Leading

2.12 (a) $p_R(t) = (359.3\cos\omega t)(35.93\cos\omega t)$

$= 6455 + 6455\cos 2\omega t$ W

(b) $p_x(t) = (359.3\cos\omega t)\left[14.37\cos(\omega t+90°)\right]$

$= 2582\cos(2\,\text{cot}+90°)$

$= -2582\sin 2\omega t$ W

(c) $P = V^2/R = \left(359.3/\sqrt{2}\right)^2/10 = 6455\,\text{W}$ Absorbed

(d) $Q = V^2/X = \left(359.3/\sqrt{2}\right)^2/25 = 2582\,\text{VARS}$ Delivered

(e) $(\beta-\delta) = \tan^{-1}(Q/P) = \tan^{-1}(2582/6455) = 21.8°$

Power factor $= \cos(\delta-\beta) = \cos(21.8°) = 0.9285$ Leading

2.13 $\bar{Z} = R - jx_c = 10 - j25 = 26.93\angle -68.2°\ \Omega$

$i(t) = (359.3/26.93)\cos(\omega t+68.2°)$

$= 13.34\cos(\omega t+68.2°)\,\text{A}$

(a) $p_R(t) = \left[13.34\cos(\omega t+68.2°)\right]\left[133.4\cos(\omega t+68.2°)\right]$

$= 889.8 + 889.8\cos\left[2(\omega t+68.2°)\right]\text{W}$

(b) $p_x(t) = \left[13.34\cos(\omega t+68.2°)\right]\left[333.5\cos(\omega t+68.2°-90°)\right]$

$= 2224\sin\left[2(\omega t+68.2°)\right]\text{W}$

(c) $P = I^2R = \left(13.34/\sqrt{2}\right)^2 10 = 889.8\,\text{W}$

(d) $Q = I^2X = \left(13.34/\sqrt{2}\right)^2 25 = 2224\,\text{VARS}$

(e) $pf = \cos\left[\tan^{-1}(Q/P)\right] = \cos\left[\tan^{-1}(2224/889.8)\right]$

$= 0.3714$ Leading

2.14 (a) $\bar{I} = 4\angle 0°\,\text{kA}$

$$\bar{V} = \bar{Z}\,\bar{I} = (2\angle -45°)(4\angle 0°) = 8\angle -45°\,\text{kV}$$

$$\upsilon(t) = 8\sqrt{2}\cos(\omega t - 45°)\,\text{kV}$$

$$p(t) = \upsilon(t)i(t) = \left[8\sqrt{2}\cos(\omega t - 45°)\right]\left[4\sqrt{2}\cos\omega t\right]$$

$$= 64\left(\frac{1}{2}\right)\left[\cos(-45°) + \cos(2\omega t - 45°)\right]$$

$$= 22.63 + 32\cos(2\omega t - 45°)\,\text{MW}$$

(b) $P = VI\cos(\delta - \beta) = 8 \times 4\cos(-45° - 0°) = 22.63\,\text{MW}$ Delivered

(c) $Q = VI\sin(\delta - \beta) = 8 \times 4\sin(-45° - 0°)$

$= -22.63\,\text{MVAR}$ Delivered $= +22.63\,\text{MVAR}$ Absorbed

(d) $pf = \cos(\delta - \beta) = \cos(-45° - 0°) = 0.707$ Leading

2.15 (a) $\bar{I} = \left[\left(4/\sqrt{2}\right)\angle 60°\right]/(2\angle 30°) = \sqrt{2}\angle 30°\,\text{A}$

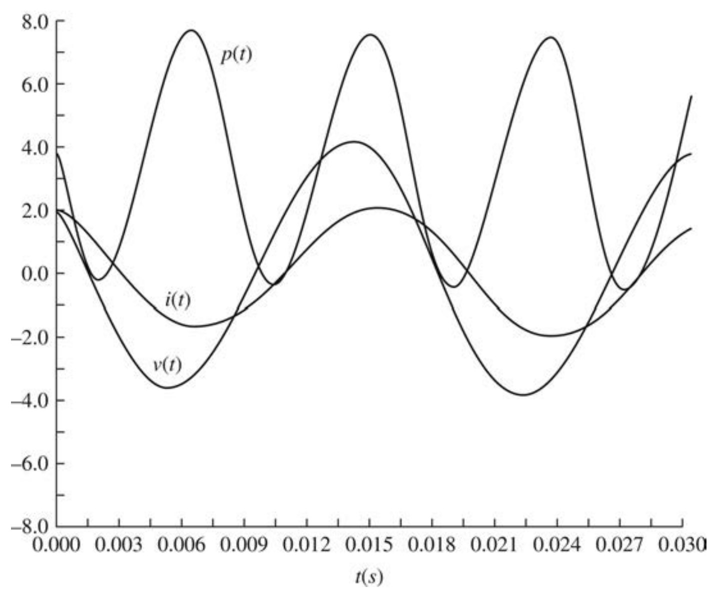
$$i(t) = 2\cos(\omega t + 30°)\,\text{A} \text{ with } \omega = 377\,\text{rad/s}$$

$$p(t) = \upsilon(t)i(t) = 4\left[\cos 30° + \cos(2\omega t + 90°)\right]$$

$$= 3.46 + 4\cos(2\omega t + 90°)\,\text{W}$$

(b) $\upsilon(t)$, $i(t)$, and $p(t)$ are plotted below: (See next page)

(c) The instantaneous power has an average value of 3.46 W, and the frequency is <u>twice</u> that of the voltage or current.



2.16 (a) $\bar{Z} = 10 + j120\pi \times 0.04 = 10 + j15.1 = 18.1\angle 56.4°\ \Omega$

$pf = \cos 56.4° = 0.553$ Lagging

(b) $\bar{V} = 120\angle 0°\ \text{V}$

The current supplied by the source is

$\bar{I} = (120\angle 0°)/(18.1\angle 56.4°) = 6.63\angle -56.4°\ \text{A}$

The real power absorbed by the load is given by

$P = 120 \times 6.63 \times \cos 56.4° = 440\ \text{W}$

which can be checked by $I^2 R = (6.63)^2\, 10 = 440\ \text{W}$

The reactive power absorbed by the load is

$Q = 120 \times 6.63 \times \sin 36.4° = 663\ \text{VAR}$

(c) Peak Magnetic Energy $= W = LI^2 = 0.04(6.63)^2 = 1.76\ \text{J}$

$Q = \omega W = 377 \times 1.76 = 663\ \text{VAR}$ is satisfied.

2.17 (a) $\bar{S} = \bar{V}\,\bar{I}^* = \bar{Z}\,\bar{I}\,\bar{I}^* = \bar{Z}\left|\bar{I}\right|^2 = j\omega L I^2$

$Q = \text{Im}[\bar{S}] = \omega L I^2 \quad \leftarrow$

(b) $\upsilon(t) = L\dfrac{di}{dt} = -\sqrt{2}\omega L\, I \sin(\omega t + \theta)$

$$p(t) = \upsilon(t) \cdot i(t) = -2\omega L\, I^2 \sin(\omega t + \theta)\cos(\omega t + \theta)$$
$$= -\omega L\, I^2 \sin 2(\omega t + \theta) \quad \leftarrow$$
$$= -Q \sin 2(\omega t + \theta) \quad \leftarrow$$

Average real power P supplied to the inductor $= 0 \quad \leftarrow$

Instantaneous power supplied (to sustain the changing energy in the magnetic field) has a maximum value of Q. $\leftarrow$

2.18 (a) $\bar{S} = \bar{V}\,\bar{I}^* = \bar{Z}\,\bar{I}\,\bar{I}^* = \text{Re}\left[\bar{Z} I^2\right] + j\,\text{Im}\left[\bar{Z} I^2\right]$

$$= P + jQ$$

$\therefore P = \bar{Z} I^2 \cos\angle\bar{Z};\ Q = \bar{Z} I^2 \sin\angle\bar{Z} \quad \leftarrow$

(b) Choosing $i(t) = \sqrt{2}\, I \cos\omega t$,

Then $\upsilon(t) = \sqrt{2}\bar{Z}\, I \cos(\omega t + \angle\bar{Z})$

$$\therefore p(t) = \upsilon(t) \cdot i(t) = \bar{Z} I^2 \cos(\omega t + \angle\bar{Z}) \cdot \cos\omega t$$
$$= \bar{Z} I^2 \left[\cos\angle\bar{Z} + \cos(2\omega t + \angle\bar{Z})\right]$$
$$= \bar{Z} I^2 \left[\cos\angle\bar{Z} + \cos 2\omega t \cos\angle\bar{Z} - \sin 2\omega t \sin\angle\bar{Z}\right]$$
$$= P(1 + \cos 2\omega t) - Q \sin 2\omega t \quad \leftarrow$$

(c) $\bar{Z} = R + j\omega L + \dfrac{1}{j\omega C}$

From part (a), $P = RI^2$ and $Q = Q_L + Q_C$

where $Q_L = \omega L I^2$ and $Q_C = -\dfrac{1}{\omega C} I^2$

which are the reactive powers into L and C, respectively.

Thus $p(t) = P\left(1+\cos 2\omega t\right) - Q_L \sin 2\omega t - Q_C \sin 2\omega t \quad \leftarrow$

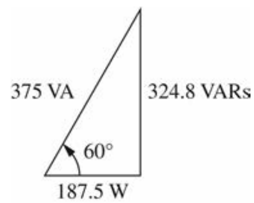
$$\left.\begin{array}{ll} \text{If } \omega^2 LC = 1, & Q_L + Q_C = Q = 0 \\ \quad\text{Then} & p(t) = P\left(1+\cos 2\omega t\right) \end{array}\right\} \leftarrow$$

2.19 (a) $\bar{S} = \bar{V}\bar{I}^* = \left(\dfrac{150}{\sqrt{2}}\angle 10°\right)\left(\dfrac{5}{\sqrt{2}}\angle -50°\right)^* = 375\angle 60°$

$= 187.5 + j324.8$

$P = \text{Re}\,\bar{S} = 187.5\,\text{W Absorbed}$

$Q = \text{Im}\,\bar{S} = 324.8\,\text{VARS Absorbed}$



(b) $pf = \cos\left(60°\right) = 0.5$ Lagging

(c) $Q_S = P\tan Q_S = 187.5\tan\left[\cos^{-1} 0.9\right] = 90.81\,\text{VARS}$

$Q_C = Q_L - Q_S = 324.8 - 90.81 = 234\,\text{VARS}$

2.20 $\bar{Y}_1 = \dfrac{1}{\bar{Z}_1} = \dfrac{1}{20\angle 30°} = 0.05\angle -30° = \left(0.0433 - j0.025\right)\text{S} = G_1 - jB_1$

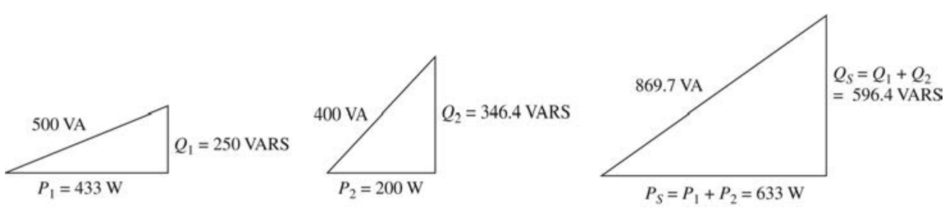
$\bar{Y}_2 = \dfrac{1}{\bar{Z}_2} = \dfrac{1}{25\angle 60°} = 0.04\angle -60° = \left(0.02 - j0.03464\right)\text{S} = G_2 + jB_2$

$P_1 = V^2 G_1 = \left(100\right)^2 0.0433 = 433\,\text{W Absorbed}$

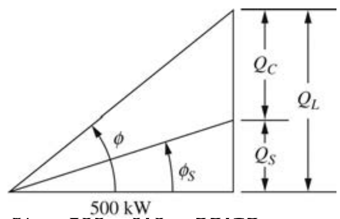
$Q_1 = V^2 B_1 = \left(100\right)^2 0.025 = 250\,\text{VARS Absorbed}$

$P_2 = V^2 G_2 = \left(100\right)^2 0.02 = 200\,\text{W Absorbed}$

$Q_2 = V^2 B_2 = \left(100\right)^2 0.03464 = 346.4\,\text{VARS Absorbed}$

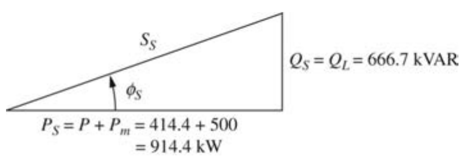


2.21 (a)



$$Q_L = P\tan\phi_L = 500\tan 53.13° = 666.7\ \text{kVAR}$$
$$\phi_S = \cos^{-1} 0.9 = 25.84°$$
$$Q_S = P\tan\phi_S = 500\tan 25.84° = 242.2\,\text{kVAR}$$
$$Q_C = Q_L - Q_S = 666.7 - 242.2 = 424.5\,\text{kVAR}$$
$$S_C = Q_C = 424.5\,\text{kVA}$$

(b) The Synchronous motor absorbs $P_m = \dfrac{(500)0.746}{0.9} = 414.4\,\text{kW}$ and $Q_m = 0\,\text{kVAR}$



Source $\text{PF} = \cos\left[\tan^{-1}(666.7/914.4)\right] = 0.808$ Lagging

2.22 (a) $\overline{Y}_1 = \dfrac{1}{\overline{Z}_1} = \dfrac{1}{(3+j4)} = \dfrac{1}{5\angle 53.13°} = 0.2\angle -53.13°$

$$= (0.12 - j0.16)\text{S}$$

$$\overline{Y}_2 = \frac{1}{\overline{Z}_2} = \frac{1}{10} = 0.1\text{S}$$

$$P = V^2(G_1 + G_2) \Rightarrow V = \sqrt{\frac{P}{G_1 + G_2}} = \sqrt{\frac{1100}{(0.12 + 0.1)}} = 70.71\ \text{V}$$

$$P_1 = V^2 G_1 = (70.71)^2\, 0.12 = 600\,\text{W}$$

$$P_2 = V^2 G_2 = (70.71)^2\, 0.1 = 500\,\text{W}$$

(b) $\overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 = (0.12 - j0.16) + 0.1 = 0.22 - j0.16$

$$= 0.272\angle -36.03°\ \text{S}$$

$$I_S = V\,Y_{eq} = 70.71(0.272) = 19.23\,\text{A}$$

2.23 $\overline{S} = \overline{V}\,\overline{I}^* = (120\angle 0°)(10\angle -30°) = 1200\angle -30°$

$$= 1039.2 - j600$$

$P = \text{Re}\,\overline{S} = 1039.2\,\text{W Delivered}$

$Q = \text{Im}\,\overline{S} = -600\,\text{VARS Delivered} = +600\,\text{VARS Absorbed}$

2.24 $\overline{S}_1 = P_1 + jQ_1 = 10 + j0;\ \overline{S}_2 = 10\angle\cos^{-1}0.9 = 9 + j4.359$

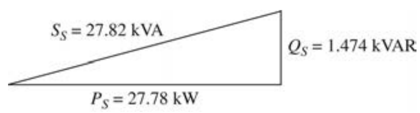
$$\overline{S}_3 = \frac{10\times 0.746}{0.85\times 0.95}\angle -\cos^{-1}0.95 = 9.238\angle -18.19° = 8.776 - j2.885$$

$\overline{S}_S = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = 27.78 + j1.474 = 27.82\angle 3.04°$

$P_S = \text{Re}(\overline{S}_S) = 27.78\,\text{kW}$

$Q_S = \text{Im}(\overline{S}_S) = 1.474\,\text{kVAR}$

$S_S = \left|\overline{S}_S\right| = 27.82\,\text{kVA}$



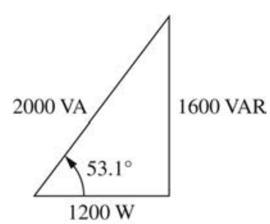
2.25 $\overline{S}_R = \overline{V}_R\overline{I}^* = R\overline{I}\,\overline{I}^* = I^2R = (20)^2 3 = 1200 + j0$

$\overline{S}_L = \overline{V}_L\overline{I}^* = (jX_L\overline{I})\overline{I}^* = jX_LI^2 = j8(20)^2 = 0 + j3200$

$\overline{S}_C = \overline{V}_C\overline{I}^* = (-j\overline{I}X_C)\overline{I}^* = -jX_CI^2 = -j4(20)^2 = 0 - j1600$

Complex power absorbed by the total load $\overline{S}_{LOAD} = \overline{S}_R + \overline{S}_L + \overline{S}_C = 2000\angle 53.1°$

Power Triangle:

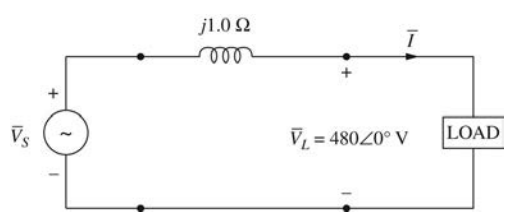


Complex power delivered by the source is

$\overline{S}_{SOURCE} = \overline{V}\,\overline{I}^* = (100\angle 0°)(20\angle -53.1°)^* = 2000\angle 53.1°$

The complex power delivered by the source is equal to the total complex power absorbed by the load.

2.26 (a) The problem is modeled as shown in figure below:



$P_L = 120\,\text{kW}$

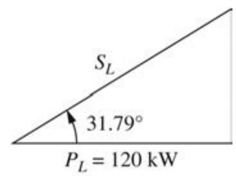
$pf_L = 0.85\,\text{Lagging}$

$\theta_L = \cos^{-1}0.85 = 31.79°$

Power triangle for the load:

$\overline{S}_L = P_L + jQ_L = 141.18\angle 31.79°\,\text{kVA}$

$I = S_L / V = 141{,}180 / 480 = 294.13\,\text{A}$



$Q_L = P_L \tan(31.79°)$

$= 74.364\,\text{kVAR}$

Real power loss in the line is zero.

Reactive power loss in the line is $Q_{LINE} = I^2 X_{LINE} = (294.13)^2\,1$

$= 86.512\,\text{kVAR}$

$\therefore \overline{S}_S = P_S + jQ_S = 120 + j(74.364 + 86.512) = 200.7\angle 53.28°\,\text{kVA}$

The input voltage is given by $V_S = S_S / I = 682.4\,\text{V (rms)}$

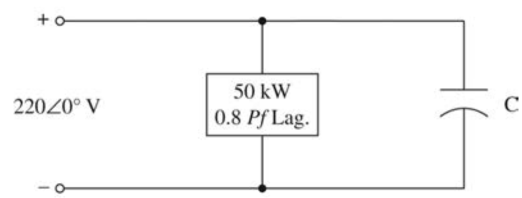
The power factor at the input is $\cos 53.28° = 0.6$ Lagging

(b) Applying KVL, $\overline{V}_S = 480\angle 0° + j1.0(294.13\angle -31.79°)$

$= 635 + j250 = 682.4\angle 21.5°\,\text{V (rms)}$

$(pf)_S = \cos(21.5° + 31.79°) = 0.6$ Lagging

2.27 The circuit diagram is shown below:



$P_{old} = 50\,\text{kW}; \cos^{-1} 0.8 = 36.87°; \theta_{OLD} = 36.87°; Q_{old} = P_{old} \tan(\theta_{old})$

$= 37.5\,\text{kVAR}$

$\therefore \overline{S}_{old} = 50{,}000 + j37{,}500$

$\theta_{new} = \cos^{-1} 0.95 = 18.19°; \overline{S}_{new} = 50{,}000 + j50{,}000 \tan(18.19°)$

$= 50{,}000 + j16{,}430$

Hence $\overline{S}_{cap} = \overline{S}_{new} - \overline{S}_{old} = -j21{,}070\,\text{VA}$

$$\therefore C = \frac{21{,}070}{(377)(220)^2} = 1155\,\mu\text{F} \quad \leftarrow$$

2.28 $\overline{S}_1 = 12 + j6.667$

$\overline{S}_2 = 4(0.96) - j4\left[\sin\left(\cos^{-1} 0.96\right)\right] = 3.84 - j1.12$

$\overline{S}_3 = 15 + j0$

$\overline{S}_{TOTAL} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = (30.84 + j5.547)\,\text{kVA}$

(i) Let $\overline{Z}$ be the impedance of a series combination of R and X

Since $\overline{S} = \overline{V}\,\overline{I}^* = \overline{V}\left(\dfrac{\overline{V}}{\overline{Z}}\right)^* = \dfrac{V^2}{\overline{Z}^*}$, it follows that

$$\overline{Z}^* = \frac{V^2}{\overline{S}} = \frac{(240)^2}{(30.84 + j5.547)10^3} = (1.809 - j0.3254)\,\Omega$$

$$\therefore \overline{Z} = (1.809 + j0.3254)\,\Omega \quad \leftarrow$$

(ii) Let $\overline{Z}$ be the impedance of a parallel combination of R and X

Then $$R = \frac{(240)^2}{(30.84)10^3} = 1.8677\,\Omega$$

$$X = \frac{(240)^2}{(5.547)10^3} = 10.3838\,\Omega$$

$$\therefore \overline{Z} = (1.8677 \| j10.3838)\,\Omega \quad \leftarrow$$

2.29 Since complex powers satisfy KCL at each bus, it follows that

$$\overline{S}_{13} = (1 + j1) - (1 - j1) - (0.4 + j0.2) = -0.4 + j1.8 \quad \leftarrow$$

$$\overline{S}_{31} = -\overline{S}_{13}^* = 0.4 + j1.8 \quad \leftarrow$$

Similarly, $\overline{S}_{23} = (0.5 + j0.5) - (1 + j1) - (-0.4 + j0.2) = -0.1 - j0.7 \quad \leftarrow$

$$\overline{S}_{32} = -\overline{S}_{23}^* = 0.1 - j0.7 \quad \leftarrow$$

At Bus 3, $\overline{S}_{G3} = \overline{S}_{31} + \overline{S}_{32} = (0.4 + j1.8) + (0.1 - j0.7) = 0.5 + j1.1 \quad \leftarrow$

2.30 (a) For load 1: $\theta_1 = \cos^{-1}(0.28) = 73.74°$ Lagging

$\overline{S}_1 = 125\angle 73.74° = 35 + j120$

$\overline{S}_2 = 10 - j40$

$\overline{S}_3 = 15 + j0$

$\overline{S}_{TOTAL} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = 60 + j80 = 100\angle 53.13°\,\text{kVA} = P + jQ$

$\therefore P_{TOTAL} = 60\,\text{kW};\ Q_{TOTAL} = 80\,\text{kVAR};\ \text{kVA}_{TOTAL} = S_{TOTAL} = 100\,\text{kVA}. \quad \leftarrow$

Supply $pf = \cos(53.13°) = 0.6$ Lagging $\leftarrow$

(b) $$\overline{I}_{TOTAL} = \frac{\overline{S}^*}{\overline{V}^*} = \frac{100\times10^3\angle -53.13°}{1000\angle 0°} = 100\angle -53.13°\,\text{A}$$

At the new *pf* of 0.8 lagging, P_{TOTAL} of 60kW results in the new reactive power Q', such that

$$\theta' = \cos^{-1}(0.8) = 36.87°$$

and $Q' = 60\tan(36.87°) = 45\,\text{kVAR}$

$\therefore$ The required capacitor's kVAR is $Q_C = 80 - 45 = 35\,\text{kVAR} \leftarrow$

It follows then $X_C = \dfrac{V^2}{\bar{S}_C^*} = \dfrac{(1000)^2}{j35000} = -j\,28.57\,\Omega$

and $$C = \frac{10^6}{2\pi(60)(28.57)} = 92.85\mu\text{F} \quad \leftarrow$$

The new current is $I' = \dfrac{\bar{S}'^*}{\bar{V}^*} = \dfrac{60{,}000 - j45{,}000}{1000\angle 0°} = 60 - j45 = 75\angle -36.87°\,\text{A}$

The supply current, in magnitude, is reduced from 100A to 75A ←

2.31 (a) $\bar{I}_{12} = \dfrac{V_1\angle\delta_1 - V_2\angle\delta_2}{X\angle 90°} = \left(\dfrac{V_1}{X}\angle\delta_1 - 90°\right) - \dfrac{V_2}{X}\angle\delta_2 - 90°$

Complex power $\bar{S}_{12} = \bar{V}_1\bar{I}_{12}^* = V_1\angle\delta_1\left[\dfrac{V_1}{X}\angle 90° - \delta_1 - \dfrac{V_2}{X}\angle 90° - \delta_2\right]$

$$= \frac{V_1^2}{X}\angle 90° - \frac{V_1V_2}{X}\angle 90° + \delta_1 - \delta_2$$

$\therefore$ The real and reactive power at the sending end are

$$P_{12} = \frac{V_1^2}{X}\cos 90° - \frac{V_1V_2}{X}\cos(90° + \delta_1 - \delta_2)$$

$$= \frac{V_1V_2}{X}\sin(\delta_1 - \delta_2) \quad \leftarrow$$

$$Q_{12} = \frac{V_1^2}{X}\sin 90° - \frac{V_1V_2}{X}\sin(90° + \delta_1 - \delta_2)$$

$$= \frac{V_1}{X}\left[V_1 - V_2\cos(\delta_1 - \delta_2)\right] \quad \leftarrow$$

Note: If $\bar{V}_1$ leads $\bar{V}_2$, $\delta = \delta_1 - \delta_2$ is positive and the real power flows from node 1 to node 2. If $\bar{V}_1$ Lags $\bar{V}_2$, δ is negative and power flows from node 2 to node 1.

(b) Maximum power transfer occurs when $\delta = 90° = \delta_1 - \delta_2 \quad \leftarrow$

$$P_{\text{MAX}} = \frac{V_1V_2}{X} \quad \leftarrow$$

2.32 4 Mvar minimizes the real power line losses, while 4.5 Mvar minimizes the MVA power flow into the feeder.