# 2

# One-Dimensional Motion

**1.** No. The path is nearly circular when viewed from the north celestial pole, indicating a two-dimensional motion.

**2.** The tracks could be used to create a motion diagram, but there are a couple limitations. The footsteps could cross, which might lead to difficulty in knowing which path is correct, and there is no information about the time between footsteps.

**3. (a)** The particle positions are:

$$\boxed{\begin{array}{l} x_A = -6\text{ m} \\ x_B = 0 \\ x_C = 4\text{ m} \end{array}}$$

**(b)** The particle positions are:

$$\boxed{\begin{array}{l} z_A = 0 \\ z_B = 6\text{ m} \\ z_C = 10\text{ m} \end{array}}$$

**4.** You can't make a position versus time plot because there is no information about the time between footsteps.

**5. (a)** The vector component is $\boxed{\vec{v} = 35.0\hat{\boldsymbol{j}}\text{ m/s}}$, the scalar component is $\boxed{v_y = 35.0\text{ m/s}}$, and the magnitude is $\boxed{v = 35.0\text{ m/s}}$.

**(b)** The vector component is $\boxed{\vec{v} = 53.0\hat{\boldsymbol{i}}\text{ m/s}}$, the scalar component is $\boxed{v_x = 53.0\text{ m/s}}$, and the magnitude is $\boxed{v = 53.0\text{ m/s}}$.

**(c)** The vector component is $\boxed{\vec{v} = -3.50\hat{\boldsymbol{k}}\text{ m/s}}$, the scalar component is $\boxed{v_z = -3.50\text{ m/s}}$, and the magnitude is $\boxed{v = 3.50\text{ m/s}}$.

**(d)** The vector component is $\boxed{\vec{v} = -5.30\hat{\boldsymbol{i}}\text{ m/s}}$, the scalar component is $\boxed{v_x = -5.30\text{ m/s}}$, and the magnitude is $\boxed{v = 53.0\text{ m/s}}$.

**6. (a)** They walk slowly away from home, quickly through the woods, and slowly past the deer. They rest on the fallen log and walk quickly through woods before resting at the ginger bread house. They run home, slowing down only once they are near home.

**(b)** In particular, the times when they are at rest are ambiguous, as we cannot determine how long they were at rest.

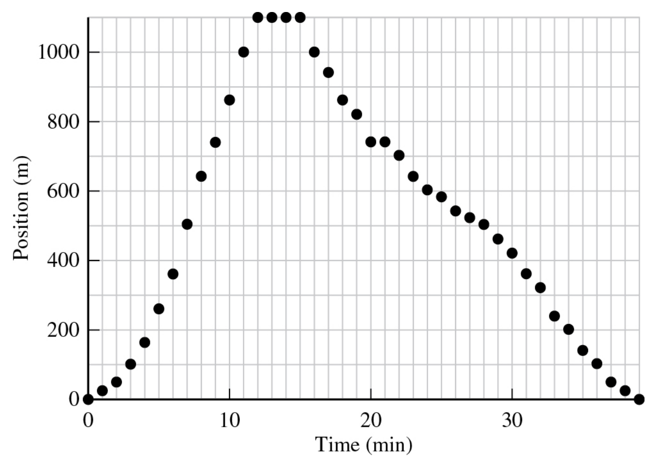



**Figure P2.6ANS**

**(c)** The plot is consistent with the description above.

**7. (a)** At time $t_1$, the slope of the graph for cadet B is less than than that for cadet A, and thus B has a lesser speed at that time.

**(b)** From just before to just after time $t_2$, the graph for cadet B is getting steeper. This means that the speed of cadet B is increasing.

**(c)** Cadet B experiences a lesser change in position during this time interval, and thus has a lower average speed. A straight line connecting $t = 0$ and $t = 60$ s on the graph for B would have a smaller slope than the graph of A.

**8. (a)** From the position versus time plot, we can determine the coordinate system used.

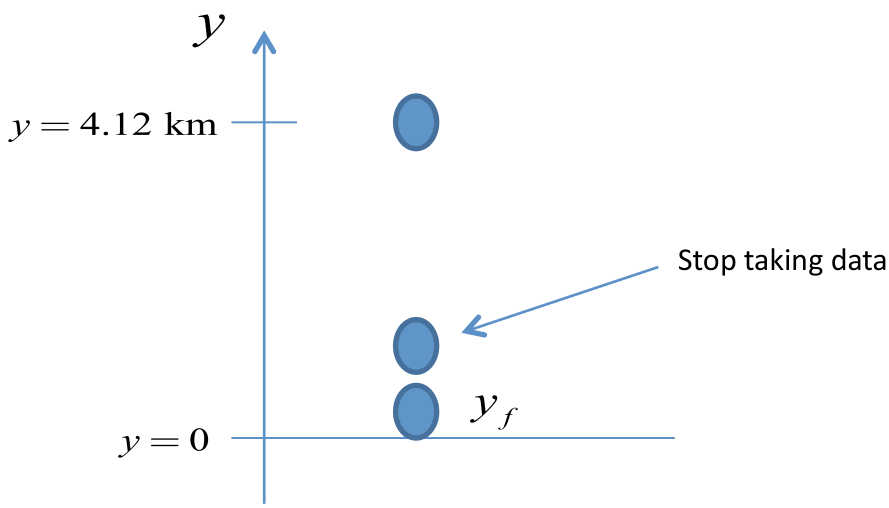



**Figure P2.8aANS**

**(b)** Her speed increases for the first few seconds, remains constant for about 55 seconds, and she finally slows down, nearly stopping, for the last 10 seconds.

**(c)** A motion diagram has points at equal time intervals. See Fig P2.8cANS.

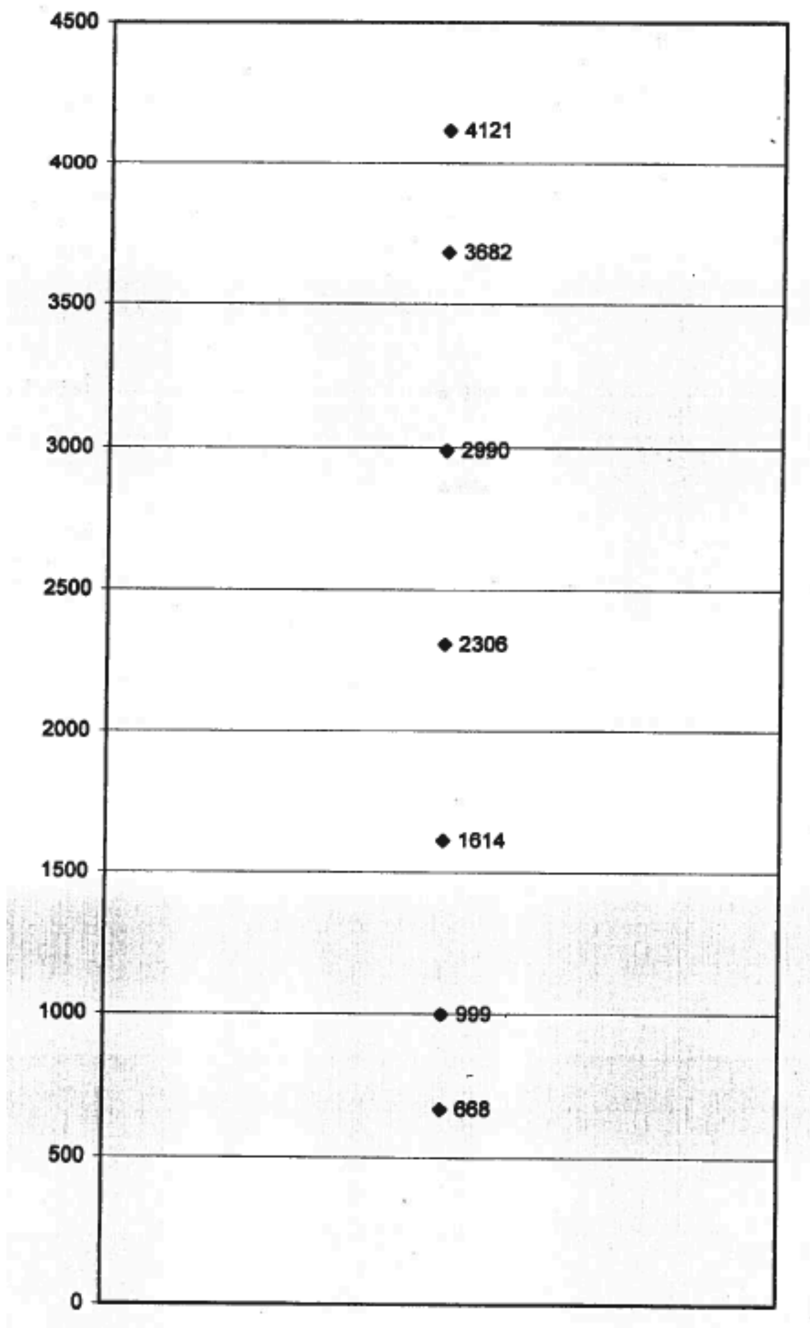



**Figure P2.8cANS**

**(d)** Yes. The first two dots are close together and the motion is slow, then there is a region of approximately equally spaced dots reflecting constant velocity, and finally a region of slower motion.

**9. (a)** A position versus time plot can be created by plotting the data in the table.

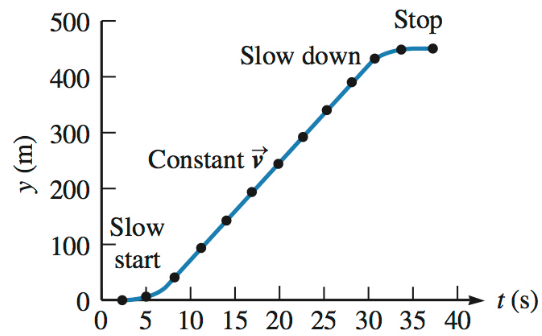



**Figure P2.9ANS**

**(b)** The elevator's speed gradually increases for the first 5 seconds, is nearly constant for about 30 seconds, and gradually decreases for the last 5 seconds.

**(c)** The highest speed occurs during the long period of constant speed indicated in the sketch. We can estimate this by selecting two points and computing the slope.

$$v_{\text{max}} = \frac{300\text{ m} - 100\text{ m}}{21\text{ s} - 9\text{ s}} = \boxed{17\ \frac{\text{m}}{\text{s}}}$$

This is around 40 mph.

**(d)** One consideration would be avoiding an uncomfortably high acceleration and deceleration at the bottom and top of the building.

**10. (a)** Yoon's origin is the same as Whipple's, but Yoon's coordinate axis points south, the opposite of Whipple's axis. Therefore, Yoon measures positions that are the opposite sign of Whipple.

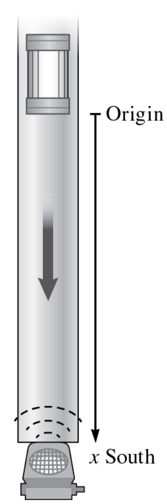



**Figure P2.10aANS**

| Label | $t$ (s) | Crall's $x$ (m) | Whipple's $x$ (m) | Yoon's $x$ (m) |
|---|---|---|---|---|
| A | 0.0 | 1.64 | 0 | 0 |
| B | 0.4 | 1.64 | 0 | 0 |
| C | 0.8 | 1.55 | –0.09 | 0.09 |
| D | 1.2 | 1.45 | –0.19 | 0.19 |
| E | 1.6 | 1.35 | –0.29 | 0.29 |
| F | 2.0 | 1.25 | –0.39 | 0.39 |
| G | 2.4 | 1.16 | –0.48 | 0.48 |
| H | 2.8 | 1.06 | –0.58 | 0.58 |
| I | 3.2 | 0.97 | –0.67 | 0.67 |
| J | 3.6 | 0.87 | –0.77 | 0.77 |
| K | 4.0 | 0.78 | –0.86 | 0.86 |
| L | 4.4 | 0.69 | –0.95 | 0.95 |
| M | 4.8 | 0.60 | –1.04 | 1.04 |
| N | 5.2 | 0.51 | –1.13 | 1.13 |
| O | 5.6 | 0.42 | –1.22 | 1.22 |
| P | 6.0 | 0.33 | –1.31 | 1.31 |

**(b)** The data from part (a) can now be plotted to create a position versus time graph.

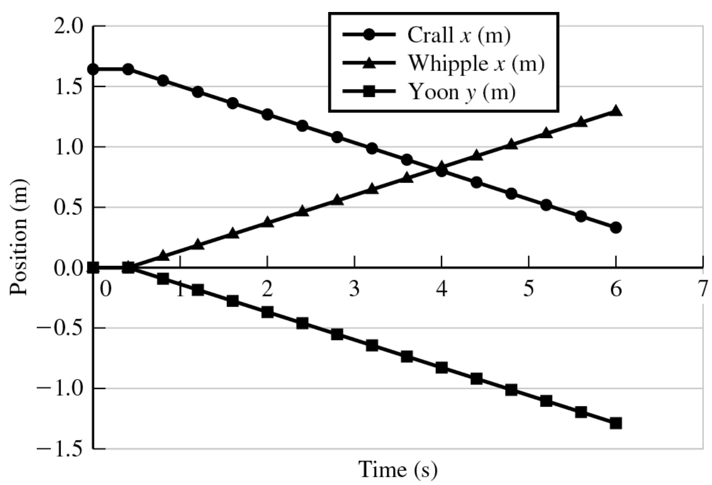



**Figure P2.10bANS**

**11.** It's *never* less. If the particle travels along a straight line always in the same direction, then they're equal. If the particle reverses direction, then the distance it travels is greater than the magnitude of its displacement.

**12. (a)** $\Delta\vec{x} = \vec{x}_C - \vec{x}_A = \left[4\hat{i} - (-6\hat{i})\right]\text{m} = \boxed{10\hat{i}\text{ m}}$

**(b)** $\Delta\vec{z} = \vec{z}_C - \vec{z}_A = \left[10\hat{i} - (0)\right]\text{m} = \boxed{10\hat{i}\text{ m}}$

**(c)** Yes. The displacement is independent of the coordinate system.

**13.** The number of times the car went around the track is the total distance traveled divided by the circumference of the track.

$$N = \frac{d}{2\pi r} = \frac{825\text{ km}}{(2\pi)(1.313\text{ km})} = 1.00\times10^{2}$$

The car makes $\boxed{100}$ loops around the track.

**14. (a)**

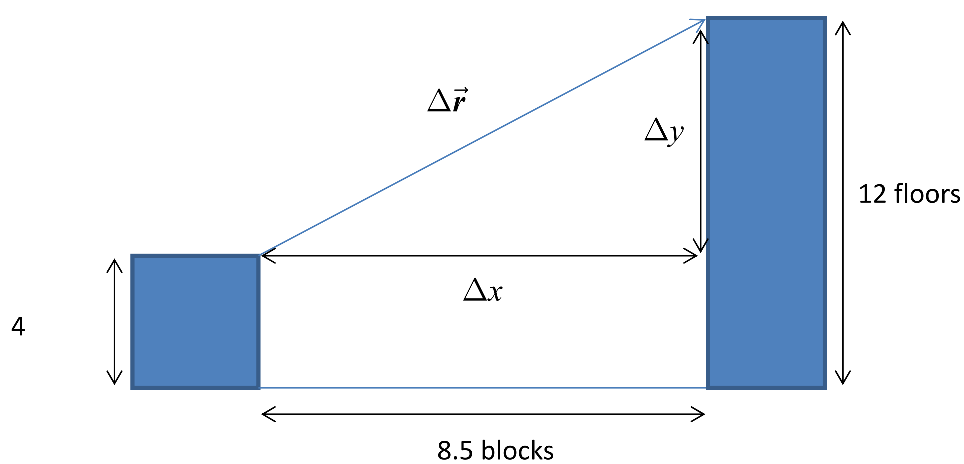



**Figure P2.14ANS**

**(b)** Traveling down to the ground: $4\times4\text{ m} = 16\text{ m}$
Walking on the street: $8.5\times146.6\text{ m} = 1246\text{ m}$
Climbing the stairs in the office building: $12\times5.5\text{ m} = 66\text{ m}$
The total distance is the sum, approximately $\boxed{1328\text{ m}}$.

**(c)** Her displacement is

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$
$$\Delta r = \sqrt{(1246\text{ m})^2 + (66\text{ m} - 16\text{ m})^2} = \boxed{1247\text{ m}}$$

Since the distance along the street is much larger than the vertical displacement, the total displacement is nearly equal to the distance traveled on the street.

**15. (a)** We first convert those magnitudes from miles into meters and then add the magnitudes.

$$d_1 = 293 \text{ miles} \times \left( \frac{1609 \text{ m}}{1 \text{ mile}} \right) = 4.71437 \times 10^5 \text{ m}$$

$$d_2 = 107 \text{ miles} \times \left( \frac{1609 \text{ m}}{1 \text{ mile}} \right) = 1.72163 \times 10^5 \text{ m}$$

$$d_{\text{tot}} = d_1 + d_2 = \boxed{6.44 \times 10^5 \text{ m}}$$

**(b)** The displacement depends only on the initial and final positions of the train. The total displacement is in the positive $x$ direction.

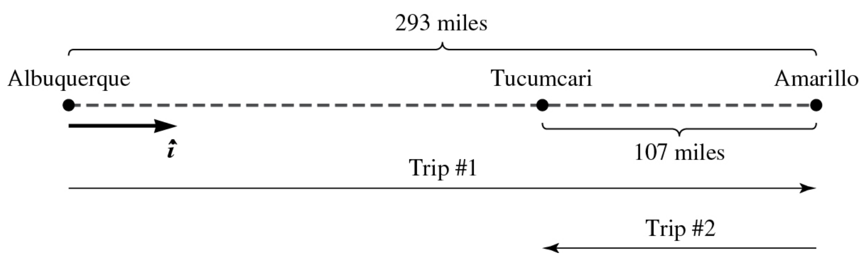



**Figure P2.15ANS**

$$\Delta x = d_1 - d_2 = 2.99 \times 10^5 \text{ m}$$

$$\Delta \vec{\boldsymbol{x}} = \boxed{2.99 \times 10^5 \hat{\boldsymbol{\imath}} \text{ m}}$$

**16.** The distance is approximately $\boxed{\text{120 miles or 200,000 m East}}$.

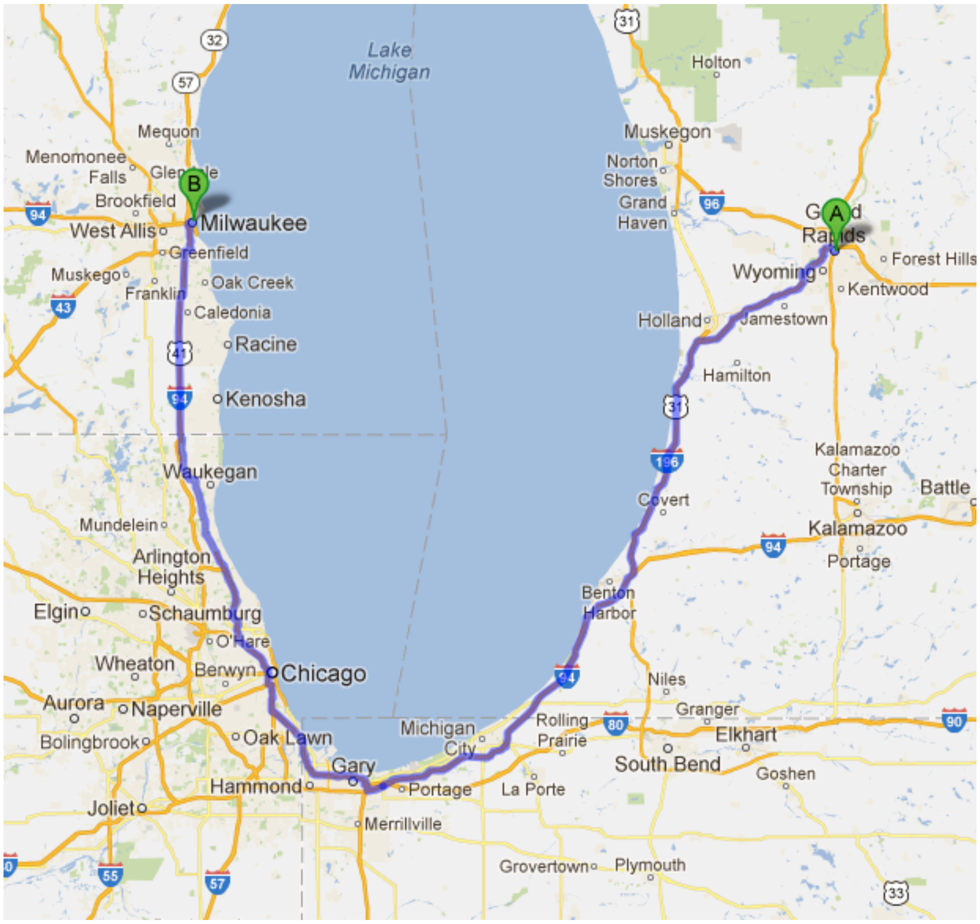



**Figure P2.16ANS**

**17. (a)** We can substitute $t = 0$ into the position equation given.

$$\vec{y}(0) = \left(y_0 \cos \omega t\right)\hat{\boldsymbol{j}} = \left(14.5 \text{ cm} \cos (0)\right)\hat{\boldsymbol{j}} = \boxed{14.5\hat{\boldsymbol{j}} \text{ cm}}$$

**(b)** We can now substitute $t = 9.0$ s into the position equation.

$$\vec{y}(9.0) = \left(y_0 \cos \omega t\right)\hat{\boldsymbol{j}} = \left(14.5 \text{ cm} \cos\left(18.85\frac{\text{rad}}{\text{s}} \times 9.0 \text{ s}\right)\right)\hat{\boldsymbol{j}} = \boxed{14.5\hat{\boldsymbol{j}} \text{ cm}}$$

**18. (a)** The $y$ axis points upwards and at $t = 0$ s, the particle is at $+y_0$.

**(b)**

$$\vec{y}(t=0) = \left(y_0 \cos 0\right)\hat{\boldsymbol{j}} = y_0\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{y}}(t=\frac{T}{2})=\left(y_0\ \cos\frac{\pi T}{T}\right)\hat{\boldsymbol{j}}=-y_0\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{y}}(t=T)=\left(y_0\ \cos\frac{2\pi T}{T}\right)\hat{\boldsymbol{j}}=y_0\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{y}}(t=\frac{3T}{2})=\left(y_0\ \cos\frac{3\pi T}{T}\right)\hat{\boldsymbol{j}}=-y_0\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{y}}(t=2T)=\left(y_0\ \cos\frac{4\pi T}{T}\right)\hat{\boldsymbol{j}}=y_0\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{y}}(t=\frac{5T}{2})=\left(y_0\ \cos\frac{5\pi T}{T}\right)\hat{\boldsymbol{j}}=-y_0\hat{\boldsymbol{j}}$$

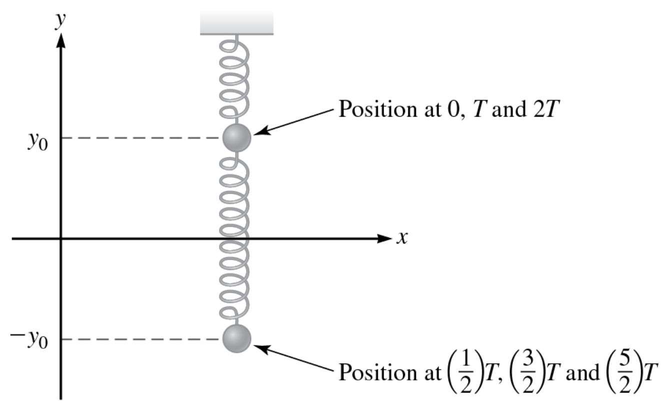



**Figure P2.18ANS**

**19. (a)** $\Delta\vec{y}=\vec{y}_{9s}-\vec{y}_{0s}=\boxed{0}$. The particle is at the same position at both times, therefore the displacement is zero.

**(b)** Each time the argument of the cosine term changes by $2\pi$, the particle has oscillated and returned to its starting position, covering a total distance of 58 cm (from +14.5 cm to –14.5 cm and back). The number of oscillations is the total number of radians traversed divided by $2\pi$ per oscillation:

$$\frac{18.85\ \text{rad/s}\times 9.0\ \text{s}}{2\pi}=27$$

The total distance traveled is $27\times 0.58\ \text{m}=\boxed{15.7\ \text{m}}$. This is the total path length, which increases as the particle moves, while the displacement depends only on the initial and final positions.

**(c)** Atoms in a solid may be modeled by particles attached by springs.

**20.** There was a moment when the particle was at rest. If the average speed is greater than the magnitude of the average velocity it means that the particle must have turned around at some point during its movement. A particle cannot reverse direction instantaneously without first coming to rest. If the particle travels along a straight line always in the same direction, then the average speed and magnitude of the average velocity are equal.

**21. (a)** The average velocity is found by dividing the distance by time. The direction of the velocity is north, along the positive $y$ direction.

$$\vec{v}_{\text{av},\,y} = \frac{\Delta\vec{y}}{\Delta t} = \frac{2.0\times10^{2}\,\hat{j}\text{ m}}{22.23\text{ s}} = \boxed{9.0\,\hat{j}\text{ m/s}}$$

**(b)** In this case, you finish at the starting position, which means that there is no displacement for the entire race. The average velocity for the race is 0 m/s.

$$\vec{v}_{\text{av},\,y} = \frac{\Delta\vec{y}}{\Delta t} = \frac{0\,\hat{j}\text{ m}}{46.38\text{ s}} = \boxed{0}$$

**22.** The average speed is always greater than or equal to the magnitude of the average velocity. If the particle travels along a straight line always in the same direction, then they're equal. If the particle reverses direction, then its average speed is greater than the magnitude of its average velocity.

**23. (a)** Let's arbitrarily assume the photon is traveling in the $x$ direction from a light that is approximately 5 meters from you.

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{c} = \frac{5\text{ m}}{3\times10^{8}\text{ m/s}} = 1.7\times10^{-8}\text{ s}$$

This is around 20 ns.

**(b)**

$$\Delta t = \frac{\Delta x}{c} = \frac{1.5\times10^{11}\text{ m}}{3\times10^{8}\text{ m/s}} = \boxed{500\text{ s}}$$

This is approximately 8.3 minutes.

**(c)** The photon travels so quickly that we would need to observe it travel a large distance to be able to easily measure the elapsed time.

**24. (a)** The time equals the distance divided by the speed of the photon.

$$\Delta t = \frac{\Delta x}{c} = \frac{8.18\times 10^{16}\ \text{m}}{3.00\times 10^{8}\ \text{m/s}} = 2.73\times 10^{8}\ \text{s}$$

Using the fact that there are $3.16 \times 10^7$ seconds in a year,

$$\Delta t = \frac{2.73\times 10^{8}\ \text{s}}{3.16\times 10^{7}\ \text{s/year}} = \boxed{8.64\ \text{years}}$$

**(b)** A light year is the distance light travels in one year.

$$\Delta x = c\Delta t = \left(3.00\times 10^{8}\ \text{m/s}\right)\left(3.16\times 10^{7}\ \text{s}\right) = 9.48\times 10^{15}\text{m}$$

Dividing the distance to Sirius by this value results in a distance of $\boxed{8.64\ \text{ly}}$.

**25. (a)** The distance the sound travels in 8 seconds can be calculated using the speed of sound.

$$\Delta x = v\Delta t = \left(343\ \text{m/s}\right)\left(8\ \text{s}\right) = 2744\ \text{m} \approx \boxed{3\times 10^{3}\ \text{m}}$$

**(b)** The speed of light can be used to determine the time for light to travel the same distance.

$$\Delta t = \frac{\Delta x}{c} = \frac{\left(343\ \text{m/s}\right)\left(8\ \text{s}\right)}{3.00\times 10^{8}\ \text{m/s}} = \boxed{9\times 10^{-6}\ \text{s}}$$

Yes, we can neglect this time as it is around $10^6$ times shorter than the time for sound to travel.

**(c)** Since the time is only known to one significant figure, knowing the speed of sound to three significant figures is unnecessary.

**26. (a)**

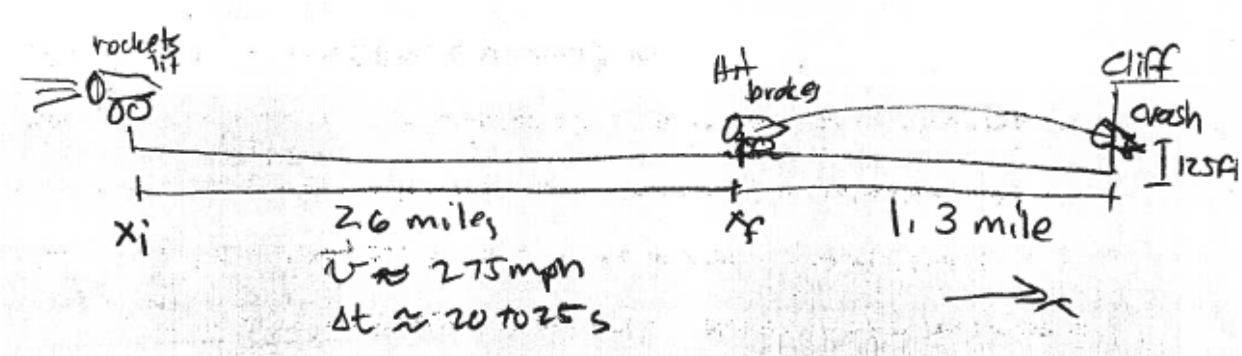



**Figure P2.26ANS**

**(b)**

$$\vec{x}_i = \boxed{0}$$
$$\vec{x}_f \approx 2.6\hat{\boldsymbol{i}}\ \text{mi} \approx \boxed{4.2\hat{\boldsymbol{i}}\ \text{km}}$$
$$\vec{v} = 275\hat{\boldsymbol{i}}\ \text{mi/h} \approx \boxed{123\hat{\boldsymbol{i}}\ \text{m/s}}$$
$$\Delta t \approx \boxed{25\ \text{s}}$$

A time of 25 seconds corresponds to the maximum time the vehicle was reportedly traveling on the ground.

**(c)** $\Delta\vec{x} = \vec{v}\Delta t = \left(123\hat{\boldsymbol{i}}\ \text{m/s}\right)\left(25\ \text{s}\right) = \boxed{3.1\times10^{3}\hat{\boldsymbol{i}}\ \text{m}}$

This displacement is smaller than the distance the vehicle reportedly covered (which was over 4 km). Therefore, they are not consistent.

**(d)** One possibility is that the actual time was larger than 25 seconds. A time of 34 seconds would be consistent with the other facts and it seems reasonable that no one would know exactly how many seconds the vehicle was driving.

**27.** We can convert 519 kilometers to inches and use $d = v\Delta t$ and solve for $\Delta t$.

$$519\ \text{km}\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)\left(\frac{1\ \text{in.}}{2.54\ \text{cm}}\right) = 2.04\times10^{7}\ \text{in.}$$

$$\Delta t = \frac{d}{v} = \frac{2.04\times10^{7}\ \text{in.}}{4.0\ \text{in./yr}} = \boxed{5.1\times10^{6}\ \text{years}}$$

**28. (a)** The distance between a person's ears is around 20 cm, or 0.2 m.

$$\Delta t_{\text{max}} = \frac{d}{v} = \frac{0.2 \text{ m}}{343 \text{ m/s}} \approx 6\times10^{-4} \text{ s} = \boxed{0.6 \text{ ms}}$$

**(b)** The higher sound speed in water leads to a smaller time for sound to travel from one ear to the other.

$$\Delta t_{\text{max}} = \frac{d}{v} = \frac{0.2 \text{ m}}{1531 \text{ m/s}} \approx 1.3\times10^{-4} \text{ s} = \boxed{0.13 \text{ ms}}$$

**(c)** Since the time difference is about five times larger in air, it is easier for your brain to distinguish the time difference in air. In water, the time difference would be much smaller and you would perceive the sound as being nearly in front or behind you.

**29. (a)** $\vec{\bar{v}}_1 = \frac{(\Delta x)_1}{(\Delta t)_1}\hat{\boldsymbol{j}} = \boxed{+\frac{L}{t_1}\hat{\boldsymbol{j}}}$

**(b)** $\vec{\bar{v}}_2 = \frac{(\Delta x)_2}{(\Delta t)_2}\hat{\boldsymbol{j}} = \boxed{-\frac{L}{t_2}\hat{\boldsymbol{j}}}$

**(c)** The total displacement is zero, therefore the average velocity is zero.

**(d)** The average speed is the total distance traveled divided by the total time.

$$\bar{v}_{\text{trip}} = \frac{|(\Delta x)_1| + |(\Delta x)_2|}{t_1 + t_2} = \boxed{\frac{2L}{t_1 + t_2}}$$

**30.** Set the derivative of the speed equal to zero and solve for $t$ to find the time that results in the maximum value of speed. Then substitute this time into the speed function.

$$\frac{dv(t)}{dt} = a\left[e^{-5t} - 5te^{-5t}\right] = 0$$

$$1 - 5t = 0$$

$$t = 0.2s$$

$v(t) = ate^{-5t} = a(0.2s)e^{-5(0.2s)} = \boxed{0.0736a}$ This speed has units of m/s.

**31. (a)** The particle is moving in the negative $y$ direction and speeding up.

**(b)** $\vec{v}_y(0) = \boxed{0 \text{ m/s}}$

$$\vec{v}_y(10.0s) = -0.758\hat{\boldsymbol{j}}\ \frac{\text{m}}{\text{s}^2} \times 10.0\text{ s} = \boxed{-7.58\hat{\boldsymbol{j}}\text{ m/s}}$$

$$\vec{v}_y(5.00\,\text{min}) = -0.758\hat{\boldsymbol{j}}\ \frac{\text{m}}{\text{s}^2} \times 300\text{ s} = \boxed{-2.27\times10^2\text{ m/s}}$$

**(c)** The speed is the magnitude of the velocity; therefore the corresponding speeds are $\boxed{0 \text{ m/s, } 7.58 \text{ m/s, and } 2.27\times10^2 \text{ m/s}}$.

**32.** The slope of a position-versus-time graph at any point in time is equal to the instantaneous velocity of the object in motion. The slope of the position-versus-time graph must start off positive, decrease until it reaches 0 m/s, and then become increasingly negative as it travels in the negative *x* direction.

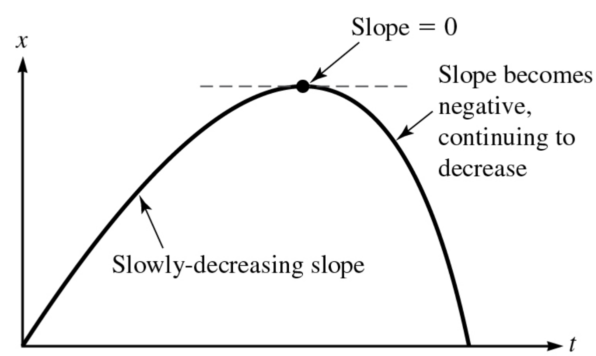



**Figure P2.32ANS**

**33. (a)** At $t = 1.00$ s, $y = 7.5$ m (point *A*), and at $t = 3.50$ s, $y = 18$ m (point *B*).

$$\vec{v}_{\text{avg}} = \frac{y_f - y_i}{t_f - t_i} = \frac{18\hat{\boldsymbol{j}}\text{ m} - 7.5\hat{\boldsymbol{j}}\text{ m}}{3.50\text{ s} - 1.00\text{ s}} = \boxed{4.2\hat{\boldsymbol{j}}\text{ m/s}}$$

**(b)** The slope of the tangent line can be found from points *C* and *D*:

$t_C = 0$, $y_C = 2.0$ m, and $t_D = 3.00$ s, $y_S = 20$ m

$$\vec{v} \approx \frac{y_D - y_C}{t_D - t_C} = \frac{20\hat{\boldsymbol{j}}\text{ m} - 2.0\hat{\boldsymbol{j}}\text{ m}}{3.00\text{ s} - 0.0\text{ s}} = \boxed{6.0\hat{\boldsymbol{j}}\text{ m/s}}$$

**(c)** The velocity is zero when the slope of the tangent line is zero. This occurs at $\boxed{t \approx 3.7\text{s}}$.

**34. (a)** The velocity is the derivative of the position with respect to time.

$$v_z(t) = \frac{dz}{dt} = \boxed{-\left(15.0\,\text{m/s}^2\right)t}$$

**(b)** The particle is speeding up. The speed is increasing linearly in time.

**(c)** At a time of 6.50 minutes, or 390 seconds,

$$z(390\text{ s}) = -\left(7.50\,\text{m/s}^2\right)(390\text{ s})^2 = \boxed{1.14\times10^6\text{ m}}$$

$$v_z(390\text{ s}) = -\left(15.0\,\text{m/s}^2\right)(390\text{ s}) = \boxed{-5.85\times10^3\text{ m/s}}$$

The speed is the magnitude of the velocity, $\boxed{5.85\times10^3\text{m/s}}$.

**35. (a)**

$$\vec{\boldsymbol{v}}(1.5\text{ s}) = -\left(15.0\hat{\boldsymbol{k}}\,\text{m/s}^2\right)(1.5\text{ s}) = \boxed{-22.5\hat{\boldsymbol{k}}\text{ m/s}}$$

$$\vec{\boldsymbol{v}}(3.5\text{ s}) = -\left(15.0\hat{\boldsymbol{k}}\,\text{m/s}^2\right)(3.5\text{ s}) = \boxed{-52.5\hat{\boldsymbol{k}}\text{ m/s}}$$

**(b)** Since the speed is increasing linearly, the average velocity during the interval is simply the average of the velocities at the start and end of the interval. (This can be confirmed by instead calculating the position of the particle at 1.5 s and 3.5 s and calculating the change in distance over time.)

$$\overline{\boldsymbol{v}} = \frac{\left(-22.5\hat{\boldsymbol{k}}\text{ m/s}\right)+\left(-52.5\hat{\boldsymbol{k}}\text{ m/s}\right)}{2} = \boxed{-37.5\hat{\boldsymbol{k}}\text{ m/s}}$$

**36. (a)** Their average velocities would always be equal since two sprinters run the same displacement during the same amount of time.

**(b)** No. One sprinter might go faster during one part of the race and slower during the other part.

**(c)** No. The final velocity is just the instantaneous velocity at the moment a sprinter crosses the finish line, which can vary between the two sprinters. Only the average velocity for the entire race must be the same.

**37.** Choosing the upward direction as positive, we write the acceleration as the change in velocity divided by the total time interval:

$$a = \frac{v_f - v_i}{\Delta t} = \frac{20.0\text{ m/s} - (-33.0\text{ m/s})}{4.00\times10^{-3}\text{ s}} = \boxed{1.33\times10^{4}\text{ m/s}^2}$$

**38. (i):** Since the velocity is downward, and the student is speeding up, the acceleration must be downward as well. This is due to Earth's gravitational pull.

**(ii):** The velocity is still downward, but the speed is now decreasing. The acceleration must be upward. This is mainly due to the upward pull of the taut bungee cord.

**(iii):** At the low point, the acceleration is upward. Just before the low point the velocity was downward; just after, it is upward.

**(iv):** Now the student has an upward velocity that is increasing in magnitude. The acceleration is thus upward as well.

**(v):** The acceleration now opposes the velocity, and must point downward.

**39. (a)** No, the advice is bad. While it is true that the velocity of an object is zero when the slope of a position versus time graph is equal to 0, the slope of the velocity versus time graph is not necessarily zero at the same time. An object might have zero velocity and non-zero acceleration, for instance at the highest point.

**(b)** For instance, consider an object initially moving upwards under constant acceleration due to gravity. Eventually, the object would reach the peak height, where the velocity is instantaneously zero, and then begin to fall, gaining an increasing downward velocity. The acceleration is always 9.81 m/s$^2$ downward.

**40. (a)**

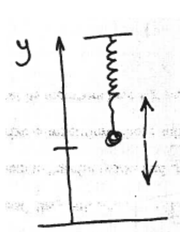



**Figure P2.40aANS**

**(b)** The given function for position can be plotted versus time.

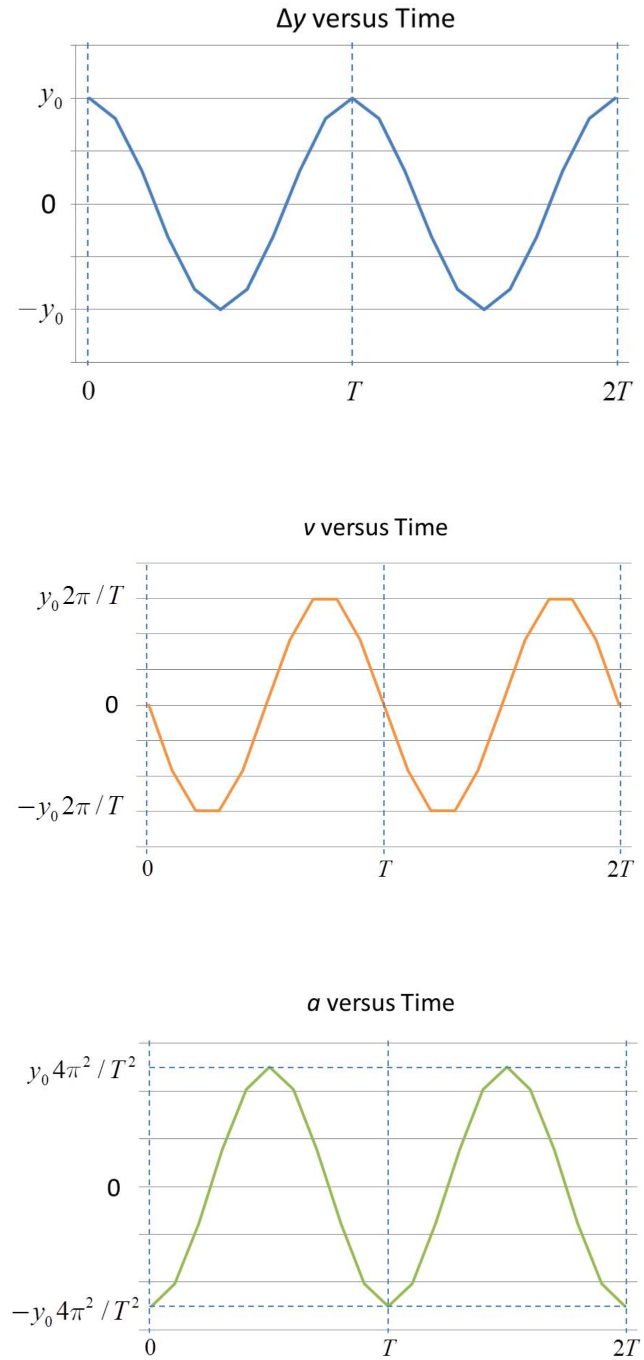



**Figure P2.40bANS**

**(c)** The velocity is calculated as the derivative of the position.

$$\vec{v} = \frac{d\vec{y}}{dt} = \left(-y_0 \frac{2\pi}{T} \sin \frac{2\pi t}{T}\right)\hat{j}$$

**(d)** The acceleration is the derivative of the velocity from part (c).

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-y_0 \frac{4\pi^2}{T^2}\cos\frac{2\pi t}{T}\right)\hat{j}$$

**(e)** The speed is maximum when $\sin\frac{2\pi t}{T} = 1$, which occurs at $t = \frac{\pi}{4}T$ and $t = \frac{3\pi}{4}T$. This occurs when $y = 0$, as the spring passes through the equilibrium position.

**(f)** The acceleration is maximum when $\cos\frac{2\pi t}{T} = 1$, which occurs at $t = 0$, $t = \frac{T}{2}$, and $t = T$. This occurs when $y = \pm y_0$, as the spring is furthest away from the equilibrium point.

**41.** The acceleration is constant and in the positive $x$ direction. The scalar component of the horizontal position can be written as $x(t) = ct^2 + dt + e$. The acceleration is the second derivative of this function $a(t) = c$, where $c$ will be a positive number given the shape of the curve.

**42. (a)** The cart leaves Crall's hand with a positive velocity, has zero velocity instantaneously at its highest point, and then has negative velocity on the way down. The acceleration is nearly constant for the cart on the incline. The velocity versus time graph will be a straight line.

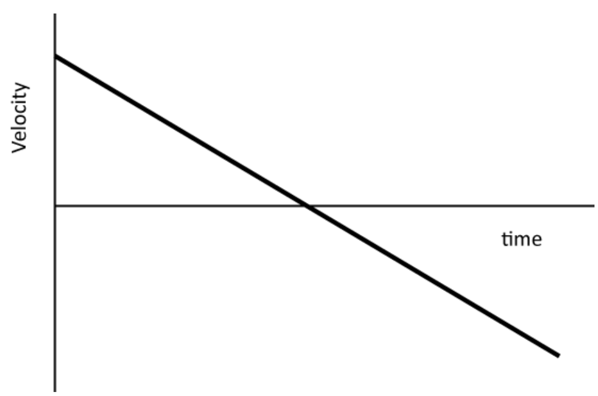



**Figure P2.42aANS**

**(b)** The acceleration is negative for the entire trajectory, analogous to an object in free fall.

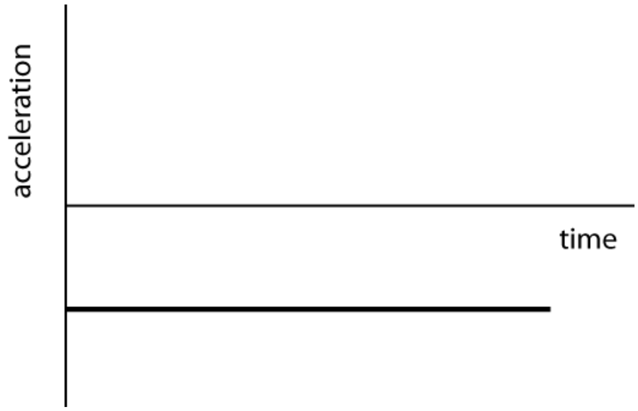



**Figure P2.42bANS**

**43. (a)** The object is slowing down; therefore the acceleration must be opposite the velocity.

**(b)** The acceleration is found by taking the derivative of the velocity function. The velocity function can be substituted into the result to find the desired form for the acceleration.

$$a_x(t) = \left|\frac{dv_x}{dt}\right| = \frac{bv_0}{\left(bt + C\right)^2} = \frac{bv_x^2}{v_0}$$

**(c)** No. The acceleration varies according to the equation in part (b), decreasing in time.

**(d)** A motor must be used to produce a force that cancels the drag force from the water, thus producing zero acceleration and constant velocity.

**44. (a)**

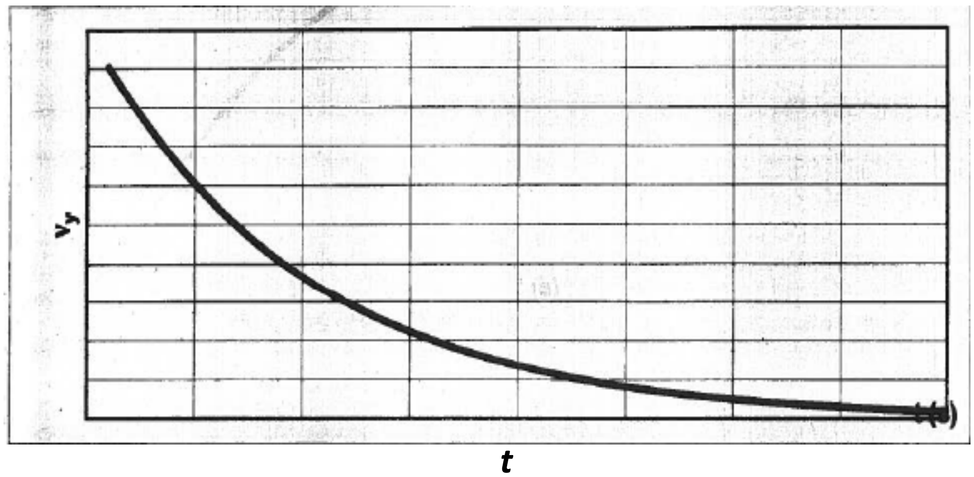



Figure P2.44aANS

**(b)** The acceleration is in the opposite direction of the velocity, or in the negative $x$ direction.

**(c)** The acceleration is found from the derivative of the velocity.

$$a_x(t) = \frac{dv_x}{dt} = \boxed{-bv_{x0}e^{-bt}}$$

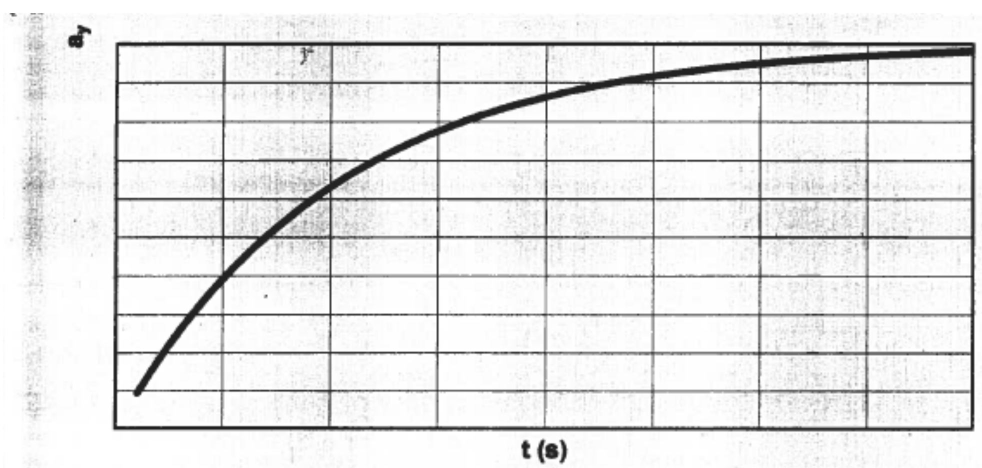



Figure P2.44cANS

**(d)** Using $v_x(t) = v_{x0}e^{-bt}$, $a_x(t) = -bv_x$.

**45. (a)** The time derivative of the position is the velocity and the time derivative of the velocity is the acceleration.

$$\vec{\boldsymbol{v}}_z(t) = \frac{d\vec{z}}{dt} = \left[(246.3\ \text{m}\cdot\text{s})\left(\frac{1}{t+2.0\ \text{s}}\right)^2\right]\hat{\boldsymbol{k}}$$

$$\vec{\boldsymbol{a}}_z(t) = \frac{d\vec{\boldsymbol{v}}_z}{dt} = \boxed{\left[-(492.6\ \text{m}\cdot\text{s})\left(\frac{1}{t+2.0\ \text{s}}\right)^3\right]\hat{\boldsymbol{k}}}$$

**(b)** The train is slowing down. To see this substitute a couple of particular times such as $t$ = 0 and 1 s. As stated in the problem, $t \geq 0$, so the velocity points in the positive z direction and the acceleration is in the negative $z$ direction. Therefore, the train is slowing down.

**(c)** No, the train does not turn around and move in the opposite direction. It is always moving forward, though its speed becomes vanishingly small as time goes on.

**46. (a)** The velocity can be found by taking the time derivative of the position. The acceleration is equal to the derivative of the velocity with respect to time.

$$\vec{\boldsymbol{v}}(t) = \frac{d\vec{y}}{dt} = \frac{2}{3}\left(R_\oplus^{3/2} + 3\sqrt{\frac{g}{2}}R_\oplus t\right)^{-1/3}\left(3\sqrt{\frac{g}{2}}R_\oplus\right)\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{v}}(t) = \frac{d\vec{y}}{dt} = \boxed{2\sqrt{\frac{g}{2}}R_\oplus\left(R_\oplus^{3/2} + 3\sqrt{\frac{g}{2}}R_\oplus t\right)^{-1/3}\hat{\boldsymbol{j}}}$$

$$\vec{\boldsymbol{a}}(t) = \frac{d\vec{\boldsymbol{v}}}{dt} = -\frac{1}{3}\left(2\sqrt{\frac{g}{2}}R_\oplus\right)\left(R_\oplus^{3/2} + 3\sqrt{\frac{g}{2}}R_\oplus t\right)^{-4/3}\left(3\sqrt{\frac{g}{2}}R_\oplus\right)\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{a}}(t) = \boxed{-\left(gR_\oplus^2\right)\left(R_\oplus^{3/2} + 3\sqrt{\frac{g}{2}}R_\oplus t\right)^{-4/3}\hat{\boldsymbol{j}}}$$

**(b)** The functions from part (a) can now be plotted.

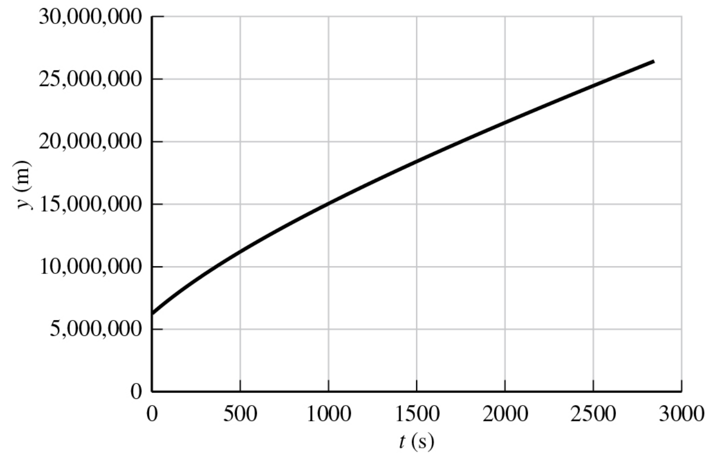



**Figure P2.46ANS (Graph 1)**

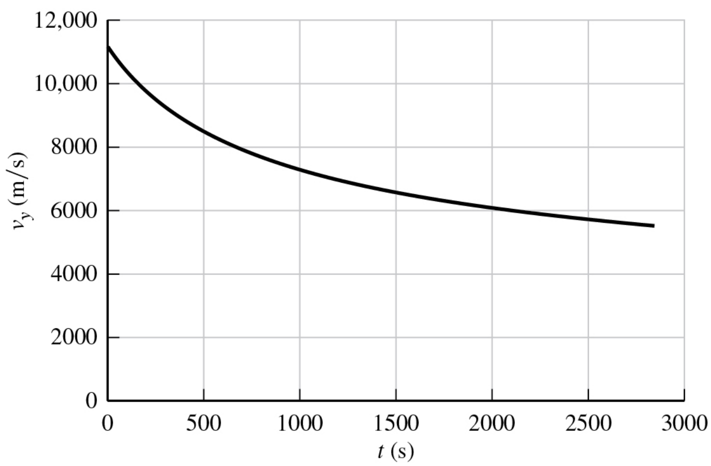



**Figure P2.46ANS (Graph 2)**

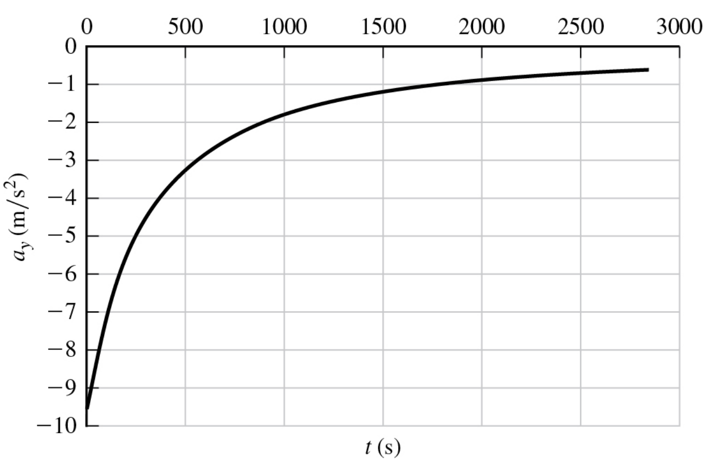



**Figure P2.46ANS (Graph 3)**

**(c)** The radius of the Earth is about 6400 km. From the graph, the rocket reaches 25,000 km around 2600 s. We can calculate this precisely as well and confirm this result.

$$4R_\oplus = \left(R_\oplus^{3/2} + 3\sqrt{\frac{g}{2}}R_\oplus t\right)^{2/3}$$

$$\left(4R_\oplus\right)^{3/2} = R_\oplus^{3/2} + 3\sqrt{\frac{g}{2}}R_\oplus t$$

$$t = \frac{4^{3/2}-1}{3}\sqrt{\frac{2R_\oplus}{g}} = \boxed{2661 \text{ s}}$$

**(d)** We can now calculate the velocity and acceleration at the time calculated in part (c), or estimate the value using the graph.

$$\vec{\boldsymbol{v}}(t) = \boxed{5590\hat{\boldsymbol{j}}\ \frac{\text{m}}{\text{s}}}$$

$$\vec{\boldsymbol{a}}(t) = \boxed{-0.613\hat{\boldsymbol{j}}\ \frac{\text{m}}{\text{s}^2}}$$

At a distance of $4R_\oplus$, the acceleration is approximately $1/16^{\text{th}}$ the value at the surface of the Earth, consistent with what we expect for the inverse square law.

**47.** We choose $t = 0$ for the instant when the rocket is 5.00 m above the ground traveling at a speed of 15.0 m/s. At $t = 1.50$ s, $x_f = 58.0$ m.

$$x_f - x_i = v_i t + \frac{1}{2}at^2$$

$$58.0 \text{ m} - 5.00 \text{ m} = (15.0 \text{ m/s})(1.50 \text{ s}) + \frac{1}{2}a(1.50 \text{ s})^2$$

$$a = \boxed{27.1 \text{ m/s}^2}$$

**48. (a)** Yes. If the acceleration is opposite the direction of the velocity, then the speed will be decreasing, as the object is decelerating.

**(b)** Yes. If the acceleration is opposite the direction of the velocity, the debris will slow down, stop instantaneously, and reverse direction.

**49. (a)** The velocity increases at a constant rate.

$$\vec{\boldsymbol{v}} = v_{0x} + a_x t = 0 \text{ m/s} + \left(6.851\hat{\boldsymbol{i}} \text{ m/s}^2\right)\left(4.55 \text{ s}\right) = \boxed{31.2\hat{\boldsymbol{i}} \text{ m/s}}$$

**(b)** The change in velocity over the given time is used to calculate the acceleration.

$$\vec{\boldsymbol{a}} = \frac{v_x - v_{x0}}{t} = \frac{0\hat{\boldsymbol{i}} \text{ m/s} - 31.2\hat{\boldsymbol{i}} \text{ m/s}}{5.62 \text{ s}} = \boxed{-5.55\hat{\boldsymbol{i}} \text{ m/s}^2}$$

**50. (a)** The graph for car A has constant slope, and thus A moves with constant acceleration. Car B has zero acceleration. Therefore, the acceleration of car A has greater magnitude than that of car B.

**(b)** According to the information given, car A is ahead of car B. During the interval from $t_1$ to $t_2$, car A is moving faster than car B and travels a greater distance than B. Car A will get even farther ahead of B during this time.

**51. (a)** The distance covered is related to the average speed and time with

$$x_f - x_i = \frac{1}{2}(v_i + v_f)t$$

$$68.0 \text{ m} = \frac{1}{2}(24.0 \text{ m/s} + v_f)(2.50 \text{ s})$$

This yields

$$v_f = \boxed{30.4 \text{ m/s}}$$

**(b)** The acceleration is the change in velocity over this time interval.

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{30.4 \text{ m/s} - 24.0 \text{ m/s}}{2.50 \text{ s}} = \boxed{2.56 \text{ m/s}^2}$$

**52. (a)** Since this graph is linear, the object moves with constant, negative acceleration. At time $t = 5$ s, as at any other instant, the acceleration of the object is negative.

**(b)** At this time, the velocity is negative. The acceleration is also negative (as it is for the entire motion). With both velocity and acceleration in the same direction, the object is speeding up.

**(c)** To find the acceleration *a*, pick two points on the velocity graph and compute the slope, for example (0 s, 3 m/s) and (10 s, –1 m/s).

$$a = (v_2 - v_1)/(t_2 - t_1) = (-1 \text{ m/s} - 3 \text{ m/s})/(10 \text{ s} - 0 \text{ s}) = -0.4 \text{ m/s}^2$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$x = 0 + (3\text{ m/s})t + \frac{1}{2}(-0.4\text{ m/s}^2)t^2$$

$$x = (3\text{ m/s})t - (0.2\text{ m/s}^2)t^2$$

**(d)** Take the time derivative of $x(t)$, set it equal to zero and solve for $t$.

$$\frac{dx}{dt} = 3\text{ m/s} - 2(0.2\text{ m/s}^2)t = 3\text{ m/s} - (0.4\text{ m/s}^2)t = 0$$

$$\boxed{t = 7.5\text{ s}}$$

Inspecting the graph, we see that $v = 0$ at $t = 7.5$ s.

**53. (a)** To find the displacement, we use $v_f^2 = v_i^2 + 2a\Delta x$ . Solving this for the displacement $\Delta x$ which gives

$$\Delta\vec{x} = \frac{v_f^2 - v_i^2}{2a}\hat{\boldsymbol{i}} = \frac{(15.0\text{ m/s})^2 - (2.00\text{ m/s})^2}{2(3.00\text{ m/s}^2)}\hat{\boldsymbol{i}} = \boxed{36.8\hat{\boldsymbol{i}}\text{ m}}$$

**(b)** Since the acceleration and the velocity of the object are in the same direction, the object is always traveling in the same (positive) direction as it is speeding up. The distance traveled is therefore also $\boxed{36.8\text{ m}}$.

**(c)** Following steps similar to those in part (a), but with $v_i = -2.00$ m/s, $\Delta\vec{x} = \boxed{36.8\hat{\boldsymbol{i}}\text{ m}}$. In this case, the object is initially moving in the negative $x$ direction, but slows down under the influence of a positive acceleration and finally moves to the right with 15.0 m/s. The total displacement is the same as in part (a).

**(d)** To calculate the total distance, we consider the motion of the particle in two parts. As it slows down from $v_i = -2.00$ m/s to $v_f = 0$ with $a = 3.00$ m/s$^2$:

$$\Delta x_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{(0.0\text{ m/s})^2 - (-2.00\text{ m/s})^2}{2(3.00\text{ m/s}^2)} = -0.666\text{ m}$$

As it accelerates from $v_i = 0$ to $v_f = 15.0$ m/s with $a = 3.00$ m/s$^2$:

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \frac{(15.0\text{ m/s})^2 - (0)^2}{2(3.00\text{ m/s}^2)} = 37.5\text{ m}$$

The total distance traveled is then $37.5\ \text{m} + 0.666\ \text{m} = \boxed{38.2\ \text{m}}$.

**54. (a)** The velocity is found by taking the derivative of the position function.

$$v_x = \frac{dx}{dt} = \boxed{0.0080\ \text{m/s} + (0.20\ \text{m/s}^2)t}$$

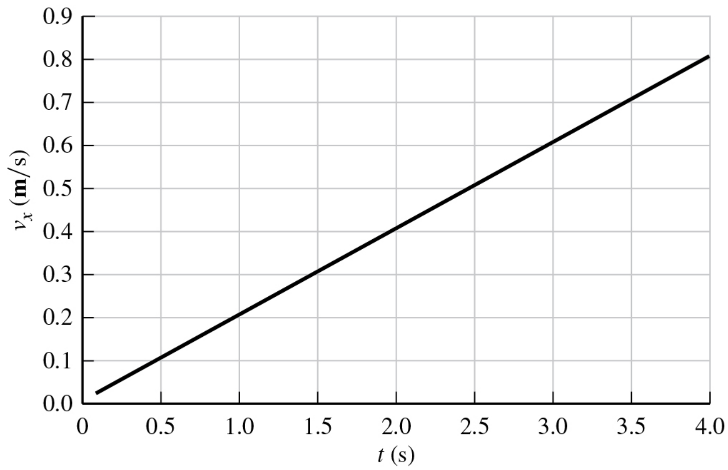



**Figure P2.54aANS**

**(b)** The plot of velocity is a straight line. Since the acceleration is the slope of the velocity, it will be constant.

**(c)** The acceleration is the derivative of velocity.

$$a_x = \frac{dv_x}{dt} = \boxed{0.20\ \text{m/s}^2}$$

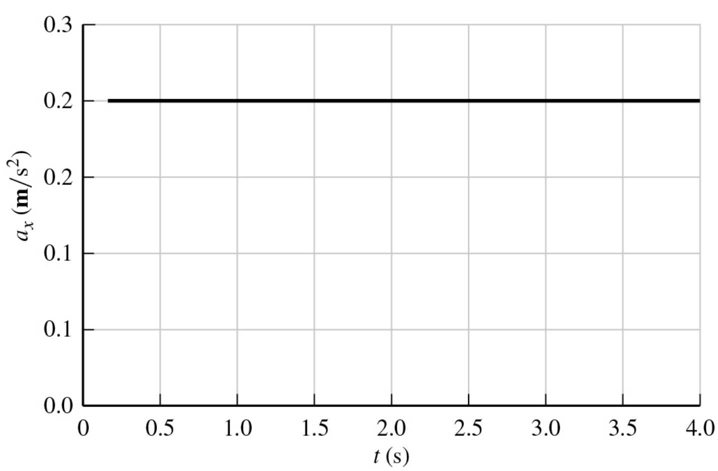



**Figure P2.54cANS**

**55. (a)** Plugging in $t = 4.00$ s into the equation for position gives

$$x = 35.0 + 20.0(4.00) + 12.0(4.00)^2 = 307 \text{ ft}$$

Because this is in the $x$ direction, we can write the final answer as $\boxed{307\hat{\boldsymbol{i}} \text{ ft}}$.

**(b)** The velocity can be determined using the derivative of position with time.

$$v = \frac{dx}{dt} = \frac{d}{dt}(35.0 + 20.0t + 12.0t^2) = 20.0 + 24.0t$$

Plugging in $t = 4.00$ s, $v = 20.0 + 24.0(4.00) = 116 \text{ ft/s}$.

Because this is in the $x$ direction, we can write the final answer as $\boxed{116\hat{\boldsymbol{i}} \text{ ft/s}}$.

**(c)** The acceleration can be determined using the derivative of velocity with time:

$a = \dfrac{dv}{dt} = \dfrac{d}{dt}(20.0 + 24.0t) = 24.0 \text{ ft/s}^2$. The acceleration is constant in time.

Because this is in the x direction, we can write the final answer as $\boxed{24.0\hat{\boldsymbol{i}} \text{ ft/s}^2}$.

**56.** Since both $v_f$ and $v_i$ are unknown, we solve the simultaneous equations

$$\left\{\begin{array}{l} v_f = v_i + at \\ x_f - x_i = \frac{1}{2}(v_i + v_f)t \end{array}\right\}$$

and plug in known values:

$$\left\{\begin{array}{l} v_f = v_i + (-1.8 \text{ m/s}^2)(12.70 \text{ s}) \\ 225.0 \text{ m} = \frac{1}{2}(v_i + v_f)(12.70 \text{ s}) \end{array}\right\}$$

Solving the first equation gives $v_i = v_f + 22.86 \text{ m/s}$. Plugging this into the second equation gives $225.0 \text{ m} = \frac{1}{2}[(v_f + 22.86 \text{ m/s}) + v_f](12.70 \text{ s})$. This yields

$$v_f = \boxed{6.29 \text{ m/s}}$$

**57.** The motion of the pebble is described by

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

Taking $y_i = 44.0$ m and $y_f = 0$:

$$0 = 44.0 \text{ m} + (-7.70 \text{ m/s})t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving for $t$ gives:

$$t = \frac{-7.70 \pm \sqrt{(-7.70)^2 - 4(4.90)(-44.00)}}{2(4.90)} = \boxed{2.31 \text{ s}}$$

**58. (a)** Given the acceleration due to gravity, the distance traveled, and the fact that the speed at the highest point is 0 m/s, we can calculate the initial velocity with the kinematic equations.

$$v_{y0}^2 = v_y^2 - 2a_y\Delta y$$

$$v_{y0} = \sqrt{v_y^2 - 2a_y\Delta y} = \sqrt{\left(0 \text{ m/s}\right)^2 - 2\left(-9.81 \text{ m/s}^2\right)\left(0.873 \text{ m}\right)} = 4.14 \text{ m/s}$$

Lastly, write the initial velocity as a vector:

$$\vec{\boldsymbol{v}}_{y0} = 4.14\hat{\boldsymbol{j}} \text{ m/s}$$

**(b)** The acceleration due to gravity is related to the change in velocity, which we can determine, divided by the time.

$$t = \frac{v_y - v_{y0}}{a_y} = \frac{0 \text{ m/s} - 4.14 \text{ m/s}}{-9.81 \text{ m/s}^2} = \boxed{0.422 \text{ s}}$$

**59. (a)** We know that $v_f = 0$ for the rocket at maximum altitude and $a = -g = 9.81\ \text{m/s}^2$.

$$v_f = v_i + at = 0$$
$$v_i = gt = (9.81\ \text{m/s}^2)(4.50\ \text{s}) = \boxed{44.1\ \text{m/s}}$$

**(b)** To find the maximum altitude, we use

$$\Delta y = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(44.1\ \text{m/s} + 0)(4.50\ \text{s}) = \boxed{99.3\ \text{m}}$$

**60. (a)**

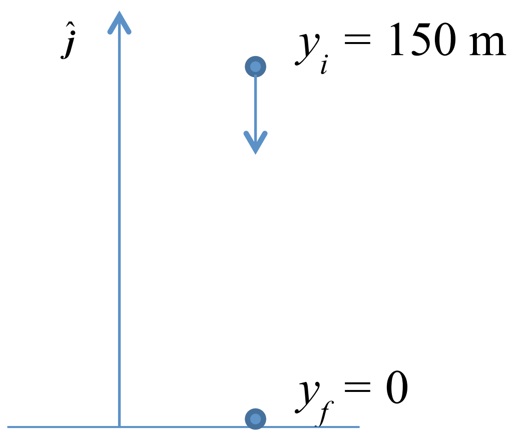



**Figure P2.60ANS**

**(b)** The Empire State Building is 102 stories, about 390 meters. Falling 40 floors corresponds to 40/102 of the total height, around 150 m. The displacement is then

$$\Delta\vec{y} = \boxed{-150\,\hat{j}\ \text{m}}$$

**(c)** $$\Delta\vec{y} = \vec{v}_{y,0}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{a} = \frac{2\Delta\vec{y}}{t^2} = \frac{2\left(-150\,\hat{j}\ m\right)}{\left(4s\right)^2} \approx -20\,\hat{j}\ \text{m/s}^2$$

**(d)** This is twice the acceleration due to gravity, so it does not make sense. The estimate for the time is likely too small.

**61. (a)** Assuming constant acceleration, we can use the definition of average velocity.

$$\Delta y = \frac{1}{2}(v_{0y} + v_y)t$$

$$v_y = \frac{2\Delta y}{t} - v_{0y} = \frac{2(-28.5)}{2.5\text{ m/s}} - 0\text{ m/s} = -23\text{ m/s}$$

$$\vec{v}_y = \boxed{-23\hat{j}\text{ m/s}}$$

**(b)** We can use the amount of time that has passed and the change in velocity that has occurred to determine the acceleration, which is in the negative *y* direction.

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{0\text{ m/s} - 23\text{ m/s}}{2.5\text{ s}} = -9.2\text{ m/s}^2$$

$$\vec{a}_y = \boxed{-9.2\hat{j}\text{ m/s}^2}$$

**62. (a)** The kinematic equations can be used.

$$\Delta y = v_i t + \frac{1}{2}at^2$$

With $\Delta y = 5.00$ m, $t = 1.80$ s, and $a = -g$:

$$5.00\text{ m} = v_i(1.80\text{ s}) - \frac{1}{2}(9.81\text{ m/s}^2)(1.80\text{ s})^2$$

Solving for the initial velocity gives $v_i = \boxed{11.6\text{ m/s}}$.

**(b)** We use the velocity equation

$$v_f = v_i + at$$

and plug in $v_i = 11.6$ m/s, $t = 1.80$ s, and $a = -g$:

$$v_f = (11.6\text{ m/s}) - (9.80\text{ m/s}^2)(1.80\text{ s}) = \boxed{-6.04\text{ m/s}}$$

A negative final velocity means that the worker on the scaffolding is catching the bricks on their way down.

**63.** The initial velocity of the rock is positive. However, the acceleration is negative and its final velocity could be positive or negative. We can use a kinematic equation to determine the times.

$$v_y = v_{0y} + a_y t$$

$$t = \frac{v_y - v_{0y}}{a_y}$$

$$t = \frac{\pm 12\,\frac{\text{m}}{\text{s}} - 24\,\frac{\text{m}}{\text{s}}}{-9.81\,\frac{\text{m}}{\text{s}^2}} = 1.2\text{ s and }3.7\text{ s}$$

The smaller time corresponds to when the rock is moving up and the larger time when it's moving down.

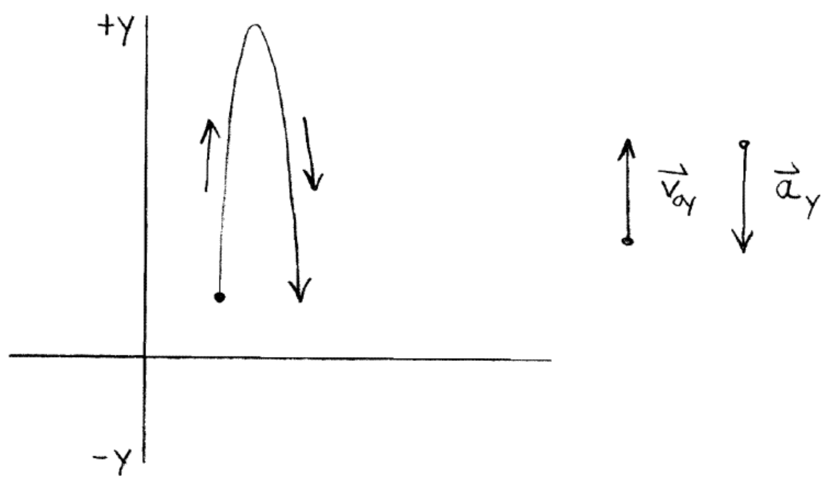



**Figure P2.63ANS**

**64.** Following the example in Problem 63, the initial velocity of the rock is positive. However, the acceleration is negative and its final velocity could be positive or negative. We can use the kinematic equations to determine the times.

$$v_y = v_{0y} + a_y t$$

$$t = \frac{v_y - v_{0y}}{a_y}$$

$$t = \frac{(\pm v/2) - v}{-g} = \frac{v}{2g}\text{ and }\frac{3v}{2g}$$

**65.** There are three stages in the motion of the rocket: the first is powered flight from the ground to an altitude of 1200 m, the second is the continued upward motion of the rocket until it reaches its maximum altitude, and the third is free fall from the maximum altitude to the ground. We treat each stage separately.

Stage 1: Using $v_i = 75.0$ m/s, $a = 5.50$ m/s$^2$, and $\Delta y = 1200$ m, we can determine the velocity at the end of this stage.

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v_f = \sqrt{v_i^2 + 2a\Delta y} = \sqrt{(75.0 \text{ m/s})^2 + 2(5.50 \text{ m/s}^2)(1200 \text{ m})} = 137 \text{ m/s}$$

This allows us to determine the time it takes for the rocket to reach an altitude of 1200 m.

$$v_f = v_i + at$$
$$t_1 = \frac{v_f - v_i}{a} = \frac{137 \text{ m/s} - 75.0 \text{ m/s}}{5.50 \text{ m/s}^2} = 11.3 \text{ s}$$

Stage 2: Now $a = -g$, $v_f = 0$, and $v_i = 137$ m/s, which allows us to determine the duration of stage 2.

$$v_f = v_i + at$$
$$t_2 = \frac{v_f - v_i}{a} = \frac{0 - 137 \text{ m/s}}{-9.81 \text{ m/s}^2} = 14.0 \text{ s}$$

We can use $v_f^2 = v_i^2 + 2a\Delta y$, with $a = -g$, $v_f = 0$, and $v_i = 137$ m/s to find the distance traveled during stage 2:

$$\Delta y_2 = \frac{v_i^2}{2g} = \frac{(137 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 958 \text{ m}$$

Stage 3: At the beginning of this stage, the rocket is at an altitude of 1200 m + 958 m = 2158 m. The rocket is now in free fall with $v_i = 0$ and $a = -g$. To find $v_f$, we use

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v_f = \sqrt{v_i^2 - 2g\Delta y} = \sqrt{(0)^2 - 2(9.81 \text{ m/s}^2)(-2158 \text{ m})} = 206 \text{ m/s}$$

This final velocity is in the downward (negative) direction. Then, from $v_f = v_i + at$, with $a = -g$,

$$t_3 = \frac{v_f - v_i}{a} = \frac{-206 \text{ m/s} - 0}{-9.80 \text{ m/s}^2} = 21.0 \text{ s}$$

**(a)** The total time of flight is $t = t_1 + t_2 + t_3 = 11.3 \text{ s} + 14.0 \text{ s} + 21.0 \text{ s} = \boxed{46.3 \text{ s}}$.

**(b)** The maximum altitude of the rocket is

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 1200 \text{ m} + 958 \text{ m} = \boxed{2.16 \times 10^3 \text{ m}}$$

**(c)** The velocity of the rocket just before it strikes the ground is

$$v_f = \boxed{206 \text{ m/s downward}}$$

**66. (a)** Since we are given the initial and final velocities of the object as well as its displacement, we use

$$v_f^2 = v_i^2 + 2a\Delta x$$

We can solve for the acceleration and plug in $\Delta x$ = 15.0 m, $v_i$ = –7.00 m/s, and $v_f$ = 10.0 m/s:

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{(10.0 \text{ m/s})^2 - (-7.00 \text{ m/s})^2}{2(15.0 \text{ m})} = \boxed{+1.70 \text{ m/s}^2}$$

**(b)** Instead of displacement, it is the total distance that is now equal to 15.0 m. Since the initial velocity is negative and the final velocity is positive, this implies that the object first has a negative displacement followed by a positive displacement, the sum of which is 15.0 m. We treat the two displacements separately.

For the first part of the motion, with $v_i$ = –7.00 m/s, and $v_f$ = 0:

$$d_1 = \Delta x_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{(0)^2 - (-7.00 \text{ m/s})^2}{2a} = \frac{(7.00)^2}{2a}$$

For the second part of the motion,

$$d_2 = \Delta x_2 = \frac{v_f^2 - v_i^2}{2a} = \frac{(10.0 \text{ m/s})^2 - (0)^2}{2a} = \frac{(10.0)^2}{2a}$$

The total distance traveled is then

$$d = d_1 + d_2 = \frac{(7.00)^2}{2a} + \frac{(10.0)^2}{2a} = \frac{(7.00)^2 + (10.0)^2}{2a}$$

Solving this for the acceleration and plugging in $d = 15.0$ m gives

$$a = \frac{(7.00)^2 + (10.0)^2}{2d} = \frac{(7.00)^2 + (10.0)^2}{2(15.0 \text{ m})} = \boxed{4.97 \text{ m/s}^2}$$

This is much higher than the result in part (a) of the problem since the total distance traveled by the object as it accelerates is much shorter here.

**67.** It is helpful to draw a sketch of the situation.

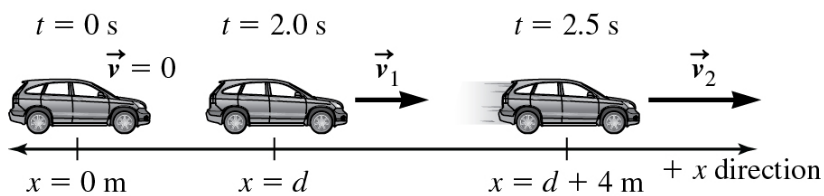



**Figure P2.67ANS**

We can express the velocity as it enters the alley, 2 s after it started from rest.

$$v_1 = a_x(2 \text{ s}) \qquad (1)$$

We can then use kinematic equations during the time that the car traverses the alley.

$$\Delta x = v_{0x}(\Delta t) + \frac{1}{2}a_x t^2$$

$$4 \text{ m} = v_1(0.5 \text{ s}) + \frac{1}{2}a_x(0.5 \text{ s})^2$$

$$4 \text{ m} = v_1(0.5 \text{ s}) + (0.13 \text{ s}^2)a_x \qquad (2)$$

We can now use equations (1) and (2) to determine the acceleration.

$$4\text{ m} = (2\text{ s})(a_x)(0.5\text{ s}) + (0.13\text{ s}^2)a_x = (1.13\text{ s}^2)a_x$$

$$a_x = 4\text{ m}/(1.13\text{ s}^2) = 3.5\text{ m}/\text{s}^2$$

or about $\boxed{4\text{ m/s}^2.}$

**68. (a)**

$$\vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{y}}}{dt} = \left(-y_0\omega\ \sin\ \omega t\right)\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{dt} = \left(-y_0\omega^2\ \cos\ \omega t\right)\hat{\boldsymbol{j}} = -\omega^2\left(y_0\ \cos\ \omega t\right)\hat{\boldsymbol{j}} = -\omega^2\vec{\boldsymbol{y}}$$

**(b)** The acceleration is not constant. It oscillates in time as a cosine function.

**(c)** We cannot use these equations since the acceleration is not constant.

**69.** Starting at time $t = 0$ s, the red car has greater speed, and thus gets farther ahead. This continues until the trooper's speed has increased to match that of the red car. The maximum distance thus occurs at the instant that the trooper's speed becomes equal to that of the red car.

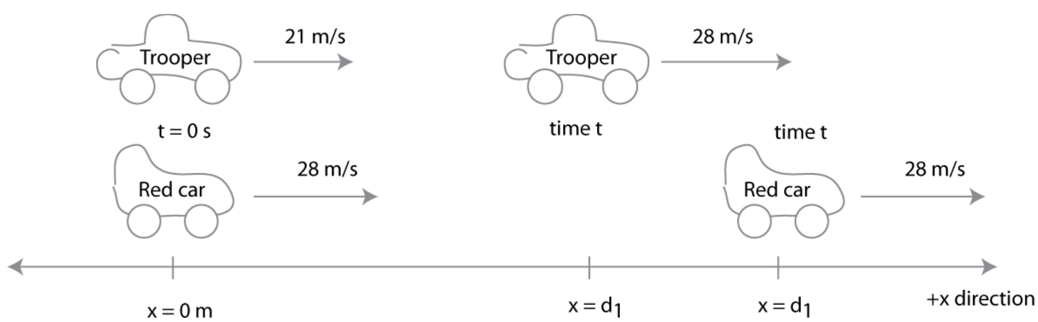



**Figure P2.69ANS**

Let $d_1$ represent the distance traveled by the trooper and $d_2$ represent the distance traveled by the red car. The difference between these two, $d_2 - d_1$, will give the maximum distance ahead of the trooper that is reached by the red car. First, find out how long it takes the trooper to change speed from 21 m/s to 28 m/s, at which point it is traveling at the same speed as the red car and the red car is not increasing the distance between them. That is, find the maximum distance between the two cars.

$$v_x = v_{0x} + a_x t$$

$$t = (28\text{ m/s} - 21\text{ m/s})/(2.0\text{ m/s}^2) = 3.5\text{ s}$$

Find the displacement of each car.

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$$

$$d_1 = (20\text{ m/s})(3.5\text{ s}) + \frac{1}{2}(2.0\text{ m/s}^2)(3.5\text{ s})^2 = 82\text{ m}$$

$$d_2 = v_{0x}\Delta t = (28\text{ m/s})(3.5\text{ s}) = 98\text{ m}$$

The maximum distance is the difference, $\boxed{16\text{ m}}$.

**70. (a)** The velocity of the dancer, as a function of time, is the time derivative of the dancer's position.

$$\vec{v}_x(t) = \frac{d\vec{x}}{dt} = \boxed{\left[\left(0.06\text{ m/s}^3\right)t^2 - \left(0.70\text{ m/s}^2\right)t + 1.75\text{ m/s}\right]\hat{i}}$$

**(b)** The function from part (a) can be plotted. The two times when the function equals zero are indicated and found to be approximately 3.6 s and 8.0 s.

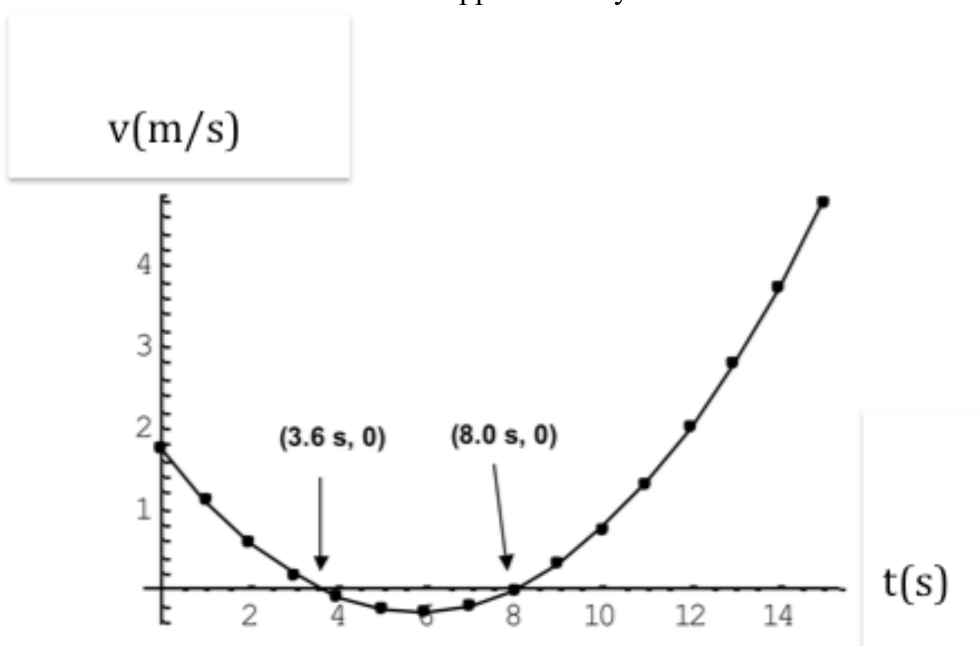



**Figure P2.70ANS**

**71.** Assuming Lina's height to be 165 cm and that the impulse travels at constant speed, the time interval is given by

$$\Delta t = \frac{\Delta x}{v} = \frac{1.65\text{ m}}{200\text{ m/s}} = \boxed{0.008\text{ s}}$$

Answers within a factor of 2 are acceptable.

**72.** The overall average speed of the second car will not be exactly $v$, but somewhat less than $v$. Since the overall distance traveled by the second car is the same, the second car will take slightly longer to cover the distance. For example, consider a trip of 10 miles, in which the first 5 miles is covered at a speed of 1 mph while the second 5 miles is covered at a speed of 1 mile per second (= 3600 mph). The total trip will take 5 hrs and 5 seconds, 5 hrs spent traveling at 1 mph and 5 s spent traveling at 1 mile per second. The overall average speed is (10 mi)/(5 hrs 5 s), or about 2 mph, much closer to 1 mph than 3600 mph.

**73. (a)** Comparing the equation for the position of the object to the general form

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$

where $y_i = -2.00$ m, $v_i = -5.00$ m/s, and $a = +12.0$ m/s$^2$. We can then plug these values into the equation for velocity and set $v_f = 0$.

$$v_f = v_i + at = -5.00 \text{ m/s} + (12.0 \text{ m/s}^2)t$$
$$0 = -5.00 \text{ m/s} + (12.0 \text{ m/s}^2)t$$
$$t = 0.417 \text{ s}$$

Plugging this time into the equation for $y$ yields

$$y = 6.00(0.417 \text{ s})^2 - 5.00(0.417 \text{ s}) - 2.00 = -3.04 \text{ m}$$

Because the motion is downwards, along the $y$ axis, we can write the final answer as

$$\boxed{-3.04\hat{\boldsymbol{j}} \text{ m}}$$

**(b)** At $t = 0$, the object was located at $y_i = -2.00$ m. To find when the object returns to this location, use $y_f - y_i = 0 = v_i t + \frac{1}{2}at^2$.

$$t = \frac{-2v_i}{a} = \frac{-2(-5.00 \text{ m/s})}{12.0 \text{ m/s}^2} = 0.833 \text{ s}$$

At this time, the particle's velocity is

$$v_f = -5.00 \text{ m/s} + (12.0 \text{ m/s}^2)(0.833 \text{ s}) = 5.00 \text{ m/s}$$

Because the motion is along the $y$ axis, we can write the final answer as $\boxed{5.00\hat{\boldsymbol{j}} \text{ m/s}}$

**74. (a)** Because the speed of the object is increasing, the average speed during the first half of the time is less than that the second half of the time. The object will cover less distance in the first half of the time than it will in the second, and at time $t = \frac{1}{2}T$ the object will have traveled a distance less than $\frac{1}{2}d$.

**(b)** Because the velocity increases uniformly from zero, the average velocity occurs halfway through the time interval. According to part (a), at time $\frac{1}{2}T$ the object has traveled a distance less than $\frac{1}{2}d$. By the time the object reaches a distance $\frac{1}{2}d$ from the start, more than half of the time has elapsed, and the object has velocity greater than $v_{av}$.

**75.** When acceleration is constant, the displacement can be determined from the average velocity and the time.

$$\Delta x = \frac{1}{2}(v_{0x} + v_x)t = \left(\frac{315 \text{ m/s} + 205 \text{ m/s}}{2}\right)(1.75 \text{ s}) = \boxed{455 \text{ m to the East}}$$

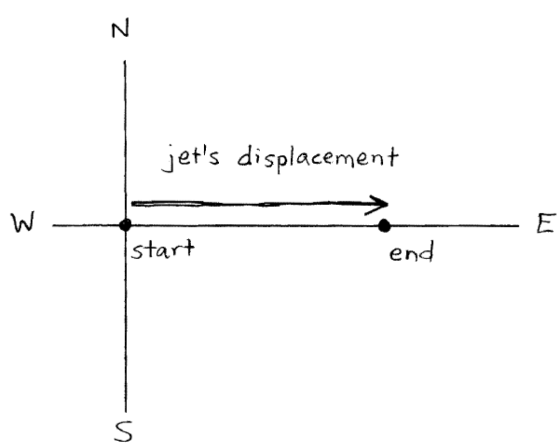



**Figure P2.75ANS**

**76. (a)** We use $v = v_i + at$ for each of the carts. To find the time(s) when the two carts have equal speeds, we set $v_1 = v_2$:

$$v_1 = v_{i1} + a_1 t = 11.8 \text{ cm/s} - (3.40 \text{ cm/s}^2)t$$
$$v_2 = v_{i2} + a_2 t = 4.30 \text{ cm/s} - (0)t = 4.30 \text{ cm/s}$$
$$11.8 \text{ cm/s} - (3.40 \text{ cm/s}^2)t = 4.30 \text{ cm/s}$$
$$t = \boxed{2.21 \text{ s}}$$

**(b)** Since cart #2 has a constant speed of 4.30 cm/s, the first cart must also have this speed for the two speeds to be equal.

**(c)** The two carts will pass one another if their positions are the same at a time *t*. To find this time, we first write the equations for the positions of the two carts and set $x_1 = x_2$.

$$x_1 = x_{i1} + v_{i1}t + \frac{1}{2}a_1t^2 = 18.0 \text{ cm} + (11.8 \text{ cm/s})t - \frac{1}{2}(3.40 \text{ cm/s}^2)t^2$$

$$x_2 = x_{i2} + v_{i2}t + \frac{1}{2}a_2t^2 = 20.0 \text{ cm} + (4.3 \text{ cm/s})t + \frac{1}{2}(0)t^2 = 20.0 \text{ cm} + (4.3 \text{ cm/s})t$$

$$18.0 \text{ cm} + (11.8 \text{ cm/s})t - \frac{1}{2}(3.40 \text{ cm/s}^2)t^2 = 20.0 \text{ cm} + (4.30 \text{ cm/s})t$$

$$t = \frac{7.50 \pm \sqrt{(7.50)^2 - 4(1.70)(2.00)}}{2(1.70)} = \boxed{4.13 \text{ s or } 0.285 \text{ s}}$$

The positions of the carts are the same and can be found by plugging these times into the position equation. At time *t* = 0.285 s, both carts are at position

$$x_1 = 18.0 \text{ cm} + (11.8 \text{ cm/s})(0.285 \text{ s}) - \frac{1}{2}(3.40 \text{ cm/s}^2)(0.285 \text{ s})^2 = \boxed{21.2 \text{ cm}}$$

At time *t* = 4.13 s, both carts are at position

$$x_1 = 18.0 \text{ cm} + (11.8 \text{ cm/s})(4.13 \text{ s}) - \frac{1}{2}(3.40 \text{ cm/s}^2)(4.13 \text{ s})^2 = \boxed{37.7 \text{ cm}}$$

**(d)** Part (a) asks us to find the time at which the velocities of the two carts are equal, whereas part (c) asks us whether the carts are at the same location. Initially, the carts are both moving in the same direction, and the first cart soon catches up with and passes the second cart, at *t* = 0.285 s. The velocities of the two carts are different at this time. Cart 1 then decelerates, so that cart 2 catches up to cart 1 at *t* = 4.13 s. The velocities of the carts are oppositely directed at this time. The two carts have the same velocity at *t* = 2.21 s.

**77. (a)** Plugging in the given times:

$x\ (t = 1.00 \text{ yr}) = 4.00(1.00)^2 - 3.00(1.00) + 5.00 = 6.00 \text{ AU}$

$x\ (t = 1.00 \text{ yr}) = 4.00(3.00)^2 - 3.00(3.00) + 5.00 = 32.0 \text{ AU}$

The average speed is then

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} = \frac{32.0 \text{ AU} - (6.00 \text{ AU})}{(3.00 - 1.00) \text{ years}} = \boxed{13.0 \text{ AU/yr}}$$

**(b)** The instantaneous velocity of the spacecraft is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}(4.00t^2 - 3.00t + 5.00) \;=\; 8.00t - 3.00$$

Plugging in $t$ = 1.00 s and $t$ = 3.00 s gives

$$v(t = 1.00 \text{ yr}) = 8.00(1.00) - 3.00 = \boxed{5.00 \text{ AU/yr}}$$

$$v(t = 3.00 \text{ yr}) = 8.00(3.00) - 3.00 = \boxed{21.0 \text{ AU/yr}}$$

**(c)** To find the time at which $v = 0$, set $v = 8.00t - 3.00 = 0$. Solving for $t$ gives

$$t = \boxed{0.375 \text{ years}}$$

**78. (a)** The velocity of car A is to right at each instant and has magnitude that decreases over time. The change in velocity is thus to the left, as is the acceleration.

**(b)** Car A experiences two units of change in velocity over each time interval while B experiences only one. Thus, the magnitude of car A's acceleration is twice that of B.

**(c)** The speed of car B is never less than that of A throughout the time intervals shown, so car B must be pulling further ahead even though the *rate* at which B is getting farther ahead of A decreases steadily until time $t_4$, when the cars have equal velocities. At this time, car A begins to close the gap and catch up to car B.

**79.** Substituting $t$ = 2.50 s, 3.50 s, and 3.60 s into $z = 5.00t^2 + 4.00t$ gives $x$ = 41.25 m, 75.25 m, and 79.20 m, respectively.

**(a)** $$\vec{v}_{\text{avg}} = \frac{\Delta \vec{z}}{\Delta t} = \frac{75.25\hat{k}\text{ m} - 41.25\hat{k}\text{ m}}{3.50\text{ s} - 2.50\text{ s}} = \boxed{34.0\hat{k}\text{ m/s}}$$

**(b)** $$\vec{v}_{\text{avg}} = \frac{\Delta \vec{z}}{\Delta t} = \frac{79.20\hat{k}\text{ m} - 75.25\hat{k}\text{ m}}{3.60\text{ s} - 3.50\text{ s}} = \boxed{39.5\hat{k}\text{ m/s}}$$

**80. (a)** The pebble falls a distance $d$ into the chasm in a time interval $\Delta t_1$ and the sound of the impact travels upward the same distance $d$ in a time interval $\Delta t_2$ before the rock climber hears it. The total time interval is $\Delta t = \Delta t_1 + \Delta t_2$ = 3.20 s. The pebble is in free fall during the time interval $\Delta t_1$ and has $v_i = 0$, so $d = \frac{1}{2}g\left(\Delta t_1\right)^2$.

The sound from the impact travels at constant speed during the time interval $\Delta t_2$, so $d = v_s \Delta t_2$ where $v_s$ = 343 m/s. Setting the two expressions for $d$ equal to one another,

$$\frac{1}{2} g\left(\Delta t_1\right)^2 = v_s \Delta t_2$$

Then, substituting $\Delta t_1 = \Delta t - \Delta t_2$ gives

$$\frac{1}{2} g\left(\Delta t - \Delta t_2\right)^2 = v_s \Delta t_2$$

Which we can rearrange as

$$\left(\Delta t_2\right)^2 - 2\left(\Delta t + \frac{v_s}{g}\right)^2 \Delta t_2 + \left(\Delta t\right)^2 = 0$$

Plugging in numbers:

$$\left(\Delta t_2\right)^2 - 2\left(3.20\text{ s} + \frac{343\text{ m/s}}{9.81\text{m/s}^2}\right)\Delta t_2 + \left(3.20\text{ s}\right)^2 = 0$$

$$\left(\Delta t_2\right)^2 - \left(76.4\right)\Delta t_2 + 10.24 = 0$$

$$\Delta t_2 = \frac{76.4 \pm \sqrt{(-76.4)^2 - 4(1)(10.24)}}{2(1)} = 0.134\text{ s}$$

Which gives $d = v_s \Delta t_2 = (343\text{ m/s})(0.134\text{ s}) = \boxed{46.1\text{ m}}$.

**(b)** Ignoring the travel time for the sound of the impact would mean using

$$d = \frac{1}{2} g\left(\Delta t\right)^2 = \frac{1}{2}(9.80\text{ m/s}^2)(3.20\text{ s})^2 = 50.2\text{ m}$$

or an error of $\boxed{8.89\%}$. Note that this error is larger for deeper chasms.

**81.** There are many good choices. Some general considerations include: (1) It is best to place an axis along the direction of motion. (2) It is sometimes helpful to choose an origin so that the position of the particle is always positive. (3) Choose positive as the direction in which the moving particle speeds up. (4) When the motion is caused by a spring, it is common to put the origin at the spring's relaxed position; because the motion switches direction, it doesn't matter which direction is chosen to be positive.

With this in mind: (A) Axis pointing to the right with origin at starting location. (B) Axis pointing up the ramp with origin at starting location. (C) Axis pointing upward with origin at relaxed position.

**82.** The average velocity is the displacement divided by time for a specific time interval.

$$\vec{\boldsymbol{v}}_y = \frac{\Delta\vec{\boldsymbol{y}}}{\Delta t} = \frac{\Delta y}{\Delta t}\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{v}}_z = \frac{\Delta\vec{\boldsymbol{z}}}{\Delta t} = \frac{\Delta z}{\Delta t}\hat{\boldsymbol{k}}$$

**83.** The instantaneous velocity is the displacement over time for a time interval that approaches zero, equivalent to the derivative of the *y* or *z* displacement with respect to time.

$$\vec{\boldsymbol{v}}_y = \lim_{\Delta t \to 0}\frac{\Delta\vec{\boldsymbol{y}}}{\Delta t} = \frac{d\vec{\boldsymbol{y}}}{dt} = \boxed{\frac{dy}{dt}\hat{\boldsymbol{j}}}$$

$$\vec{\boldsymbol{v}}_z = \lim_{\Delta t \to 0}\frac{\Delta\vec{\boldsymbol{z}}}{\Delta t} = \frac{d\vec{\boldsymbol{z}}}{dt} = \boxed{\frac{dz}{dt}\hat{\boldsymbol{k}}}$$

**84.** The average acceleration is the change in velocity divided by time for a specific time interval.

$$\vec{\boldsymbol{a}}_y = \frac{\Delta\vec{\boldsymbol{v}}_y}{\Delta t} = \boxed{\frac{\Delta v_y}{\Delta t}\hat{\boldsymbol{j}}}$$

$$\vec{\boldsymbol{a}}_z = \frac{\Delta\vec{\boldsymbol{v}}_z}{\Delta t} = \boxed{\frac{\Delta v_z}{\Delta t}\hat{\boldsymbol{k}}}$$

**85.** The instantaneous acceleration is the change in velocity over time for a time interval that approaches zero, equivalent to the derivative of the *y* or *z* velocity with respect to time.

$$\vec{\boldsymbol{a}}_y = \lim_{\Delta t \to 0}\frac{\Delta\vec{\boldsymbol{v}}_y}{\Delta t} = \frac{d\vec{\boldsymbol{v}}_y}{dt} = \boxed{\frac{dv_y}{dt}\hat{\boldsymbol{j}}}$$

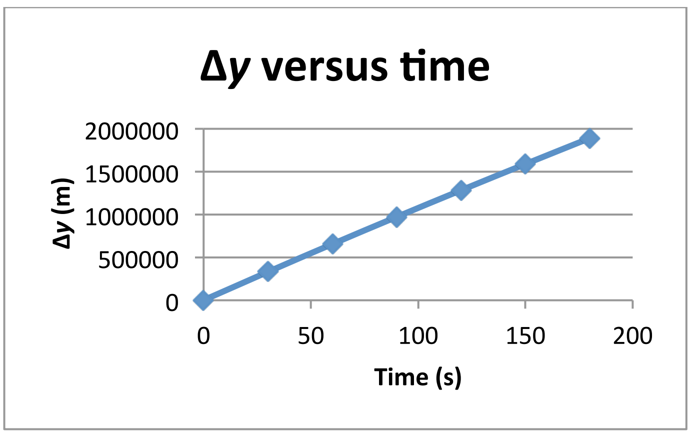
$$\vec{\boldsymbol{a}}_z = \lim_{\Delta t \to 0}\frac{\Delta\vec{\boldsymbol{v}}_z}{\Delta t} = \frac{d\vec{\boldsymbol{v}}_z}{dt} = \boxed{\frac{dv_z}{dt}\hat{\boldsymbol{k}}}$$

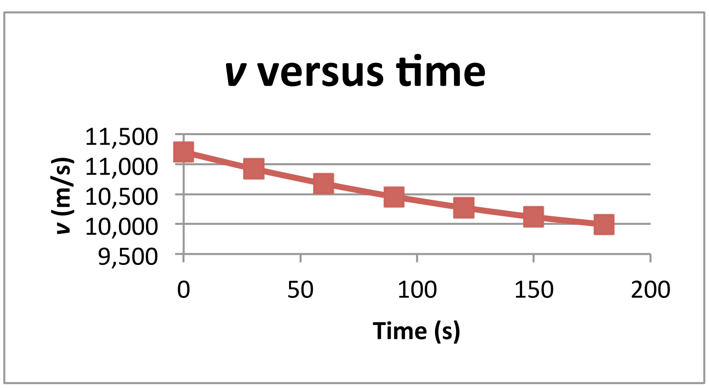
**86.** From Example 2.6,

$$\Delta\vec{y}(t) = \left(Bt - Ct^2 + Dt^3\right)\hat{j}$$
$$\vec{v}_y(t) = \left(B - 2Ct + 3Dt^2\right)\hat{j}$$
$$\vec{a}_y(t) = \left(-2C + 6Dt\right)\hat{j}$$

with $A = 6.37\times10^6$ m, $B = 1.12\times10^4$ m/s, $C = 4.90$ m/s$^2$, $D = 5.73\times10^{-3}$ m/s$^3$.
We can now calculate the desired values and plot these quantites.

| time (s) | $\Delta y$ (m) | $v$ (m/s) | $a$ (m/s$^2$) |
|---|---|---|---|
| 0 | 0.00 | 11,200 | –9.80 |
| 30 | 331,745 | 10,921 | –8.77 |
| 60 | 655,598 | 10,674 | –7.74 |
| 90 | 972,487 | 10,457 | –6.71 |
| 120 | 1,283,341 | 10,272 | –5.67 |
| 150 | 1,589,089 | 10,117 | –4.64 |
| 180 | 1,890,657 | 9,993 | –3.61 |



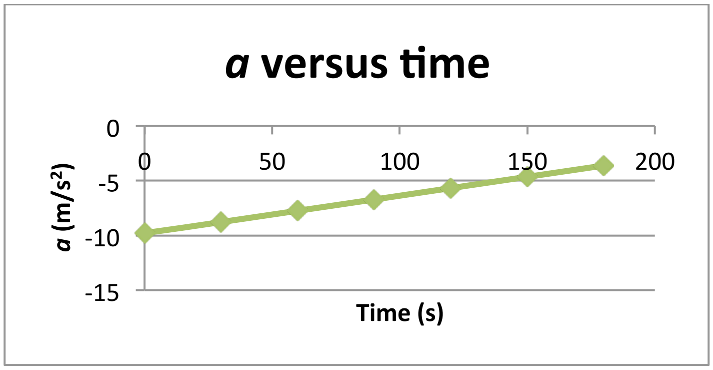



**Figure P2.86ANS**

**87.** Following the lead of Example 2.7A, using a speed of 39.24 mph (= 17.4 m/s) and an initial distance of 6 miles (= 9660 m).

$$a_x = \frac{v_x{}^2 - v_{0x}{}^2}{2\ \Delta x}$$

$$a_x = \frac{(17.4^2\ \text{m/s})^2 - 0^2}{2(9.66\times 10^3\ \text{m})} = 0.0157\ \text{m/s}^2$$

$$\vec{\boldsymbol{a}}_x = \boxed{0.0157\hat{\boldsymbol{i}}\ \text{m/s}^2}$$

Following Example 2.7B, the time for the timed mile is

$$t = \frac{\Delta x}{v_x} = \frac{1.61\times 10^3\ \text{m}}{17.4\ \text{m/s}} = \boxed{92.6\ \text{s}}$$

Following Example 2.7C, the deceleration in the last six miles is the opposite of the initial acceleration.

$$\vec{\boldsymbol{a}}_x = \boxed{-0.0157\hat{\boldsymbol{i}}\ \text{m/s}^2}$$

ThrustSSC's acceleration is nearly 400 times greater than Jeantaud's, and Jeantaud's timed mile was about five times longer than ThrustSSC's.

**88.** The two simultaneous equations for the ball and the dart are

$$y_{Bf} = \left(8.0 - \frac{9.81}{2}t^2\right)\ \text{m}$$

and

$$y_{Df} = \left(11.5t - \frac{9.81}{2}t^2\right)\ \text{m}$$

We want the position and time to be the same at the collision, or $y_{Bf} = y_{Df}$.

$$\left(8.0 - \frac{9.81}{2}t^2\right) = \left(11.5t - \frac{9.81}{2}t^2\right)$$
$$11.5t = 8.0$$
$$t = \boxed{0.70 \text{ s}}$$

At this time, $y_{Bf} = \left(8.0 - \frac{9.81}{2}(0.70)^2\right) \text{ m} = \boxed{5.6 \text{ m}}$.

This agrees with what was determined graphically.

**89.** When an object is experiencing a constant acceleration, we can write that the derivative of the velocity as a function of time is a constant, $a_x$.

$$\frac{dv_x}{dt} = a_x$$
$$dv_x = a_x dt$$

Integrate both sides of the equation, where the velocity at the time $t = 0$ is $v_{0x}$.

$$\int_{v_{0x}}^{v_x} dv_x = \int_0^t a_x\, dt$$
$$v_x - v_{0x} = a_x t$$
$$v_x = v_{0x} + a_x t$$

Similar to before, we can write the derivative of the position as a function of time, which is equal to the velocity.

$$\frac{dx}{dt} = v_x$$
$$dx = v_x dt$$

Integrate both sides of the equation, using the result for $v_x$.

$$\int_{x_0}^{x} dx = \int_0^t v_x\, dt = \int_0^t \left(v_{0x} + a_x t\right) dt$$
$$\Delta x = \int_0^t v_{0x}\, dt + \int_0^t \left(a_x t\right) dt$$
$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2$$