# Chapter 2
## Applying Time Value Concepts

## ■ Chapter Overview

Albert Einstein, the renowned physicist whose theories of relativity formed the theoretical base for the utilization of atomic energy, called the time value of money principle one of the strongest forces on earth. Chapter 2 discusses the importance of the time value of money. The concepts of simple and compound interest are introduced in the chapter. Simple interest refers to interest on a loan computed as a percentage of the loan amount. Compound interest refers to the process of earning interest on interest.

In addition, chapter 2 also discusses the time value of money as it is applied to two types of cash flows: a single dollar amount (or lump sum) and an annuity. An annuity is a stream of equal payments paid over equal intervals of time. The use of present and future value tables and formulas to aid calculations is explained in the chapter. In addition, the chapter explains how to use a financial calculator to make time value calculations. Example calculations show the inputs required using the TI BA II Plus calculator.

In discussing the future and present value of an annuity, the chapter differentiates between an ordinary annuity, for which payments occur at the end of the period, and an annuity due, for which payments occur at the beginning of the period. Annuities are illustrated through the use of timelines. As with the single dollar calculations, present and future value of an annuity tables are provided within the chapter, as are instructions for using a financial calculator. Throughout the chapter, practical uses for each type of calculation are described.

The chapter concludes with a discussion on how to convert a nominal interest rate to an effective interest rate and vice versa. The nominal interest rate is the stated, or quoted, rate of interest. It is the rate of interest that is used in time value of money calculations. The effective interest rate is the actual rate of interest that you earn, or pay, over a period of time. Effective interest rates can be compared with each other; whereas nominal interest rates cannot be directly compared in situations where the compounding period between interest rates is different.

## ■ Chapter Objectives

The objectives of this chapter are to:

1. Explain the difference between simple interest and compound interest,
2. Calculate the future value of a single dollar amount that you save today,
3. Calculate the present value of a single dollar amount that will be received in the future,
4. Calculate the future value of an annuity,
5. Calculate the present value of an annuity, and
6. Convert a nominal interest rate to an effective interest rate.

## ■ Teaching Tips

1. The classic example of the power of compound interest is to ask students whether they would rather have $500,000 right now or one cent that you would double each day for the next thirty days. Many students will choose the $500,000 at first. However, the one cent would grow to $10,737,417.65 at the end of thirty days. While this represents a 100% daily interest rate which cannot be obtained, it is a powerful example. This chapter concerns the "time" value of money and it is a good idea to emphasize time over deposit amounts or rates of return. Have the students calculate, either using the tables or with a financial calculator, a single sum at a given interest rate, changing only the length of time of the investment. For instance, a one-time $5,000 investment at 10% compounded annually would return $87,247 after 30 years, but only $54,174 after 25 years. Only 5 years difference in time amounts to a difference of $33,073, or an average loss of $6,614 per year. After 20 years the investment would return only $33, 637, and after 10 years it would return only $12,969. Emphasize that all of this is based on the initial amount of $5,000 with no additional investment by the investor.

2. It is sometimes said that those who understand compound interest collect it while those who don't understand it pay it. Discuss the fact that while compound interest works to our advantage when we save and invest, it works to our detriment when we are in debt. Suppose you bought a $2,000 home entertainment system on a credit card that charges19.99% annual interest, compounded daily. Using minimum payments of 3% of the outstanding balance for each month, it will take 15 years and 3 months to pay off the debt. You would have paid $2,238.13 in interest, making the total payments $4,238.13, the true cost of the entertainment system when purchased on credit. The moral of this example is to never pay minimum payments on high interest credit cards. Paying $60 a month would pay off the same debt in 4 years and 2 months, and reduce the interest paid to $942.82. Paying $100 a month would pay the debt off in 2 years 1 month, and would cost only $452.92 in interest. Make the calculations before making the purchase to ensure that you can make payments that will minimize the amount of interest you will pay.

3. Demonstrate the difference between simple and compound interest. If you deposit $2,000 per year for 40 years and earn 10% compounded annually, but withdraw the interest and spend it, the $2,000 deposit annually would be worth $80,000 in 40 years. By allowing the interest to compound with the deposits, the investment would be worth $885,185.

4. Class exercise: Have the class answer the following question:
   You have two investment choices:
   Option 1:
   Receive $500,000 cash today.
   Option 2:
   Receive 1 penny today, 2 pennies tomorrow, 4 pennies the next day, 8 pennies the next day…doubling every day for thirty days.

   Which option do you choose?

As shown in the table below, option 2 is the better choice.

| SUNDAY | MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $0.01 | $0.02 | $0.04 | $0.08 | $0.16 | $0.32 | $0.64 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| $1.32 | $2.56 | $5.12 | $10.24 | $20.48 | $40.96 | $81.92 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| $163.84 | $327.68 | $655.36 | $1,310.72 | $2,621.44 | $5,242.88 | $10,485.76 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| $20,971.52 | $41,943.04 | $83,886.08 | $167,772.16 | $335,544.32 | $671,088.64 | $1,342,177.20 |
| 29 | 30 | **TOTAL** | | | | |
| $2,684,354.40 | $5,368,708.80 | **$10,737,417.65** | | | | |

5. Online/Team exercise—review of TVM problems. Generally, students will have a variety of backgrounds on this topic. The use of a team exercise gives those with some expertise a chance to help those with little or no background. It is a good review for those with the expertise and makes the others more comfortable with peer help.

## ■ Suggested Answers to Chapter Overview Questions

1. $125 x 52 = $6,500.
2. $6,500 x .25 = $$1,625. If Haroon invested this money at 7 percent compounded annually, he would have $22,451.73 after 10 years, calculated as follows:

   P/Y = 1, C/Y = 1, N = 10, I/Y = 7, PV = 0, PMT = 1625, **FV = ?**

   After 20 years, he would have $66,617.68, calculated as follows:

   P/Y = 1, C/Y = 1, N = 20, I/Y = 7, PV = 0, PMT = 1625, **FV = ?**

3. Haroon could adopt a cash budget wherein he only withdraws the amount that he has budgeted for that week. For example, he could reduce his spending by 25 percent by giving himself a weekly allowance of $93,75; which represents 75 percent of what he is currently spending. Haroon could also spend less on each purchase or reduce the number of purchases he makes. For example, he could purchase a less expensive brand or type of coffee and reduce the frequency with which he makes these types of purchases. Finally, Haroon could make a commitment to himself to bring lunch from home once per week, thereby reducing his purchases when he is at college.

## ■ Answers to End-of-Chapter Review Questions

1. The time value of money is a powerful principle that can be used to explain how money grows over time. When you spend money, you incur an opportunity cost of what you could have done with that money had you not spent it. For example, if you spent $2000 on a vacation rather than saving it, you would have incurred an opportunity cost of the alternative ways that you could have used the money. You can use the time value of money to compute the actual cost of the opportunity.

2. Interest is the rent charged for the use of money. Depending on whether you have borrowed or loaned money, you will either pay or receive interest, respectively. Simple interest is interest on a loan computed as a percentage of the loan amount, or principal. The interest earned or paid is not reinvested. Simple interest is measured by multiplying the principal, the interest rate applied to the principal, and the loan's time to maturity (in years). Compound interest refers to the process of earning interest on interest.

3. For simple problems a time value of money table may be used to calculate the future or present value of a single dollar amount. Other methods that may be used to solve time value of money problems include time value of money formulas and financial calculators.

4. The time value of money is most commonly applied to two types of cash flows: a single dollar amount (also referred to as a lump sum) and an annuity. An annuity refers to the payment of a series of equal cash flow payments at equal intervals of time.

5. The inputs required when calculating the future value, FV, of a single dollar amount using a formula are the present future value of an investment (PV), the annual interest rate, i, (expressed as a decimal), the number of compounding periods per year (n), and time, t, (in years).

6. The future value interest factor (*FVIF*), is a factor multiplied by today's savings to determine how the savings will accumulate over time. The factor is determined based on an annual interest rate where the number of compounding periods is one. The formula for determining the future value of a single dollar amount when using the future value interest factor is:

$$FV = PV \times FVIF_{i,n}$$

   In order to find the correct future value interest factor, you must know the interest rate and the number of years the money is invested.

7. Clear the existing TVM values in the calculator's TVM worksheet by entering 2ND CLR TVM.

8. A cash inflow (for example, income received from an investment) should be entered as a positive number. A cash outflow (for example, an investment amount) should be entered as a negative number. The +/- key on the TI BA II Plus is used to convert a positive number to a negative number, and vice versa.

9. There are 12 compounding periods in a year when an investment compounds interest monthly. An investment that compounds interest quarterly has 4 compounding periods. An investment that compounds interest daily has 365 compounding periods.

10. Discounting is the process of obtaining present values.

11. Suppose you need $20 000 to purchase a car in 3 years. You may want to determine how much money you need to invest today to achieve the $20 000 in three years. Another instance where determining the present value is useful would be if you want to pay off a loan today that will, for example, be paid over 3 years. In this case, you want to know the present value of these future payments.

12. The formula for the present value of a single dollar amount is:

$$PV = \frac{FV}{(1 + \frac{i}{n})^{nt}}$$

13. The present value interest factor is a factor multiplied by the future value to determine the present value of that amount. The formula for determining the present value of a single dollar amount when using the present value interest factor is:

$$PV = FV \times PVIF_{i,n}$$

14. An annuity refers to the payment of a series of equal cash flow payments at equal intervals of time. An ordinary annuity is a stream of equal payments that are received or paid at equal intervals in time at the end of a period. An annuity due is a series of equal cash flow payments that occur at the beginning of each period. Thus, an annuity due differs from an ordinary annuity in that the payments occur at the beginning instead of the end of the period. The most important thing to note about an annuity is that if the payment changes over time, the payment stream does not reflect an annuity.

15. The formula used to determine the future value of an annuity is:

$$FV = PMT \text{ x } [\frac{(1 + i)^n - 1}{i}]$$

16. The future value interest factor for an annuity, FVIFA, is a factor multiplied by the periodic savings level (annuity) to determine how the savings will accumulate over time. The formula for the future value interest factor for an annuity, when using a table, is:

$$FVA = PMT \times FVIFA_{i,n}$$

17. An annuity formula or table will provide the future value for an ordinary annuity. In order to adjust your calculation for an annuity due, you would multiply the annuity payment generated by multiplying the value from the table by $(1 + i)$.

18. The formula used to determine the present value of an annuity is:

$$PV = PMT \text{ x } \left[\frac{1 - [\frac{1}{(1 + i)^n}]}{i}\right]$$

19. The present value interest factor for an annuity, PVIFA, is a factor multiplied by a periodic savings level (annuity) to determine the present value of the annuity. The formula for the present value interest factor for an annuity, when using a table, is:

$$PVA = PMT \times PVIFA_{i,n}$$

20. 60.

21. The nominal interest rate is the stated, or quoted, rate of interest. It is also known as the annual percentage rate (APR). The effective interest rate is the actual rate of interest that you earn, or pay, over a period of time. It is also known as the effective yield (EY). When comparing two or more interest rates, the nominal interest rate is not useful because it does not take into account the effect of compounding. In order to make objective investment decisions regarding loan costs or investment returns over different compounding frequencies, the effective interest rate has to be determined. The effective interest rate allows for the comparison of two or more interest rates because it reflects the effect of compound interest.

22. The present value of a single sum

23. The future value of an annuity

24. The future value of a single sum

25. The present value of an annuity

## ■ Answers to Financial Planning Problems

1. P/Y = 1, C/Y = 1, N = 5, I/Y = 4, PV = -1000, PMT = 0, **FV = ?**

   **Rodney will have $1,216.65 in five years to put down on his car.**

2. P/Y = 12, C/Y = 1, N = 60, I/Y = 3, PV = 0, PMT = -50, **FV = ?**

   **Michelle's balance in five years will be $3,229.05.**

3. Jessica: P/Y = 1, C/Y = 1, N = 10, I/Y = 10, PV = 0, PMT = -2000, **FV = ?**

   Jessica will have $31,874.85 after 10 years. (assuming ordinary annuity)

   Jessica: P/Y = 1, C/Y = 1, N = 30, I/Y = 10, PV = 31874.85, PMT = 0, **FV = ?**

   **Jessica will have $556,197.07 at retirement. She contributed $20,000 in total.**

   Joshua: P/Y = 1, C/Y = 1, N = 30, I/Y = 10, PV = 0, PMT = -2000, **FV = ?**

   **Joshua will have $328,988.05 at retirement. He contributed $60,000 in total.**

4. P/Y = 1, C/Y = 12, N = 3, I/Y = 4, **PV = ?**, PMT = 0, FV = 2000

   **Cheryl must deposit $1,774.19 now in order to have the money she needs in three years.**

5. P/Y = 12, C/Y = 2, N = 4, I/Y = 8, PV = 0, **PMT = ?**, FV = 7000

   **Amy and Vince must save $124.56 each month to have the money they need.**

6. P/Y = 12, C/Y = 1, N = 300, I/Y = 12, PV = 0, PMT = -400, **FV = ?**

   **Judith's employer contributes $200 per month. She will have $674,482.60 in her retirement plan at retirement**

7. P/Y = 12, C/Y = 4, N = 360, I/Y = 11, PV = 0, PMT = -300, **FV = ?**

   **Stacey will not reach her retirement goal since she will only be able to accumulate $823,358.63 by the time she retires in 30 years.**

8. P/Y = 1, C/Y = 1, N = 18, I/Y = 7, **PV = ?**, PMT = 0, FV = 10000

   **Juan must deposit $2,958.64 now in order to achieve his goal.**

9. P/Y = 1, C/Y = 12, N = 20, I/Y = 4, PV = 0, PMT = -100, **FV = ?**

   **$3,000.82 will be in the account in 20 years.**

10. P/Y = 1, C/Y = 4, N = 3, I/Y = 9, PV = -3000, PMT = 0, **FV = ?**

    **Earl will have $3,918.15 to spend on his trip to Belize.**

11. Lump-sum payment: P/Y = 1, C/Y = 12, N = 20, I/Y = 8, PV = 312950, PMT = 0, **FV = ?**

    **The lump-sum payment will be worth $1,541,842.93 after 20 years.**

    Annual payment: P/Y = 1, C/Y = 1, N = 20, I/Y = 6, PV = 0, PMT = 50000, **FV =?**

    **The annual payment will be worth $1,839,279.56 after 20 years. Jesse should choose the annual payment.**

12. P/Y = 1, C/Y = 4, N = 20, I/Y = 7, **PV = ?**, PMT = 0, FV = 6000000

    **The cash option payout would be $1,497,607.85.**

13. P/Y = 52, C/Y = 12, N = 260, I/Y = 10, PV = 0, PMT = -10, **FV = ?**

    **She will have $3,366.34 in five years.**

14. Invest It: P/Y = 1, C/Y = 4, N = 3, I/Y = 5, PV = -1000, PMT = 0, **FV = ?**

    **Investing the income tax refund will give him $1,160.75 at the end of three years.**

    Purchase Stereo: P/Y = 12, C/Y = 365, N = 36, I/Y = 4, PV = 0, PMT = -30, **FV = ?**

    **Purchasing the stereo and investing $30 per month will give him $1,145.56 at the end of three years.**

15. P/Y = 12, C/Y = 52, N = 36, I/Y = 10, PV = 0, PMT = -75, **FV = ?**

    **You will have $3,135.17.**

16. The equivalent effective interest rate is 8.71%.

## ■ Answers to Challenge Questions

1. P/Y = 12, C/Y = 12, N = 144, I/Y = 5, PV = 0, PMT = 300, **FV = ?**

   **The future value of their education savings is $59,029.12.**

   P/Y = 12, C/Y = 12, N = 144, I/Y = 7, PV = 0, PMT = 300, **FV = ?**

   **If they could earn 7 percent a year instead of 5 percent, the future value of their education savings would be $67,408.50. In other words, they would have an additional $8,379.38 in education savings.**

   P/Y = 12, C/Y = 12, N = 144, I/Y = 5, PV = 0, PMT = 400, **FV = ?**

   **If they could save $400 per month at 5 percent, the future value of their education savings would be $78,705.49.**

   P/Y = 12, C/Y = 12, N = 144, I/Y = 7, PV = 0, PMT = 400, **FV = ?**

   **If they could save $400 per month at 7 percent, the future value of their education savings would be $89,877.99.**

2. Cash flow stream A:

   P/Y = 1, C/Y = 4, N = 6, I/Y = 6.5, **PV = ?**, PMT = 1500, FV = 0

   **The present value of cash flow stream A is $7225.44**

   Cash flow stream B:

   P/Y = 1, C/Y = 4, N = 4, I/Y = 6.5, **PV = ?**, PMT = 2500, FV = 0

   **At the end of year 2, the value of cash flow stream B is $8,533.44.**

   P/Y = 1, C/Y = 4, N =3, I/Y = 6.5, **PV = ?**, PMT = 0, FV = 6,601.74

   **The present value of cash flow stream B is $7032.59.**

   **Cash flow stream is more attractive since it has a greater present value.**

## ■ Suggested Answers to Ethical Dilemma Questions

(a) This question will hopefully spark a lively discussion between those students who believe that a salesperson’s first obligation is to sell products or services and those students who believe that a salesperson’s first obligation is to assist the customer.

(b) Two hundred dollars per month at 6 percent compounded annually will grow to $194,902.59 in 30 years. Two hundred and forty dollars per month at 6 percent compounded annually will grow to $162,309.35 in 25 years. Therefore, Herb is incorrect in his calculation.

## ■ Answers to Mini-Case Questions

MINI-CASE 1:

**Employer Pension Plan Participation**: Since her employer matches employee contributions by up to 6 percent of their salary, Jenny's total contribution to her pension plan will increase by $2,400, calculated as $40,000 x 6 percent. Assuming an interest rate of 6 percent, compounded annually, the value of Jenny's employer contributions may be calculated as follows assuming Jenny retires at age 60, her employer makes monthly contributions, and she receives no further increases in her annual salary.

P/Y = 12, C/Y = 1, N = 384, I/Y = 6, PV = 0, PMT = 200, **FV = ?**

**The future value of employer contributions at retirement would be $224,071.09.**

**Down Payment on a New Car:** The amount that Jenny needs to save each year in order to make a down payment of $8,000 towards the purchase of a new car is calculated as follows.

P/Y = 1, C/Y = 1, N = 2, I/Y = 6, PV = 0, **PMT = ?,** FV = 8,000

**Jenny needs to save $3,883.50 each year for the down payment on a new car.**

**Future Value of Trust Fund:** The future value of one half of Jenny's trust fund at age 60 is calculated as follows. Note: The calculation assumes that Jenny is able to maintain her investment tin bonds at a rate of return of 7 percent, compounded quarterly.

P/Y = 1, C/Y = 4, N = 30, I/Y = 7, PV = 25 000, PMT = 0, **FV = ?**

**The value of Jenny's trust fund at age 60 will be $200,479.59.**

**Repay Student Loans:** Paul's annual payment on his student loan is calculated as follows.

P/Y = 1, C/Y = 1, N = 5, I/Y = 7.75, PV = 40 000, PMT = ?, **FV = 0**

**In order to pay off his student loan within 5 years, Paul will need to make annual payments of $9.886.59.**

MINI-CASE 2:

**Will James be able to withdraw enough from his other investments?**

James' remaining monthly expenses will be **$2,000**, calculated as $5800 **-** $3800. If James leaves his retirement bonus in his bank account, he will have a total of $250 000 in his bank account. At an interest rate of 2 percent per year, compounded annually, James will be able to generate a monthly income of **$921.79** (see TVM calculation below) during the next 30 years from his bank account.

P/Y = 12, C/Y = 1, N = 360, I/Y = 2, PV = 250 000, **PMT = ?,** FV = 0

In addition, at an interest rate of 5 percent per year, compounded annually, James will be able to generate a monthly income of **$795.08** (see TVM calculation below) during the next 30 years from his retirement savings account.

P/Y = 12, C/Y = 1, N = 360, I/Y = 5, PV = 150 000, **PMT = ?,** FV = 0

As a result, James will have a monthly income shortfall of **$283.13**, calculated as $2,000**-** (921.79 + 795.08).

**How long will James be able to withdraw income from his other investments?**
If James withdraws the required income of $2,000 from the account that earns the lower interest rate, i.e. his bank account, he will be able to cover his remaining expenses for approximately 140 months, or 11 years and 8 months (see TVM calculation below).

P/Y = 12, C/Y = 1, **N = ?,** I/Y = 2, PV = (250 000), PMT = 2000, FV = 0

While he is using his bank account to fund his retirement, James' retirement savings account will continue to grow at 5 percent per year, compounded annually. As a result, he will have $265,032.88 (see TVM calculation below) in his retirement savings account when he runs out of savings in his bank account.

P/Y = 12, C/Y = 1, N =140, I/Y = 5, PV = 150 000, PMT = 0, **FV = ?**

James will be able to withdraw money from his retirement savings account to cover his remaining expenses of $2,000 for approximately 190 months, or 15 years and 10 months (see TVM calculation below).

P/Y = 12, C/Y = 1, **N = ?,** I/Y = 5, PV = (265 032.88), PMT = 2000, FV = 0

In total, James will be able to withdraw $2,000 of monthly income from his other investments for approximately 27 years and 6 months.

**What rate of return on his investments will allow James to cover his income shortfall during his retirement years?**
After James has withdrawn all of his savings from his bank account, he will still have 220 months, or 18 years and 4 months of retirement to cover. In order to cover this period of time, his retirement savings account would need to generate a rate of return of 6.24 percent per year.

P/Y = 12, C/Y = 1, N = 220, **I/Y = ?,** PV = (265 032.88), PMT = 2000, FV = 0

## ■ Answers to Questions in The Sampson Family: A Continuing Case

1. *Savings Accumulated Over the Next 12 Years (Based on Plan to Save $300 per month)*

| | | |
|---|---|---|
| Amount Saved Per Month | $300 | $300 |
| Interest Rate | 5% | 7% |
| Years | 12 | 12 |
| Future Value of Savings | $59,029 | $67,408 |

*Savings Accumulated Over the Next 12 Years (Based on Plan to Save $400 per month)*

| | | |
|---|---|---|
| Amount Saved Per Year | $400 | $400 |
| Interest Rate | 5% | |
| | 7% Years | 12 |
| | 12 | |
| Future Value of Savings | $78,705 | $89,878 |

2. If the Sampsons save $300 per month, the higher interest rate would result in an extra accumulation in savings of more than $8,000. If they save $400 per month, the higher interest rate would result in an extra accumulation in savings of more than $11,000 per year.

3. Using a 5% interest rate, saving $400 per month instead of $300 would increase their total savings by more than $19,000. Using a 7% interest rate, saving $400 per month instead of $300 would increase their total savings by more than $22,000.

4. To achieve a goal of $70,000 over 12 years, they would need to save an annual amount (annuity) as determined below:

P/Y = 1, C/Y = 12, N = 12, I/Y = 5, PV = 0, **PMT = ?**, FV = 70000

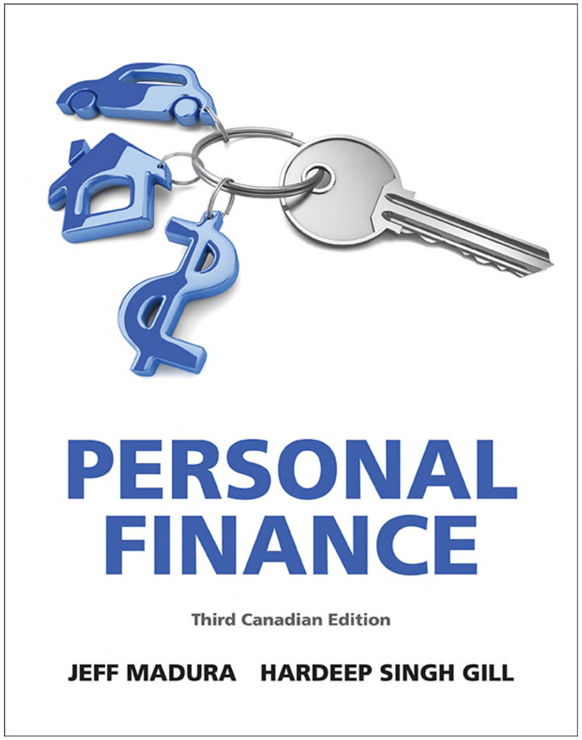
Thus, they would have to invest $4,368 by the end of each year to accumulate $70,000 in twelve years.

## ■ Answers to Myth or Fact Margin Questions

<table>
<tr><th>Page</th><th>Myth or Fact</th></tr>
<tr><td>23</td><td>The interest rate that you are quoted on an investment or loan represents the amount of interest that you will earn or pay.</td></tr>
<tr><td colspan="2">Myth. The interest rate quoted, i.e. the nominal rate of interest, may not be the same as the interest earned or paid, i.e. the effective or real rate of interest. It is important to know the number of compounding periods associated with a loan or investment.</td></tr>
<tr><td>31</td><td>All financial calculators calculate the time value of money in the same manner.</td></tr>
<tr><td colspan="2">Myth. Financial calculators produced by different manufacturers will involve different steps when performing a time value of money calculation.</td></tr>
<tr><td>33</td><td>Future value interest factors (FVIF) and a financial calculator will generate different answers to a question.</td></tr>
<tr><td colspan="2">Fact. Due to rounding error, each method will provide a slightly different answer.</td></tr>
</table>

# Chapter 2

# Applying Time Value Concepts



# Chapter Objectives

- Explain the difference between simple interest and compound interest
- Calculate the future value of a single dollar amount that you save today
- Calculate the present value of a single dollar amount that will be received in the future

# Chapter Objectives (cont'd)

- Calculate the future value of an annuity
- Calculate the present value of an annuity
- Convert a nominal interest rate to an effective interest rate

# The Importance of the Time Value of Money

- Money grows over time when you receive a return on your investment
- Interest: the rent charged for the use of money
- There are two ways of computing interest – simple interest and compound interest

# The Importance of the Time Value of Money (cont'd)

- Simple interest: interest on a loan computed as a percentage of the loan amount, or principal
  - Interest earned, or paid, is not reinvested
  - Measured using the principal (P), the interest rate applied to the principal (r), and the loan's time to maturity (t)

# The Importance of the Time Value of Money (cont'd)

$$I = P \times r \times t$$

where $I$ = interest earned

$P$ = principal, or present value

$r$ = annual interest rate expressed as a decimal or percent

$t$ = time (in years)

# The Importance of the Time Value of Money (cont'd)

Farah makes a deposit of $1000 in a high-interest savings account paying 3 percent interest annually. At the end of year one, the bank will credit Farah's chequing account with $30, calculated as:

$$I = P \times r \times t$$
$$\$30 = \$1000 \times 0.03 \times 1$$

In year two, the initial principal of $1000 will again earn $30 in interest. This amount will also be credited to Farah's chequing account.

# The Importance of the Time Value of Money (cont'd)

- Compound interest: the process of earning interest on interest
- Solving TVM problems:
  - TVM formulas
  - TVM tables
  - Financial calculator (e.g. TI BA II Plus)

# The Importance of the Time Value of Money (cont'd)

Samantha makes an initial deposit of $1000 in a compound interest savings account paying 3 percent interest annually. At the end of year one, the bank will credit Samantha's account with $30 ($1000 × 0.03 × 1). Compound interest *increases the principal amount on which Samantha earns interest in the second year, by the amount of interest that is earned in the first year.* In other words, in year two, she would earn 3 percent interest on the $1000 of original principal plus the $30 of interest earned in year one. This process will repeat itself in subsequent years.

| Year | Principal | Interest | Accumulated Interest | Account Balance |
|---|---|---|---|---|
| 1 | $1000 | $30 | $30 | $1030 |
| 2 | $1030 | $30.90 | $60.90 | $1060.90 |
| 3 | $1060.90 | $31.83 | $92.73 | $1092.73 |

# The Importance of the Time Value of Money (cont'd)

- Time value of money is most commonly applied to two types of cash flows – a single dollar amount and an annuity
- Annuity: the payment of a series of equal cash flow payments at equal intervals of time

# Future Value of a Single Dollar Amount

$$FV = PV\left(1 + \frac{i}{n}\right)^{nt}$$

where $FV$ = future value of an investment

$PV$ = present value of an investment

$i$ = annual interest rate (as a decimal or percent)

$n$ = number of compounding periods per year

$t$ = time (in years)

# The Importance of the Time Value of Money (cont'd)

Based on her initial deposit of $1000, Samantha now asks you to determine how much money she would have in her compound interest savings account if she keeps it open for the next 20 years. Since it would be tedious and time-consuming to create a table to calculate this amount, you have decided to use the future value formula for a single dollar amount. Using the formula above, you determine that she should have $1806.11 in her account after 20 years.

$$FV = PV(1 + i)^t = \$1000(1 + 0.03)^{20} = \$1806.11$$

# The Importance of the Time Value of Money (cont'd)

- The number of compounding periods per year for different compounding periods:

| Compounding Period | Number of Compounding Periods per Year (n) |
|---|---|
| Annually (every year) | 1 |
| Semi-annually (every 6 months) | 2 |
| Quarterly (every 3 months) | 4 |
| Monthly (every month) | 12 |
| Weekly (every week) | 52 |
| Daily (every day) | 365 |

# Future Value of a Single Dollar Amount (cont'd)

- Future Value Interest Factor (FVIF): a factor multiplied by today's savings to determine how the savings will accumulate over time.
  - Depends on the interest rate and the number of years the money is invested
- *FVIF* Table
  - Columns list an interest rate
  - Rows list a time period

# Future Value of a Single Dollar Amount (cont'd)

Suppose that you want to know how much money you will have in five years if you invest \$5000 now and earn an annual return of 9 percent. The present value of money (*PV*) is the amount invested, or \$5000. The *FVIF* for an interest rate of 9 percent and a time period of five years is 1.539 (look down the column for 9 percent, and across the row for five years). Thus, the future value (*FV*) of the \$5000 in five years will be:

$$FV = PV \times FVIF_{i,n}$$

$$FV = PV \times FVIF_{9\%,5}$$

$$= \$5000 \times 1.539$$

$$= \$7695$$

# Using A Financial Calculator

- Basic TVM function keys are located in the 3rd row of the TI BA II Plus keyboard
- Important to clear the existing TVM values in the calculator's TVM worksheet
- Enter a value for four of the five TVM keys (N, I/Y, PV, PMT, FV)

# Using A Financial Calculator (cont'd)

- Cash outflows, such as an investment amount, are entered as a negative number
- Cash inflows, such as investment income, are entered as a positive number
- Specify the number of payments per year (P/Y) and the number of compounding periods per year (C/Y)

# Using A Financial Calculator (cont'd)

Suppose you have $5687 to invest in the stock market today. You like to invest for the long term and plan to choose your stocks carefully. You will invest your money for 12 years in certain stocks on which you expect a return of 10 percent compounded annually. The inputs for the TI BA II Plus are shown in the diagram in the margin. Note that other financial calculators can vary slightly in their setup.

The calculator key strokes are as follows:

2ND CLR TVM
2ND P/Y 1 ENTER
↓ 1 ENTER
2ND QUIT
1 2 N
1 0 I/Y
5 6 8 7 +/− PV
0 PMT
CPT FV

# Using A Financial Calculator (cont'd)

The *PV* is a negative number here, reflecting the outflow of cash to make the investment. The calculator computes the future value to be $17 848.24, which indicates that you will have $17 848.24 in your brokerage account in 12 years if you achieve a return of 10 percent compounded annually on your $5687 investment.

Use a financial calculator to determine the future value of $5000 invested at 9 percent compounded annually for five years. (This is the previous example used for the *FVIF* table.) Your answer should be $7693.12. Any difference in answers using the *FVIF* table versus using a financial calculator is due to rounding.

# Present Value of a Single Dollar Amount

- Discounting: the process of obtaining present values

$$PV = \frac{FV}{(1 + \frac{i}{n})^{nt}}$$

# Future Value of a Single Dollar Amount (cont'd)

## Example

Suppose you want to know how much money you need to invest today to achieve $20 000 in three years. (NOTE: $n$ = 1)

$$PV = \frac{FV}{(1 + \frac{i}{n})^{nt}}$$

$$\$18\,302.83 = \frac{\$20\,000}{(1 + .03)^3}$$

# Future Value of a Single Dollar Amount (cont'd)

- Present Value Interest Factor (PVIF): a factor multiplied by the future value to determine the present value of that amount.
  - Depends on the interest rate and the number of years the money is invested
- *PVIF* Table
  - Columns list an interest rate
  - Rows list a time period

# Future Value of a Single Dollar Amount (cont'd)

Loretta Callahan would like to accumulate $500 000 by the time she retires in 20 years. If she can earn an 8.61 percent return compounded annually, how much must she invest today to have $500 000 in 20 years? Since the unknown variable is the present value (*PV*), the calculator input will be as shown at left.

The calculator key strokes are as follows:

2ND | CLR TVM

2ND | P/Y | 1 | ENTER

↓ | 1 | ENTER

2ND | QUIT

2 | 0 | N

8 | – | 6 | 1 | I/Y

5 | 0 | 0 | 0 | 0 | 0 | FV

0 | PMT

CPT | PV

Therefore, Loretta would have to invest $95 845.94 today to accumulate $500 000 in 20 years if she really earns 8.61 percent compounded annually.

# Future Value of an Annuity

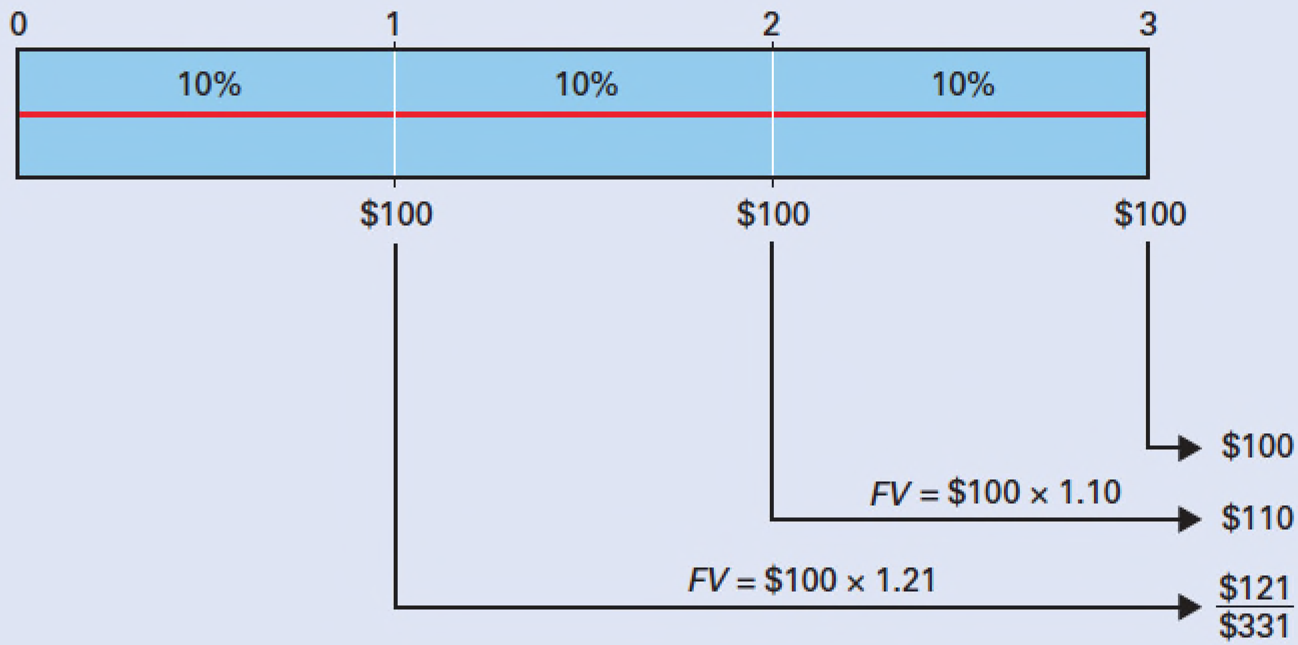
- Ordinary annuity: a stream of equal payments that are received or paid at equal intervals in time at the end of a period
- Annuity due: a series of equal cash flow payments that occur at the beginning of each period

# Future Value of an Ordinary Annuity

- If the payment changes over time the payment stream does not reflect an annuity
- The best way to illustrate the future value of an ordinary annuity or an annuity due is through the use of timelines
- Timelines: diagrams that show payments received or paid over time

# Future Value of an Ordinary Annuity (cont'd)

You plan to invest $100 at the end of every year for the next three years. You expect to earn an annual interest rate of 10 percent compounded annually on the funds you invest. Using a timeline, the cash flows from this ordinary annuity can be represented as follows:



# Future Value of an Ordinary Annuity (cont'd)

- For an ordinary annuity, the future value can be determined using the formula:

$$FV = PMT \ x \ [\frac{(1+i)^n - 1}{i}]$$

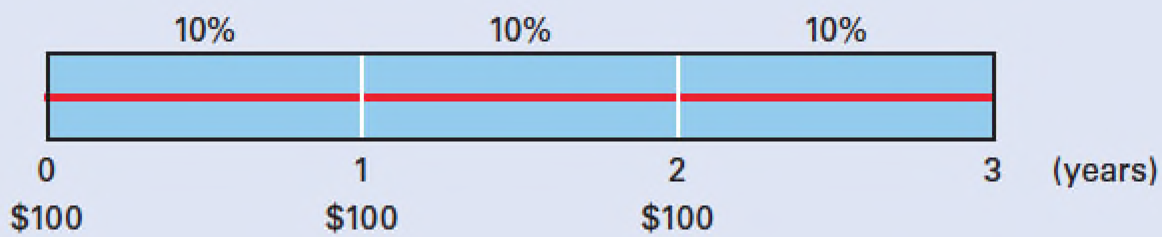
# Future Value of an Ordinary Annuity (cont'd)

- Using the information from the timeline example:

$$\$331 = \$100\ x\ [\frac{(1 + 0.10)^3 - 1}{0.10}]$$

# Future Value of an Annuity Due

In contrast, consider the situation where you plan to invest $100 at the beginning of every year for the next three years. Assuming that you earn the same interest rate, the cash flows from this annuity due can be represented as follows:




The diagram above reflects the cash flows that would be paid from an annuity due.

# Future Value of an Annuity Due (cont'd)

- For an annuity due, the future value can be determined by multiplying the future value of an annuity formula by (1+$i$)

$$FV = PMT\ x\ \left[\frac{(1+i)^n - 1}{i}\right] x\ (1+i)$$

# Future Value of an Annuity Due (cont'd)

- Referring again to the timeline example, the value of your investment if you invest at the beginning of the year is:

$$\$331\ x\ (1 + i)$$

$$\$331\ x\ (1 + 0.10) = \$364.10$$

# Future Value of an Annuity

- Future Value Interest Factor for an Annuity(FVIFA): a factor multiplied by the periodic savings level (annuity) to determine how the savings will accumulate over time.
  - Depends on the interest rate and the number of years the money is invested
- *FVIFA* Table
  - Columns list an interest rate
  - Rows list a time period

# Future Value of an Annuity (cont'd)

Example

Suppose that you have won the lottery and will receive $150 000 at the end of every year for the next 20 years. The payments represent an ordinary annuity and will be invested at your bank at an interest rate of 7 percent.

# Future Value of an Annuity (cont'd)

Example

$$FVA = PMT \ x \ FVIFA_{i,n}$$

$$FVA = PMT \ x \ FVIFA_{7\%,20}$$

$$\$6\ 149\ 250 = \$150\ 000\ x\ 40.995$$

# Using A Financial Calculator

You have instructed your employer to deduct $80 from your paycheque at the end of every month and automatically invest the money at an annual interest rate of 5 percent compounded annually. You intend to use this money for your retirement in 30 years. How much will be in the account at that time?

This problem differs from the others we have seen so far, in that the payments are received on a monthly (not annual) basis; as a result, $P/Y = 12$. You would like to obtain the future value of the annuity and consequently need the number of periods, the interest rate, the present value, and the payment. Because there are 12 months in a year, there are $30 \times 12 = 360$ periods. The number of compounding periods, $n$, can always be calculated as the number of years times the number of payments per year, $P/Y$. The interest rate is 5. Also, note that to determine the future value of an annuity, most financial calculators require an input of zero for the present value. The payment in this problem is −$80, since the deduction represents a cash outflow. The input for the calculator would be as shown at left.

The calculator key strokes are as follows:

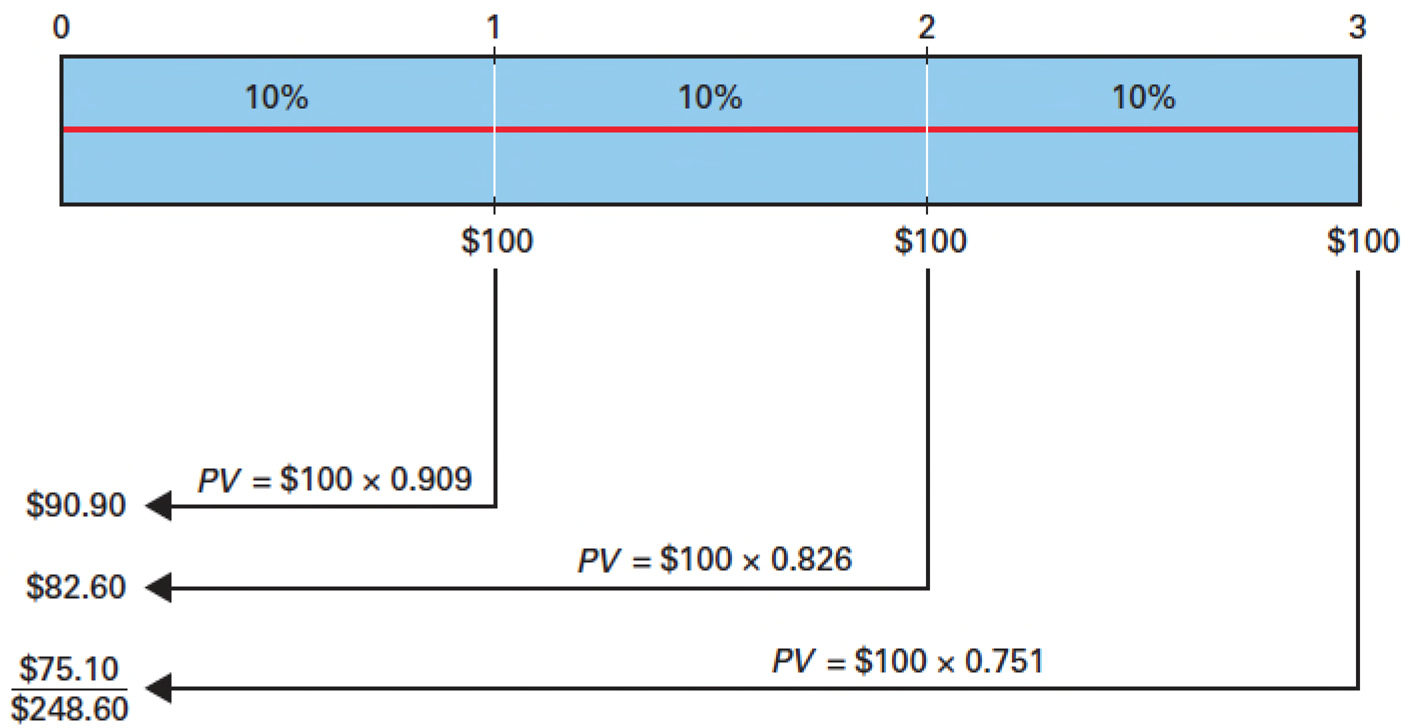
2ND CLR TVM
2ND P/Y 1 2 ENTER
↓ 1 ENTER
2ND QUIT
3 6 0 N
5 I/Y
0 PV
8 0 +/− PMT
CPT FV

# Present Value of an Annuity

- The present value of an annuity can be obtained by discounting the individual cash flows of the annuity and totaling them

# Present Value of an Ordinary Annuity

- Referring to our earlier example:



# Present Value of an Annuity (cont'd)

- The present value of an ordinary annuity can be determined using the formula:

$$PV = PMT\ x\ [\frac{1 - [\frac{1}{(1+i)^n}]}{i}]$$

# Present Value of an Annuity (cont'd)

- Using the information from the timeline example:

$$\$248.69 = \$100\, x\, [\frac{1 - [\frac{1}{(1 + 0.10)^3}]}{0.10}]$$

- If payments are at the beginning of each period i.e. an annuity due, you would multiply your answer by $(1 + i)$

# Present Value of an Annuity (cont'd)

- Present Value Interest Factor for an Annuity (PVIFA): a factor multiplied by a periodic savings level (annuity) to determine the present value of the annuity.
  - Depends on the interest rate and the number of years the money is invested
- *PVIFA* Table
  - Columns list an interest rate
  - Rows list a time period

# Present Value of an Annuity (cont'd)

## Example

You have just won the lottery. As a result of your luck, you will receive \$82 000 at the end of every year for the next 25 years. A financial firm offers you a lump sum of \$700 000 in return for these payments. If you can invest your money at an annual interest rate of 9 percent, should you accept the offer?

# Present Value of an Annuity (cont'd)

Example

$$PVA = PMT \; x \; PVIFA_{i,n}$$

$$PVA = PMT \; x \; PVIFA_{9\%,25}$$

$$\$805\,486 = \$82\,000 \; x \; 9.823$$

# Using A Financial Calculator

Dave Buzz, a recent retiree, receives his $600 pension at the end of each month. He will receive this pension for 20 years. If Dave can invest his funds at an interest rate of 10 percent compounded annually, he should be just as satisfied receiving this pension as receiving a lump sum payment today of what amount?

This problem requires us to determine the present value of the pension annuity. Since payments are received monthly, $P/Y = 12$. Because there are $20 \times 12 = 240$ months in 20 years, $n = 240$. The $600 monthly pension is a cash inflow to Dave, so this amount is entered as a positive number. Using these inputs with a financial calculator, we obtain the inputs shown at left.

# Using A Financial Calculator (cont'd)

The calculator key strokes are as follows:

2ND | CLR TVM

2ND | P/Y | 1 | 2 | ENTER

↓ | 1 | ENTER

2ND | QUIT

2 | 4 | 0 | N

1 | 0 | I/Y

0 | FV

6 | 0 | 0 | PMT

CPT | PV

The present value is $64 059. If Dave is offered a lump sum of $64 059 today, he should accept it if he can invest his funds at a minimum of 10 percent compounded annually. What would be the present value of Dave's pension if he received it at the beginning of each month? You should get $64 570.

# Interest Rate Conversion

- Nominal interest rate: the stated or quoted, rate of interest
  - Also known as the annual percentage rate (APR)
- When comparing two or more interest rates, the nominal interest rate is not useful because it does not take into account the effect of compounding

# Interest Rate Conversion (cont'd)

- Effective interest rate: the actual rate of interest that you earn, or pay, over a period of time
  - Also known as the effective yield (EY)
- Allows for the comparison of two or more interest rates because it reflects the effect of compound interest

# Interest Rate Conversion (cont'd)

- Convert nominal interest rates to effective interest rates using the following formula:

$$EY = \left[\left(1 + \frac{i}{n}\right)^{n}\right] - 1$$

# Interest Rate Conversion (cont'd)

- Effective yield of 10 percent compounded semi-annually is:

$$.1025 = \left[\left(1 + \frac{0.10}{2}\right)^2\right] - 1$$

# Interest Rate Conversion (cont'd)

- Equivalent effective interest rates for various nominal interest rates:

| Nominal Interest Rate | Effective Interest Rate |
|---|---|
| 10% compounded annually | 10.00% |
| 10% compounded semiannually | 10.25% |
| 10% compounded quarterly | 10.38% |
| 10% compounded monthly | 10.47% |
| 10% compounded weekly | 10.51% |
| 10% compounded daily | 10.52% |

# Using A Financial Calculator

These values demonstrate two important points: (1) The nominal and effective interest rates are equivalent for annual compounding, and (2) the effective annual interest rate increases with increases in compounding frequency.

The effective yield can also be calculated using the TI BA II Plus calculator.

Marco has $4000 invested at 4.5 percent compounded semi-annually at his bank. In order to make a comparison with another financial institution, he needs to know the effective interest rate at his bank. What is the effective annual interest rate? For this problem, the number of compounding periods is two. Use the interest conversion worksheet in the TI BA II Plus calculator to solve as follows:

2ND ICONV 4 . 5 ENTER (the nominal interest rate)

↑ 2 ENTER (number of compounding periods)

↑ CPT (calculate the effective interest rate)

The effective yield is 4.55 percent.