Chapter 2

Lecture Notes and Teaching Tips

This chapter includes chapter overviews, section-by-section lecture notes, and teaching tips for the lecture notes. The lecture notes include homework assignments.

The teaching tips for the lecture notes describe typical student difficulties and what an instructor can do to help students overcome these obstacles.

CHAPTER 1 OVERVIEW

Chapter 1 contains a large number of concepts. If you cannot afford the time to address them all, focus on Sections 1.1, 1.2, and 1.7 because they contain important algebra concepts.

Of these three sections, Section 1.7 is the most important because it contains the key concepts exponents, square roots, order of operations, and scientific notation. Exponents and scientific notation are presented so early because they are necessary to interpret calculator outputs when finding normal probabilities in Section 5.4. I considered postponing these two concepts until Chapter 5, but I was concerned that doing so would break the flow of students learning statistics in Chapters 2–5.

Proportions (Section 1.3) should be discussed because they will be used throughout Chapters 2–5. Converting units (Section 1.3) will come in handy for a few exercises in subsequent chapters. The concept change in a quantity (Section 1.5) should be discussed because this concept lays the foundation for slope of a line and other concepts. Ratios and percents (Section 1.6) should also be addressed.

I strongly advise obtaining data about your students by anonymously surveying them. Although there are plenty of current, compelling, authentic data sets to choose from in the textbook, students will get a kick out of analyzing data about themselves. This data can first be used in Chapter 3. Collecting it now will buy you time to input it electronically, although StatCrunch and other technologies can do that automatically.

Here are questions I typically include in my survey:

1. How many units are you taking right now?
2. How many classes are you taking right now?
3. What is your major? (If you don't have one, write "undecided.")
4. How many hours do you work per week during this semester on average?
5. What is your gender? (male or female)
6. What is the total number of people living in your household at this time (include yourself)?
7. Do you exercise? If yes, how many minutes do you spend exercising per day, on average?
8. What is your favorite genre of music? (hip hop, rap, electronic, rock, country, classical, folk, etc)

9. How many novels do you read per year for fun (not for a class)?
10. Which political party do you tend to be in line with (e.g. Republican, Democrat, Independent, etc)?
11. What is your religious affiliation (e.g. Catholic, Protestant, Mormon, etc)
12. How many friends did you have during your senior year in high school? How many of those people are you still in touch with and consider to still be friends with?
13. How many cigarettes did you smoke in the last month?
14. Do you drive a car? If yes, have you ever intentionally entered an intersection when the light was red ("run" a red light)?
15. How many hours do you spend online on a weekday?
16. How many hours do you spend watching TV shows, movies, and videos on a weekday?
17. What is your age?
18. What is your height?
19. How many languages do you speak?
20. What is the total number of cars owned by people in your household?
21. How many tattoos do you have?
22. Estimate your professor's (Jay Lehmann's) age.
23. Is your mother alive? If yes, what is her age?
24. How many states have you lived in?
25. How many times do you not tell the truth per day?
26. How often do you go to church? (never, once a month, once a week, twice a week, daily)
27. How many times do you go to a movie theater each month?
28. Do you have a Facebook account? If yes, how many friends do you have on Facebook?
29. Circle the social media that you use:

    Facebook Twitter Instagram Pinterest Vine Snapchat

30. How many texts do you send on a weekday?
31. How many e-mails do you send on a weekday?
32. How many tweets do you send on a weekday?
33. How many times do you eat at a restaurant (including fast-food) per week?
34. How many times do you eat at fast-food restaurants per week?
35. If you could have one superpower, what would it be?
36. How many alcoholic drinks do you have per week?

Also consider adding another dimension to your course by finding data about your college, the college's neighborhood, and breaking news about your state. Analysis of such data will impress upon students that statistics is truly relevant.

SECTION 1.1 LECTURE NOTES

Objectives

1. Describe the meaning of *variable* and *constant*.
2. Describe the meaning of *counting numbers*, *integers*, *rational numbers*, *irrational numbers*, *real numbers*, *positive numbers*, and *negative numbers*.
3. Use a number line to describe numbers.
4. Graph data on a number line.
5. Plot points on a coordinate system.
6. Describe the meaning of *inequality symbols* and *inequality*.
7. Graph an inequality and a compound inequality on a number line.
8. Use inequality notation, interval notation, and graphs to describe possible values of a variable for an authentic situation.
9. Describe a concept or procedure.

OBJECTIVE 1

Definition *Variable*

A **variable** is a symbol that represents a quantity that can vary.

1. Let p be the price (in dollars) to see a Modest Mouse concert. What is the meaning of $p = 60$?
2. Let t be the number of years since 2010. What is the meaning of $t = 7$? of $t = -5$?
3. Choose a symbol to represent the number of students in a class. Explain why the symbol is a variable. Give two numbers that the variable can represent and two numbers that it cannot represent.

Definition *Constant*

A **constant** is a symbol that represents a specific number (a quantity that does *not* vary).

4. A rectangle has an area of 16 square feet. Let W be the width (in feet), L be the length (in feet), and A be the area (in square feet).

   a. Sketch three possible rectangles of area 16 square feet.

b. Which of the symbols W, L, and A are variables? Explain.

c. Which of the symbols W, L, and A are constants? Explain.

OBJECTIVE 2

- The **counting numbers**, or **natural numbers**, are the numbers 1, 2, 3, 4, 5, ...
- The **integers** are the numbers . . . , -3, -2, -1, 0, 1, 2, 3, ...
- The **positive integers** are the numbers 1, 2, 3, ...
- The **negative integers** are the numbers -1, -2, -3, ...
- The number 0 is neither positive nor negative.
- The **rational numbers** are the numbers that can be written in the form $\frac{n}{d}$, where n and d are integers and d is nonzero.
- The numbers represented on the number line that are *not* rational are called **irrational numbers**.
- The **real numbers** are all of the numbers represented on the number line.

OBJECTIVE 3

5. Graph the integers between -4 and 2, inclusive, on one number line.

6. Graph the integers between -4 and 2 on one number line.

7. Graph all of the numbers 3, -5, $\frac{7}{2}$, 0, -2.7, and -1.3 on one number line.

OBJECTIVE 4

8. The average student debts per borrower are shown in the following table for the five states with the largest per-student debts.

State	Average Student Debt (thousands of dollars)
Georgia	30.4
Maryland	30.0
Virginia	28.5
South Carolina	28.3
Florida	27.9

Source: *U.S. Department of Education*

Let d be a state's average student debt (in thousands of dollars). Graph the average debts shown in the table on a number line.

When we write numbers on a number line, they should increase by a fixed amount and be equally spaced.

OBJECTIVE 2 (revisited)

- The **negative numbers** are the real numbers less than 0.
- The **positive numbers** are the real numbers greater than 0.

9. Let T be the temperature (in degrees Fahrenheit). What value of T represents the temperature 20 degrees Fahrenheit below 0? Graph the number on a number line.

OBJECTIVE 5

10. Plot the points $(2, 5)$, $(-3, 1)$, $(-2, -4)$, and $(6, -3)$ on a coordinate system.

OBJECTIVE 6

Here are the definitions of **inequality symbols**:

Symbol	**Meaning**
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to

An **inequality** contains one of the symbols $<$, $\leq$, $>$, and $\geq$ with a constant or variable on one side and a constant or variable on the other side.

11. Decide whether the inequality statement is true or false.

a. $4 < 7$ **b.** $3 \leq 3$ **c.** $-4 > -4$ **d.** $-6 \geq -2$

OBJECTIVE 7

Write the inequality in interval notation, and graph the values of x.

12. $x < 4$ **13.** $x \geq -3$ **14.** $-2 < x \leq 5$

OBJECTIVE 8

15. A person held his breath for at least 50 seconds. Let t be the length of time (in seconds) that he held his breath. Describe the length of time he held his breath using inequality notation, interval notation, and a graph.

16. Let t be the time (in minutes) it takes for a student to drive to college. Interpret and graph the inequality $19 < t < 23$.

OBJECTIVE 9

Here are some guidelines on writing a good response:

- Create an example that illustrates the concept or outlines the procedure. Looking at examples or exercises may jump-start you into creating your own example.
- Using complete sentences and correct terminology, describe the key ideas or steps of your example. You can review the text for ideas, but write your description in your own words.

- Describe also the concept or the procedure in general without referring to your example. It may help to reflect on several examples and what they all have in common.
- In some cases, it will be helpful to point out the similarities and the differences between the concept or the procedure and other concepts or procedures.
- Describe the benefits of knowing the concept or the procedure.
- If you have described the steps in a procedure, explain why it's permissible to follow these steps.
- Clarify any common misunderstandings about the concept, or discuss how to avoid making common mistakes when following the procedure.

17. Describe how inequality notation, interval notation, and graphs can be used to describe possible values of a variable for an authentic situation.

HW 1, 3, 5, 15, 21, 29, 31, 47, 51, 55, 71, 81, 85, 99, 107

SECTION 1.1 TEACHING TIPS

This section contains a lot of objectives, but students have an easy time with this section, so I advise not getting too bogged down. Aside from the obvious importance of variables, it's a good idea to focus on inequalities because these are used so much with probability (Chapter 5). Students' greatest challenge in this section tends to be using interval notation correctly.

OBJECTIVE 1 COMMENTS

Problem 2 is good preparation for working with time-series data because students will sometimes get a negative value when using a model to make an estimate for the explanatory variable. Most students think that time cannot be negative and are surprised that $t = -5$ represents the year 2005.

In Problem 3, discussing unreasonable values of the variable is a nice primer for the concept of model breakdown (Section 6.3).

After completing Problems 1–3, I quickly mention several more examples of variables, pointing out that variables "are all around us." I suggest that throughout the rest of the day, my students should notice quantities that can be represented by variables. I say that a variable is a key building block of algebra and statistics.

In Section 2.1, a key distinction will be made between how variables are defined and used in algebra and statistics. But for now I restrict my discussion about variables to the scope of algebra.

When discussing constants, I give π and a number such as 5 as examples.

Before completing Problem 4, I discuss the meaning of the area of a flat surface. Most students do not know that the area (in square inches) is the number of square inches that it takes to cover the surface. I remind my students that the area of a rectangle is equal to the rectangle's length times its width and explain this by breaking up a rectangle into unit squares. Although these are simple concepts, they are critical development for the frequent use of density histograms in Chapters 3–5.

OBJECTIVE 2 COMMENTS

When I define counting numbers and integers, I make sure that students understand the meaning of the ellipsis. I quickly define the various types of numbers, giving especially light treatment to defining rational and irrational numbers because even though these terminologies are used in the textbook, a superficial understanding of them will suffice.

Comparing the differences between the integers and the real numbers can help prepare students to learn about discrete and continuous variables (Section 3.3).

OBJECTIVE 3 COMMENTS

Graphing numbers on the number line is good preparation for constructing dotplots, time-series plots, and scatterplots. Discussing how to graph decimal numbers is especially important for work with authentic data (Problem 8).

Most students do not understand how Problems 5 and 6 differ. This issue will keep coming up throughout the course (e.g. "Find the probability that a randomly selected test score is between 80 and 89 points, inclusive.").

OBJECTIVE 4 COMMENTS

For Problem 8, I emphasize that the number-line sketch should include a title, the variable "d", and the units of d. I also emphasize that when we write numbers on a number line, they should increase by a fixed amount and be equally spaced. Without giving such a warning several times, many students will make related errors when constructing number lines and various types of statistical diagrams.

To further engage students when discussing Problem 8, I ask them to identify in which region(s) the states are located. And I ask for some possible reasons why the average student debt is higher in those five states. Then I generalize, suggesting that students should be this engaged when working with data sets throughout the course.

GROUP EXPLORATION: Reasonable Values of a Variable

The exploration is a good primer for the concept of model breakdown (Section 6.3).

OBJECTIVE 2 (REVISITED) COMMENTS

I emphasize that 0 is neither negative nor positive.

OBJECTIVE 5 COMMENTS

When plotting a point, I remark that it's similar to looking up a date on a calendar. This skill is a precursor to constructing time-series plots (Section 3.3) and scatterplots (Section 6.1).

OBJECTIVE 6 COMMENTS

Students tend to have difficulty with Problem 11(b) and sometimes 11(c).

OBJECTIVE 7 COMMENTS

Even if students have seen interval notation before, they tend to have forgotten it. Understanding the meaning of the inequalities in Problems 12–15 will be good preparation for probability problems (Sections 5.1–5.3) and normal distribution problems (Sections 5.4 and 5.5). Problem 14 also provides a lead-in to confidence intervals, whose foundation will be further developed in Section 8.5.

OBJECTIVE 8 COMMENTS

Throughout the course, I look for every opportunity to have students work with the phrases *less than*, *at most*, *greater than*, and *at least* because many students struggle with their meanings in introductory statistics. Problem 15 is a great way to get that process started.

A subtle issue to explore for Problem 15 is that an answer of $(50, \infty)$ does not imply that we believe the person could hold their breath for an extremely long time such as a 1000 minutes. Rather, it is impossible to know what upper limit to choose. For example, it may surprise you or your students that Stig Severinsen set a Guinness World Record by holding his breath for 22 minutes. Here's a link at the Guinness World Records site: http://www.guinnessworldrecords.com/world-records/24135-longest-time-breath-held-voluntarily-male.

OBJECTIVE 9 COMMENTS

When asked to describe concepts, students tend to write a single sentence or two. They need me to model a thorough response: describing a concept's meaning, listing benefits and drawbacks, comparing, contrasting, stating exceptions, and so on. Most students are astounded by how much there is to say.

I will model this for students several times throughout the course, but in the meantime I tell them to read Example 12 in the textbook and the preceding "Guidelines on Writing a Good Response" on page 12 of the textbook at home. Students will be asked to explain concepts or procedures in most homework sections. This is very much in line with introductory statistics, where students will have to describe statistical practices and interpret concepts and results.

SECTION 1.2 LECTURE NOTES

Objectives

1. Describe the meaning of *expression* and *evaluate an expression*.
2. Use expressions to describe authentic quantities.
3. Evaluate expressions.
4. Translate English phrases to and from mathematical expressions.
5. Evaluate expressions with more than one variable.

OBJECTIVES 1–3

1. A person is driving 3 miles per hour over the speed limit. For each speed limit shown, find the diving speed.

   a. 55 mph **b.** 70 mph **c.** s mph

We call $s + 3$ is an *expression*.

Definition *Expression*

An **expression** is a constant, a variable, or combination of constants, variables, operation symbols, and grouping symbols, such as parentheses.

Here are some examples of expressions: $x + 9$, 5, $x - 7$, π, $\dfrac{20}{x}$, xy.

2. Substitute 65 for s in the expression $s + 3$ and discuss the meaning of the result (see Problem 1).

We say we have *evaluated the expression* $s + 3$ at $s = 65$.

Definition *Evaluate an expression*

We **evaluate an expression** by substituting a number for each variable in the expression and then calculating the result. If a variable appears more than once in the expression, the same number is substituted for that variable each time.

3. A certain type of pen costs $3.

   a. Complete the following table to help find an expression that describes the total cost (in dollars) of n pens. Show the arithmetic to help see the pattern.

Number of Pens	Total Cost (dollars)
5	
6	
7	
8	
n	

b. Evaluate your result for $n = 10$. What does it mean in this situation?

- We avoid using $\times$ for the multiplication operation.
- Each of the following expressions describes multiplying 3 by n: $3n$, $3 \cdot n$, $3(n)$, $(3)n$, and $(3)(n)$.

OBJECTIVE 4

Mathematics is a language.

> **Definition** *Product, factor, and quotient*
>
> Let a and b be numbers. Then
>
> - The **product** of a and b is ab. We call a and b **factors** of ab.
> - The **quotient** of a and b is $a \div b$, where b is not zero.

4. Use phrases such as "2 plus x" and sentences such as "Add 2 and x." to complete the second column of the following table.

Mathematical Expression	English Phrase or Sentence
$2 + x$	
$2 - x$	
$2 \cdot x$	
$2 \div x$	

Warning: Subtracting 2 from 7 is $7 - 2$, not $2 - 7$.

5. Let x be a number. Translate the English phrase into a mathematical expression or vice versa, as appropriate:

a. The difference of the number and 9

b. The product of 5 and the number

c. $7 + x$

d. $x \div 4$

6. Let x be a number. Translate the sentence "Subtract the number from 10." into a mathematical expression. Evaluate the expression at $x = 7$.

OBJECTIVE 5

Recall that the area of a rectangle is equal to the length times the width of the rectangle.

7. Let W be the width (in feet) and L be the length (in feet) of a rectangle. Evaluate the expression LW for $L = 7$ and $W = 5$. What does your result mean in this situation?

8. Write the phrase "the quotient of x and y" as a mathematical expression, and then evaluate the result for $x = 24$ and $y = 3$.

HW 1, 3, 11, 13, 19, 21, 23, 29, 35, 49, 53, 59, 61, 63, 67

SECTION 1.2 TEACHING TIPS

The most important skill in this section is evaluating expressions, which students pick up on quickly. Emphasizing translating English to and from mathematics helps build engaged reading, which is a key ingredient for success in introductory statistics.

OBJECTIVES 1–3 COMMENTS

For Part (a) of Problem 3, students want to simply write the answers, but I emphasize that we want to show the arithmetic to help see the pattern. Insisting that students do this now will form good habits so that students will be able to find more challenging expressions of the form $mx + b$ in Section 1.7. The form of students' answers will vary for Part (a), and they will be reassured to know that $3n = n(3)$. The commutative law for multiplication will be formally discussed in Section 8.1.

When discussing Problem 3, I emphasize that we can describe the number of pens and their total cost by the variable n and the expression $3n$, respectively. So, by the abstraction of algebra we are able to relate two quantities. This is the fundamental idea between relations (Section 7.4), functions (Section 7.4), and ultimately regression (Chapters 9 and 10).

GROUP EXPLORATION: Expressions Used to Describe a Quantity

In this exploration, students work backwards and find an authentic quantity that can be described by the given expression. Ideally, this prompts students to reflect on how expressions connect to their world. In fact, as a follow-up or replacement for the exploration, you could have groups of students create their own expressions that describe authentic quantities that are important or interesting to them. This might stump them, so be prepared to model how to do this.

OBJECTIVE 4 COMMENTS

I emphasize that mathematics is a language. Most students have never thought of mathematics in this way.

Most students do not know the meanings of *product*, *quotient*, and *ratio*. These terminologies will be used extensively in the textbook. In particular, devoting some attention to ratios will serve as a good primer for rate of change and slope (Section 7.2). Computing ratios about your class (e.g. ratio of nursing majors to psychology majors) and comparing them to your college's student body can be interesting and instructive.

Although students won't have to translate English phrases to and from mathematical expressions in introductory statistics, they will have to read passages carefully for meaning. This important process can be fostered in part by practicing translating.

I emphasize that "Subtracting 2 from 7" is $7 - 2$, not $2 - 7$. I point out that if you take 2 CDs *from* 7 CDs, then 5 CDs remain. This may help students see why we subtract the numbers "out of order."

OBJECTIVE 5 COMMENTS

Students have an easy time with this objective. I advise working with the area of a rectangle such as in Problem 7 to continue to prepare students for density histograms, which will be emphasized in Chapters 3–5.

SECTION 1.3 LECTURE NOTES

Objectives

1. Describe the meaning of a fraction.
2. Explain why division by zero is undefined.
3. Describe the rules for $a \cdot 1$, $\frac{a}{1}$, and $\frac{a}{a}$.
4. Perform operations with fractions.
5. Find the prime factorization of a number.
6. Simplify fractions.
7. Find proportions.
8. Convert units of quantities.

OBJECTIVE 1

1. Shade a portion of a circle to illustrate the meaning of $\frac{5}{8}$ of a pizza.
2. Use two pizzas with 4 slices each to show that $\frac{8}{4} = 8 \div 4 = 2$.

The fraction $\frac{a}{b}$ means $a \div b$.

OBJECTIVE 2

Use the concept that division is repeated subtraction to show why division by 0 is undefined (see the top of page 24 of the textbook).

Division by Zero

The fraction $\frac{a}{b}$ is undefined if $b = 0$. Division by 0 is undefined.

OBJECTIVE 3

- $a \cdot 1 = a$
- $\frac{a}{1} = a$
- $\frac{a}{a} = 1$, where a is nonzero

- When we write statements such as $a \cdot 1 = a$, we mean that if we evaluate $a \cdot 1$ and a for *any* value of a in both expressions, the results will be equal.
- We say the expressions $a \cdot 1$ and a are **equivalent expressions.**

OBJECTIVE 4

Multiplying Fractions

If b and d are nonzero, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

3. $\frac{3}{5} \cdot \frac{7}{2}$

4. $\frac{4}{9} \cdot \frac{5}{7}$

OBJECTIVE 5

5. Prime factor 24.

6. Prime factor 36.

OBJECTIVE 6

Use a drawing of a pizza to show that $\frac{6}{8} = \frac{3}{4}$.

Simplifying a Fraction

To simplify a fraction.

1. Find the prime factorizations of the numerator and the denominator.
2. Find an equal fraction in which the numerator and the denominator do not have common positive factors other than 1 by using the property

$$\frac{ab}{ac} = \frac{a}{a} \cdot \frac{b}{c} = 1 \cdot \frac{b}{c} = \frac{b}{c}$$

where a and c are nonzero.

7. Simplify $\frac{6}{8}$.

8. Simplify $\frac{24}{18}$.

9. Simplify $\frac{7}{49}$.

OBJECTIVE 4 (revisited)

10. $\frac{4}{25} \cdot \frac{35}{8}$

11. $\frac{14}{8} \cdot \frac{10}{21}$

Dividing Fractions

If b, c, and d are nonzero, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

12. $\frac{9}{2} \div \frac{3}{4}$

13. $\frac{45}{16} \div \frac{35}{8}$

When you use a calculator to check work with fractions, enclose each fraction in parentheses.

14. Use a drawing of a pizza to show that $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$.

Adding Fractions with the Same Denominator

If b is nonzero, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

15. $\frac{7}{12} + \frac{2}{12}$ **16.** $\frac{5}{6} + \frac{7}{2}$ **17.** $\frac{4}{5} + \frac{2}{3}$ **18.** $\frac{5}{6} + \frac{3}{8}$

Use a drawing of a pizza to show that $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$.

Subtracting Fractions with the Same Denominator

If b is nonzero, then $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$.

19. $\frac{5}{6} - \frac{1}{6}$ **20.** $\frac{7}{6} - \frac{2}{3}$ **21.** $\frac{5}{8} - \frac{3}{10}$

Adding (or Subtracting) Fractions with Different Denominators

To add (or subtract) two fractions with different denominators, use the fact that $\frac{a}{a} = 1$, where a is nonzero, to write an equal sum (or difference) of fractions for which each denominator is the LCD.

Throughout the course, if a result is a fraction, it must be simplified.

OBJECTIVE 7

Definition *Proportion*

In statistics, a **proportion** is a fraction of the whole. A proportion can also be written as a decimal number.

A proportion is always between 0 and 1, inclusive.

Sum of the Proportions Equals 1

If an object is made up of two or more parts, then the sum of their proportions equals 1.

We will use capital letters to emphasize the mathematical meanings of "AND," "OR," and "NOT."

22. About $\frac{3}{20}$ of 1.2 million people surveyed think that Five Guys has the best burger out of all fast food chains (Source: *YouGov*). What proportion of those surveyed do NOT think Five Guys has the best burger out of all fast food chains?

Proportion of the Rest

Let $\frac{a}{b}$ be the proportion of the whole that has a certain characteristic. Then the proportion of the whole that does NOT have that characteristic is

$$1 - \frac{a}{b}$$

23. Of all bribes made in the multinational industry, $\frac{1}{10}$ of the bribes are made in extraction (of oil, coal, and so on) and $\frac{3}{20}$ of the bribes are made in construction (Source: *OECD*). What proportion of the bribes are made in sectors other than extraction and construction?

24. A total of 1012 Americans were asked whether God plays a role in determining which team wins a sporting event. Their responses are summarized in the following table.

	God Plays a Role in Sporting Events					
	Completely Agree	Mostly Agree	Mostly Disagree	Completely Disagree	Don't Know/Refused	Total
Number of People	101	162	203	516	30	1012

Source: *Public Religion Research Institute*

Find the proportion (rounded to the third decimal) of those surveyed who

a. completely agreed.

b. did NOT completely agree.

c. mostly disagreed OR completely disagreed.

OBJECTIVE 8

Make the indicated unit conversions. Round approximate results to two decimal places.

25. The Sears Tower in Chicago is 1450 feet tall. What is its height in yards?

26. A jug contains 5 gallons of water. How many cups of water does it contain?

27. A car travels at a speed of 65 miles per hour. What is the car's speed in feet per second?

Converting Units

To convert the units of a quantity,

1. Write the quantity in the original units.
2. Multiply by fractions equal to 1 so that the units you want to eliminate appear in one numerator and one denominator.

HW 1, 3, 21, 25, 33, 41, 49, 67, 73, 81, 87, 93, 95, 99, 103, 107

SECTION 1.3 TEACHING TIPS

Because some students have difficulty with this material, this is a good time to remind them of your availability during office hours and other forms of college support such as math center tutoring. Students' greatest difficulty is usually with adding and subtracting fractions with different denominators. So, it is wise to go quickly through the rest of the material so you have adequate time to focus on this more difficult skill.

OBJECTIVE 1 COMMENTS

Some students will need a reminder about the meaning of a fraction (see Problem 1).

OBJECTIVE 2 COMMENTS

I use repeated subtraction to show that division by zero is undefined. Most students really appreciate this explanation, because they have never been told *why* division by zero is undefined.

OBJECTIVE 3 COMMENTS

Although these rules are very basic, they are the foundation for performing operations with fractions.

OBJECTIVE 4 COMMENTS

After we have discussed how to simplify fractions (Objective 6), we will discuss how to multiply fractions which can be simplified.

OBJECTIVE 5 COMMENTS

In arithmetic, students learn a variety of ways to prime factor a number. Although I allow students to use any method, I encourage them to use the method shown in Example 2 on page 26 of the textbook, because this method clearly portrays that the factorization is equal to the original number.

OBJECTIVE 6 COMMENTS

Most students can successfully simplify a fraction, but most are not aware of the concepts involved. I make sure they understand that we use the property $\frac{a}{a} = 1$, where $a \neq 0$, to simplify a fraction. The textbook avoids using the terminology "reduce," because many professors think that this terminology suggests that reducing a fraction makes it smaller. The textbook also avoids using the technique of "canceling," because students misapply this technique in a myriad of ways.

OBJECTIVE 4 (REVISITED) COMMENTS

Once my students know how to simplify fractions, I discuss how to multiply fractions in which the result needs to be simplified.

Using a drawing of a pizza to show that $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ helps students understand why it doesn't make sense to add the denominators.

When adding fractions with different denominators, I list multiples of the denominators to find the LCD. When adding fractions such as $\frac{5}{6}$ and $\frac{3}{8}$ (Problem 18), I show at least once that we are multiplying fractions by 1 to get each denominator to be the LCD:

$$\begin{aligned}\frac{5}{6} + \frac{3}{8} &= \frac{5}{6} \cdot 1 + \frac{3}{8} \cdot 1 \\ &= \frac{5}{6} \cdot \frac{4}{4} + \frac{3}{8} \cdot \frac{3}{3} \\ &= \frac{20}{24} + \frac{9}{24} \\ &= \frac{29}{24}\end{aligned}$$

GROUP EXPLORATION: Illustrations of Simplifying Fractions and Operations with Fractions

This exploration is a nice way for students to learn to visualize their work with fractions.

OBJECTIVE 7 COMMENTS

The property about the sum of the proportions equals 1 is good preparation for the fact that the sum of the relative frequencies of all the categories of a categorical variable equals 1. It also prepares students for the similar property about numerical variables and ultimately that the area under the normal curve is 1. A quick discussion may suffice to explain this concept or you could demonstrate it by working with a categorical variable that describes your class.

Although it is a bit jarring to read the words *AND*, *OR*, and *NOT* in all uppercase letters, this is a helpful signal to students that the words are being used in a mathematical way. The word *AND* will not be used in the homework until Chapter 3, but it is discussed briefly in this section for completeness. The words *AND* and *OR* will be more carefully described and compared in Section 3.1 (see page 137 of the textbook). If you postpone discussing *AND* and *OR* until then, be sure to avoid assigning Exercise 93(aiii) and Exercise 94(aiii) because they both include the word *OR*.

The word *NOT* and the proportion of the rest property are emphasized in the homework and should be discussed now. Before introducing the boxed general statement of the proportion of the rest property, I first have students solve Problem 22 because they are capable of solving it without my help. Then it's easy to bridge to the general statement.

Of course, the proportion of the rest property is very similar to the complement rule but I don't refer to it as such because the complement rule has to do with probabilities. Nonetheless, students will have lots of practice with the proportion of the rest property in Chapters 1, 3, and 4, readying them for the complement rule (Section 5.2), which will be especially helpful when working with normal distributions (Sections 5.4 and 5.5).

There are several ways to solve Problem 24(b). Consider having groups of students solve the problem and share their methods with the class.

OBJECTIVE 8 COMMENTS

The skill of converting units will come in handy several times in the text when students work with units with which they are not as familiar. For example, in Exercise 22 of Homework 6.2, students will convert Earth's escape velocity from 11.2 km/s to units of mi/h.

Just as with Objective 7, I prefer to problem solve first and then present the boxed summary statement because I don't believe the summary will mean much until I've given some specific examples.

I emphasize that we perform conversions by multiplying by fractions equal to 1, so a conversion describes the same quantity. Some equivalent units are shown in the margin on page 34 of the textbook.

SECTION 1.4 LECTURE NOTES

Objectives

1. Find the opposite of a number.
2. Find the absolute value of a number.
3. Add two real numbers with the same sign.
4. Add two real numbers with different signs.
5. Add real numbers pertaining to authentic situations.

OBJECTIVE 1

- Two number are called **opposites** of each other if they are the same distance from 0 on the number line, but are on opposite sides of 0.
- To find the opposite of a number, we reflect the number across 0 on the number line.
- The opposite of 5 is -5. The opposite of -5 is 5. [Draw a figure.]

Use reflections with a number line to show that $-(-a) = a$.

1. $-(-6)$

2. $-(-(-5))$

We use parentheses to separate two opposite symbols or an operation symbol and an opposite symbol.

OBJECTIVE 2

Definition *Absolute value*

The **absolute value** of a number is the distance that the number is from 0 on the number line. [Draw a figure.]

3. $|4|$

4. $|-7|$

5. $-|8|$

6. $-|-2|$

OBJECTIVE 3

7. **a.** A person has a credit card balance with a 0 dollar balance. If she uses her credit card to make two purchases for $3 and $4, what is the new balance?

b. Write a sum that is related to the computation in Part (a).

c. Use a number line to illustrate the sum found in Part (b).

Adding Two Numbers with the Same Sign

To add two numbers with the same sign,

1. Add the absolute values of the numbers.
2. The sum of the original numbers has the same sign as the sign of the original numbers.

8. $-3 + (-8)$

9. $-2.57 + (-6.84)$

10. $-\frac{5}{8} + \left(-\frac{1}{8}\right)$

OBJECTIVE 4

11. **a.** A brother owes his sister $5. If he then pays her back $2, how much does he still owe her?

b. Write a sum that is related to your work in Part (a).

c. Use a number line to illustrate the sum you found in Part (b).

Adding Two Numbers with Different Signs

To add two numbers with different signs:

1. Find the absolute values of the numbers. Then subtract the smaller absolute value from the larger absolute value.
2. The sum of the original numbers has the same sign as the original number with the larger absolute value.

12. $-9+2$

13. $7+(-4)$

14. $-\frac{7}{6}+\frac{5}{6}$

15. $-7+(-4)$

16. $-5.31+1.98$

17. $329+(-838)$

18. $\frac{3}{10}+\left(-\frac{5}{6}\right)$

OBJECTIVE 5

19. A person bounces several checks and is charged service fees such that the balance of the checking account is -78 dollars. The person then deposits \$250. Find the balance.

20. Two hours ago, the temperature was $-9°$F. If the temperature has increased by $4°$F in the last two hours, what is the current temperature?

HW 1, 3, 5, 15, 23, 25, 39, 45, 53, 59, 63, 67, 69, 71, 75

SECTION 1.4 TEACHING TIPS

In this section, students add real numbers and find expressions to model authentic quantities. They will use these skills throughout the course.

OBJECTIVE 1 COMMENTS

Although finding the opposite of a number is an easy topic, I discuss reflecting a number across 0 on the number line to set the stage for Objective 2.

The concept of finding the opposite of the opposite will help students subtract numbers in Section 1.5:

$$a-(-b)=a+[-(-b)]=a+b$$

OBJECTIVE 2 COMMENTS

Absolute value comes in handy when discussing not only how to add signed numbers but also to interpret a correlation coefficient.

OBJECTIVES 3 and 4 COMMENTS

Most of my students prefer to think of adding real numbers in terms of money (as opposed to the number line or absolute value). I quickly go over the rules and spend more time having my students do mixed sets of problems such as Problems 12–18.

GROUP EXPLORATION: Adding a Number and Its Opposite

This exploration is a nice way for students to learn that $a+(-a)=0$.

OBJECTIVE 5 COMMENTS

When solving Problems 19 and 20, I emphasize that the results must include units.

SECTION 1.5 LECTURE NOTES

Objectives

1. Find the change in a quantity.
2. Subtract real numbers.
3. Find a change in elevation.
4. Determine the sign of the change for an increasing or decreasing quantity.

OBJECTIVE 1

1. **a.** If the price of a basket of strawberries increases from \$2 to \$5, find the change in the price.
b. Write a difference that is related to the computation in Part (a).

Change in a Quantity

The change in a quantity is the ending amount minus the beginning amount:

$$\text{Change in the quantity} = \text{Ending amount} - \text{Beginning amount}$$

2. The revenues of Facebook are shown in the following table for various years.

Year	Facebook Revenue (billions of dollars)
2009	0.78
2010	1.97
2011	3.71
2012	5.09
2013	7.87

Source: *Facebook*

a. For the period 2009–2013, find each of the changes in annual revenue from one year to the next.
b. From which year to the next did the annual revenue increase the most?

OBJECTIVE 2

3. **a.** The volume of gasoline in a car's gasoline tank decreases from 6 gallons to 2 gallons. What is the change in the volume of gasoline in the tank?
b. Write a difference that is related to the computation in Part (a).
c. From Part (b), we found that $2 - 6 = -4$. Find $2 + (-6)$. Explain why we can conclude that $2 - 6 = 2 + (-6)$.

Subtracting a Real Number

$$a - b = a + (-b)$$

4. $3 - 7$

5. $-2 - 8$

6. $\frac{2}{9} - \frac{8}{9}$

7. **a.** The temperature increases from -3°F to 2°F. Find the change in temperature.

b. Write a difference that is related to the work in Part (a).

c. Find the difference obtained in Part (b) by using the rule $a - b = a + (-b)$.

8. $5 - (-2)$

9. $-6 - (-1)$

10. $\frac{2}{7} - \left(-\frac{3}{7}\right)$

11. Translate the sentence "Subtract the number from 5." into a mathematical expression. Then evaluate the expression for $x = -3$.

12. $-5 + (-4)$

13. $4 - 11$

14. $-9 - 7$

15. $-\frac{2}{5} - \left(-\frac{7}{10}\right)$

16. $2.5 + (-7.1)$

17. $62 - (-95)$

18. $\frac{7}{12} - \frac{11}{15}$

19. $46 + (-11)$

OBJECTIVE 3

20. The lowest elevation in Louisiana is in Eastern New Orleans (-8 ft), and the highest elevation is at the top of Driskill Mountain (535 ft). Find the change in elevation from Eastern New Orleans to Driskill Mountain.

OBJECTIVE 4

Refer to Problem 2 to motivate the following properties.

Changes of Increasing and Decreasing Quantities

- An increasing quantity has a positive change.
- A decreasing quantity has a negative change.

21. The number of movies shot in Georgia are shown in the following table for various years.

Year	Number of Movies Shot in Georgia
2004	10
2005	9
2006	12
2007	9
2008	11
2009	17
2010	17
2011	23
2012	24
2013	22

Source: *Georgia Film, Music, and Digital Entertainment Office*

a. Find the change in the number of movies shot in Georgia from 2006 to 2007.

b. In 2008, Georgia increased the tax incentives to shoot movies in the state. Find the change in the number of movies shot in Georgia from 2008 to 2013.

HW 1, 3, 5, 15, 19, 33, 47, 53, 57, 61, 65, 73, 79, 81, 85

SECTION 1.5 TEACHING TIPS

In this section, students subtract real numbers, a skill that they will use throughout the course. In this section, students find the change in a quantity. Unfortunately, this important concept is overlooked in most arithmetic and algebra textbooks. This is a central concept needed to understand that in going from a point (x_1, y_1) to a point (x_2, y_2), $y_2 - y_1$ is the vertical change and $x_2 - x_1$ is the horizontal change. This gives meaning to the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Another key piece of the puzzle about slope will be to discuss the meaning of ratios in Section 1.6. Understanding how to find the change in a quantity will also help students make sense of the techniques we use to subtract real numbers.

OBJECTIVE 1 COMMENTS

Problem 1 nicely motivates the change in a quantity property. Note that the results of Part (a) of Problem 2 are rates of change (Section 7.2) for various years.

OBJECTIVE 2 COMMENTS

Problem 3 suggests the property $a - b = a + (-b)$. Problem 7 suggests how to subtract a negative number. Problems 12–19 serve as a summary of Sections 1.4 and 1.5.

GROUP EXPLORATION: Subtracting Numbers

This activity is a nice way for groups of students to discover that $a - b = a + (-b)$. Some groups will need help with Problems 4 and 5 of this exploration.

OBJECTIVE 3 COMMENTS

After solving Problem 20, I sketch a figure to illustrate why the result makes sense.

OBJECTIVE 4 COMMENTS

Although I address the concepts about the significance of signs of change in Objectives 1 and 2, it is nice to reinforce these important concepts at the end of this section. Problem 21 serves as good preparation for work with time-series plots (Section 3.3), scatterplots (Section 6.1), and rates of change (Section 7.2).

SECTION 1.6 LECTURE NOTES

Objectives

1. Find the ratio of two quantities.
2. Describe the meaning of *percent*.
3. Convert percentages to and from decimal numbers.
4. Use proportions and percentages to describe authentic situations.
5. Find the percentage of a quantity.
6. Multiply and divide real numbers.
7. Describe which fractions with negative signs are equal to each other.

OBJECTIVE 1

- Suppose that there are 8 women and 4 men on a softball team. The ratio of women to men is

$$\frac{8 \text{ women}}{4 \text{ men}} = \frac{2 \text{ women}}{1 \text{ men}}$$

- This means that there are 2 women to 1 man. This ratio is called a *unit ratio*.
- A **unit ratio** is a ratio written as $\frac{a}{b}$ or as $a : b$ with $b = 1$.

1. The blink of an eye takes 400 milliseconds. One beat of a hummingbird's wings takes 20 milliseconds. Find the unit ratio of the time it takes for the blink of an eye to the time it takes for one beat of a hummingbird's wings. What does your result mean in this situation?

2. The median sales prices of existing homes and the median household incomes in 2011 are shown in the following table for four regions of the United States.

Region	Median Sales Price of Existing Homes (dollars)	Median Household Income (dollars)
Northeast	237,500	51,862
Midwest	135,400	49,549
South	135,200	42,590
West	201,300	53,367

Sources: *U.S. Census; Federal Reserve Bank of St. Louis*

a. Find the unit ratio of the median sales price of existing homes to the median household income in the Northeast. What does the result mean?

b. For each of the four regions, find the unit ratio of the median sales price of existing homes to the median household income. Taking into account the median household income of each region, list the regions in order of affordability of existing homes, from greatest to least.

c. A person believes that existing homes in the South are more affordable than in the Midwest, because the median price of existing homes is lower in the South than in the Midwest. What would you tell that person?

OBJECTIVE 2

- If 57 of 100 songs are hip hop, then 57% of the songs are hip hop.
- **Percent** means "for each hundred": $a\% = \frac{a}{100}$.
- A percent is a ratio.

OBJECTIVE 3

Converting Percentages to and from Decimal Numbers

- To write a percentage as a decimal number, remove the percent symbol and divide the number by 100 (move the decimal point two places to the left).
- To write a decimal number as a percentage, multiply the number by 100 (move the decimal point two places to the right) and insert a percent symbol.

3. Write 8% as a decimal number.

4. Write 0.145 as a percent.

Warning: 8% is equal to 0.08, not 0.8.

OBJECTIVE 4

5. About 63% of Americans feel the distribution of money and wealth in the United States should be more evenly distributed (Source: *CBS News Poll*). Use a proportion to describe this situation.

6. The proportion of Yahoo employees who are Asian is 0.39 (Source: *Yahoo*). Use a percentage to describe this situation.

7. Of 905 surveyed drivers, 163 drivers said it takes them at least three tries to parallel park (Source: *Hankook Tire Fall Gauge Index survey*). Use a percentage to describe the situation.

OBJECTIVE 5

Finding the Percentage of a Quantity

To find the percentage of a quantity, multiply the decimal form of the percentage and the quantity.

8. Of 4672 surveyed students in grades 3–5, 85% knew how to use a keyboard to type answers to questions (Source: *Idaho's Smarter Balanced Field Test survey*). How many of the students knew how to use a keyboard?

9. A person buys some take-out food for $8 (not including state meals tax) in Boston, Massachusetts, which has a state meals tax of 6.25%. How much money is the state meals tax?

One Hundred Percent of a Quantity

One hundred percent of a quantity is *all* of the quantity.

OBJECTIVE 6

Use repeated addition to show that $4(-3) = -12$.

Multiplying Two Numbers with Different Signs

The product of two number that have different signs is negative.

10. **a.** Find the products $3(-4) = -12$, $2(-4) = -8$, $1(-4) = -4$, $0(-4) = 0$.

b. Explain why the results of Part (a) suggest that $-1(-4) = 4$, $-2(-4) = 8$, and $-3(-4) = 12$.

c. Form a theory about the sign of the product of two numbers that have the same sign.

Multiplying Two Numbers with the Same Sign

The product of two numbers that have the same sign is positive.

11. $3(-8)$ **12.** $-5(-9)$ **13.** $-0.2(-0.1)$ **14.** $-\frac{4}{9} \cdot \frac{3}{2}$

The rules for dividing numbers are similar to those for multiplying numbers, because to divide by a number, we multiply by the reciprocal of the number.

Multiplying or Dividing Real Numbers

The product or quotient of two numbers that have different signs is negative. The product or quotient of two numbers that have the same sign is positive.

15. $-12 \div (-2)$ **16.** $-35 \div 7$ **17.** $475 \div -25$ **18.** $-\frac{5}{49} \div \left(-\frac{25}{21}\right)$

19. A person has credit card balances of -3580 dollars on a Discover® account and -1590 dollars on a MasterCard® account.

a. Find the unit ratio of the Discover balance to the MasterCard balance.

b. If the person wishes to gradually pay off both accounts in the same amount of time, describe how the result in Part (a) can help guide the person in making his next payment.

20. $-4-(-7)$ **23.** $(-7)(-10)$ **26.** $-\frac{27}{20} \div \frac{9}{8}$

21. $8 \div (-2)$ **24.** $-5.8+2.9$ **27.** $-\frac{7}{4} - \frac{3}{2}$

22. $-15+11$ **25.** $-0.2(-0.3)$

OBJECTIVE 7

Compare the results of $\frac{-6}{3}$, $\frac{6}{-3}$, and $-\frac{6}{3}$ to motivate the following property.

Equal Fractions with Negative Signs

If $b \neq 0$, then

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

28. $\frac{-32}{20}$ **29.** $\frac{7}{9} + \frac{-4}{9}$ **30.** $\frac{7}{6} - \left(\frac{5}{-4}\right)$

HW 1, 3, 9, 11, 15, 19, 27, 29, 35, 45, 55, 65, 69, 77, 83, 87, 89, 97

SECTION 1.6 TEACHING TIPS

In this section, students work with ratios, proportions, percents, and multiplying and dividing real numbers. It may be tempting to skip or deemphasize ratios, but this concept, together with the concept of change, will be very helpful in introducing rate of change and the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ (Section 7.2). Most students have little experience working with ratios.

OBJECTIVE 1 COMMENTS

Most students have an easy time computing ratios but have a harder time thinking about the significance of comparing ratios such as in Problem 2. Part (c) addresses the extremely important concept in introductory statistics that it usually is more helpful to compare unit ratios (or proportions) than it is to compare quantities. The textbook will continue to emphasize this concept in Chapters 3 and 5.

OBJECTIVE 2 COMMENTS

Most students have a weak understanding of percents, which is important for work with proportions (Chapter 3) and probabilities (Chapter 5).

OBJECTIVE 3 COMMENTS

I use the definition of percent to suggest how to convert percentages to and from decimal numbers.

For Problem 3, some students think that the result is 0.8.

OBJECTIVE 4 COMMENTS

Problems 5 and 6 can be a bit challenging for some students because they must change the structure of the sentences.

OBJECTIVE 5 COMMENTS

I first use the definition of percent to find the percentage of a quantity and then streamline the process. For Problems 8 and 9, I remind students to include units in their results.

Students will need to know that one hundred percent of a quantity is all of the quantity for many reasons, including recognizing when model breakdown has occurred when making predictions with linear and exponential models (Chapters 6–10).

OBJECTIVE 6 COMMENTS

Students have an easy time with these skills.

Writing $4(-3) = (-3) + (-3) + (-3) + (-3) = -12$ suggests the rule for multiplying two numbers with different signs. Problem 10 suggests the rule for multiplying two numbers with the same sign.

For Problem 13, many students think that $-0.2(-0.1) = 0.2$. For Problem 14, some students need to be reminded to simplify their result.

The textbook includes a detailed explanation about why the sign rules for division are the same as for multiplication (see pages 59 and 60 of the textbook). Due to time constraints, I usually just say the rules for dividing numbers are similar to those for multiplying numbers because to divide by a number, we multiply by the reciprocal of the number.

Problem 19 ties ratios in nicely with dividing two negative numbers. Many students who have credit card debt could directly benefit from learning how to do such work with credit card balances.

I budget plenty of time for students to complete Problems 20–27, which serve as a good summary of Sections 1.3–1.6.

GROUP EXPLORATION: Finding the Product of a Positive Number and a Negative Number

Groups of students can discover how to multiply two numbers with different signs by completing this exploration. I tell students that the more they can understand why properties make sense, the better they will be able to keep them straight.

OBJECTIVE 7 COMMENTS

I emphasize the property $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$, where $b \neq 0$, because it will be useful when working with slope. It seems that no amount of emphasis is too much because some students completely forget this property by the time we need to use it for slope (Sections 7.2 and 7.3).

I tell my students to write their results which are negative fractions in the form $-\frac{a}{b}$ rather than $\frac{-a}{b}$ or $\frac{a}{-b}$.

SECTION 1.7 LECTURE NOTES

Objectives

1. Describe the meaning of *exponent*.
2. Describe the meaning of the exponent zero.
3. Describe the meaning of a negative-integer exponent.
4. Describe the meaning of *square root* and *principal square root*.
5. Approximate a principal square root.
6. Use the rules for order of operations to perform computations and evaluate expressions.
7. Use the rules for order of operations to make predictions.
8. Find the area of part of an object.
9. Use scientific notation.

OBJECTIVE 1

- $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$
- $4^3 = 4 \cdot 4 \cdot 4 = 64$

Definition *Exponent*

For any counting number n,

$$b^n = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{n \text{ factors of } b}$$

We refer to b^n as the **power**, the ***n*th power of *b***, or ***b* raised to the *n*th power**. We call b the **base** and n the **exponent**.

1. 3^4 **2.** $(-2)^5$ **3.** $(-5)^2$ **4.** -5^2

For an expression of the of the form $-b^n$, we calculate b^n before taking the opposite.

OBJECTIVE 2

Find the powers 2^4, 2^3, 2^2, and 2^1 and have students guess the result for 2^0.

Definition *Zero exponent*

For nonzero b,

$$b^0 = 1$$

5. 269.72^0 **6.** $(-9)^0$

OBJECTIVE 3

Find the powers 2^2, 2^1, 2^0, and have students guess the result for 2^{-1}, 2^{-2}, and 2^{-3}.

> **Definition** *Negative-integer exponent*
>
> If n is a counting number and b is nonzero, then
>
> $$b^{-n} = \frac{1}{b^n}$$

7. 4^{-2}

8. 3^{-4}

OBJECTIVE 4

> **Definition** *Principal square root*
>
> If a is a nonnegative number, then $\sqrt{a}$ is the nonnegative number we square to get a. We call $\sqrt{a}$ the **principal square root** of a.

- The symbol "$\sqrt{}$" is called a **radical sign**.
- An expression under a radical sign is called a **radicand**.
- A radical sign together with a radicand is called a **radical**.
- An expression that contains a radical is called a **radical expression**.
- **A square root of a negative number is not a real number.**

Find the square root.

9. $\sqrt{25}$

10. $\sqrt{-25}$

11. $-\sqrt{25}$

12. $-\sqrt{-25}$

OBJECTIVE 5

State whether the square root is rational or irrational. If the square root is rational, find the (exact) value. If the square root is irrational, estimate its value by rounding to the second decimal place.

13. $\sqrt{144}$

14. $\sqrt{24}$

OBJECTIVE 6

- We do operations that lie within grouping symbols before we perform other operations.
- The order of operations does matter:

$$(4+2) \cdot 5 = 6 \cdot 5 = 30$$
$$4 + (2 \cdot 5) = 4 + 10 = 14$$

- For a fraction such as $\frac{2+5}{9-4}$, the following use of parentheses is assumed:

$$\frac{2+5}{9-4} = \frac{(2+5)}{(9-4)} = \frac{7}{5}$$

- For a radical expression such as $\sqrt{16+9}$, the following use of parentheses are assumed:

$$\sqrt{16+9} = \sqrt{(16+9)} = \sqrt{25} = 5$$

Warning: $\sqrt{16+9}$ does *not* equal $\sqrt{16}+\sqrt{9}$.

15. $(4-8)(9-2)$

16. $\frac{2-8}{3-7}$

Order of Operations

We perform operations in the following order:

1. First, perform operations within parentheses or other grouping symbols, starting with the innermost group.
2. Then perform exponentiations.
3. Next, perform multiplications and divisions, going from left to right.
4. Last, perform additions and subtractions, going from left to right.

17. $3 - 8 \div 4$

18. $4 + (-2)^3$

19. $2(5-8) - (4-9)$

20. $3 - [6 - 2(3+4)]$

21. $3^3 - 2(3-5)^2 \div (-4)$

22. $\frac{9}{10} - \frac{3}{5} \div \frac{2}{7}$

23. $\frac{9+4+7+1+9}{5}$

24. $\frac{4-(-2)^3}{2+4^2}$

Operation	**Computation with 10s**
Exponentiation	$10^{10} = 10,000,000,000$
Multiplication	$10 \cdot 10 = 100$
Addition	$10 + 10 = 20$

Order of Operations and the Strengths of Operations

After we have performed operations in parentheses, the order of operations goes from the most powerful operation, exponentiation, to the next-most-powerful operations, multiplication and division, to the weakest operations, addition and subtraction.

25. $\frac{(1-3)^2 + (2-3)^2 + (6-3)^2}{3-1}$

26. $(44-35) - 4\sqrt{\frac{4^2}{2} + \frac{7^2}{49}}$

27. Use a calculator to perform the indicated operations for

$$\sqrt{\frac{(2.3-5.4)^2 + (5.6-5.4)^2 + (8.3-5.4)^2}{3-1}}$$

28. Evaluate $\dfrac{a-b}{c-d}$ for $a = -3$, $b = 5$, $c = 4$, and $d = -6$.

29. Evaluate $\bar{x} - t\dfrac{s}{\sqrt{n}}$ for $\bar{x} = 12$, $t = 3$, $s = 10$, and $n = 25$.

30. Let x be a number. Translate the sentence "Subtract 5 from the product of the number and -3." Then evaluate the expression for $x = -2$.

OBJECTIVE 7

31. The number of knee replacement surgeries among Medicare participants was about 244 thousand in 2010 and has increased by approximately 8 thousand surgeries per year until 2014 (Source: *Journal of the American Medical Association*). Complete the following table to help find an expression that stands for the number of such surgeries at t years since 2010. Show the arithmetic to help see a pattern.

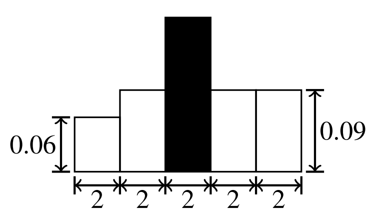
Years Since 2010	Number of Knee Replacement Surgeries Among Medicare Participants (in thousands)
0	
1	
2	
3	
t	

Evaluate your result for $t = 4$ and discuss what the result means in this situation.

32. The number of age-related retirements of U.S. pilots was 226 retirements in 2010, and it increased by 20.8% in 2011 (Source: *KitDarby.com Aviation Consulting*). What was the number of age-related retirements in 2011?

OBJECTIVE 8

33. Use the fact that the total area of the following object is 1 to find the area of the shaded bar. The object has not been drawn to scale.



OBJECTIVE 9

Definition *Scientific notation*

A number is written in **scientific notation** if it has the form $N \times 10^k$, where k is an integer and the absolute value of N is between 1 and 10 or is equal to 1.

Examples: 6.74×10^7, 9.2×10^{-6}

Write the number in standard decimal notation.

34. 5.36×10^3 **35.** 8.192×10^6 **36.** 4.6×10^{-3} **37.** 2.99×10^{-5}

Converting from Scientific Notation to Standard Decimal Notation

To write the scientific notation $N \times 10^k$ in standard decimal notation, we move the decimal point of the number N as follows:

- If k is *positive*, we multiply N by 10 k times and hence move the decimal point k places to the *right*.
- If k is *negative*, we divide N by 10 k times and hence move the decimal point k places to the *left*.

Write the number in scientific notation.

38. The first evidence of life on Earth dates back to 3.6×10^9 years ago.

39. The wavelength of violet light is about 0.00000047 meter.

Converting from Standard Decimal Notation to Scientific Notation

To write a number in scientific notation, count the number k of places that the decimal point must be moved so that the absolute value of the new number N is between 1 and 10 or is equal to 1.

- If the decimal point is moved to the left, then the scientific notation is written as $N \times 10^k$.
- If the decimal point is moved to the right, then the scientific notation is written as $N \times 10^{-k}$.

- Some calculators represent 7.29×10^{26} as 7.29 E 26.
- Some calculators represent 5.96×10^{-23} as 5.96 E -23.

HW 1, 3, 5, 19, 25, 39, 45, 53, 67, 71, 73, 79, 81, 85, 87, 91, 97, 103, 105, 109, 117, 123, 129, 131

SECTION 1.7 DETAILED COMMENTS

In this section, students work with exponents and use the rules for order of operations to perform computations and evaluate expressions. Students will use this material throughout the course.

OBJECTIVE 1 COMMENTS

It will be especially helpful for students to know the meaning of *exponent*, *base*, and *power* when working with exponential models in Chapter 10. Many students think that *power* means the same thing as *exponent*. Explain that b^n is a power of b and that n is the exponent.

For Problem 4, most students are surprised that -5^2 is not equal to 25. I tell students that for an expression of the of the form $-b^n$, we calculate b^n before taking the opposite. It also helps to point out that the base of $(-5)^2$ is -5 and that the base of -5^2 is 5.

OBJECTIVE 2 COMMENTS

Many students are surprised to learn that b^0 is not equal to 0.

OBJECTIVE 3 COMMENTS

The reason negative-integer exponents are included in this section is to prepare students for scientific notation, which will be displayed by calculators when students compute probabilities for the normal distribution (Section 5.4).

OBJECTIVE 4 COMMENTS

Square roots are included in this section because students will work with square roots when computing standard deviation (Section 4.2). To keep things moving, I skip all the definitions for principal square root, radical sign, radicand, and so on and just do Problems 9–12.

OBJECTIVE 5 COMMENTS

If students are using a variety of calculators, it can be challenging to tell them all how to access the square root command. If students have trouble, I tell them to wait until they next work in groups so I can then go about the classroom helping students individually.

OBJECTIVE 6 COMMENTS

When discussing the order of operations, I emphasize that multiplication and division are on the same level of the hierarchy; some students think that "going from left to right" means that all multiplications should be done before all divisions, because the word "multiplications" appears to the left of the word "divisions" in the instructions for the order of operations. The same goes for addition and subtraction.

Sometimes it seems that the only things students have learned from past math courses are the acronyms "PEMDAS" and "FOIL." I dislike "PEMDAS" because it lacks details needed for students to use it correctly. Even with commas, the acronym "P,E,MD,AS" lacks necessary details. Nonetheless, I present "PE,MD,AS" because if I don't, students will ask about it anyway. Then I emphasize how the acronym is incomplete.

Even though order of operations is a review topic from arithmetic, some students are challenged by Problems 17–24. Problems 23, 25, and 27 are good preparation for calculating the mean (Section 4.1), variance (Section 4.2), and standard deviation (Section 4.3), respectively. Problem 28 relates to computing slope of a line (Section 7.2). Finally, Problems 26 and 29 relate to finding confidences intervals, although confidences intervals are not addressed in this course.

When students practice Problem 27, I encourage them to save time by computing the entire numerator rather than computing and recording the values of the three terms before adding.

GROUP EXPLORATION: Order of Operations

In this activity, groups of students discover that it matters in which order we add and multiply.

OBJECTIVE 7 COMMENTS

In Problem 31, students continue their work with building expressions by recognizing patterns in their arithmetic. Although students will use a different method to find linear models in Section 7.3, the two methods have some overlap and to the degree that they differ they can serve as checks of each other.

OBJECTIVE 8 COMMENTS

I have groups of students work on Problem 33, which is excellent preparation for the development of density histograms, which will be emphasized in Chapters 3–5. Although students have already worked with the proportion of the rest property in Section 1.3, they tend to have a tough time with this problem. For groups that get stuck, I ask them whether they can find the areas of all the unshaded rectangles. When they say yes, I remind them of the proportion of the rest property, which usually is a big enough hint.

OBJECTIVE 9 COMMENTS

Deemphasizing scientific notation is one way to handle getting through this long section. However, if your students are using TI-84 graphing calculators, make sure that they know how to interpret their calculators' way of expressing scientific notation, because students will run into this notation when finding probabilities for normal distributions (Section 5.4).

One way to motivate scientific notation is to ask students to use their graphing calculators to compute 2^{50} or 2^{-50}.

For converting scientific notation to and from standard decimal notation, students are not sure whether to move the decimal point to the right or left. First, I demonstrate that to multiply by 10, we move the decimal point one place to the right, and to divide by 10, we move the decimal point one place to the left. Then I bridge to powers of 10. For example, for 3.56×10^{-8}, 3.56 is divided by 10^8 (a product of eight 10's), so we need to move the decimal point 8 places to the left.

CHAPTER 2 OVERVIEW

How many times have you questioned the validity of conclusions drawn by the media or corporations because of nonsampling error or some other bad practice of statistics? It would be a powerful life lesson for your students to enhance their ability to do this. I believe that should be the highest goal of this chapter. In fact, one could argue that this is the most important objective of the entire course because some students may not use statistics much in their careers but we all are subject to bad practices of statistics by the media and advertisements.

This chapter is also hugely important because it provides the big picture for the rest of the course and even for the subsequent course in introductory statistics.

In my opinion, the single most important concept of the chapter is randomness. Not only do many students have misconceptions about this concept, they also have much to gain by learning about concepts related to randomness such as random sampling, random selection, and probability.

A close second in importance is the distinction made between experiments and observational studies in Section 2.3. It is this distinction that allows students to discern, for the most part, whether a study can determine that there is causality. This concept will be needed for many parts of exercises in Chapters 3–5.

Some of the Hands-On Research exercises and the Hands-on Project assignments require students to perform surveys or physical experiments. The sampling methods discussed in Section 2.2 will expand students' options of collecting such data. And the descriptions of various types of bias in Section 2.1 will serve as warnings for collecting meaningful data.

In Section 2.3, be prepared to coach students on how to read descriptions of studies. I provide some tips in this manual for that section. The higher reading level of the examples and the exercises in Section 2.3 is the natural result of trying to express complex ideas about authentic studies. If students can learn to comprehend such descriptions, they will be well prepared to digest similar descriptions in introductory statistics courses as well as in their careers.

If you have not already surveyed your students, this would be good time to do so because the data will come in handy at the start of Chapter 3, where the class will begin to construct statistical diagrams.

SECTION 2.1 LECTURE NOTES

Objectives

1. Identify the *individuals*, the *variables*, and the *observations* of a study.
2. Describe the five steps of *statistics*.
3. Identify the *sample* and the *population* of a study.
4. Identify *descriptive statistics* and *inferential statistics*.
5. Select a *simple random sample*.
6. Identify the *sampling bias*, the *nonresponse bias*, and the *response bias* of a study.
7. Identify sampling and nonsampling error.

OBJECTIVE 1

Individuals are the people or objects we want to learn about.

Definition *Variable*

In statistics, a **variable** is a characteristic of the individuals to be measured or observed.

Observations are data that we observe for a variable.

1. The number of listens of certain songs during the week ending Sunday, February, 2015 on Last.fm are shown in the following table.

Song	Band	Genre	Number of Listens
"Breezeblocks"	Alt-J	Alternative	1436
"10 Bands"	Drake	Hip-Hop	5817
"Backseat Freestyle"	Kendrick Lamar	Hip-Hop	1466
"Chandelier"	Sia	Pop	3256
"Blue Moon"	Beck	Alternative	5360

Source: *Last.fm*

a. Identify the individuals.

b. Identify the variables.

c. Identify the observations for each variable.

d. Of the songs listed in the table, which one was listened to the most?

OBJECTIVE 2

Definition *Statistics*

Statistics is the practice of the following five steps:

1. **Raise a precise question about one or more variables.**
2. **Create a plan to answer the question.**
3. **Collect the data.**
4. **Analyze the data.**
5. **Draw a conclusion about the question.**

2. *Motivational enhancement therapy* (*MET*) is a counseling process that aims to motivate drug abusers to stop using drugs. Some researchers wanted to test whether MET really works. Out of 70 drug abusers who participated in the study, 35 received the standard drug treatment provided at a hospital. The other 35 individuals received MET in addition to the standard drug treatment. After receiving treatment for 12 weeks, all 70 individuals took a questionnaire which measures a drug abuser's motivation to stop using drugs. The researchers concluded that MET effectively motivates drug abusers to stop using drugs. (Source: Motivational Enhancement Therapy for Substance Abusers: A Quasi Experimental Study, *Seema Rani et al.*). Describe the five steps of the study. If no details are given about a step, say so.

OBJECTIVE 3

Definition *Population*

A **population** is the *entire* group of individuals about which we want to learn.

A **census** is a study in which data are collected about *all* members of the population.

> **Definition** *Sample*
>
> A **sample** is the part of a population from which data are collected.

Collecting data from a sample, rather than the entire population, saves time, money, and effort.

3. In a poll of 1000 American adults, 13% of respondents said that concerns about global warming are unwarranted. The study then made a conclusion about all American adults (Source: *NBC News/Wall Street Journal*).

a. Define a variable for the study.

b. Identify the sample.

c. Identify the population.

OBJECTIVE 4

- **Descriptive statistics** is the practice of using tables, graphs, and calculations about a sample to draw conclusions about only the sample.
- **Inferential statistics** is the practice of using information from a sample to draw conclusions about the *entire* population. When we perform inferential statistics, we call the conclusions **inferences**.

4. Researchers wanted to find out how a restricted diet would affect a Labrador Retriever's lifespan. Of 48 Labrador Retrievers being observed, 24 were fed a normal diet and 24 were fed a restricted diet. The dogs' lifetimes were recorded. The study concluded that a typical Labrador Retriever on a restricted diet tends to live longer than a typical Labrador Retriever on a normal diet (Source: Effects of Diet Restriction on Life Span and Age-Related changes in Dogs*, Richard D. Kealy, PhD, et al.*).

a. What question were the researchers trying to answer?

b. Identify the population

c. Identify the sample.

d. Identify the conclusion. Is it part of descriptive or inferential statistics?

OBJECTIVE 5

When a study makes use of a sample, it is important that the sample represents the population well.

> **Definition** *Simple random sample*
>
> A process of selecting a sample of size n is **simple random sampling** if every sample of size n has the same chance of being chosen. A sample selected by such a process is called a **simple random sample**.

- If we allow an individual to be selected more than once, then we are sampling with replacement.
- If we do not allow an individual to be selected more than once, then we are sampling without replacement.
- A **frame** is a numbered list of all the individuals in the population.

5. **a.** Create a frame of all the students in your class.

   b. Use technology to randomly select 6 students without replacement.

6. In 2013, 1523 out of 4270 colleges were 2-year colleges (Source: *U.S. Department of Education*).

   a. What proportion of the colleges were 2-year colleges?

   b. By using the numbers 1 through 1523 to represent the 2-year colleges and the numbers 1524 through 4270 to represent the other colleges, the author used technology to randomly select 500 colleges without replacement. There were 173 2-year colleges in the sample. What proportion of the sample was 2-year colleges?

   c. If you did not know the proportion of the population that are 2-year colleges and used the result you found in Part (b) to estimate it, would that be part of descriptive statistics or inferential statistics? Explain.

Definition *Sampling error*

Sampling error is the error involved in using a sample to estimate information about a population due to randomness in the sample.

7. Find the sampling error for the estimation you made in Problem 6.

If random sampling is used to select a large enough sample, the sampling error will not be too large.

OBJECTIVE 6

Definition *Bias*

A sampling method that consistently underemphasizes or overemphasizes some characteristic(s) of the population is said to be **biased**.

If a sampling method is biased, then any inferences made will be misleading.
There are three types of bias:

1. **Sampling bias** occurs if the sampling technique favors one group of individuals over another.
2. **Nonresponse bias** happens if individuals refuse to be part of the study or if the researcher cannot track down individuals identified to be in the sample.
3. **Response bias** occurs if surveyed people's answers do not match with what they really think.

Guidelines for Constructing Survey Questions

When constructing a survey question:

- Do not include judgmental words.
- Avoid asking a yes/no question.
- If the question includes two or more choices, switch the order of the choices for different respondents.
- Address just one issue.

For each study, identify possible forms of bias. Also, discuss sampling error.

8. A liberal radio station conducts a call-in survey, asking callers, "Should Congress do the right thing and increase the budget for education?" Of the 1140 callers, 969 answered yes. The station concludes that 85% of all Americans think that the budget for education should be increased.

9. A group of students surveys other students who enter and exit the gym and finds that 67 students out of 100 students exercise every day. The group concludes that 67% of all students at the college exercise every day.

OBJECTIVE 7

Definition *Nonsampling error*

Nonsampling error is error from using biased sampling, recording data incorrectly, and analyzing data incorrectly.

10. A hotel randomly selects some customers who stayed at the hotel in the past month, asking whether they had taken clothes hangers and/or towels from their room. The proportion of the sampled customers who said they had taken clothes hangers and/or towels was 0.01. But the population proportion is actually 0.12. Identify whether sampling error, nonsampling error, or both have likely occurred.

HW 1, 3, 7, 11, 13, 19, 21, 23, 27, 33, 35, 41, 43, 58, 59

SECTION 2.1 TEACHING TIPS

One could argue that the most important topic of this section is simple random sampling because it is the first of several sampling methods that will be addressed in this chapter. Plus, the concept of randomness will also come up when discussing probability (Chapter 5). This section lays the foundation for Section 2.3, which describes key features of an experiment.

OBJECTIVE 1 COMMENTS

Students often confuse the meanings of *individuals*, *variables*, and *observations*. I prefer to introduce the terminologies as I work on Problem 1 so students can see concrete examples. Then I present the general definitions.

As time passes, students tend to slip into thinking that the terminology *individuals* refers to people and not objects, so it will help to remind them of the correct definition in a few days.

OBJECTIVE 2 COMMENTS

The five steps of statistics lays the foundation not only for this course, which addresses descriptive statistics, but also for introductory statistics courses, which address both descriptive and inferential statistics. I quickly discuss the steps generally and then have groups of students complete Problem 2, which they have an easy time with.

OBJECTIVE 3 COMMENTS

Students are quick to pick up on the basic idea of the meanings of *population* and *sample*, but there are subtleties that can be challenging.

For example, some students think that individuals in the control group are not part of the sample. It is helpful to point out that such individuals are part of the sample because data are collected from them. (Control groups will be defined in Section 2.3.)

When reading statistical reports, it can be difficult for students to determine the population because it is often implied. For example, in Exercise 22 of Homework 2.1, a student might conclude that the population is all sexually confident women. Actually, a comparison is being made between women who are sexually confident and those who are not, so the population is all women. Also, if all the women in the sample were randomly selected from the United States, then the population would be all American women rather than all women in the world.

Another issue is that some students understand how to identify the sample and population, but they give vague references to each. I tell them that stating the number of individuals in the sample (if provided) and using the word "all" when describing the population can help clarify which individuals are in which groups.

I prefer to assign the simpler exercises about sample and population in Homework 2.1 so that students gain some confidence. Once we reach Section 2.3, the exercises that continue to address population and sample will raise students' understanding to a higher level.

OBJECTIVE 4 COMMENTS

Some students struggle with distinguishing descriptive statistics from inferential statistics. I'm not sure why. To help clarify, I give a lot of examples of each. Although Chapters 3–10 will focus on descriptive statistics, there are examples of inferences sprinkled throughout the text in which students must identify them as such.

OBJECTIVE 5 COMMENTS

One could argue that simple random sampling is the most important concept of this section because it is the key idea of forming samples, performing random assignment (Section 2.3), and probability (Chapter 5).

Many students have misconceptions about randomness. Some students believe six flips of a fair coin would result in three heads and three tails. Some students believe that flips of a fair coin would not result in streaks. To dismantle these misconceptions, I flip a coin in class 20 times and record the results on the board. Then we identify the longest streak and also compare the frequencies of heads and tails. If there happen to be 10 of each, we compare the frequencies of heads and tails after the first n flips, where n is even and the numbers of heads and tails are not equal.

After discussing how to randomly select people using names in a hat, I discuss how to use technology. For exams, I tell students what number to use as a seed so that it won't be too cumbersome to grade, but the fact that all students then generate the same random numbers muddies the waters about the meaning of randomness. Pointing out that technology *simulates* random selection can help clarify.

Another thing that is curious about a seed is that before we can use technology to randomly select some numbers, we must randomly select the seed. The textbook uses the current date, time, or temperature to select a seed, but students might think a seed must always be selected in those ways. Tossing a 10-sided die several times to generate the digits of a seed might be the best way to go.

It's a good idea to point out that in most cases, samples are formed using sampling without replacement.

OBJECTIVE 6 COMMENTS

I emphasize the meaning of *sampling bias*, *nonresponse bias*, and *response bias* because so many bad practices of statistics stem from these misuses. Most textbooks address these concepts in a single chapter and do not revisit them in subsequent chapters on descriptive statistics. In *Pathway*, these concepts are addressed many times in Chapters 2–5. Chapters 6–10 will focus on the prevalent misunderstanding that a strong association implies causation.

Discussing bias at great length will better prepare students for some of the Hands-On Research exercises as well as some of the Hands-On Projects that require students to perform surveys. Due to students' limited time and money, their surveys will have bias, but they are required to describe the bias (see Problems 7 and 8 of the Survey about Proportions Project on page 123 of the textbook).

Before discussing sampling, nonresponse, and response bias, I have groups of students identify sampling issues in Problems 8 and 9. Then I generalize their comments by providing the definitions. Having students first work with concrete examples prepares them for digesting the general definitions.

When some students learn about sampling bias, they have distorted ideas about how to avoid it. For a survey about people's favorite sport, some students may see the pitfall in polling people at a basketball game but may think that polling people at various sporting events would be fine. Even after an in-depth classroom discussion about sampling bias, some students will not understand that the only way to avoid sampling bias is to randomly select the sample. As a result, I make sure I revisit this concept several times throughout the course.

I spend a disproportionate amount of time discussing response bias because it has many facets and it is the bias that students will be able to control the most when performing surveys by carefully constructing their questions.

Sampling will be further developed in Section 2.2, which describes systematic, stratified, and cluster sampling.

OBJECTIVE 7 COMMENTS

Students often confuse the meaning of *sampling error* with the meaning of *nonsampling error* probably because they sound so similar. Once students can keep straight which is which, they have an easy time with nonsampling error. Sampling error is a much more challenging concept to grasp. Students tend to think that the value of a statistic is equal to the value of the parameter. The media has introduced or at least reinforced this misconception. Grappling with sampling error is at the heart of statistics, but it is difficult to address this concept in the midst of descriptive statistics.

Using technology to solve an example similar to Example 6 in the textbook is a nice way to explore sampling error, but I emphasize that normally we do not know the value of a parameter. Example 6 is also misleading because it suggests that in statistics we use point estimates to estimate parameters when in fact we use confidence intervals. For this reason, I seriously considered introducing a light treatment of confidence intervals in Section 2.1, but I decided that it would likely create too many misconceptions about confidence intervals.

Despite these issues, Example 6 does a great job of clarifying sampling error because it is so concrete.

GROUP EXPLORATION: Using Sampling Methods

Telling students how to construct good survey questions is a good first step. But having students construct their own questions and getting feedback from you will greatly enhance their understanding. This is a good time to warn students to avoid inappropriate questions.

Providing a diagram of the names of students and where they are seated can move things along. I emphasize that groups should record the seed they selected. Having them carry out their surveys can be chaotic and inefficient unless you provide some structure. Each group could select one member to be the surveyor. The surveyors could circulate the classroom while the other students remain seated. Class time could be preserved by having the students participate in the surveys online, but this would require time and effort on your part to coordinate posting students' questions and responses.

This exploration is good preparation for the Survey about Proportions Project on page 123 of the textbook.

SECTION 2.2 LECTURE NOTES

Objectives

1. Identify and explain *systematic sampling*.
2. Identify and explain *stratified sampling*.
3. Identify and explain *cluster sampling*.
4. Compare sampling methods.
5. Describe why *convenience sampling* and *voluntary response sampling* should never be used.

OBJECTIVE 1

Definition *Systematic sampling*

To perform **systematic sampling**, we randomly select an individual out of the first k individuals and also select every kth individual after the first selected individual.

Warning: Once k is chosen, the researcher must make sure that selecting every kth individual does not match with some pattern in the population.

1. A Target manager wants to survey customers as they leave the store on a certain day. There are about 925 customers each day. The manager would like to get feedback from at least 75 customers. From past experience, the manager knows that if 100 customers are approached, about 75 of them will participate in the survey.

a. Given that the manager wants to survey customers as they leave the store, explain why it would not be possible to perform simple random sampling.

b. The manager decides to perform systematic sampling. If every kth customer is approached, find k so that 100 customers are approached.

c. To find which customer should be first approached, use 822 as the seed to find a random number between 1 and k, inclusive.

d. List the first 5 customers who should be approached.

We should always round down when calculating k for systematic sampling.
A key benefit of systematic sampling is that a frame is not required.

OBJECTIVE 2

Definition *Stratified sampling*

To perform **stratified sampling**, a population is divided into subgroups called **strata** (singular: **stratum**) and simple random sampling is performed on each stratum. In each stratum, the individuals share some characteristic. The ratios of the number of individuals selected from each stratum are equal to the corresponding ratios of the stratum sizes.

Benefits of stratified sampling:

- Individuals in small strata are not excluded.
- A smaller sample can be selected than by using simple random sampling. This saves time, money, and effort.

2. Emory University wants to survey a sample of 250 students from the class of 2015, asking them whether they feel like they belong. The numbers of students are shown in the following table for various ethnicities. How many students of each ethnicity should be included in the study?

Ethnicity	Number of Students
Caucasian	1826
Asian American	800
African American	425
Latino	507
Multi-Racial	155
Native American	1
International	855
Did Not Identify	261

Source: *Emory University*

3. Western Illinois University, Quad Cities, wants to survey its students about their first-semester experience. Because female undergraduates, male undergraduates, female transfer students, male transfer students, female graduate students, and male graduate students may have had different experiences, the university plans to use these six types of students as strata. The stata sizes are shown in the following table. If the university wants to survey 150 students, how many students in each strata should be surveyed?

Gender	Freshmen	Undergraduate Transfer	First-Time Graduate	
Female	154	234	115	503
Male	130	186	52	368
Total	284	420	167	871

Source: *Western Illinois University*

OBJECTIVE 3

Definition *Cluster sampling*

To perform **cluster sampling**, a population is divided into subgroups called **clusters**. Then simple random sampling is used to select some of the clusters. All of the individuals in those selected clusters form the sample.

Benefits of cluster sampling:

- It does not require constructing a frame of individuals.
- It saves time, money, and effort.

4. Boston Mayor Marty Walsh wants data collectors to conduct in-person interviews with some Boston residents. If each city block is treated as a cluster, describe how cluster sampling could be accomplished.

OBJECTIVE 4

Sampling Method	Requirement	Benefits
Simple Random	A frame of all individuals.	Works fine for telephone and e-mail surveys in which there is little risk of excluding anyone.
Systematic	Selecting every kth individual must not match with some pattern in the population.	No frame is required.
Stratified	In each strain, individuals are similar. A frame for individuals in each strata.	Individuals in small strata are not excluded from the sample. Can save time, money, and effort.
Cluster	A frame of clusters.	No frame of individuals is required. Can save time, money, and effort.

OBJECTIVE 5

Definition *Convenience sampling*

To perform **convenience sampling**, we gather data that are easy to collect and do not bother with collecting them randomly.

Although convenience sampling is easy to perform, it should never be done because the sample will usually not represent the population well.

Definition *Voluntary response sampling*

To perform **voluntary response sampling**, we let individuals choose to be in the sample.

Never perform voluntary response sampling or the more general convenience sampling.

Identify whether the sampling method is simple random, systematic, stratified, cluster, or convenience. Explain.

5. In a study at a university, 250 students are selected from each of the classes freshmen, sophomores, juniors, and seniors.

6. Home Depot creates a frame of all of its 340 thousand employees and randomly selects some of the employees.

7. A blog hosts an online survey, asking respondents whether they go to the movies at least once per month.

8. Twenty Burger King locations in the United States are randomly selected and all the employees at those locations are surveyed.

9. A police unit stops every fourth car on a highway and tests whether the driver is driving under the influence of alcohol.

HW 1, 3, 5, 7, 9, 11, 13, 19, 27, 29, 31, 33, 39, 43, 45

SECTION 2.2 TEACHING TIPS

Students are challenged by having to distinguish between simple random sampling and the five sampling methods introduced in this section. Some of the difficulty stems from using some of the same words to describe different methods. And even though the different words *strata* and *cluster* are used to describe subgroups for stratified and cluster sampling methods, students still tend to struggle to keep the two methods straight because similar steps are taken but in a different order.

OBJECTIVE 1 COMMENTS

Students are so accustomed to the usual routine of rounding that I make sure they understand that when determining k for systematic sampling, we should always round down. This practice involves some nice critical thinking and will also lead to correct answers for homework exercises, but it is not important for many actual studies because the maximum number of individuals that might be surveyed or measured is unclear. For example, the total number of people that might visit a store in one day is unknown.

Although using systematic sampling for quality control of products on an assembly line might not impress students, they do tend to find it compelling that the Supreme Court ruled that sobriety checkpoints are legal, but police departments must file a plan that describes how they will randomly select drivers for sobriety tests. In fact, Exercise 43 describes how a motorist arrested for drunk driving contested and won his court case because the police did not perform systematic sampling correctly.

Such authentic stories will make the course come alive for your students. Consider researching how your local police department has carried out sobriety checkpoints.

OBJECTIVE 2 COMMENTS

Instead of using the ethnic breakdown at Emory University in Problem 2 to discuss stratified sampling, consider researching the ethnic breakdown at your college. Or you could research the academic-major breakdown. It is ideal if one or more of the categories has a percentage close to zero to illustrate that stratified sampling can ensure that such categories are represented in the sample.

Although the calculations for Problem 2 suggest that the Native American should not be included in the sample, you could point out that it might be a good idea to include the student to hear the student's possibly unique perspective.

For fear of creating more confusion than understanding, I did not state in the textbook that stratified sampling can be used to survey a disproportionate amount of categories and that this can be corrected by using a weighted mean or other measure. This way, the true composition of categories with small percentages are well represented. It is probably not worth the trouble to include weighting in your lesson plan–this form of stratified sampling is not even addressed in many introductory statistics textbooks–but if you want to discuss it, I advise waiting until you reach Section 4.1, which introduces the concept of arithmetic mean.

For Problem 3, students tend to need clarification why we find the appropriate proportions by dividing the cells by the grand total 871 rather than the subtotals. Some reviewers feel that Problem 3 is too difficult for students, but I've found my students can handle it. The real question is whether working with a *single* variable such as ethnicity suffices to get across the idea of stratified sampling. One benefit of working with two-way tables is that it will prime the pump for students' work with two-way tables in Chapters 3–5.

OBJECTIVE 3 COMMENTS

I prefer to discuss Example 3 in the textbook to get across the idea of cluster sampling because students can relate to the situation. I emphasize why we divide the 1000 students by the *smallest* class size. This concept, as well as why we round k down for stratified sampling, makes for good conceptual questions on quizzes and exams. I also emphasize that the benefit of cluster sampling is that no frame of individuals is required and it can save time, money, and effort.

OBJECTIVE 4 COMMENTS

Students often confuse stratified sampling with cluster sampling. Referring to Figs. 8 and 9 on page 105 of the textbook or creating your own diagrams can help you compare and contrast the two methods. Table 14 on page 105 of the textbook provides a nice comparison of the requirements and benefits of the four sampling methods.

Even for students who are not into sports, comparing cluster and stratified sampling by referring to football teams (or some other sport's teams) as clusters/strata seems to get across the difference in the two sampling methods.

Even once students can identify the various types of sampling methods, their understanding tends to slip away with time. It's a good idea to revisit this topic near the end of Chapter 2 and to encourage students to review the methods thoroughly.

GROUP EXPLORATION: Performing Sampling Methods

Providing a diagram of the names of students and where they are seated can move things along. I emphasize that they should record the seed that they selected. Having groups carry out their surveys can be chaotic and inefficient, unless you provide some structure. Each group could select one member to be the surveyor. The surveyors could circulate the classroom while the other students remain seated. Class time could be preserved by having the students participate in the surveys online, but this would require time and effort on your part to coordinate posting questions, responding to questions, and posting responses.

Although this exploration may be too time-consuming, the physical act of carrying out the three types of sampling methods would enhance students' understanding and retention of the three methods.

OBJECTIVE 5 COMMENTS

I hesitated in constructing definition boxes for *convenience sampling* and *voluntary response sampling* for fear that students would think that these techniques are valid sampling methods. For the same reason, I was also hesitant to include exercises that require students to identify such bad sampling practices. However, reviewers suggested I include definition boxes so there would be a clear way to refer to such bad sampling methods. I felt this was a good point, so in the end, I included the definition boxes. Hopefully the boldface warnings on page 106 of the textbook and your verbal warnings will impress upon students that these methods should never be used.

Groups of students are quick to point out the problems with using a specific form of voluntary sampling or the more general convenience sampling.

SECTION 2.3 LECTURE NOTES

Objectives

1. Identify the following components of a good study: *treatment group*, *control group*, *placebo*, *single-blind and double-blind*, *random assignment*, and large-enough sample size.
2. Identify *experiments* and *observational studies*.
3. Identify *explanatory variables*, *response variables*, *association*, and *causation*.
4. Identify *lurking variables* and *confounding variables*.
5. Redesign an observational study into an experiment.

OBJECTIVE 1

Definition *Treatment group and control group*

Each **treatment group** in a study is a collection of individuals who receive a certain treatment (or have a certain characteristic of interest). The **control group** is a collection of individuals who do not receive any treatment (or do not have the characteristics of any treatment group).

When designing a study, it is important that each treatment group be as similar to the control group as possible, except for the characteristic of interest.

1. A researcher wants to investigate whether a new drug lowers the blood pressure of patients who have high blood pressure. Describe the design of a possible study, which includes a treatment group and a control group.

- The **placebo effect** occurs when the characteristic of interest changes in individuals due to the individuals believing the characteristic should change.
- A **placebo** is a fake drug or procedure administered to the control group.
- In a **single-blind study**, individuals do not know whether they are in the treatment group(s) or the control group.
- In a **double-blind study**, neither the individuals nor the researcher in touch with the individuals know who is in the treatment group(s) and the control group.

2. Explain how the design of the study you described in Problem 1 could be modified so that it would be double-blind and use a placebo.

Definition *Random assignment*

Random assignment is the process of assigning individuals to the treatment group(s) and the control group randomly.

3. A researcher wants to test whether a high-protein diet improves college male sprinters' performances. The researcher randomly selects the 10 college male sprinters shown in the following table. Use the seed 892 to randomly select 5 of the sprinters for the treatment group. The other 5 students will be in the control group.

Benny	David	Nico	Austin	Daniel
Ryan	Matt	Marcos	Cameron	Randell

Components of a Well-Designed Study

In a well-designed study,

- There should be a control group and at least one treatment group.
- Individuals should be randomly assigned to the control group and the treatment group(s).
- The sample size should be large enough.
- A placebo should be used when appropriate.
- The study should be double-blind when possible. If this is impossible, the study should be single-blind if possible.

OBJECTIVE 2

Definition *Experiment and observational study*

In an **experiment**, researchers determine which individuals are in the treatment groups(s) and the control group, often by using random assignment. In an **observational study**, researchers do not determine which individuals are in the treatment group(s) and the control group.

Identify whether the study is an experiment or an observational study. Discuss whether the components of a well-designed study were used.

4. In a study to determine whether the drug latrepirdine improves the mental impairment of patients with Huntington disease, researchers randomly assigned 403 patients with mild to moderate Huntington disease to a treatment group and a control group. For 3 times daily for 26 weeks, latrepirdine was orally administered to the treatment group and a placebo was orally administered to the control group. The study concluded that latrepirdine does not improve the mental impairment of patients with Huntington disease (Source: A randomized, Double-Blind, Placebo-Controlled Study of Latrepirdine in Patients with Mild to Moderate Huntington Disease*, Kieburtz et al.*).

5. In a study to determine whether elite athletes have unbalanced postures, researchers measured the posture of the following elite athletes: 29 rugby players, 29 bikers (motorcyclists), 10 skiers, and 10 judokas. The postures of 71 amateur athletes were also measured. The study found that elite rugby players, elite skiers, and elite judokas had unbalanced postures but elite bikers had balanced postures (Source: Assessment of Body Plantar Pressure in Elite Athletes: an Observational Study*, G. Gobbi et al.*).

OBJECTIVE 3

Definition *Explanatory and response variables*

In a study about whether a variable x explains (affects) a variable y,

- We call x the **explanatory variable** (or **independent variable**).
- We call y the **response variable** (or **dependent variable**).

An explanatory variable may or may not turn out to affect (explain) the response variable.

6. In a double-blind study, 40 healthy foreign students were randomly assigned to take pills of the natural substance Rhodiola rosea extract SHR-5 or a placebo for 20 days during an examination period. The study concluded that the substance improves people's mental fatigue caused by stress (Source: A Double-Blind, Placebo-Controlled Pilot Study of the Stimulating and Adaptogenic Effect of Rhodiola Rosea SHR-5 Extract On the Fatigue of Students Caused by Stress During an Examination Period with a Repeated Low-Dose Regimen, *Spasov et al*).

a. Describe the treatment and control groups.

b. Is the study an experiment or an observational study? Explain.

c. Describe the sample and the population.

d. What are the explanatory and response variables?

e. What does it mean that the study is double-blind? How could that be accomplished?

- There is an **association** between the explanatory and response variables if the response variable changes as the explanatory variable changes.
- If the change in the explanatory variable *causes* a change in the response variable, we say there is **causality**.

Determining Causality

- A well-designed experiment can determine whether there is causality between the explanatory and the response variables.
- Most observational studies *cannot* determine whether there is causality between the explanatory and response variables. They can only determine whether there is an association between the two variables.

OBJECTIVE 4

Definition *Lurking variable*

A **lurking variable** is a variable that causes both the explanatory and the response variables to change during the study.

Lurking variables must be avoided. We can usually do so by using random assignment.

Definition *Confounding variable*

A **confounding variable** is a variable other than the explanatory variable that causes or helps cause the response variable to change during the study.

A well-designed experiment requires careful planning so that there are as few confounding variables as possible.

OBJECTIVE 5

7. A researcher wants to determine whether watching television and playing video games causes attention deficit hyperactivity disorder (ADHD) in children. The researcher randomly selects 200 children and asks their parents how much television their children watch daily and whether their children have ADHD. The researcher analyzes their responses and concludes that watching television and playing video games causes ADHD in children.

a. Describe some problems with the observational study. Include in your description at least one possible lurking or confounding variable and identify which type it is.

b. Redesign the study so that it is a well-designed experiment.

HW 1, 3, 5, 7, 11, 13, 15, 17, 19, 23, 29, 33, 41

SECTION 2.3 TEACHING TIPS

This is the most important section of Chapter 2 because it addresses so many important features of a well-designed study and it emphasizes the concept of causation, which is misused so often by the media and businesses.

If you are tight on time, it is probably not worth distinguishing lurking variables from confounding ones. It is probably enough for students at this level to learn that a well-designed experiment can avoid the impact of such variables.

OBJECTIVE 1 COMMENTS

After defining *treatment group* and *control group*, I discuss the importance of control groups. I also explain that individuals in the control group are considered part of the sample.

When first describing the components of a good study, I keep things simple and stick to describing studies that have just one treatment group and a control group. Later, I explain the benefits of using multiple treatment groups.

Students are quick to pick up on the meaning and benefit of a placebo. Students tend to already know that sugar pills are used as placebos, but most students don't know that placebos include fake procedures. In fact, many students are shocked to learn that fake knee surgeries were used in the study that is described near the top of page 112 of the textbook. I make sure that I share this study with students because when they have such a strong emotional response, they are that much more likely to retain the concept.

Random assignment is a crucial concept because it is often what distinguishes experiments from observations. Unfortunately, students often confuse random assignment with random sampling, so I make sure I compare and contrast these two techniques and remind students of the distinction between the methods several times during the course. It is worth pointing out that in many experiments, the individuals were not randomly selected but that is okay provided the researchers performed random assignment.

Having groups of students first randomly select a sample from a population and then randomly assign members of the sample to a treatment group and a control group would be a worthwhile way for students to learn the difference between selecting a random sample and performing random assignment. If you cannot afford the time, consider displaying a diagram similar to Fig. 12 on page 113 of the textbook.

The textbook's discussion about sample size is purposely vague. It is only once students study inferential statistics that they can quantify how the sample size is related to the error bound. At this point in the course, I say that the larger the sample size, the more information we will have so the lower the error will be; therefore, a relatively large sample size is desirable. Because we have not discussed the arithmetic mean yet, I do not bother to say that "relatively large" can mean quite different things for estimating, say, the population mean versus estimating, say, the population proportion.

Once I have discussed all the components of a good study, I like to display an actual study which consists of several pages on an overhead. I describe how the study is structured and point out the components that we have discussed in general. I emphasize that students may someday have to read such studies for their careers. This drives home the point that the concepts we discuss in class are truly relevant and important.

The greatest challenge with this objective is reading comprehension. The homework exercises are at a higher reading level than most of the rest of the exercises in the book. This is due to numerous reasons, including the large number of aspects of a study, the intricacies of controlling for confounding, and complicated names of drugs. I urge students to read descriptions of studies many times. I also suggest that after reading each sentence, students should pause and reflect on its meaning and determine why that aspect of the study is important. Most of my students do not know how to perform engaged reading, and I have to devote much time to coaching them on this. It can be helpful to model it. For example, I read a sentence out loud and then say out loud all the thoughts and questions I have in response to that information. Then I do the same for the next sentence and the next, until I reach the end of the description. Most students are amazed at the level of engagement I have with reading.

Learning how to read in an engaged way can benefit students in all of their courses and later in their career. For some students, it might be the most powerful learning experience of the semester.

OBJECTIVE 2 COMMENTS

Initially, students have an easy time distinguishing between experiments and observational studies because they can scan the description of a study to see whether the phrase "random assignment" is included. However, students tend to forget that random assignment is the key distinction unless they are reminded many times. If an observational study involves explanatory and response variables in which causation seems likely, students tend to assume there *is* causation.

Students struggle to learn why random assignment allows researchers to determine whether there is causation. To help them understand, I give a lot of examples of what can go wrong (i.e. lurking variables) if random assignment is not used. Explaining this concept can serve as a good conceptual question on a quiz or exam.

OBJECTIVE 3 COMMENTS

Once students understand that the explanatory variable involves the treatment that the treatment group receives and the response variable involves some response due to the treatment, students have a fairly easy time identifying the two variables. The biggest stumbling block is the same as with Objective 1: reading comprehension. The suggestions I gave regarding that objective apply here, too.

To get across the idea that the presence of an association between two variables does not guarantee causality, I describe the firefighter situation illustrated by Fig. 13 on page 116 of the textbook. This brings the classroom discussion full circle to the fact that we can use experiments to determine whether there is causality because experiments employ random assignment.

The reason the textbook states that *most* observational studies cannot determine whether there is causation is because in some cases, causality can be determined from "Big Data." However, to avoid confusion, I don't explain this to students. Even without this complication, it is an uphill battle to make sure that all students learn the pitfalls of observational studies commonly referred to by the media. One could argue that this is the most important statistical insight students can gain from the course.

OBJECTIVE 4 COMMENTS

Students have trouble distinguishing between lurking and confounding variables, which is understandable. Although I sketch diagrams such as those in Figs. 14 and 15 on page 117 of the textbook, I don't spend much time making the distinction. What really matters is that students understand that both types of variables can wreak havoc. I emphasize that to contend with these variables, researchers must use random assignment and take great care in designing an experiment.

OBJECTIVE 5 COMMENTS

When discussing how to redesign an observational study into a well-designed experiment, I emphasize that a key step is to introduce random assignment. Then I suggest students read a list of components of a well-designed experiment and check whether each of the components is present in the study. Of course, it is impossible to employ components such as double-blind in all studies, but discussing why a component cannot be used can be a learning experience, too.

GROUP EXPLORATION: Designing an Experiment

Because designing an experiment is more challenging than planning a survey, this exploration is more challenging than the ones in Sections 2.1 and 2.2. At this point in the course, most students have only learned about a few experiments and it will be difficult for them to think of an experiment on their own. Plus, many or all of the experiments you have discussed likely involve equipment or medication that students won't have access to. Distributing a handout with examples of simple experiments may help students think of an experiment. If you can't afford the time for students to brainstorm ideas, you could allow students to choose from your examples, but this would rob them of a great deal of critical thinking.

If you facilitated the exploration in Section 2.2, students should have an easy time carrying out the cluster sampling.

CHAPTER 3 OVERVIEW

By now you have probably surveyed your students, but if you haven't yet, I advise doing so very soon. You can use the data as early Section 3.1 when constructing bar graphs. And having access to raw data may reveal response and/or nonresponse bias that you can refer to as a reminder of Chapter 2 material.

It is easy to become bogged down with this chapter because it includes so many types of tables and diagrams, but it is not necessary to have students learn about all of them for several reasons.

First, students only need to know about two-way tables (Section 3.2) and histograms (Section 3.4) to do the vast majority of exercises in subsequent chapters. Two-way tables are an excellent way for students to grapple with probability (Sections 5.1–5.3) and density histograms are a great way to prepare students for normal distributions (Sections 5.4 and 5.5). For Chapters 6–10, students will work extensively with scatterplots, but those diagrams are not included in Chapter 3. (They are introduced in Section 6.1.)

Second, aside from a quick run-through of many types of tables and diagrams, the main tables and diagrams used in introductory statistics are once again two-way tables, histograms, and scatterplots.

Third, I do not believe the point of *Pathway* is to address every single descriptive statistics topic that is handled in introductory statistics. Rather, *Pathway* should focus on key concepts that lay a foundation so students can have sure footing in introductory statistics courses as they learn complex concepts such as the sampling distribution and hypothesis testing.

For example, it is far more important that students have an in-depth understanding of the normal curve than it is for them to know how to construct a myriad of types of tables and diagrams. That is why the textbook emphasizes the interpretation of density histograms (Section 3.4).

Whatever tables and diagrams you include in the course, make sure the emphasis is on interpreting them. It is the interpretation of concepts and results, not the construction of tables and diagrams, that challenges most introductory statistics students. That's why the four characteristics (Sections 3.3 and 3.4) of a numerical variable are introduced in this chapter rather than in Chapter 4.

Other topics that are key to this chapter are the ways diagrams can be used to persuade or mislead (Section 3.5). Some students will read statistical reports in their careers. Few may perform statistics. But all students can benefit from becoming more critical of diagrams displayed by businesses and the media.

SECTION 3.1 LECTURE NOTES

Objectives

1. Identify *categorical variables* and *numerical variables*.
2. Construct and interpret *frequency tables* and *relative frequency tables*.
3. Construct and interpret *frequency bar graphs* and *relative frequency bar graphs*.
4. Describe the meanings of *AND* and *OR*.
5. Use a relative frequency bar graph to find proportions.
6. Interpret *multiple bar graphs*.

OBJECTIVE 1

Definition *Categorical variable*

A **categorical variable** (or **qualitative variable**) consists of names or labels of groups of individuals.

Definition *Numerical variable*

A **numerical variable** (or **quantitative variable**) consists of measurable quantities that describe individuals.

1. Identify whether the variable is categorical or numerical.

a. The salary (in dollars) of a person

b. The state where a person lives

c. The ZIP code of a home in Illinois

d. A variable consisting of zeroes and ones, where 0 stands for a full-time student and 1 stands for a part-time student

OBJECTIVE 2

Definition *Frequency of a category*

The **frequency of a category** is the number of observations in that category.

A **frequency table** for a categorical variable is a table that lists all the categories and their frequencies.

2. The music genres of the 20 top-selling singles during the week ending on September 5, 2015, are shown in the following table. Construct a frequency table.

R & B	Dance	Electronic	Pop	Pop	Dance	Pop
Pop	Pop	Pop	Alternative	Hip-Hop/Rap	Dance	Dance
Hip-Hop/Rap	Pop	Pop	Pop	Alternative	Pop	

Source: *AT40*

Definition *Frequency distribution of a categorical variable*

The **frequency distribution of a categorical variable** is the categories of the variable together with their frequencies.

The frequency table we constructed in Problem 2 describes the frequency distribution of the categorical variable music genre.

Definition *Relative frequency*

The **relative frequency of a category** is give by

$$\frac{\text{frequency of the category}}{\text{total number of observations}}$$

- For Problem 2, the relative frequency of dance category is $\frac{4}{20} = \frac{1}{5}$.
- **A relative frequency is a proportion.**

- A **relative frequency table** of a categorical variable is a list of the categories and their relative frequencies.

3. Refer to the frequency table we constructed in Problem 2.

a. What is the relative frequency of the hip-hop/rap category?

b. Construct a relative frequency table.

c. Which category has the largest relative frequency? What is that relative frequency? What does it mean in this situation?

We agree to round relative frequencies and proportions to the third decimal place.

Definition *Relative frequency distribution of a categorical variable*

The **relative frequency distribution of a categorical variable** is the categories of the variable together with their relative frequencies.

4. Use StatCrunch to construct a frequency and relative frequency table of the music distribution.

Sum of Relative Frequencies

For a categorical variable, the sum of the relative frequencies of all the categories is equal to 1.

OBJECTIVE 3

A **frequency bar graph** is a graph that uses heights of bars to describe the frequencies of categories.

5. Construct a bar graph of the music distribution.

A **relative frequency bar graph** is a graph that uses heights of bars to describe the relative frequencies of categories.

6. Construct a relative frequency bar graph of the music distribution.

OBJECTIVE 4

- When "**AND**" is used with two categories, this means to consider the observations that the categories have in common.
- When "**OR**" is used with two categories, this means to consider the observations in the categories all together.

7. The dates of September 2016 are shown in the following table.

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28			

a. Find the dates that are in the fourth week.

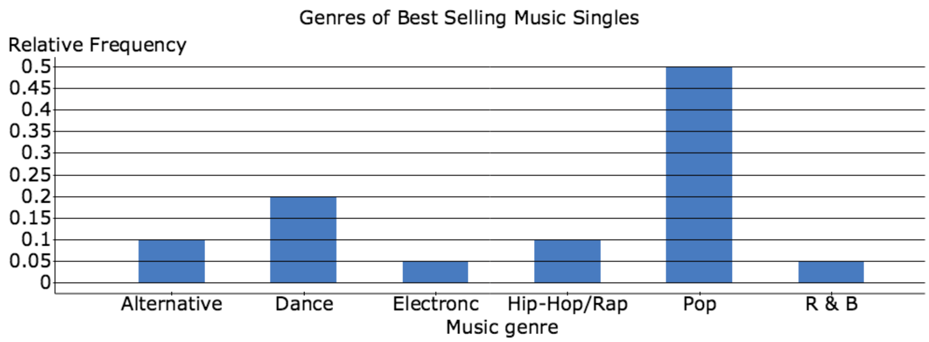
b. Find the dates that are Wednesdays.

c. Find the dates that are in the fourth week AND are Wednesdays.

d. Find the dates that are in the fourth week OR are Wednesdays.

OBJECTIVE 5

8. A relative frequency bar graph of the music distribution is shown in the following figure.



Source: *AT40*

Find the proportion of the observations that

a. fall in the electronic category.

b. do NOT fall in electronic category.

c. fall in the alternative category OR fall in the pop category.

d. fall in the alternative category AND fall in the pop category.

Note that in Problem 8(b), we used the proportion of the rest property (Section 1.3).

Proportion of the Rest

Let $\frac{a}{b}$ be the proportion of the whole that has a certain characteristic. Then the proportion of the whole that does NOT have that characteristic is

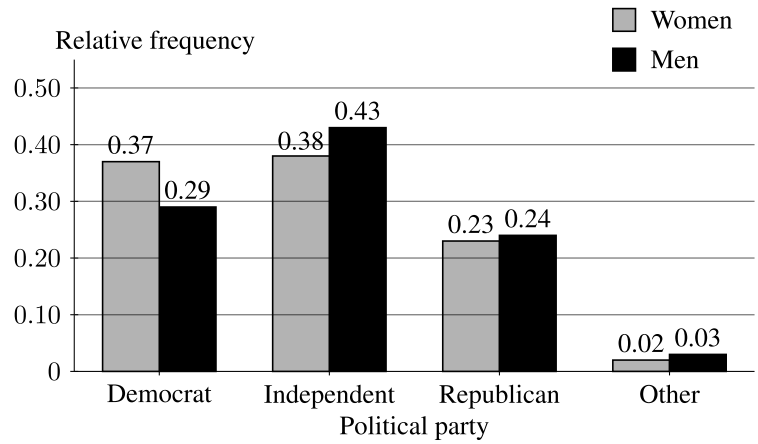
$$1 - \frac{a}{b}$$

OBJECTIVE 6

A **multiple bar graph** is a graph that has two or more bars for each category of the variable described on the horizontal axis.

9. In a survey in 2012, 1960 adults were asked the following question: "Generally speaking, do you usually think of yourself as a Republican, Democrat, Independent, or what?" The results of the survey are described by the multiple bar graph in the following figure.

Political Party, by Gender



Source: *General Social Survey*

a. What proportion of women thought of themselves as Democrats?

b. To which political party did the greatest proportion of men choose?

c. Compare the proportion of women who thought of themselves as Independents to the proportion of men who thought of themselves as Independents.

d. A total of 1081 women and 879 men responded to the survey. Were there more women or men who thought of themselves as Independents? How is this possible, given there was a smaller proportion of women who thought of themselves as Independents than men?

e. On the basis of the multiple bar graph, a student concludes that 24% of *all* American men are Republicans. What would you tell the student?

For a multiple bar graph, it is more meaningful for the vertical axis to describe relative frequencies rather than frequencies.

HW 1, 3, 9, 11, 13, 21, 23, 27, 29, 31, 33, 35, 39, 43, 51

SECTION 3.1 TEACHING TIPS

One could argue that the key objective of this section is for students to be able to use relative frequency bar graphs to find proportions. This skill will lay the foundation for work with density histograms (Section 3.4) and probability (Chapter 5). A close second would be the meanings of AND and OR, which will be addressed in the rest of Chapter 3 as well as Chapter 5.

OBJECTIVE 1 COMMENTS

Students are quick to pick up on the difference between categorical and numerical variables, but I make sure they understand that numbers are sometimes used to represent the categories of a categorical variable. Problems 1(c) and 1(d) are examples of this.

In this section, the textbook does not explain that discrete and continuous variables are two types of numerical variables because this section focuses on categorical variables. Discrete and continuous variables are described in Section 3.3, which focuses on numerical variables.

Distinguishing between discrete and continuous variables is important for many reasons, including that it is a key step in determining which types of statistical diagrams are appropriate to use.

OBJECTIVE 2 COMMENTS

Because students have an easy time constructing frequency and relative frequency tables, I move quickly through this material. This is a good opportunity to use data collected from surveying students because they are usually interested in learning about their classmates. Plus, hopefully they will make the connection that if frequency and relative frequency tables can describe categorical variables about themselves, then such tables could be used to describe most any categorical variable.

For starters, I write on the board a subset (about 20) of my students' responses to a question about their favorite music genres, using the category "other" so that there are not too many categories. Next, I construct a frequency and relative frequency table. Then I demonstrate how to use StatCrunch to construct such a table for all the students in the class. Finally, I demonstrate how to use StatCrunch to construct such a table for a data set consisting of hundreds of observations. This progression displays the power of technology.

Each time a new table or diagram is introduced in this chapter, I require students to construct a couple of them by hand and from then on allow students to use technology. This way, students get the idea of how to construct the various diagrams but they are not turned off to statistics by the tedium of constructing lots of them.

I describe frequency and relative frequency distributions of a categorical variable carefully, because the word *distribution* is used so often in the course. It's a good idea to remind students of its meaning each time it is used in a new context (e.g. frequency distribution of a numerical variable in Section 3.3) because many students will otherwise forget.

If you taught from Chapter 1, then your students should already have a pretty good understanding of proportions. Now you can emphasize that a relative frequency is a proportion. In the future, you can stress how both proportions and relative frequencies are related to percentiles (Section 3.3), areas of bars of density histograms (Section 3.4), and probabilities (Chapter 5).

I also emphasize that the sum of the relative frequencies of all the categories of a categorical variable is equal to 1. This dovetails with the concept that the sum of all the parts is equal to the whole (Section 1.3). These concepts set the stage for the complement rule, which will be informally used in this section and subsequent ones and formally introduced in Section 5.2. I advise that students check that the sum of the relative frequencies sum to 1 whenever they construct a relative frequency table.

When going over Problem 8(b), I first find the probability by adding the probabilities for all the categories except electronics. Then I ask the class if there's an easier way. Students are quick to suggest using the proportion-of-the-rest property (Section 1.3). As much as possible, I prompt students to be active learners in the classroom in hopes of minimizing memorization without understanding.

Tell students that they will need to round relative frequencies (and proportions) to the third decimal place for their answers to match the ones in the back of the book (and most likely in MyMathLab).

OBJECTIVE 3 COMMENTS

Students have an easy time constructing frequency and relative frequency bar graphs for the most part, but there are details that they need coaching on. If you taught from Section 1.1, students will hopefully remember that they should use uniform scaling on the (vertical) axis. In addition, you should tell them to write "frequency" or "relative frequency" on the vertical axis, the categorical variable and its categories below the horizontal axis, and a title above the graph. Ideally, a bar graph should convey the complete "story" of the data.

I advise students to read instructions carefully, especially on tests. Weaker students tend to construct frequency bar graphs when asked to construct relative frequency graphs, or vice-versa. Students have sometimes even constructed a frequency bar graph when requested to construct a frequency table.

After students practice constructing one or two bar graphs by hand, I allow them to use StatCrunch to construct them.

In Chapters 3 and 4, the homework exercises are structured so that there is first at least a pair of exercises with about 10 observations, next at least a pair of exercises with about 25 observations, and then at least a pair of exercises with at least 50 observations. This structure is meant to offer instructors flexibility in having students construct diagrams and/or compute measurements with or without statistical technology. For example, I would have students construct a bar graph for the 15 observations in Exercise 35 by hand, the 24 observations in Exercise 37 by hand, and the 50 observations in Exercise 39 using technology.

The purple icon "DATA" flags exercises whose data sets can be downloaded from MyMathLab and from the Pearson Downloadable Student Resources for Math & Statistics website:

http://www.pearsonhighered.com/mathstatsresources.

OBJECTIVE 4 COMMENTS

Although it is a bit jarring to read the words *AND* and *OR* in all uppercase letters, this is a helpful signal to students that the words are being used in a mathematical way. Drawing a diagram similar to Fig. 5 on page 137 of the textbook can help clarify the meanings of *AND* and *OR*. Such a diagram is a gentle introduction to Venn diagrams without calling them as such because students do not need to keep track of sample spaces and events yet. Venn diagrams will be formally introduced in Section 5.2.

I also demonstrate the meanings of *AND* and *OR* by having one or more students raise their hands if they are in the, say, second row OR in the, say, third column. Then I have one or more students raise their hands if they sit in the second row AND the third column. Before I begin the demonstration, I make it clear that the entire class is responsible for the correct students to raise their hands so that everyone is involved and also so that the student who sits in the second row AND the third column does not feel put on the spot. I try to pick a row and column in which the student who sits in the intersection is good-humored and won't mind being singled out.

Using a calendar such as the one used in Problem 7 is yet another nice way to work with *AND* and *OR*. Students will work with calendars in a similar manner when finding probabilities in Sections 5.2 and 5.3.

Students will continue to work with *AND* and *OR* many times in Chapters 3–5, including the important use of *AND* with the multiplicative rule for independent events (Section 5.3).

OBJECTIVE 5 COMMENTS

When finding proportions using a relative frequency bar graph, I emphasize using the proportion-of-the-rest property (Section 1.3). Of course, the property is equivalent to the complement rule, which will be formally introduced in Section 5.2. Note that Problem 8(c) is really an application of the addition rule for disjoint events. Applying both probability rules now (without calling them as such) is good preparation for solving probability problems in Chapter 5, including working with the normal distribution.

GROUP EXPLORATION: Relative Frequencies

This exploration could be used directly after describing how to construct relative frequency bar graphs and discussing the meaning of *OR*.

OBJECTIVE 6 COMMENTS

When working with multiple bar graphs, I have students reflect on the difference between comparing frequencies versus comparing relative frequencies (Problem 9d). Students tend to draw conclusions by comparing frequencies when in many of those cases it is more meaningful to compare relative frequencies. Directly following such a discussion, I point out that for multiple bar graphs, it is more meaningful for the vertical axis to describe relative frequencies than frequencies.

Because the issue of comparing frequencies versus comparing relative frequencies is so important, it is addressed in many exercises throughout the textbook.

I also have students reflect on concepts from Chapter 2 (Problem 9e). Exercises scattered throughout Chapters 3–5 continue to require student to apply concepts from Chapter 2 so that students remember this foundational material.

SECTION 3.2 LECTURE NOTES

Objectives

1. Construct and interpret *pie charts*.
2. Construct and interpret *two-way tables*.
3. Use a two-way table to compute proportions.

OBJECTIVE 1

The distribution of a categorical variable can be described by a **pie chart**, which is a disk where slices represent the categories. The proportion of the total area for one slice is equal to the relative frequency for the category represented by the slice. The relative frequencies are usually written as percentages.

1. Some Americans at least 12 years of age were surveyed about the medium they first use to find out about new music. The mediums and the percentages of respondents who first use them are shown in the following table.

Medium	Percent
Internet	44
Radio	32
Television	9
Newspaper	1
Other	14

Source: *Edison Research*

a. Use StatCrunch to construct a pie chart.

b. Find the proportion of the observations that fall in the Internet category.

c. Find the proportion of the observations that do NOT fall in the Internet category.

d. Find the proportion of the observations that fall in the radio category OR fall in the television category.

e. On the basis of the pie chart, a student concludes that 44% of all Americans at least 12 years of age first use the Internet to find out about new music. What woud you tell the student?

OBJECTIVE 2

A **two-way table** is a table in which frequencies correspond to two categorical variables. The categories of one variable are listed vertically on the left side of the table, and the categories of the other variable are listed along the top.

2. In a survey of one of the author's statistics classes, 27 students recorded whether they have a Snapchat account. A few of the responses are shown in the following table.

Gender	Have a Snapchat account?
Female	No
Male	No
Female	Yes
Male	Yes
Male	Yes
Male	No
Female	Yes
Female	Yes

Source: *J. Lehmann*

Construct a two-way table by hand that compares the responses and grades of the individuals.

3. Use StatCrunch to construct a two-way table that compares the responses and grades of the individuals.

OBJECTIVE 3

4. The following table summarizes the responses from all 27 students who participated in the survey about whether they have a Snapchat account.

Gender	Do Not Have Account	Have Account	Total
Female	7	7	14
Male	5	8	13
Total	12	15	27

Source: *J. Lehmann*

a. How many of the students have a Snapchat account?

b. What proportion of the students do NOT have a Snapchat account?

c. What proportion of the men have a Snapchat account?

d. What proportion of the students who have a Snapchat account are male?

5. In 2012, a total of 1824 adults were asked the following question: "Do you favor or oppose the death penalty for persons convicted of murder?" The following table compares the adults' responses with their ethnicities.

Favor or Oppose the Death Penalty	African American	Caucasian	Other	Total
Favor	128	953	108	1189
Oppose	140	414	81	635
Total	268	1367	189	1824

Source: *General Social Survey*

a. Find the proportion of adults in the survey who oppose the death penalty OR are Caucasian.

b. Find the proportion of adults in the survey who oppose the death penalty AND are Caucasian.

c. Find the proportion of Caucasians who oppose the death penalty.

d. Find the proportion of African Americans who oppose the death penalty.

e. A student says that Caucasians in the study are more likely to oppose the death penalty than African Americans in the study because more of the Caucasians (414) oppose the death penalty than the African Americans (140). What would you tell the student?

HW 1, 3, 5, 9, 13, 15, 17, 19, 21, 23, 25, 30, 34

SECTION 3.2 TEACHING TIPS

The most important objective of this section is to use a two-way table to compute proportions. This skill will be extremely important when finding probabilities (Sections 5.2 and 5.3).

OBJECTIVE 1 COMMENTS

Instructors who do not have many contact hours with students should consider skipping pie charts because categorical variables are more likely to be described by relative frequency bar graphs (Section 3.2) in introductory statistics. One small advantage of pie charts is that they visually convey how a category's relative frequency compares to the whole better than relative frequency bar graphs do. Another reason to devote at least brief coverage to pie charts is that students will use pie charts to find a few probabilities in Sections 5.1 and 5.2.

In general, I think it is a good idea for students to construct a couple of each type of statistical diagram by hand so that they understand what's involved, but pie charts are the exception. I don't want to trouble students to obtain a protractor, and, besides, the meaning of pie charts is fairly obvious.

When working with pie charts, I continue to use the complement rule and the addition rule for disjoint events without calling them as such.

OBJECTIVE 2 COMMENTS

It is important that students know how to convert data displayed in columns such as the data displayed in Problem 2 to a two-way table because data are often stored in columns and then converted by technology to two-way tables. The best way to convey the connection between the two formats is to have students convert a very small data set once or twice by hand.

GROUP EXPLORATION: Association versus Causation

This activity could be used directly after discussing how to construct a two-way table or it could be used to introduce two-way tables. Problem 5 is a nice reminder that a strong association does not guarantee causation, which is a common student misunderstanding.

OBJECTIVE 3 COMMENTS

Working with two-way tables now is good preparation for using two-way tables to find probabilities in Sections 5.2 and 5.3. Just as with multiple bar graphs, I emphasize that comparing relative frequencies is more meaningful than comparing frequencies in most cases. I also point out that both multiple bar graphs and two-way tables can be used to describe the association between two categorical variables.

Students tend to have trouble knowing which rows or columns to work with. To clarify, I make sure to compare and contrast the results for Parts (c) and (d) of Problem 4.

Part of the reason why two-way tables are an effective format for students to find probabilities is that the intersection of events described by rows and columns is apparent. For example, Problem 5(a) is good preparation for learning the general addition rule in Section 5.2. When going over Problem 5(a), I let students tell me that we should not count the frequency 414 twice.

SECTION 3.3 LECTURE NOTES

Objectives

1. Identify *discrete variables* and *continuous variables*.
2. Construct and interpret *dotplots*.
3. Identify *outliers* of a distribution.
4. Find percentiles of a distribution.
5. Estimate the center of a distribution.
6. Construct and interpret *stemplots*.
7. Construct and interpret *time-series plots*.

OBJECTIVE 1

Definition *Discrete variable*

A **discrete variable** is a variable that has gaps between successive, possible values.

Definition *Continuous variable*

A **continuous variable** is a variable that can take on any value between two possible values.

Identify whether the variable is discrete or continuous.

1. the number of students in a prestatistics course

2. the volume (in gallons) of water in a lake

3. a person's height (in inches)

4. the price (in dollars) of a candy bar

When identifying a variable as discrete or continuous, we consider the possible values of the variable *before* rounding.

OBJECTIVE 2

To construct a **dotplot**, for each observation, we plot a dot above the number line, stacking dots as necessary.

5. The endorsements of the 15 athletes with the highest 2015 endorsements are shown in the following table.

Athlete	Sport	Endorsement (millions of dollars)
Tiger Woods	Golf	50
Cristiano Ronaldo	Soccer	27
Usain Bolt	Track and Field	21
Lebron James	Basketball	44
Kobe Bryant	Basketball	26
Kevin Durant	Basketball	35
Rory Mcllroy	Golf	32
Rafael Nadal	Tennis	28
Mahendra Singh Dhoni	Cricket	27
Roger Federer	Tennis	58
Maria Sharapova	Tennis	23
Novak Djokovic	Tennis	31
Lionel Messi	Soccer	22
Phil Mickelson	Golf	44
Neymar da Silva Santos Junior	Soccer	17

Source: *Opendorse*

a. Construct a dotplot of the endorsements.
b. What observation(s) occurred the most?
c. How many observations are at least $40 million?
d. What proportion of the observations are at most $30 million?

We tend to use dotplots to describe data values of discrete variables, but they can be used to describe data values of continuous variables, too.

Definition *Frequency of an observation*

The **frequency of an observation** of a numerical variable is the number of times the observation occurs in the group of data.

Definition *Frequency distribution of a numerical variable*

The **frequency distribution of a numerical variable** is the observations together with their frequencies.

OBJECTIVE 3

An **outlier** is an observation that is quite a bit smaller or larger than the other observations.

6. The maximum recorded lifetimes of animals that have the 10 largest maximum recorded lifetimes are shown in the following table.

Animal	Maximum Recorded Lifetime (years)
Ocean Quahog	400
Bowhead Whale	211
Rougheye Rockfish	205
Red Sea Urchin	200
Galapagos Tortoise	177
Shortracker Rockfish	157
Lake Sturgeon	152
Aldabra Giant Tortoise	152
Orange Roughy	149
Warty Orea	140

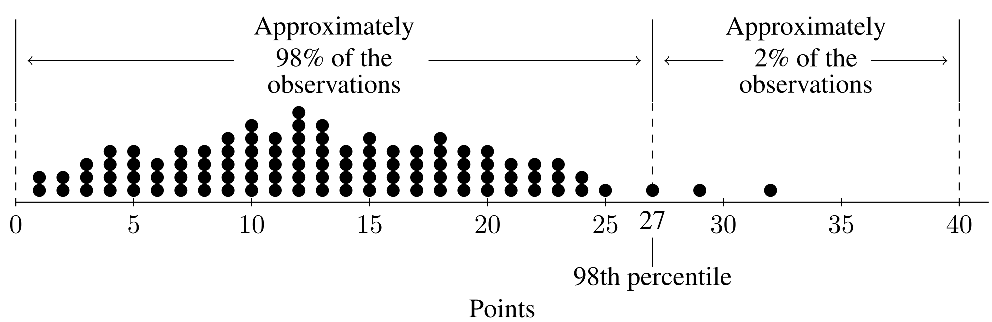
Source: *Discovery News*

a. Construct a dotplot.

b. Identify any outliers.

OBJECTIVE 4

For the data described by the following dotplot, 27 points is at the 98th percentile.



Definition *Percentile*

The ***k*th percentile** of some data is a value (not necessarily a data value) that is greater than or equal to approximately $k\%$ of the observations and is less than approximately $(100 - k)\%$ of the observations.

7. For the endorsement data in Problem 5, find the percentile of Maria Sharapova.

8. For the endorsement data, find the 90th percentile.

OBJECTIVE 5

For this chapter, we will always use the 50th percentile to measure the center of a distribution.

9. For the endorsement data in Problem 5, use the 50th percentile to measure the center.

OBJECTIVE 6

A **stemplot** (or **stem-and-leaf plot**) breaks up each data value into two parts: the **leaf**, which is the rightmost digit, and the **stem**, which is the other digits.

10. Construct a stemplot for the endorsement distribution.

A **split stem** is a stemplot which lists the leaves from 0 to 4 in one row and lists the leaves from 5 to 9 in the next.

11. Construct a split stem by hand for the endorsement distribution.

12. Use StatCrunch to construct a split stem for the endorsement distribution.

Stemplots work best with a small number of observations whose variable is discrete or continuous with rounded values

OBJECTIVE 7

To construct a **time-series plot**, we plot points in a coordinate system where the horizontal axis represents time and the vertical axis represents some other quantity, and we draw line segments to connect each pair of successive dots.

13. The number of new apps submitted to Apple's App Store per month are shown in the following table for various years.

Year	Number of Apps (thousands per month)
2009	5
2010	13
2011	13
2012	20
2013	25
2014	34
2015	40

Source: *pocketgamer.biz*

a. Construct a time-series plot of the data.

b. Describe how the number of new apps submitted to Apple's App store per month has changed over the years.

c. Find the change in the number of new apps submitted to Apple's App store per month from 2009 to 2015. If the number of new apps submitted to Apple's App store per month changes by that same amount from 2015 to 2021, what will it be in 2021? Do you have much faith that this will turn out to be true? Explain.

If a variable has increased (or decreased) throughout a period, we cannot assume it will continue to increase (or decrease) before or after that period.

HW 1, 3, 5, 7, 13, 15, 17, 21, 23, 25, 29, 31, 35, 37, 39

SECTION 3.3 TEACHING TIPS

Students tend to have any easy time constructing the diagrams discussed in this section, so I strive to keep the emphasis on interpreting the diagrams, including measuring the center and identifying outliers, which are two of four characteristics (center, spread, shape, and outliers) of numerical variables that are emphasized in Chapters 3 and 4.

I spend a good deal of time discussing percentiles because students tend to have more trouble with this concept than the other concepts in this section. This work will lay good groundwork for their study of the normal distribution (Sections 5.4 and 5.5).

OBJECTIVE 1 COMMENTS

Students tend to have trouble getting the idea of discrete and continuous variables. Although I present the formal definitions, students tend to learn the distinction between the two variables better by seeing several examples. For some students, discussing whether there are gaps between successive possible values works fine. For other students, it helps to discuss whether a possible value can have any number of decimal places.

The distinction between these two types of variables will not matter when constructing histograms in this course because the textbook presents a one-size-fits-all approach (Section 3.4). As I've mentioned before, the textbook's philosophy is to have students construct a couple of each type of diagram so they have the general idea and then allow them to use technology from then on. However, the distinction *will* matter when finding percentiles from density histograms (Section 3.4).

OBJECTIVE 2 COMMENTS

When constructing dotplots, I point out that the process is similar to plotting points on number lines (Section 1.1). I remind students that they should use uniform scaling on the horizontal axis, write the variable below the horizontal axis, and title the diagram.

When working with dotplots, I take every opportunity to find frequencies and relative frequencies that involve the phrases *less than*, *greater than*, *at least*, and *at most* because students have such difficulty with these phrases in introductory statistics courses. In particular, students find the phrases *at least* and *at most* to be the most challenging.

This is a good time to remind students of the difference in meaning between phrases such as *between 10 and 20* and *between 10 and 20, inclusive* (Section 1.1).

As usual, I have students construct one or two dotplots by hand and from then on allow them to use technology. Recall that in Chapters 3 and 4, the homework exercises are structured so that there is first at least a pair of exercises with about 10 observations, next at least a pair of exercises with about 25 observations, and then at least a pair of exercises with at least 50 observations. This structure is meant to offer instructors flexibility in having students construct diagrams and/or compute measurements with or without statistical technology. For example, I would have students construct a dotplot for the 12 observations in Exercise 19 by hand, the 24 observations in Exercise 21 by hand, and the 50 observations in Exercise 23 by using technology.

The purple icon "DATA" flags exercises whose data sets can be downloaded from MyMathLab and from the Pearson Downloadable Student Resources for Math & Statistics website:

http://www.pearsonhighered.com/mathstatsresources.

Just as with frequency distributions of categorical variables, I emphasize the meaning of *frequency distribution of a numerical variable* because the terminology *distribution* is used so much in statistics.

OBJECTIVE 3 COMMENTS

At this point in the course, the textbook informally defines an outlier to be an observation that is quite a bit smaller or larger than the other observations. A precise definition is given in Section 4.3, where the IQR and fences are defined. Exercises 21(a) and 35 of Homework 3.3 are meant to prepare students for the explanation about what should be done with outliers that is included in the green box on page 185 of the textbook.

A dotplot is a better diagram to illustrate an outlier than a histogram or a stemplot because the outlier is both singled out (by a dot) and its distance from the other observations is displayed.

For Problem 6, students are surprised that an animal can live as long as 400 years. The ocean quahog is a clam that is native to the North Atlantic Ocean.

OBJECTIVE 4 COMMENTS

To introduce percentiles, I skip the challenging abstract definition and give examples. Dotplots are a convenient way to introduce percentiles because counting dots is so concrete. Displaying a dotplot with 100 observations such as the one included in the lecture notes is a good way to get across the concept of percentiles.

Even after such an introduction, students initially have a tough time with Problem 8. Some students mistakenly think they should find 90% of the largest endorsement ($58 million) rather than the number of observations (15 endorsements).

Finding percentiles is a great primer for finding a value of a normal variable for a given probability when working with a normal curve (Section 5.5).

OBJECTIVE 5 COMMENTS

Computing the 50th percentile is an excellent way to get students thinking about the center of a distribution. I toyed with postponing a discussion about center until the mean, median, and mode are introduced in Section 4.1, but I really wanted students to be engaged with the material in Chapter 3 at a higher level than just constructing diagrams. This is achieved in a number of ways, but a key one is having students identify the four characteristics of a numerical variable: outliers (this section), shape (Section 3.4), center (this section), and spread (Section 3.4). The four characteristics are summarized in the green box on page 185 of the textbook.

One technical difficulty with the 50th percentile is that it is not well defined for really small data sets with an odd number of observations. For example, consider the set of numbers 2, 3, 5, 8, and 9. According to the textbook's definition for percentile, which is fairly standard, the numbers in the interval $[3, 5)$ are at the 40th percentile and the numbers in the interval $[5, 8)$ are at the 60th percentile. The median is 5, so it is a bit of a stretch to say that the 50th percentile and the median are the same thing. The root of the problem is that percentiles are meant for data sets that involve much more than 5 observations.

I allude to this issue on page 218 of the textbook, where the median is introduced, but I don't go into details because I'm more interested in getting across that there are various ways to measure the center and we must determine the most appropriate measure for a particular data set. Besides, the median and the 50th percentile are extremely close or equal for large data sets, which is when measuring the center is most helpful.

OBJECTIVE 6 COMMENTS

Instructors who do not have many contact hours with students should consider skipping stemplots because histograms are usually used to describe numerical variables in introductory statistics.

For the most part, students have an easy time constructing and interpreting stemplots. The one task they find challenging is determining to which decimal place the data should be rounded before constructing the stemplot. Rounding some data values to, say, the second decimal place and constructing a stemplot, and then rounding the same data values to, say, the first decimal place and constructing a stemplot will help students understand.

Recall that to prepare students for introductory statistics, I take every opportunity to solve problems with the phrases *less than*, *greater than*, *at least*, and *at most*.

To save time, I do not bother showing students how to construct split stems, but I do demonstrate how to use StatCrunch to construct them and we interpret the diagrams.

OBJECTIVE 7 COMMENTS

After having constructed so many other diagrams, students have an easy time constructing time-series plots. This is good preparation for all the work with scatterplots to come in Chapters 6–10. I emphasize that for times-series plots, time is always described by the horizontal axis. Because of all the work students have done on other diagrams, after I draw the two axes, students are quick to tell me to use uniform scaling, indicate the units on the axes, and to title the diagram.

When working with time-series plots, I warn students that we cannot extrapolate, although I wait to introduce the terminology *extrapolate* until Section 6.3, where the concept is described in great detail.

GROUP EXPLORATION: Determining Which Table or Diagram to Use

This exploration has merit because it involves a form of critical thinking similar to determining which type of confidence interval or hypothesis test to use for a particular data set. A variation on this exploration would be to present groups of students with actual data sets and have them determine which diagrams could be used to describe them. Or you could ask the groups to go ahead and construct one or more diagrams for each data set.

SECTION 3.4 LECTURE NOTES

Objectives

1. Construct and interpret *frequency and relative frequency tables*.
2. Construct and interpret *frequency histograms* and *relative frequency histograms*.
3. Interpret *density histograms*.
4. Describe the *shape* of a distribution.
5. Describe the *spread* of a distribution.
6. Describe the meaning of *model*.

OBJECTIVE 1

1. The endorsements of the 15 athletes with the highest 2015 endorsements are shown in the following table.

Athlete	Sport	Endorsement (millions of dollars)
Tiger Woods	Golf	50
Cristiano Ronaldo	Soccer	27
Usain Bolt	Track and Field	21
Lebron James	Basketball	44
Kobe Bryant	Basketball	26
Kevin Durant	Basketball	35
Rory Mcllroy	Golf	32
Rafael Nadal	Tennis	28
Mahendra Singh Dhoni	Cricket	27
Roger Federer	Tennis	58
Maria Sharapova	Tennis	23
Novak Djokovic	Tennis	31
Lionel Messi	Soccer	22
Phil Mickelson	Golf	44
Neymar da Silva Santos Junior	Soccer	17

Source: *Opendorse*

Construct a frequency and relative frequency table for the endorsement distribution.

For a class $[a, b]$, the **lower class limit** is a and the **upper class limit** is b. To find the **class width** of a class, we subtract the lower class limit of a class from the lower class limit of the next class.

Definition *Frequency of a class and relative frequency of a class*

The **frequency of a class** is the number of observations in the class. The **relative frequency of a class** is the proportion of the observations in the class.

When using classes, the **frequency distribution of a numerical variable** are the classes together with their frequencies and the **relative frequency distribution of a numerical variable** are the classes together with their relative frequencies.

Sum of Relative Frequencies

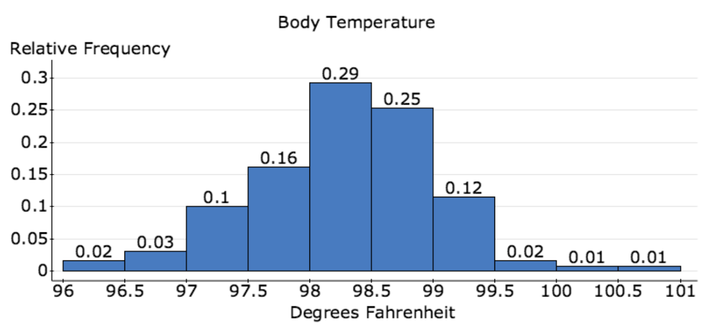
For a numerical variable, the sum of the relative frequencies of all the classes is equal to 1.

OBJECTIVE 2

2. Construct a frequency histogram by hand for the endorsement distribution in Problem 1.
3. Construct a relative frequency histogram by hand for the endorsement distribution.
4. Use technology to construct a frequency histogram and a relative frequency histogram for the endorsement distribution.

- Histograms can be used with any numerical variable (discrete or continuous) and any number of observations.
- Histogram bars can touch, but bar-graph bars never touch.

5. The body temperatures of 130 adults are described by the following relative frequency histogram.



Source: *American Statistical Association*

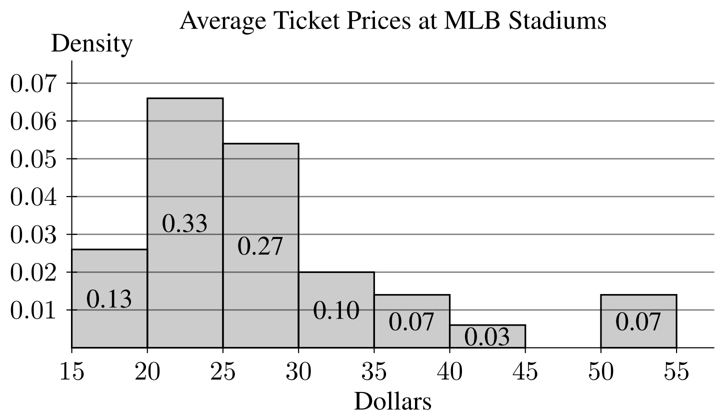
Estimate the proportion of observations that are

a. greater than or equal to 98 degrees Fahrenheit AND less than 99 degrees Fahrenheit.
b. less than 97.5 degrees Fahrenheit.
c. at least 99 degrees Fahrenheit.

OBJECTIVE 3

For a **density histogram**, the vertical axis has units called **density** so that the *area* of each bar is the relative frequency of the bar's class.

6. The average prices of 2014 Major League Baseball® (MLB) tickets at the stadiums are described by the following density histogram.



Source: *Team Marketing Report*

a. Find the proportion of stadiums whose average price of 2014 MLB tickets are . . .

i. between \$30 and \$39.99, inclusive.

ii. less than \$20.

iii. at least \$20.

b. Estimate the percentile of a \$25 average ticket price. Round to the nearest dollar.

c. The average ticket price at Nationals Park, home of the Washington Nationals, is at the 83rd percentile. Estimate the average ticket price. Round to the nearest dollar.

d. Identify the class which contains the 50th percentile, which is a reasonable measure of the center.

OBJECTIVE 4

Definition *Unimodal, bimodal,* and *multimodal distributions*

A distribution is **unimodal** if it has one mound, **bimodal** if it has two mounds, and **multimodal** if it has more than two mounds.

For a unimodal distribution, the **left tail** is the part of the histogram to the left of the 50th percentile and the **right tail** is the part of the histogram to the right of the 50th percentile

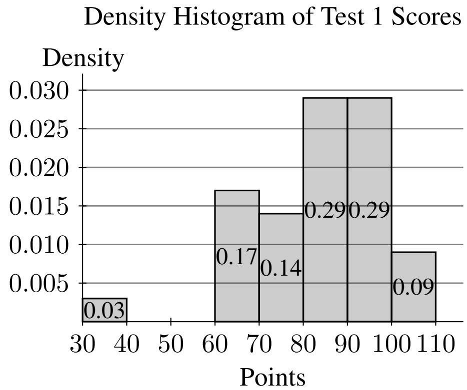
Definition *Skewed-left, skewed-right,* and *symmetric distributions*

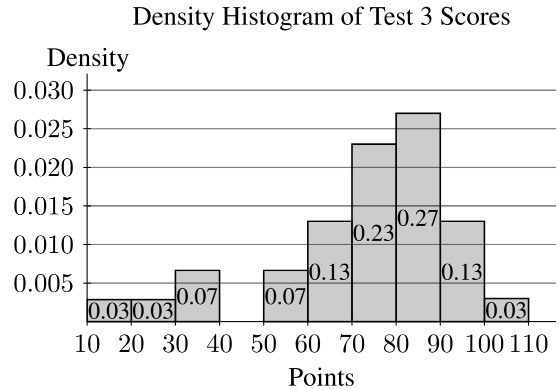
- If the left tail of a unimodal distribution is longer than the right tail, then the distribution is **skewed left**.
- If the right tail of a unimodal distribution is longer than the left tail, then the distribution is **skewed right**.
- If the left tail of a distribution is roughly the mirror image of the right tail, the distribution is **symmetric**.

An observation at the 50th percentile (the approximate center) of a unimodal distribution tends to be a typical observation.

OBJECTIVE 5

7. The following two density histograms describe calculus students' test scores (Source: *J. Lehmann*). The left histogram describes the scores of 35 students on Test 1. The right histogram describes the scores of 30 remaining students on Test 3.





a. For the two distributions identify any outliers.

b. Compare the shapes of the two distributions.

c. Compare the centers of the two distributions. What does that mean in this situation?

d. Compare the spreads of the two distributions. What does that mean in this situation?

e. Give three possible reasons why the distributions are different.

Order of Determining the Four Characteristics of a Distribution with a Numerical Variable

We often determine the four characteristics of a distribution with a numerical variable in the following order:

1. Identify all outliers.

  a. For outliers that stem from errors in measurement or recording, correct the errors, if possible. If the errors cannot be corrected, remove the outliers.

  b. For other outliers, determine whether they should be analyzed in a separate study.

2. Determine the shape. If the distribution is bimodal or multimodal, determine whether subgroups of the data should be analyzed separately.

3. Estimate and interpret the center.

4. Describe the spread.

OBJECTIVE 6

Definition *Model*

A **model** is a mathematical description of an authentic situation. We say the description *models* the situation.

Diagram	Types of Variables	Benefits
Frequency Bar Graph	One categorical variable	Compare frequencies of categories.
Relative Frequency Bar Graph	One categorical variable	Compare a part to the whole.
Multiple Bar Graph	Two categorical variables	Compare a part to the whole.
Pie Chart	One categorical variable	Compare a part to the whole.
Two-Way Table	Two categorical variables	Compare a part to the whole.
Dotplot	One numerical variable	Describe individual values for a small or medium number of observations.
Stemplot	One numerical variable	Describe individual values for a small number of observations.
Frequency Histogram	One numerical variable	Compare the frequencies of classes.
Relative Frequency Histogram	One numerical variable	Compare a part to the whole.
Density Histogram	One numerical variable	Compare a part to the whole.
Time-Series Plot	Two numerical variables	Find the association between two variables.

HW 1, 3, 5, 9, 10, 11, 13, 17, 19, 23, 25, 27, 33, 43, 45, 49

SECTION 3.4 TEACHING TIPS

Although Objectives 1–5 are all very important, I emphasize interpreting density histograms the most because it is such good preparation for the normal curve (Sections 5.4 and 5.5).

OBJECTIVE 1 COMMENTS

When constructing a frequency and relative frequency table for a numerical variable, I point out that the process is similar to constructing such a table for a categorical variable, where the classes act as categories.

For the data in Table 46 on page 174 of the textbook, it is helpful to compare Table 47 (the frequency and relative frequency table) with the stemplot shown in Fig. 62.

Because of the similarities described in the previous two paragraphs, students have a fairly easy time constructing histograms.

Some textbooks give complex instructions about how to determine the classes, but I believe there is little benefit with having students get so caught up in the details. The main point is to have students construct one or two histograms by hand so they can better interpret them. Technology can take care of the rest.

Each time students are required to construct a table and/or histogram in the exercises, the lower class limit and the class width are given. This way, their table and/or histogram will match the one in the answer section. Supplying the lower class limit and the class width on a test makes it easier to grade students' work.

Whenever students construct a relative frequency table, I advise that they check that the sum of the relative frequencies sum to 1. I explain that the sum equals 1 for the same reason that the relative frequencies sum to 1 when working with a categorical variable.

OBJECTIVE 2 COMMENTS

Because histograms will be used more than any other diagram in Chapters 4 and 5 (and in introductory statistics), I make sure students have a solid understanding of how to construct and interpret histograms. In addition to drawing the comparison between constructing bar graphs and histograms, I demonstrate that a stemplot rotated counterclockwise 90 degrees is similar to a histogram (see Figs. 63 and 65 on page 176 of the textbook).

To impress upon students the importance of histograms, I show them histograms from a wide variety of mediums, including newspapers, blogs, and statistical reports. It is visceral to have students interpret a histogram that has to do with an event that has just occurred in the past week or so.

Because of the way histograms are constructed in the textbook, I tend to solve problems with the phrases *less than* and *at least* (rather than *greater than* and *at most*). Recall that this is good preparation for introductory statistics.

OBJECTIVE 3 COMMENTS

Consider a step function that is related to the tops of a density histogram. That step function is a probability density function. That is why interpreting density histograms is excellent preparation for working with normal distributions (Sections 5.4 and 5.5). For that reason, I emphasize them more than any other diagram in Chapter 3. But I don't mention any of this to students, except to say that density histograms will be very helpful in Chapters 4 and 5.

Introductory statistics textbooks tend to give short shrift to density histograms, which is unfortunate, but this creates the opportunity for *Pathway* to offer a different perspective to students. Learning a concept from two perspectives is better than from one.

It is key that students understand why the area of a bar is equal to the relative frequency of the bar's class. To convey this, I compute the area of a bar similar to the one in Fig. 73 on page 179 of the textbook. Then I build on this fact to argue that the total area of the bars of a density histogram is equal to 1 and make great use of the addition rule for disjoint events and the complement rule (without calling them as such). Both rules will be formally introduced in Section 5.2.

The homework exercises do not require students to construct density histograms but they do have students interpret them (see Exercises 13–20). Select exercises in Chapters 4 and 5 will continue to require students to work with density histograms.

OBJECTIVE 4 COMMENTS

When determining the shape of a distribution, I tell students to notice the general pattern rather than focus on small deviations from the overall pattern. For example, for almost all authentic unimodal distributions we would consider symmetric, the left half of the histogram is not the exact mirror image of the right half.

I emphasize that if a distribution is bimodal or multimodal, it's a good idea to determine whether subgroups of the data should be analyzed separately.

OBJECTIVE 5 COMMENTS

Until measures of variation are introduced in Section 4.2, the best way to have students reflect on spread is to have them compare the spreads of two or more distributions such as the test-score distributions described in Problem 7. Students are quick to see that the distribution of Test 3 scores is more spread out than the distribution of Test 1 scores.

With the introduction of spread, students now know the four characteristics of a numerical distribution (outliers, shape, center, and spread). The four characteristics will be further developed and emphasized in Chapter 4.

OBJECTIVE 6 COMMENTS

To convey the idea of a model, I show students several histograms that represent a single data set, where each histogram has a different class width (see the group exploration on page 186 of the textbook). This sets the stage for the concept that a normal distribution is a model (it's extremely rare for an authentic distribution to be exactly normal). The concept of a model is the most important theme that runs through all of Chapters 3–10.

I advise students to carefully examine the overview of tables and diagrams (models) included in Table 48 on page 186 of the textbook. As I mentioned earlier, the skill of identifying which tables and diagrams describe a distribution well (Exercises 43–48) is similar to the key skill of identifying which types of confidence intervals and hypothesis tests are appropriate to draw inferences about a population.

GROUP EXPLORATION: Comparing Histograms with Different Class Widths

The main point of this exploration is that the perspective we gain from a histogram depends in part on the class size we choose. This reinforces the concept that a histogram is a model. The exploration is also a good primer for Example 1 of Section 3.5, which shows that certain class widths can be misleading.

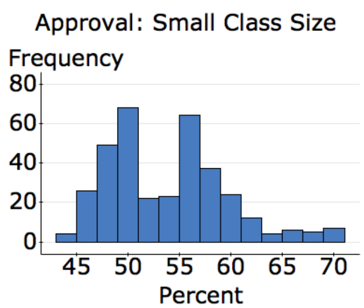
SECTION 3.5 LECTURE NOTES

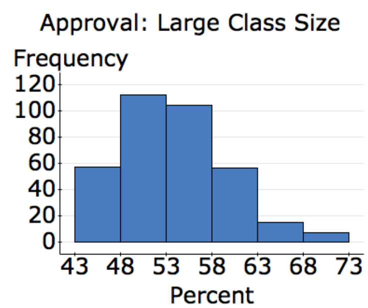
Objectives

1. Explain why a certain class width of a histogram can emphasize or de-emphasize an aspect of a distribution.
2. Explain why the starting value of the vertical axis of a bar graph or a time-series plot can emphasize or de-emphasize an aspect of a distribution.
3. Explain why nonuniform scaling can be misleading.
4. Explain why *three-dimensional graphs* can be misleading.

OBJECTIVE 1

1. The daily approval ratings of President Obama are described by the following histograms. The left histogram describes the daily approval ratings in 2009 using a smaller class width (2 percentage points), and the right histogram describes the same ratings using a larger class width (5 percentage points). Which of these histograms is misleading? Explain.



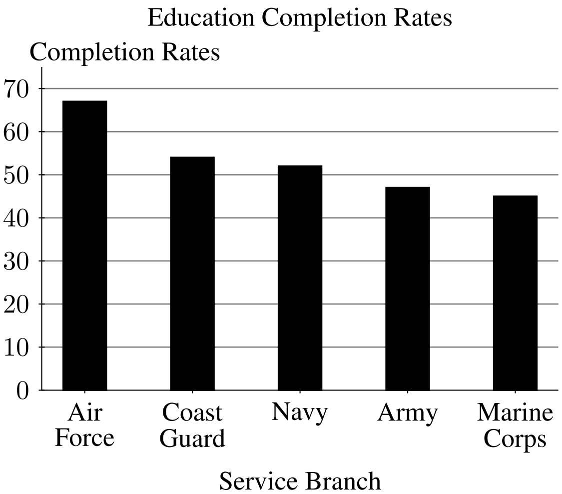


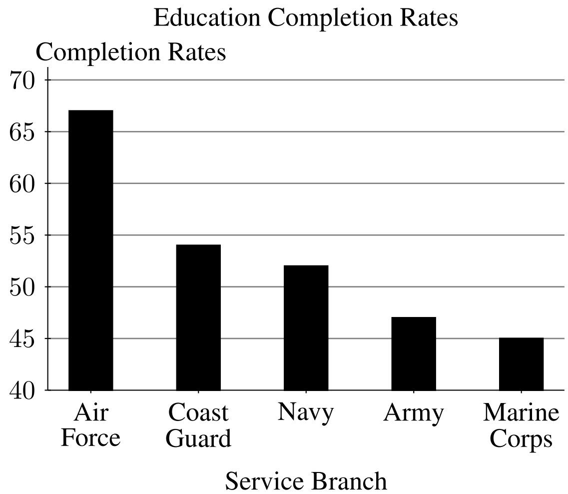
Source: *Rasmussen Reports*

When viewing a histogram, keep in mind that a certain choice of class width can emphasize or de-emphasize certain aspects of the distribution.

OBJECTIVE 2

2. The percentages of veterans who used GI Bill benefits from 2002 through 2013 to complete educations ranging from vocational training to post-graduate are described by the following bar graphs for various service branches.





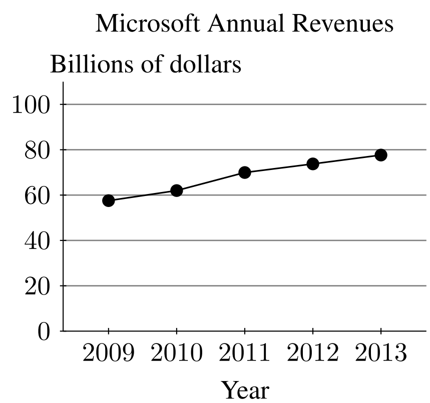
Source: *Student Veterans of America*

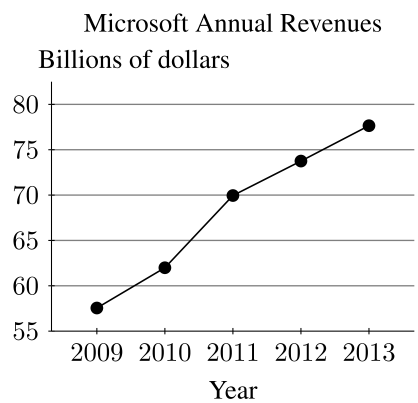
a. If the Air Force wants to convince potential recruits how much better the education completion rate is for Air Force veterans than for veterans of the other four service branches, which bar graph would it display? Explain.

b. If the Navy wants to convince potential recruits that the education completion rate for Navy veterans is very close to the rate for Coast Guard veterans, which bar graph would it display? Explain.

c. From which bar graph can you better estimate the education completion rate for Coast Guard veterans? Explain. Estimate the rate.

3. The annual revenues of Microsoft® are described by the following time-series plots for various years. If Microsoft wants to emphasize the growth of its revenue, which time-series plot would it want to display? Explain.

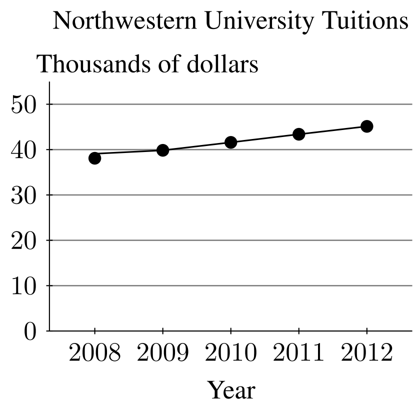


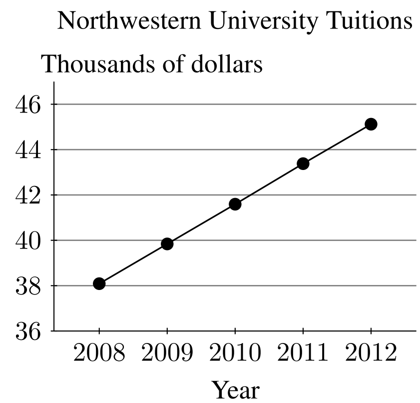


Source: *Microsoft Corporation*

If the vertical axis of a time-series plot does not start at 0, the changes in the variable described by that axis are being emphasized.

4. Northwestern University tuitions are described by the following time-series plots for various years, where the starting year of the academic year is shown on the horizontal axis. If Northwestern wants to de-emphasize how much its tuition has increased, which time-series plot would it want to display?



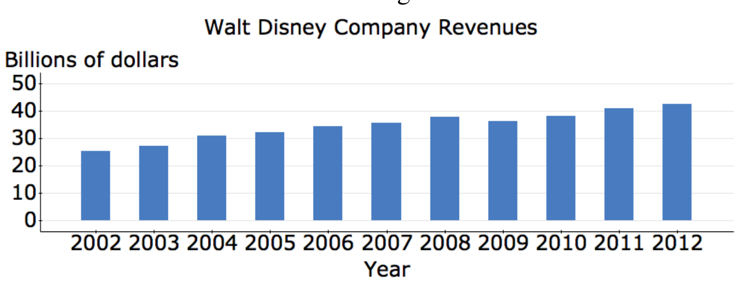


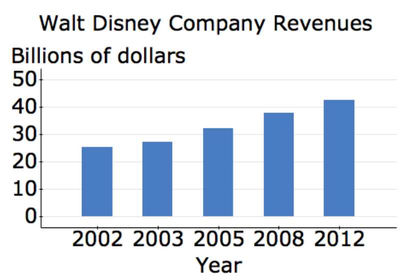
Source: *Northwestern University*

If the vertical axis of a time-series plot starts at 0, be aware that the changes in the variable described by that axis are being de-emphasized.

OBJECTIVE 3

5. The revenues (in billions of dollars) of the Walt Disney Company® are described by the following bar graphs for various years. If Walt Disney wanted to emphasize its growth in revenue, which bar graph would be more convincing? Why is that bar graph misleading? What type of graph would contain the same information but not be misleading?



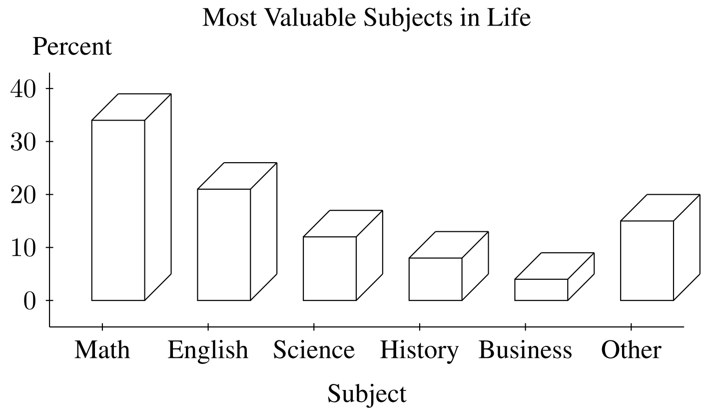


Source: *Walt Disney Company*

If the categories for a bar graph are various years and those years do not increase by the same amount, the bar graph can be misleading. A time-series plot would be a better graph to use.

OBJECTIVE 4

6. A total of 2059 randomly selected adults were asked which school subject has been the most valuable to them in their lives. Their responses are described by the following three-dimensional graph.



Source: *The Gallup Organization*

a. What is confusing about this graph?

b. Determine a better type of graph and draw it.

c. Which subject received the most votes? What percentage of adults voted for it?

HW 1, 3, 5, 7, 9, 11, 13, 15, 17, 20, 22

SECTION 3.5 TEACHING TIPS

One could argue that this is the most important section of the text because the media and businesses often use statistical diagrams in misleading ways.

OBJECTIVE 1 COMMENTS

To get across how a histogram is impacted by its class width, you could use StatCrunch to display histograms with various class widths for the same data set. Or you could have groups of students complete the exploration on page 186 of the textbook or Problem 1 in the lecture notes.

In fact, you could have groups of students work on all of Problems 1–6 because they are so intuitive.

OBJECTIVE 2 COMMENTS

The impact of the starting value on the vertical axis of a bar graph or time-series plot might be the most important concept of this section because corporations and the media use this tactic so often. I point out that even though the practice may be used to persuade or even mislead, there is nothing mathematically wrong with it. I emphasize that students should carefully inspect both axes to make sure they interpret a diagram correctly.

GROUP EXPLORATION: Using the Vertical Axis to Persuade

Students tend to really engage in this exploration, probably because they see they can directly benefit from being critical consumers of statistics. Problems 3–5 are a nice introduction to linear modeling issues that will be further developed in Chapters 6–9. Problems 3–5 are also good preparation for Exercises 9(d), 10(d), 11(d), and 12(d).

OBJECTIVE 3 COMMENTS

When I lecture on Problem 5 or facilitate group work on it, I tell students that the media often uses bar graphs when time-series plots would be more appropriate. Similar to my warning regarding Objective 2, I emphasize that students should carefully inspect the horizontal axis of a bar graph.

OBJECTIVE 4 COMMENTS

Consider finding three-dimensional graphs in your local newspaper or other media to show your students. Have them determine whether the graphs are confusing, misleading, or both.

CHAPTER 4 OVERVIEW

The main path through Chapters 2–5 culminates in the normal distribution (Sections 5.4 and 5.5). And the introduction of the mean (Section 4.1), the standard deviation (Section 4.2), and the Empirical Rule (Section 4.2) are three significant steps toward that goal.

No matter how much time is devoted to measures of variability in introductory statistics courses, students tend to have a sketchy understanding of this concept, so I advise facilitating plenty of activities that enhance their understanding of it.

This chapter offers copious amounts of critical-thinking opportunities, especially in the form of comparisons:

- Students compare the mean with the median for skewed-left, symmetric, and skewed-right distributions.
- Students identify which of the measures mean, median, and mode is the best measure of the center for a given data set.
- Students identify which of the measures standard deviation, IQR, and range is the best measure of the variability for a given data set.
- Students compare and interpret the centers of two groups of data.
- Students compare and interpret the variability of two groups of data.

Many of these concepts can be discovered by having groups of students complete the explorations or by having them find solutions for textbook examples.

It's a good idea to continue using data about your students. If they seemed to especially enjoy a certain data set that you worked with in Chapter 3, consider returning to that data set to now measure the center, measure the variability, and construct a boxplot (Section 4.3).

Constructing boxplots is a bit more technical than the diagrams discussed in Chapter 3, so make sure that students do not get so involved with the calculations that they lose sight of a boxplot's many advantages: to visualize the center (median), the variability (the IQR and range), and outliers.

SECTION 4.1 LECTURE NOTES

Objectives

1. Compute the *median* of some data.
2. Describe the meaning of *sigma notation*.
3. Compute the *arithmetic mean* of some data.
4. Compare the means of two groups of data.
5. Compare the mean and the median of some data.
6. Find the means and medians of bimodal and multimodal distributions.
7. Find the *mode* of some data.

OBJECTIVE 1

Definition *median*

The **median** of some data is the 50th percentile.

Finding the Median of a Distribution

To find the median of some data values, first list the observations from smallest to largest.

- If the number of observations is odd, then the median is the middle observation.
- If the number of observations is even, then the median is the average of the two middle observations.

1. The percentages of adults who exercise frequently are shown in the following table for Northeastern states and Mountain states.

Northeastern State	Percent	Mountain State	Percent
Connecticut	54	Arizona	53
Maine	55	Colorado	60
Massachusetts	53	Idaho	58
New Hampshire	54	Montana	60
New Jersey	48	Nevada	55
New York	49	New Mexico	57
Pennsylvania	50	Utah	54
Rhode Island	48	Wyoming	54
Vermont	65		

Source: *The Gallup Organization*

a. Find the median percent for the Northeastern states. What does it mean in this situation?

b. Find the median percent for the Mountain states. What does it mean in this situation?

c. Compare the results you found in Parts (a) and (b). What does this mean in this situation?

d. A student says that the result you found in Part (a) should be greater than the result you found in Part (b) because there are more Northeastern states than Mountain states. What would you tell the student?

OBJECTIVE 2

Definition *Summation Notation*

Let $x_1, x_2, x_3, \ldots, x_n$ be some data values. The **summation notation** $\boldsymbol{\Sigma x_i}$ stands for the sum of the data values:

$$\Sigma x_i = x_1 + x_2 + x_3 + \cdots + x_n$$

2. The TV series with the top five most-watched finales and the finale viewerships are shown in the following table. Let x_i be the finale viewership (in millions) listed in the ith row of the table.

a. Find the values of x_1, x_2, x_3, x_4, and x_5.

b. Find Σx_i. What does it mean in this situation?

TV Series	Finale Viewership (in millions)
M*A*S*H	105.9
Cheers	80.4
Seinfeld	76.3
Friends	52.5
Magnum, p.i.	50.7

Source: *USA Today, Reuters, TV Guide, ABC*

OBJECTIVE 3

Definition *Mean*

The **arithmetic mean** (or **mean**) of n data values $x_1, x_2, x_3, \ldots, x_n$ is given by

$$\bar{x} = \frac{\Sigma x_i}{n}$$

- **Remember to always include the units of the mean.**
- A **formula** is an equation that contains two or more variables.

3. The numbers of teaspoons of sugar in 12 fluid ounces of various beverages are shown in the following table. Find the mean number of teaspoons of sugar in the beverages. Then construct a dotplot and verify that the mean is a reasonable measure of the center.

Beverage	Number of Teaspoons of Sugar
Mountain Dew®	11
Pepsi®	9
Ame Orange and Grape®	7
Dr. Pepper®	9
Fanta Grape Flavored Drink®	11
Coca-Cola®	9
Club Orange®	12
Tesco Original Cola®	8

Source: *The companies*

Mean is a Measure of the Center

If a distribution is unimodal and approximately symmetric, the mean is a reasonable measure of the center. In this case, we say the mean is a typical value.

4. The students in one of the author's prestatistics classes estimated his age. Their estimates are shown in the following table.

48	45	45	41	40	44	47	44
50	35	45	54	38	40	46	50
38	42	47	43	49	45	43	43
49							

Source: *J. Lehmann*

a. Determine whether the mean should be a reasonable measure of the center of the distribution.

b. Compute the mean. What does it tell us in this situation?

c. A student calculates the mean estimate to be 44.4 years and concludes that a typical estimate by all students at the college where the survey was performed is 44.4 years. What would you tell the student?

When computing the mean or other measures of data, we will round to one more decimal place than the data.

OBJECTIVE 4

5. The prices of hot dogs at each of the stadiums for the baseball teams in the National League Central (NLC) and the National League West (NLW) are shown in the following table.

NLC Team	Hot Dog Price (dollars)	NLW Team	Hot Dog Price (dollars)
Chicago Cubs	5.50	Arizona Diamondbacks	2.75
Cincinnati Reds	1.00	Colorado Rockies	4.75
Milwaukee Brewers	3.50	Los Angeles Dodgers	5.50
Pittsburgh Pirates	3.25	San Diego Padres	4.00
St. Louis Cardinals	4.25	San Francisco Giants	5.25

Source: *Team Marketing Report*

a. Determine whether the mean hot dog price of the NLC distribution should be a reasonable measure of the center of the distribution. Do the same for the NLW distribution.

b. Find the mean hot dog price for the NLC.

c. Find the mean hot dog price for the NLW.

d. What do your results in Parts (b) and (c) mean in this situation?

6. **a.** Compute the mean for the numbers 3, 4, and 5.

b. Compute the mean for the numbers 3, 4, 5, and 4.

c. A students says that the result you found in Part (b) should be larger than the result you found in Part (a) because there are more observations in Part (b). What would tell the student? Include in your explanation a discussion about division as well as a comparison of dotplots of the two data sets.

OBJECTIVE 5

7. The following data are the savings (in thousands of dollars) of five adults: 15, 9, 11, 6, and 3.

a. Find the mean savings.

b. Find the median savings.

c. Suppose that the adult who had $15 thousand in savings wins $800 thousand (after taxes) from the lottery. So, the adult's new savings is $815 thousand. Find the mean savings of the five adults after the lottery win.

d. Find the median savings of the five adults after the lottery win.

e. Describe how much the outlier $815 thousand affected the mean and the median.

The Effect of Outliers on the Mean and the Median

- The mean is sensitive to outliers.
- The median is resistant to outliers.

8. The top-paid female CEOs and their compensations are shown in the following table.

CEO	Company	Compensation (in billions of dollars)
Irene Rosenfeld	Mondelez	21.0
Virginia Rometty	IBM	19.3
Safra Ada Catz	Oracle	37.7
Phebe Novakovic	General Dynamics	19.3
Carol Meyrowitz	TJK	28.7
Indra Nooyi	PepsiCo	22.5
Meg Whitman	Hewlett-Packard	19.6
Debra Reed	Sempra	16.9
Marissa Mayer	Yahoo	42.1
Ursula Burns	Xerox	22.2
Marillyn Hewson	Lockheed Martin	33.7
Mary Barra	GM	16.2

Source: *S&P Capital IQ, USA Today, Bespoke Investment Group*

a. Find the mean compensation.

b. Find the median compensation.

c. Construct a frequency histogram and indicate the mean and the median on it. Describe the shape.

d. Which measures the center better, the mean or the median? Explain.

e. Explain why it makes sense that the mean is larger than the median.

How the Shape of a Distribution Affects the Mean and the Median

- If a distribution is skewed left, the mean is usually less than the median and the median is usually a better measure of the center.
- If a distribution is symmetric, the mean is approximately equal to the median and both are reasonable measures of the center.
- If a distribution is skewed right, the mean is usually greater than the median and the median is usually a better measure of the center.

OBJECTIVE 6

9. **a.** Consider how many days each president of the United States has served in office. What shape would the distribution have?

b. The numbers of days served by the presidents are described by the dotplot in Fig. 2.1. The mean and median are shown in Fig. 2.2. How do the mean and the median relate to the distribution? How well do they describe a typical observation?

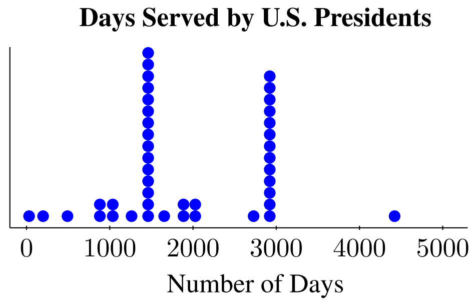


Figure 2.1: Numbers of days served by U.S. presidents (**Source:** *World Almanac and Book of Facts 2014*)

Summary statistics:

Column	Mean	Median
Length (days)	1910.5581	1461

Figure 2.2: Numbers of days served by U.S. presidents

c. What would be a more useful way to analyze this situation?

OBJECTIVE 7

Definition *Mode*

The **mode** of some data is an observation with the greatest frequency. There can be more than one mode, but if all the observations have frequency 1, then there is no mode.

10. For each data set, find the mode.

a. The author surveyed the students in one of his prestatistics classes about the number of states they have lived in. Some of their responses are shown in the following table.

1	10	1	2	1	1
2	1	1	1	4	1

Source: *J. Lehmann*

b. The author surveyed the students in one of his prestatistics classes about what superpower they wished they had. Some of their responses are shown in the following table.

fly	teleport	mind read	fly	fly
invisible	fly	teleport	invisible	clone myself

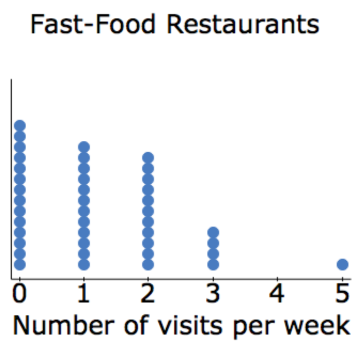
Source: *J. Lehmann*

c. The tuitions of the five most expensive colleges are shown in the following table.

College	Tuition, Fees, Room, and Board (dollars)
Sarah Lawrence College	61,236
New York University	59,837
Harvey Mudd College	58,913
Columbia University	58,742
Wesleyan University	58,202

Source: *CampusGrotto.com*

11. The author surveyed students in one of his statistics classes about the how many times per week they eat at fast-food restaurants. A dotplot and the mean, median, and mode are shown in the following figure. Explain why the median and the mean (rounded to the ones place) are better measures of the center than the mode.



Summary statistics:

Column	n	Mean	Median	Mode
Visits	42	1.2142857	1	0

HW 1, 3, 5, 7, 15, 17, 19, 23, 27, 29, 35, 37, 43, 49, 53, 59, 63

SECTION 4.1 TEACHING TIPS

This is an extremely important section because central tendency is at the heart and soul of statistics. The mean and the median are both important measures of the center, but the mode can be de-emphasized or skipped because it is rarely used in statistics.

Although students have a fairly easy time computing means, medians, and modes, they often confuse one with another. I advise warning your students that they should devote some of their study time to keeping the three measures straight in their minds.

OBJECTIVE 1 COMMENTS

In Chapter 3, the textbook uses the 50th percentile to measure the center. In Chapter 4, the textbook uses the median instead of the 50th percentile to measure the center.

One technical difficulty with the 50th percentile is that it is not well defined for really small data sets with an odd number of observations. For example, consider the set of numbers 2, 3, 5, 8, and 9. According to the textbook's definition for percentile, which is fairly standard, the numbers in the interval $[3, 5)$ are at the 40th percentile and the numbers in the interval $[5, 8)$ are at the 60th percentile. The median is 5, so it is a bit of a stretch to say that the 50th percentile and the median are the same thing. The root of the problem is that percentiles are meant for data sets that involve much more than 5 observations.

I allude to this issue on page 218 of the textbook, where the median is introduced, but I don't go into details because I'm more interested in getting across that there are various ways to measure the center and we must determine the most appropriate measure for a particular data set. Besides, the median and the 50th percentile are extremely close or equal for large data sets, which is when measuring the center is most helpful.

When finding a median, I emphasize that the data values must be listed from smallest to largest before identifying the middle observation or the average of the two middle observations. I also emphasize that the median is a measure of the center and that students must indicate its units.

OBJECTIVE 2 COMMENTS

Once students realize that $\sum x_i$ represents the sum of the observations x_i, they can easily find correct results, but I try to make sure they understand the role of the subscripts in hopes of better preparing them for the standard deviation formula, which is introduced in Section 4.2. Just like with the median, I emphasize that students should indicate the units for the sum of some data values.

Much more complicated summations are included in Section 8.4, which focuses on statistics formulas. If you do not plan to use Chapter 8, consider having students work a bit with the summations in Section 8.4 to better prepare students for introductory statistics.

OBJECTIVE 3 COMMENTS

Because many students have computed averages in previous classes, they are relieved to know that the mean is the same measure.

After completing Problem 3, I say that if we imagine that the horizontal axis of the dotplot is a seesaw and the dots are bowling balls that all have the same weight, the mean (9.5 teaspoons) would be the balance point. I emphasize that if a distribution is unimodal and symmetric, the mean is a reasonable measure of the center and we say it is a typical value. I also emphasize that students should indicate the mean's units.

It can be instructive to use technology to construct a histogram that describes a uniform, symmetric distribution that involves a large number of observations and have students estimate the center by inspection. Then you can use technology to compute the mean and compare it with the students' estimates.

Tell students that they will need to round the mean and other measures of data to one more decimal place than the data so that their answers match the ones in the back of the book (and most likely on MyMathLab).

OBJECTIVE 4 COMMENTS

Measuring the center of a data set is interesting, but comparing the means of *two data sets* provides extra punch:

- Which car model tends to have better gas mileages?
- Students of which section of prestatistics tend to perform better on tests.
- Which ethnic group tends to spend more money on hip-hop music?

I remind students that before we compare the means of two or more data sets, we should check that each distribution is unimodal and symmetric. I also remind them that we cannot draw conclusions about individuals not in the data sets (Chapter 2).

Even after I have discussed with students at great length that for unimodal, symmetric distributions the mean is a good measure of the center and a reasonable typical value, some students think that the mean weight of 20,000 randomly selected cats would be greater than the mean weight of five randomly selected human adults (Exercise 63). It seems that such students think of the mean as a sum of observations, perhaps because the formula for the mean includes a summation. Or maybe they don't read the exercise carefully. In any case, this misconception certainly deserves some attention! Problem 6 is a nice way to address this issue. Problem 6 could be followed by the comparison of cats and adult humans or see Exercise 64 for a comparison of high school and NBA basketball players' heights.

OBJECTIVE 5 COMMENTS

The fundamental concepts for this objective are that the mean is sensitive to outliers and the median is resistant to outliers. Problem 7 suggests that these concepts are true, but after going over such an example, I discuss *why* the concepts make sense (see the two paragraphs directly after Example 6).

The two concepts lead to the conclusions about how the mean and the median compare for skewed-left, symmetric, and skewed-right distributions (see the solution to Problem 5 of Example 7 and the paragraphs directly following Example 7). I take my time making these connections because they involve so many terminologies/concepts. Students will need to apply this material many times in Sections 4.1 and 4.2.

GROUP EXPLORATION: Comparing the Mean and the Median

Because groups of students have an easy time with this exploration and it addresses important concepts, I advise using this exploration to introduce Objective 5.

OBJECTIVE 6 COMMENTS

Problem 9 serves as a nice reminder that we often separate bimodal and multimodal distributions into two or more groups and then find the mean and/or other measures of each group.

OBJECTIVE 7 COMMENTS

I was tempted to exclude the mode from the textbook because it is used so much less often than the mean and the median in statistics, and for good reason. However, some reviewers wanted its inclusion, so I did so for completeness. Nonetheless, if you do include the topic, I advise you to give it light treatment. The only homework sections in the textbook that include the mode are Homework 4.1 (Exercises 15, 16, 27, 28, and 31), Chapter 4 Review Exercises (Exercises 3 and 4), and Chapter 4 Test (Exercise 4).

The paragraphs directly following Example 9 give a nice example in which the mode is not a reliable measure of the center.

SECTION 4.2 LECTURE NOTES

Objectives

1. Compute the *range* of some data.
2. Compute the *standard deviation* of some data.
3. Compare the means and standard deviations of two groups of data.
4. Apply the *Empirical Rule* to some data.
5. Determine whether an observation is unusual.
6. Compare the range and the standard deviation.
7. Compute the *variance* of some data.

OBJECTIVE 1

Definition *Range*

The **range** of some data values is given by

$$R = \text{largest number} - \text{smallest number}$$

1. The prices of 8 randomly selected calculators are shown in the following table.

132.99	110.63	132.00	60.00
49.99	91.50	137.81	138.82

Source: *Amazon.com*

Find the range in prices and explain what it means in this situation.

OBJECTIVE 2

Definition *Standard deviation*

The **standard deviation** of n data values $x_1, x_2, x_3, \ldots, x_n$ is given by

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

The standard deviation measures the spread. The greater the spread, the greater the standard deviation will be.

2. The highway gas mileages of different versions of the Audi R8 Spyder are 23, 20, 22, and 19, all in miles per gallon. Find the standard deviation of the gas mileages.

OBJECTIVE 3

3. A household suffers from *food insecurity* if at some point in the year the household eats less, goes hungry, or eats less nutritional meals because there is not enough money for food. The percentages of households suffering from food insecurities are shown in the following table for the West North Central and Pacific states.

West North Central State	Food Insecurity (percent of households)	Pacific State	Food Insecurity (percent of households)
Iowa	12.7	Alaska	14.0
Kansas	14.8	California	16.2
Minnesota	10.7	Hawaii	14.2
Missouri	17.1	Oregon	16.7
Nebraska	13.4	Washington	15.0
North Dakota	7.7		
South Dakota	12.3		

Source: *Feeding America; U.S. Department of Agriculture*

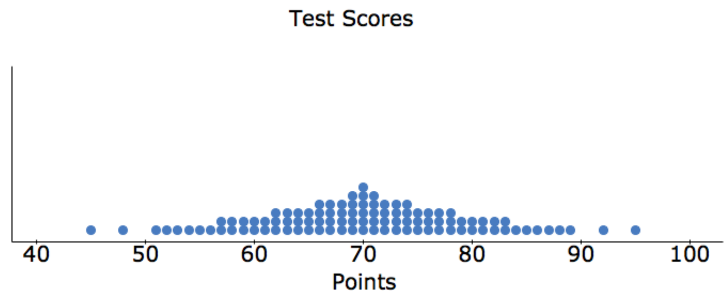
a. Compare the mean food insecurities for the West North Central states and for the Pacific states. What does your comparison mean in this situation?

b. Compare the standard deviations of the food insecurities for the West North Central states and for the Pacific states. What does your comparison mean in this situation?

c. Even though the mean food insecurity for the West North Central states is less than the mean food insecurity for the Pacific states, one of the West North Central states (Missouri) has a larger percentage of households with food insecurity than every one of the Pacific states. Explain why this is not surprising by referring to the standard deviation of food insecurities for the West North Central states and the Pacific states.

OBJECTIVE 4

4. Suppose that 100 statistics students' scores (in points) on a test are described by the following dotplot. The mean is 70 points and the standard deviation is approximately 10 points.



a. Describe the distribution.

b. Find the percentage of the scores that are within one standard deviation of the mean.

c. Find the percentage of the scores that are within two standard deviations of the mean.

d. Find the percentage of the scores that are within three standard deviations of the mean.

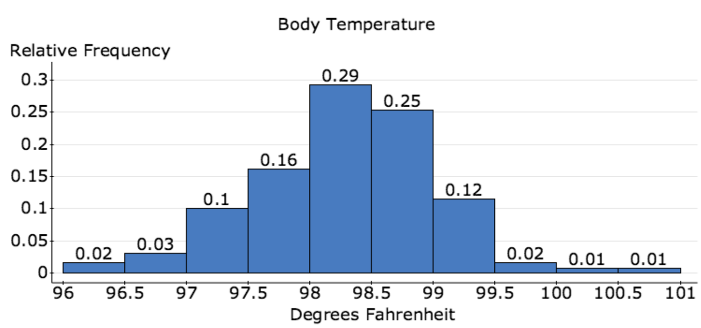
Empirical Rule

If a distribution is unimodal and symmetric, then the following statements are true.

- Approximately 68% of the observations lie within one standard deviation of the mean. So, approximately 68% of the observations lie between $\bar{x} - s$ and $\bar{x} + s$.
- Approximately 95% of the observations lie within two standard deviations of the mean. So, approximately 95% of the observations lie between $\bar{x} - 2s$ and $\bar{x} + 2s$.
- Approximately 99.7% of the observations lie within three standard deviations of the mean. So, approximately 99.7% of the observations lie between $\bar{x} - 3s$ and $\bar{x} + 3s$.

Before applying the Empirical Rule to a particular distribution, we must make sure the distribution is unimodal and symmetric. Other distributions can have quite different percentages.

5. The body temperatures of 130 adults are described by the following relative frequency histogram. The mean is 98.25°F and the standard deviation is 0.73°F (Source: *American Statistical Association*).



Source: *American Statistical Association*

a. Explain why the Empirical Rule can be applied.

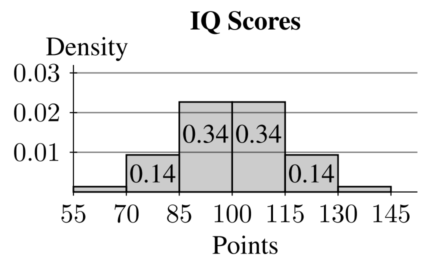
b. Apply the Empirical Rule.

c. Use the Empirical Rule to estimate the *number* of the adults who have body temperatures between 97.52°F and 98.98°F. Verify your result by referring to the histogram.

d. Use the Empiriical Rule to estimate the percentage of the adults who have body temperatures less than $96.79°$F OR greater than $99.71°$F?

e. Use the Empirical Rule to estimate the *number* of the adults who have body temperatures greater than $99.71°$F.

6. The Wechsler IQ test measures a person's intelligence. The IQs (in points) of some people are described by the following density histogram. Some of the bars have been left blank on purpose.



a. Estimate M.

b. Estimate $\bar{x}$.

c. Estimate s.

OBJECTIVE 5

If an observation is more than two standard deviations away from the mean, we refer to the observation as unusual.

7. In Problem 5, the 130 adults have a mean body temperature of $98.25°$F with standard deviation $0.73°$F. Is a body temperature of $96.3°$F unusual?

OBJECTIVE 6

Summary of Measuring Center and Spread

- If a distribution is unimodal and symmetric, then we usually use the mean to measure the center and the standard deviation to measure the spread.
- If a distribution is skewed, then we usually use the median to measure the center and the range to measure the spread.

OBJECTIVE 7

Definition *Variance*

The **variance** is the square of the standard deviation.

8. In Problem 2, we found that the standard deviation of the gas mileages of different versions of the Audi R8 Spyder is 1.8257 miles per gallon. Find the variance.

The meanings and symbols of the measures we have discussed in Sections 4.1 and 4.2 are shown in the following table.

Measure	What It Measures	Symbol
mean	center	$\bar{x}$
median	center	M
mode	center	(no standard symbol)
range	spread	R
standard deviation	spread	s
variance	spread	s^2

HW 1, 3, 7, 9, 13, 19, 21, 23, 27, 29, 31, 33, 35, 37, 43, 45, 47, 55

SECTION 4.2 TEACHING TIPS

Standard deviation is an extremely important foundational concept. In particular, it will be used a great deal when working with normal distributions (Sections 5.4 and 5.5). Although variance serves a role in an introductory statistics course, it will not be used in subsequent chapters of this text.

OBJECTIVE 1 COMMENTS

Students have an easy time with finding the range. As usual, I emphasize that students should state the units.

OBJECTIVE 2 COMMENTS

At the bottom of page 241 of the textbook, there is a nice example of the shortcoming of using the range to measure spread. To prepare students for the forthcoming standard deviation formula, I emphasize that when we subtract two numbers, the result is the distance between the numbers or the opposite of the distance, depending on which of the two numbers is larger.

Students have a tough time following the derivation for the standard deviation formula. The first two paragraphs on page 242 of the textbook are a nice way to get across why we square the differences. Rather than go into detail about why we divide by $n - 1$ rather than n, which would confuse students more than help, I simply say that it turns out to be more useful to divide by $n - 1$.

To compute a standard deviation, I first demonstrate how to use a table (see Table 22 on page 243 of the textbook). Then I show how to avoid using a table by pressing all the appropriate buttons on a calculator. Some students prefer the table approach and others prefer the latter approach. I warn students that on the next test they will have to use one of these two methods to find the standard deviation for a data set containing about four observations.

With all the necessary calculations, some students forget that standard deviation measures the spread of some observations, so I repeat this many times, reminding them of the geometric meaning of differences.

After having students calculate one or two standard deviations, I show them how to compute standard deviation using StatCrunch. As usual, I emphasize that they should include units in their result.

OBJECTIVE 3 COMMENTS

By comparing the standard deviation of two data sets, students start to see how standard deviation can be helpful. Not only does Problem 3 demonstrate making such a comparison, it also demonstrates the subtle point that we must consider the means and the standard deviations of two groups of data if we want some hint toward how the largest (or smallest) observations compare.

OBJECTIVE 4 COMMENTS

The Empirical Rule is a highlight of the chapter and will be used frequently in Section 4.2 and will obviously be an important concept when discussing the normal distribution (Sections 5.4 and 5.5).

To prepare students for the geometric significance of expressions such as $\bar{x} - s$ and $\bar{x} + s$ in the Empirical

Rule, I sketch a figure similar to Fig. 51 on page 245 of the textbook to get across the geometric significance of the numbers 7 ± 3. This is a simple idea, but many students lack the number sense to sort this out on their own.

GROUP EXPLORATION: Empirical Rule

This activity is meant to introduce the Empirical Rule. I advise using the exploration because the Empirical Rule is so important. Because the calculations are somewhat involved, I like to list the results of the exploration in stages so that groups of students can make sure they're on the right track.

If you choose to lecture on the Empirical Rule, I advise working with a data set consisting of 100 observations so computing percentages will be convenient. I also advise constructing a data set with a mean and a standard deviation that are easy to perform calculations with, such as the ones in Example 4: $\bar{x} = 70$ and $s \approx 10$. For a first example, it is helpful to display the data with a dotplot (as opposed to a histogram) so that individual observations can be counted. Subtracting the number of observations not in an interval from 100 can move things along.

Once students understand how the Empirical Rule connects with a dotplot of data, it is important to move on to density histograms. This should go smoothly due to students' work with density histograms in Sections 3.4 and 4.1. The connection between the Empirical Rule and density histograms will set a solid foundation for the normal distribution.

Problem 5 is a nice application of the Empirical Rule. It is harder than the exercises, but it will prime students for tackling such harder problems when working with the normal curve (Sections 5.4 and 5.5).

Problem 6 requires students to go backward: from probabilities to finding the mean and the standard deviation. When a concept is important, students will gain a better understanding by having to go forward and backward. The Empirical Rule is certainly worth that emphasis.

OBJECTIVE 5 COMMENTS

For now, the textbook defines an observation to be unusual if it is more than 2 standard deviations away from the mean. Once students can find probabilities for the normal curve with greater precision, the textbook will use the more precise cutoff of 1.96 standard deviations (Section 5.5).

OBJECTIVE 6 COMMENTS

After establishing that we should use the range to measure the spread for a skewed distribution, I remind students that the range is a crude way to measure spread and say that in Section 4.3 we will learn another measure of spread (the IQR) that will help complete the picture.

OBJECTIVE 7 COMMENTS

After calculating a standard deviation, it is easy for students to find the variance.

Because of the preferred units of standard deviation and the usefulness in using standard deviation to find z-scores, variance will not be used in subsequent sections.

SECTION 4.3 LECTURE NOTES

Objectives

1. Find *quartiles* of some data.
2. Find the *interquartile range* of some data.
3. Construct a *boxplot* to describe a distribution without outliers.
4. Identify outliers.
5. Construct a *boxplot* to describe a distribution with one or more outliers.
6. Compare the boxplots of two groups of data.

OBJECTIVE 1

Definition *First quartile, second quartile,* and *third quartile*

The *first quartile*, *second quartile*, and *third quartile* are the 25th, 50th, and 75th percentiles, respectively.

We use the symbols Q_1, Q_2, and Q_3 to stand for the first, second, and third quartiles, respectively.

1. Students in one of the author's statistics classes were surveyed about the number of hours they spending watching videos, movies, and television shows on a weekday. Here are the anonymous responses (in hours) of ten of the students: 4, 10, 3, 2, 0, 2, 1, 2, 4, 7 (Source: *J. Lehmann*).

- **a.** Construct a histogram to determine whether the mean or the median is the better measure of the center. Explain.
- **b.** Find Q_2 (the median).
- **c.** Find Q_1.
- **d.** Find Q_3.
- **e.** Use technology to verify the results you found in Parts (b), (c), and (d).
- **f.** Describe the meaning of the results you found in Parts (b), (c), and (d).

If some data have an odd number of data values, do not include the median in the lower half of the data when finding Q_1. Likewise, do not include the median in the upper half of the data when finding Q_3.

OBJECTIVE 2

Definition *Interquartile Range*

The **interquartile range** (**IQR**) is given by

$$\text{IQR} = Q_3 - Q_1$$

The IQR measures the spread of the middle 50% of the observations (from Q_1 to Q_3).

2. In Problem 1, we found $Q_1 = 2$, $Q_2 = 2.5$, and $Q_3 = 4$ for the data about the number of hours students spend watching videos, movies, and television shows per weekday: 4, 10, 3, 2, 0, 2, 1, 2, 4, 7.

- **a.** Calculate the IQR.
- **b.** Calculate the range.
- **c.** What do the IQR and the range mean in this situation?
- **d.** Discuss whether the IQR and the range are sensitive to the outlier 10 hours.

The IQR is resistant to outliers and the range is sensitive to outliers.

OBJECTIVE 3

3. The number of annual shark attacks in Hawaii are shown in the following table for the years 2001–2014.

3	6	5	3	4	3	7
1	3	4	3	10	13	7

Source: *Florida Museum of Natural History*

a. Construct a boxplot.

b. Which is longer, the left part of the box or the right part? Which is longer, the left whisker or the right whisker? Assuming the distribution is unimodal, is the distribution skewed left, skewed right, or symmetric?

c. Refer to the boxplot to estimate the range. What does it mean in this situation?

d. Refer to the boxplot to estimate the IQR. What does it mean in this situation?

If one whisker is longer than the other, that does *not* mean more observations are represented by the longer whisker. It means that the observations represented by the longer whisker are more spread out than the approximately equal number of observations represented by the shorter whisker.

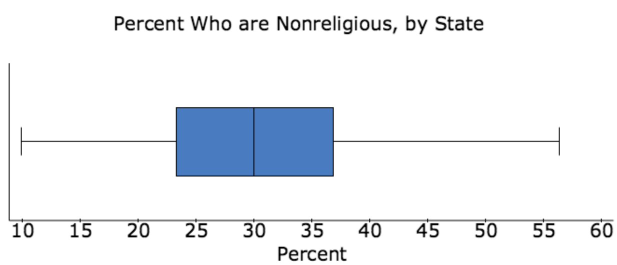
Meaning of Boxplots That Describe Distributions without Outliers

- A boxplot that describes a distribution without outliers consists of four parts: the left whisker, the left part of the box, the right part of the box, and the right whisker. Each part represents 25% of the observations.
- The full length of the box is equal to the IQR.

The following two statements apply to unimodal distributions:

- If the left part of the box is at least as long as the right part and the left whisker is quite a bit longer than the right whisker, then the distribution is skewed left.
- If the right part of the box is at least as long as the left part, and the right whisker is quite a bit longer than the left whisker, then the distribution is skewed right.

4. The percentages of residents who are nonreligious are described by the boxplot for the fifty states.



Source: *The Gallup Organization*

a. It turns out the distribution is unimodal. On the basis of the boxplot, is the distribution symmetric, skewed left, or skewed right? Explain.

b. Estimate the range. What does it mean in this situation?

c. Estimate the IQR. What does it mean in this situation?

OBJECTIVE 4

Definition *Fences*

The **left fence** and the **right fence** of some data values are given by

- left fence $= Q_1 - 1.5$ IQR
- right fence $= Q_3 + 1.5$ IQR

Definition *Outlier*

An **outlier** is an observation that is less than the left fence or greater than the right fence. We say that the outlier lies *outside* the fences.

5. In Problems 1 and 2, we found $Q_1 = 2$, $Q_3 = 4$, and IQR $= 2$ for the data about the number of hours students spend watching videos, movies, and television shows per weekday: 4, 10, 3, 2, 0, 2, 1, 2, 4, 7. Identify and interpret any outliers.

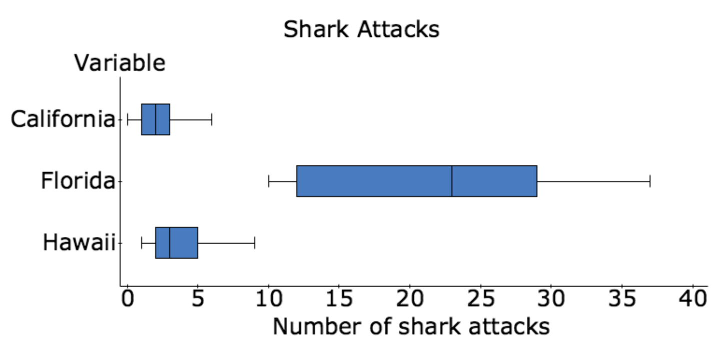
OBJECTIVE 5

6. Construct a boxplot that describes the data about the number of hours students spend watching videos, movies, and television shows per weekday: 4, 10, 3, 2, 0, 2, 1, 2, 4, 7.

7. Which is longer, the left part of the box or the right part? Which is longer the left whisker or the right whisker? What do your responses tell you about the shape of the distribution?

OBJECTIVE 6

8. The number of annual shark attacks in California, Florida, and Hawaii are described by the following boxplots for the years 2001–2014.



Source: *Florida Museum of Natural History*

a. Compare the centers of the three distributions. What does this mean in this situation?

b. Compare the spreads of the three distributions. What does this mean in this situation?

c. Compare the IQR of Florida's distribution to the range of California's distribution. What does this mean in this situation?

HW 1, 3, 5, 11, 13, 15, 17, 21, 23, 25, 29, 33, 41, 45, 47

SECTION 4.3 TEACHING TIPS

This section could be skipped because boxplots will not be used in subsequent chapters, but boxplots are useful in introductory statistics. In particular, they are a convenient way to identify outliers.

OBJECTIVE 1 COMMENTS

Some students think that the distances between the minimum observation and Q_1, Q_1 and Q_2, Q_2 and Q_3, and so on, are all equal. The diagram in Fig. 79 on page 260 of the textbook makes it clear that this is not true.

In general, students find it easy to find the quartiles, but they do need to be reminded that for an odd number of observations, we do not include the median in the lower or upper half when finding Q_1 or Q_3, respectively. Although the TI-84 follows this practice for any odd number of observations, StatCrunch follows this practice only when the number of observations is one of the following odd numbers: 3, 7, 11, 15, 19, ...

OBJECTIVE 2 COMMENTS

Once students understand how to compute the IQR, I emphasize that if a distribution is unimodal and symmetric, standard deviation is a better measure of variability than the IQR because standard deviation takes into account all the observations but the IQR takes into account only two observations.

For a distribution that is not unimodal and symmetric, I suggest that students compute both the IQR and the range to measure the variability because each measure involves only two observations. This motivates constructing boxplots.

A simple but good critical-thinking question is to ask students why the IQR is resistant to outliers but the range is sensitive to outliers.

OBJECTIVE 3 COMMENTS

Constructing boxplots of distributions that do not have outliers is really just a steppingstone to constructing boxplots of distributions that do have outliers, which is Objective 5.

I tell students that the boxplot is the only diagram we have discussed whose features allow us to visualize all three of following: the median, the IQR, and the range.

Once students understand how to find the quartiles, it is easy for them to construct a boxplot. It is the interpretation of boxplots that challenges some students. In particular, some students think that if one part of the boxplot, say the left whisker, is shorter than another part, say the right whisker, then fewer observations are represented by the left whisker. Drawing a boxplot above a dotplot such as in Fig. 85 on page 263 of the textbook can dispel this misconception, although it will be even clearer if you work with the following contrived data because none of the observations are equal to the quartiles: 0, 4, 6, 19, 21, 49, 51, and 100. Or you can facilitate the exploration, which addresses this issue.

Comparing a boxplot and a histogram of the same data can also be instructive (see Figs. 85 and 86 on page 263 of the textbook). This can be followed up by having students match histograms with boxplots (Exercises 5–8).

As usual, after students have constructed one or two boxplots by hand, I allow them to use technology.

OBJECTIVE 4 COMMENTS

Once students are comfortable calculating the fences and identifying the outliers, I try to make sure they understand why the process makes sense. In particular, I discuss that the left fence is a distance of 1.5IQR to the left of Q_1 and the right fence is a distance of 1.5IQR to the right of Q_3.

After displaying several boxplots for distributions that have just one or two outliers, it's fun to display a boxplot for the distribution of interest rates at the website Lending Club, which has 186 outliers (Exercise 11). Another option is to display boxplots for the distributions of highway and city gas mileages of cars (Exercise 28).

OBJECTIVE 5 COMMENTS

When introducing how to construct a boxplot of a distribution with outliers, I first construct a boxplot as if there were no outliers. Then I erase the appropriate whisker(s), draw the fences, draw the shorter whisker(s), and plot the outlier(s). This progression helps students build on what they already learned about boxplots in Objective 3.

I tell students that even if I already have a sense of a distribution by viewing a diagram other than a boxplot, I often still use technology to construct a boxplot to check for outliers. Then I generalize, explaining that we can gain much insight by constructing several types of diagrams of one distribution. And this can be done very quickly with the use of technology.

OBJECTIVE 6 COMMENTS

Problem 5 demonstrates a meaningful comparison of boxplots.

GROUP EXPLORATION: Determining the Meaning of a Boxplot

This is a nice way to convey to students that the lengths of the parts of a boxplot indicate the variability of the observations and not the number of observations.

Usually, I'm a fan of having students use technology when completing explorations so they can focus on the concepts and not get bogged down with constructing diagrams or computing values, but for this exploration it helps students to construct the boxplots by hand to see what's going on.

CHAPTER 5 OVERVIEW

This chapter draws heavily on concepts addressed in Chapters 1–4. The foundation of probability (Chapter 5) is randomness, which was discussed in Chapter 2. The complement rule, addition rule for disjoint events, and the general addition rule (all in Section 5.2) have all been addressed without calling them as such when working with proportions in Chapters 1 and 3. The normal curve (Section 5.4 and 5.5) relies on central tendency concepts (Section 4.1), variation concepts (Section 4.2), and density histograms (Section 3.4).

Because so much of the groundwork has been established in previous chapters, you will be pleased how quickly students pick up on many concepts in this chapter.

If you are tight on time, consider skipping Sections 5.3 and 5.5. Sections 5.1 and 5.2 provide important development for Section 5.4, which is the grand finale of the first five chapters. Upon completion of Section 5.4, students should have a solid foundation for quite a bit of an introductory statistics course, including the crucial sampling distribution.

SECTION 5.1 LECTURE NOTES

Objectives

1. Compute and interpret *probabilities*.
2. Use *simulation* to estimate a probability.
3. Use the *equally likely probability formula* to find probabilities.
4. Describe properties of probability.
5. Use a pie chart to find probabilities.
6. Use a density histogram to find probabilities.
7. Use the *Empirical Rule* to find probabilities.

OBJECTIVE 1

- Recall from Section 2.1 that **if we randomly select one item from a group of items, each item has the same chance of being selected.**
- We call a process that selects items randomly a **random experiment**.

Definition *Probability*

The **probability** of an outcome from a random experiment is the relative frequency of the outcome if the experiment were run an infinite number of times.

1. An experiment consists of rolling a six-sided die once. Find and interpret the probability of rolling a 5.

A probability does not tell us what to expect if we run an experiment a small number of times.

OBJECTIVE 2

Using technology to imitate a random experiment is called a **simulation**.

2. Use technology to simulate flipping a coin the given number of times. Discuss how well the relative frequency of tails estimates the probability of getting tails.

a. 5 times **b.** 100 times **c.** 1100 times **d.** 10,100 times

By running a random experiment a large number of times, we can use relative frequency to estimate the probability.

OBJECTIVE 3

3. An experiment consists of rolling a six-sided die once. Find and interpret the probability of rolling an odd number.

Definition *Sample space*

The **sample space** of a random experiment is the group of all possible outcomes.

Definition *Event*

An **event** is some of the outcomes in the sample space, all of them, or none of them.

Equally Likely Probability Formula

If the sample space of a random experiment consists of n equally likely outcomes and an event E consists of m of those outcomes, then

$$P(E) = \frac{m}{n}$$

If the outcomes of a random experiment are not equally likely, then we cannot use the equally likely probability formula.

4. For a group of 10 students in one of the author's prestatistics classes, there are 4 sociology majors, 2 graphic design majors, 3 communications majors, and 1 art major. None of the students are double-majors. Assume one student is randomly selected from the group. Use S for sociology, G for graphic design, C for communications, and A for art.

a. Construct a frequency and relative frequency table of the students' majors.

b. Find $P(S)$, $P(G)$, $P(C)$, and $P(A)$. How do your results tie into the table you constructed in Part (a)?

c. Find $P(\text{math})$.

d. Find the probability of randomly selecting a sociology, graphic design, communications, OR art major.

e. Find $P(S) + P(G) + P(C) + P(A)$. Why does the result make sense?

OBJECTIVE 4

- An **impossible event** does not contain any outcomes of the sample space.
- A **sure event** (or **certain event**) contains all the outcomes in the sample space.
- An event that contains just one outcome is a **single-outcome event**.

Probability Properties

- The probability of an impossible event is equal to 0.
- The probability of a sure event is equal to 1.
- The probability of an event is between 0 and 1, inclusive.
- The sum of the probabilities of all the single-outcome events in the sample space is equal to 1.

If you ever calculate a probability to be negative or greater than 1, you should realize that you've made an error.

5. A student surveys other students about their favorite sport. For each sport, the student calculates the probability of randomly selecting that sport from the responses (see the following table). What would you tell the student?

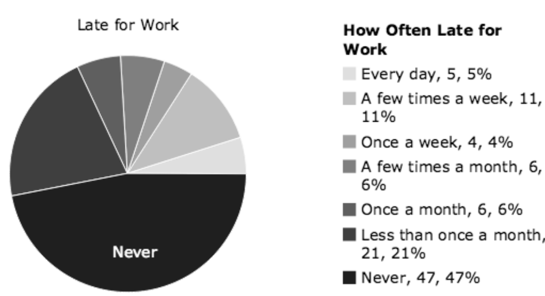
Sport	Probability
Basketball	0.3
Football	0.5
Baseball	0.2
Soccer	0.2
Other	0.3

6. A student calculates that the probability of randomly selecting a car that has gas mileage of 25 miles per gallon is -0.32. What would you tell the student?

If a situation involves a proportion or probability, it can be interpreted to involve the other as well.

OBJECTIVE 5

7. Some adult employees were surveyed about how often they are late for work. The percentages for various responses are described by the following pie chart (Source: *YouGov*).



Find the probability that a person randomly selected from the survey is

a. never late to work.

b. late to work at least once a month.

c. late to work at most once a month.

> **Definition** *Random variable*
>
> A **random variable** is a numerical measure of an outcome from a random experiment. We often use a capital letter such as X to stand for a random variable.

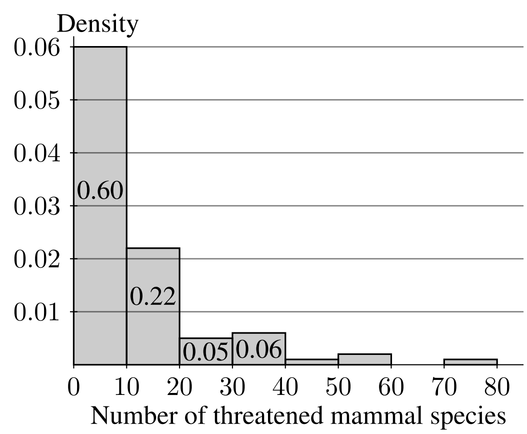
8. Let X be the outcome of rolling a six-sided die once. Find the given probability.

- **a.** $P(X = 5)$.
- **b.** $P(X \leq 3)$
- **c.** $P(X > 3)$
- **d.** $P(2 \leq X \leq 5)$
- **e.** $P(2 < X < 5)$

OBJECTIVE 6

The following density histogram describes the number of mammal species threatened in each country in the world. Some countries have at least 80 threatened mammal species, but the bars would not be visible.

Threatened Mammal Species, by Country



Let X be the number of threatened mammal species in a randomly selected country. Find the given probability.

1. $P(10 \leq X \leq 40)$

2. $P(X < 30)$

3. $P(X \geq 30)$

> **Interpreting the Area of a Bar of a Density Histogram**
>
> We can interpret the area of a bar of a density histogram as
>
> - the proportion of observations that lie in the bar's class.
> - the probability of randomly selecting an observation that lies in the bar's class.

OBJECTIVE 7

9. The Wechsler IQ test measures a person's intelligence. The distribution of IQ scores is unimodal and symmetric. The test is designed so that the mean score is 100 points and the standard deviation is 15 points (Source: Essentials of WAIS-IV Assessment, *Elizabeth Lichtenberger et al.*). Find the probability of randomly selecting a person who has IQ between 55 and 145 points.

HW 1, 3, 5, 7, 15, 19, 23, 29, 31, 35, 41, 45, 49, 53, 55, 57, 59, 65, 73

SECTION 5.1 TEACHING TIPS

A key concept of this chapter is randomness. If you skipped Section 2.1, I advise addressing the randomness concepts included in Section 2.1 now.

OBJECTIVE 1 COMMENTS

For Problem 1, students are quick to say that the probability is $\frac{1}{6}$. But on a test, some students tend to think the probability is $\frac{5}{6}$ (and other wrong answers), so it is worth revisiting this basic type of problem several times once other concepts in this section have been addressed.

OBJECTIVE 2 COMMENTS

I believe completing a simulation like in Problem 2 is a really effective way to get across how relative frequencies can estimate probabilities.

GROUP EXPLORATION: Performing a Simulation

Instead of going over Problem 2, you could have groups complete this exploration. Problem 4 of the exploration can clarify a typical student misconception.

OBJECTIVE 3 COMMENTS

Completing Problem 3 is a great primer for defining *sample space* and *event*. It is also a nice way to introduce the equally likely probability formula.

The discussion about the spinner in Fig. 6 on page 288 of the textbook is a great way to underscore the importance that outcomes must be equally likely to apply the equally likely probability formula.

Solving Problem 4 will be great preparation for discussing probability properties (the next objective).

OBJECTIVE 4 COMMENTS

When discussing the probability properties, I refer to the results we found for Problem 4. I emphasize that because of what we know about relative frequencies, it makes sense that the sum of the probabilities of all the single-outcome events in the sample space is equal to 1.

I also emphasize that a probability must be between 0 and 1, inclusive. Otherwise, students tend to miss catching that they've made such errors on tests.

OBJECTIVE 5 COMMENTS

Problem 8 is a good reminder about inequality notation (Section 1.1), which some students struggle with.

OBJECTIVE 6 COMMENTS

Some students continue to struggle with inequality notation, but these issues are largely resolved upon completion of this objective.

OBJECTIVE 7 COMMENTS

Problem 9 is a nice reminder about the Empirical Rule (Section 4.2), which is good preparation for the normal curve (Section 5.4).

SECTION 5.2 LECTURE NOTES

Objectives

1. Use the *complement rule* to find probabilities.
2. Use the addition rule for disjoint events to find probabilities.
3. Use the general addition rule to find probabilities.

OBJECTIVE 1

1. For a group of 10 students in one of the author's prestatistics classes, there are 4 sociology majors, 2 graphic design majors, 3 communications majors, and 1 art major. None of the students are double-majors. Assume one student is randomly selected from the group. Use S for sociology, G for graphic design, C for communications, and A for art.

   a. Find the probability of randomly selecting a communications major.

   b. Find the probability of NOT randomly selecting a communications major.

Definition *Complement*

The **complement** of an event E is the event that consists of all the outcomes not in E. We write "NOT E" to stand for the complement of E.

A **Venn diagram** consists of one or more shaded circles that are all bordered by a rectangle. The region inside the rectangle represents the sample space and the region inside each circle represents an event.

Complement Rule

For any event E,

$$P(\text{NOT } E) = 1 - P(E)$$

2. Consider the experiment of randomly selecting an American. The probability that the person is African American is 0.13 (Source: *U.S. Census Bureau*). Find the probability of selecting a person who is NOT African American.

OBJECTIVE 2

Definition *Disjoint events*

Two events are **disjoint** (or **mutually exclusive**) if they have no outcomes in common.

3. A committee consists of 3 Republicans, 5 Democrats, and 2 Independents. One person is randomly selected from the committee. Use R for Republican, D for Democrat, and I for Independent.

- **a.** Are the events R and I disjoint? Explain.
- **b.** Find $P(R)$.
- **c.** Find $P(I)$.
- **d.** Find $P(R \text{ OR } I)$.
- **e.** Find $P(R) + P(I)$.
- **f.** Compare the results of Parts (d) and (e).

Addition Rule for Disjoint Events

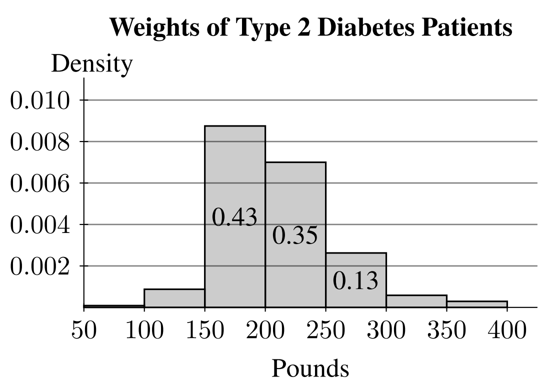
If E and F are disjoint events, then

$$P(E \text{ OR } F) = P(E) + P(F)$$

4. A ten-sided die is rolled once. Use F to stand for an outcome less than 4 and E to stand for an outcome greater than 8. Find the given probabilities.

- **a.** $P(F)$
- **b.** $P(E)$
- **c.** $P(F \text{ OR } E)$

5. The weights (in pounds) of type 2 diabetes patients from 2006 to 2007 are described by the following density histogram. The weights are rounded to the ones place.



- **a.** Find the probability that a patient randomly selected from the study would weigh between 150 and 299 pounds, inclusive.
- **b.** Find the probability that a patient randomly selected from the study would weigh less than 150 pounds OR at least 300 pounds.
- **c.** Find the probability that a patient randomly selected from the study would weigh between 150 and 199 pounds, inclusive, OR between 250 and 299 pounds, inclusive.

OBJECTIVE 3

6. A total of 1287 randomly selected adults were asked the following question. "Do you agree or disagree? Gay couples should have the right to marry one another." Their responses and their religious preferences are summarized in the following table.

	Protestant	Catholic	Jewish	Other	None	Total
Agree	230	171	12	46	167	626
Neither Agree Nor Disagree	71	36	1	22	24	154
Disagree	308	94	2	60	43	507
Total	609	301	15	128	234	1287

Source: *General Social Survey*

An adult is randomly selected from those surveyed. Use A to stand for an adult who agrees and N to stand for an adult who has no religious preference. Find the given probability. Round to the third decimal place.

a. $P(A)$ **b.** $P(N)$ **c.** $P(A \text{ AND } N)$ **d.** $P(A \text{ OR } N)$

General Addition Rule

For any events E and F,

$$P(E \text{ OR } F) = P(E) + P(F) - P(E \text{ AND } F)$$

7. A six-sided die is rolled once. Find the probability of rolling a number that is at most 3 OR is odd.

8. We return to the survey about gay couples' right to marry versus religious preference (see the following table). Find the probability of randomly selecting a surveyed adult who disagrees OR is Catholic. Round to the third decimal place.

	Protestant	Catholic	Jewish	Other	None	Total
yellow Agree	230	171	12	46	167	626
Neither Agree Nor Disagree	71	36	1	22	24	154
Disagree	308	94	2	60	43	507
Total	609	301	15	128	234	1287

Source: *General Social Survey*

If the general addition rule is used with two disjoint events, the result will be the same as using the addition rule for disjoint events.

HW 1, 3, 5, 9, 13, 17, 23, 27, 35, 43, 47, 53, 55, 59, 65

SECTION 5.2 TEACHING TIPS

The complement rule and the addition rule for disjoint events are extremely important preparation for the normal distribution (Section 5.4), which is a highlight of the course.

OBJECTIVE 1 COMMENTS

After having worked so much with the proportion-of-the-rest property, students are quick to find the solution to

Problem 1(b). From their perspective, the complement rule is simply another name for the proportion-of-the-rest property.

Because students are so quick to pick up on the complement rule, they don't really need to see it illustrated with a Venn diagram, but I show them one any way to get them used to viewing such diagrams.

OBJECTIVE 2 COMMENTS

When working with density histograms and pie charts in Chapter 3, students were basically applying the addition rule for disjoint events without realizing it, so students can now easily learn this rule.

Having groups of students work on Problem 3 is an excellent way to introduce the addition rule for disjoint events. Students are quick to say that $P(R \text{ OR } I)$ and $P(R) + P(I)$ are "the same thing."

Because of students' past work with density histograms, they have an easy time with Problem 5.

OBJECTIVE 3 COMMENTS

When working with two-way tables in Chapter 3, students were basically applying the general addition rule without realizing it, so they easily learn this rule now. Even though students already have the idea, I emphasize that we subtract P(E AND F) to account for the double-counting and illustrate with a Venn Diagram.

GROUP EXPLORATION: General Addition Rule

This exploration can be used to introduce the general addition rule. Even if some groups of students are unable to complete the exploration, whatever portion they are able to complete will serve as a good primer as you go over the rest of the exploration and then discuss the general addition rule in general.

SECTION 5.3 LECTURE NOTES

Objectives

1. Find and interpret *conditional probabilities*.
2. Determine whether two events are *independent* or *dependent*.
3. Use the multiplication rule for independent events to find probabilities.

OBJECTIVE 1

Definition *Conditional probability*

If E and F are events, then the **conditional probability** $P(E \mid F)$ is the probability that E occurs, given that F occurs.

It is important to keep straight the difference in meanings of $P(E \text{ AND } F)$ and $P(E \mid F)$.

1. A total of 989 randomly selected adults were asked whether they are interested in international issues. The following table compares their responses and their ages.

Table 2.1: Interest in International Issues versus Age

	Age Group (years)				
	18–30	31–40	41–55	56–89	Total
Very interested	17	36	67	106	226
Moderately interested	81	69	128	184	462
Not at all interested	77	65	79	80	301
Total	175	170	274	370	989

Source: *General Social Survey*

Suppose a person is randomly selected from the survey.

a. Find $P(\text{not at all interested})$. What does this mean in this situation?

b. Find $P(\text{not at all interested} \mid \text{ages 18–30 years})$. What does this mean in this situation?

c. Compare the results you found in Parts (a) and (b). What does your comparison mean in this situation?

2. In Problem 1, we worked with the data shown in the following table.

	Age Group (years)				
	18–30	31–40	41–55	56–89	Total
Very interested	17	36	67	106	226
Moderately interested	81	69	128	184	462
Not at all interested	77	65	79	80	301
Total	175	170	274	370	989

Source: *General Social Survey*

a. Find the probability that an adult randomly selected from the survey is between 56 and 89 years of age, given the person is not at all interested in international issues.

b. Find the probability that an adult randomly selected from the survey is between 31 and 40 years of age, given the person is not at all interested in international issues.

c. Compare the results you found in Problems 1 and 2. What does your comparison mean in this situation?

In Problem 1, we compared two conditional probabilities, which was quite instructive. In Problem 2, we compared two other conditional probabilities about the same situation, but the comparison was not instructive. The main point is that care should be taken to understand the meaning of probabilities before jumping to conclusions by comparing them.

3. A person rolls a six-sided die once. Find the probability that the outcome is

a. a 5, given the number is odd.

b. an odd number, given the number is at most 3.

OBJECTIVE 2

Definition *Independent events* and *dependent events*

Two events E and F are **independent** if $P(E \mid F) = P(E)$. The events are **dependent** if $P(E \mid F)$ and $P(E)$ are *not* equal.

Determine whether the given events E and F are independent.

1. Experiment: A college student who is taking prestatistics is randomly chosen.
   Event E: The student attempts every exercise in the textbook.
   Event F: The student passes the course.

2. Experiment: A person flips a coin and rolls a six-sided die.
   Event E: The coin lands on heads.
   Event F: The person rolls a 2.

If two events are dependent, this does not necessarily mean that one event causes the other.

4. A total of 1310 adults were surveyed about whether they had a great deal of confidence, only some confidence, or hardly any confidence in the press (newspapers and magazines). The following table compares their responses and their ages.

Confidence	Caucasian	African American	Hispanic	Other	Total
A Great Deal	56	27	17	16	116
Only Some	370	90	83	33	576
Hardly Any	439	83	75	21	618
Total	865	200	175	70	1310

Source: *General Social Survey*

   a. Find the probability that a person randomly selected from the survey
      i. had a great deal of confidence in the press.
      ii. had a great deal of confidence in the press, given the person is Hispanic.
   b. Determine whether having a great deal of confidence in the press is independent of being Hispanic. What does that mean in this situation?

OBJECTIVE 3

Multiplication Rule for Independent Events

If E and F are independent events, then

$$P(E \text{ AND } F) = P(E) \cdot P(F)$$

5. **a.** A person flips a coin and rolls a six-sided die. What is the probability that the person rolls a 5 AND the coin lands on tails?
   b. Three coins are flipped. What is the probability that all the outcomes are heads?

6. A person flips a coin 8 times.
   a. Find the probability of alternating between getting heads and tails, beginning with tails: THTHTHTH.
   b. Find the probability of getting all tails.
   c. Compare the results you found in Parts (a) and (b).

- Suppose a person flips a coin 7 times and gets all tails. What is the probability of getting another tails? There are two ways to interpret the question:
   1. What is the probability of getting 8 tails in a row?
   2. What is the probability of getting a tail, given that the first 7 outcomes were all tails?

Many students confuse interpretations (1) and (2). It is important to understand the meaning of each.

- **Before using the multiplication rule for independent events, we must check that the events are truly independent.**

7. For the spring semester 2014, 43% of students at Bellevue College were male (Source: *Bellevue College*). If 4 students were randomly selected from the college during that semester, what is the probability that at least 1 of the students would have been male?

Summary of Probability Rules

Let E and F be events.

• $P(\text{NOT } E) = 1 - P(\text{E})$	Complement rule
• If E and F are disjoint, then $P(E \text{ OR } F) = P(E) + P(F)$	Addition rule for disjoint events
• $P(E \text{ OR } F) = P(E) + P(F) - P(E \text{ AND } F)$	General addition rule
• If E and F are independent, then $P(E \text{ AND } F) = P(E) \cdot P(F)$	Multiplication rule for independent events

HW 1, 3, 5, 7, 11, 15, 19, 23, 25, 27, 29, 33, 35, 39, 45, 51, 55, 57

SECTION 5.3 TEACHING TIPS

If you are tight on time, you could skip this section because it is not needed for subsequent sections. One could argue that the concepts in this section are at a higher level than the rest of the concepts in the text and should be first taught in an introductory statistics course.

OBJECTIVE 1 COMMENTS

For Problem 1, some students think that $P(\text{not at all interested} \mid \text{ages 18–30 years})$ must be smaller than $P(\text{not at all interested})$ because the sample space of $P(\text{not at all interested} \mid \text{ages 18–30 years})$ is just adults ages 18–30 years in the study whereas the sample space of $P(\text{not at all interested})$ is so much larger (all adults in the study).

For Problem 2, I emphasize that it is not meaningful to compare the results of Parts (a) and (b) and explain why.

Even if all groups of students get correct answers for Problem 3(a), I emphasize the difference between $P(5 \mid \text{odd})$, $P(\text{odd} \mid 5)$, $P(5)$, and $P(\text{odd})$ because students often confuse these when taking a test.

GROUP EXPLORATION: Conditional Probability

Because this activity is hands-on, students would likely gain a better understanding of conditional probability than by other activities. In Problem 5, make sure students give careful thought to why their result is a conditional probability. Otherwise, they might miss the whole point.

OBJECTIVE 2 COMMENTS

Most students believe that if two events are dependent, then one event causes the other. To dismantle this belief, I point out that even if it turns out that people who drink orange juice every day get sick less often, that does not mean that orange juice causes good health. It might be that people who drink orange juice tend to take better care of themselves in all sorts of ways (exercise, eat healthy, get enough sleep, etc) and it's really because of these other behaviors that they get sick less often.

OBJECTIVE 3 COMMENTS

A person's cell-phone battery goes dead right at the moment they get stuck in an elevator. A typical response would be, "What are the chances of that!" Taking the question literally, it all comes down to how we define the probability. The bullet point just after Problem 6 addresses this issue. I discuss this carefully with my students.

Once students learn the multiplication rule, they tend to want to use it regardless of whether two events are independent. I caution them to always check whether events are independent before using this rule.

SECTION 5.4 LECTURE NOTES

Objectives

1. Describe the meaning of a *normal curve* and a *normal distribution*.
2. Compute and interpret z-scores.
3. Use z-scores and tables to find probabilities for a normal distribution.
4. Use technology to find probabilities for a normal distribution.

OBJECTIVE 1

- When a normal curve describes a distribution perfectly, we say the relationship is **normal** or **normally distributed**.
- When a normal curve describes a distribution fairly well (but not exactly), we say the distribution is **approximately normal** or **approximately normally distributed**.

Properties of a Normal Curve

- A normal curve is unimodal and symmetric.
- The mean is equal to the median. Both are the center of the curve.

Area-Probability Equality Property

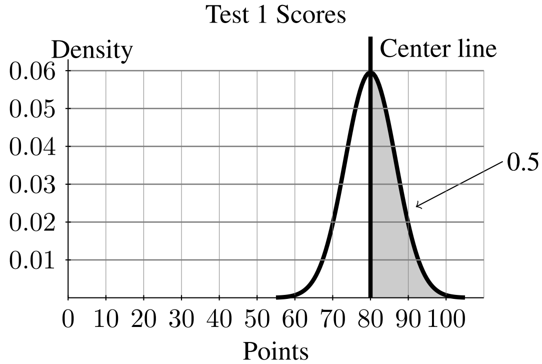
The area under a normal curve for an interval is equal to the probability of randomly selecting an observation that lies in the interval.

Total Area under the Normal Curve

The total area under a normal curve is equal to 1.

The probability of a single value of a normally distributed variable is 0.

1. Suppose that a prestatistics class takes a first test and their scores are described well by the following normal curve.



a. What is the probability that a randomly selected student scored more than 80 points?

b. On Test 2, the mean score is 20 points less than the mean score on Test 1, but the standard deviation is the same. Assuming the scores are still normally distributed, sketch the curve that describes the scores.

c. On Test 3, the mean score is the same as on Test 2, but the standard deviation is larger. Assuming the scores are still normally distributed, sketch a curve that describes the scores.

Comparing Normal Curves with Different Standard Deviations

A normal curve for a distribution with larger standard deviation will be wider and flatter than a normal curve for a distribution with smaller standard deviation. [Draw a figure.]

Empirical Rule

If a distribution is normal, then the probability that a randomly selected observation lies within

- one standard deviation of the mean is approximately 0.68. So, about 68% of the data lie between $\bar{x} - s$ and $\bar{x} + s$. [Draw a figure.]
- two standard deviations of the mean is approximately 0.95. So, about 95% of the data lie between $\bar{x} - 2s$ and $\bar{x} + 2s$. [Draw a figure.]
- three standard deviations of the mean is approximately 0.997. So, about 99.7% of the data lie between $\bar{x} - 3s$ and $\bar{x} + 3s$. [Draw a figure.]

OBJECTIVE 2

Definition *z-Score*

The ***z*-score** of an observation is the number of standard deviations that the observation is from the mean. If the observation lies to the left of the mean, then its z-score is negative. If the observation lies to the right of the mean, then its z-score is positive.

z-Score Formula

The z-score of an observation x is given by

$$z = \frac{x - \bar{x}}{s}$$

2. The 2014 draft picks for NBA basketball teams have heights that are approximately normally distributed with mean 79.1 inches and standard deviation 3.0 inches (Source: *nbadraft.net*). Shabazz Napier was the shortest 2014 draft pick with a height of 72 inches. Find the z-score for 72 inches. What does it mean?

Standard Normal Distribution

If a distribution is normally distributed, then the distribution of the observations' z-scores is also normally distributed with mean 0 and standard deviation 1. We call the distribution of z-scores the **standard normal distribution**.

OBJECTIVES 3 and 4

3. Find the probability that a z-score randomly selected from the standard normal distribution is

- **a.** less than -2.3.
- **b.** greater than 1.6.
- **c.** between 0.5 and 2.8

The entries in the standard normal distribution table are areas to the *left* of a value. We have to subtract an entry from 1 to find the area to the right.

4. Recall that scores on the Wechsler IQ test are normally distributed with mean 100 points and standard deviation 15 points. Let X be the IQ (in points) of a randomly selected person. Find the probability that a randomly selected person has an IQ

- **a.** less than 90 points.
- **b.** greater than 125 points.
- **c.** between 90 and 125 points.

5. Recall that the 2014 draft picks for NBA basketball teams have heights that are approximately normally distributed with mean 79.1 inches and standard deviation 3.0 inches. Find the probability that a randomly selected draft pick has height

- **a.** less than 72 inches.
- **b.** greater than 81 inches.
- **c.** between 77 inches and 87 inches.

Interpreting the Area Under a Normal Curve for an Interval

We can interpret the area A under the normal curve for an interval as

- the probability that a randomly selected observation will lie in the interval.
- the proportion of observations that lie in the interval.

HW 1, 3, 7, 9, 13, 23, 25, 29, 35, 43, 47, 49, 53, 55, 61, 65

SECTION 5.4 TEACHING TIPS

This is a crucial section because it lays a foundation for the sampling distribution, which is a key foundational concept in an introductory statistics course.

OBJECTIVE 1 COMMENTS

Because of past work with the shape of distributions, students are quick to observe that the normal curve is unimodal, symmetric, and the mean is equal to the median. Because of past work with density histograms and probability, students are also quick to say that the area under the normal curve for an interval is equal to a probability. They can also observe that the area under the entire normal curve must be equal to 1 because the sum of the areas of all the bars of a density histogram is equal to 1.

Because of past work with density histograms, students have an easy time with Parts (a) and (b) of Problem 1, but they often still need coaching with Part (c).

Because of past work with the Empirical Rule, a quick discussion of this rule in the context of probability will suffice.

OBJECTIVE 2 COMMENTS

To circumvent students from using the z-score formula without understanding, I carefully connect the formula with our past work with the Empirical Rule. The use of z-scores to compare observations from two different distributions will be introduced in Section 5.5.

OBJECTIVES 3 and 4 COMMENTS

I emphasize that the entries given in the standard normal distribution table for z-scores are areas to the *left* of a value.

Because of past work with density histograms, students are quick to pick up on how to apply the complement rule for Part (b) of Problems 4 and 5. However, they still have a tough time with Part (c) with both problems, partly because the subtraction does not involve the number 1 and partly because of the complexity of having to compute two z-scores.

Even after some practice, some students tend to skip certain crucial steps. For example, some students will skip using the standard normal distribution table, thinking a z-score is a probability.

To reinforce that students should think geometrically when working with a normal curve, for each exercise, I require that students sketch a normal curve, record relevant values on the X-axis and the z-axis, and write appropriate areas under the curve.

GROUP EXPLORATION: Symmetry of a Normal Curve

This exploration reinforces the importance of sketching a normal curve and considering its symmetry to find probabilities.

SECTION 5.5 LECTURE NOTES

Objectives

1. Find an observation from its z-score.
2. Use z-scores to the find value of a normally distributed variable from a probability.
3. Use technology to find the value of a normally distributed variable from a probability.
4. Use z-scores to compare the relative size of two observations from different groups of data.
5. Identify unusual observations.
6. Investigate whether a claim is true.

OBJECTIVE 1

Finding the Value of an Observation from Its z-Score

An observation x with z-score z is given by

$$x = \bar{x} + zs$$

1. Recall that the scores on the Wechsler IQ test are normally distributed with mean 100 points and standard deviation 15 points. James Woods is reported to have an approximate z-score of $z = 5.33$ (Source: The Chicago Tribune). What is his approximate IQ score?

OBJECTIVES 2 and 3

2. A professor gives a test to her prestatistics students. The scores are approximately normally distributed with mean 74 points and standard deviation 8 points. The professor decides to give As to approximately 10% of the students but not less than 10%. Use the standard normal distribution table or technology to find the cutoff score for an A.

- **Remember that whenever you use the standard normal distribution table, you must look for the area to the *left* of a value.**
- **Remember that whenever you use invNorm, you must enter the area to the *left* of a value.**

3. A researcher measured the lifetimes of some Activair® 13 HPX batteries when used for hearing aids. The mean lifetime was 252.4 hours and the standard deviation was 18.4 hours (Source: Measurement with Various Loads on Three Sizes of Zinc-Air Batteries for Hearing Aids, *Björn Hagerman*). Assume all Activair 13 HPX batteries are normally distributed with the same mean and same standard deviation as those tested.

   a. Explain why it would be unwise for the company to advertise that their batteries last 252.4 hours.

   b. What should the company advertise the lifetime of their batteries to be so that 99% of the batteries would last at least that long?

OBJECTIVE 4

Jenny and Eric took tests on probability in two different sections of prestatistics. Jenny scored 92 points on a test with mean 78 points and standard deviation 7 points. Eric scored 79 points on a test with mean 67 points and standard deviation 3 points. The test scores on each test are approximately normally distributed.

1. Find the z-score for Jenny's test score. What does it mean in this situation?

2. Find the z-score for Eric's test score. What does it mean in this situation?

3. Assuming a typical student in one class knows the material as well as a typical student in the other class, determine whether Jenny did relatively better than Eric.

OBJECTIVE 5

Definition *Unusual observation*

Assume that some data are normally or approximately normally distributed. An observation is **unusual** if it is more than 1.96 standard deviations away from the mean.

4. In a study of 1541 randomly selected young men (ages 20–29 years), the young men's HDL cholesterol levels are approximately normally distributed with mean 47 mg/dl and standard deviation 12.5 mg/dl (Source: *NHANES*).

a. Suppose that a young man in the study has an HDL cholesterol level of 19 mg/dl. Is that an unusual observation?

b. Find the probability of randomly selecting a young man from the study with an HDL cholesterol less than or equal to 19 mg/dl.

c. Explain why it makes sense that your result in Part (b) is less than 0.025.

Determining Whether an Observation is Unusual

An observation of a normal or an approximately normal distribution is unusual if either of the following equivalent statements is true:

- The observation's z-score is less than -1.96 OR greater than 1.96.
- The probability of randomly selecting the observation OR one on the same side of the mean but even more extreme is less than 0.025.

OBJECTIVE 6

5. A company claims that its low-salt potato chips contain only 83 mg of sodium per serving, on average. An independent company randomly selects and tests a bag of the low-salt potato chips, which turns out to contain 102 mg of sodium per serving.

a. Assuming the sodium levels per serving of the low-salt chips are approximately normally distributed with mean 83 mg and standard deviation 6 mg, find the probability that a randomly selected bag would have at least 102 mg of sodium per serving.

b. If the assumptions you made in Part (a) are true, is the bag containing 102 mg of sodium per serving an unusual event?

c. Describe a scenario in which the bag containing 102 mg of sodium per serving is not an unusual event.

d. On the basis of the bag containing 102 mg of sodium per serving, do the assumptions you made in Part (a) seem reasonable? Should a more thorough investigation occur?

HW 1, 3, 7, 9, 13, 15, 17, 19, 21, 23, 25, 29, 31, 37, 43

SECTION 5.5 TEACHING TIPS

If you are tight on time, you might consider skipping this section, which contains concepts that will not be needed in subsequent sections. If you rush through this material, it may muddy the waters and do more harm than good. However, a thorough treatment of this section could lay an even stronger foundation for the normal curve, which is so important in an introductory statistics course.

OBJECTIVE 1 COMMENTS

To introduce the formula, $x = \bar{x} + zs$, I first ask students what test score corresponds to a z-score of 2 if the mean score is 70 points and the standard deviation is 10 points. Then I use the computation $90 = 70 + 2(10)$ to introduce the formula.

OBJECTIVES 2 and 3 COMMENTS

When using the standard deviation normal table to solve Problem 2, students tend to look up the area 0.1 in the middle of the table rather than the correct area 0.9 even though they learned in Section 5.4 that each entry in the table is an area to the *left* of a value. Students tend to make a similar error when using invNorm with a TI-84.

After completing Problem 3, I point out that it would be beneficial to the company (and consumers) if the standard deviation were lower.

GROUP EXPLORATION: Grading on a Curve

Students tend to ask instructors whether they grade on a curve, but most of them have no idea what this truly means. Perhaps the biggest benefit of this exploration is that students can finally learn what's involved. They tend to be surprised to learn that grading on a curve can result in students earning lower grades than by the grade cutoffs used by most instructors. The most challenging part of this exploration is that students must realize they need to add the given percentages in Problem 2 to find the grade cutoffs.

OBJECTIVE 4 COMMENTS

I emphasize this concept because it is important that students make meaningful comparisons between two quantities not only in statistics but in their lives.

OBJECTIVE 5 COMMENTS

Section 4.2 states that an observation is unusual if it is more than 2 standard deviations away from the mean. But this objective refines the cutoff to 1.96 standard deviations because we can now determine this more precise cutoff by using the standard normal distribution table and/or technology.

OBJECTIVE 6 COMMENTS

This is a nice objective to finish up Chapter 5 because it inches up quite close to hypothesis testing, which is a key technique used in introductory statistics.

A key concept related to Problem 5 is that we cannot conclude the company's claim is false simply because the randomly selected bag contains 102 mg of sodium per serving; the standard deviation (and probability) must be considered.

CHAPTER 6 OVERVIEW

This chapter provides the big picture for linear modeling, which will be addressed in Chapters 6–9. Chapters 7 and 8 introduce the necessary algebra so that linear modeling can be explored at a deeper level in Chapter 9.

The key theme of the four characteristics of an association is introduced in this chapter and will be developed further in Chapters 7–10.

Many concepts in this chapter can be tied to concepts included in Chapter 2: explanatory and response variables, associations, warnings about causation, lurking variables, and time-series plots. Concerns about extrapolation are similar to the concerns about making inferences. And the four characteristics of an association are similar to the center, spread, shape, and outliers of a distribution of one numerical variable. Because of these similarities and the relative easiness of the material, you will be able to traverse this chapter at a faster pace than preceding ones.

SECTION 6.1 LECTURE NOTES

Objectives

1. Identify explanatory and response variables.
2. Construct a scatterplot.
3. Determine the direction of an association.

OBJECTIVE 1

Here we review the meanings of *explanatory variable* and *response variable* (Section 2.3):

Definition *Explanatory and response variables*

In a study about whether a variable x explains (affects) a variable y,

- We call x the **explanatory variable** (or **independent variable**).
- We call y the **response variable** (or **dependent variable**).

Recall that calling a variable the explanatory variable does not mean for sure that the variable affects (explains) the response variable. After all, we'd have to carry out the study to find out.

1. For each situation, identify the explanatory variable and the response variable.
   a. Let d be the number of miles a person runs per day, and let T (in seconds) be the person's best mile run time.
   b. Let n be the number of adults who want to buy an iPhone that costs d dollars.
   c. Let s be an employee's salary (in thousands of dollars) at a company, and let n be the number of years the employee has worked for the company.

For an ordered pair (a, b), we write the value of the explanatory variable in the first (left) position and the value of the response variable in the second (right) position.

2. Let s be the sales (in thousands) of a video game, and let b be the game's advertising budget (in thousands of dollars). A video game with a \$50-thousand-dollar advertising budget has sales of 742 thousand games. Express this as an ordered pair.

3. Let n be the mean number of hours that a student studies prestatistics per week, and let p be percentage of points the student earns on the first test. A student who studies 15 hours per week earns 93% of the points on the first test. Express this as an ordered pair.

4. Let s be the mean number of grams of sugar that an adult consumes daily, and let n be the mean number of cavities the person gets per decade. What does the ordered pair $(76, 1.5)$ mean in this situation?

5. Let p be the percentage of Americans at age A years who listen to talk radio. What does the ordered pair $(59, 23)$ mean in this situation?

Columns of Tables and Axes of Coordinate Systems

Assume that an authentic situation can be described by using two numerical variables. Then

- For tables, the values of the explanatory variable are listed in the first column and the values of the response variable are listed in the second column. [Give an example.]
- For coordinate systems, the values of the explanatory variable are described by the horizontal axis and the values of the response variable are described by the vertical axis. [Draw a figure.]

OBJECTIVE 2

A coordinate system with plotted ordered pairs is called a **scatterplot**.

6. The fatality rates from automobile crashes are shown in the following table for various speeds. Let r be the fatality rate (deaths per 1000 crashes) for a speed of s mph.

Speed Group (mph)	Speed Used to Represent Speed Group (mph)	Fatality Rate (deaths per 1000 crashes)
0–30	15	2.5
30–40	35	3.5
40–50	45	6.1
50–60	55	15.3
over 60	70	16.9

Source: *National Highway Transportation Safety Administration*

   a. Construct a scatterplot.

   b. Which of the points in your scatterplot is lowest? What does that mean in this situation?

   c. Which of the points in your scatterplot is highest? What does that mean in this situation?

- Recall that there is an **association** between the explanatory and response variables if the response variable changes as the explanatory variable changes.
- An association of *numerical* explanatory and response variables is often called a **correlation**.
- If there is an association between two variables that does *not* necessarily mean there is causation.

OBJECTIVE 3

Definition *Positive and negative association*

Assume two numerical variables are the explanatory and response variables of a study.

- If the response variable tends to increase as the explanatory variable increases, we say the variables are **positively associated** (or **positively correlated**) and that there is a **positive association** (or **positive correlation**). [Draw a figure.]
- If the response variable tends to decrease as the explanatory variable increases, we say the variables are **negatively associated** (or **negative correlated**) and that there is a **negative association** (or **negative correlation**). [Draw a figure.]

We describe the **direction** of an association by determining whether the association is positive, negative, or neither.

7. Percentages of adults surveyed who plan to attend a Halloween party this year are shown in the following table for various ages groups.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
18–24	21.0	44
25–34	29.5	34
35–44	39.5	25
45–54	49.5	14
55–64	59.5	10
65 or over	70.0	6

Source: *International Mass Retail Association*

Let p be the percentage of adults at age a years who plan to attend a Halloween party this year.

a. Construct a scatterplot.

b. Determine whether the association is positive, negative, or neither.

8. Let t be the number of years since 2000. Find the values of t that represent the years 2000, 2005, 2010, and 2015.

9. Let n be the number (in thousands) of new apps submitted to Apple's App Store per month at t years since 2010. In 2015, 40 thousand new apps were submitted to Apple's App Store (Source: *pocketgamer.biz*). Express this as an ordered pair.

10. Let p be the percentage of adults ages 18–24 who live with their parents at t years since 2005. What does the ordered pair $(7, 56)$ mean in this situation?

11. The numbers of Internet searches using Google are shown in the following table for various years.

Year	Number of Searches (in billions)
2008	83
2009	107
2010	122
2011	133
2012	139

Source: *Google*

Let r be the annual number (in billions) of searches at t years since 2005.

a. Construct a scatterplot.

b. Determine whether the association is positive, negative, or neither.

HW 1, 3, 9, 17, 23, 31, 35, 39, 41, 43, 47, 51, 55, 59, 69

SECTION 6.1 TEACHING TIPS

Students tend to have an easy time with this section because many of the concepts have been addressed in previous chapters but in different contexts. Constructing scatterplots is a foundational skill that students will use frequently throughout the rest of the course.

OBJECTIVE 1 COMMENTS

Although students learned about explanatory and response variables in Chapter 2, they tend to need a refresher at this point in the course. I emphasize the new information that values of the explanatory variable are described by the first coordinate in an ordered pair, the first column of a table, and the horizontal axis of a coordinate system. Even after completing the entire chapter, there tend to be a few students who continue to struggle with this concept, so you can't emphasize it enough.

OBJECTIVE 2 COMMENTS

I point out to my students that scatterplots are similar to time-series plots, except we do not "connect the dots" with scatterplots.

Students tend to construct excellent scatterplots, probably due to all the practice they have had constructing other types of diagrams in previous chapters. I remind students that units and variables should be displayed on each axis and a title should also be displayed.

Rather than introduce the terminology *correlation*, to keep the amount of terminology to a minimum, I stick with using *association* because we have already introduced this terminology in Chapter 2.

Even though the distinction between association and causation was already addressed in Chapter 2, I emphasize this distinction now because it is such an important concept. This can be illustrated by the Internet association described in Problem 11. In Section 6.2, this topic will be revisited by comparing the U.S. per-person consumption of margarine and the divorce rate in Maine (Figure 50 on page 389 of the textbook).

OBJECTIVE 3 COMMENTS

Students have an easy time identifying the direction of an association.

Working with definitions such as t is the number of years since 2000 is new content and it's worth taking time to carefully describe how to interpret such definitions because some students struggle with this initially.

GROUP EXPLORATION: Analyzing Points below, above, and on a Line Containing Points with Equal Coordinates

Although not crucial for the course, this exploration requires some good critical thinking. Problem 6 provides a hint of what's to come with linear regression (Section 9.3).

SECTION 6.2 LECTURE NOTES

Objectives

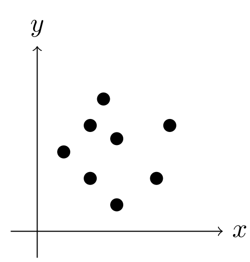
1. Determine the shape of a distribution.
2. Determine the strength of a distribution.
3. Compute and interpret the correlation coefficient.
4. Determine whether there are any outliers and what to do with them.
5. Determine the four characteristics of an association in the correct order.
6. Explain why strong (or weak) association does not guarantee causation.

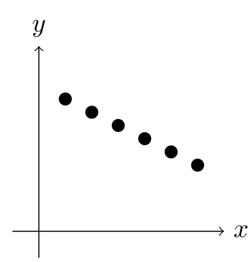
OBJECTIVE 1

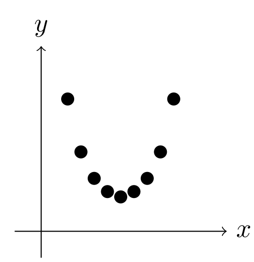
- If the points of a scatterplot lie close to (or on) a line, we say the variables are **linearly associated** and that there is a **linear association**. [Draw a figure.]
- If the points of a scatterplot lie close to (or on) a curve that is not a line, we say there is a **nonlinear association**. [Draw a figure.]
- If no curve comes close to all the points of a scatterplot, we say there is **no association**. [Draw a figure.]

For any pair of numerical variables, there is either a linear association, a nonlinear association, or no association. We will refer to the **shape** of an association as being one of these three types.

1. For each of the following scatterplots, determine whether there is a linear association, nonlinear association, or no association.







OBJECTIVE 2

- If a curve passes through all the points of a scatterplot, we say there is an **exact** association with respect to the curve. [Draw a figure.]
- If a curve comes close to all the points, we say there is a **strong association** with respect to the curve. [Draw a figure.]
- If a curve comes somewhat close to all the points, we say there is a **weak association** with respect to the curve. [Draw a figure.]

2. The carbohydrates, calories, and fat for 29 pizzas made by six of the leading pizza companies are shown in the following table.

Carbohydrates (g)	Calories	Fat (g)	Carbohydrates (g)	Calories	Fat (g)
39	368	17	35	342	16
36	308	13	37	359	17
20	259	15	26	297	16
39	348	15	32	305	13
36	311	13	53	499	21
20	249	14	64	549	21
40	357	16	42	410	19
27	279	13	63	579	24
28	246	9	43	395	17
30	276	11	29	260	9
37	338	15	37	277	9
41	367	16	30	269	11
35	320	14	29	286	14
36	329	15	33	309	14
26	266	13			

Source: *Domino's, Little Ceasar's, Papa John's, Pizza Hut, DiGiorno Frozen, Kashi Frozen*

a. Construct a scatterplot that compares carbohydrates and calories.

b. Construct a scatterplot that compares carbohydrates and fat.

c. For each of the scatterplots you constructed, describe the shape of the association.

d. Compare the strengths of the two associations.

e. For each of the scatterplots you constructed, identify whether the association is positive, negative, or neither.

f. Use an appropriate scatterplot to estimate the caloric contents of the pizzas with 20 g of carbohydrates.

g. Use an appropriate scatterplot to estimate the fat content of the pizzas with 20 g of carbohydrates.

OBJECTIVE 3

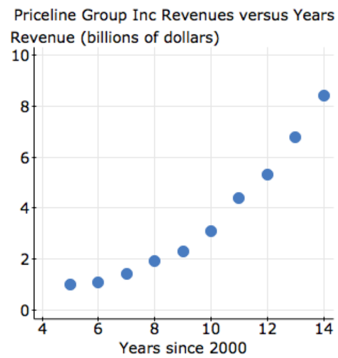
Properties of the Linear Correlation Coefficient

Assume r is the linear correlation coefficient for the association between two numerical variables. Then

- The values of r are between -1 and 1, inclusive.
- If r is positive, then the variables are positively associated.
- If r is negative, then the variables are negatively associated.
- If $r = 0$, there is no *linear* association.
- The larger the value of $|r|$, the stronger the linear association will be.
- If $r = 1$, then the points lie exactly on a line and the association is positive.
- If $r = -1$, then the points lie exactly on a line and the association is negative.

[Draw figures to illustrate all of the above concepts.]

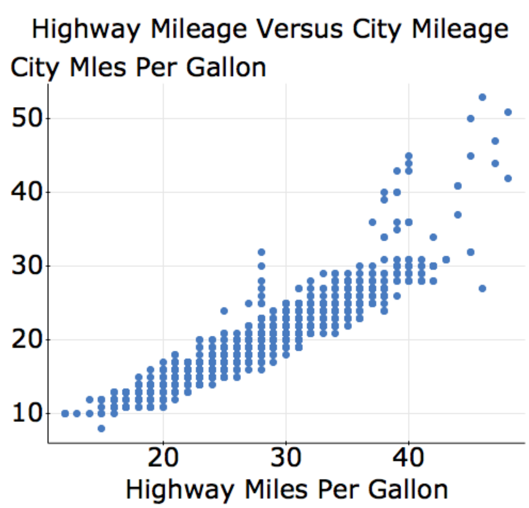
- If $r = 0$, then there is no linear association but there may be a *nonlinear* association.
- To determine the shape and strength of an association, we should inspect a scatterplot of the data as well as compute r. For example, even though $r \approx 0.96$ for the data described by the following scatterplot, the points lie much closer to a curve than they do to a line.



OBJECTIVE 4

- For an association between two numerical variables, a point is an outlier if it does not fit the overall pattern of the points in the scatterplot. [Draw a figure.]
- The correlation coefficient is very sensitive to outliers. [Draw scatterplots to show how r can change from 0 to approximately 0.95 by adding an outlier. See Figs. 43 and 44 on page 387 of the textbook.]

3. A scatterplot comparing the highway mileages and city mileages of 1195 types of cars is displayed in the following figure.



a. Identify the outliers. Should they be removed? If yes, for what purpose?

b. Assume the outliers have been removed. For the data that remain,

- **i.** determine the shape of the association.
- **ii.** determine the strength of the association.
- **iii.** determine whether the association is positive, negative, or neither.

OBJECTIVE 5

Order of Determining the Four Characteristics of an Association

We determine the four characteristics of an association in the following order:

1. Identify all outliers.
   a. For outliers that stem from errors in measurement or recording, correct the errors if possible. If the errors cannot be corrected, remove the outliers.
   b. For other outliers, determine whether they should be analyzed in a separate study.
2. Determine the shape of the association.
3. If the shape is linear, then on the basis of r and the scatterplot, determine the strength. If the shape is nonlinear, then on the basis of the scatterplot, determine the strength.
4. Determine the direction. In other words, determine whether the association is positive, negative, or neither.

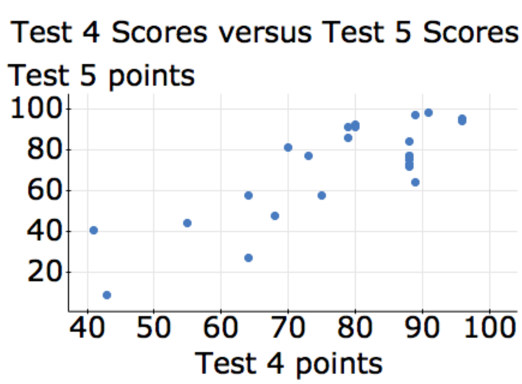
OBJECTIVE 6

Definition *Lurking variable*

A **lurking variable** is a variable that causes both the explanatory and response variables to change during the study.

A strong association between two variables does not guarantee that a change in the explanatory variable will cause a change in the response variable.

4. The scatterplot in the following figure compares the scores of Test 4 and Test 5 for a calculus course taught by the author.



   a. Describe the four characteristics of the association.
   b. Does a higher score on Test 4 cause a higher score on Test 5? If no, describe at least one possible lurking variable.
   c. On the basis of the fairly strong, positive association between Test 4 and Test 5 scores, a student concludes that there must have been a fairly strong, positive association between any pair of tests for the course. What would you tell the student?

HW 1, 3, 7, 9, 11, 15, 17, 19, 21, 23, 29, 33, 35, 37, 44, 47

SECTION 6.2 TEACHING TIPS

Although this section contains a lot of material, you can move fairly quickly because students do not tend to have much trouble with the concepts.

OBJECTIVE 1 COMMENTS

Students have an easy time determining the shape of an association.

OBJECTIVE 2 COMMENTS

Students are quick to pick up on describing the strength of an association, although they tend to be more picky at what qualifies as a strong association. An association that a statistician would call strong, a student would call weak. This is not surprising because aside from this course, students have little or no experience working with (data) points that are not colinear.

If you don't want to deal with all the data in Problem 2, you could work with a subset. And if you are tight on time you could just have students complete Parts (a), (b), and (d); Part (d) is the main point.

OBJECTIVE 3 COMMENTS

Rather than list all the bulleted statements in the box shown for Objective 3, I draw a bunch of scatterplots along with the appropriate values of r and verbally point out the properties as they emerge.

Students' intuition about the value of r becomes strong quite quickly. Throughout the rest of the chapter, each time I display a scatterplot, I ask students to guess the value of r and their guesses are quite good. At least once, we calculate the mean (or median) of several guesses and compare it to the actual value, which is a nice reminder of the benefit of the measures of central tendency.

Displaying the Priceline Group Inc scatterplot (in the lecture notes for Objective 3) and observing that the association is nonlinear even though $r \approx 0.96$ is a nice way to emphasize that we need to observe a scatterplot as well compute r to make a reliable determination of the shape and strength of an association.

GROUP EXPLORATION: Impact of Transforming Data on the Correlation Coefficient

This exploration would have more pedagogical value if students understood how r is calculated, but they still enjoy this activity, which increases their sense of r.

OBJECTIVE 4 COMMENTS

Similar to the issue of determining the strength of an association, students tend to identify too many data points as outliers.

OBJECTIVE 5 COMMENTS

Although I discuss the order of determining the four characteristics as shown in the boxed property statement for this objective, I'm quick to add that some of these determinations can be made simultaneously and the order is not set in stone. For example, we might find ourselves determining the shape and direction of an association simultaneously.

OBJECTIVE 6 COMMENTS

Students tend to remember the meaning of lurking variables, which was discussed in Chapter 2. Students can draw on their own personal experiences when responding to Problem 4(b).

SECTION 6.3 LECTURE NOTES

Objectives

1. Use a line to model an association between two numerical variables.
2. Use a linear model to make estimates and predictions.
3. Describe the meaning of *input* and *output*.
4. Find errors in estimations.
5. Describe the meaning of *interpolate*, *extrapolate*, and *model breakdown*.
6. Find intercepts of a line.
7. Find intercepts of a linear model.
8. Modify a model.

OBJECTIVE 1

1. Let n be the number of bank robberies per 100 branches at t years since 1995.

Year	Robberies per 100 Branches	Year	Robberies per 100 Branches
1997	10.6	2007	6.0
1999	8.8	2009	6.0
2001	10.3	2011	5.3
2003	8.7	2012	4.5
2005	7.7		

Source: *FBI*

   a. Construct a scatterplot.
   b. Is there a linear association, a nonlinear association, or no association?
   c. Compute r. On the basis of r and the scatterplot, determine the strength of the association.
   d. Draw a line that comes close to the points of the scatterplot.

Definition *Model*

A **model** is a mathematical description of an authentic situation. We say that the description *models* the situation.

Definition *Linear model*

A **linear model** is a nonvertical line that describes the association between two quantities in an authentic situation.

OBJECTIVE 2

2. Use the robbery model to estimate the number of bank robberies per 100 branches in 2004.

3. Use the model to estimate when there were 5.6 bank robberies per 100 branches.

- We construct a scatterplot of data to determine whether there is a linear association. If so, we draw a line that comes close to the data points and use the line to make estimates and predictions.

- It is a common error to try to find a line that contains the greatest number of points. However, our goal is to find a line that comes close to *all* of the data points.

OBJECTIVE 3

Definition *Input, output*

An **input** is a permitted value of the *explanatory* variable that leads to at least one **output**, which is a permitted value of the *response* variable.

4. In Problem 2, identify the input and the output.

OBJECTIVE 4

The **error** in an estimate is the amount by which the estimate differs from the actual value.

- For an overestimate, the error is positive.

- For an underestimate, the error is negative.

5. Use the robbery model to estimate the number of bank robberies per 100 branches in 2001. Find the error in the estimate.

6. Use the robbery model to estimate the number of bank robberies per 100 branches in 2007. Find the error in the estimate.

OBJECTIVE 5

Refer to the estimates and predictions that you made in Problems 2, 3, 5, and 6 as interpolations or extrapolations.

Definition *Interpolation, extrapolation*

For a situation that can be described by a model whose explanatory variable is x,

- We perform **interpolation** when we use a part of the model whose x-coordinates are between the x-coordinates of two data points.

- We perform **extrapolation** when we use a part of the model whose x-coordinates are not between the x-coordinates of any two data points.

7. Use the robbery model to estimate the number of bank robberies per 100 branches in 2002. Did you perform interpolation or extrapolation? Explain.

8. Use the robbery model to estimate the number of bank robberies per 100 branches in 2014. Did you perform interpolation or extrapolation? Explain.

9. Use the robbery model to estimate the number of bank robberies per 100 branches in 2030. Did you perform interpolation or extrapolation? Explain.

Definition *Model breakdown*

When a model gives a prediction that does not make sense or an estimate that is not a good approximation, we say that **model breakdown** has occurred.

OBJECTIVE 6

Definition *Intercepts of a line*

An **intercept** of a line is any point where the line and an axis (or axes) of a coordinate system intersect. There are two types of intercepts of a line sketched on a coordinate system with an x-axis and a y-axis:

- An $\boldsymbol{x}$**-intercept** of a line is a point where the line and the x-axis intersect. [Draw a figure.]
- A $\boldsymbol{y}$**-intercept** of a line is a point where the line and the y-axis intersect. [Draw a figure.]

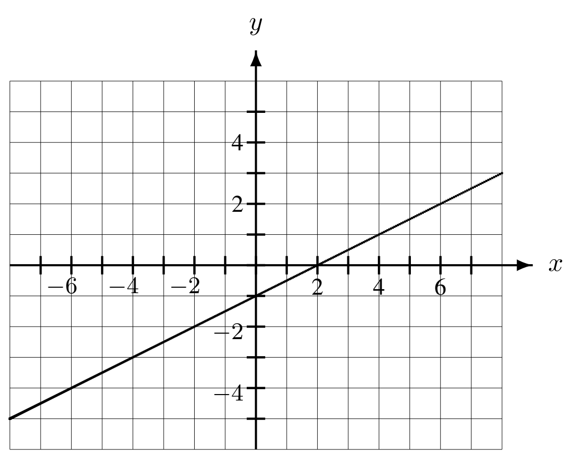
10. Refer to the following figure.

a. Find the x-intercept of the line.

b. Find the y-intercept of the line.

c. Find y when $x = -6$.

d. Find x when $y = -3$.



OBJECTIVE 7

11. Find the given intercept of the robbery model. What does it mean in this situation? Did you perform interpolation or extrapolation? Explain.

a. n-intercept

b. t-intercept

12. The percentages of cell phone users who send or receive text messages multiple times per day are shown in the following table for various age groups.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
18–24	21.0	76
25–34	29.5	63
35–44	39.5	42
45–54	49.5	37
55–64	59.5	17

Source: *Edison Research and Arbitron*

Let p be the percentage of cell phone users at age a years who send or receive text messages multiple times per day.

a. Draw a linear model that describes the association between and a and p.

b. Predict the percentage of 25-year-old cell phone users who send or receive text messages multiple times per day. Did you perform interpolation or extrapolation? Explain.

c. Find the p-intercept. What does it mean in this situation? Did you perform interpolation or extrapolation? Explain.

d. Find the a-intercept. What does it mean in this situation? Did you perform interpolation or extrapolation? Explain.

OBJECTIVE 8

13. Additional research about cell phone users who send or receive text messages multiple times per day yields the data shown in the first and last rows of the following table. Use this data and the following assumptions to modify the model you found in Problem 12:

- Children 3 years old and younger do not send or receive text messages multiple times per day.
- The percentage of cell phone users who send or receive text messages multiple times per day levels off at 5% for users over 80 years in age.
- The age of the oldest cell phone users is 116 years.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
12–17	14.5	75
18–24	21.0	76
25–34	29.5	63
35–44	39.5	42
45–54	49.5	37
55–64	59.5	17
over 64	70.0	7

Source: *Edison Research and Arbitron*

HW 1, 3, 5, 7, 17, 21, 25, 29, 31, 35, 37, 39, 43, 47, 51

SECTION 6.3 TEACHING TIPS

This section completes the big picture of linear modeling. In particular, students will sketch linear models and use the models to make predictions.

In Chapters 7 and 8, students will learn the necessary algebra so they can revisit linear modeling in a more meaningful way in Chapter 9.

OBJECTIVE 1 COMMENTS

For Problem 1 of the lecture notes, some students may need a reminder of the meaning of the phrase "at t years since 1995." It is important that the scaling on each axis begin at 0 because students will find the intercepts in Problems 11 and 12.

When describing the meaning of a model, I give intuitive examples such as the textbook's example about airplane models (middle of page 403 of the textbook).

In Problem 1(d) of the lecture notes, I demonstrate that there are many acceptable linear models. Many students try to find a line that contains the greatest number of data points rather than a line that comes close to all of the data points. To clarify this confusion, I compare a reasonable linear model that comes close to the data points but does not contain any data points with the terribly inaccurate linear model that contains the data points $(4, 8.8)$ and $(6, 10.3)$.

GROUP EXPLORATION: Modeling a Linear Association

This exploration is a great collaborative activity for students to discover the fundamental ideas of this section. Some students will likely disregard the instruction in Problem 2 and draw a "zigzag" curve rather than a line, unless you warn them not to. Waiting for students to make this error and then clarifying the meaning of "line" might make a more lasting impression.

OBJECTIVE 2 COMMENTS

When completing Problems 2 and 3, I draw input-output arrows similar to those in Fig. 80 on page 404 of the textbook. I require my students to use such arrows so that I can better evaluate their work on quizzes and exams. It is difficult to award any partial credit if no arrows are displayed.

I warn my students that even though their answers may differ from those in the answers section of the textbook, their answers may be correct. I explain that it's possible to draw reasonable linear models that are different than the reasonable models that the textbook used. Although I want my students to carefully draw scatterplots and linear models, I don't want them to be anxious about getting the exact results.

OBJECTIVE 3 COMMENTS

This objective is setting the stage for the concept function, which is discussed in Section 7.4. Chapters 8–10 use function notation to a limited extent so as to accommodate both instructors who use Section 7.4 and those that don't.

OBJECTIVE 4 COMMENTS

According to the textbook's definition of error, the error of an estimate is the opposite of a residual. I struggled with what to do with this discrepancy. In the end, I decided to let the discrepancy exist because I believe it makes more sense for the error to be positive when the model leads to an overestimate and for the error to be negative when the model leads to an underestimate.

For many students, the distinction between a model and reality is foggy. For example, many students tend to think that the answers to Problems 5 and 6 are the actual 10.3 robberies per 100 branches and 6.0 robberies per 100 branches, respectively, even if the linear model does not contain the data points $(6, 10.3)$ and $(12, 6.0)$. Making such a distinction while completing Problems 5 and 6 pays big dividends both in the short and long run. For an intuitive explanation, I tell my students that a model is fiction, which we hope is very close to the truth.

OBJECTIVE 5 COMMENTS

I emphasize that we have little or no faith when we extrapolate. One could argue that this is the most important concept of the section because extrapolation is so often misused by the media.

Problem 9 is a nice way to drive home the point of the dangers of extrapolation.

The concept of model breakdown is important, because most models do "break down." I tell my students that when model breakdown occurs, they must say so on quizzes and exams. Even if a student believes that no bank robberies will occur in the year represented by the t-intercept in Problem 11(b), the student should explain why that is reasonable. Regardless if students think the result is reasonable, they should also say they have little or no faith in it because they performed extrapolation.

When I discuss the model breakdown in Problem 9, I ask my students for suggestions of graphs of (nonlinear) curves that might turn out to be better models, which is nice preparation for Objective 8. In general, such questions can generate some fun and interesting discussions.

GROUP EXPLORATION: Identifying Types of Modeling Errors

If you do not plan to assign most (or all) of the explorations in the textbook, it would probably be good to omit assigning this exploration or to assign it for homework. Many of these ideas can be touched on as you proceed through this section.

OBJECTIVE 6 COMMENTS

When students find intercepts, I require them to include both coordinates as an ordered pair to reinforce that intercepts are points.

OBJECTIVE 7 COMMENTS

Many students have trouble finding intercepts of models for a variety of reasons. Some students get off on the wrong foot by writing the variables on the wrong axes when constructing a scatterplot. Some students write the coordinates of an intercept in the wrong order, and others write the coordinates of an intercept in the correct order but interpret them as if they were written in the other order.

OBJECTIVE 8 COMMENTS

In many cases in which we can determine model breakdown has occurred, it is unclear how to modify the model. Problem 13 is nice because it includes additional facts that suggest how to modify the linear model sketched in Problem 12(a).

Discussing this objective is important because students should be aware that even though scientists may currently be using a certain model, discoveries made at a later point may warrant using a modified (or even a completely different) model.

CHAPTER 7 OVERVIEW

In Sections 7.1–7.3, students will learn how to graph linear equations and how to compute rates of change. They will also discover that these two tasks are very much connected. All of this work will lay a crucial foundation for work with regression lines in Section 9.3.

The chapter closes with investigating functions in Section 7.4. Because not all departments will have time for this concept, I have included exercises that involve functions in subsequent chapters but not so much that it bogs down homework sections for courses that do not address functions.

Despite most students' poor experiences in previous algebra courses, you might be surprised how well they do in this chapter. Chapters 1–6 tend to build quite a bit of mathematical maturity in students. Plus, graphing equations of the form $y = mx + b$ is fairly easy. The more challenging task of graphing linear equations in two variables in which y is not isolated will be addressed in Section 8.4. And students will be ready for that challenge by then.

SECTION 7.1 LECTURE NOTES

Objectives

1. For an equation in two variables, determine the meaning of *solution*, *satisfy*, and *solution set*.
2. Describe the meaning of the *graph* of an equation.
3. Graph equations of the form $y = mx + b$.
4. Describe the meaning of b in equations of the form $y = mx + b$.
5. Graph equations of the form $y = b$ and $x = a$.
6. Describe the Rule of Four for equations.
7. Graph an equation of a linear model.

OBJECTIVE 1

1. Show that the equation $y = x + 3$ becomes a true statement when 2 is substituted for x and 5 is substituted for y.

We say $(2, 5)$ *satisfies* $y = x + 3$ and that $(2, 5)$ is a *solution* of $y = x + 3$.

Definition *Solution, satisfy,* and *solution set* of a equation in two variables

An ordered pair (a, b) is a **solution** of an equation in terms of x and y if the equation becomes a true statement when a is substituted for x and b is substituted for y. We say (a, b) **satisfies** the equation. The **solution set** of an equation is the set of all solutions of the equation.

2. Is $(2, 7)$ a solution of $y = 4x - 1$?
3. Is $(3, 5)$ a solution of $y = 4x - 1$?

OBJECTIVE 2

4. **a.** Find five solutions of $y = 2x - 3$ and plot them in the same coordinate system.
b. Do the five points lie on a line? If so, sketch the line.
c. Select another point on the line and show that the corresponding ordered pair satisfies $y = 2x - 3$.
d. Select a point that doesn't lie on the line and show that the corresponding ordered pair does not satisfy $y = 2x - 3$.

We call the line that we sketched in Problem 4 the *graph* of $y = 2x - 3$.

Definition *Graph*

The **graph** of an equation in two variables is the set of points that correspond to all solutions of the equation.

- Every point on the graph of an equation represents a solution of the equation.
- Every point *not* on the graph represents an ordered pair that is *not* a solution.

OBJECTIVE 3

The equation $y = 2x - 3$ is of the form $y = mx + b$.

Graph of $y = mx + b$

The graph of an equation of the form $y = mx + b$, where m and b are constants, is a line.

Here are some equations whose graphs are lines:

$$y = 3x + 7 \qquad y = -6x + 2 \qquad y = 2x \qquad y = x - 4 \qquad y = 1$$

Graph the equation by hand. Also, find the y-intercept.

5. $y = 3x - 5$ **6.** $y = -2x + 4$ **7.** $y = -3x$ **8.** $y = \frac{2}{3}x + 1$

OBJECTIVE 4

Substitute 0 for x into the equation $y = mx + b$:

$$y = m(0) + b = 0 + b = b$$

The y-intercept of the graph of $y = mx + b$ is $(0, b)$.

y-Intercept of the Graph of $y = mx + b$

The graph of an equation of the form $y = mx + b$ has y-intercept $(0, b)$.

For $y = 3x - 5$, the y-intercept is $(0, -5)$ and for $y = -2x + 4$, the y-intercept is $(0, 4)$. See Problems 5 and 6.

OBJECTIVE 5

Find five solutions of the given equation. Then graph the equation by hand.

9. $x = 2$ **10.** $y = 3$

Equations for Horizontal and Vertical Lines

If a and b are constants, then

- The graph of $y = b$ is a horizontal line. [Show the graph.]
- The graph of $x = a$ is a vertical line. [Show the graph.]

Graph the equation by hand.

11. $x = 4$ **12.** $y = -2$ **13.** $x = 0$ **14.** $y = 0$

Equations Whose Graphs Are Lines

If an equation can be put into the form $y = mx + b$ or $x = a$, where m, a, and b are constants, then the graph of the equation is a line. We call such an equation a **linear equation in two variables**.

Here are some examples of linear equations in two variables:

$$y = -5x + 9 \qquad y = 3x - 8 \qquad y = 4 \qquad x = -2$$

OBJECTIVE 6

Rule of Four for Solutions of an Equation

We can describe some or all of the solutions of an equation in two variables with:

1. an equation,

2. a table,

3. a graph, or

4. words.

These four ways to describe solutions are known as the **Rule of Four**.

15. **a.** List some solutions of $y = 4x - 3$ by using a table.
b. Describe the solutions of $y = 4x - 3$ by using a graph.
c. Describe the solutions of $y = 4x - 3$ by using words.

16. Consider the equation $y = x + 2$.

a. Substitute 3 for x and show that the input $x = 3$ leads to the output $x = 5$.
b. Graph $y = x + 2$. Then use arrows to show that the input $x = 3$ leads to the output $x = 5$.

OBJECTIVE 7

17. In the United Kingdom, nervous dental patients are sometimes given a sedative to calm them. To administer the sedative, the dentist performs a *cannulation*, which involves inserting a thin tube into a vein in a patient's hand. In an experiment involving 20 individuals, researchers tested whether the amount of pain the individuals experienced during cannulation explains the level of anxiety that the individuals predicted they would experience by undergoing cannulation again. After undergoing cannulations, the individuals rated their experiences of pain from 0 to 100 and their predicted anxieties about undergoing cannulations again from 0 to 100. The patients' ratings are shown in the following table.

Pain Rating	Anxiety Rating	Pain Rating	Anxiety Rating	Pain Rating	Anxiety Rating	Pain Rating	Anxiety Rating
37	50	23	13	29	35	69	68
63	70	38	35	34	30	14	3
8	2	60	20	41	48	12	10
10	10	38	30	75	70	49	43
70	81	42	42	8	32	48	43

Source: *Pain-anxiety scatterplot (**Source:*** Anaesthetics: A Randomised, Double-Blind, Placebo-Controlled, Comparative Study of Topical Skin Analgesics and the Anxiety and Discomfort Associated with Venous Cannulation*, A. F. Speirs et al.)*

Let a be the anxiety rating a patient predicts for undergoing another cannulation after having undergone one with a pain rating of p.

- **a.** Identify the explanatory and response variables.
- **b.** Construct a scatterplot.
- **c.** Describe and interpret the direction of the association.
- **d.** Graph the model $a = 0.91p + 1.71$ on the scatterplot.
- **e.** Does the model come close to the data points?
- **f.** Use the model to predict the anxiety rating a patient would predict about undergoing another cannulation after having undergone one with a pain rating of 30.

HW 1, 3, 7, 9, 19, 25, 27, 35, 39, 43, 47, 49, 51, 77, 81

SECTION 7.1 TEACHING TIPS

In this section, students will graph linear equations, which will lay the foundation for the rest of the chapter and ultimately prepare them to graph regression lines in Section 9.3. The last objective in this section–graphing equations of linear models–will connect nicely with what students learned in Section 6.3.

OBJECTIVE 1 COMMENTS

I emphasize these terminologies, because I use these words so much throughout the course.

OBJECTIVE 2 COMMENTS

Problem 4 is important, because it helps students understand that every point on the graph represents a solution of the equation and that every point not on the graph represents an ordered pair that is not a solution. This important concept is used when finding equations of linear equations (Sections 9.1 and 9.2).

OBJECTIVE 3 COMMENTS

For Problem 5, I suggest that students use three points, because the third point can serve as a check. Most students are challenged by Problem 8. Students will have an easier time graphing such an equation once they have learned about slope.

GROUP EXPLORATION: Solutions of an Equation

Although this exploration addresses the meaning of a graph (Objective 2), it is better to use this activity once students are comfortable graphing linear equations (Objective 3). That way, students will be able to efficiently do Problem 1 of the exploration and get to the main point of the exploration.

OBJECTIVE 4 COMMENTS

Students will need to be able to determine the y-intercept of the graph of an equation of the form $y = mx + b$ when they perform graphing in Section 7.3.

OBJECTIVE 5 COMMENTS

For Problems 9 and 10, many students are thrown for a loop, because only one variable appears in these equations. One way to address this issue is to begin with an application. Let n be the number of states in the United States at t years since 1960. Next, create a table of nonnegative values of t and n. Then determine that the model is $n = 50$ and graph the model. Finally, discuss for which years model breakdown occurs. (Hawaii became the 50th state on August 21, 1959.)

Another way to address the issue, when finding solutions of the equation $x = 2$ (Problem 9), is to use the following scenario:

> Suppose that a family always has exactly 2 cars and another family has various numbers of cars at various times.

This scenario helps students see that the equation $x = 2$ is restricting the value of x to be 2, but the value of y can be any real number.

For students who confuse the graphs of equations of the form $x = a$ and $y = b$, I suggest that they build a table of (at least two) solutions to tip them off whether they should graph a horizontal or vertical line.

OBJECTIVE 6 COMMENTS

The Rule of Four will be revisited when working with functions in Section 7.4.

Problem 16 is important, because it helps students see the connection between finding inputs and outputs symbolically and graphically.

OBJECTIVE 7 COMMENTS

In Problem 17, some students tend to wonder how the equation $a = 0.91p + 1.71$ was found. Even after I say that we will learn how to find such an equation in Section 9.3 after we have discussed the necessary algebra, some students are not satisfied; they want to know now, which I take to be a good sign that they so much want to understand the reasoning behind things. Despite their discomfort, it is a good idea to graph models now to keep students thinking about linear modeling.

When completing Problem 17(f), I emphasize that the equation $a = 0.91p+1.71$ allows us to make predictions without having to draw arrows, which is time consuming and imprecise.

SECTION 7.2 LECTURE NOTES

Objectives

1. Calculate the rate of change of a quantity with respect to another quantity.
2. Explain the connection between constant rate of change and an exact linear association.
3. Use a run and the corresponding rise of a linear model to find a rate of change.
4. Describe the connection between the direction of an association and rate of change.
5. Calculate the slope of a line.
6. Explain why the slope of an increasing line is positive and the slope of a decreasing line is negative.
7. Explain why the absolute value of the slope of a line measures the steepness of the line.
8. Explain why the slope of a horizontal line is zero and the slope of a vertical line is undefined.

OBJECTIVE 1

1. If the temperature increased by 12°F over 3 hours, estimate how much the temperature increased per hour.
2. If the temperature increased from 60°F at 7 A.M. to 68°F at 11 A.M., estimate how much the temperature increased per hour.

Formula for Rate of Change

Suppose that a quantity y changes steadily from y_1 to y_2 as a quantity x changes steadily from x_1 to x_2. Then the **rate of change** of y with respect to x is the ratio of the change in y to the change in x:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

3. The sales of cigarettes in the United States deceased approximately linearly from 400 billion cigarettes in 2003 to 280 billion cigarettes in 2013 (Source: *Euromonitor International*). Find the approximate rate of change of cigarettes sales.
4. A person of height 132.5 centimeters should use a ski pole of length 90 centimeters. A person of height 153.0 centimeters should use a ski pole of length 110 centimeters. Find the approximate rate of change of ski pole length with respect to a person's height.

OBJECTIVE 2

5. Suppose that a student travels at 60 miles per hour on a road trip. Let d be the distance (in miles) that the student can drive in t hours.

   a. Identify the explanatory and response variables.

   b. Construct a table to describe the association between t and d for driving times of 0, 1, 2, 3, 4, and 5 hours.

c. Construct a scatterplot for driving times of 0, 1, 2, 3, 4, and 5 hours.

d. Find the slope of the linear model.

e. Describe the four characteristics of the association. Compute and interpret r as part of your analysis. If there is an association, draw an appropriate model on the scatterplot.

Constant Rate of Change Implies an Exact Linear Association

If the rate of change of one variable with respect to another variable is constant, then there is an exact linear association between the variables.

6. The amounts of money a student is paid are shown in the following table for various numbers of hours worked. Let p be the student's pay (in dollars) for working t hours.

Time (hours)	Pay (dollars)
0	0
1	15
2	30
3	45
4	60
5	75

a. Identify the explanatory and response variables.

b. Describe the four characteristics of the association. Compute and interpret r as part of your analysis.

c. Find the rate of change of pay from 2 to 3 hours.

d. Find the rate of change of pay from 0 to 5 hours. Compare the result with the result you found in Part (c).

An Exact Linear Association Implies Constant Rate of Change

If there is an exact linear association between two variables, then the rate of change of one variable with respect to the other is constant.

OBJECTIVE 3

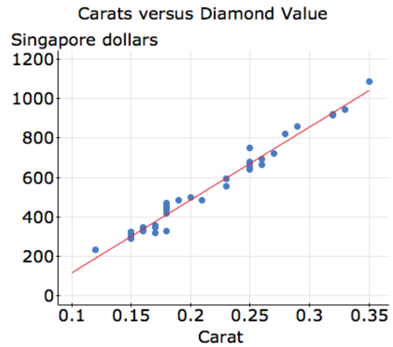
For any linear model, the **run** is the horizontal change and the **rise** is the vertical change in going from one point on the line to another point on the line.

Using Run and Rise to Find Rate of Change

Assume there is an exact linear association between an explanatory variable x and a response variable y and (x_1, y_1) and (x_2, y_2) are two distinct points of the linear model. Then the rate of change of y with respect to x is the ratio of the rise to the run in going from point (x_1, y_1) to point (x_2, y_2):

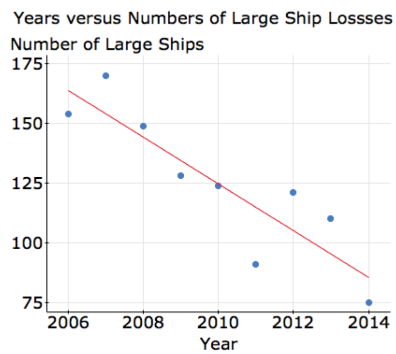
$$\text{rate of change of } y \text{ with respect to } x = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

7. A *carat* is a unit of weight for precious stones and pearls. The scatterplot and the model in the following figure describe the association between the weights (in carats) of some diamonds and their value (in Singapore dollars). Is the association positive, negative, or neither? Estimate the rate of change of value with respect to number of carats.



In Problem 7, you likely found that if one diamond weighs 1 carat more than another diamond, it will cost approximately \$3720 more than the other diamond. This does *not* mean that the diamonds cost \$3720 per carat.

8. The scatterplot and the model in the following figure describe the association between years and the number of large ships lost at sea due to fire, collision, storm, and machine breakdown. Is the association positive, negative, or neither? Estimate the rate of change of number of large ship losses.



OBJECTIVE 4

Connection between the Direction of an Association and Rate of Change

- If an association between two variables is positive, then the approximate rate of change of one variable with respect to the other is positive (see Problem 7).
- If an association between two variables is negative, then the approximate rate of change of one variable with respect to the other is negative (see Problem 8).

OBJECTIVE 5

Definition *Slope of a nonvertical line*

Assume (x_1, y_1) and (x_2, y_2) are two distinct points of a nonvertical line. The **slope** of the line is the rate of change of y with respect to x. In symbols:

$$m = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

9. Find the slope of the line that contains the points $(1, 5)$ and $(4, 3)$.

It is a common error to make incorrect substitutions into the slope formula. Carefully consider why the middle and right-hand formulas are incorrect:

Correct	**Incorrect**	**Incorrect**
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	$m = \dfrac{y_2 - y_1}{x_1 - x_2}$	$m = \dfrac{x_2 - x_1}{y_2 - y_1}$

Find the slope of the line that contains the given points.

10. $(1, 3)$ and $(4, 9)$

11. $(-4, 1)$ and $(2, -3)$

12. $(-5, -1)$ and $(4, 2)$

13. $(-6, -3)$ and $(-2, -5)$

When working with negative coordinates, it can help to first write

$$\frac{(\ \ ) - (\ \ )}{(\ \ ) - (\ \ )}$$

and then insert the coordinates into the appropriate parentheses.

OBJECTIVE 6

- A line that goes upward from left to right is **increasing**. [Draw a figure.]
- A line that goes downward from left to right is **decreasing**. [Draw a figure.]

Slopes of Increasing or Decreasing Lines

- An increasing line has positive slope. [Draw a figure. Also refer to Problems 10 and 12.]
- A decreasing line has negative slope. [Draw a figure. Also refer to Problems 11 and 13.]

14. Find the approximate slope of the line that contains the points $(-2.8, 5.9)$ and $(-1.1, -3.7)$. Round the result to the second decimal place. State whether the line is increasing or decreasing.

OBJECTIVE 7

15. Compare the steepness of two lines with slopes 2 and 3.

Measuring the Steepness of a Line

The absolute value of the slope of a line measures the steepness of the line. The steeper the line, the larger the absolute value of its slope will be.

OBJECTIVE 8

- Draw a horizontal line and find its slope $\left(\dfrac{0}{\text{run}} = 0\right)$.
- Draw a vertical line and explain why its slope, $\dfrac{\text{rise}}{0}$, is undefined.

Slopes of Horizontal and Vertical Lines

- A horizontal line has slope equal to zero. [Draw a figure.]
- A vertical line has undefined slope. [Draw a figure.]

For Problems 16–19, find the slope of the line that contains the given points. Determine whether the line is increasing, decreasing, horizontal, or vertical.

16. $(2, -3)$ and $(2, 1)$

17. $(-3, -2)$ and $(2, 8)$

18. $(-1, -4)$ and $(3, -4)$

19. $(-5, -2)$ and $(-1, -4)$

20. Sketch an increasing line, a decreasing line, a horizontal line, and a vertical line. For each line, determine whether the slope is positive, negative, zero, or undefined.

HW 1, 3, 7, 9, 13, 17, 21, 23, 25, 31, 39, 43, 53, 59, 63, 65, 67, 77, 83, 97

SECTION 7.2 TEACHING TIPS

This section begins with rate of change, which is intuitive and easy to motivate–there are so many applications! And the transition to slope is smooth because finding rates of change of linear models leads nicely into discussing rates of change of lines.

Rate of change and slope of a line are crucial topics for linear regression, which will be fully addressed in Section 9.3.

OBJECTIVE 1 COMMENTS

If you discussed the change of a quantity in Section 1.5 and unit ratios in Section 1.6, your students should be well prepared for this objective. If you didn't discuss these concepts then, spend some time discussing them now.

I tell students that 50 miles per hour is an example of a rate of change. I say a rate of change is a description of how quickly a quantity changes in relation to another quantity changing. I explain that we often use the word "per" to describe a rate of change. I give some more examples of rates of change such as

- A student earns \$8 per hour.
- The altitude of an airplane decreases by 2200 feet per minute.
- A college charges \$80 per unit. (You could use the charge per unit (hours or credits) at your college.)

Then I discuss the material in the lecture notes. When going over Problems 1–4, I emphasize that we should include units when computing changes in the explanatory variable and changes in the response variable. Including units throughout the calculations will help students write the correct units for their result. I emphasize that we divide the change in the response variable by the change in the explanatory variable. Students tend to ask me whether this is true before I mention it, which shows how adept they have become in thinking in terms of the explanatory and response variables.

GROUP EXPLORATION: Finding the Mean of Rates of Change

Although this activity does not address a crucial topic of the course, it does require some good critical thinking. Students tend to be intimidated by Problem 6; telling them it isn't as difficult as it first looks can help. A stronger hint would be to direct their attention to each of the denominators that equal 1. Students tend to do well on the other problems, but the challenge is for them to take in the main point of the exploration (Problem 7).

OBJECTIVE 2 COMMENTS

This objective ties rate of change to linear models, which is an important transition, but I'm usually tight on time so instead of having students work on all parts of Problems 5 and 6, I just complete Part 5(c) and point out that the data points lie on a straight line.

OBJECTIVE 3 COMMENTS

For Problems 7 and 8, I perform the same types of calculations as in Problems 2–4, but now I emphasize the graphical interpretation. In particular, the concepts rise and run are introduced.

OBJECTIVE 4 COMMENTS

To address this objective, I ask students why the rate of change in Problem 7 is positive and why the rate of change in Problem 8 is negative. Students are quick to explain.

In general, it is important to look for such opportunities to have students come up with an explanation for the next concept to be discussed. This increases the chances they will understand the material because they came up with the explanation rather than having passively received it.

OBJECTIVE 5 COMMENTS

For Problem 9, I find the slope by using the slope formula but also plot the points and label the rise and the run to keep students thinking about the connection between the computations and the rise and run.

For Problem 9, I also demonstrate that when we use the slope formula with two points on a line, it doesn't matter which point we choose to be (x_1, y_1) and which point we choose to be (x_2, y_2).

When some or all coordinates are negative (Problems 11–14), a common student error is to omit at least one subtraction symbol from the slope formula or negative sign from the coordinates. The suggestion in the lecture notes to first write the parentheses addresses this type of error.

GROUP EXPLORATION: For a Line, Rise over Run is Constant

This exploration has students discover that the slope of a line is independent of which pair of points on the line are chosen to calculate the slope. Most students won't understand this concept unless they take part in such an activity.

GROUP EXPLORATION: Graphical Significance of m for $y = mx$

This activity assists students in discovering the graphical significance of m for an equation of the form $y = mx$, which is a key concept of the course. After students complete the exploration, I facilitate a classroom discussion in which students share their responses for Problem 4 of the exploration. There is a lot of information in this exploration, and students have trouble seeing how it all fits together.

This exploration could be used before discussing Objectives 6–8 so students discover the concepts. However, one could argue that our earlier work with rate of change has already laid the groundwork for those objectives so this exploration should be used to sum things up at the end of the section. One difficulty with either placement is that the connection between slope and the coefficient of x for an equation of the form $y = mx$ will not be established until Section 7.3. So, you may prefer to use this exploration in that section.

OBJECTIVE 6 COMMENTS

This objective follows directly from the sign analysis of rate of change for Objective 4. To reinforce the concepts, I perform a sign analysis of rise and run as shown in each of Figs. 44 and 45 on pages 451 and 452 of the textbook. Or, if you prefer, you could show a similar sign analysis earlier when discussing rate of change.

A common error when computing the slope is for the sign to be incorrect. To check the sign, a student can quickly plot the points and determine whether the line is increasing or decreasing.

OBJECTIVE 7 COMMENTS

Even though this objective follows logically from the earlier taught fact that slope is a rate of change, I prefer to sketch two lines like in Fig. 46 on page 452 of the textbook and compare their slopes and steepness because this concept is so important.

OBJECTIVE 8 COMMENTS

After demonstrating that a horizontal line has a slope equal to zero, I point out that this matches with what we know about rate of change. Once students have seen how to show that a horizontal line has slope equal to zero, they can discover that a vertical line has undefined slope.

SECTION 7.3 LECTURE NOTES

Objectives

1. Use the slope and the y-intercept of a line to sketch the line.
2. Describe the meaning of m for an equation of the form $y = mx + b$.
3. Graph an equation of the form $y = mx + b$ by using the line's slope and y-intercept.
4. Find an equation of a line from its graph.
5. Graph an equation of a linear model by using the model's slope and vertical intercept.
6. Use a linear model's slope and vertical intercept to find its equation.

OBJECTIVE 1

Sketch the line that has the given slope and y-intercept.

1. $m = \frac{2}{5}, (0, -3)$ **2.** $m = -\frac{3}{2}, (0, 4)$ **3.** $m = -2, (0, -1)$

OBJECTIVE 2

4. **a.** Use the method discussed in Section 3.1 to graph $y = 2x + 1$.
b. What is the slope of the line $y = 2x + 1$?
c. Compare the slope with the number multiplied times x in the equation $y = 2x + 1$.

Finding the Slope and y-Intercept from a Linear Equation

For a linear equation of the form $y = mx + b$,

- the slope of the line is m and
- the y-intercept of the line is $(0, b)$.

We say this equation is in **slope-intercept form**.

The graph of the equation $y = -3x + 8$ is a line with slope -3 and y-intercept $(0, 8)$.

OBJECTIVE 3

Sketch the graph of the equation by hand.

5. $y = \frac{3}{5}x - 1$ **6.** $y = -\frac{4}{3}x + 5$ **7.** $y = 3x - 4$ **8.** $y = -2x - 3$

Graphing an Equation in Slope-Intercept Form

To graph an equation of the form $y = mx + b$,

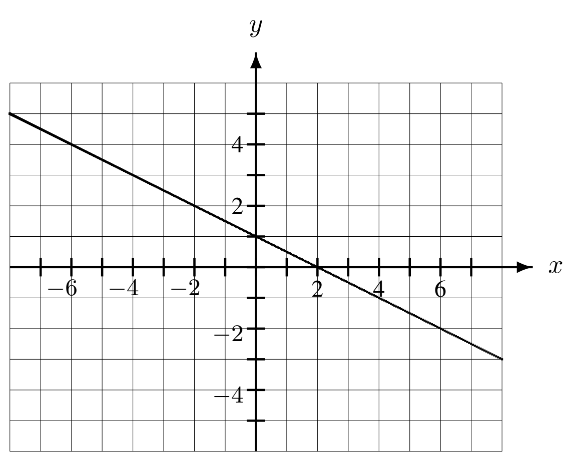
1. Plot the y-intercept $(0, b)$.
2. Use $m = \frac{\text{rise}}{\text{run}}$ to plot a second point. For example, if $m = \frac{2}{3}$, then count 3 units to the right (from the y-intercept) and 2 units up to plot another point.
3. Sketch the line that passes through the two plotted points.

9. Graph an equation of the form $y = mx + b$ where m is postive and b is negative.

OBJECTIVE 4

10. Find an equation of a line that has slope $-\frac{5}{7}$ and y-intercept $(0, 2)$.

11. Find an equation of the line sketched below.



Finding an Equation of a Line from a Graph

To find an equation of a line from a graph.

1. Determine the slope m and the y-intercept $(0, b)$ from the graph.
2. Substitute your values for m and b into the equation $y = mx + b$.

OBJECTIVE 5

12. Let p be the percentage of Americans who watch a greater number of TV programs on channels' websites (rather than on TV) at t years since 2005 (see the following table). A reasonable model of the situation is $p = 3.7t - 0.8$.

Year	Amount of Money (in billions of dollars)
2008	10
2009	14
2010	17
2011	21
2012	27
2013	27

Source: *GfK*

a. Graph the model by hand.

b. Predict in which year 36% of Americans will watch a greater number of TV programs on channels' websites. Did you perform interpolation or extrapolation? Do you have much faith in the prediction? Explain.

OBJECTIVE 6

13. A person earns a starting salary of $30 thousand at a company. Each year, she receives a $2 thousand raise. Let s be the person's salary (in thousands of dollars) after she has worked at the company for t years.

a. Is there an exact linear association between t and s? Explain.

b. Find the s-intercept of a linear model. What does it mean in this situation?

c. Find the slope of the linear model. What does it mean in this situation?

d. Find an equation of a linear model.

14. Let T be the total one-semester cost (in dollars) of tuition plus parking fee for u units (credits or hours) of classes at your college. [You could replace the parking fee with any one-time charge such as a student-services fee at your college.]

a. Is there an exact linear association between u and T? If so, find the slope.

b. Find an equation of the model.

c. Graph the model.

d. Select a total one-semester cost and predict the corresponding number of units.

15. Let s be U.S. retail sales (in billions of dollars) of vitamins and dietary supplements at t years since 2000 (see the following table). A reasonable model is $s = 1.23t + 8.33$.

Year	U.S. Retail Sales of Vitamins and Dietary Supplements (billions of dollars)
2007	17.1
2008	17.9
2009	19.3
2010	20.7
2011	22.1
2012	22.9

Source: *Euromonitor International*

a. Use technology to draw a scatterplot and the model in the same viewing window. Check whether the line comes close to the data points.

b. What is the slope of the model? What does it mean in this situation?

c. Find the rates of change in the number of retails sales from one year in the table to the next one listed. Compare the rates of change with the result you found in Part (b).

d. What is the s-intercept? What does it mean in this situation?

e. Use the model to estimate the retail sales in 2012. Did you perform interpolation or extrapolation? Compute the error.

HW 1, 3, 7, 11, 19, 29, 41, 43, 51, 55, 63, 65, 69, 73, 77, 81, 85, 97

SECTION 7.3 TEACHING TIPS

In this section, students use the slope and the y-intercept to graph equations in slope-intercept form. After students learn how to solve equations in Sections 8.2 and 8.3, they will graph linear equations not in slope-intercept form in Section 8.4. Students tend to do better with this organization, rather than having to contend with graphing linear equations of all forms in just one section.

OBJECTIVE 1 COMMENTS

Problems 1–3 provide the “big picture” of the section.

Some students tend to use the run and the rise starting from the origin rather than from the y-intercept. I remind them that the run and the rise have to do with going from one point *on* a line to another point on the line.

For Problem 2, some students are unsure what to do with the negative sign of the slope. They want to know whether they should work with the slope $\frac{-3}{2}$ or $\frac{3}{-2}$. I demonstrate that both ways give the same result; I emphasize that our main goal is to graph the line, not find two points on the line.

A common error is to write $-\frac{3}{2} = \frac{-3}{-2}$. I point out that this is incorrect because $-\frac{3}{2}$ is negative but $\frac{-3}{-2}$ is positive.

For Problem 3, some students need a reminder that $a = \frac{a}{1}$.

OBJECTIVE 2 COMMENTS

You can address this objective by completing Problem 4 or having groups of students work on the following exploration.

GROUP EXPLORATION: The Meaning of m in the Equation $y = mx + b$

This activity is a great way for students to discover that for an equation of the form $y = mx + b$, the slope of the equation is m. Many students will need help with Problem 4(e) of the exploration.

OBJECTIVE 3 COMMENTS

This objective follows nicely from objective 1.

GROUP EXPLORATION: Drawing Lines with Various Slopes

This activity is a fun way to drive home the point of the meaning of slope. Most students will use only integer values for the slope. I tell them to also try values between -1 and 1 (in addition to 0).

OBJECTIVE 4 COMMENTS

Students have an easy time with this objective. I point out that this task is the reverse of graphing. I remind my students about the Rule of Four for equations. I say we can go from any one of these four ways to another. (Students will learn how to go from tables to equations in Section 9.2.)

GROUP EXPLORATION: Finding an Equation of a Line from Its Graph

Students tend to have fun with this activity.

OBJECTIVE 5 COMMENTS

For Problem 12, I tell my students that they will learn to find linear models such as $p = 3.7t - 0.8$ in Section 9.2. I demonstrate how to use the method in this section to sketch the model $p = 3.7t - 0.8$, but I also point out that it is often useful to find a point far off to the right such as $(10, 36.2)$ by substituting 10 for t. Finding such a third point can help students sketch an accurate line, especially when the run of their first two chosen points is small compared to the largest value of the scaling on the horizontal axis.

OBJECTIVE 6 COMMENTS

Although students have found equations of linear models by recognizing patterns of arithmetic (Section 1.7), they will initially have trouble with finding such an equation using the concepts in this section.

For Problem 13, the first thing I do is use the starting salary of $30 thousand to plot the s-intercept on a coordinate system. Then I use the $2 thousand raise per year to draw the linear model. The graph helps clarify how to apply the concepts of Objective 4 to complete Parts (b), (c), and (d).

SECTION 7.4 DETAILED COMMENTS

Objectives

1. Describe the meanings of *relation*, *domain*, *range*, and *function*.
2. Identify functions by using the *vertical line test*.
3. Describe the meaning of *linear function*.
4. Describe the Rule of Four for functions.
5. Use the graph of a function to find the function's domain and range.
6. Use function notation.
7. Use function notation to make predictions.
8. Determine the domain and range of a model.

OBJECTIVE 1

Refer to the ordered pairs $(1, 4)$, $(2, 3)$, $(2, 5)$, and $(3, 1)$ to introduce the concepts *relation*, *domain*, *range*, *input*, and *output*.

> **Definition** *Relation, domain, and range*
>
> A **relation** is a set of ordered pairs. The **domain** of a relation is the set of all values of the explanatory variable, and the **range** of the relation is the set of all values of the response variable.

- Each member of the domain is an **input**.
- Each member of the range is an **output**.

> **Definition** *Function*
>
> A **function** is a relation in which each input leads to exactly one output.

Determine whether the given relation is a function.

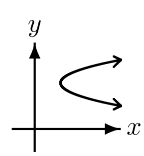
1. $y = x + 5$ **2.** $y = 2x$ **3.** $y = \pm x$

4. Some ordered pairs of four relations are listed in the following table. Which of these relations could be functions? Explain.

Relation 1		Relation 2		Relation 3		Relation 4	
x	y	x	y	x	y	x	y
0	13	2	20	5	3	7	1
1	18	3	18	6	3	7	2
2	23	3	16	7	3	7	3
3	28	4	14	8	3	7	4
4	33	5	12	9	3	7	5

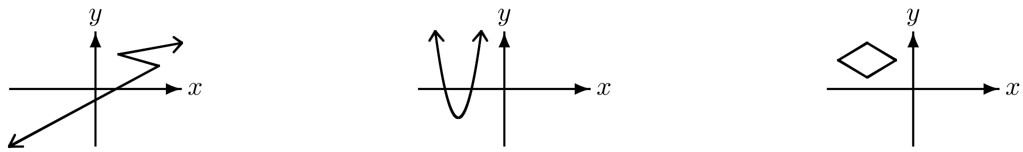
OBJECTIVE 2

Use arrows to show that there is an input that has two outputs for the relation graphed below. Conclude that the relation is not a function.



> **Vertical Line Test**
>
> A relation is a function if and only if each vertical line intersects the graph of the relation at no more than one point. We call this requirement the **vertical line test**.

Determine whether the following graph represents a function. Explain.



OBJECTIVE 3

> **Definition** *Linear Function*
>
> A **linear function** is a relation whose equation can be put into the form
>
> $$y = mx + b$$
>
> where m and b are constants.

Determine whether the given relation is a linear function.

5. $y = -\frac{2}{5}x + 3$ **6.** $x = 2$ **7.** $y = -4$ **8.** $x = y^2$

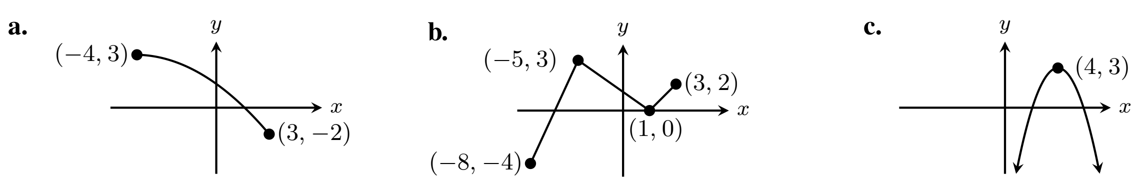
OBJECTIVE 4

> **Rule of Four for Functions**
>
> We can describe some or all of the input-output pairs of a function by means of
>
> **1.** an equation,
> **2.** a graph,
> **3.** a table, or
> **4.** words.

9. **a.** Is the relation $y = 3x - 2$ a function?
b. List some input-output pairs of $y = 3x - 2$ by using a table.
c. Describe the input-output pairs of $y = 3x - 2$ by using a graph.
d. Describe the input-output pairs of $y = 3x - 2$ by using words.

OBJECTIVE 5

10. Use the graph of the function to determine the function's domain and range.



OBJECTIVE 6

- To use "f" as the name of the function $y = 4x + 2$, we use "$f(x)$" to represent y: $y = f(x)$.

- We can substitute $f(x)$ for y in the equation $y = 4x + 2$: $f(x) = 4x + 2$.
- To find $f(3)$, we say we **evaluate** the function f at 3.
- Warning: "$f(x)$" does not mean f times x.

Evaluate $f(x) = 3x - 5$ at the given values of x.

11. $f(4)$ **12.** $f(0)$ **13.** $f\left(-\frac{1}{3}\right)$

For $f(x) = -2x + 6$, $g(x) = 3x^2 - 4x$, $h(x) = \frac{5x-4}{3x+2}$, and $k(x) = 5$, find the following:

14. $f(4)$ **15.** $g(2)$ **16.** $h(6)$ **17.** $k(3)$

18. Let $f(x) = -8.73x - 256$. Find x when $f(x) = 96.15$. Round your result to the second decimal place.

Some input-output pairs of a function f are shown in the following table.

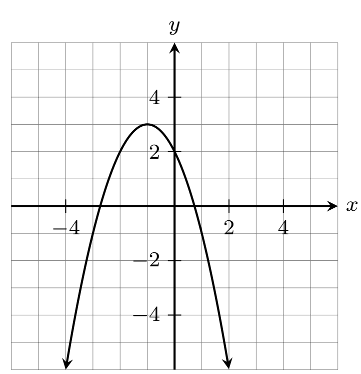
19. Find $f(3)$.

20. Find x when $f(x) = 3$.

x	y
1	4
2	3
3	2
4	3
5	4

21. A function f is graphed in the figure below.

a. Find $f(-1)$.
b. Find x when $f(x) = -1$.
c. Find the domain of f.
d. Find the range of f.



22. Recall that we can describe some or all of the input-output pairs of a function by means of an equation, a graph, a table, or words. Let $f(x) = -4x + 7$.

a. Describe five input-output pairs of f by using a table.

b. Describe the input-output pairs of f by using a graph.

c. Describe the input-output pairs of f by using words.

OBJECTIVE 7

Definition *Function notation*

The response variable of a function f can be represented by the expression formed by writing the explanatory variable name within the parentheses of $f(\ )$:

$$\text{response variable} = f(\text{explanatory variable})$$

23. Let T be the total fall semester cost (in dollars) of tuition and fees for part-time students who took C credits (units or hours) of courses at Centenary College (Source: *Centenary College*). The situation can be modeled by the equation $T = 575C + 15$.

a. Rewrite the equation $T = 575C + 15$ with the function name f.

b. Find $f(6)$. What does it mean in this situation?

24. The percentages of American adults who smoke are shown in the following table for various years.

Year	Percent
1970	37.4
1980	33.2
1990	25.3
2000	23.1
2010	19.4
2012	18.0

Source: *National Center for Health Statistics*

A model is $p = -0.45t + 36.83$, where p is the percentage of Americans who smoke at t years since 1970.

a. Verify that the graph of $p = -0.45t + 36.83$ comes close to the data points.

b. Rewrite the equation $p = -0.45t + 36.83$ with the function name g.

c. Estimate the percentage of American adults who smoked in 2005.

d. Find the p-intercept of the model. What does it mean in this situation?

25. Let p be the percentage of gambling revenue in Nevada that is from penny slot machines at t years since 2008. The percentage in 2008 was 14% and increased by 2.8 percentage points per year until 2013 (Source: *Nevada State Gaming Control Board*).

a. Find an equation of a model that describes this situation.

b. Rewrite your equation using the function name f.

c. Find $f(5)$. What does it mean in this situation?

26. The number of households with cable TV subscriptions was 61.8 million in 2009 and decreased by about 1.5 million per year until 2014 (Source: *IHS*). Let $n = g(t)$ be the number (in millions) of households with cable TV subscriptions at t years since 2009.

a. Find an equation of g.

b. Find $g(4)$. What does it mean in this situation?

OBJECTIVE 8

For the **domain** and **range** of a model, we consider input-output pairs only when both the input and output make sense in the situation.

27. A store is open from 9 A.M. to 5 P.M., Mondays through Saturdays. Let $I = f(t)$ be an employee's weekly income (in dollars) from working t hours each week at $10 per hour.

a. Find an equation of the model f.

b. Find the domain and range of the model f.

HW 1, 3, 5, 13, 19, 29, 33, 39, 47, 55, 57, 61, 65, 79, 81, 85, 89, 93, 95, 101

SECTION 7.4 TEACHING TIPS

Students learn about the concepts of relation and function in this section. Concepts related to functions will be needed throughout the rest of the textbook. Students tend to have an easy time with function concepts, although they struggle initially with finding the domain and range of a function. They also have growing pains learning about function notation, especially with confusing the instructions such as "Find $f(5)$" and "Find x when $f(x) = 5$.

OBJECTIVE 1 COMMENTS

To get across the concepts of relation and function, I like to draw lots of "relation machines" such as the ones in Figs. 102 and 103 on page 477 of the textbook as well as tables such as Tables 39 and 40 on pages 476 and 478 of the textbook. I make sure that students understand that a function may have two inputs that share the same output.

OBJECTIVE 2 COMMENTS

Before discussing the vertical line test, I use arrows similar to those in Fig. 105 on page 478 of the textbook to get students accustomed to thinking visually of inputs being sent to outputs. I find that even calculus students lack this essential perspective. I also use arrows with tables and their corresponding graphs to help students see the connection between tables and graphs in terms of inputs being sent to outputs.

Once students see how to draw such arrows with graphs, students are quick to identify graphs of functions. They can do this without having been told about the vertical line test. I prefer having students draw arrows, because the vertical line test strips away the key idea of a function. Once students have mastered using arrows to identify graphs of functions, I'll sometimes briefly discuss the vertical line test. Most students persist in drawing arrows on quizzes and exams, which I take to be a good sign.

GROUP EXPLORATION: Use Before Vertical Line Test

This activity nicely motivates the vertical line test. By completing the exploration, students will see not only how to use the vertical line test, but also *why* it works.

OBJECTIVE 3 COMMENTS

Students have an easy time with this objective.

OBJECTIVE 4 COMMENTS

There are exercises scattered throughout the textbook that require concepts related to the Rule of Four for functions. For example, see Exercises 25 and 26 of this section and see Exercises 59 and 60 of Homework 9.1.

OBJECTIVE 5 COMMENTS

To find the domain and range of a function from its graph, I draw arrows indicating about 10 inputs being sent to corresponding outputs (see Figures 112 and 113 on page 481 of the textbook). Most students will then be quick to pick up on the domain and range of the function. I find that students tend to have much more success with this approach than with other standard approaches.

For Problem 10(c), students will wonder whether the domain is really the set of all real numbers. It will help to explain the meaning of the two displayed arrows at the "ends" of the curve.

In the answers section, inequality notation (Section 1.1) is used to describe domains and ranges of functions. In the second edition, I will likely add interval notation (Section 1.1) to these answers.

OBJECTIVE 6 COMMENTS

Many students wonder why we would want to represent "y" by the more complicated notation "$f(x)$." I emphasize that functional notation allows us to conveniently name a function. I explain that being able to name a function is especially helpful when working with two models, such as the model described in Problem 25 and the model that describes the percentage of gambling revenue in Nevada that is from blackjack at t years since 2008.

Students tend to be reassured when I tell them that $f(x)$ simply stands for y. Later (when discussing Objective 7) I explain that, in general, response variable $= f($explanatory variable$)$.

I spend a good chunk of time discussing how to find values of x or $f(x)$, where a function f is described by a table, a graph, or an equation. I also tie in function notation with function-machine figures such as the one in Fig. 114 on page 482 of the textbook. Students need to view functions from multiple perspectives to really understand them.

Many students think that the notation $f(4)$ means that y is 4 or that the equation $f(x) = 4$ means that x is 4. Comparing the work for Problem 19 with the work for Problem 20 and comparing the work for Problem 21(a) with the work for Problem 21(b) should help address these issues.

GROUP EXPLORATION: Formula for Slope

This activity does not address a crucial concept of the course, but it does shed a different light on the slope formula.

OBJECTIVE 7 COMMENTS

Students deal with function notation well in Problems 23–26, although some students tend to continue to struggle to find equations of models in Problems 25(a) and 26(a). They will be challenged by function notation to a much larger degree when they must distinguish between evaluating a function and solving for an explanatory variable in Sections 8.3, 9.2, and 9.3.

OBJECTIVE 8 COMMENTS

Due to time limitations, I usually skip or give light treatment to this objective. In most situations, it's very difficult to determine a precise domain and, hence, range of a model. It's far more important to get across the meanings of interpolation and extrapolation and why we have little or no faith when we extrapolate.

CHAPTER 8 OVERVIEW

In this chapter, students will simplify linear expressions and solve linear equations in one variable, formulas, and linear inequalities. The skill of solving equations will prepare students to find equations of lines and linear models in Sections 9.1 and 9.2. This skill will also prepare students to learn the derivations of sample-size formulas in introductory statistics. The skill of solving inequalities will prepare students to learn about the derivation of confidence intervals in introductory statistics.

One could argue that Section 8.4 (solving formulas) is the most important section because students will work with so many formulas in introductory statistics.

Students tend to be greatly challenged by Sections 8.3 (solving more complicated linear equations and linear modeling) and 8.4 (solving formulas), so be sure to allocate enough time for these sections.

SECTION 8.1 LECTURE NOTES

Objectives

1. Describe the *commutative law*.
2. Describe the *associative law*.
3. Describe the *distributive law*.
4. Describe the meaning of *equivalent expressions*.
5. *Combine like terms.*
6. *Simplify expressions.*

OBJECTIVE 1

The equations $3 + 7 = 7 + 3$ and $3 \cdot 7 = 7 \cdot 3$ suggest the following laws.

Commutative Laws for Addition and Multiplication

Commutative law for addition: $a + b = b + a$
Commutative law for multiplication: $ab = ba$

Use the commutative law for addition to write the expression in another form.

1. $5 + x$ **2.** $2 + 7k$

Use the commutative law for multiplication to write the expression in another form.

3. xy **4.** $-3c$ **5.** $x \cdot 5 + 9$

- There is no commutative law for subtraction.
- A **term** is a constant, variable, or a product of a constant and one or more variables raised to powers.
- Here are some terms: $6x$, -5, w, $-3pt$, $\frac{4y^5}{w^2}$.
- **Variable terms** are terms that contain variables.

- **Constant terms** are terms that do not contain variables.
- We usually write a sum of both variable and constant terms with the variable terms to the left of the constant terms.

6. Write $7 - 3x$ so that the variable term is to the left of the constant term.

OBJECTIVE 2

The equations $(3 + 2) + 5 = 3 + (2 + 5)$ and $(3 \cdot 2) \cdot 5 = 3 \cdot (2 \cdot 5)$ suggest the following laws.

Associative Laws for Addition and Multiplication

Associative law for addition: $a + (b + c) = (a + b) + c$
Associative law for multiplication: $a(bc) = (ab)c$

Use an associative law to write the expression in another form.

7. $(x + 4) + y$ **8.** $b + (5 + c)$ **9.** $8(xy)$ **10.** $(kp)w$

11. Rearrange the terms of $7 - 5x + 3 - 9$ so that the numbers can be added.

12. Use the commutative and associative laws to remove the parentheses. Also, multiply and add numbers when possible.

a. $-2(4p)$
b. $(3 + 5x) + 7$

OBJECTIVE 3

The equation $3(2 + 4) = 3 \cdot 2 + 3 \cdot 4$ suggests the following law.

Distributive Law

$$a(b + c) = ab + ac$$

Find the product.

13. $2(x + 3)$ **15.** $7(w - 5)$ **17.** $(x - 9)(2)$ **19.** $3(2p - 4t + 6)$
14. $4(2x + 8)$ **16.** $3(4x - 7y)$ **18.** $-4(7 - x)$

- In applying the distributive law to an expression such as $2(x + 3)$, remember to distribute the 2 to *every* term in the parentheses.
- Avoid confusing the associative law with the distributive law. We *cannot* apply the distributive law to the expression $a(bc)$ to get $(ab)(ac)$.

Multiplying a Number by -1

$$-1a = -a$$

Remove the parentheses.

20. $-(x+5)$

21. $-(8x-3y+6)$

OBJECTIVE 4

22. Evaluate both of the expressions $2(x+3)$ and $2x+6$ for the values $x=0$, $x=1$, $x=2$, $x=3$, $x=4$, and $x=5$. (After performing a couple of evaluations by hand, use technology.)

> **Definition** *Equivalent expressions*
>
> Two or more expressions are **equivalent expressions** if, when each variable is evaluated for *any* real number (for which all the expressions are defined), the expressions all give equal results.

OBJECTIVE 5

- The **coefficient** of a term is the constant factor of the term.
- **Like terms** are either constant terms or variable terms that contain the same variable(s) raised to exactly the same power(s).
- If terms are not like terms, we say that they are **unlike terms**.
- When we write a sum or difference of like terms as one term, we say we have **combined like terms**.

Combine like terms.

23. $3x+5x$

24. $9w-6w$

25. $4b+b$

> **Combining Like Terms**
>
> To combine like terms, add the coefficients of the terms and keep the same variable factors.

OBJECTIVE 6

- We **simplify an expression** by removing parentheses and combining like terms.
- The result of simplifying an expression is a **simplified expression**, which is equivalent to the original expression.

Simplify.

26. $5x-2y+3x+4-6y-1$

27. $7-5(b+4)$

28. $-4(2w-5y)-2(w+y)$

29. $3(k-5)-(k+2)$

30. Let x be a number. Translate the phrase "3, minus 2 times the difference of twice the number and 5" to an expression. Then simplify the expression.

31. Let x be a number. Translate the expression $3x-2(x-5)$ into an English phrase. Then simplify the expression.

HW 1, 3, 7, 11, 17, 27, 37, 47, 55, 63, 67, 69, 75, 83, 91, 97, 107

SECTION 8.1 TEACHING TIPS

In this section, students use the commutative, associative, and distributive laws to simplify expressions. These concepts will be used throughout the rest of the course. Most students will have an easy time with this section.

OBJECTIVES 1–3 COMMENTS

When I am tight on time, I give light treatment to the commutative and associative laws and emphasize the distributive law. Either from past experiences with algebra or by way of intuition, students seem to know how to apply the commutative and associative laws to simplify expressions. The main thing that students need to know in relation to the commutative and associative laws is that they can rearrange terms.

Students tend to have difficulty applying the distributive law when the expression includes negative coefficients and/or constant terms.

For expressions such as the ones in Problems 20 and 21, most textbooks suggest to change the signs of the terms in the parentheses, but students don't seem to understand this technique. Plus, it is not clear to students how that technique is tied to the laws of operations, whereas students can see that the technique that uses the fact $-1a = -a$ is tied to the distributive law. When using this technique, I point out that the distributive law says that we can distribute a number such as -1, not an opposite symbol.

GROUP EXPLORATION: Laws of Operations

This activity has students discover the three laws of operations discussed in this section. Students have an easy time with this exploration.

OBJECTIVE 4 COMMENTS

For Problem 22, I emphasize that when we simplify $2(x+3)$ and write $2(x+3) = 2x+6$ we mean that the expressions $2(x+3)$ and $2x+6$ are equivalent. Most students think of simplifying an expression as a task to complete; they do not know that the result and the original expression are equivalent.

OBJECTIVE 5 COMMENTS

To introduce how to combine like terms for $2x+3x$, I use the distributive law to write $2x+3x = (2+3)x = 5x$, but I also demonstrate that $2x+3x = (x+x)+(x+x+x) = x+x+x+x+x = 5x$. The latter technique might be the clearer way for students to see that $4b+b = 5b$ (Problem 25).

OBJECTIVE 6 COMMENTS

In Problem 27, for the first step in trying to simplify $7-5(b+4)$, some students mistakenly write $2(b+4)$. To point out their error, I remind them of the order of operations.

It is with more complicated expressions like those in Problems 27–29, that students are more likely to forget to distribute a factor to all terms inside parentheses.

GROUP EXPLORATION: Simplifying Expressions

This activity essentially warns groups of students of three types of common student errors.

SECTION 8.2 LECTURE NOTES

Objectives

1. Describe the meaning of *linear equation in one variable*.
2. For an equation in one variable, describe *satisfy*, *solution*, *solution set*, and *solve*.
3. Describe *equivalent equations*.
4. Describe the *addition property of equality*.
5. Describe the *multiplication property of equality*.
6. Solve a linear equation in one variable.
7. Solve a percentage problem.
8. Use graphing to solve a linear equation in one variable.
9. Use a table to solve a linear equation in one variable.

OBJECTIVE 1

Definition *Linear equation in one variable*

A **linear equation in one variable** is an equation that can be put into the form

$$mx + b = 0$$

where m and b are constants and $m \neq 0$.

OBJECTIVE 2

1. Show that the equation $x + 3 = 7$ becomes a true statement if 4 is substituted for x.

Definition *Solution, satisfy, solution set,* and *solve* for an equation in one variable

A number is a **solution** of an equation in one variable if the equation becomes a true statement when the number is substituted for the variable. We say the number **satisfies** the equation. The set of all solutions of the equation is called the **solution set** of the equation. We **solve** the equation by finding its solution set.

2. Is 2 a solution of the equation $2 - 4x = 3(x - 4)$?
3. Is 5 a solution of the equation $2 - 4x = 3(x - 4)$?

OBJECTIVE 3

4. Show that the following equations all have the same solution set: $x = 6$, $x + 2 = 6 + 2$, and $x + 2 = 8$.

Equivalent equations are equations that have the same solution set.

OBJECTIVE 4

Addition Property of Equality

If A and B are expressions and c is a number, then the equations $A = B$ and $A + c = B + c$ are equivalent.

5. Solve $x - 4 = 2$.

- After solving an equation, check that all of your results satisfy the equation.
- Our strategy in solving linear equations in one variable will be to use properties to get the variable alone on one side of the equation.

Solve.

6. $x - 1.5 = 8.2$ **7.** $x + 3 = 7$ **8.** $b + 5 = 0$

OBJECTIVE 5

9. Show that the following equations all have the same solution set: $x = 4$, $3 \cdot x = 3 \cdot 4$, and $3x = 12$.

Multiplication Property of Equality

If A and B are expressions and c is a nonzero number, then the equations $A = B$ and $Ac = Bc$ are equivalent.

OBJECTIVE 6

Solve.

10. $\frac{5}{2}x = 3$

11. $-15 = -5x$

12. $-t = 7$

13. $\frac{5}{2}w = \frac{3}{4}$

14. $3x - 5 = 7$

15. $2w + 4 - 5w = 19$

16. $-x + 2 = 3x - 18$

OBJECTIVE 7

17. In a survey, 70% of adults answered yes to the following question (Source: *The Gallup Organization*): "When a person has an incurable disease, should doctors be allowed to help end a patient's life by some painless means if the patient requests it?" If the number of people in the survey who said this is 717, what is the total number of adults surveyed?

OBJECTIVE 8

Using Graphing to Solve an Equation in One Variable

To use graphing to solve an equation $A = B$ in one variable, x, where A and B are expressions,

1. Graph the equations $y = A$ and $y = B$ on the same coordinate system. (For example, if the original equation is $2x + 5 = 4x - 9$, then we would graph the equations $y = 2x + 5$ and $y = 4x - 9$.)
2. Find all intersection points.
3. The x-coordinates of those intersection points are the solutions of the equation $A = B$.

For Problems 18 and 19, graph $y = \frac{3}{5}x + 1$ by hand. Use the graph to solve the given equation.

18. $\frac{3}{5}x + 1 = 4$

19. $\frac{3}{5}x + 1 = -2$

20. Use "intersect" on a TI-84 to solve $\frac{2}{3}x + \frac{5}{6} = \frac{3}{2}x - \frac{5}{3}$.

OBJECTIVE 9

21. Use a table of values of $y = 4x - 3$ to solve $4x - 3 = 5$.

HW 1, 3, 7, 11, 27, 39, 41, 55, 59, 65, 69, 71, 75, 79, 93, 95, 111

SECTION 8.2 TEACHING TIPS

In this section, students use symbolic methods, graphing, and tables to solve linear equations in one variable. They will use this material throughout the rest of Chapter 8 as well as Chapter 9.

OBJECTIVE 1 COMMENTS

I remind my students that in Chapter 3 we graphed equations in two variables. I say we will now work with linear equations in *one* variable. I provide a few examples and then define *linear equation in one variable*.

Another option is to introduce the definition after you have solved a few equations so that students know what "put into the form" means.

OBJECTIVE 2 COMMENTS

For Problem 1, I write $x + 3 = 7$ on the board and ask students what x can be. Students are quick to say x is equal to 4. I say 4 is called a *solution* and that it *satisfies* the equation. Then I define the words in the boxed statement and say these words mean about the same thing when applied to equations in two variables.

OBJECTIVE 3 COMMENTS

This objective is setting the stage for Objective 5.

OBJECTIVE 4 COMMENTS

For the addition property of equality, most students think that the entire point is that "we can add a number to both sides of an equation." Most students do not understand that a solution of an equation will satisfy *all* of the equations that lead to finding the solution. At various times in the course, I return to this topic to deepen students' understanding.

For Problem 5, I say our objective in solving an equation is to isolate x on one side of the equation.

For Problem 7, I point out that we can still use the addition property of equality, because subtracting 3 from both sides of the equation is the same as adding -3 to both sides.

GROUP EXPLORATION: Locating an Error in Solving an Equation
This activity is a good reminder of the meaning of equivalent equations.

OBJECTIVE 5 COMMENTS
Students are quick to pick up on this concept because it is so similar to the addition property of equality.

OBJECTIVE 6 COMMENTS
For Problem 10, I first show that the product of a fraction and its reciprocal is equal to 1. Then I use the multiplication property of equality to solve the equation $\frac{5}{2}x = 3$.

For Problem 11, I point out that we can still use the multiplication property of equality, because dividing both sides by -5 is the same as multiplying both sides by $-\frac{1}{5}$.

When solving $-t = 7$ (Problem 12), I show students how to solve the equation by dividing both sides of the equation by -1 as well as by multiplying both sides by -1. Almost all students prefer dividing by -1 rather than multiplying by -1.

For Problem 16, I ask students why the equation is different than the ones in Problems 10–15. When they say the equation has variable terms on both sides of the equation, I say that a good first step is to get the variables terms on just one side of the equation.

OBJECTIVE 7 COMMENTS
Perhaps the hardest part of Problem 17 is realizing that it helps to define a variable (for the number of adults in the survey). Such percentage problems are good preparation for more challenging ones in Section 8.3.

OBJECTIVE 8 COMMENTS
A discussion about inputs and outputs can be used to explain how to use graphing to solve an equation in one variable. Nonetheless, most students will be challenged by such an explanation.

For Problem 20, I point out the advantages of using technology to solve such an equation. It is important that students learn symbolic methods of the course, but it is also important that students learn appropriate uses of technology.

OBJECTIVE 9 COMMENTS
Although students are quick to pick up on how to solve problems such as Problem 21, they might not really understand what is going on. A discussion about inputs and outputs can help.

SECTION 8.3 LECTURE NOTES

Objectives

1. Use combinations of properties of equality to solve a linear equation in one variable.
2. Compare the meanings of *simplify an expression* and *solve an equation*.
3. Use graphing to solve a linear equation in one variable.
4. Use two tables to solve a linear equation in one variable.
5. Find the input of a linear function for a given output.
6. Make estimates and predictions by solving linear equations.

OBJECTIVE 1

Solve.

1. $5 - 4(2x - 3) = 3(4 - x)$

2. $2(3x + 4) - (4x - 1) = 3(x + 2)$

3. $\frac{5}{6}k - \frac{3}{2} = \frac{7}{3}$

4. $\frac{4w - 7}{3} = \frac{2w + 1}{4}$

- It is wise to check that a result (or the results) of solving an equation does indeed satisfy the equation.
- A key step in solving an equation that contains fractions is to multiply both sides of the equation by the LCD so that there are no fractions on either side of the equation.

5. Solve $1.96 = \frac{x - 21.9}{3.5}$. Round any solutions to two decimal places.

OBJECTIVE 2

6. Simplify $\frac{3}{8}x + \frac{7}{6} - \frac{7}{4}x$.

7. Solve $\frac{3}{8}x + \frac{7}{6} = \frac{7}{4}x$.

8. Compare the first steps of your work in Problems 6 and 7.

It is incorrect to say "solve an expression." It is incorrect to say "simplify an equation."

Results of Simplifying an Expression and Solving an Equation

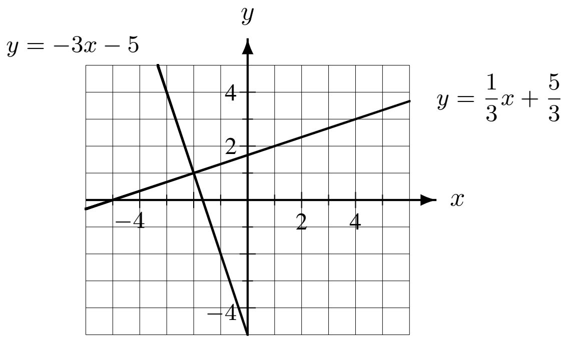
The result of simplifying an expression is an expression. The result of solving a linear equation in one variable is a number.

Multiplying Expressions and Both Sides of Equations by Numbers

When simplifying an expression, the only number that we can multiply the expression by or part of it by is 1. When solving an equation, we can multiply both sides of the equation by *any* number except 0.

OBJECTIVE 3

For Problems 9 and 10, solve the given equation by referring to the graphs of $y = \frac{1}{3}x + \frac{5}{3}$ and $y = -3x - 5$ shown below.



9. $\frac{1}{3}x + \frac{5}{3} = -3x - 5$

10. $\frac{1}{3}x + \frac{5}{3} = 2$

OBJECTIVE 4

11. Use tables to solve the equation $3x - 2 = 6 - x$. [The solution is 2.]

OBJECTIVE 5

Let $f(x) = \frac{3}{4}x - 2$.

12. Find $f(8)$.

13. Find x when $f(x) = 8$.

Note that in Problem 12, we are to find a value of y, but in Problem 13, we are to find a value of x.

OBJECTIVE 6

14. The Total U.S. debit card purchases was $1.6 trillion in 2010 and has increased by $0.13 trillion per year in 2014 (Source: *The Nilson Report*).

- **a.** Find a model of the situation. Explain what your variables represent.
- **b.** Estimate total U.S. debit card purchases in 2014.
- **c.** Estimate when the total annual U.S. debit card purchases was $2.0 trillion.

Using an Equation of a Model to Make Predictions

- When making a prediction about the response variable of a linear model, substitute a chosen value for the explanatory variable in the model. Then solve for the response variable.
- When making a prediction about the explanatory variable of a linear model, substitute a chosen value for the response variable in the model. Then solve for the explanatory variable.

15. The number of tobacco farms was 16,228 farms in 2007, and it decreased by about 1495 farms per year until 2015 (Source: *U.S. Department of Agriculture National Agricultural Statistics*). Estimate when there were 6000 tobacco farms.

16. In 2013, the student-teacher ratio in U.S. public schools was 15.1, down 5.6% from 2000 (Source: *National Center for Education Statistics*). What was the ratio in 2000?

17. Many appliances such as televisions and electronic gadgets such as cell phones contain rare Earth metals. Annual U.S. electronics and appliance stores sales and annual worldwide production of rare Earth metals are shown in the following table for the period 1992–2012.

Annual Electronics and Appliance Store Sales (billions of dollars)	Annual Worldwide Production of Rare Earth Metals (thousand metric tons)	Annual Electronics and Appliance Store Sales (billions of dollars)	Annual Worldwide Production of Rare Earth Metals (thousand metric tons)
42.631	50.1	86.689	97.1
48.614	46.7	94.416	102
57.266	55.1	101.340	122
64.770	74.3	107.989	137
68.363	79.7	110.673	124
70.061	68.3	108.663	132
74.527	77.1	98.030	135
78.977	86.6	99.128	126
82.206	90.9	100.792	113
80.240	94.5	102.998	110
83.740	98.2		

Source: *U.S. Geological Survey*

Let $p = f(s)$ be the annual worldwide production (in thousand of metric tons) of rare Earth metals, and let s be the annual U.S. electronics and appliance stores sales (in billions of dollars) in the same year.

a. Construct a scatterplot.

b. Describe the four characteristics of the association.

c. Graph the model $f(s) = 1.34s - 16.31$ in the scatterplot. Does it come close to the data points?

d. Find $f(100)$. What does it mean in this situation?

e. Find s when $f(s) = 100$. What does it mean in this situation?

HW 1, 3, 5, 9, 15, 21, 31, 35, 41, 43, 51, 71, 77, 81, 85, 91, 95, 101, 107

SECTION 8.3 TEACHING TIPS

The key idea of this section is solving linear equations and using this skill with linear models to make predictions. Most students are greatly challenged by this section due to solving equations with fractions, finding equation of linear models, and working with function notation.

OBJECTIVE 1 COMMENTS

Most students are challenged by Problems 3 and 4. Some students try to write all of the fractions with a common denominator, which is not incorrect but is inefficient, and usually such students make other errors along the way. Some students have trouble finding the LCD. Other students have trouble simplifying both sides of the equation after multiplying both sides of the equation by the LCD.

GROUP EXPLORATION: Any Linear Equation in One Variable Has Exactly One Solution

This activity not only has students determine the number of solutions of a linear equation but also has them work with a formula, which is good warm-up for Section 8.4 (solving formulas).

OBJECTIVE 2 COMMENTS

Students usually have no trouble distinguishing when to apply simplifying techniques and when to apply solving techniques for linear expressions and equations, respectively, except when it comes to those with fractional coefficients and constant terms.

OBJECTIVES 3 and 4 COMMENTS

Students usually have no trouble with these two objectives because they build on similar objectives addressed in Section 8.2.

OBJECTIVE 5 COMMENTS

For Problem 13, students tend to substitute for 8 for x rather than for $f(x)$. Drawing a distinction between problems similar to Problems 12 and 13 several times in the next few days is a good idea because students tend to forget.

OBJECTIVE 6 COMMENTS

For Problem 14(a), students tend to need a reminder about how to find such a linear model (Section 7.3). This part poses an extra challenge because students must define the relevant variables.

For Problem 16, students tend to think that what's required is to calculate 5.6% of the ratio in 2013 (15.1). Encourage your students to take the necessary time to understand the situation before trying to perform calculations or solve equations.

Problem 17 serves as a good reminder of the four characteristics of an association. It also helps students see how the current material fits into the big picture of linear modeling.

SECTION 8.4 LECTURE NOTES

Objectives

1. Find a quantity by substituting values for all but one variable in a formula and then solving for the remaining variable.
2. Find a quantity by using a formula with summation notation.
3. Solve a formula for one of its variables.
4. Solve a formula with a square root.
5. Graph a linear equation in two variables by solving for y.

OBJECTIVE 1

Recall that a **formula** is an equation that contains two or more variables (Section 4.1).

1. At Apple, 7% of employees are African American and 18% of employees are African American OR Hispanic (Source: *Apple*). Find the probability of randomly selecting an Apple employee who is Hispanic.

To find a single value of a variable in a formula, we often substitute numbers for all of the other variables and then solve for the remaining variable.

2. Recall that the scores on the Wechsler IQ test are normally distributed with mean 100 points and standard deviation 15 points. Actress Sharon Stone is reported to have an IQ of 154 points (Source: *Chicago Tribune*). Use the formula $x = \bar{x} + zs$ to find the z-score for an IQ of 154 points.

OBJECTIVE 2

3. Substitute the following values in the formula $\mu = \sum x_i P(x_i)$ and solve for the remaining variable:

$$x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 3,\ x_5 = 4,$$
$$P(x_1) = 0.0625,\ P(x_2) = 0.25,\ P(x_3) = 0.375,\ P(x_4) = 0.25,\ P(x_5) = 0.0625$$

4. Substitute the following values in the formula $\text{MSE} = \dfrac{\sum[(n_i - 1)s_i^2]}{n-k}$ and solve for the remaining variable:

$$n_1 = 22,\ n_2 = 28,\ n_3 = 21,\ s_1 = 2,\ s_2 = 5,\ s_3 = 4,\ n = 71, \text{ and } k = 3$$

Round the result to the first decimal place.

OBJECTIVE 3

5. **a.** Solve the formula $P(A \text{ OR } H) = P(A) + P(H)$ for $P(H)$.

b. Substitute 0.18 for $P(A \text{ OR } H)$ and 0.07 for $P(A)$ in the formula found in Problem 1.

Solving for a variable in a formula will not change the association between the variables in the formula.

6. **a.** Solve the formula $x = \bar{x} + zs$ for z.

b. For students who graduated from high school in 2013, their SAT math scores were approximately normally distributed with a mean of 514 points and a standard deviation of 118 points (Source: *College Board*). Use the formula $x = \bar{x} + zs$ or the formula you found in Part (a) to find the z-scores for the following students' SAT scores (all in points): 621, 457, 555, 748, 242, and 562.

c. Which of the students' scores given in Part (b) is most unusual? Explain.

To find several values of a variable in a formula, we usually solve the formula for that variable before we make any substitutions.

7. Let F be the Fahrenheit reading corresponding to a Celsius reading of C degrees. A formula that describes the association between F and C is $C = \dfrac{5}{9}(F - 32)$.

a. Solve the Fahrenheit-Celsius formula for F.

b. Convert 10°C to the equivalent Fahrenheit temperature.

c. Use technology to convert 10°C, 15°C, 20°C, 25°C, and 30°C to the equivalent Fahrenheit temperatures.

8. Solve $u_r = \dfrac{2n_1n_2}{n} + 1$ for n_1.

OBJECTIVE 4

Squaring a Principal Square Root

If x is nonnegative, then

$$\left(\sqrt{x}\right)^2 = x$$

9. In introductory statistics, you will learn about a *margin of error formula* $E = z \cdot \dfrac{s}{\sqrt{n}}$, where E has to do with the error in making a certain type of estimate. Solve the formula for n, which is the number of individuals in a random sample.

OBJECTIVE 5

Determine the slope and the y-intercept. Use the slope and the y-intercept to graph the equation by hand.

10. $3x + 5y = 10$ **11.** $2(x - 2y) + 1 = 9$ **12.** $5 - 2(3x - y) = 1 - 3(x - 2y)$

HW 1, 3, 5, 13, 23, 29, 41, 43, 47, 61, 63, 65, 75, 87, 95, 101

SECTION 8.4 TEACHING TIPS

In this section, students will solve formulas and use this skill to graph linear equations in which y is not isolated on one side of the equation. The skill of solving formulas can be used in Sections 9.1 and 10.4, but it is not essential. The skill is important in introductory statistics for deriving sample-size and z-score formulas.

This section serves as a nice reminder of past formulas used. It will also enhance students' familiarity with summation notation (Objective 2).

OBJECTIVE 1 COMMENTS

Some textbooks seem to suggest that we always want to first solve a formula for a variable before making substitutions; this hardly makes sense if we want to find only one value of a quantity.

OBJECTIVE 2 COMMENTS

Although students have been exposed to summation notation when the formulas for the mean and the standard deviation were introduced, they tend to memorize the procedures without really understanding the meaning of summation notation. This objective will force students to sort out its meaning.

OBJECTIVE 3 COMMENTS

Most students have a tough time with Problems 5–8. It can help to solve each formula alongside a similar equation in one variable. For example, see the solutions of Examples 5 and 6 on pages 547 and 548, respectively, of the textbook.

Note that Problem 7(a) is essentially having students find the inverse function of the model $C = \frac{5}{9}(F - 32)$. When completing Part (c), I point out that it is advantageous to first solve for F before substituting the five values for C; if we substituted before solving, we'd have five equations to solve. I say that first solving for F would become a huge advantage if we wanted to make hundreds of substitutions for C.

GROUP EXPLORATION: Determining when It's Better to Solve a Formula for a Variable

This is a lengthy activity, but it might be the best way to drive home the value of solving a formula before substituting values for the variables because it's one thing to tell students how this practice saves time but it's more convincing for them to *experience* it.

OBJECTIVE 4 COMMENTS

The textbook gives light treatment to this objective (Example 10 and Exercises 53 and 54), but the objective has been included to give students a bit of exposure so they'll have a better chance of following derivations of sample-size formulas in introductory statistics.

OBJECTIVE 5 COMMENTS

This objective is not needed in introductory statistics because linear regression equations are already in the form $\hat{y} = mx + b$. But some reviewers wanted this objective included in the textbook so I have done so. Perhaps the best reason for teaching this material is that it provides a review of graphing (Section 7.3).

SECTION 8.5 LECTURE NOTES

Objectives

1. Describe the *addition property of inequalities.*
2. Describe the *multiplication property of inequalities*.
3. Describe the meaning of *satisfy*, *solution*, and *solution set* for a *linear inequality in one variable*.
4. Solve a linear inequality in one variable, and graph the solution set.
5. Substitute values for variables in a compound inequality.
6. Solve a *compound inequality in one variable*, and graph the solution set.
7. Use linear inequalities to make predictions about authentic situations.

OBJECTIVE 1

Show what happens when we add a number to both sides of $5 < 7$.

Addition Property of Inequalities

If $a < b$, then $a + c < b + c$.

Similar properties hold for $\leq$, $>$, and $\geq$.

Illustrate the addition property of inequalities by using a number line.

OBJECTIVE 2

Show what happens when we multiply both sides of the inequality $5 < 7$ by a number.

Multiplication Property of Inequalities

- For a *positive* number c, if $a < b$, then $ac < bc$.
- For a *negative* number c, if $a < b$, then $ac > bc$.

Similar properties hold for $\leq$, $>$, and $\geq$.

Illustrate that if $a < b$, then $-a > -b$ by using a number line.

OBJECTIVE 3

Definition *Linear inequality in one variable*

A **linear inequality in one variable** is an inequality that can be put into one of the forms

$$mx + b < 0, \qquad mx + b \leq 0, \qquad mx + b > 0, \qquad mx + b \geq 0$$

where m and b are constants and $m \neq 0$.

We say a number **satisfies** an inequality in one variable if the inequality becomes a true statement after we have substituted the number for the variable.

1. Does the number 2 satisfy the inequality $4x - 3 < 8$?

2. Does the number 5 satisfy the inequality $4x - 3 < 8$?

Definition *Solution, solution set,* and *solve* for an inequality in one variable

We say a number is a **solution** of an inequality in one variable if it satisfies the inequality. The **solution set** of an inequality is the set of all solutions of the inequality. We **solve** an inequality by finding its solution set.

OBJECTIVE 4

Solve the inequality. Describe the solution set as an inequality, in interval notation, and as a graph.

3. $3x - 5 \le 7$

4. $-4x < -12$

5. $4x < -12$

6. $-3x + 1 < -11$

7. $3k + 3 \ge 8k - 4$

8. $-2.9x + 3.2 < -8.98$

9. $4m + 3 > 2(3m - 4)$

10. $\frac{2}{9} - \frac{5}{3}x \le \frac{7}{3}$

11. $\frac{2p-1}{3} - \frac{7p+3}{4} \ge \frac{5}{6}$

OBJECTIVE 5

12. Substitute 35.2 for $\bar{x}$, 1.761 for t, 4.7 for s, and 14 for n in the compound inequality

$$\bar{x} - t \cdot \frac{s}{\sqrt{n}} < \mu < \bar{x} + t \cdot \frac{s}{\sqrt{n}}$$

to find a compound inequality that describes the variable μ. Also describe the solution set in interval notation and as a graph. Round to the first decimal place.

OBJECTIVE 6

Solve the inequality. Describe the solution set as an inequality, in interval notation, and in a graph.

13. $-1 \le 3x - 7 \le 5$

14. $2 < 8 - 2w < 10$

15. $\frac{1}{2} < 3 - \frac{5}{4}x \le \frac{7}{2}$

OBJECTIVE 7

16. Let B be the total annual box office grosses (in billions of dollars) in the United States and Canada at t years since 1980 (see the following table).

Year	Total Box Office Grosses in the United States and Canada (billions of dollars)
1987	4.25
1990	5.02
1995	5.27
2000	7.51
2005	8.82
2010	10.58

Sources: *AC Nielsen EDI, Rentrak Corporation*

A reasonable model is $B = 0.28t + 1.98$. In which years was the annual box office gross greater than \$9.5 billion?

HW 1, 3, 7, 17, 37, 47, 55, 59, 61, 65, 73, 77, 81, 85, 89

SECTION 8.5 TEACHING TIPS

This section is good preparation for introductory statistics if the instructor plans to derive confidence interval formulas. Students will not need to solve inequalities in the rest of this course.

OBJECTIVES 1 and 2 COMMENTS

When going over these objectives, I emphasize the graphical interpretation because I believe it is an ideal way to convey that adding a number to both sides of an inequality or multiplying both sides by a positive number preserves order.

I also emphasize that when we multiply or divide both sides of an inequality by a negative number, we reverse the inequality symbol. Students need several reminders about this.

GROUP EXPLORATION: Properties of Inequalities

This activity is very accessible to students.

OBJECTIVE 3 COMMENTS

I point out to students that the meanings of the words *satisfy*, *solution*, and *solution set* are similar for equations and inequalities.

OBJECTIVE 4 COMMENTS

I tell students that solving a linear inequality is similar to solving a linear equation, except that care must be taken to reverse the inequality symbol when appropriate. When saying something like, “Here we *divide* both sides by *negative* 5,” I’ll raise my voice dramatically when I say “divide” and “negative.” Although this is cheesy, my students get the point that they need to be vigilant about reversing the inequality symbol when appropriate.

Many students do not understand the significance of the solution set of an inequality in one variable. For example, consider the inequality $-3x + 1 < -11$ (Problem 6), whose solution is $x > 4$. It is instructive for students to see that numbers greater than 4 such as 4.1, 5, and 6 satisfy the inequality and numbers less than or equal to 4 such as 2, 3, and 4 do not satisfy the inequality. Exercises 87 and 88 address this issue.

GROUP EXPLORATION: Meaning of the Solution Set of an Inequality

Although students have an easy time with this activity, it emphasizes a concept that students tend to lose sight of.

OBJECTIVE 5 COMMENTS

This objective is meant to prepare students for the complexity of confidence interval formulas.

OBJECTIVE 6 COMMENTS

Students will have an easy time solving compound inequalities. However, students will need some coaching to see that an inequality such as $a > x > b$ is equivalent to $b < x < a$. This issue naturally arises in Problem 14, where the inequality $2 < 8 - 2x < 10$ has solution $3 > x > -1$, which is equivalent to $-1 < x < 3$.

OBJECTIVE 7 COMMENTS

To solve Problem 16, I first translate the phrase "annual box office gross greater than \$9.5 billion:"

$$B > 9.5$$

Then I substitute $0.28t + 1.98$ for B:

$$0.28t + 1.98 > 9.5$$

Some students have trouble interpreting the meaning of an inequality in terms of the situation. For Problem 16, some students would say the inequality $t > 26.9$ means that the annual box office gross will be more than $9.5 billion *in* 2007, rather than *after* 2007. With repeated warnings about this issue, far fewer students tend to make this error on exams. It also helps to sketch a graph of the model and discuss the situation in detail.

CHAPTER 9 OVERVIEW

In Section 9.1, students find the equation of a line that contains two points. Then in Section 9.2, they use this skill to find an equation of a linear model. Finally, in Section 9.3, students use technology to find a linear regression model.

If you are pressed for time, you could skip Sections 9.1 and 9.2 because the method of using two points does not generalize to finding linear regression models. However, Sections 9.1 and 9.2 can help build students' understanding of the connection between scatterplots and the equations of linear models. And Section 9.2 can enhance students' ability to draw conclusions about linear models and how to use them to make predictions.

Section 9.3 contains a large amount of material and serves as a great review and extension of Chapters 6–8. Most reviewers said that this section would be the last one they would include in the course.

SECTION 9.1 LECTURE NOTES

Objectives

1. Find an equation of a line by using the slope-intercept form of a linear equation.
2. Find an equation of a line by using the point-slope form of a linear equation.

OBJECTIVE 1

Recall that an equation of a line can be put in slope-intercept form $y = mx + b$ (Section 7.1).

Use the slope-intercept form to find an equation of the line that has the given slope and contains the given point.

1. $m = 3$, $(-4, 5)$
2. $m = -2$, $(-3, -6)$
3. $m = \frac{2}{5}$, $(-3, 4)$
4. $m = -\frac{4}{3}$, $(5, -2)$
5. $m = 0$, $(-3, 2)$
6. m is undefined, $(-5, 1)$

Finding an Equation of a Line by Using the Slope, a Point, and the Slope-Intercept Form

To find an equation of a line by using the slope and a point,

1. Substitute the given value of the slope m into the equation $y = mx + b$.
2. Substitute the coordinates of the given point into the equation you found in step 1 and solve for b.
3. Substitute the value of b you found in step 2 into the equation you found in step 1.
4. Check that the graph of your equation contains the given point.

Use the slope-intercept form to find an equation of the line that contains the two given points.

7. $(2, 3)$ and $(4, 7)$ [slope is an integer]
8. $(-3, 4)$ and $(2, -6)$ [slope is an integer]
9. $(-5, -3)$ and $(2, 1)$ [slope is a fraction]
10. $(-6, -2)$ and $(-3, -7)$ [slope is a fraction]

Finding an Equation of a Line by Using Two Points and the Slope-Intercept Form

To find an equation of the line that passes through two given points whose x-coordinates are different,

1. Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the slope of the line containing the two points.
2. Substitute the m value you found in step 1 into the equation $y = mx + b$.
3. Substitute the coordinates of one of the given points into the equation you found in step 2 and solve for b.
4. Substitute the b value you found in step 3 into your equation you found in step 2.
5. Check that the graph of your equation contains the two given points.

11. Find an approximate equation of the line that contains the points $(-3.25, 8.11)$ and $(2.67, -1.39)$. Round the slope and the constant term to two decimal places.

12. Find an equation of the line that contains the given points.

    a. $(4, -3)$ and $(4, 2)$ [slope is undefined]
    b. $(-5, -2)$ and $(3, -2)$ [slope is 0]

OBJECTIVE 2 (optional)

Let (x_1, y_1) and (x, y) be two distinct points on a line with slope m. Use the true statement $\dfrac{y - y_1}{x - x_1} = m$ to derive the point-slope form.

Point-Slope Form

If a nonvertical line has slope m and contains the point (x_1, y_1), then an equation of the line is

$$y - y_1 = m(x - x_1)$$

Repeat any of Problems 1–5, 7–11, and 12(b). Compare your results with your earlier results.

HW 1, 3, 7, 15, 19, 25, 29, 43, 47, 51, 55, 57, 63, 69, 73

SECTION 9.1 TEACHING TIPS

This section discusses how to find equations of lines, both by using the slope-intercept form and the point-slope form. In subsequent sections, only the slope-intercept form will be used to find such equations. So, the point-slope form is optional.

I have chosen to emphasize finding linear equations using the slope-intercept form for several reasons:

- My students have better success with this method than with using the point-slope form, especially when the slope is a fraction.
- The notion of substituting a point to find the parameter b of $y = mx + b$ generalizes to finding parameters for exponential functions (Section 10.4).
- Students have a better sense of why this method "works" than with using the point-slope form.

- Using two forms for linear equations confuses the issue for many students.

The skill of finding an equation of a line that contains two given points will also be necessary for Section 9.2, where students will find equations of linear models.

OBJECTIVE 1 COMMENTS

For Problem 1, when I substitute the ordered pair $(-4, 5)$ into the equation $y = 3x + b$ to find b, I remind students that an ordered pair that corresponds to a point on a graph satisfies an equation of the graph. So, making such a substitution will give a true statement, one that will allow us to find the value of b.

For Problems 3, 4, 9, and 10, I prefer that students use fractions, not decimals, when finding linear equations to remind them of how to perform operations of fractions. When solving an equation with fractions (to find b of $y = mx + b$), students tend to prefer that we multiply both sides of the equation by the LCD to clear the equation of fractions.

However, one could argue that it suffices to allow students to use decimals rather than fractions because that's all they will need to do when finding equations of linear models in Section 9.2. For that reason, Problem 11 is a good primer for Section 9.2.

GROUP EXPLORATION: Deciding which Points to Use to Find an Equation of a Line

This activity responds to the frequent student query, "Does it matter which ordered pair is used to find b?" and "Does it matter which two points I choose from the graph of a line to find its equation?" I make sure that students understand the instructions of Problem 2 of the exploration, and that they imagine a line that is *not* parallel to either axis. This is an easy exploration for most students, and it could be assigned as homework.

GROUP EXPLORATION: Finding Equations of Lines

This exploration is a fun critical-thinking activity. Groups of students can complete this exploration in a timely fashion if they divide up the work.

OBJECTIVE 2 COMMENTS

This concept is optional. See my opening comments for the teaching tips for this section.

SECTION 9.2 LECTURE NOTES

Objectives

1. Use two points to find an equation of a linear model.

OBJECTIVE 1

1. Let A be the annual U.S. consumption (in billions of pounds) of avocados at t years since 2000. Find an equation of a linear model to describe the data in the following table.

Year	Consumption (billions of pounds)
2003	0.7
2005	0.9
2007	1.1
2009	1.2
2011	1.2
2012	1.6

Source: *California Avocado Commission*

It is a common error to skip constructing a scatterplot when we find an equation of a model.

Finding an Equation of a Linear Model

To find an equation of a linear model, given some data,

1. Construct a scatterplot of the data.
2. Determine whether there is a line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find an equation of a linear model.
3. Find an equation of the line.
4. Use technology to verify that the graph of your equation contains the two chosen points and comes close to all of the data points of the scatterplot.

2. The number of new-car dealerships are shown in the following table for various years. Let n be the number (in thousands) of new-car dealerships at t years since 1900.

Year	Number of New-Car Dealerships (thousands)
1990	24.8
1995	22.8
2000	22.3
2005	21.6
2010	18.5
2011	17.7

Source: *NADA Industry Analysis Division*

a. Construct a scatterplot.
b. Describe the four characteristics of the association. Compute and interpret r as part of your analysis.
c. Find an equation of a linear model to describe the data.
d. Rewrite the equation with the function name f.
e. Find $f(108)$. What does it mean in this situation?

3. Researchers compared the mean sulfur dioxide concentrations (air pollution) and the mean deterioration rates of marble tombstones in 21 U.S. cities for the period 1893–1993 (see the following table). A microgram (μg) is equal to 0.000001 gram.

Mean Sulfur Dioxide Concentration (μg/m^3)	Mean Tombstone Deterioration Rate (mm per century)	Mean Sulfur Dioxide Concentration (μg/m^3)	Mean Tombstone Deterioration Rate (mm per century)
180	1.53	239	2.51
12	0.27	48	0.84
197	2.71	94	1.21
142	1.01	102	1.09
234	1.61	142	1.90
117	1.72	91	1.78
20	0.14	178	1.98
323	3.16	20	0.33
122	1.18	224	2.41
244	2.15	92	1.08
46	0.81		

Source: Marble Tombstone Weathering and Air Pollution in North America*, T.C. Meierding*

Let s be the mean sulfur dioxide concentration (in μg/m^3) and d be the mean deterioration rate (in mm per century) of marble tombstones, both for the same city.

a. Describe the four characteristics of the association. Compute and interpret r as part of your analysis.

b. Find an equation of a model to describe the situation.

c. Estimate the mean deterioration rate of marble tombstones during the period 1893–1993 in a city where the mean sulfur dioxide concentration was 150 μg/m^3) for that period.

d. A city's mean deterioration of marble tombstones for the period 1893–1993 is 2 mm. Estimate the city's mean sulfur dioxide concentration for that period.

e. Because the association is strong, a student concludes that air pollution causes tombstone deterioration. What would you tell the student?

HW 1, 3, 7, 9, 11, 17, 21, 25, 27, 29, 31, 33

SECTION 9.2 DETAILED COMMENTS

In this section, students use two points to find an equation of a linear model. This section is optional because the algorithm does not generalize to the method of minimizing the sum of squared residuals (Section 9.3). But this section helps students see the connection between data points and the equation of a linear model, which is loosely relevant to Section 9.3.

OBJECTIVE 1 COMMENTS

To find an equation of a linear model, I first construct a scatterplot. Then I number the data points and ask my students which pairs of data points lie on a line that comes close to the other data points and which pairs do not. I really like the avocado data set (Problem 1) because it illustrates that using the "first and last points," $(3, 0.7)$ and $(12, 1.6)$, does not generate an ideal model. Many students tend to blindly use the first and last data points without bothering to construct a scatterplot, so hopefully this example will serve as a cautionary tale.

Students could use two nondata points to find a linear model, but I don't bring up this option for four reasons. First, because using data points works just fine. Second, it is easier to use coordinates of data points than to estimate coordinates of nondata points. Third, skipping this option saves me a bit of class time. Fourth, by not discussing how to use nondata points, I reduce the amount of time I spend grading exams because there is less variation in students' derived equations of models.

Once the class has determined two "good points," I derive an equation of the line that contains the two points. As I've done in previous sections, I start by writing the equation $y = mx + b$ and then replace the variables x and

y with the appropriate variables. When finding m and b, I emphasize that it's okay to use decimals; many students think they need to use fractions because they were required to do so when finding equations of (non-model) lines in Section 9.1. I explain that it's okay to round when finding a model because there are already various types of errors built into the process, anyway (see the exploration "Identifying types of modeling errors" on page 412 of the textbook). I say that when modeling, we round the values of m and b to the second decimal place, unless values are close to 0 such as $m = 0.003$.

When verifying a model's equation, explain that the line should contain the two chosen points (and come close to the other data points). Encourage students to take the time to distinguish between their chosen two points and the other data points. (By the way, most students do not realize that a point that lies close to, but not on a line, may appear to lie *on* the line.)

For quizzes and exams, if your students are using technology to verify their models, you can have them copy their technology's display onto their test papers; if you don't require this (by making it worth points), some students will resist ever learning how to use technology to verify their models.

The textbook's answers for this section consist of regression equations because it's impossible to anticipate which two "good" points students will choose. So, when assigning this section's homework, warn students that their answers will vary from the ones in the textbook. Remind them that they can verify their work using technology, if the technology you are using has this capability.

GROUP EXPLORATION: Adjusting the Fit of a Model

This activity is a fun reminder of the graphical significance of slope and the vertical intercept. More generally, the connection between a function's equation and graph is reinforced.

SECTION 9.3 LECTURE NOTES

Objectives

1. Compute and interpret residuals.
2. Compute and interpret the sum of squared residuals.
3. Find an equation of a linear regression model and use it to make predictions.
4. Interpret residual plots.
5. Use a residual plot to help determine whether a regression line is an appropriate model.
6. Identify influential points.
7. Compute and interpret the coefficient of determination.

OBJECTIVE 1

1. The ages and asking prices for 8 Honda Accords at dealerships in the Boston area are shown in the following table.

Age (in years)	Price (thousands of dollars)
8	13.0
7	9.2
12	7.0
9	9.9
5	13.0
11	10.0
3	17.0
5	14.2

Source: *Edmunds.com*

Let x be the age (in years) and y be the price (in thousands of dollars), both for a Honda Accord.

a. Describe the four characteristics of the association. Compute and interpret r as part of your analysis.

b. By viewing a scatterplot, we can determine that the line that contains the data points $(9, 9.9)$ and $(5, 13.0)$ comes fairly close to the data points. By using the technique discussed in Section 9.2, we can find that an equation of the line is $y = -0.78x + 16.88$ (try it). Verify that the line comes fairly close to the data points.

c. Use the model to predict the price of an 8-year-old Honda Accord.

d. Find the difference of the actual price and the predicted price for an 8-year-old Honda Accord.

e. Find the difference of the actual price and the predicted price for a 7-year-old Honda Accord.

For a data point (x, y), the **observed value of y** is y and the **predicted value of y** (written $\hat{y}$ or $\hat{f(x)}$) is the value obtained by using a model to predict y.

Definition *Residual*

For a given data point (x, y), the **residual** is the difference of the observed value of y and the predicted value of y:

$$\text{Residual} = \text{Observed value of } y - \text{Predicted value of } y = y - \hat{y}$$

Residuals for Data Points Above, Below, or on a Line

Suppose some data points are modeled by a line.

- A data point on the line has residual equal to 0.
- A data point above the line has positive residual. [Draw a figure.]
- A data point below the line has negative residual. [Draw a figure.]

OBJECTIVE 2

Sum of Squared Residuals

We measure how well a line fits some data points by calculating the sum of the squared residuals:

$$\sum(y_i - \hat{y}_i)^2$$

The smaller the sum of squared residuals, the better the line will fit the data. If the sum of squared residuals is 0, then there is an exact linear association.

2. Here we continue to work with the Honda Accord data (see the following table). Let x be the age (in years) and y be the price (in thousands of dollars), both for a Honda Accord. Find the sum of squared residuals for the linear model we worked with in Example 1, which is $\hat{y} = -0.78x + 16.88$.

Age (in years)	Price (thousands of dollars)
8	13.0
7	9.2
12	7.0
9	9.9
5	13.0
11	10.0
3	17.0
5	14.2

Source: *Edmunds.com*

OBJECTIVE 3

Definition *Linear regression function, line, equation, and model*

For a group of points, the **linear regression function** is the linear function with the least sum of squared residuals. Its graph is called the **regression line** and its equation is called the **linear regression equation**, written

$$\hat{y} = b_1x + b_0$$

where b_1 is the slope and $(0, b_0)$ is the y-intercept. The **linear regression model** is the linear regression function for a group of *data* points.

3. a. Use technology to find the linear regression equation for the Honda Accord association.

b. Predict the price of a 10-year-old Honda Accord.

c. What is the slope? What does it mean in this situation?

d. What is the y-intercept? What does it mean in this situation?

OBJECTIVE 4

A **residual plot** is a graph that compares data values of the explanatory variable with the data points' residuals.

4. a. Construct a residual plot for the Honda Accord linear regression model.

b. How many dots are above the zero residual line of the residual plot? What do they mean in this situation?

c. How many points are below the zero residual line of the residual plot? What do they mean in this situation?

d. Which point is farthest from the zero residual line? What does that mean in this situation?

e. Does the residual plot support that the regression line is a reasonable model? Explain.

OBJECTIVE 5

Using a Residual Plot to Help Determine Whether a Regression Line Is an Appropriate Model

The following statements apply to a residual plot for a regression line.

- If the residual plot has a pattern where the dots do not lie close to the zero residual line, then there is either a nonlinear association between the explanatory and response variables or there is no association.
- If a dot lies much farther away from the zero residual line than most or all of the other dots, then the dot corresponds to an outlier. If the outlier is neither adjusted nor removed, the regression line may *not* be an appropriate model.
- The *vertical* spread of the residual plot should be about the same for each value of the explanatory variable.

OBJECTIVE 6

If the slope of a regression line is greatly affected by the removal of a data point, we say the data point is an **influential point**.

5. **a.** The scatterplot and the regression line in Fig. 2.3 compare the hand lengths and heights of 75 women. The residual plot of the observations is shown in Fig. 2.4. Identify all outliers for the given situation.

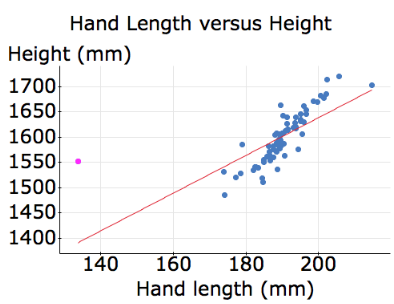


Figure 2.3: Scatterplot for hand-height data (**Source:** *S.G. Sani, E.D. Kizilkanat, N. Boyan, et al. (2005). "Stature Estimation Based on Hand Length and Foot Length," Clinical Anatomy, Vol. 18, pp. 589-596.*)

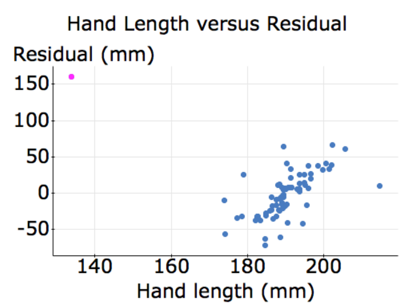


Figure 2.4: Residual plot for hand-height data

b. The outlier you likely found in Part (a) has been removed and the remaining data points are described by the scatterplot in Fig. 2.5. Is the removed outlier an influential point? Explain.

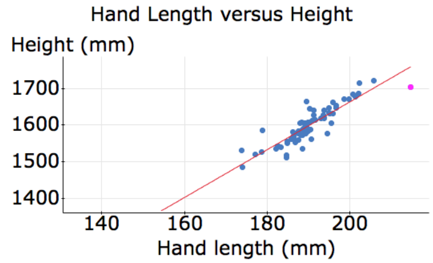


Figure 2.5: Scatterplot with outlier removed

6. **a.** The lengths of cruise ships and the sizes of the crews are compared by the scatterplot and the regression line shown in Fig. 2.6. The residual plot of the observations is shown in Fig. 2.7. Identify all outliers for the given situation.

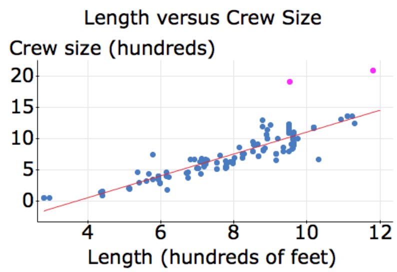


Figure 2.6: Scatterplot for cruise data (**Source:** *True Cruise*)

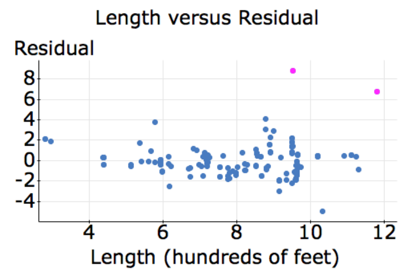


Figure 2.7: Residual plot for cruise data

b. The outliers you likely found in Part (a) have been removed and the remaining data points are described by the scatterplot in Fig. 2.8. Are the removed outliers influential points? Explain.

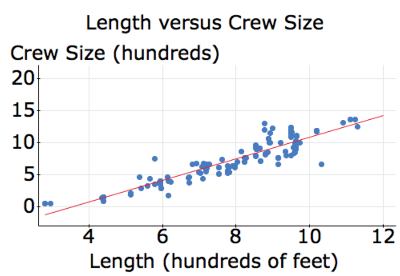


Figure 2.8: Scatterplot with outliers removed

Outliers tend to be influential points when they are horizontally far from the other data points.

OBJECTIVE 7

Coefficient of Determination

The coefficient of determination, r^2, is the proportion of the variation in the response variable that is explained by the regression line.

7. The temperatures and relative humidities in Phoenix, Arizona, are shown in the following table for various times on June 1, 2014.

Temperature (Fahrenheit degrees)	Relative Humidity (percent)	Temperature (Fahrenheit degrees)	Relative Humidity (percent)
89.06	11	102.92	7
84.92	14	100.94	7
82.04	16	105.98	5
80.06	20	104	5
78.98	22	102.92	5
80.96	21	102.2	6
84.02	19	100.4	5
87.98	16	98.06	6
93.02	12	91.94	9
98.06	8	91.94	9
100.04	8	87.98	3

Source: *Weatherbase*

a. Construct a scatterplot.

b. Find the linear regression model. Graph it on the scatterplot.

c. Compute the coefficient of determination. What does it mean in this situation?

HW 1, 3, 5, 9, 13, 17, 21, 23, 25, 29, 33, 35, 43, 45, 47, 49, 51, 61

SECTION 9.3 TEACHING TIPS

This section is the grand finale of Chapters 6–9, which addresses linear regression. The foundational concept is the residual, which leads toward the sum of squared residuals, the regression line, and residual plots.

OBJECTIVE 1 COMMENTS

When introducing the concept of residual, I point out that a residual is the opposite of how error was defined in Section 6.3. (See the lecture tips of that section for my reasoning in defining error to be the opposite of a residual.) Because of students' past work with computing errors in predictions, they quickly pick up on computing and interpreting residuals.

OBJECTIVE 2 COMMENTS

For Problem 2, as I build a table similar to Table 32 on page 602 of the textbook, I refer to the scatterplot that we constructed in Problem 1 to make sure students can picture the graphical significance of the calculations in each column of the table. I point out that we are finding squares of differences for reasons similar to why we find squares of differences to compute the variance.

OBJECTIVE 3 COMMENTS

After finding that the sum of squared residuals is equal to 21.20 for the model we worked with in Problem 2, I remind students that our result 21.20 measures how well the model fits the data points. Then I sketch the model in Fig. 28 on page 602 of the textbook and share with students that the sum of squared residuals is 82.36. I point out that it makes sense that the result 82.36 is larger than 21.20 because the line in Fig. 28 does not fit the data as well as the model in Problem 2. I then sketch the linear regression model, explaining that it has the least sum of squared residuals, 17.01, and conclude that it must fit the data points the best (see Fig. 30 on page 603 of the textbook).

Parts (b), (c), and (d) of Problem 3 serve as a nice review of concepts already addressed in Sections 7.1–7.3.

OBJECTIVE 4 COMMENTS

When constructing the residual plot for Problem 4(a), I refer back to the scatterplot we constructed in Problem 1(a) and make sure students see the connection between the residuals in the scatterplot and in the residual plot.

GROUP EXPLORATION: Meaning of Residual Plots

This activity helps students discover how a residual plot relates to a scatterplot and the relevant regression line. Students tend to be surprised that the residual plots for the data sets described by the scatterplots in Figs. 54 and 55 are the same.

OBJECTIVE 5 COMMENTS

For each of the three bulleted statements for this objective, I display both a scatterplot with a model and the corresponding residual plot to make sure students see the connection between the two diagrams.

OBJECTIVE 6 COMMENTS

Students have an easy time learning about outliers and influential points because they are such intuitive, visual concepts.

OBJECTIVE 7 COMMENTS

The most challenging concept in this section is the coefficient of determination. Many introductory statistics textbooks try to convey the idea by labeling the explained and unexplained deviations of just one data point in a scatterplot, but I don't believe this approach is effective because it doesn't get across a sense of variation. The detailed description on pages 611 and 612 of the textbook tries to convey the essence of the coefficient of determination by illustrating relevant variations.

CHAPTER 10 OVERVIEW

In this chapter, students simplify expressions with exponents, graph exponential functions, and perform modeling with exponential functions.

Although the material in this chapter is very challenging for most students, it creates the opportunity to revisit many topics addressed in Chapters 6–9: constructing scatterplots and residual plots; finding equations of models; graphing models; computing residuals, correlation coefficients, and coefficients of determination; identifying outliers and influential points; and making predictions.

It also creates the opportunity to compare linear and exponential models, which sheds light on many modeling concepts, including determining which model best fits some data points.

Although exponential regression is probably not part of your department's introductory statistics course, one could argue there is another reason to include this chapter in a prestatistics course. Assuming that students who take this course are bypassing algebra, we must ask ourselves if there is any algebra content that they absolutely must see to have a well-rounded education. I believe there are two such concepts: linear and exponential models. These are the two models most applicable to our world.

SECTION 10.1 LECTURE NOTES

Objectives

1. Describe the *product property* for exponents.
2. Simplify expressions involving nonnegative-integer exponents.
3. Describe the following properties for exponents: quotient, raising a product to a power, raising a quotient to a power, and raising a power to a power.
4. Use combinations of properties of exponents to simplify expressions involving nonnegative-integer exponents.
5. Simplify expressions involving negative-integer exponents.
6. Describe the meaning of *exponential function*.
7. Work with models whose equations involve integer exponents.

OBJECTIVE 1

Show that $b^2 \cdot b^4 = b^{2+4}$:

$$\begin{aligned} b^2 \cdot b^4 &= (b \cdot b)(b \cdot b \cdot b \cdot b) \\ &= b \cdot b \cdot b \cdot b \cdot b \cdot b \\ &= b^6 \end{aligned}$$

Product Property for Exponents

If m and n are counting numbers, then

$$b^m b^n = b^{m+n}$$

For example, $b^3 b^5 = b^{3+5} = b^8$. Also, $b^4 b = b^4 b^1 = b^{4+1} = b^5$.

Find the product.

1. $3b(7b^6)$ **2.** $4b^5(-6b^3)$ **3.** $-5p^3q(-3p^2q^5)$

OBJECTIVE 2

Simplifying an Expression Involving Nonnegative-Integer Exponents

An expression involving nonnegative-integer exponents is simplified if

1. It includes no parentheses.
2. Each variable or constant appears as a base as few times as possible. For example, for nonzero b, we write b^2b^3 as b^5.
3. Each numerical expression (such as 3^2) has been calculated, and each numerical fraction has been simplified.

OBJECTIVE 3
Quotient Property for Exponents

Here we simplify $\frac{b^5}{b^3}$:

$$\begin{aligned}
\frac{b^5}{b^3} &= \frac{b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} && \text{Write quotient without exponents.} \\
&= \frac{b \cdot b \cdot b}{b \cdot b \cdot b} \cdot \frac{b \cdot b}{1} && \frac{ac}{bd} = \frac{a}{b} \cdot \frac{c}{d} \\
&= 1 \cdot b \cdot b && \text{Simplify: } \frac{b \cdot b \cdot b}{b \cdot b \cdot b} = 1. \\
&= b^2 && \text{Simplify.}
\end{aligned}$$

This suggests we can subtract the exponents: $\frac{b^5}{b^3} = b^{5-3} = b^2$.

Quotient Property for Exponents

If m and n are counting numbers and b is nonzero, then

$$\frac{b^m}{b^n} = b^{m-n}$$

Simplify.

4. $\frac{6b^7}{9b^4}$ **5.** $\frac{4b^9c^5}{6b^7c^2}$ **6.** $\frac{20a^8b^5}{15a^6b}$

Raising a Product to a Power

We can write $(bc)^4$ as a product of powers:

$$\begin{aligned}
(bc)^4 &= (bc)(bc)(bc)(bc) && \text{Write power without exponents.} \\
&= (b \cdot b \cdot b \cdot b)(c \cdot c \cdot c \cdot c) && \text{Rearrange factors.} \\
&= b^4c^4 && \text{Simplify.}
\end{aligned}$$

We can get the same result by raising each factor of the base bc to the 4th power:

$$(bc)^4 = b^4c^4$$

Raising a Product to a Power

If n is a counting number, then

$$(bc)^n = b^nc^n$$

Perform the indicated operation.

7. $(bc)^5$

8. $(2b)^4$

9. $(-4bc)^3$

Warning: The expression $(3b)^2$ is equivalent to $9b^2$, not $3b^2$.

Raising a Quotient to a Power

Here we write $\left(\frac{b}{c}\right)^3$ in another form:

$$\begin{aligned}\left(\frac{b}{c}\right)^3 &= \frac{b}{c}\cdot\frac{b}{c}\cdot\frac{b}{c} && \text{Write power expression without exponents.}\\ &= \frac{b\cdot b\cdot b}{c\cdot c\cdot c} && \text{Multiply numerators; multiply denominators.}\\ &= \frac{b^3}{c^3} && \text{Simplify.}\end{aligned}$$

This suggests that both the numerator and the denominator should be raised to the 3rd power: $\left(\frac{b}{c}\right)^3 = \frac{b^3}{c^3}$

Raising a Quotient to a Power

If n is a counting number and c is nonzero, then

$$\left(\frac{b}{c}\right)^n = \frac{b^n}{c^n}$$

Simplify.

10. $\left(\frac{b}{c}\right)^5$

11. $\left(\frac{b}{4}\right)^3$

Raising a Power to a Power

Here we write $(b^2)^4$ in another form:

$$\begin{aligned}(b^2)^4 &= b^2\cdot b^2\cdot b^2\cdot b^2 && \text{Write expression without exponent 4.}\\ &= b^{2+2+2+2} && \text{Add exponents: } b^mb^n = b^{m+n}.\\ &= b^8 && \text{Simplify.}\end{aligned}$$

This suggests we can multiply the exponents: $(b^2)^4 = b^{2\cdot 4} = b^8$.

Raising a Power to a Power

If m and n are counting numbers, then

$$(b^m)^n = b^{mn}$$

Simplify.

12. $(b^3)^5$

13. $(b^4)^7$

OBJECTIVE 4

Recall from Section 1.7 that if b is nonzero, then $b^0 = 1$. For example, $7^0 = 1$.

Simplify.

14. $(3b^4c^7)^2$

15. $5b^5(2b^7)^3$

16. $\dfrac{14b^3b^5}{21b^8}$

17. $\left(\dfrac{3b^9}{2c^6}\right)^4$

18. $\dfrac{(2b^5c)^4}{b^9c}$

OBJECTIVE 5

In Section 1.7, we defined a negative-integer exponent.

Definition *Negative-integer exponent*

If n is a counting number and $b \neq 0$, then

$$b^{-n} = \frac{1}{b^n}$$

For example, $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$.

Simplifying an expression involving negative-integer exponents includes writing it so that each exponent is positive.

Simplify.

19. $6b^{-4}$

20. $2^{-1} + 5^{-1}$

Here we write $\dfrac{1}{b^{-n}}$ in another form:

$\dfrac{1}{b^{-n}} = 1 \div b^{-n}$	$\dfrac{a}{b} = a \div b$
$= 1 \div \dfrac{1}{b^n}$	Write power so exponent is positive: $b^{-n} = \dfrac{1}{b^n}$
$= 1 \cdot \dfrac{b^n}{1}$	Multiply by reciprocal of $\dfrac{1}{b^n}$, which is $\dfrac{b^n}{1}$.
$= b^n$	Simplify.

So, $\dfrac{1}{b^{-n}} = b^n$.

Negative-Integer Exponent in a Denominator

If n is a counting number and $b \neq 0$, then

$$\frac{1}{b^{-n}} = b^n$$

Simplify.

21. $\frac{1}{4^{-3}}$ **22.** $\frac{8}{b^{-9}}$ **23.** $\frac{b^{-7}}{c^4}$ **24.** $\frac{b^5}{c^{-3}}$ **25.** $\frac{7a^{-4}b^9}{3c^{-2}}$

Properties of Exponents

If m and n are integers, $b \neq 0$, and $c \neq 0$, then

- $b^m b^n = b^{m+n}$ Product property for exponents
- $\frac{b^m}{b^n} = b^{m-n}$ Quotient property for exponents
- $(bc)^n = b^n c^n$ Raising a product to a power
- $\left(\frac{b}{c}\right)^n = \frac{b^n}{c^n}$ Raising a quotient to a power
- $(b^m)^n = b^{mn}$ Raising a power to a power

Simplifying Expressions Involving Exponents

An expression involving integer exponents is simplified if

1. It includes no parentheses.
2. Each variable or constant appears as a base as few times as possible.
3. Each numerical expression (such as 3^2) has been calculated, and each numerical fraction has been simplified.
4. Each exponent is positive.

Simplify.

26. $3^{1007}3^{-1005}$

27. $(b^5)^{-2}$

28. $(6b^{-9}c^4)(-2b^5c^{-3})$

29. $\frac{b^3}{b^8}$

30. $\frac{9b^{-7}}{7b^{-2}}$

31. $(4b^{-4})^{-3}$

32. $\frac{18b^{-5}c^{-4}}{27b^3c^{-6}}$

33. $\left(\frac{b^5}{2c^{-3}}\right)^{-4}$

34. $\frac{(2b^4c)^3}{(3b^{-3}c^2)^4}$

35. $\left(\frac{2b^{-4}c^4}{3b^2c^{-5}}\right)^{-3}$

OBJECTIVE 6

Definition *Exponential function*

An **exponential function** is a function whose equation can be put into the form

$$f(x) = ab^x$$

where $a \neq 0$, $b > 0$, and $b \neq 1$. The constant b is called the **base**.

For $f(x) = 2^x$ and $g(x) = 2(3)^x$, find the following.

36. $f(5)$ **37.** $f(-4)$ **38.** $g(3)$ **39.** $g(-2)$ **40.** $g(0)$

41. For $E(x) = 2^x$ and $L(x) = 2x$, compare $E(5)$ with $L(5)$.

OBJECTIVE 7

42. The weight (in pounds) $f(d)$ of an astronaut when d thousand miles from the center of Earth is described by the equation $f(d) = 3060d^{-2}$.

- **a.** Simplify the right-hand side of the equation.
- **b.** When at sea level, the astronaut is about 4 thousand miles from Earth's center. How much does the astronaut weigh at sea level?
- **c.** Find $f(6)$. What does the result mean in this situation?

HW 1, 3, 7, 13, 25, 33, 35, 55, 61, 67, 75, 87, 91, 97, 103, 105, 111

SECTION 10.1 DETAILED COMMENTS

Students learn many definitions and properties of exponents in this section and Section 10.2, which will help them use two points to find exponential models in Section 10.4.

Although students may know the properties of exponents from a previous class, they probably do not know why they are true, which can help with knowing when and how to apply a property.

Students are challenged by this section because so much symbol manipulation is required.

OBJECTIVE 1 COMMENTS

Students have an easy time with the product property in isolation, but once the raising a power to a power property has been introduced, some students confuse the two properties. So, when working with an expression such as b^4b^3, I encourage students to imagine strings of b factors to help ground them with the concept that the exponents need to be added.

For Problems 2 and 3, I emphasize that the expressions are products, not differences.

OBJECTIVE 2 COMMENTS

I go over this objective quickly because it will become clear as we simplify expressions throughout the section.

OBJECTIVE 3 COMMENTS

I also go over this objective quickly because students tend to have an easy time simplifying expressions when only one property is required. It is when a combination of properties is required (Objective 4) that students tend to have trouble, especially if they must work with negative exponents (Objective 5).

For Problem 4, I clarify how the quotient property for exponents can be used by breaking up the fraction $\frac{6b^7}{9b^4}$ into the product $\frac{6}{9} \cdot \frac{b^7}{b^4}$.

For Problem 8, students tend to incorrectly simplify the expression $(2b)^4$ to $2b^4$. I emphasize that the correct result is $16b^4$. It is interesting that students tend to simplify $(bc)^5$ (Problem 7) correctly but have trouble simplifying $(2b)^4$ correctly. I point out that 2 is a factor of the base $2b$ of $(2b)^4$ just like b is a factor of the base bc of $(bc)^5$.

After introducing the raising a power to a power property, I compare simplifying $(b^3)^2$ with simplifying b^3b^2. As I mentioned earlier, students tend to confuse the raising a power to a power property with the product property.

OBJECTIVE 4 COMMENTS

For Problems 14–18, students tend to skip steps, which leads to errors or fuzzy thinking that gets them into trouble when the problems are more difficult. I encourage students to write all the steps until they can picture steps that they skip.

GROUP EXPLORATION: Properties of Positive-Integer Exponents

This activity addresses common student errors when simplifying expressions with positive-integer exponents.

OBJECTIVE 5 COMMENTS

Even though negative integers were already introduced in Section 1.7 and they likely came up again in Sections 5.4 and 5.5, students need a refresher by this point in the course.

For Problem 26, it is good for students to find out that most calculators give incorrect results. In particular, it is instructive for them to reason why many calculators give the incorrect result 0 for 3^{-1005}.

For Problem 30, some students will mistakenly write $\frac{9b^{-7}}{7b^{-2}}$ as $\frac{9b^{-7-2}}{7}$, rather than correctly write $\frac{9b^{-7-(-2)}}{7}$.

There are many ways to simplify quotients of powers of the form $\frac{b^m}{b^n}$. One way is to use the property $\frac{b^m}{b^n} = b^{m-n}$ and then simplify so that there are no negative exponents. Another way is to "move" the power with the smaller exponent across the fraction bar (and change the sign of the exponent). Showing more than one method usually confuses students. It is best to select one method and use it whenever appropriate.

Students tend to be intimidated by problems as complicated as Problem 35, but with practice, they can master them.

You may prefer to introduce the property $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ and use it to simplify certain expressions, but I have not included this property in the textbook in an attempt to avoid overloading students with too many options.

GROUP EXPLORATION: Properties of Negative-Integer Exponents

This activity addresses common student errors when simplifying expressions with negative-integer exponents.

OBJECTIVE 6 COMMENTS

After completing Problem 41, consider comparing input-output tables of E and L to help students begin to distinguish exponential functions from linear functions. Or, you could assign Exercise 89 of Homework 10.1 and have such a discussion at the next class meeting.

OBJECTIVE 7 COMMENTS

Most students know that an astronaut is weightless in outer space, but few students know that an astronaut's weight declines *continuously* as the astronaut's distance from the center of Earth increases.

SECTION 10.2 LECTURE NOTES

Objectives

1. Describe the meaning of *rational exponents*.
2. Simplify expressions involving rational exponents.
3. Describe properties of rational exponents.

OBJECTIVE 1

Argue that

$$\left(16^{1/2}\right)^2 = 16^{\frac{1}{2}\cdot 2} = 16^1 = 16$$

suggests that $16^{1/2} = 4$. (Note that $16^{1/2}$ is *not* equal to 8, which is half of 16.)

Argue that

$$\left(8^{1/3}\right)^3 = 8^{\frac{1}{3}\cdot 3} = 8^1 = 8$$

suggests that $8^{1/3} = 2$.

Definition $\boldsymbol{b^{1/n}}$

For the counting n, where $n \neq 1$,

- If n is odd, then $\boldsymbol{b^{1/n}}$ is the number whose nth power is b, and we call $b^{1/n}$ the ***n*th root of *b***.
- If n is even and $b \geq 0$, then $\boldsymbol{b^{1/n}}$ is the nonnegative number whose nth power is b, and we call $b^{1/n}$ the **principal *n*th root of *b***.
- If n is even and $b < 0$, then $b^{1/n}$ is not a real number.

$\boldsymbol{b^{1/n}}$ may be represented by $\sqrt[n]{b}$.

OBJECTIVE 2

Simplify.

1. $49^{1/2}$
2. $27^{1/3}$
3. $(-27)^{1/3}$
4. $32^{1/5}$
5. $-81^{1/4}$
6. $(-81)^{1/4}$
7. $64^{-1/2}$
8. $(-64)^{-1/3}$

Show that $16^{3/4} = 16^{\frac{1}{4}\cdot 3} = \left(16^{1/4}\right)^3 = 2^3 = 8$

Definition *Rational exponent*

For the counting numbers m and n, where $n \neq 1$, and b is any real number for which $b^{1/n}$ is a real number,

- $\boldsymbol{b^{m/n}} = \left(b^{1/n}\right)^m = \left(b^m\right)^{1/n}$
- $\boldsymbol{b^{-m/n}} = \dfrac{1}{b^{m/n}}, b \neq 0$

A power of the form $b^{m/n}$ or $b^{-m/n}$ is said to have a **rational exponent**.

Simplify.

9. $25^{3/2}$ **10.** $8^{5/3}$ **11.** $(-27)^{4/3}$ **12.** $(-81)^{3/4}$ **13.** $9^{-3/2}$ **14.** $(-32)^{-2/5}$

For $f(x) = 2(27)^x$, find the following.

15. $f\left(\dfrac{2}{3}\right)$

16. $f\left(-\dfrac{1}{3}\right)$

OBJECTIVE 3

The properties of exponents that we discussed in Section 10.1 are valid for *rational exponents*.

> **Properties of Rational Exponents**
>
> If m and n are rational numbers and b and c are any real numbers for which b^m, b^n, and c^n are real numbers, then
>
> - $b^m b^n = b^{m+n}$ Product property for exponents
> - $\dfrac{b^m}{b^n} = b^{m-n}, b \neq 0$ Quotient property for exponents
> - $(bc)^n = b^n c^n$ Raising a product to a power
> - $\left(\dfrac{b}{c}\right)^n = \dfrac{b^n}{c^n}, c \neq 0$ Raising a quotient to a power
> - $(b^m)^n = b^{mn}$ Raising a power to a power

Simplify. Assume that b and c are positive.

17. $\left(8b^6\right)^{4/3}$

18. $\dfrac{b^{-5/9}}{b^{-2/9}}$

19. $b^{3/5}b^{7/8}$

20. $(7b^2c^5)^{4/9}(7b^2c^5)^{5/9}$

21. $\left[\left(2b^3\right)^2\left(4b^2c^6\right)\right]^{1/4}$

22. $\dfrac{(8b^{-6}c^3)^{1/3}}{(16b^4c^{-12})^{1/4}}$

23. $\left(\dfrac{49b^5c^{-8}}{64b^{-3}c^{-2}}\right)^{-1/2}$

24. $\dfrac{\left(81b^8c^4\right)^{3/4}}{(64bc^7)^{1/3}}$

HW 1, 3, 11, 17, 27, 33, 41, 43, 53, 59, 67, 69, 75, 77, 79, 81

SECTION 10.2 DETAILED COMMENTS

In this section, students learn the meaning of rational exponents. They will use this concept throughout the chapter. Students also simplify expressions involving rational exponents in this section.

OBJECTIVE 1 COMMENTS

When introducing rational exponents, I write $16^{1/2}$ on the board and say that we will find its value, which I will call “hand.” Next, I write $\left(16^{1/2}\right)^2$ on the board and point out that this expression is “hand squared,” and I place my hand over just the base to illustrate. Then I remove my hand and perform the following calculation:

$$\left(16^{1/2}\right)^2 = 16^{\frac{1}{2}\cdot 2} = 16^1 = 16$$

Then I cover $16^{1/2}$ (on the farthest left side of the equation) and observe that “hand squared is equal to 16.” I ask my students, “So, what is ‘hand’ equal to?” Someone always sees that “hand” is equal to 4 (no one ever mentions -4, which we won’t include in our definition, anyway). Finally, we discuss that $16^{1/2}$ means to find the square root of 16.

Then I use “hand” again to find the value of $8^{1/3}$. After that, students are ready for me to discuss the meaning of $b^{1/n}$ in general.

For powers with positive rational exponents, I encourage my students to practice computing them mentally, without writing all of the steps. This will keep the amount of writing to a more reasonable level when they are simplifying complicated expressions.

Most students have difficulty computing powers with negative rational exponents, especially after some time has passed (see Problems 7, 8, 13, 14, and 16).

I delay discussing how to use a calculator to compute powers with rational exponents until I'm confident that students can compute such powers without a calculator. When using a calculator to find a power such as $\left(\frac{9}{2}\right)^{3/4}$, I emphasize that two sets of parentheses must be used.

GROUP EXPLORATION: Definition of $b^{1/n}$

Another way to introduce rational exponents is to assign this activity. Students may get more out of this exploration than from a lecture because they are more involved with each step of investigating the meaning of rational exponents.

OBJECTIVE 3 COMMENTS

Most students have a tough time simplifying expressions with rational exponents. They will continue to be challenged by determining which order to apply the properties and definitions of exponents. Further, students tend to need reminders on how to perform operations with fractions (e.g. add fractions with different denominators). Sometimes students get psyched out when trying to simplify such expressions. I try to ease their overwhelm by pointing out that we are still performing the same types of steps as in Section 10.1; it is mostly performing operations of fractions that extends the time to simplify expressions involving rational exponents.

Problems 22–24 are quite difficult and certainly not needed for introductory statistics. If you are tight on time, you could omit discussing these problems and assigning these types of exercises.

SECTION 10.3 LECTURE NOTES

Objectives

1. Graph an exponential function.
2. Describe the *base multiplier property* and the *increasing or decreasing property*.
3. Find the y-intercept of the graph of an exponential function.
4. Describe the *reflection property for exponential functions*.
5. Graph an equation of an exponential model.
6. Interpret the coefficient and the base of an exponential model.

OBJECTIVE 1

1. Create a table of solutions of $f(x) = 2^x$ and graph f by hand.
2. Create a table of solutions of $g(x) = 8\left(\frac{1}{2}\right)^x$ and graph g by hand.

OBJECTIVE 2

Refer to your tables of solutions of $f(x) = 2^x$ and $g(x) = 8\left(\frac{1}{2}\right)^x$ to motivate the base multiplier property.

Base Multiplier Property

For an exponential function of the form $y = ab^x$, if the value of x increases by 1, then the value of y is multiplied by b.

Describe the base multiplier property as applied to the given function.

3. $f(x) = 5(4)^x$

4. $g(x) = 7\left(\frac{2}{3}\right)^x$

Use the base multiplier property to explain the following connections:

- $f(x) = 2^x$ is an increasing function and $b = 2 > 1$ (see Problem 1).
- $g(x) = 8\left(\frac{1}{2}\right)^x$ is a decreasing function and $b = \frac{1}{2} < 1$ (see Problem 2).

Increasing or Decreasing Property

Let $f(x) = ab^x$, where $a > 0$. Then

- If $b > 1$, then the function f is increasing. We say the function **grows exponentially**. [Draw a figure.]
- If $0 < b < 1$, then the function f is decreasing. We say the function **decays exponentially**. [Draw a figure.]

OBJECTIVE 3

Substitute 0 for x in the general equation $y = ab^x$ to show the following property.

y-Intercept of an Exponential Function

For the graph of an exponential function of the form

$$y = ab^x,$$

the y-intercept is $(0, a)$.

Find the y-intercept of the graph of the function.

5. $y = 8(5)^x$

6. $y = -4\left(\frac{2}{7}\right)^x$

Warning: The y-intercept of the graph of a function of the form $y = b^x$ is $(0, 1)$, *not* $(0, b)$.

Find the y-intercept of the graph of the given function. Then use the base multiplier property to graph the function by hand.

7. $f(x) = 2(3)^x$

8. $g(x) = 12\left(\frac{1}{3}\right)^x$

OBJECTIVE 4

9. Sketch and compare the graphs of $f(x) = -4(2)^x$ and $g(x) = 4(2)^x$.

> **Reflection Property**
>
> The graphs of $f(x) = -ab^x$ and $g(x) = ab^x$ are reflections of each other across the x-axis.

Graph the function by hand.

10. $f(x) = -3(4)^x$

11. $g(x) = -16\left(\frac{1}{4}\right)^x$

- The graph of an exponential function does not have any x-intercepts and the x-axis is a horizontal asymptote.
- The domain of any exponential function $f(x) = ab^x$ is the set of all real numbers.
- The range of an exponential function $f(x) = ab^x$ is the set of all positive real numbers if $a > 0$, and the range is the set of all negative real numbers if $a < 0$.

Find the domain and range of the function.

12. $f(x) = 3(2)^x$

13. $g(x) = -10\left(\frac{1}{2}\right)^x$

OBJECTIVE 5

> **Definition** *Exponential model*
>
> An **exponential model** is an exponential function, or its graph, that describes an authentic association.

14. The prices of one ounce of gold on the New York Stock Exchange (NYSE) at closing on January 2 are shown in the following table for various years. If the NYSE was closed on January 2, then the closing price on the first date it was open was used.

Year	Price (dollars per ounce)
2000	289
2002	278
2004	417
2006	519
2008	856
2010	1102
2012	1587

Source: *Kitco Metals, Inc.*

Let y be the price (in dollars per ounce) of gold on January 2 at x years since 2000.

a. Construct a scatterplot.

b. Describe the shape and the direction of the association.

c. Graph the model $\hat{y} = 289(1.15)^x$ on the scatterplot. Does the model come close to the data points?

d. What is the coefficient of the model $\hat{y} = 289(1.15)^x$? What does it mean in this situation?

e. What is the base of the model $\hat{y} = 289(1.15)^x$? What does it mean in this situation?

If the points of a scatterplot lie close to (or on) an exponential curve, we say the variables are **exponentially associated** and that there is an **exponential association**.

OBJECTIVE 6

Meaning of the Coefficient of an Exponential Model

Assume that an association between the variables x and y can be described exactly by the exponential model $\hat{y} = ab^x$. Then the coefficient a equals the quantity y when the quantity x is 0.

Meaning of the Base of an Exponential Model

Assume that an association between the variables x and y can be described exactly by the exponential model $\hat{y} = ab^x$, where $a > 0$. Then the following statements are true.

- If $b > 1$, then the quantity y grows exponentially at a rate $b - 1$ percent (in decimal form) per unit increase of the quantity x.
- If $0 < b < 1$, then the quantity y decays exponentially at a rate of $1 - b$ percent (in decimal form) per unit increase of the quantity x.

15. Malathion is an insecticide that has been sprayed from helicopters on suburban areas to control mosquitoes and Mediterranean fruit flies. The California Department of Health Services concluded that some people may be sensitive to malathion (see the source shown in the following table). In an experiment conducted in 1993, researchers sprayed some tomatoes with malathion and recorded how much of the insecticide persisted. The mean malathion concentrations on the surfaces of four groups of tomatoes are shown in the following table for various amounts of time.

Time (hours)	Mean Concentration (micrograms per centimeter squared)
0	0.169
12	0.167
24	0.178
48	0.120
96	0.073
239	0.070
504	0.008

Source: Assessment of Malathion and Malaoxon Concentration and Persistence in Water, Sand, Soil and Plant Matrices Under Controlled Exposure Conditions, *Neal et al.*

Let y be the mean malathion concentration (in micrograms per centimeter squared) on the tomatoes at x hours.

a. Construct a scatterplot.

b. Describe the shape and the direction of the association.

c. Graph the model $\hat{y} = 0.18(0.994)^x$ on the scatterplot. Then describe the strength of the association.

d. What is the coefficient of the model $\hat{y} = 0.18(0.994)^x$? What does it mean in this situation?

e. What is the base of the model $\hat{y} = 0.18(0.994)^x$? What does it mean in this situation?

f. Predict the percentage of the malathion that remained on the tomatoes 3 days after spraying.

HW 1, 3, 5, 9, 15, 19, 25, 31, 33, 39, 51, 53, 59, 79, 87

SECTION 10.3 DETAILED COMMENTS

Students learn to sketch graphs of exponential functions in this section. Concepts related to graphing exponential functions will be very helpful throughout the rest of this chapter. Students will also learn how to interpret the coefficient and the base of an exponential model.

OBJECTIVE 1 COMMENTS

For Problem 1, I evaluate f at -2, -1, 0, 1, 2, 3, and 4 to find points to plot. Students can observe that the graph of f is not a line. Then I point out the base multiplier property without yet calling it as such to find other points (without evaluating f) and sketch the exponential curve. For example, to plot a point with x-coordinate -3, I plot a point above $(-3, 0)$ that is half as high above the x-axis as the point $(-2, \frac{1}{4})$. By thinking in terms of halving distances visually each time we move to the left one unit, students can see that the curve is getting closer and closer to the x-axis, but never reaching it. I then introduce the terminology of a horizontal asymptote.

For Problem 2, I evaluate g at 0, 1, 2, 3, and 4 to find points to plot. Students can see that the graph of g is not a line. We can again use the base multiplier property (without naming it yet) to find other points (without evaluating g) and sketch the exponential curve.

OBJECTIVE 2 COMMENTS

I refer to Problems 1 and 2 to introduce the base multiplier property. For Problems 3 and 4, I do not graph the functions; I wait to do more graphing until I've discussed finding the y-intercept of an exponential function.

I show students how the increasing or decreasing property follows directly from the base multiplier property. The concepts of exponential growth and exponential decay will come in handy when performing modeling in Sections 10.4 and 10.5.

OBJECTIVE 3 COMMENTS

For an exponential function of the form $f(x) = b^x$, most students think that the y-intercept of the graph of f is $(0, b)$. By writing $f(x) = 1(b^x)$, students see that the y-intercept is $(0, 1)$. It is usually necessary to remind students of this at least a couple of times.

After discussing the y-intercept, I show students how to efficiently graph an exponential function. First, I plot the y-intercept. Then I use the base multiplier property to determine other points to plot. By this method, I never have to evaluate the function. Students tend to resist this method—they initially prefer evaluating the function. However, with some encouragement, most students will switch to using the base multiplier method. I prefer this method not only because it is more efficient, but also because it reinforces the base multiplier property and has students think more geometrically than the other method.

GROUP EXPLORATION: Numerical Significance of a and b for $f(x) = ab^x$

Students have an easy time with this activity.

OBJECTIVE 4 COMMENTS

The base multiplier method can also be used to graph the functions in Problem 9, which leads to discussing the reflection property. To get across the idea of reflection, I say that if a mirror were placed along the x-axis, the graph of $g(x) = 4(2)^x$ would "see" the graph of $f(x) = -4(2)^x$ as its reflection in the mirror. I also say that if we painted the graph of g on graph paper and folded the paper along the x-axis, the blotted image would be the graph of f.

Due to students' work with domain and range in Section 7.4, most students have a fairly easy time finding the domain and range of an exponential function. A reminder that the graph of an exponential function $f(x) = ab^x$ does not intersect the x-axis helps students find the range of such functions.

GROUP EXPLORATION: Graphical Significance of a and b for $y = ab^x$

This activity is a great way to address the key concepts of this section. Once groups of students are done, you can have a classroom discussion about Problems 4 and 5 to get closure. Functions of the form $y = ab^x + c$ will not be needed for modeling in the textbook, so this exploration omits investigation of vertical translation. If you want your students to explore the graphical significance of c, you could ask them to explore its significance at the end of the exploration. A good follow-up question is to ask students to describe the y-intercept of the graph of a function of the form $y = ab^x + c$.

GROUP EXPLORATION: Drawing Families of Exponential Curves

This activity is a fun way for students to reflect on the graphical significance of parameters a and b for an exponential function of the form $y = ab^x$. Even if students have explored the graphical significance of the base b, students may still need a hint to use values of b between 0 and 1 for Problem 2.

OBJECTIVE 5 COMMENTS

Students have an easy time with Parts (a) and (b) of Problem 14 because of earlier work with linear modeling.

Even though the meanings of the coefficient and the base of an exponential model are formally discussed in boxed statements for Objective 6, they are included in Parts (d) and (e) of Problem 14 to prepare them for the formal, general statements in the property boxes.

In Problem 14(d), students have a bit more trouble than expected to interpret the coefficient of an exponential model, perhaps because they are so used to working with linear models.

Students have even more trouble with Problem 14(e). It is challenging for them to follow the work included on page 664 of the textbook for the solution to Problem 5 of Example 6. Nonetheless, they follow that work better than simply saying that multiplying by 1.15 is equivalent to finding 115% of the price from the previous year, so the price increases by 15% per year.

OBJECTIVE 6 COMMENTS

Although students struggle with finding both exponential growth and decay rates, they have an especially hard time determining decay rates. This is partly because students lose sight that the "1" in the expression $1 - b$ is the decimal form of 100%.

Because of the previous work in this section, students have an easy time with all parts of Problem 15 except Part (e).

SECTION 10.4 LECTURE NOTES

Objectives

1. Solve an equation of the form $ab^n = k$ for the base b.
2. Use two points to find an equation of an exponential curve.
3. Use two points to find an equation of an exponential model.

OBJECTIVE 1

Find all real-number solutions.

1. $b^2 = 9$
2. $b^3 = 27$
3. $3b^4 = 48$
4. $5b^5 = 95$
5. $b^4 = -67$

Discuss how Problems 1–5 suggest how to solve equations of the form $b^n = k$ for b.

Solving Equations of the Form $b^n = k$ for b

To solve an equation of the form $b^n = k$ for b,

1. If n is odd, the real-number solution is $k^{1/n}$.
2. If n is even and $k \geq 0$, the real-number solutions are $\pm k^{1/n}$.
3. If n is even and $k < 0$, there is no real-number solution.

Find all real-number solutions. Round all results to the second decimal place.

6. $2.7b^4 - 5.3 = 351.9$
7. $\frac{5}{6}b^4 - \frac{3}{8} = \frac{7}{12}$
8. $\frac{b^7}{b^2} = \frac{83}{4}$

OBJECTIVE 2

Find an approximate equation $y = ab^x$ of the exponential curve that contains the given pair of points. Round the value of b to two decimal places.

9. $(0, 7)$ and $(6, 273)$
10. $(0, 95)$ and $(3, 17)$

Show that if we divide the left sides and divide the right sides of the equations $5 = 5$ and $7 = 7$, we obtain the true statement $\frac{5}{7} = \frac{5}{7}$.

Dividing Left Sides and Right Sides of Two Equations

If $a = b$, $c = d$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{c} = \frac{b}{d}$$

Find an approximate equation $y = ab^x$ of the exponential curve that contains the given pair of points. Round the values of a and b to two decimal places.

11. $(3, 6)$ and $(7, 85)$
12. $(4, 47)$ and $(9, 6)$

OBJECTIVE 3

13. The tuition rates at Princeton University are shown in the following table for the academic years ending in the displayed year.

Year	Tuition Rate (thousands of dollars)
1950	0.6
1960	1.5
1970	2.4
1980	5.6
1990	14.4
2000	24.6
2010	35.3
2015	41.8

Source: *Princeton University*

Let y be the tuition rate (in thousands of dollars) at x years since 1950.

a. Construct a scatterplot.

b. Find an equation of a model.

c. What is the base b of the model $\hat{y} = ab^x$? What does it mean in this situation?

d. Estimate the tuition in 2013.

14. The numbers of viewers of the Major League Baseball (MLB) All-Star Game are shown in the following table for various years.

Year	Numbers of Viewers (millions)
1982	34
1985	28
1990	24
1995	20
2000	15
2005	12
2010	12
2014	11

Source: *Nielsen Media Research, Inc*

Let y be the number (in millions) of viewers at x years since 1980.

a. Construct a scatterplot.

b. Is the association positive, negative, or neither? What does this mean in this situation?

c. Find an equation of a model.

d. What is the coefficient a of the model $\hat{y} = ab^x$? What does it mean in this situation?

e. What is the base b of the model $\hat{y} = ab^x$? What does it mean in this situation?

HW 1, 3, 11, 15, 21, 33, 35, 45, 53, 55, 59, 61, 63, 73

SECTION 10.4 DETAILED COMMENTS

In this section, students will find an equation of an exponential model that contains two given points. Although this method does not generalize to finding the exponential regression equation (Section 10.5), interpreting the meaning of a model and using it to make predictions is good preparation for Section 10.5.

OBJECTIVE 1 COMMENTS

Problems 1–5 relate nicely to previous work done in Section 10.2 on rational exponents. Students tend to forget that there can sometimes be two solutions, so I remind them about this several times. For Problems 3, 4, 6, and 7, I emphasize that we need to isolate the power b^n to one side of the equation. Problem 8 is excellent preparation for the work required to find an exponential equation that contains two given points in which neither point is the y-intercept.

OBJECTIVE 2 COMMENTS

Students have an easy time finding an exponential equation if one of the two given points is the y-intercept.

Although it will take some practice, students can become very successful at finding equations in which neither of the given points is the y-intercept. I like this method, because students have the rare opportunity to witness a meaningful application of the quotient property for exponents.

For Problem 11, I tell students to write the system

$$\begin{aligned} 85 &= ab^7 \qquad \text{Equation 1} \\ 6 &= ab^3 \qquad \text{Equation 2} \end{aligned}$$

so that the equation with the larger exponent, $85 = ab^7$, is on top. That way, when students later subtract the exponents, the difference of the exponents will be positive. When many students simplify the sides of the equation $\frac{85}{6} = \frac{ab^7}{ab^3}$, they want to perform the same operation with the 85 and 6 as with the exponents 7 and 3. So, they either divide 85 by 6 and divide 7 by 3, or they subtract 6 from 85 and subtract 3 from 7.

There is a way to avoid these pitfalls. Instead of dividing the left sides and dividing the right sides of equations (1) and (2), you can solve equation (2) for a: $a = \frac{6}{b^3}$. Next, substitute $\frac{6}{b^3}$ for a in equation (1): $85 = \frac{6}{b^3} \cdot b^7$ and solve for b. Note that solving equation (2) for a is preferable, because if we had solved equation (1) for a, the substitution would've led to the equation $6 = 85b^{-4}$, which contains a negative exponent.

Students usually need a reminder on how to use technology to verify their work.

It is a good idea to compare and contrast equations of the form $ab^n = c$ (e.g. $5b^7 = 95$) with exponential functions (e.g. $y = 5(7)^x$).

GROUP EXPLORATION: Using Trial and Error to Find an Equation of a Model

This activity serves as a nice review of how to interpret the coefficient and base of an exponential model. It also provides the big picture of this section and helps students discover how they can make adjustments to models found throughout this section.

OBJECTIVE 3 COMMENTS

When finding equations of exponential models, it can be challenging to select two "good points" because it is hard to imagine an exponential curve from just two points that it contains. Using the trial and error method described in this section's exploration can help fine-tune a found equation.

When preparing a lecture, I often "cheat" by determining a "good" pair of points by graphing the regression exponential model and selecting two data points that lie closest (or on) the curve. However, as I've mentioned before, I require my students to derive (nonregression) equations of models by hand, because I want students to know those fundamental skills of the course.

Just as with linear modeling, some students will want to skip constructing a scatterplot and simply use the first and last data points. I remind students that there are two problems with this strategy:

- The association might not be exponential.
- If the association is exponential, the first and last data points may not be the best pair of points to use to find an equation of a model.

In Problem 13, students tend to be surprised how much Princeton's tuition has increased since 1950.

Students tend to know how to interpret the base of an increasing exponential model [Problem 13(c)] by this section but continue to struggle with doing so for a decreasing exponential model [Problem 14(e)].

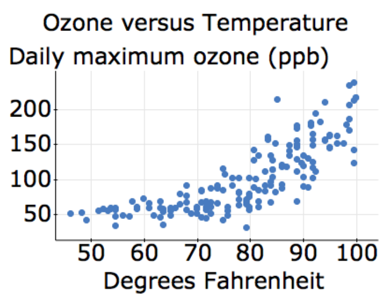
SECTION 10.5 LECTURE NOTES

Objectives

1. Compute and interpret the *exponential correlation coefficient*.
2. Compute and interpret residuals when working with an exponential model.
3. Find the sum of squared residuals for an exponential model.
4. Find an equation of an *exponential regression model*.
5. Use a residual plot to help determine whether an exponential regression curve is an appropriate model.
6. Identify influential points for an exponential regression model.
7. Compute and interpret the *exponential coefficient of determination*.

OBJECTIVE 1

1. The following scatterplot describes the association of daily maximum temperatures in New York City versus daily maximum ozone levels, which are measured in parts per billion (ppb). Let y be the maximum ozone (in ppb) for a day in New York City with maximum temperature x degrees Fahrenheit. Describe the four characteristics of the association (Source: *NAST*).



Warning: Because the study is observational, the positive ozone association does *not* mean that an increase in temperature *causes* an increase in ozone level.

Properties of the Exponential Correlation Coefficient

Assume r is the exponential correlation coefficient for the association between two numerical variables. Then

- The values of r are between -1 and 1, inclusive.
- If r is positive, then the variables are positively associated.
- If r is negative, then the variables are negatively associated.
- If $r = 0$, there is no exponential association.
- The larger the value of $|r|$, the stronger the exponential association will be.
- If $r = 1$, then the data points lie exactly on an exponential curve and the association is positive.
- If $r = -1$, then the data points lie exactly on an exponential curve and the association is negative.

- If the exponential correlation coefficient is 0, then there is no exponential association, but there may be some other type of association.
- Even if an association is strongly linear, the exponential coefficient might be quite close to 1. So, to determine the strength and type of an association, we should inspect the scatterplot for the data as well as compute one or more correlation coefficients.

OBJECTIVE 2

2. The numbers of teachers with various years of experience are shown in the following table.

Years of Experience	Number of Teachers (thousands)
5	185
10	145
15	90
20	63
25	54
30	50
35	38

Source: *R. Ingersoll and E. Merrill, University of Pennsylvania, original analyses for NCTAF of the Schools and Staffing Survey*

Let y be the number (in thousands) of teachers with x years of experience.

a. Describe the four characteristics of the association. Compute and interpret an appropriate correlation coefficient r as part of your analysis.

b. We can use the data points $(10, 145)$ and $(25, 54)$ to find the exponential model $\hat{y} = 280(0.936)^x$. How well does the model fit the data?

c. Use the model to predict the number of teachers who have 5 years experience.

d. Find the residual for the prediction you made in Part (c). What does it mean in this situation? What does it mean about the positions of the data point and the exponential curve?

Residuals for Data Points Above, Below, or On a Line

Suppose that some data points are modeled by an exponential curve.

- A data point on the curve has residual equal to 0.
- A data point above the curve has positive residual. [Draw a figure.]
- A data point below the curve has negative residual [Draw a figure.]

OBJECTIVE 3

3. In Problem 2, we modeled the teacher association with the function $\hat{y} = 280(0.936)^x$. Find the sum of squared residuals for the model.

OBJECTIVE 4

Definition *Exponential Regression Model*

For a group of points, the **exponential regression function** is the exponential function with the least sum of squared residuals of all exponential functions. Its graph is called the **exponential regression curve** and its equation is called the **exponential regression equation**, written

$$\hat{y} = ab^x$$

where $a \neq 0$, $b > 0$, and $b \neq 1$. The **exponential regression model** is the exponential regression function for a group of *data* points.

4. Pointing a laser at aircraft, which can temporarily blind pilots, is a serious offense with a maximum punishment of 20 years in prison and a $250,000 fine. The numbers of laser incidents involving aircraft are shown in the following table for various years.

Year	Number
2005	283
2006	446
2007	675
2008	988
2009	1527
2010	2836

Source: *Federal Aviation Administration*

Let y be the number of laser incidents involving aircraft in the year that is x years since 2000.

a. Construct a scatterplot.

b. Find a regression equation to describe the data. How well does the model fit the data points?

c. What is the y-intercept? What does it mean in this situation?

d. Estimate the percentage increase in laser incidents per year.

OBJECTIVE 5

Using a Residual Plot to Help Determine Whether an Exponential Regression Function is an Appropriate Model

The following statements apply to a residual plot for an exponential regression function.

- If the residual plot has a pattern where the dots do not lie close to the zero residual line, then there is either a non-exponential association between the explanatory and response variables or there is no association. [Draw a figure.]
- If a dot lies much farther away from the zero residual line than most or all of the other dots, then the dot corresponds to an outlier. If the outlier is neither adjusted nor removed, the exponential regression function may *not* be an appropriate model. [Draw a figure.]
- The vertical spread of the residual plot should be about the same for each value of the explanatory variable. [Draw a figure.]

OBJECTIVE 6

5. The fatality rates from automobile accidents are shown in the following table for various years.

Table 2.2: Years versus Fatality Rates From Automobile Accidents

Year	Fatality Rate (deaths per 100 million miles)
1921	24.1
1930	15.1
1940	10.9
1950	7.2
1960	5.1
1970	4.7
1980	3.3
1990	2.1
2000	1.5
2010	1.1

Source: *National Highway Traffic Safety Administration*

Let y be the fatality rate (number of deaths per 100 million miles driven) at x years since 1900.

a. Construct a scatterplot.

b. Find the equation of a model. Does the model fit the data points well?

c. Determine whether there are any outliers or influential points.

d. Find the exponential decay rate in fatality rates.

OBJECTIVE 7

Exponential Coefficient of Determination

Let r be the exponential correlation coefficient for a group of data points. The exponential coefficient of determination, r^2, is the proportion of the variation in the response variable that is explained by the exponential regression curve.

When we report a coefficient of determination, it is important that we state whether it is for a linear regression line or an exponential regression curve.

6. The University of Michigan offers a $500,00 life insurance policy. Monthly rates for nonsmoking faculty are shown in the following table for various ages.

Age Group (years)	Age Used to Represent Age Group (years)	Monthly Rate (dollars)
30–34	32	15.00
35–39	37	18.50
40–44	42	26.00
45–49	47	46.00
50–54	52	75.50
55–59	57	118.00
60–64	62	195.50
65–69	67	326.60

Source: *University of Michigan*

Let y be the monthly rate (in dollars) for a nonsmoking faculty member at x years of age.

a. Construct a scatterplot.

b. Find a regression equation. Does the graph come close to the data points?

c. Describe the four characteristics of the association. Compute and interpret an appropriate correlation coefficient as part of your analysis.

d. Compute and interpret an appropriate coefficient of determination.

e. Predict the monthly rate for a 40-year-old nonsmoking faculty member.

HW 1, 3, 5, 11, 13, 17, 21, 23, 25, 29, 31, 33, 35, 39, 45, 49, 51, 61

SECTION 10.5 TEACHING TIPS

In this section, students use exponential regression functions to model data. Although this material is probably not included in your introductory statistics course, there are so many concepts that are similar to those addressed in Section 9.3 (linear regression) that this section serves as a great reinforcer of modeling.

OBJECTIVE 1 COMMENTS

Problem 1 serves as a nice review of the four characteristics of an association and more importantly introduces the concept of an exponential model. Because there are so many data points, the values are not provided. But you could quickly plot a bunch of points that roughly fit the pattern and have students do the same. Then you could sketch a curve that resembles the exponential regression curve shown in Fig. 45 on page 683 of the textbook.

Or you could use data from an exercise in the homework section. If you would like to work with a large data set, you could download the data for Exercise 49 or 50, and analyze it using technology.

When discussing the exponential correlation coefficient, I emphasize the importance of distinguishing it from the linear correlation coefficient, but I also point out the similarities of the properties of the two coefficients.

Up until now, students tend to think that if the linear correlation coefficient is equal to 0, then there is no association despite all warnings I have given that all we really know is that there is no *linear* association. Now that two types of models are on the playing field, this is a good opportunity to revisit this issue.

Another issue is that an association might be approximately linear and yet the values of the linear and exponential correlation coefficients are both close to 1. The same is true for some associations that are approximately exponential.

The previous two paragraphs underscore the importance of considering the scatterplot as well as the correlation coefficients.

OBJECTIVES 2–4 COMMENTS

Students have a fairly easy time with these objectives because of their previous work with linear regression models (Section 9.3).

GROUP EXPLORATION: Comparing a Linear Model with an Exponential Model

This exploration is a nice activity for students to compare the growth predicted by linear and exponential models. The exploration also sends a subtle message (which you could emphasize) that it is foolish to extrapolate from just a few data points, especially just two data points!

OBJECTIVE 5 COMMENTS

When discussing the bulleted statements in the box for this objective, I sketch figures such as those in Figs. 66–68 on page 691 of the textbook. To add some context to the discussion you could go over Example 5 on pages 692 and 693 of the textbook.

OBJECTIVE 6 COMMENTS

Students quickly pick up on the concept of influential points for exponential models because of their previous work with influential points for linear models (Section 9.3).

OBJECTIVE 7 COMMENTS

Most students struggle with interpreting the linear coefficient of determination (Section 9.3) and resort to memorizing an interpretation. As a result, most students also struggle with the meaning of the exponential coefficient of determination. But because of the repetition, students tend to gain at least a bit more comprehension of this challenging concept.

Chapter 2
Designing Observational Studies and Experiments

Homework 2.1

2. A sample is the part of a population from which data are collected.

4. A sampling method that consistently underemphasizes or overemphasizes some characteristic(s) of the population is said to be biased.

6. a. Andrew Brannan, Roger Collins, Jerry Heidler, Warren Hill, and Darryl Scott.

b. County of conviction, race, age (in years), and time served (in years).

c. County of conviction: Laurens, Houston, Toombs, Lee, and Chatham. Race: Caucasian, African American, Caucasian, African American, and Caucasian. Age, all in years: 66, 55, 37, 54, and 31. Time served, all in years: 14, 37, 15, 23, and 7.

d. Andrew Brannan: $66-14=52$ years. Roger Collins: $55-37=18$ years. Jerry Heidler: $37-15=22$ years. Warren Hill: $54-23=31$ years. Darryl Scott: $31-7=24$ years.

e. The youngest when convicted was Roger Collins, at age 18 years.

8. a. Mars Polar Lander, Opportunity, Mars Reconnaissance Orbiter, Yinghuo-1, and Mars Orbiter Mission.

b. Mission, outcome, cost (in millions of dollars), and launch mass (in pounds).

c. Mission: lander, rover, orbiter, orbiter, and orbiter. Outcome: failure, success, success, failure, and success. Cost, all in millions of dollars: 110, 400, 720, 163, and 74. Mass, all in pounds: 640, 408, 4810, 29,100, and 2948.

d. Mars Polar Lander, $110/640=0.172$; Opportunity, $400/408=0.980$; Mars Reconnaissance Orbiter, $720/4810=0.150$; Yinghuo-1, $163/29{,}100=0.006$; Mars Orbiter Mission, $74/2948=0.025$; all in millions of dollars per pound.

e. Yinghuo-1, 0.006 million dollars per pound.

10. a. The variable is whether people think the Affordable Care Act goes too far.

b. The sample is the 1000 likely voters who were polled.

c. The population is all likely voters.

12. a. The variable is whether people think the United States does too much in solving the world's problems.

b. The sample is the 1501 adults who were surveyed.

c. The population is all American adults.

14. a. The variable is whether people believe that police can protect them from violent crime.

b. The sample is the 776 Caucasians who were surveyed.

c. The population is all Caucasians.

16. a. The variable is whether parents will limit their children's choices of college based on cost.

b. The sample is the 1000 parents who were surveyed.

c. The population is all parents with college-bound teenagers ages 16 to 18 years.

18. a. The researchers were trying to answer whether simvastatin heals ulcers.

b. The sample is the 66 ulcer patients who were tested.

c. The population is all patients with ulcers.

d. The researchers concluded that the drug heals ulcers. The study is part of inferential statistics because it uses sample data to draw a conclusion about a population.

20. **a.** The researchers were trying to answer whether autistic adults are less able to process social rewards than monetary rewards.

b. The sample is the 20 adults who were in the study.

c. The population is all adults.

d. The conclusion is that adults with autism are less able to process social rewards than adults without autism. It is part of inferential statistics because it uses data from a sample to make a statement about the population.

22. **a.** The researchers were trying to answer whether women who are more sexually confident are also more likely to achieve sexual satisfaction.

b. The sample is the 45 women who took the online survey.

c. The population is all women.

d. The conclusion is that women who are more sexually confident are also more likely to achieve sexual satisfaction. It is part of inferential statistics because it draws a conclusion about a population, based on data taken from a sample.

24. **Using a TI-84:** Mariah, Rani, May, and Brenton. **Using StatCrunch:** Rani, Brenton, Kali, and Shea.

26. **a.** **Using a TI-84:** Samuel, Paola, Joshua, Win, and Phoebe. **Using StatCrunch:** Win, Taja, Nathan, Samuel, and Jeffrey.

b. **Using a TI-84:** $3 \div 5 = \frac{3}{5}$. **Using StatCrunch:** $2 \div 5 = \frac{2}{5}$.

Using this result to describe the sample is part of descriptive statistics because it does not draw conclusions about a larger group.

c. **Using a TI-84:** Samuel, Jeffrey, Phoebe, Win, and Arnold. **Using StatCrunch:** Karen, Win, Monique, Arnold, and Jeffrey.

d. **Using a TI-84:** $3 \div 5 = \frac{3}{5}$. **Using StatCrunch:** $2 \div 5 = \frac{2}{5}$.

e. **Using a TI-84:** Yes. **Using StatCrunch:** Yes. However, not all randomly selected samples of size 5 will give the same results. Because of the randomness of choosing the sample, different samples could be collected.

28. **a.** **Using a TI-84:** Dimitrios, Aksana, Jessica, Luis, Fan, Chris, and Gauri. **Using StatCrunch:** Gauri, Chris, Aksana, Fadi, Devin, Jose, and Julia.

b. **Using a TI-84:** $4 \div 7 = \frac{4}{7}$. **Using StatCrunch:** $5 \div 7 = \frac{5}{7}$.

c. $9 \div 14 = \frac{9}{14}$

d. **(Either technology)** No. The difference between the answers in b. and c. is due to sampling error.

e. For many random samples of size 7, the proportion of students who think it is more important to improve student success would not be the same on each sample and would not all be the same as the proportion for all 14 students. This is due to sampling error.

30. **a.** $10571 \div 20329 = 0.520$

b. $523 \div 1000 = 0.523$

c. No, the result from part (b) does not equal the result from part (a). This is due to sampling error.

d. It would be inferential statistics because it draws a conclusion about a population based on data from a sample.

32. Do you access Facebook every day or not access Facebook every day?

34. Do you have a regular exercise program or not have one?

36. The method favors students who take evening classes, so it has sampling bias.

38. The wording of the question is not clear (since it asks whether they post daily and also if they like Facebook), so it has response bias.

40. Because 9 out of 12 subjects did not respond, the method has nonresponse bias. It also has response bias because an adult who neglects their children is unlikely to say "yes."

42. Because 93% of those who were contacted did not give a response, the method has nonresponse bias.

44. The method has response bias because the scale of numbers for the response is not consistent.

46. This method has sampling bias because it favors cars that pass by during the morning rush hour.

48. The method has response bias because the question addresses more than one issue. It also has sampling bias, because it excludes people who do not watch this TV show.

50. **a.** The survey is likely to have nonresponse bias because participation is voluntary. It probably also has sampling bias, since it favors diners who want to complain about their experience.

b. The survey likely has less nonresponse bias because of the incentive. It likely has less sampling bias because of the incentive. It may have more response bias, since the future discount likely improves the customer's satisfaction with the restaurant.

52. Using samples involves less time, less money, and less labor than taking a census.

54. Answers may vary.

56. Answers may vary.

58. Sampling error refers to the random nature of the sample; nonsampling error refers to the design of the sampling process.

Homework 2.2

2. We should always round down when calculating k for systematic sampling.

4. False. Convenience sampling should never be used because such samples usually do not represent the population well.

6. Cluster sampling is the method because the 40 blocks are randomly selected, but every adult resident of each block is surveyed.

8. Systematic sampling is the method because every 100^{th} car fuel tank after the first selected tank is tested.

10. Convenience sampling is the method because the employee only surveys the Americans whom she can contact easily.

12. Stratified sampling is the method because registered voters are randomly sampled within each of three strata: Republicans, Democrats, and Independents.

14. Simple random sampling is the method because sample members are selected at random from the whole population.

16. The method is systematic sampling because the pollster surveys every 10^{th} person after the first to be selected.

18. The method is simple random sampling because members are randomly selected from all the paying guests in the past month.

20. **a.** $420 \div 50 = 8.4$; round down to 8.

b. **Using a TI-84:** 4. **Using StatCrunch:** 5.

c. **Using a TI-84:**
$4, 4+8=12, 12+8=20, 20+8=28,$
$28+8=36$

Using StatCrunch:
$5, 5+8=13, 13+8=21, 21+8=29,$
$29+8=37.$

22. **a.** $47,756 \div 150 \approx 318.4$; round down to 318.

b. **Using a TI-84:** 130. **Using StatCrunch:** 168.

c. **Using a TI-84:**
$130, 130+318=448, 448+318=766,$
$766+318=1084, 1084+318=1402$

Using StatCrunch:
$168, 168+318=486, 486+318=804,$
$804+318=1122, 1122+318=1440.$

24. From the police department, survey $0.62(70)=43$ employees. From the fire department, survey $0.29(70)=20$ employees. From the judicial department, survey $0.09(70)=6$ employees. Since $43+20+6=69,$ one more person should be selected at random from one of the three departments, also selected at random, to meet the goal of a sample size of 70.

26. The total number of students in the four schools is 1936 + 1466 + 899 + 83 = 4384. The proportions are: Franklin High School, $1936 \div 4384 = 0.442;$ Centennial High School, $1466 \div 4384 = 0.334;$ Fred J. Page High School, $899 \div 4384 = 0.205;$ Middle College High School, $83 \div 4384 = 0.019.$

The numbers of students in the sample from each high school, respectively, are:
$0.442(50) \approx 22;$ $0.334(50) \approx 17;$
$0.205(50) \approx 10;$ $0.019(50) \approx 1.$

28. The total number of applicants to the five graduate business majors is 85 + 368 + 109 + 90 + 83 = 735. The proportions are: Accounting, $85 \div 735 = 0.116;$ Finance, $368 \div 735 = 0.501;$ Information Risk and Operations Management, $109 \div 735 = 0.148;$ Management, $90 \div 735 = 0.122;$ Marketing, $83 \div 735 = 0.113.$

The numbers of applicants in the sample from each major, respectively, are:
$0.116(100) \approx 12;$ $0.501(100) \approx 50;$
$0.148(100) \approx 15;$ $0.122(100) \approx 12;$
$0.113(100) \approx 11.$

30. The proportions of each of the strata: Female undergraduate, $10,588 \div 29,135 = 0.363;$ female graduate, $4475 \div 29,135 = 0.154;$ female professional, $1421 \div 29,135 = 0.049;$ male undergraduate, $7762 \div 29,135 = 0.266;$ male graduate, $3736 \div 29,135 = 0.128;$ male professional, $1153 \div 29,135 = 0.040.$

The numbers of students in the sample from each of the strata, respectively:
$0.363(1200) \approx 436;$ $0.154(1200) \approx 185;$
$0.049(1200) \approx 59;$ $0.266(1200) \approx 319;$
$0.128(1200) \approx 154;$ $0.040(1200) = 48.$ Because of rounding, the sample would actually have 1201 students.

32. **Using a TI-84:** Republicans Reagan, Farnsworth, Biggs, Yarbrough, Yee; Democrats Tovar, Bedford, Bradley, McGuire.

Using StatCrunch: Republicans Farnsworth, Crandell, Melvin, Yee, Worsley; Democrats Bradley, Hobbs, McGuire, Gallardo.

34. The number of clusters is $75 \div 25 = 3.$ **Using a TI-84:** Red Sox, Royals, Athletics. **Using StatCrunch:** Royals, Tigers, Indians.

36. Stratified sampling is being used, where the strata are farmers and city or suburban residents because each of the two strata is sampled separately.

38. Cluster sampling would require the least money and effort because surveying each resident on a selected block involves less travel time than a simple random sample. The city would decide on a sample size, identify a frame of all the blocks in Los Angeles, then divide the desired sample size by the smallest number of residents per block. The required number of blocks would be randomly selected, and then every resident on the selected blocks would be surveyed.

40. Simple random sampling is the best method, since Barnes & Noble® has a frame and the surveying can be done using e-mail. The company would choose a desired sample size, then randomly select that many online customers from the frame.

42. **a.** If each city block is treated as a cluster, the city would decide on a sample size, identify a frame of all the city blocks in Kansas City, and then divide the desired sample size by the smallest number of residents per block. That many blocks would be randomly selected, and every resident on the selected blocks would be interviewed in person.

b. To conduct stratified sampling, the data collectors would first choose a total sample size, then identify what proportions of registered voters are Democrats, Republicans, Independents, and so on, and compute the sample size for each of the strata by multiplying the total sample size by the respective proportions. The required number for each of the strata would then be randomly selected.

c. Cluster sampling would be easier than stratified because the data collectors would only need to visit the selected blocks in person.

d. Stratified sampling would probably give better results if the sample size is small because it is more likely to get a sample that represents the whole city.

44. a. The police used systematic sampling when the traffic was heavier because they stopped every fourth car.

b. Sampling every third and fourth car is not systematic sampling because it violates the pattern of selecting every *k*th person, animal, or thing.

c. In lighter traffic, the police could have pulled over every other car. They would still be stopping two cars out of every four, but they would be using systematic sampling.

46. Answers may vary.

48. Answers may vary.

50. Answers may vary.

Homework 2.3

2. In a double-blind study, neither the individuals nor the researcher in touch with the individuals know who is in the treatment group(s) and who is in the control group.

4. A lurking variable is a variable that causes both the explanatory and response variables to change during the study.

6. a. The treatment groups are the second and third groups because they receive training in addition to that which the first group receives.

b. The study is an experiment because each participant is assigned to one of the treatment and control groups.

c. Random assignment means that the researchers use random sampling to decide which participants are in which groups. For example, the researchers could create a frame of all 50 older adults, randomly choose 17 of them to be in the second group, randomly choose another 17 for the third group, and assign the other 16 to the first group.

d. The sample is the 50 older adults in the study. The population is all older adults.

8. a. The explanatory variable is the type of training that participants did. The response variable is walking speed when an older person is performing a mental task at the same time.

b. The researchers concluded that the training methods for improving walking speed that include both physical and mental tasks are more effective than those used in the first group when an older adult is performing a mental task at the same time. Causality can be concluded because the participants were randomly assigned to the treatment and control groups.

c. The first group's confidence may have increased because the group's training was easier than that of the second and third groups.

10. a. It makes sense that the study is observational because the researchers cannot randomly assign anyone to have a major bone fracture.

b. It would be unethical to randomly assign an older adult to "treatment" when that treatment requires a major bone fracture.

c. The sample is the people whose records were studied. The population is all adults over 60.

d. The explanatory variable is whether or not the person had a major bone fracture. The response variable is the death rate.

e. The conclusion is that the death rate for older adults who have had a major fracture is higher than the death rate for older adults who have never had a major fracture. Only an association can be concluded because there was no random assignment to treatment or control groups.

12. a. The study is observational because there is no random assignment.

b. Since a placebo has no proven medical effect, it would be unethical for the doctors to administer it instead of prescribing an effective remedy for an acute cough.

c. The sample is the 241 children in the study. The population is all children with an acute cough.

d. The researchers concluded that children who took levodroprophizine recovered better from coughs than children who took other cough syrups. Only an association can be concluded because there was no random assignment to treatment and control groups.

e. Researchers could be influenced, consciously or unconsciously, by earning a salary from the company that manufactures levodroprophizine. The two researchers' disclosing that they work for the company encourages other researchers who do not work for the company to repeat the experiment and see if they get similar results.

14. a. The treatment group is the one that received a gift card plus monetary rewards based on their class work. The control group is the one that received only a gift card.

b. The study is an experiment because the researchers randomly assigned students to the treatment and control groups.

c. Random assignment means that the researchers chose some students at random to receive the treatment and others to be the control group.

d. The sample is the 1019 students in the study. The population is all low-income community college students who are parents.

16. a. It would be impossible to use a placebo for a monetary reward. A participant would quickly discover whether they have real or fake money.

b. In order to be double-blind, the participants would have to not know whether they will receive monetary rewards, which would remove the incentive to earn more credits.

c. The explanatory variable is whether or not students would receive an additional monetary reward based on the credits they earn. The response variable is the number of credits the students earned.

d. The researchers concluded that monetary rewards increase the number of credits earned by low-income community college students who are parents. Causality can be concluded because students were randomly assigned to the treatment and control groups.

e. Mistakenly giving monetary rewards to some of the control group introduces a possible lurking variable; believing that they will be rewarded no matter how many classes they pass could decrease students' motivation to do well.

18. a. This is an observational study because there is no differentiation into treatment and control groups and no random assignment.

b. The sample is the 210 motorists whose behavior was observed. The population is all motorists in Chicago.

c. The explanatory variable is whether the crosswalk was marked or unmarked. The response variable is whether or not the motorist stopped.

d. The researchers concluded that motorists are more likely to follow the Must Stop law at marked than at unmarked crosswalks. Only association can be concluded because there was no random assignment into treatment and control groups.

e. We cannot assume the conclusion is also true in Prairie City because the habits of motorists could be very different in a small farm town than in a large city. Motorists are also more likely to be acquainted with the pedestrians in a small town, which could influence their behavior.

20. **Using a TI-84:** Lenovo ThinkPad X240, Dell Latitude 7440, Dell XPS 13, Acer Aspire S7. **Using StatCrunch:** Lenovo ThinkPad X240, Acer Aspire S7, Samsung ATIV Book 9, HP Spectre 13 Ultrabook.

This is an example of random assignment because the ultrabooks were randomly assigned to the treatment and control groups.

22. **Using a TI-84:** Palm Beach State College, Virginia Military Institute, Kean University, SUNY College at Oneonta, Angelo State University, Langston University. **Using StatCrunch:** Eastern Illinois University, Lander University, Boise State College, Langston University, Kean University, Oakland University.

This is an example of random assignment because the colleges were randomly assigned to the treatment and control groups.

24. a. Because this is an observational study, the student cannot conclude causality. There is likely to be response bias; people may overstate the amount of exercise they get. Also, motivation to stay healthy could be a lurking variable affecting whether or not people smoke (or quit smoking) and whether or not they exercise.

b. The student could find people who currently smoke and randomly assign the smokers to one of two groups, treatment or control. The treatment group would exercise on a regular basis, while the control group would not. It would be impossible for the study to be double-blind since the participants know what treatment they have; however, it could be a blind study if the researcher(s) in contact with the participants do not know which is in each group.

26. a. Although there is an explanatory variable, there is no control group (or random assignment). This is an observational study, and causality cannot be concluded. There is also a confounding variable since the reward is not just monetary. Students could also be motivated to write a better project for the public honor of appearing in the newspaper.

b. The students could be randomly assigned to one of two groups, treatment or control. The control group would have the same assignment but no prize for the best project. The treatment group will be told that the best project will win a $25 prize. It could be a blind study if the projects are graded by another teacher who does not know which group the students were in.

28. a. Although there is an explanatory variable, there is no control group (or random assignment). This is an observational study and causality cannot be concluded. There could be sampling bias and response bias because the survey is online; it could favor people with greater incomes who could afford newer cars, and voluntary response could favor people whose mileage improved after using the additive. The financial state of the respondents could also be a lurking variable, since it could influence both the ability of car owners to respond online and their ability to keep the car in good running condition.

b. The magazine could select a random sample of drivers and randomly assign them to either a treatment group or a control group. The treatment group would be given the additive and asked to use it, while the control group would not. The gas mileage of each car would be recorded at the beginning of the study, and again a month afterwards.

30. Researchers could recruit a sample of volunteers who suffer from insomnia and randomly assign them to either a treatment or a control group. Volunteers in the treatment group would receive the drug, and volunteers in the control group would receive a sugar pill. The study could be double-blind, with one researcher labeling pill vials but not giving the code to the researchers in contact with the patients. The extent of insomnia would be measured at the beginning of the study and again a month later.

32. A sample of students who have the same class with the same professor could be randomly assigned to a treatment or control group. Each student could be given a recording of the professor's lectures, but only the treatment group would be instructed to listen to the lectures while they sleep. The professor (or someone else who grades the tests) could be blind to which students are in which group.

34. With random assignment, the frame is a sample, and sampling divides the individuals into treatment group(s) and a control group. Stratified sampling does not involve treatment and control groups; it defines groups with similar characteristics (strata) that already exist in the population and creates frames for each of the strata.

36. The key difference in the designs of an experiment and an observational study is the presence of both treatment group(s) and a control group to which individuals are randomly assigned. Random assignment to one of the groups makes it possible to isolate the effects of the treatment from other factors.

38. Answers may vary.

40. The student is correct. The objects do not know whether they are in a treatment or control group.

Chapter 2 Review Exercises

1. a. The individuals are the countries: Bahrain, Iraq, Israel, Kuwait, and Saudi Arabia.

b. The variables are government, population (in millions), 2012 military expenditure (in billions of dollars), and oil production (in billions of barrels per day).

c. For the variable government: monarchy, republic, republic, monarchy, and monarchy. For the variable population, all in millions: 1.3, 31.9, 7.7, 2.7, and 26.9. For 2012 military expenditure, all in billions of dollars: 0.92, 5.69, 15.54, 5.95, and 54.22. For oil production, all in billions of barrels per day: 0.05, 2.99, 0.20, 2.69, and 9.90.

d. Bahrain, $0.92 \div 0.05 = 18.4$; Iraq, $5.69 \div 2.99 = 1.903$; Israel, $15.54 \div 0.20 = 77.7$; Kuwait, $5.95 \div 2.69 = 2.212$; Saudi Arabia, $54.22 \div 9.90 = 5.477$; all in dollars per barrel per day.

e. Israel has the greatest ratio of 2012 military expenditure to oil production, 77.7 dollars per barrel per day.

2. a. The variable is whether American adults experience a lot of happiness and enjoyment.

b. The sample is the 500 American adults who were telephoned.

c. The population is all American adults.

3. a. **Using a TI-84:** Antoine, Jacob, Ruben, Sandra, Dante, Jose. **Using StatCrunch:** Mario, Sandra, Antoine, Jacob, Alyssa, John.

b. **Using a TI-84:** $3 \div 6 = \frac{1}{2}$. **Using StatCrunch:** $3 \div 6 = \frac{1}{2}$. Using this result to describe the sample is part of descriptive statistics because it does not generalize the results of the sample to describe the population.

c. The proportion who prefer comedies is $\frac{5}{12} = 0.417$.

d. **Using a TI-84 or Using StatCrunch:** No, the sample proportion who prefer comedies does not equal the population proportion who prefer comedies. The difference is due to sampling error.

e. No, the proportion in each sample would not equal the proportion of all 12 students who prefer comedies. The difference is due to sampling error.

f. If two researchers perform the same study with different simple random samples of the same size, their inferences will not necessarily be the same because the sample data are not the same.

4. Choose the number of questions you usually ask during one hour of your prestatistics class: 0, 1, 2, 3, or more than 3.

5. The method has sampling bias; the sampling favors people who visit the militia group site.

6. The method has sampling bias, response bias, and nonresponse bias. The sampling favors people who are often in the financial district; some people may exaggerate their salary; 55 of those who were approached declined to answer.

7. Answers may vary.

8. The method is cluster sampling because the researcher selects 50 blocks at random and then surveys each adult resident of those blocks.

9. The method is simple random sampling because Human Resources creates a frame of all U.S. employees and selects at random from that frame.

10. The method is convenience sampling because the pollster only surveys people who are easy to find, without attention to any random selection.

11. The method is stratified sampling because the researchers identify two strata (people with landlines and people with cell phones), and randomly select numbers in each of the strata.

12. The method is systematic sampling because the manager surveys every eighth person leaving the store after the first person is randomly selected.

13. **a.** $105{,}000 \div 800 = 131.25$; round down to 131.

b. **Using a TI-84:** 57. **Using StatCrunch:** 47.

c. **Using a TI-84:** 57, $57+131=188$; $188+131=319$; $319+131=450$; $450+131=581$. **Using StatCrunch:** 47, $47+131=178$; $178+131=309$; $309+131=440$; $440+131=571$.

14. The total number of employees (in thousands) is 82 + 57 + 30 + 19 + 8 + 3 = 199. The proportions are: commercial airplanes, $82 \div 199 \approx 0.412$; defense, space, and security, $57 \div 199 \approx 0.286$; corporate, $30 \div 199 \approx 0.151$; engineering, operations, and technology, $19 \div 199 \approx 0.095$; shared services group, $8 \div 199 \approx 0.040$; other, $3 \div 199 \approx 0.015$.

The numbers of employees in the sample from each group, respectively, are: $0.412(80) \approx 33$; $0.286(80) \approx 23$; $0.151(80) \approx 12$; $0.095(80) \approx 8$; $0.040(80) \approx 3$; $0.015(80) \approx 1$.

15. **Using a TI-84:** Democrats Gerratana, Ayala, and Duff; Republicans Frantz and Linares. **Using StatCrunch:** Democrats Looney, Stillman, and Bartolomeo; Republicans Guglielmo and Welch.

16. If the clusters are city blocks, cluster sampling would require the least time and effort. The city would create a frame of all the blocks in the city, select some at random, and then survey each resident of the selected blocks.

17. **a.** The treatment groups are the three groups who receive the drug; the control group is the group who receives a placebo.

b. The study is an experiment because the researchers randomly assigned participants to one of the three treatment groups or to the control group.

c. Random assignment means that the researchers randomly assigned patients to one of the four groups. To accomplish the random assignment, create a frame of the 560 MDD adults. For each treatment group, randomly select 140 different MDD adults to be in the group. The remaining 140 MDD adults are the control group.

d. The sample is the 560 MDD adults in the study. The population is all adults with MDD.

18. **a.** The placebo could be a sugar pill.

b. Neither the study participants nor the researchers in contact with them know which treatment (or placebo) the participants receive. One researcher could have labeled the pill vials with numbers to identify the pills, but not told the code to another researcher who was in contact with the participants.

c. The explanatory variable is the dosage of the drug the person receives. The response variable is the person's HRSD score.

d. The conclusion of the study is that Lu AA21004 successfully lowers MDD adults' HRSD scores. The researchers can conclude causality because adults were randomly assigned to the treatment and control groups.

e. The researchers concluded that the drug tends to lower MDD adults' HRSD scores, but that might not mean that the drug tends to reduce depression in MDD adults.

19. a. The study is observational because mothers were not randomly assigned to the group with eating disorders or the group without eating disorders.

b. It would be impossible to use random assignment in this study because mothers could not start having an eating disorder, or stop having an eating disorder, due to a researcher telling them to.

c. The sample is the mothers who were observed. The population is all mothers with first-born infants.

d. The explanatory variable is whether or not the mother has an eating disorder. The response variable is the level of negative emotions expressed toward the infants during mealtimes.

e. The conclusion of the study is that mothers with eating disorders express more negative emotions toward their first-born infants during mealtimes than mothers without eating disorders. It only describes an association; an observational study cannot conclude causality.

20. Using a TI-84: Elon University, Campbell University, Wellesley College, Columbia College, University of Mount Union. **Using StatCrunch:** Nichols College, Columbia College, Mills College, Rider University, Villanova University. Yes, it is an example of random assignment because the colleges were randomly assigned to the two groups.

21. a. The coordinator did not use random assignment, so she cannot conclude causality. Also, the attendance should include a time requirement because a student who attended the math center for only five minutes once in the entire semester should not be considered a student who used the center. Motivation could be a lurking variable: students who attend the math center might be more motivated, and study harder, than other students.

b. The coordinator could randomly assign some students to a treatment group and others to a control group. The students in the treatment group would, for example, attend the math center for one hour per weekday during the entire semester, while the students in the control group would not attend the math center. After the semester is over, the coordinator could compare the proportion of the treatment group who passed their math classes that semester with the proportion of the control group who passed their math classes that semester.

22. The company could randomly assign some bald people to a treatment group and some to a control group. The treatment group would take the drug and the control group would take a sugar pill. The study could be double-blind. The company would then measure the extent of the individuals' hair growth after 8 months.

Chapter 2 Test

1. a. The individuals are Delaware, Hawaii, Mississippi, Texas, and Wisconsin.

b. The variables are region, number of workers (in thousands), and number of workers in unions (in thousands).

c. Region: East, West, South, South, and Midwest. Number of workers: 370, 549, 1040, 10,877, and 2569, all in thousands. Number of workers in unions: 38, 121, 38, 518, and 317, all in thousands.

d. Delaware, $38 \div 370 \approx 0.1027 = 10.3\%$;
Hawaii, $121 \div 549 \approx 0.2204 = 22.0\%$;
Mississippi, $38 \div 1040 \approx 0.0365 = 3.7\%$;
Texas, $518 \div 10{,}877 \approx 0.04762 = 4.8\%$;
Wisconsin, $317 \div 2569 \approx 0.1234 = 12.3\%$.

e. Hawaii has the largest percentage of workers in unions, 22.0%.

2. **a.** The variable is whether an adult intends to buy wearable technology in the next 12 months.

b. The sample is the 2011 American adults who were surveyed.

c. The population is all American adults.

3. The study has response bias and nonresponse bias. The complex wording of the question may lead customers to give an answer that is not consistent with their opinion (response bias), and the nonresponse rate of 92% indicates nonresponse bias.

4. **Using a TI-84:** Jamie, Jared, Isabel, Lisa. **Using StatCrunch:** Jamie, Brianna, Dan, Michael.

5. The method is cluster sampling. The farmer randomly selects 8 subsections, then measures the total yield from each of those subsections.

6. **a.** $k = 500 \div 80 = 6.25$; round down to 6.

b. **Using a TI-84:** 1. **Using StatCrunch:** 2.

c. **Using a TI-84:** 1; 1+ 6 = 7; 7 + 6 = 13; 13 + 6 = 19; 19 + 6 = 25. **Using StatCrunch:** 2, 2 + 6 = 8, 8 + 6 = 14, 14 + 6 = 20, 20 + 6 = 26.

7. The total number of students is given: 21,471. The proportions are: female undergraduates, $4833 \div 21,471 \approx 0.225$; female graduate students, $1792 \div 21,471 \approx 0.083$; male undergraduates, $9725 \div 21,471 \approx 0.453$; male graduate students, $5121 \div 21,471 \approx 0.239$.

The numbers of students in the sample from each of the strata, respectively, are: $0.225(500) \approx 113$; $0.083(500) \approx 42$; $0.453(500) \approx 226$; $0.239(500) \approx 119$.

8. **a.** The treatment groups are the 4 groups taking different drug dosages; the control group is the group receiving a placebo.

b. The study is an experiment because patients were randomly assigned to the treatment and control groups.

c. Random assignment means that the researchers randomly assigned the patients to the groups. To accomplish this, create a frame of the 361 patients. Then for each of the 4 treatment groups, randomly select 72 patients to be in the group. The remaining 73 adults should be in the control group.

d. The sample is the 361 patients in the study. The population is all Japanese adults with type 2 diabetes.

9. **a.** The placebo could be a sugar pill.

b. Neither the patients nor the researcher(s) in contact with the patients knew which patients were in each group; one researcher could have labeled the pill vials with numbers to identify the pills, but not tell the code to another researcher who was in contact with the individuals.

c. The explanatory variable is the dosage of the drug. The response variable is the glycated hemoglobin level.

d. The conclusion of the study is that the drug successfully lowers glycated hemoglobin levels in Japanese patients with type 2 diabetes. Because treatments and control were randomly assigned, the researchers can claim causality.

e. Researchers could be influenced, consciously or unconsciously, by earning a salary from the company that manufactures ipragliflozin. Reporting that they work for the company encourages other researchers who do not work for the company to repeat the experiment and see if they get similar results.

10. **a.** The researcher did not use random assignment. The players who run every day may also practice basketball longer and harder than players who do not run every day; motivation may be a lurking variable.

b. The researcher could randomly assign players to a treatment group and a control group. Players in the treatment group would run for one hour every day, and players in the control group would not run. The researcher could have an assistant oversee the training, so the researcher would be blind to which players are in which group. After the players have run daily for one month, the researcher would compare the scoring of the two groups in the next month, while the players in the treatment group continued to run daily.

11. The owner could randomly assign the sales force to a treatment group and a control group. The treatment group would attend a workshop about emotions for a weekend. The control group would not attend the workshop. The owner could be blind to which employees attended the workshop. One month later, the owner would compare the monthly sales by the treatment and control groups.