CHAPTER

2

Choice Sets and Budget Constraints

The even-numbered solutions to end-of-chapter exercises are provided for use by instructors. (Solutions to odd-numbered end-of-chapter exercises are provided here as well as in the *Study Guide* that is available to students.)

Solutions may be shared by an instructor with his or her students at the instructor's discretion.

They may not be made publicly available.

If posted on a course web-site, the site must be password protected and for use only by the students in the course.

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In most colleges, it is a violation of the student honor code for a student to share solutions to problems with peers that take the same class at a later date.

- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student* Study Guide.

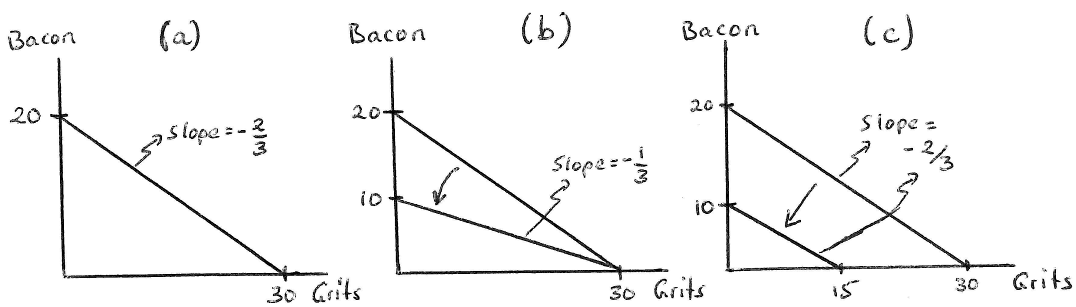
Exercise 2.1

Any good Southern breakfast includes grits (which my wife loves) and bacon (which I love). Suppose we allocate $60 per week to consumption of grits and bacon, that grits cost $2 per box and bacon costs $3 per package.

A: *Use a graph with boxes of grits on the horizontal axis and packages of bacon on the vertical to answer the following:*

(a) *Illustrate my family's weekly budget constraint and choice set.*

Answer: The graph is drawn in panel (a) of Exercise Graph 2.1.



Exercise Graph 2.1 : (a) Answer to (a); (b) Answer to (c); (c) Answer to (d)

(b) *Identify the opportunity cost of bacon and grits and relate these to concepts on your graph.*

Answer: The opportunity cost of grits is equal to 2/3 of a package of bacon (which is equal to the negative slope of the budget since grits appear on the horizontal axis). The opportunity cost of a package of bacon is 3/2 of a box of grits (which is equal to the inverse of the negative slope of the budget since bacon appears on the vertical axis).

(c) *How would your graph change if a sudden appearance of a rare hog disease caused the price of bacon to rise to $6 per package, and how does this change the opportunity cost of bacon and grits?*

Answer: This change is illustrated in panel (b) of Exercise Graph 2.1. This changes the opportunity cost of grits to 1/3 of a package of bacon, and it changes the opportunity cost of bacon to 3 boxes of grits. This makes sense: Bacon is now 3 times as expensive as grits — so you have to give up 3 boxes of grits for one package of bacon, or 1/3 of a package of bacon for 1 box of grits.

(d) *What happens in your graph if (instead of the change in (c)) the loss of my job caused us to decrease our weekly budget for Southern breakfasts from $60 to $30? How does this change the opportunity cost of bacon and grits?*

Answer: The change is illustrated in panel (c) of Exercise Graph 2.1. Since relative prices have not changed, opportunity costs have not changed. This is reflected in the fact that the slope stays unchanged.

B: *In the following, compare a mathematical approach to the graphical approach used in part A, using x_1 to represent boxes of grits and x_2 to represent packages of bacon:*

(a) *Write down the mathematical formulation of the budget line and choice set and identify elements in the budget equation that correspond to key features of your graph from part 2.1A(a).*

Answer: The budget equation is $p_1x_1 + p_2x_2 = I$ can also be written as

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.1.i}$$

With $I = 60$, $p_1 = 2$ and $p_2 = 3$, this becomes $x_2 = 20 - (2/3)x_1$ — an equation with intercept of 20 and slope of $-2/3$ as drawn in Exercise Graph 2.1(a).

(b) *How can you identify the opportunity cost of bacon and grits in your equation of a budget line, and how does this relate to your answer in 2.1A(b).*

Answer: The opportunity cost of x_1 (grits) is simply the negative of the slope term (in terms of units of x_2). The opportunity cost of x_2 (bacon) is the inverse of that.

(c) *Illustrate how the budget line equation changes under the scenario of 2.1A(c) and identify the change in opportunity costs.*

Answer: Substituting the new price $p_2 = 6$ into equation (2.1.i), we get $x_2 = 10 - (1/3)x_1$ — an equation with intercept of 10 and slope of $-1/3$ as depicted in panel (b) of Exercise Graph 2.1.

(d) *Repeat (c) for the scenario in 2.1A(d).*

Answer: Substituting the new income $I = 30$ into equation (2.1.i) (holding prices at $p_1 = 2$ and $p_2 = 3$, we get $x_2 = 10 - (2/3)x_1$ — an equation with intercept of 10 and slope of $-2/3$ as depicted in panel (c) of Exercise Graph 2.1.

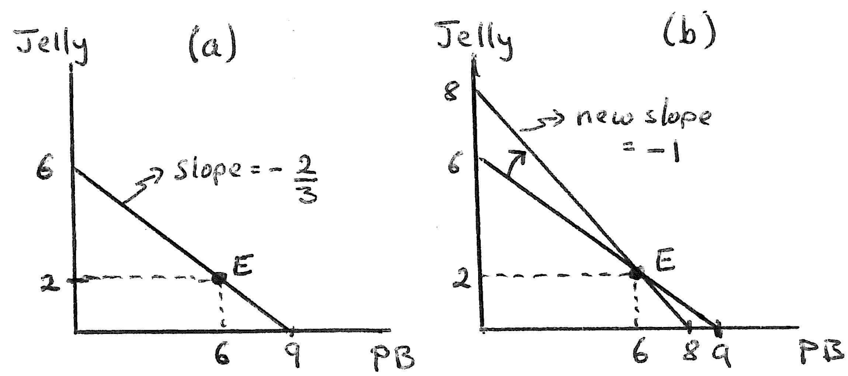
Exercise 2.2

Suppose the only two goods in the world are peanut butter and jelly.

A: *You have no exogenous income but you do own 6 jars of peanut butter and 2 jars of jelly. The price of peanut butter is \$4 per jar, and the price of jelly is \$6 per jar.*

(a) *On a graph with jars of peanut butter on the horizontal and jars of jelly on the vertical axis, illustrate your budget constraint.*

Answer: This is depicted in panel (a) of Exercise Graph 2.2. The point E is the endowment point of 2 jars of jelly and 6 jars of peanut butter (PB). If you sold your 2 jars of jelly (at a price of \$6 per jar), you could make \$12, and with that you could buy an additional 3 jars of PB (at the price of \$4 per jar). Thus, the most PB you could have is 9, the intercept on the horizontal axis. Similarly, you could sell your 6 jars of PB for \$24, and with that you could buy 4 additional jars of jelly to get you to a maximum total of 6 jars of jelly — the intercept on the vertical axis. The resulting budget line has slope $-2/3$, which makes sense since the price of PB (\$4) divided by the price of jelly (\$6) is in fact $2/3$.



Exercise Graph 2.2 : (a) Answer to (a); (b) Answer to (b)

(b) *How does your constraint change when the price of peanut butter increases to \$6? How does this change your opportunity cost of jelly?*

Answer: The change is illustrated in panel (b) of Exercise Graph 2.2. Since you can always still consume your endowment E, the new budget must contain E. But the opportunity costs have now changed, with the ratio of the two prices now equal to 1. Thus, the new budget constraint has slope -1 and runs through E. The opportunity cost of jelly has now fallen from $3/2$ to 1. This should make sense: Before, PB was cheaper than jelly and so, for every jar of jelly you had to give up more than a jar of peanut butter.

Now that they are the same price, you only have to give up one jar of PB to get 1 jar of jelly.

B: *Consider the same economic circumstances described in 2.2A and use x_1 to represent jars of peanut butter and x_2 to represent jars of jelly.*

(a) *Write down the equation representing the budget line and relate key components to your graph from 2.2A(a).*

Answer: The budget line has to equate your wealth to the cost of your consumption. Your wealth is equal to the value of your endowment, which is $p_1e_1 + p_2e_2$ (where e_1 is your endowment of PB and e_2 is your endowment of jelly). The cost of your consumption is just your spending on the two goods — i.e. $p_1x_1 + p_2x_2$. The resulting equation is

$$p_1e_1 + p_2e_2 = p_1x_1 + p_2x_2. \tag{2.2.i}$$

When the values given in the problem are plugged in, the left hand side becomes $4(6) + 6(2) = 36$ and the right hand side becomes $4x_1 + 6x_2$ — resulting in the equation $36 = 4x_1 + 6x_2$. Taking x_2 to one side, we then get

$$x_2 = 6 - \frac{2}{3}x_1, \tag{2.2.ii}$$

which is exactly what we graphed in panel (a) of Exercise Graph 2.2 — a line with vertical intercept of 6 and slope of $-2/3$.

(b) *Change your equation for your budget line to reflect the change in economic circumstances described in 2.2A(b) and show how this new equation relates to your graph in 2.2A(b).*

Answer: Now the left hand side of equation (2.2.i) is $6(6) + 6(2) = 48$ while the right hand side is $6x_1 + 6x_2$. The equation thus becomes $48 = 6x_1 + 6x_2$ or, when x_2 is taken to one side,

$$x_2 = 8 - x_1. \tag{2.2.iii}$$

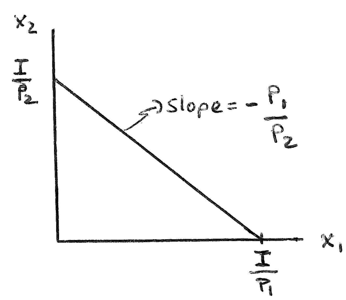
This is an equation of a line with vertical intercept of 8 and slope of -1 — exactly what we graphed in panel (b) of Exercise Graph 2.2.

Exercise 2.3

Consider a budget for good x_1 (on the horizontal axis) and x_2 (on the vertical axis) when your economic circumstances are characterized by prices p_1 and p_2 and an exogenous income level I.

A: *Draw a budget line that represents these economic circumstances and carefully label the intercepts and slope.*

Answer: The sketch of this budget line is given in Exercise Graph 2.3.



Exercise Graph 2.3 : A budget constraint with exogenous income I

The vertical intercept is equal to how much of x_2 one could by with I if that is all one bought — which is just I/p_2. The analogous is true for x_1 on the horizontal intercept. One way to verify the slope is to recognize it is the "rise" (I/p_2) divided by the "run" (I/p_1) — which gives p_1/p_2 — and that it is negative since the budget constraint is downward sloping.

(a) *Illustrate how this line can shift parallel to itself without a change in I.*

Answer: In order for the line to shift in a parallel way, it must be that the slope $-p_1/p_2$ remains unchanged. Since we can't change I, the only values we can change are p_1 and p_2 — but since p_1/p_2 can't change, it means the only thing we can do is to multiply both prices by the same constant. So, for instance, if we multiply both prices by 2, the ratio of the new prices is $2p_1/(2p_2) = p_1/p_2$ since the 2's cancel. We therefore have not changed the slope. But we have changed the vertical intercept from I/p_2 to $I/(2p_2)$. We have therefore shifted in the line without changing its slope.

This should make intuitive sense: If our money income does not change but all prices double, then I can by half as much of everything. This is equivalent to prices staying the same and my money income dropping by half.

(b) *Illustrate how this line can rotate clockwise on its horizontal intercept without a change in p_2.*

Answer: To keep the horizontal intercept constant, we need to keep I/p_1 constant. But to rotate the line clockwise, we need to increase the vertical intercept I/p_2. Since we can't change p_2 (which would be the easiest way to do this), that leaves us only I and p_1 to change. But since we can't change I/p_1, we can only change these by multiplying them by the same constant. For instance, if we multiply both by 2, we don't change the horizontal intercept since $2I/(2p_1) = I/p_1$. But we do increase the vertical intercept from I/p_2 to $2I/p_2$. So, multiplying both I and p_1 by the same constant (greater than 1) will accomplish our goal.

This again should make intuitive sense: If you double my income and the price of good 1, I can still afford exactly as much of good 1 if that is all I buy with my income. (Thus the unchanged horizontal intercept). But, if I only buy good 2, then a doubling of my income without a change in the price of good 2 lets me buy twice as much of good 2. The scenario is exactly the same as if p_2 had fallen by half (and I and p_1 had remained unchanged.)

B: *Write the equation of a budget line that corresponds to your graph in 2.3A.*

Answer: $p_1x_1 + p_2x_2 = I$, which can also be written as

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.3.i}$$

(a) *Use this equation to demonstrate how the change derived in 2.3A(a) can happen.*

Answer: If I replace p_1 with αp_1 and p_2 with αp_2 (where α is just a constant), I get

$$x_2 = \frac{I}{\alpha p_2} - \frac{\alpha p_1}{\alpha p_2}x_1 = \frac{(1/\alpha)I}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.3.ii}$$

Thus, multiplying both prices by α is equivalent to multiplying income by $1/\alpha$ (and leaving prices unchanged).

(b) *Use the same equation to illustrate how the change derived in 2.3A(b) can happen.*

Answer: If I replace p_1 with βp_1 and I with βI, I get

$$x_2 = \frac{\beta I}{p_2} - \frac{\beta p_1}{p_2}x_1 = \frac{I}{(1/\beta)p_2} - \frac{p_1}{(1/\beta)p_2}x_1. \tag{2.3.iii}$$

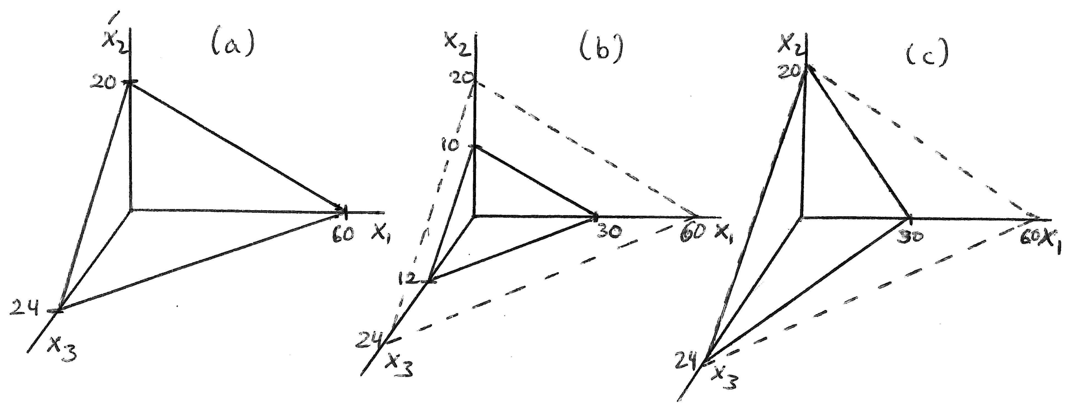
Thus, this is equivalent to multiplying p_2 by $1/\beta$. So long as $\beta > 1$, it is therefore equivalent to reducing the price of good 2 (without changing the other price or income).

Exercise 2.4

Suppose there are three goods in the world: x_1, x_2 and x_3.

A: *On a 3-dimensional graph, illustrate your budget constraint when your economic circumstances are defined by $p_1 = 2$, $p_2 = 6$, $p_3 = 5$ and $I = 120$. Carefully label intercepts.*

Answer: Panel (a) of Exercise Graph 2.4 illustrates this 3-dimensional budget with each intercept given by I divided by the price of the good on that axis.



Exercise Graph 2.4 : Budgets over 3 goods: Answers to 2.4A,A(b) and A(c)

(a) *What is your opportunity cost of x_1 in terms of x_2? What is your opportunity cost of x_2 in terms of x_3?*

Answer: On any slice of the graph that keeps x_3 constant, the slope of the budget is $-p_1/p_2 = -1/3$. Just as in the 2-good case, this is then the opportunity cost of x_1 in terms of x_2 — since p_1 is a third of p_2, one gives up 1/3 of a unit of x_2 when one chooses to consume 1 unit of x_1. On any vertical slice that holds x_1 fixed, on the other hand, the slope is $-p_3/p_2 = -5/6$. Thus, the opportunity cost of x_3 in terms of x_2 is 5/6, and the opportunity cost of x_2 in terms of x_3 is the inverse — i.e. 6/5.

(b) *Illustrate how your graph changes if I falls to \$60. Does your answer to (a) change?*

Answer: Panel (b) of Exercise Graph 2.4 illustrates this change (with the dashed plane equal to the budget constraint graphed in panel (a).) The answer to part (a) does not change since no prices and thus no opportunity costs changed. The new plane is parallel to the original.

(c) *Illustrate how your graph changes if instead p_1 rises to \$4. Does your answer to part (a) change?*

Answer: Panel (c) of Exercise Graph 2.4 illustrates this change (with the dashed plane again illustrating the budget constraint from part (a).) Since only p_1 changed, only the x_1 intercept changes. This changes the slope on any slice that holds x_3 fixed from $-1/3$ to $-2/3$ — thus doubling the opportunity cost of x_1 in terms of x_2. Since the slope of any slice holding x_1 fixed remains unchanged, the opportunity cost of x_2 in terms of x_3 remains unchanged. This makes sense since p_2 and p_3 did not change, leaving the tradeoff between x_2 and x_3 consumption unchanged.

B: *Write down the equation that represents your picture in 2.4A. Then suppose that a new good x_4 is invented and priced at $1. How does your equation change? Why is it difficult to represent this new set of economic circumstances graphically?*

Answer: The equation representing the graphs is $p_1x_1 + p_2x_2 + p_3x_3 = I$ or, plugging in the initial prices and income relevant for panel (a), $2x_1+6x_2+5x_3 = 120$. With a new fourth good priced at 1, this equation would become $2x_1 + 6x_2 + 5x_3 + x_4 = 120$. It would be difficult to graph since we would need to add a fourth dimension to our graphs.

Exercise 2.5

Everyday Application: *Watching a Bad Movie*: *On one of my first dates with my wife, we went to see the movie "Spaceballs" and paid $5 per ticket.*

A: *Halfway through the movie, my wife said: "What on earth were you thinking? This movie sucks! I don't know why I let you pick movies. Let's leave."*

(a) *In trying to decide whether to stay or leave, what is the opportunity cost of staying to watch the rest of the movie?*

Answer: The opportunity cost of any activity is what we give up by undertaking that activity. The opportunity cost of staying in the movie is whatever we would choose to do with our time if we were not there. The price of the movie tickets that got us into the movie theater is NOT a part of this opportunity cost — because, whether we stay or leave, we do not get that money back.

(b) *Suppose we had read a sign on the way into theater stating "Satisfaction Guaranteed! Don't like the movie half way through — see the manager and get your money back!" How does this change your answer to part (a)?*

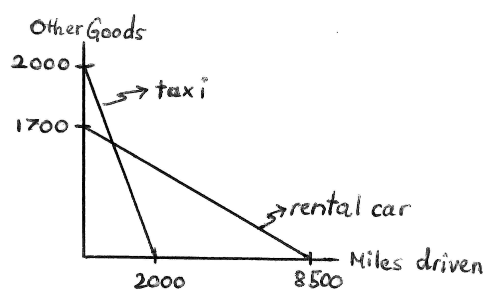
Answer: Now, in addition to giving up whatever it is we would be doing if we weren't watching the movie, we are also giving up the price of the movie tickets. Put differently, by staying in the movie theater, we are giving up the opportunity to get a refund — and so the cost of the tickets is a real opportunity cost of staying.

Exercise 2.6

Everyday Application: *Renting a Car versus Taking Taxis*: *Suppose my brother and I both go on a week-long vacation in Cayman and, when we arrive at the airport on the island, we have to choose between either renting a car or taking a taxi to our hotel. Renting a car involves a fixed fee of $300 for the week, with each mile driven afterwards just costing 20 cents — the price of gasoline per mile. Taking a taxi involves no fixed fees, but each mile driven on the island during the week now costs $1 per mile.*

A: *Suppose both my brother and I have brought $2,000 on our trip to spend on "miles driven on the island" and "other goods". On a graph with miles driven on the horizontal and other consumption on the vertical axis, illustrate my budget constraint assuming I chose to rent a car and my brother's budget constraint assuming he chose to take taxis.*

Answer: The two budget lines are drawn in Exercise Graph 2.6. My brother could spend as much as $2,000 on other goods if he stays at the airport and does not rent any taxis, but for every mile he takes a taxi, he gives up $1 in other good consumption. The most he can drive on the island is 2,000 miles. As soon as I pay the $300 rental fee, I can at most consume $1,700 in other goods, but each mile costs me only 20 cents. Thus, I can drive as much as 1700/0.2=8,500 miles.



Exercise Graph 2.6 : Graphs of equations in exercise 2.6

(a) *What is the opportunity cost for each mile driven that I faced?*

Answer: I am renting a car — which means I give up 20 cents in other consumption per mile driven. Thus, my opportunity cost is 20 cents. My opportunity cost does not include the rental fee since I paid that before even getting into the car.

(b) *What is the opportunity cost for each mile driven that my brother faced?*

Answer: My brother is taking taxis — so he has to give up $1 in other consumption for every mile driven. His opportunity cost is therefore $1 per mile.

B: *Derive the mathematical equations for my budget constraint and my brother's budget constraint, and relate elements of these equations to your graphs in part A. Use x_1 to denote miles driven and x_2 to denote other consumption.*

Answer: My budget constraint, once I pay the rental fee, is $0.2x_1 + x_2 = 1700$ while my brother's budget constraint is $x_1 + x_2 = 2000$. These can be rewritten with x_2 on the left hand side as

$$x_2 = 1700 - 0.2x_1 \text{ for me, and} \tag{2.6.i}$$

$$x_2 = 2000 - x_1 \text{ for my brother.} \tag{2.6.ii}$$

The intercept terms (1700 for me and 2000 for my brother) as well as the slopes (−0.2 for me and −1 for my brother) are as in Exercise Graph 2.6.

(a) *Where in your budget equation for me can you locate the opportunity cost of a mile driven?*

Answer: My opportunity cost of miles driven is simply the slope term in my budget equation — i.e. 0.2. I give up $0.20 in other consumption for every mile driven.

(b) *Where in your budget equation for my brother can you locate the opportunity cost of a mile driven?*

Answer: My brother's opportunity cost of miles driven is the slope term in his budget equation — i.e. 1; he gives up $1 in other consumption for every mile driven.

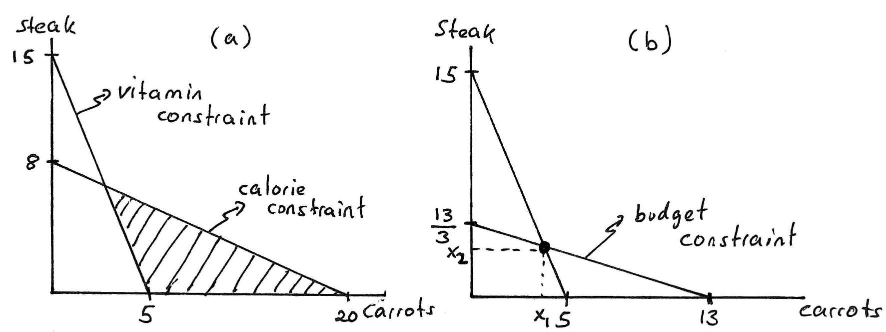
Exercise 2.7

Everyday Application: *Dieting and Nutrition: On a recent doctor's visit, you have been told that you must watch your calorie intake and must make sure you get enough vitamin E in your diet.*

A: *You have decided that, to make life simple, you will from now on eat only steak and carrots. A nice steak has 250 calories and 10 units of vitamins, and a serving of carrots has 100 calories and 30 units of vitamins Your doctor's instructions are that you must eat* no more *than 2000 calories and consume* at least *150 units of vitamins per day.*

(a) *In a graph with "servings of carrots" on the horizontal and steak on the vertical axis, illustrate all combinations of carrots and steaks that make up a 2000 calorie a day diet.*

Answer: This is illustrated as the "calorie constraint" in panel (a) of Exercise Graph 2.7. You can get 2000 calories only from steak if you eat 8 steaks and only from carrots if you eat 20 servings of carrots. These form the intercepts of the calorie constraint.



Exercise Graph 2.7 : (a) Calories and Vitamins; (b) Budget Constraint

(b) *On the same graph, illustrate all the combinations of carrots and steaks that provide exactly 150 units of vitamins.*

Answer: This is also illustrated in panel (a) of Exercise Graph 2.7. You can get 150 units of vitamins from steak if you eat 15 steaks only or if you eat 5 servings of carrots only. This results in the intercepts for the "vitamin constraint".

(c) *On this graph, shade in the bundles of carrots and steaks that satisfy both of your doctor's requirements.*

Answer: Your doctor wants you to eat no more than 2000 calories — which means you need to stay underneath the calorie constraint. Your doctor also wants you to get at least 150 units of vitamin E — which means you must choose a bundle *above* the vitamin constraint. This leaves you with the shaded area to choose from if you are going to satisfy both requirements.

(d) *Now suppose you can buy a serving of carrots for \$2 and a steak for \$6. You have \$26 per day in your food budget. In your graph, illustrate your budget constraint. If you love steak and don't mind eating or not eating carrots, what bundle will you choose (assuming you take your doctor's instructions seriously)?*

Answer: With \$26 you can buy 13/3 steaks if that is all you buy, or you can buy 13 servings of carrots if that is all you buy. This forms the two intercepts on your budget constraint which has a slope of $-p_1/p_2 = -1/3$ and is depicted in panel (b) of the graph. If you really like steak and don't mind eating carrots one way or another, you would want to get as much steak as possible given the constraints your doctor gave you and given your budget constraint. This leads you to consume the bundle at the intersection of the vitamin and the budget constraint in panel (b) — indicated by (x_1, x_2) in the graph. It seems from the two panels that this bundle also satisfies the calorie constraint and lies inside the shaded region.

B: *Continue with the scenario as described in part A.*

(a) *Define the line you drew in A(a) mathematically.*

Answer: This is given by $100x_1 + 250x_2 = 2000$ which can be written as

$$x_2 = 8 - \frac{2}{5}x_1. \tag{2.7.i}$$

(b) *Define the line you drew in A(b) mathematically.*

Answer: This is given by $30x_1 + 10x_2 = 150$ which can be written as

$$x_2 = 15 - 3x_1. \tag{2.7.ii}$$

(c) *In formal set notation, write down the expression that is equivalent to the shaded area in A(c).*

Answer:

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid 100x_1 + 250x_2 \leq 2000 \text{ and } 30x_1 + 10x_2 \geq 150\right\} \tag{2.7.iii}$$

(d) *Derive the exact bundle you indicated on your graph in A(d).*

Answer: We would like to find the most amount of steak we can afford in the shaded region. Our budget constraint is $2x_1 + 6x_2 = 26$. Our graph suggests that this budget constraint intersects the vitamin constraint (from equation (2.7.ii)) within the shaded region (in which case that intersection gives us the most steak we can afford in the shaded region). To find this intersection, we can plug equation (2.7.ii) into the budget constraint $2x_1 + 6x_2 = 26$ to get

$$2x_1 + 6(15 - 3x_1) = 26, \tag{2.7.iv}$$

and then solve for x_1 to get $x_1 = 4$. Plugging this back into either the budget constraint or the vitamin constraint, we can get $x_2 = 3$. We know this lies on the vitamin constraint as well as the budget constraint. To check to make sure it lies in the shaded region, we just have to make sure it also satisfies the doctor's orders that you consume fewer than 2000 calories. The bundle (x_1, x_2)=(4,3) results in calories of $4(100) + 3(250) = 1150$, well within doctor's orders.

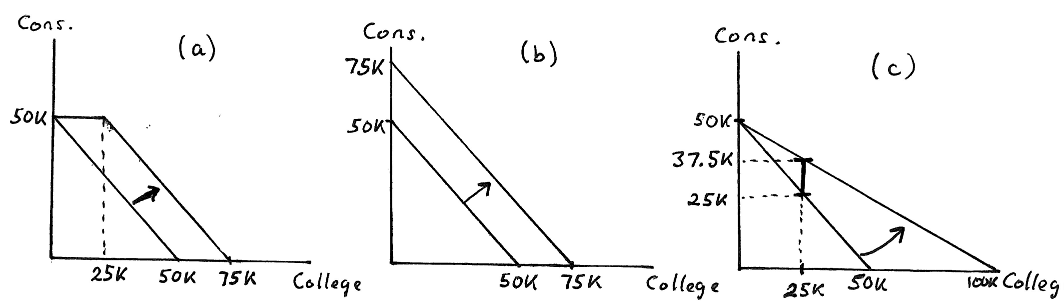
Exercise 2.8

Everyday Application: *Setting up a College Trust Fund: Suppose that you, after studying economics in college, quickly became rich — so rich that you have nothing better to do than worry about your 16-year old niece who can't seem to focus on her future. Your niece currently already has a trust fund that will pay her a nice yearly income of $50,000 starting when she is 18, and she has no other means of support.*

A: *You are concerned that your niece will not see the wisdom of spending a good portion of her trust fund on a college education, and you would therefore like to use $100,000 of your wealth to change her choice set in ways that will give her greater incentives to go to college.*

(a) *One option is for you to place $100,000 in a second trust fund but to restrict your niece to be able to draw on this trust fund only for college expenses of up to $25,000 per year for four years. On a graph with "yearly dollars spent on college education" on the horizontal axis and "yearly dollars spent on other consumption" on the vertical, illustrate how this affects her choice set.*

Answer: Panel (a) of Exercise Graph 2.8 illustrates the change in the budget constraint for this type of trust fund. The original budget shifts out by $25,000 (denoted $25K), except that the first $25,000 can only be used for college. Thus, the maximum amount of other consumption remains $50,000 because of the stipulation that she cannot use the trust fund for non-college expenses.



Exercise Graph 2.8 : (a) Restricted Trust Fund; (b) Unrestricted; (c) Matching Trust Fund

(b) *A second option is for you to simply tell your niece that you will give her $25,000 per year for 4 years and you will trust her to "do what's right". How does this impact her choice set?*

Answer: This is depicted in panel (b) of Exercise Graph 2.8 — it is a pure income shift of $25,000 since there are no restrictions on how the money can be used.

(c) *Suppose you are wrong about your niece's short-sightedness and she was planning on spending more than $25,000 per year from her other trust fund on college education. Do you think she will care whether you do as described in part (a) or as described in part (b)?*

Answer: If she was planning to spend more than $25K on college anyhow, then the additional bundles made possible by the trust fund in (b) are not valued by her. She would therefore not care whether you set up the trust fund as in (a) or (b).

(d) *Suppose you were right about her — she never was going to spend very much on college. Will she care now?*

Answer: Now she will care — because she would actually choose one of the bundles made available in (b) that is not available in (a) and would therefore prefer (b) over (a).

(e) *A friend of yours gives you some advice: be careful — your niece will not value her education if she does not have to put up some of her own money for it. Sobered by this advice, you decide to set up a different trust fund that will release 50 cents to your niece (to be spent on whatever she wants) for every dollar that she spends on college expenses. How will this affect her choice set?*

Answer: This is depicted in panel (c) of Exercise Graph 2.8. If your niece now spends $1 on education, she gets 50 cents for anything she would like to spend it on — so, in effect, the opportunity cost of getting $1 of additional education is just 50 cents. This "matching" trust fund therefore reduces the opportunity cost of education whereas the previous ones did not.

(f) *If your niece spends $25,000 per year on college under the trust fund in part (e), can you identify a vertical distance that represents how much you paid to achieve this outcome?*

Answer: If your niece spends $25,000 on her education under the "matching" trust fund, she will get half of that amount from your trust fund — or $12,500. This can be seen as the vertical distance between the before and after budget constraints (in panel (c) of the graph) at $25,000 of education spending.

B: *How would you write the budget equation for each of the three alternatives discussed above?*

Answer: The initial budget is $x_1 + x_2 = 50,000$. The first trust fund in (a) expands this to a budget of

$$x_2 = 50,000 \text{ for } x_1 \leq 25,000 \text{ and } x_1 + x_2 = 75,000 \text{ for } x_1 > 25,000, \quad (2.8)$$

while the second trust fund in (b) expands it to $x_1 + x_2 = 75,000$. Finally, the last "matching" trust fund in (e) (depicted in panel (c)) is $0.5x_1 + x_2 = 50,000$.

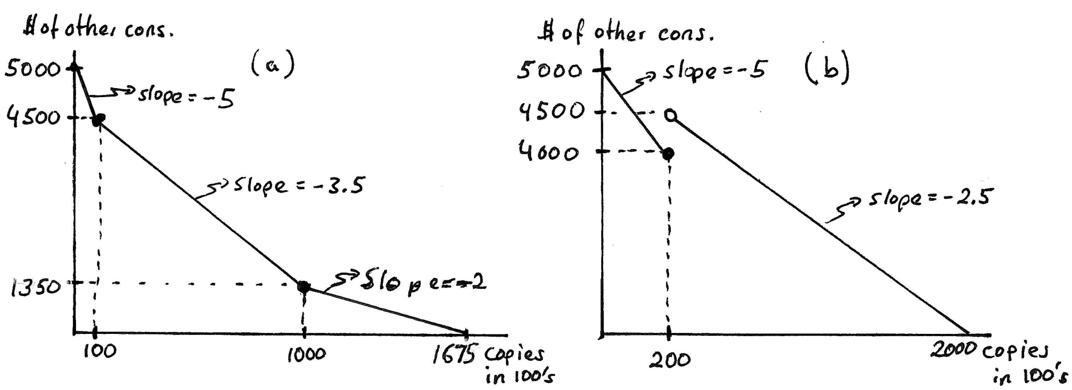
Exercise 2.9

Business Application: *Pricing and Quantity Discounts*: *Businesses often give quantity discounts. Below, you will analyze how such discounts can impact choice sets.*

A: *I recently discovered that a local copy service charges our economics department $0.05 per page (or $5 per 100 pages) for the first 10,000 copies in any given month but then reduces the price per page to $0.035 for each additional page up to 100,000 copies and to $0.02 per each page beyond 100,000. Suppose our department has a monthly overall budget of $5,000.*

(a) *Putting "pages copied in units of 100" on the horizontal axis and "dollars spent on other goods" on the vertical, illustrate this budget constraint. Carefully label all intercepts and slopes.*

Answer: Panel (a) of Exercise Graph 2.9 traces out this budget constraint and labels the relevant slopes and kink points.



Exercise Graph 2.9 : (a) Constraint from 2.9A(a); (b) Constraint from 2.9A(b)

(b) *Suppose the copy service changes its pricing policy to $0.05 per page for monthly copying up to 20,000 and $0.025 per page for* all *pages if copying exceeds 20,000 per month. (*Hint*: Your budget line will contain a jump.)*

Answer: Panel (b) of Exercise Graph 2.9 depicts this budget. The first portion (beginning at the x_2 intercept) is relatively straightforward. The second part arises for the following reason: The problem says that, if you copy more than 2000 pages, *all* pages cost only $0.025 per page — including the first 2000. Thus, when you copy 20,000 pages per month, you total bill is $1,000. But when you copy 2001 pages, your total bill is $500.025.

(c) *What is the marginal (or "additional") cost of the first page copied after 20,000 in part (b)? What is the marginal cost of the first page copied after 20,001 in part (b)?*

Answer: The marginal cost of the first page after 20,000 is -\$499.975, and the marginal cost of the next page after that is 2.5 cents. To see the difference between these, think of the marginal cost as the increase in the total photo-copy bill for each additional page. When going from 20,000 to 20,001, the total bill falls by \$499.975. When going from 20,001 to 20,002, the total bill rises by 2.5 cents.

B: *Write down the mathematical expression for choice sets for each of the scenarios in 2.9A(a) and 2.9A(b) (using x_1 to denote "pages copied in units of 100" and x_2 to denote "dollars spent on other goods").*

Answer: The choice set in (a) is

$$\begin{aligned}\{(x_1,x_2)\in\mathbb{R}^2_+ \mid \quad & x_2 = 5000-5x_1 && \text{for } x_1 \le 100 \text{ and}\\ & x_2 = 4850-3.5x_1 && \text{for } 100 < x_1 \le 1000 \text{ and}\\ & x_2 = 3350-2x_1 && \text{for } x_1 > 1000\,\}.\end{aligned} \tag{2.9.i}$$

The choice set in (b) is

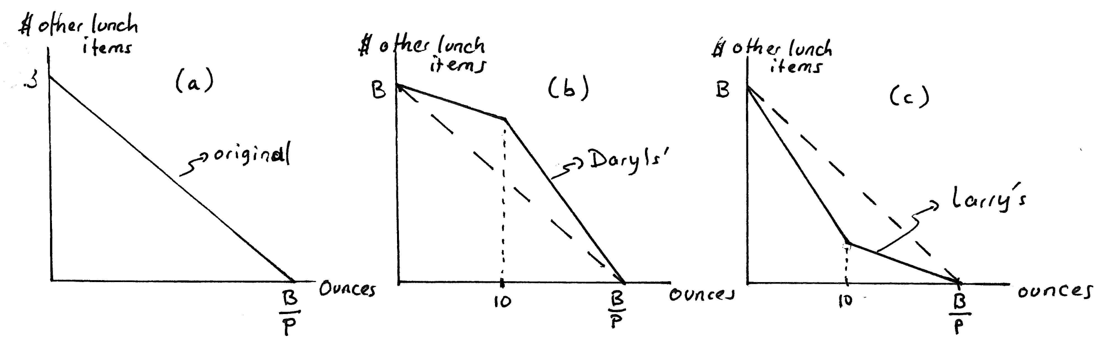
$$\begin{aligned}\{(x_1,x_2)\in\mathbb{R}^2_+ \mid \quad & x_2 = 5000-5x_1 && \text{for } x_1 \le 200 \text{ and}\\ & x_2 = 5000-2.5x_1 && \text{for } x_1 > 200\,\}.\end{aligned} \tag{2.9.ii}$$

Exercise 2.10

Business Application: *Supersizing: Suppose I run a fast-food restaurant and I know my customers come in on a limited budget. Almost everyone that comes in for lunch buys a soft-drink. Now suppose it costs me virtually nothing to serve a medium versus a large soft-drink, but I do incur some extra costs when adding items (like a dessert or another side-dish) to someone's lunch tray.*

A: *Suppose for purposes of this exercise that cups come in all sizes, not just small, medium and large; and suppose the average customer has a lunch budget B. On a graph with "ounces of soft-drink" on the horizontal axis and "dollars spent on other lunch items" on the vertical, illustrate a customer's budget constraint assuming I charge the same price p per ounce of soft-drink no matter how big a cup the customer gets.*

Answer: Panel (a) of Exercise Graph 2.10 illustrates the original budget, with the price per ounce denoted p. The horizontal intercept is the money budget B divided by the price per ounce of soft drink; the vertical intercept is just B (since the good on the vertical axis is denominated in dollars — with the price of "$'s of lunch items" therefore implicitly set to 1.



Exercise Graph 2.10 : (a) Original Budget; (b) The Daryls' proposal; (c) Larry's proposal

(a) *I have three business partners: Larry, his brother Daryl and his other brother Daryl. The Daryls propose that we lower the price of the initial ounces of soft-drink that a consumer buys and then, starting at 10 ounces, we increase the price. They have calculated that our average customer would be able to buy exactly the same number of ounces of soft-drink (if that is all he bought on his lunch budget) as under the current single price. Illustrate how this will change the average customer's budget constraint.*

Answer: Panel (b) illustrates the Daryls' proposal. The budget is initially shallower (because of the initial lower price and then becomes steeper at 10 ounces because of the new higher price.) The intercepts are unchanged because nothing has been done to allow the average customer to buy more of non-drink items if that is all she buys, and because the new prices have been constructed so as to allow customers to achieve the same total drink consumption in the event that they do not buy anything else.

(b) *Larry thinks the Daryls are idiots and suggests instead that we raise the price for initial ounces of soft-drink and then, starting at 10 ounces, decrease the price for any additional ounces. He, too, has calculated that, under his pricing policy, the average customer will be able to buy exactly the same ounces of soft-drinks (if that is all the customer buys on his lunch budget). Illustrate the effect on the average customer's budget constraint.*

Answer: Larry's proposal is graphed in panel (c). The reasoning is similar to that in the previous part, except now the initial price is high and then becomes low after 10 ounces.

(c) *If the average customer had a choice, which of the three pricing systems — the current single price, the Daryls' proposal or Larry's proposal — would he choose?*

Answer: Customers would surely prefer the Daryls' proposal — since the choice set it forms contains all the other choice sets.

P.S: If you did not catch the reference to Larry, his brother Daryl and his other brother Daryl, I recommend you rent some old versions of the 1980's Bob Newhart Show.

B: *Write down the mathematical expression for each of the three choice sets described above, letting ounces of soft-drinks be denoted by x_1 and dollars spend on other lunch items by x_2.*

Answer: The original budget set in panel (a) of Exercise Graph 2.10 is simply $px_1 + x_2 = B$ giving a choice set of

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid x_2 = B - px_1\right\}. \tag{2.10.i}$$

In the Daryls' proposal, we have an initial price $p' < p$ for the first 10 ounces, and then a price $p'' > p$ thereafter. We can calculate the x_2 intercept of the steeper line following the kink point in panel (b) of the graph by simply multiplying the x_1 intercept of B/p by the slope p'' of that line segment to get Bp''/p. The choice set from the Daryls' proposal could then be written as

$$\{(x_1, x_2) \in \mathbb{R}^2_+ \mid \quad \begin{aligned} x_2 &= B - p'x_1 && \text{for } x_1 \leq 10 \text{ and} \\ x_2 &= \tfrac{Bp''}{p} - p''x_1 && \text{for } x_1 > 10 \text{ where } p' < p < p'' \end{aligned} \} \tag{2.10.ii}$$

We could even be more precise about the relationship of p', p and p''. The two lines intersect at $x_1 = 10$, and it must therefore be the case that $B - 10p' = (Bp''/p) - 10p''$. Solving this for p', we get that

$$p' = \frac{B(p - p'')}{10p} + p''. \tag{2.10.iii}$$

Larry's proposal begins with a price $p'' > p$ and then switches at 10 ounces to a price $p' < p$ (where these prices have no particular relation to the prices we just used for the Daryl's proposal). This results in the choice set

$$\{(x_1, x_2) \in \mathbb{R}^2_+ \mid \begin{array}{ll} x_2 = B - p''x_1 & \text{for } x_1 \leq 10 \text{ and} \\ x_2 = \frac{Bp'}{p} - p'x_1 & \text{for } x_1 > 10 \text{ where } p' < p < p'' \end{array}\} \quad (2.10.iv)$$

We could again derive an analogous expression for p' in terms of p and p''.

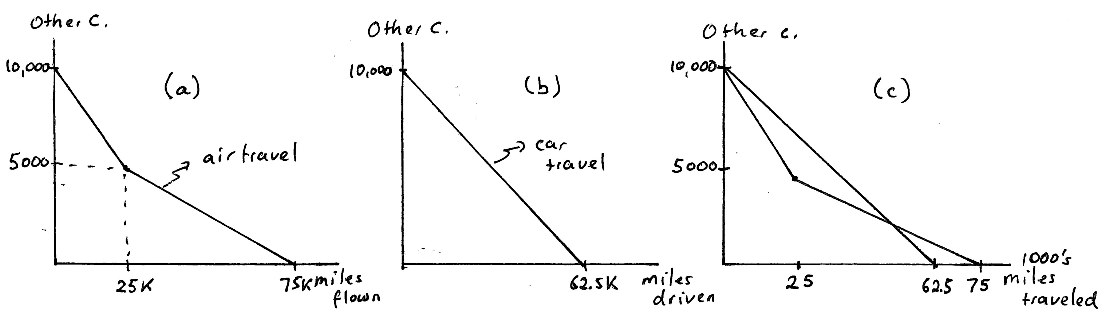
Exercise 2.11

Business Application: *Frequent Flyer Perks: Airlines offer frequent flyers different kinds of perks that we will model here as reductions in average prices per mile flown.*

A: *Suppose that an airline charges 20 cents per mile flown. However, once a customer reaches 25,000 miles in a given year, the price drops to 10 cents per mile flown for each additional mile. The alternate way to travel is to drive by car which costs 16 cents per mile.*

(a) *Consider a consumer who has a travel budget of $10,000 per year, a budget which can be spent on the cost of getting to places as well as "other consumption" while traveling. On a graph with "miles flown" on the horizontal axis and "other consumption" on the vertical, illustrate the budget constraint for someone who only considers flying (and not driving) to travel destinations.*

Answer: Panel (a) of Exercise Graph 2.11 illustrates this budget constraint.



Exercise Graph 2.11 : (a) Air travel; (b) Car travel; (c) Comparison

(b) *On a similar graph with "miles driven" on the horizontal axis, illustrate the budget constraint for someone that considers only driving (and not flying) as a means of travel.*

Answer: This is illustrated in panel (b) of the graph.

(c) *By overlaying these two budget constraints (changing the good on the horizontal axis simply to "miles traveled"), can you explain how frequent flyer perks might persuade some to fly a lot more than they otherwise would?*

Answer: Panel (c) of the graph overlays the two budget constraints. If it were not for frequent flyer miles, this consumer would never fly — because driving would be cheaper. With the frequent flyer perks, driving is cheaper initially but becomes more expensive per additional miles traveled if a traveler flies more than 25,000 miles. This particular consumer would therefore either not fly at all (and just drive), or she would fly a lot because it can only make sense to fly if she reaches the portion of the air-travel budget that crosses the car budget. (Once we learn more about how to model tastes, we will be able to say more about whether or not it makes sense for a traveler to fly under these circumstances.)

B: *Determine where the air-travel budget from A(a) intersects the car budget from A(b).*

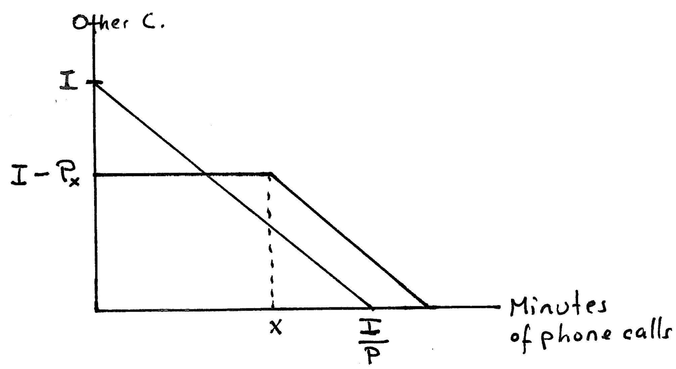
Answer: The shallower portion of the air-travel budget (relevant for miles flown above 25,000) has equation $x_2 = 7500 - 0.1x_1$, where x_2 stands for other consumption and x_1 for miles traveled. The car budget, on the other hand, has equation $x_2 = 10000 - 0.16x_1$. To determine where they cross, we can set the two equations equal to one another and solve for x_1 — which gives $x_1 = 41,667$ miles traveled. Plugging this back into either equation gives $x_2 = \$3,333$.

Exercise 2.12

Business Application: *Choice in Calling Plans: Phone companies used to sell minutes of phone calls at the same price no matter how many phone calls a customer made. (We will abstract away from the fact that they charged different prices at different times of the day and week.) More recently, phone companies, particularly cell phone companies, have become more creative in their pricing.*

A: *On a graph with "minutes of phone calls per month" on the horizontal axis and "dollars of other consumption" on the vertical, draw a budget constraint assuming the price per minute of phone calls is p and assuming the consumer has a monthly income I.*

Answer: Exercise Graph 2.12 gives this budget constraint as the straight line with vertical intercept I.



Exercise Graph 2.12 : Phone Plans

(a) *Now suppose a new option is introduced: You can pay $\$P_x$ to buy into a phone plan that offers you x minutes of free calls per month, with any calls beyond x costing p per minute. Illustrate how this changes your budget constraint and assume that P_x is sufficiently low such that the new budget contains some bundles that were previously unavailable to our consumer.*

Answer: The second budget constraint in the graph begins at $I - P_x$ — which is how much monthly income remains available for other consumption once the fixed fee for the first x minutes is paid. The price per additional minute is the same as before — so after x calls have been made, the slope of the new budget is the same as the original.

(b) *Suppose it actually costs phone companies close to p per minute to provide a minute of phone service so that, in order to stay profitable, a phone company must on average get about p per minute of phone call. If all consumers were able to choose calling plans such that they always use exactly x minutes per month, would it be possible for phone companies to set P_x sufficiently low such that new bundles become available to consumers?*

Answer: If the phone company needs to make an average of p per minute of phone calls, and if all consumers plan ahead perfectly and choose calling plans under which they use all their free minutes, then the company would have to set $P_x = px$. But that would mean that the kink point on the new budget would occur exactly on the original budget — thus making no new bundles available for consumers.

(c) *If some fraction of consumers in any given month buy into a calling plan but make fewer than x calls, how does this enable phone companies to set P_x such that new bundles become available in consumer choice sets?*

Answer: If some consumers do not in fact use all their “free minutes”, then the phone company could set $P_x < px$ and still collect an average of p per minute of phone call. This would cause the kink point of the new budget to shift to the right of the original budget — making new bundles available for consumers. Consumers who plan ahead well are, in some sense, receiving a transfer from consumers who do not plan ahead well.

B: *Suppose a phone company has 100,000 customers who currently buy phone minutes under the old system that charges p per minute. Suppose it costs the company c to provide one additional minute of phone service but the company also has fixed costs FC (that don't vary with how many minutes are sold) of an amount that is sufficiently high to result in zero profit. Suppose a second identical phone company has 100,000 customers that have bought into a calling plan that charges $P_x = kpx$ and gives customers x free minutes before charging p for minutes above x.*

(a) *If people on average use half their “free minutes” per month, what is k (as a functions of FC, p, c and x) if the second company also makes zero profit?*

Answer: The profit of the second company is its revenue minus its costs. Revenue is

$$100,000(P_x) = 100,000(kpx). \tag{2.12.i}$$

Each customer only uses $x/2$ minutes, which means the cost of providing the phone minutes is $100,000(cx/2) = 50,000cx$. The company also has to cover the fixed costs FC. So, if profit is zero for the second company (as it is for the first), then

$$100000(kpx) - 50000(cx) - FC = 0. \tag{2.12.ii}$$

Solving this for k, we get

$$k = \frac{FC}{100000px} + \frac{c}{2p}. \tag{2.12.iii}$$

(b) *If there were no fixed costs (i.e. $FC = 0$) but everything else was still as stated above, what does c have to be equal to in order for the first company to make zero profit? What is k in that case?*

Answer: $c = p$ and $k = 1/2$. This should make intuitive sense: Under the simplified scenario, the fact that people on average use only half their "free minutes" implies that the second company can set its fixed fee of x minutes at half the price that the other company would charge for consuming that many minutes.

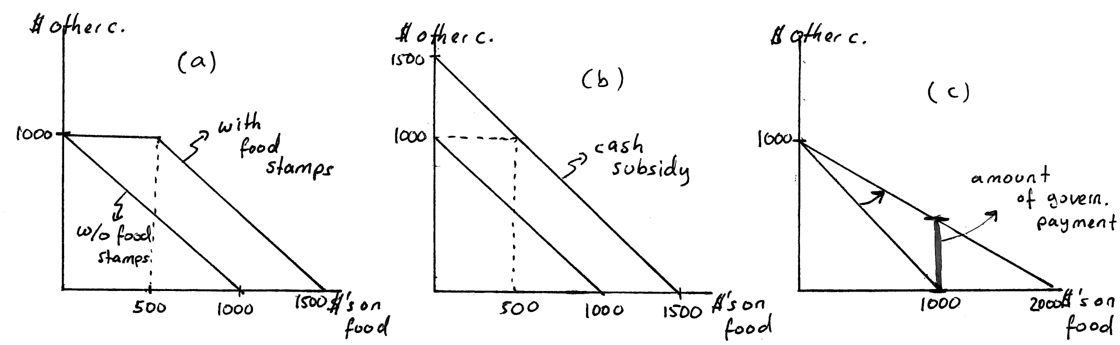
Exercise 2.13

Policy Application: *Food Stamp Programs and other Types of Subsidies: The U.S. government has a food stamp program for families whose income falls below a certain poverty threshold. Food stamps have a dollar value that can be used at supermarkets for food purchases as if the stamps were cash, but the food stamps cannot be used for anything other than food.*

A: *Suppose the program provides $500 of food stamps per month to a particular family that has a fixed income of $1,000 per month.*

(a) *With "dollars spent on food" on the horizontal axis and "dollars spent on non-food items" on the vertical, illustrate this family's monthly budget constraint. How does the opportunity cost of food change along the budget constraint you have drawn?*

Answer: Panel (a) of Exercise Graph 2.13 illustrates the original budget — with intercept 1,000 on each axis. It then illustrates the new budget under the food stamp program. Since food stamps can only be spent on food, the "other goods" intercept does not change — owning some food stamps still only allows households to spend what they previously had on other goods. However, the family is now able to buy $1,000 in other goods even as it buys food — because it can use the food stamps on the first $500 worth of food and still have all its other income left for other consumption. Only after all the food stamps are spent — i.e. after the family has bought $500 worth of food — does the family give up other consumption when consuming additional food. As a result, the opportunity cost of food is zero until the food stamps are gone, and it is 1 after that. That is, after the food stamps are gone, the family gives up $1 in other consumption for every $1 of food it purchases.



Exercise Graph 2.13 : (a) Food Stamps; (b) Cash; (c) Re-imburse half

(b) *How would this family's budget constraint differ if the government replaced the food stamp program with a cash subsidy program that simply gave this* *family \$500 in cash instead of \$500 in food stamps? Which would the family prefer, and what does your answer depend on?*

Answer: In this case, the original budget would simply shift out by \$500 as depicted in panel (b). If the family consumes more than \$500 of food under the food stamp program, it would not seem like anything really changes under the cash subsidy. (We can show this more formally once we introduce a model of tastes). If, on the other hand, the family consumes \$500 of food under the food stamps, it may well be that it would prefer to get cash instead so that it can consume more other goods instead.

(c) *How would the budget constraint change if the government simply agreed to reimburse the family for half its food expenses?*

Answer: In this case, the government essentially reduces the price of \$1 of food to 50 cents because whenever \$1 is spent on food, the government reimburses the family 50 cents. The resulting change in the family budget is then depicted in panel (c) of the graph.

(d) *If the government spends the same amount for this family on the program described in (c) as it did on the food stamp program, how much food will the family consume? Illustrate the amount the government is spending as a vertical distance between the budget lines you have drawn.*

Answer: If the government spent \$500 for this family under this program, then the family will be consuming \$1,000 of food and \$500 in other goods. You can illustrate the \$500 the government is spending as the distance between the two budget constraints at \$1,000 of food consumption. The reasoning for this is as follows: On the original budget line, you can see that consuming \$1,000 of food implies nothing is left over for "other consumption". When the family consumes \$1,000 of food under the new program, it is able to consume \$500 in other goods because of the program — so the government must have made that possible by giving \$500 to the family.

B: *Write down the mathematical expression for the choice set you drew in 2.13A(a), letting x_1 represent dollars spent on food and x_2 represent dollars spent on non-food consumption. How does this expression change in 2.13A(b) and 2.13A(c)?*

Answer: The original budget constraint (prior to any program) is just $x_2 = 1000 - x_1$, and the budget constraint with the \$500 cash payment in A(b) is $x_2 = 1500 - x_1$. The choice set under food stamps (depicted in panel (a)) then is

$$\{(x_1, x_2) \in \mathbb{R}^2_+ \mid \quad \begin{array}{ll} x_2 = 1000 & \text{for } x_1 \leq 500 \text{ and} \\ x_2 = 1500 - x_1 & \text{for } x_1 > 500\,\}, \end{array} \tag{2.13.i}$$

while the choice set in panel (b) under the cash subsidy is

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid x_2 = 1500 - x_1\right\}. \tag{2.13.ii}$$

Finally, the choice set under the re-imbursement plan from A(c) is

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid x_2 = 1000 - \frac{1}{2}x_1\right\}. \tag{2.13.iii}$$

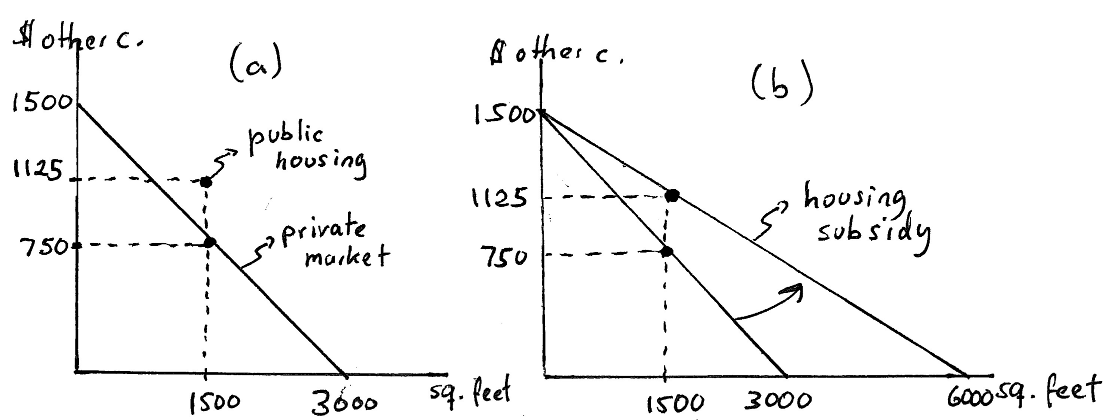
Exercise 2.14

Policy Application: *Public Housing and Housing Subsidies: For a long period, the U.S. government focused its attempts to meet housing needs among the poor through public housing programs. Eligible families could get on waiting lists to apply for an apartment in a public housing development and would be offered a particular apartment as they moved to the top of the waiting list.*

A: *Suppose a particular family has a monthly income of $1,500 and is offered a 1,500 square foot public housing apartment for $375 in monthly rent. Alternatively, the family could choose to rent housing in the private market for $0.50 per square foot.*

(a) *Illustrate all the bundles in this family's choice set of "square feet of housing" (on the horizontal axis) and "dollars of monthly other goods consumption" (on the vertical axis).*

Answer: The full choice set would include all the bundles that are available through the private market plus the bundle the government has made available. In panel (a) of Exercise Graph 2.14, the private market constraint is depicted together with the single bundle that the government makes available through public housing. (That bundle has $1,125 in other monthly consumption because the government charges $375 for the 1,500 square foot public housing apartment.)



Exercise Graph 2.14 : (a) Public Housing; (b) Rental Subsidy

(b) *In recent years, the government has shifted away from an emphasis on public housing and toward providing poor families with a direct subsidy to allow them to rent more housing in the private market. Suppose, instead of offering the family in part (a) an apartment, the government offered to pay half of the family's rental bill. How would this change the family's budget constraint?*

Answer: The change in policy is depicted in panel (b) of the graph.

(c) *Is it possible to tell which policy the family would prefer?*

Answer: Since the new budget in panel (b) contains the public housing bundle from panel (a) but also contains additional bundles that were previously not available, the housing subsidy must be at least as good as the public housing program from the perspective of the household.

B: *Write down the mathematical expression for the choice sets you drew in 2.14A(a) and 2.14A(b), letting x_1 denote square feet of monthly housing consumption and x_2 denote dollars spent on non-housing consumption.*

Answer: The public housing choice set (which includes the option of not participating in public housing and renting in the private market instead) is given by

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid (x_1, x_2) = (1500, 1125) \text{ or } x_2 \leq 1500 - 0.5x_1\right\}. \tag{2.14.i}$$

The rental subsidy in panel (b), on the other hand, creates the choice set

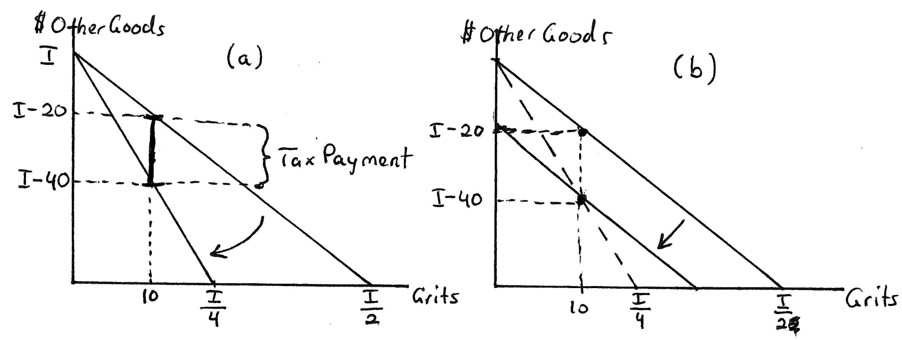
$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid x_2 \leq 1500 - 0.25x_1\right\}. \tag{2.14.ii}$$

Exercise 2.15

Policy Application: *Taxing Goods* versus *Lump Sum Taxes*: *I have finally convinced my local congressman that my wife's taste for grits are nuts and that the world should be protected from too much grits consumption. As a result, my congressman has agreed to sponsor new legislation to tax grits consumption which will raise the price of grits from \$2 per box to \$4 per box. We carefully observe my wife's shopping behavior and notice with pleasure that she now purchases 10 boxes of grits per month rather than her previous 15 boxes.*

A: *Putting "boxes of grits per month" on the horizontal and "dollars of other consumption" on the vertical, illustrate my wife's budget line before and after the tax is imposed. (You can simply denote income by I.)*

Answer: The tax raises the price, thus resulting in a rotation of the budget line as illustrated in panel (a) of Exercise Graph 2.15. Since no indication of an income level was given in the problem, income is simply denoted I.



Exercise Graph 2.15 : (a) Tax on Grits; (b) Lump Sum Rebate

(a) *How much tax revenue is the government collecting per month from my wife? Illustrate this as a vertical distance on your graph. (*Hint*: If you know how much she is consuming after the tax and how much in other consumption this leaves her with, and if you know how much in other consumption she would have had if she consumed that same quantity before the imposition of the tax, then the difference between these two "other consumption" quantities must be equal to how much she paid in tax.)*

Answer: When she consumes 10 boxes of grits after the tax, she pays \$40 for grits. This leaves her with $(I-40)$ to spend on other goods. Had she bought 10 boxes of grits prior to the tax, she would have paid \$20, leaving her with $(I-20)$. The difference between $(I-40)$ and $(I-20)$ is \$20 — which is equal to the vertical distance in panel (a). You can verify that this is exactly how much she indeed must have paid — the tax is \$2 per box and she bought 10 boxes, implying that she paid \$2 times 10 or \$20 in grits taxes.

(b) *Given that I live in the South, the grits tax turned out to be unpopular in my congressional district and has led to the defeat of my congressman. His replacement won on a pro-grits platform and has vowed to repeal the grits tax. However, new budget rules require him to include a new way to raise the same tax revenue that was yielded by the grits tax. He proposes to simply ask each grits consumer to pay exactly the amount he or she paid in grits taxes as a monthly lump sum payment. Ignoring for the moment the difficulty of gathering the necessary information for implementing this proposal, how would this change my wife's budget constraint?*

Answer: In panel (b) of Exercise Graph 2.15, the previous budget under the grits tax is illustrated as a dashed line. The grits tax changed the opportunity cost of grits — and thus the slope of the budget (as illustrated in panel (a)). The lump sum tax, on the other hand, does not alter opportunity costs but simply reduces income by \$20, the amount of grits taxes my wife paid under the grits tax. This change is illustrated in panel (b).

B: *State the equations for the budget constraints you derived in A(a) and A(b), letting grits be denoted by x_1 and other consumption by x_2.*

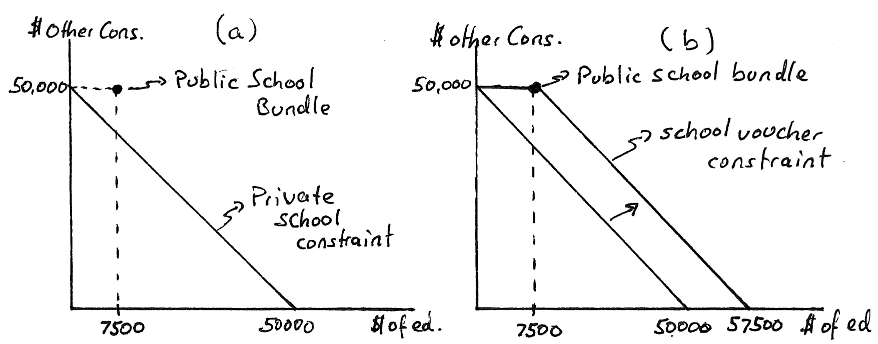
Answer: The initial (before-tax) budget was $x_2 = I - 2x_1$ which becomes $x_2 = I - 4x_1$ after the imposition of the grits tax. The lump sum tax budget constraint, on the other hand, is $x_2 = I - 20 - 2x_1$.

Exercise 2.16

Policy Application: *Public Schools and Private School Vouchers: Consider a simple model of how economic circumstances are changed when the government enters the education market.*

A: *Suppose a household has an after-tax income of $50,000 and consider its budget constraint with "dollars of education services" on the horizontal axis and "dollars of other consumption" on the vertical. Begin by drawing the household's budget line (given that you can infer a price for each of the goods on the axes from the way these goods are defined) assuming that the household can buy any level of school spending on the private market.*

Answer: The budget line in this case is straightforward and illustrated in panel (a) of Exercise Graph 2.16 as the constraint labeled "private school constraint".



Exercise Graph 2.16 : (a) Public Schools; (b) Private School Voucher

(a) *Now suppose the government uses its existing tax revenues to fund a public school at $7,500 per pupil; i.e. it funds a school that anyone can attend for free and that provides $7,500 in education services. Illustrate how this changes the choice set.(*Hint*: One additional point will appear in the choice set.)*

Answer: Since public education is free (and paid for from existing tax revenues — i.e. no new taxes are imposed), it now becomes possible to consume a public school that offers $7,500 of educational services while still consuming $50,000 in other consumption. This adds an additional bundle to the choice set — the bundle (7,500, 50,000) denoted "public school bundle" in panel (a) of the graph.

(b) *Continue to assume that private school services of any quantity could be purchased but only if the child does not attend public schools. Can you think of how the availability of free public schools might cause some children to receive more educational services than before they would in the absence of public schools? Can you think of how some children might receive fewer educational services once public schools are introduced?*

Answer: If a household purchased less than $7,500 in education services for a child prior to the introduction of the public school, it seems likely that the household would jump at the opportunity to increase both consumption of other goods and consumption of education services by switching to the public education bundle. At the same time, if a household purchased more than $7,500 in education services prior to the introduction of public schools, it is plausible that this household will also switch to the public school bundle — because, while it would mean less eduction service for the child, it would also mean a large increase in other consumption. (We will be able to be more precise once we introduce a model of tastes.)

(c) *Now suppose the government allows an option: either a parent can send her child to the public school or she can take a voucher to a private school and use it for partial payment of private school tuition. Assume that the voucher is worth $7,500 per year; i.e. it can be used to pay for up to $7,500 in private school tuition. How does this change the budget constraint? Do you still think it is possible that some children will receive less education than they would if the government did not get involved at all (i.e. no public schools and no vouchers)?*

Answer: The voucher becomes equivalent to cash so long as at least $7,500 is spent on education services. This results in the budget constraint depicted in panel (b) of Exercise Graph 2.16. Since one cannot use the voucher to increase other consumption beyond $50,000, the voucher does not make any private consumption above $50,000 possible. However, it does make it possible to consume any level of education service between 0 and $7,500 without incurring any opportunity cost in terms of other consumption. Only once the full voucher is used and $7,500 in education services have been bought will the household be giving up a dollar in other consumption for every additional dollar in education services.

It is easy to see how this will lead some parents to choose more education for their children (just as it was true that the introduction of the public school bundle gets some parents to increase the education services consumed by their children.) But the reverse no longer appears likely — if someone choses more than $7,500 in education services in the absence of public schools and vouchers, the effective increase in household income implied by the voucher/public school combination makes it unlikely that such a household will reduce the education services given to her child. (Again, we will be able to be more precise once we introduce tastes — and we will see that it would take unrealistic tastes for this to happen.)

B: *Letting dollars of education services be denoted by x_1 and dollars of other consumption by x_2, formally define the choice set with just the public school (and a private school market) as well as the choice set with private school vouchers defined above.*

Answer: The first choice set (in panel (a) of the graph) is formally defined as

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid x_2 \leq 50000 - x_1 \text{ or } (x_1, x_2) = (7500, 50000)\right\}, \tag{2.16.i}$$

while the introduction of vouchers changes the choice set to

$$\{(x_1, x_2) \in \mathbb{R}^2_+ \mid \begin{array}{ll} x_2 = 50000 & \text{for } x_1 \leq 7500 \text{ and} \\ x_2 = 57500 - x_1 & \text{for } x_1 > 7500 \,\}. \end{array} \tag{2.16.ii}$$

Exercise 2.17

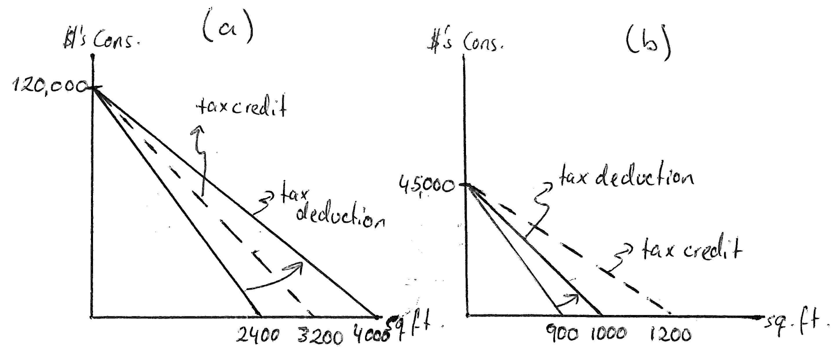
Policy Application: *Tax Deductions and Tax Credits*: *In the U.S. income tax code, a number of expenditures are "deductible". For most tax payers, the largest tax deduction comes from the portion of the income tax code that permits taxpayers to deduct home mortgage interest (on both a primary and a vacation home). This means that taxpayers who use this deduction do not have to pay income tax on the portion of their income that is spent on paying interest on their home mortgage(s). For purposes of this exercise, assume that the entire yearly price of housing is interest expense.*

A: True or False*: For someone whose marginal tax rate is 33%, this means that the government is subsidizing roughly one third of his interest/house payments.*

Answer: Consider someone who pays $10,000 per year in mortgage interest. When this person deducts $10,000, it means that he does not have to pay the 33% income tax on that amount. In other words, by deducting $10,000 in mortgage interest, the person reduces his tax obligation by $3,333.33. Thus, the government is returning 33 cents for every dollar in interest payments made — effectively causing the opportunity cost of paying $1 in home mortgage interest to be equal to 66.67 cents. So the statement is true.

(a) *Consider a household with an income of $200,000 who faces a tax rate of 40%, and suppose the price of a square foot of housing is $50 per year. With square footage of housing on the horizontal axis and other consumption on the vertical, illustrate this household's budget constraint with and without tax deductibility. (Assume in this and the remaining parts of the question that the tax rate cited for a household applies to all of that household's income.)*

Answer: As just demonstrated, the tax deductibility of home mortgage interest lowers the price of owner-occupied housing, and it does so in proportion to the size of the marginal income tax rate one faces.



Exercise Graph 2.17 : Tax Deductions versus Tax Credits

Panel (a) of Exercise Graph 2.17 illustrates this graphically for the case described in this part. With a 40 percent tax rate, the household could consume as much as 0.6(200,000)=120,000 in other goods if it consumed no housing. With a price of housing of $50 per square foot, the price falls to (1 − 0.4)50 = 30 under tax deductibility. Thus, the budget rotates out to the solid budget in panel (a) of the graph. Without deductibility, the consumer pays $50 per square foot — which makes 120,000/50=2,400 the biggest possible house she can afford. But with deductibility, the biggest house she can afford is 120,000/30=4,000 square feet.

(b) *Repeat this for a household with income of $50,000 who faces a tax rate of 10%.*

Answer: This is illustrated in panel (b). The household could consume as much as $45,000 in other consumption after paying taxes, and the deductibility of house payments reduces the price of housing from $50 per square foot to (1 − 0.1)50 = $45 per square foot. This results in the indicated rotation of the budget from the lower to the higher solid line in the graph. The rotation is smaller in magnitude because the impact of deductibility on the after-tax price of housing is smaller. Without deductibility, the biggest affordable house is 45,000/50=900 square feet, while with deductibility the biggest possible house is 45,000/45=1,000 square feet.

(c) *An alternative way for the government to encourage home ownership would be to offer a tax* credit *instead of a tax* deduction. *A tax credit would allow all taxpayers to subtract a fraction k of their annual mortgage payments directly from the tax bill they would otherwise owe. (Note: Be careful — a tax credit is deducted from tax* payments *that are due, not from the taxable income.) For the households in (a) and (b), illustrate how this alters their budget if $k = 0.25$.*

Answer: This is illustrated in the two panels of Exercise Graph 2.17 — in panel (a) for the higher income household, and in panel (b) for the lower income household. By subsidizing housing through a credit rather than a deduction, the government has reduced the price of housing by the same amount (k) for everyone. In the case of deductibility, the government had made the price subsidy dependent on one's tax rate — with those facing higher tax rates also getting a higher subsidy. The price of housing how falls from $50 to (1 − 0.25)50 = $37.50 — which makes the largest affordable house for the wealthier household 120,000/37.5=3,200 square feet and, for the poorer household, 45,000/37.5=1,200 square feet. Thus, the poorer household benefits more from the credit when $k = 0.25$ while the richer household benefits more from the deduction.

(d) *Assuming that a tax deductibility program costs the same in lost tax revenues as a tax credit program, who would favor which program?*

Answer: People facing higher marginal tax rates would favor the deductibility program while people facing lower marginal tax rates would favor the tax credit.

B: *Let x_1 and x_2 represent square feet of housing and other consumption, and let the price of a square foot of housing be denoted p.*

(a) *Suppose a household faces a tax rate t for all income, and suppose the entire annual house payment a household makes is deductible. What is the household's budget constraint?*

Answer: The budget constraint would be $x_2 = (1-t)I - (1-t)px_1$.

(b) *Now write down the budget constraint under a tax credit as described above.*

Answer: The budget constraint would now be $x_2 = (1-t)I - (1-k)px_1$.

INSTRUCTORS

0

Preparing a Syllabus for the Course in Three Quick Steps

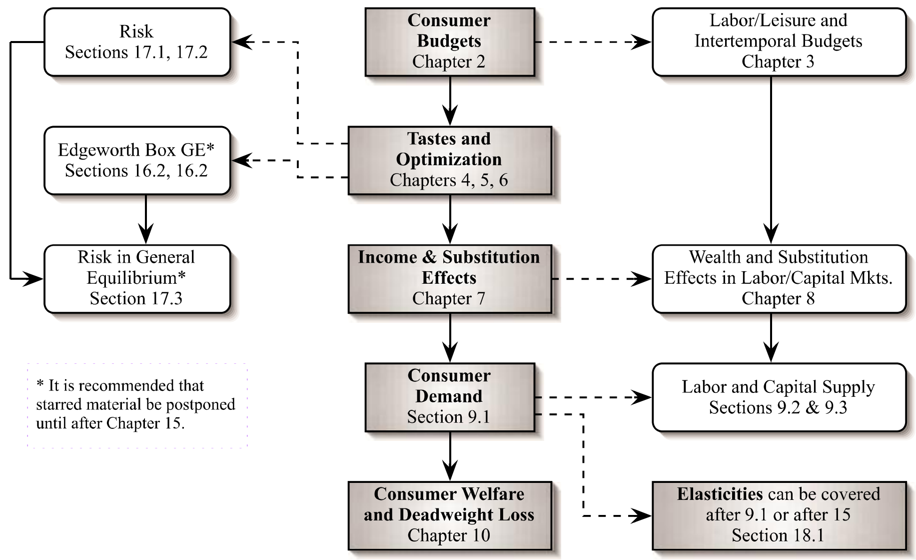
This textbooks clearly has far too much material for a one-semester course in microeconomics and in fact was written as a relatively comprehensive two-semester book. Part A in each chapter is non-mathematical but rigorous, written with an eye toward making the math that is added in part B make sense. One decision you have to make is how much use you will make of A-parts versus B-parts. In some universities, students might be taking a second intermediate microeconomics course following a non-mathematical course earlier — in which case a heavy emphasis on the B-parts is clearly warranted. In other universities, microeconomics is taught across multiple semesters, with each semester containing an equal emphasis on math and intuition. In such cases, you will probably use both A and B parts but will restrict yourself to only half the topics in the text. (Which half will probably depend on whether you go first or second.) Or you might be in a university where you are teaching a relatively non-mathematical intermediate microeconomics course and you'll rely most heavily on just the A-parts. (If you wish to use none of the mathematical B-parts, you may want to use the non-mathematical spin-off book entitled *Microeconomics: An Intuitive Approach*.)

I'll say little more about the balance between the intuitive and math sections — you are clearly in the best position to make that judgment. Instead, what follows is an outline for a three-step process for constructing the syllabus — where the overall aim for a one-semester (or one-quarter course) should be to plan to cover roughly half the material (or a bit less) in the textbook. The three steps consist of:

1. Include all the **foundational material** on consumer theory, producer theory and competitive equilibrium — and decide which other topics that relate to this foundational material you want to add.

2. Choose a **path through the second half** of the book depending on your focus and the types of tools beyond the foundational tools you want your students to know.

3. Decide how to conclude the course.



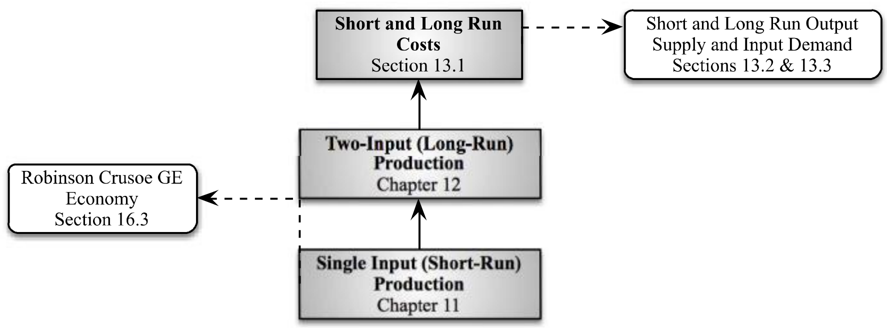
Graph 0.1: Building the Consumer Model Portion of the Syllabus

Step 1: Building Foundational Material into the Beginning of the Syllabus

The text begins with the consumer model, then proceeds to the producer model before putting them together to discuss competitive equilibrium

Graph 0.1 begins with a guide for constructing the **Consumer Model Portion of the Syllabus**. The **shaded boxes represent material that is foundational** — with solid arrows indicating the manner in which the material builds. The non-shaded boxes are optional in the sense that later material rarely if ever depends on it (and when it does, it is contained in small sub-sections that can be skipped). Dashed arrows indicate at what point these optional materials can be covered. Of course you can choose any point after that in the syllabus to cover these optional materials — the dashed arrows simply indicate the earliest that the students are prepared to tackle them. While the first two sections of the chapter on Risk (top left of the Graph) might, for instance, reasonably be covered early in the course, the general equilibrium section is probably best saved for after Chapter 15 (as is the Edgeworth Box material).

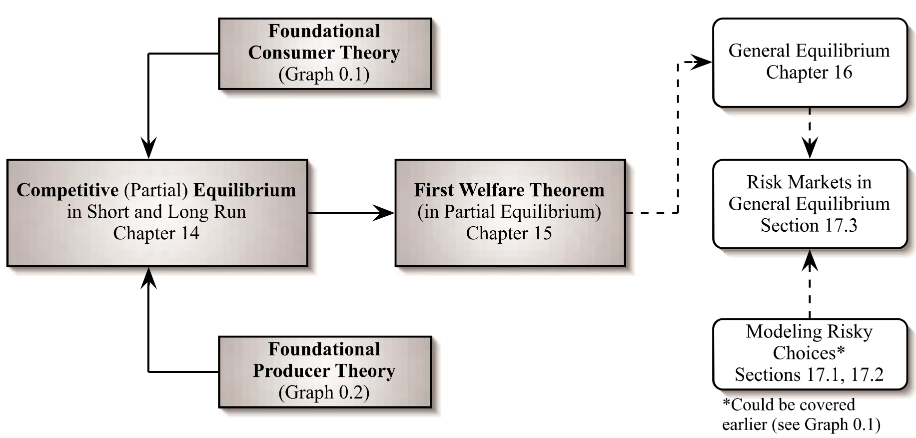
My own preference is to develop the worker/saver model (on the right hand side of Graph 0.1) jointly with the standard consumer model (in the center of the Graph) because all the same tools are used and students can therefore get a feeling for how we really are doing the same thing in all these cases, but this is a matter of preference. I also know that some instructors like to cover elasticities earlier than where they appear in the text, and this is possible as indicated in Graph 0.1



Graph 0.2: Building the Producer Model Portion of the Syllabus

(although my own preference is to cover elasticities right before I start using the concept extensively in Chapter 18).

Graph 0.2 presents a similar flow chart for the producer theory portion of the course (with the arrows this time pointing up). While in principle it might be possible to cover later material without going into two-input production (Chapter 12) at all, I doubt many instructors who adopt this book will opt for this — and I have therefore indicated Chapter 12 as foundational even though one could envision glancing over some of that material and still making it through much of the rest of the book. The optional sections of Chapter 13 take some time to develop, and I suspect that most instructors will opt to skip them. However, each of these sections (13.2 and 13.3) comes with an intuitive introductory section that could easily be incorporated without going into the detail that follows.



Graph 0.3: Building the Final Foundational Pieces of the Syllabus

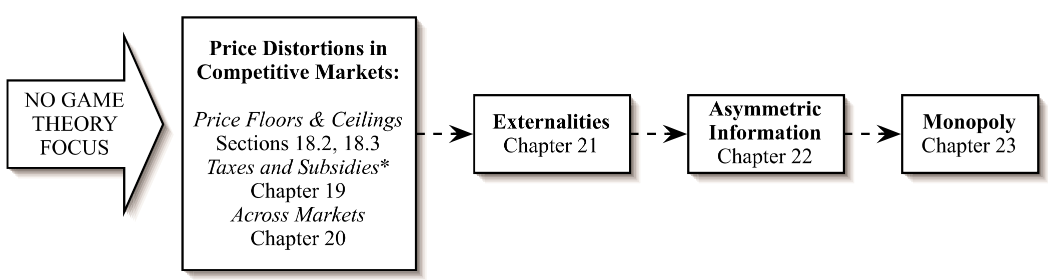
The foundational elements of consumer theory (Graph 0.1) and producer theory (Graph 0.2) then combine to build up to the treatment of competitive equilibrium as illustrated in Graph 0.3. Chapter 14 describes the competitive partial equilibrium and Chapter 15 illustrates its welfare properties. These two chapters are, as indicated by the shading, foundational for the rest of the text. But you may wish to elaborate on them further by investigating the general equilibrium material in the next two chapters. These are not, however, foundational to the rest of the text, and you could, in principle, also cover them in the consumer theory section (see Graph 0.1), although my recommendation would be to hold off until after the partial equilibrium and its welfare properties have been developed.

Step 2: Choosing a Path Following the Foundational Material in the Middle of the Syllabus

If you are teaching both the A- and B-parts of each of the chapters, you will probably have covered a semester's worth of material by the time you make it through the consumer theory (Graph 0.1), producer theory (Graph 0.2) and Competitive Equilibrium (Graph 0.3). Your syllabus should therefore be pretty much done, with perhaps one or two chapters from the second half of the book added.

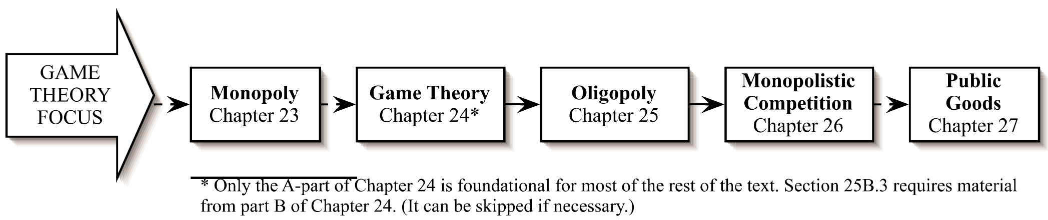
For those teaching primarily the A- or the B-parts of the foundational material, a choice now has to be made as to which path to choose through the rest of the book. Even if you cover only the shaded core material in Graph 0.9 of the Appendix (which summarizes some of the previous flow charts), you probably do not have enough time to cover Chapters 18 through 30 in their entirety. Several obvious options emerge from Graph 0.10 which is the second flow chart in the Appendix (and which illustrates the various paths outlined below):

1. A **Traditional Theory Focus without Game Theory** would take you from Chapters 18 through 23 — covering all the major violations of the first welfare theorem without any formal use of game theory. This is illustrated in Graph 0.4 which would follow in your syllabus immediately after the choices you made in Graph 0.3.



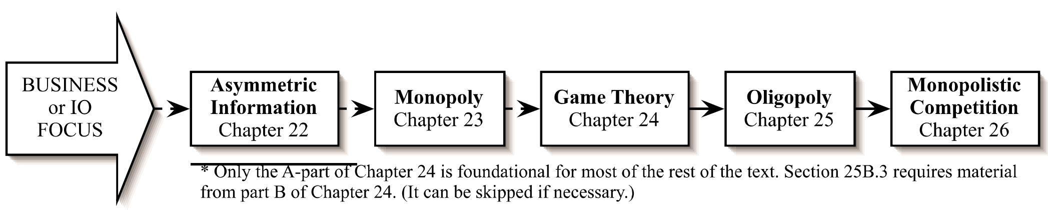
Graph 0.4: Building No-Game-Theory-Path into the Syllabus

2. A **Game Theory Focus** could skip all but the first section of Chapter 18 through Chapter 22, introduce some initial strategic thinking in the context of Monopolies in Chapter 23 before covering Game Theory in Chapter 24. Note that the game theory chapter covers Nash and subgame perfect equilibrium in part A and the Bayesian analogs — Bayesian Nash and perfect Bayesian Nash equilibrium — in part B. The remainder of the game theory-based chapters, however, draw primarily on part A of Chapter 24, with the only sequenced chapters (indicated by the solid arrows in Graph 0.5) involving Chapters 24 and 25.



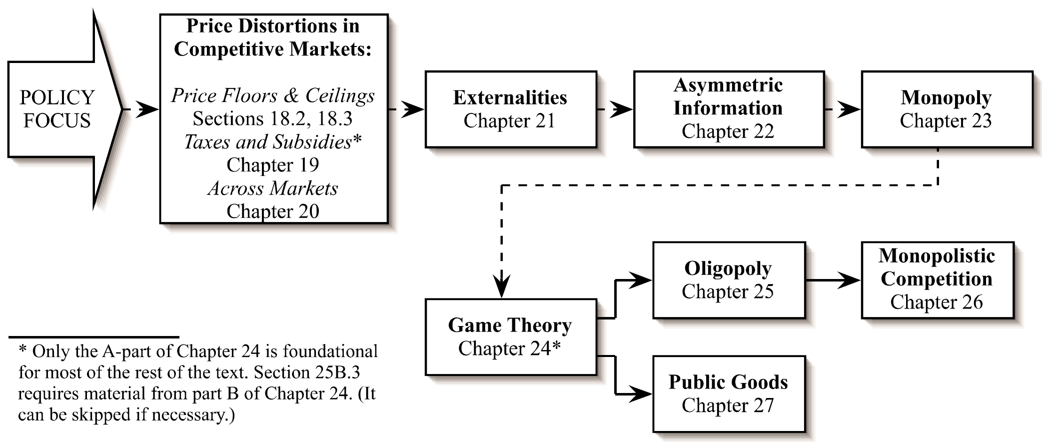
Graph 0.5: Building a Heavy Game Theory Emphasis into the Syllabus

3. A **Business** or **Industrial Organization (IO) Focus** would share a lot in common with the game theory path outlined in Graph 0.5 but with some slight modifications outlined in Graph 0.6.



Graph 0.6: Building a Business or IO Emphasis into the Syllabus

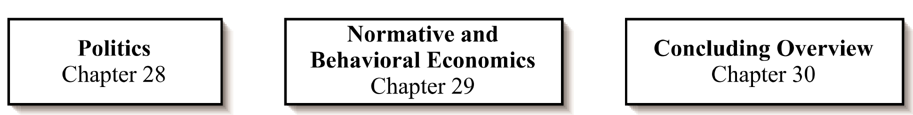
4. Finally, a **Policy Focus** can take a variety of forms, depending primarily on whether you decide to cover game theory or not. If you do not, then the Policy Focus looks a lot like the No-Game-Theory Focus in Graph 0.4; otherwise, you have the flexibility to build in some game theory features by covering part A of Chapter 24 and then either some additional material on market power (Chapters 25 and/or 26) or on public goods and the free rider problem (Chapter 27). These options are illustrated in Graph 0.7.



Graph 0.7: Options for a Policy Emphasis in the Syllabus

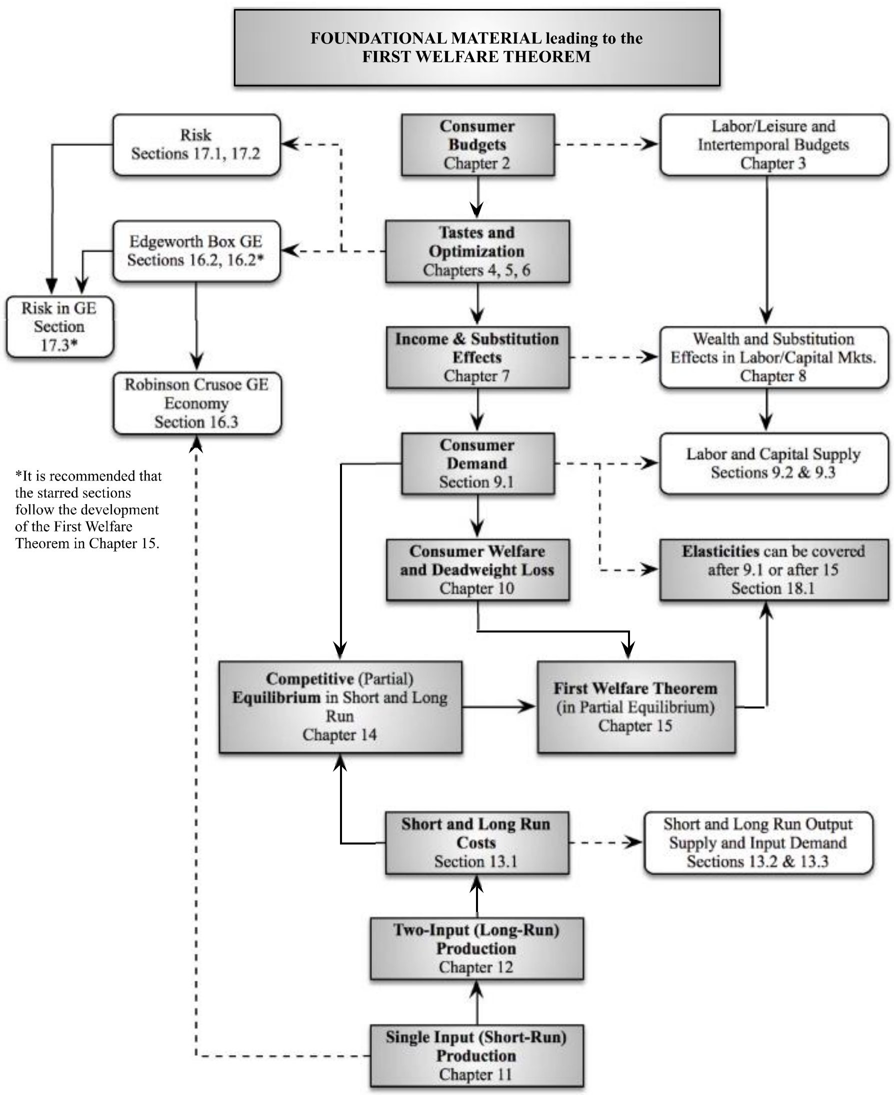
Step 3: Choosing a Conclusion for the Syllabus

The last part to finishing up your syllabus is to simply choose how to conclude your course (with the understanding that the last few planned parts of a course often tend to be short-changed as we rush to finish up a semester). The options, laid out in Graph 0.8, include a chapter on Politics which, in my experience, students enjoy a great deal. (The best students — or those in a very mathematical course — are sometimes quite taken by part B, a proof of Arrow's Impossibility Theorem.) Chapter 29 covers normative issues in the first section before providing an overview to behavioral economics. Both these chapters have a way of emphasizing that economics is really about more than what students might think, and so they tend to open up horizons. The concluding Chapter 30 is very brief and simply tries to get back to the big picture theme flowing throughout the book. (I have tried to write this chapter, as well as the rest of the book, in a very non-ideological way — my hope is that you will find it laying out a *way of thinking* that can fit with a variety of ideological predisposition.)

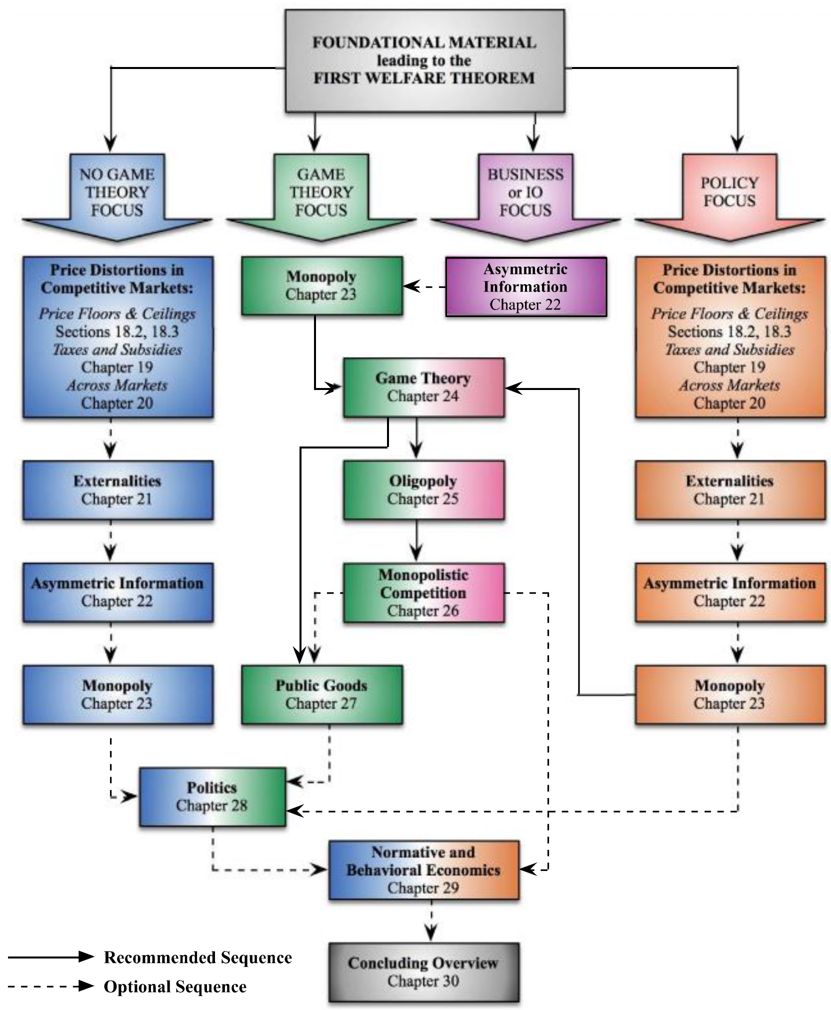


Graph 0.8: Options for Concluding the Syllabus

Appendix: Overview in Two Pages



Graph 0.9: Tools Building up to First Welfare Theorem



Graph 0.10: Exploring what Lies Beneath the First Welfare Theorem

Microeconomics: An Intuitive Approach, 2E

Chapter 2 – A Consumer's Economic Circumstances

Consumer Choice Sets and Budget Constraints

People make the best choices they can *given their circumstances*

"Constraints"

- Current Income
- Assets (from Savings)
- Skills
- Time
- Future Income (for Borrowing)
- (Psychological – Ch. 29)

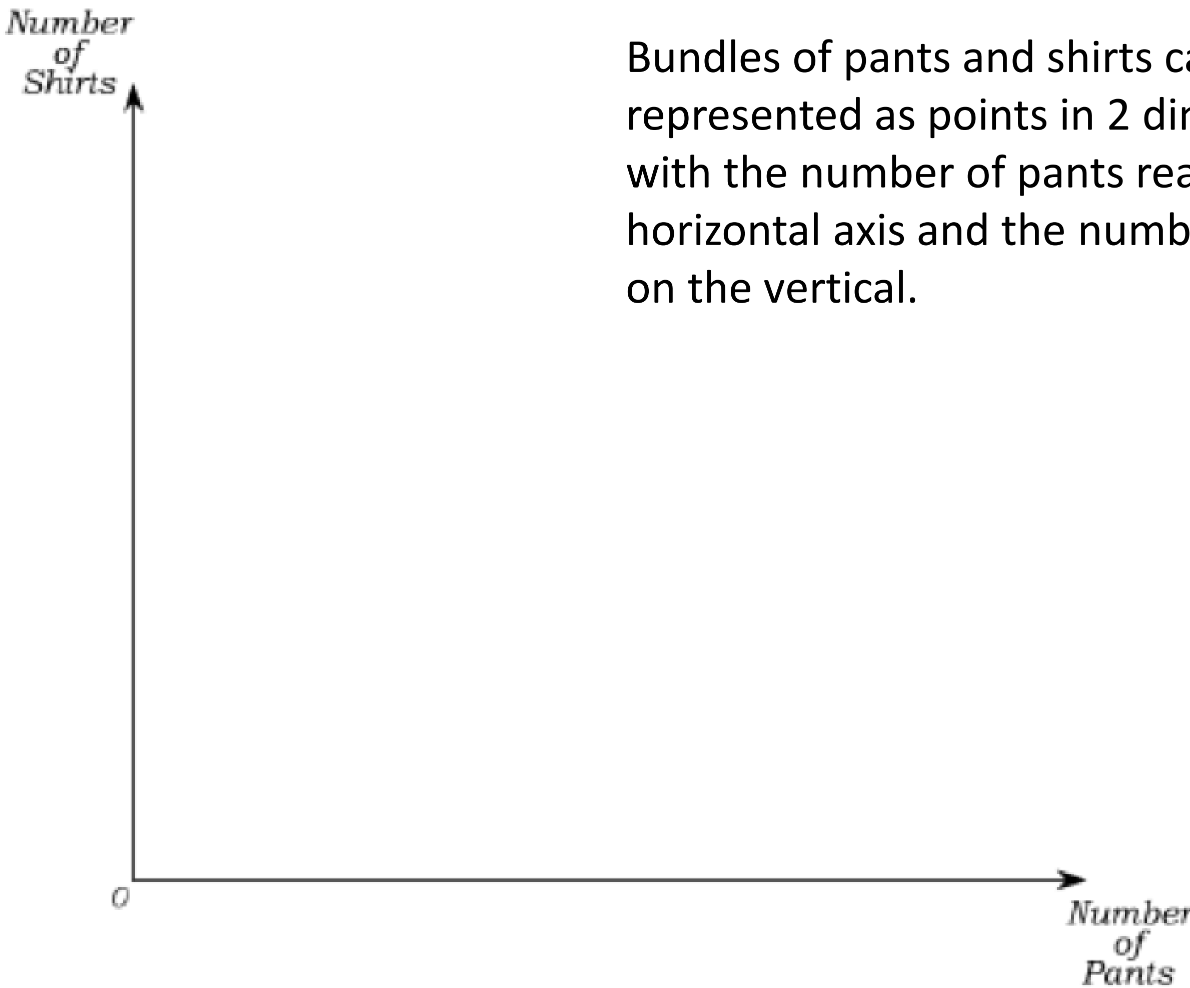
Consumer Choice Sets and Budget Constraints

- Consumers are "small", have no power over prices ➡ ***"price-takers"***
- We begin by assuming that decisions leading to currently available income are in the past
  - ➢ The choice that remains is how to spend this available income
  - ➢ Such income that the consumer takes "as given" is called *"exogenous"*

Consumer Choice Sets… -
Shopping on a Fixed (Exogenous) Income

- Suppose I have $200 to spend on pants and shirts. (I can spend less, but never more).
- The price of pants is $20 and the price of shirts is $10.
- Given those restrictions, I can buy any "bundle" of pants and shirts that I want.
- The set of all "bundles" of shirts and pants that I could buy is then called my ***"Choice Set"***

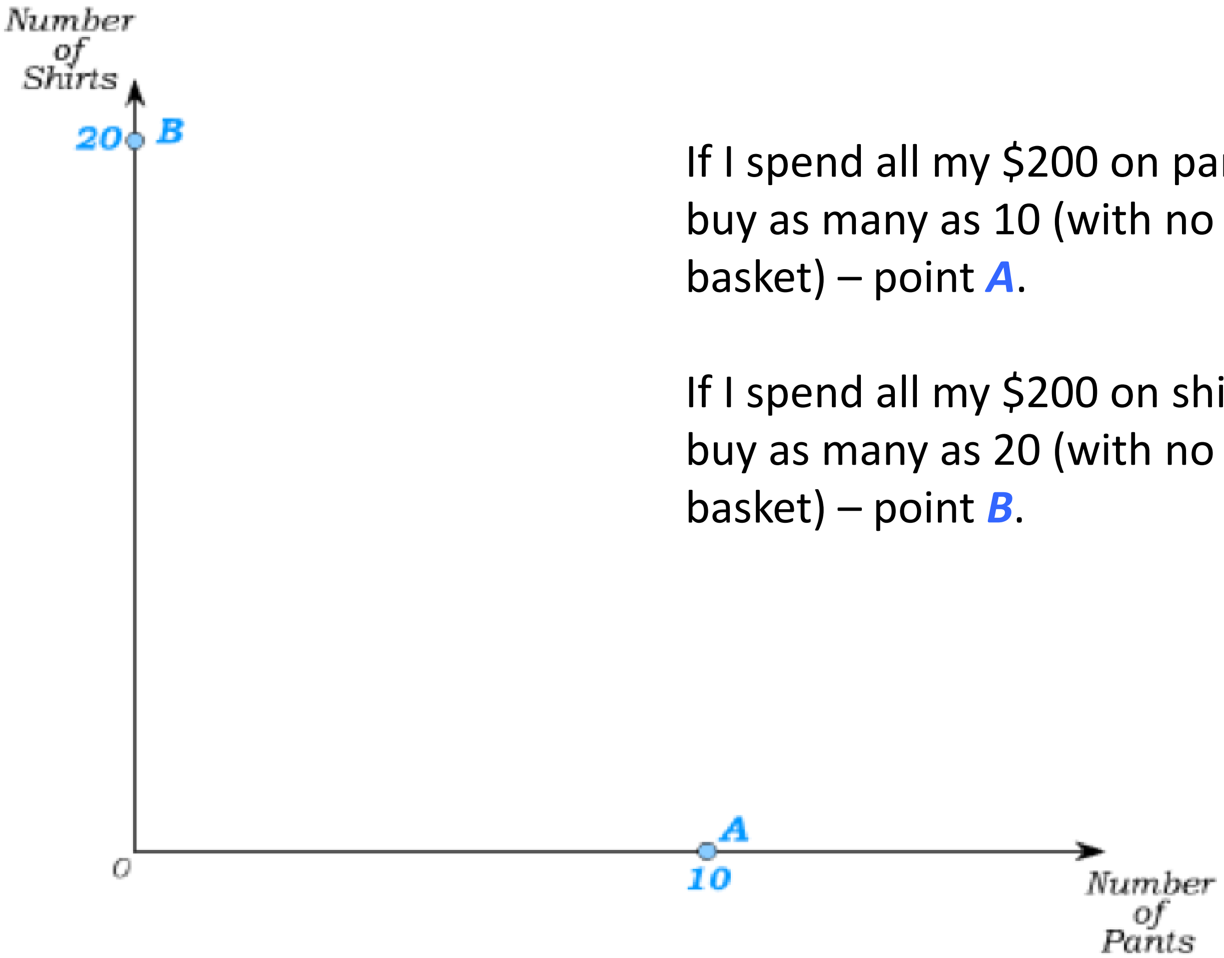
Consumer Choice Sets… - Shopping on a Fixed (Exogenous) Income



Bundles of pants and shirts can then be represented as points in 2 dimensions with the number of pants read on the horizontal axis and the number of shirts on the vertical.

Go to Math

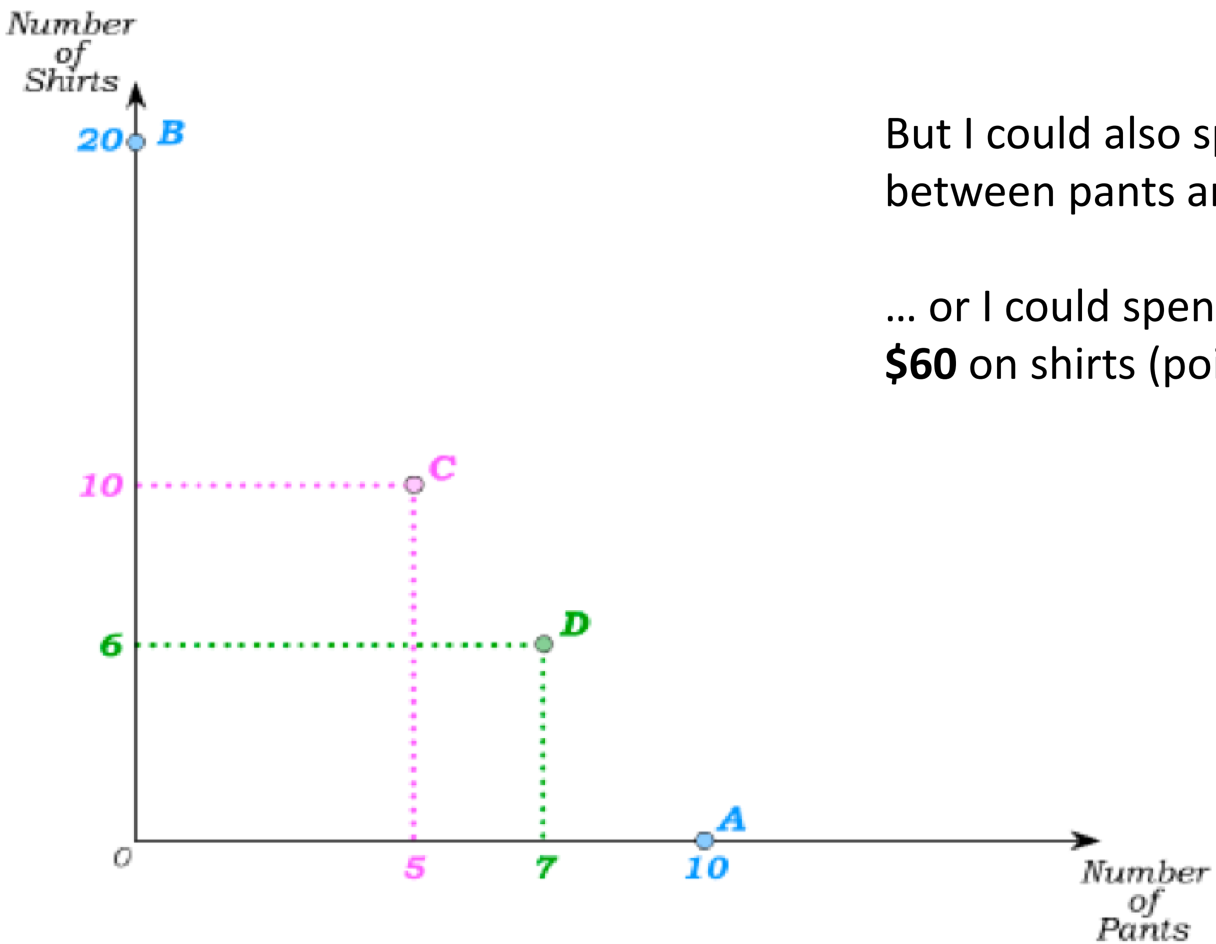
Consumer Choice Sets… - Shopping on a Fixed (Exogenous) Income



If I spend all my $200 on pants, I can buy as many as 10 (with no shirts in my basket) – point ***A***.

If I spend all my $200 on shirts, I can buy as many as 20 (with no pants in my basket) – point ***B***.

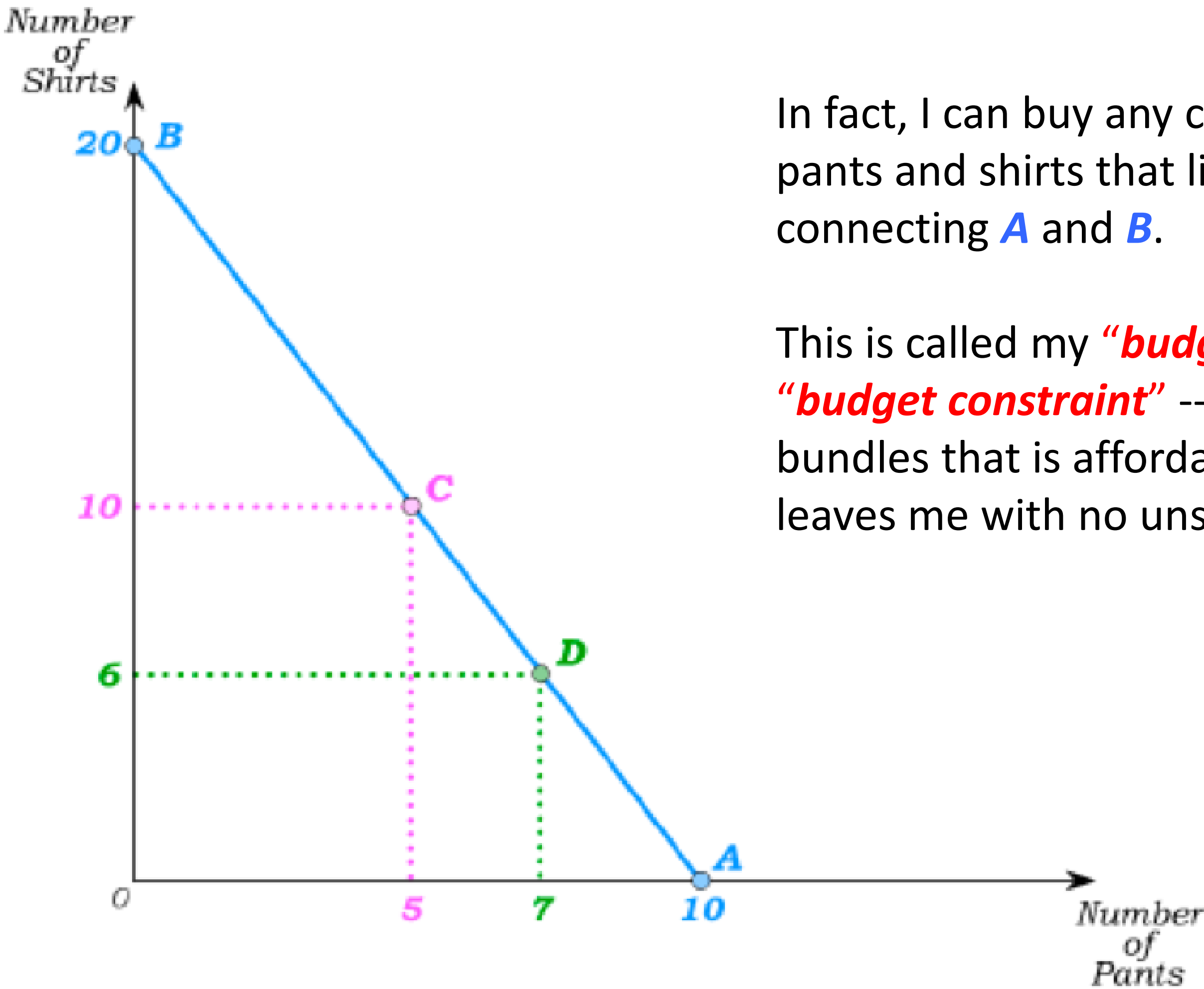
Consumer Choice Sets… - Shopping on a Fixed (Exogenous) Income



But I could also split the **$200** evenly between pants and shirts (point ***C***) …

… or I could spend **$140** on pants and **$60** on shirts (point ***D***).

Consumer Choice Sets…- Shopping on a Fixed (Exogenous) Income

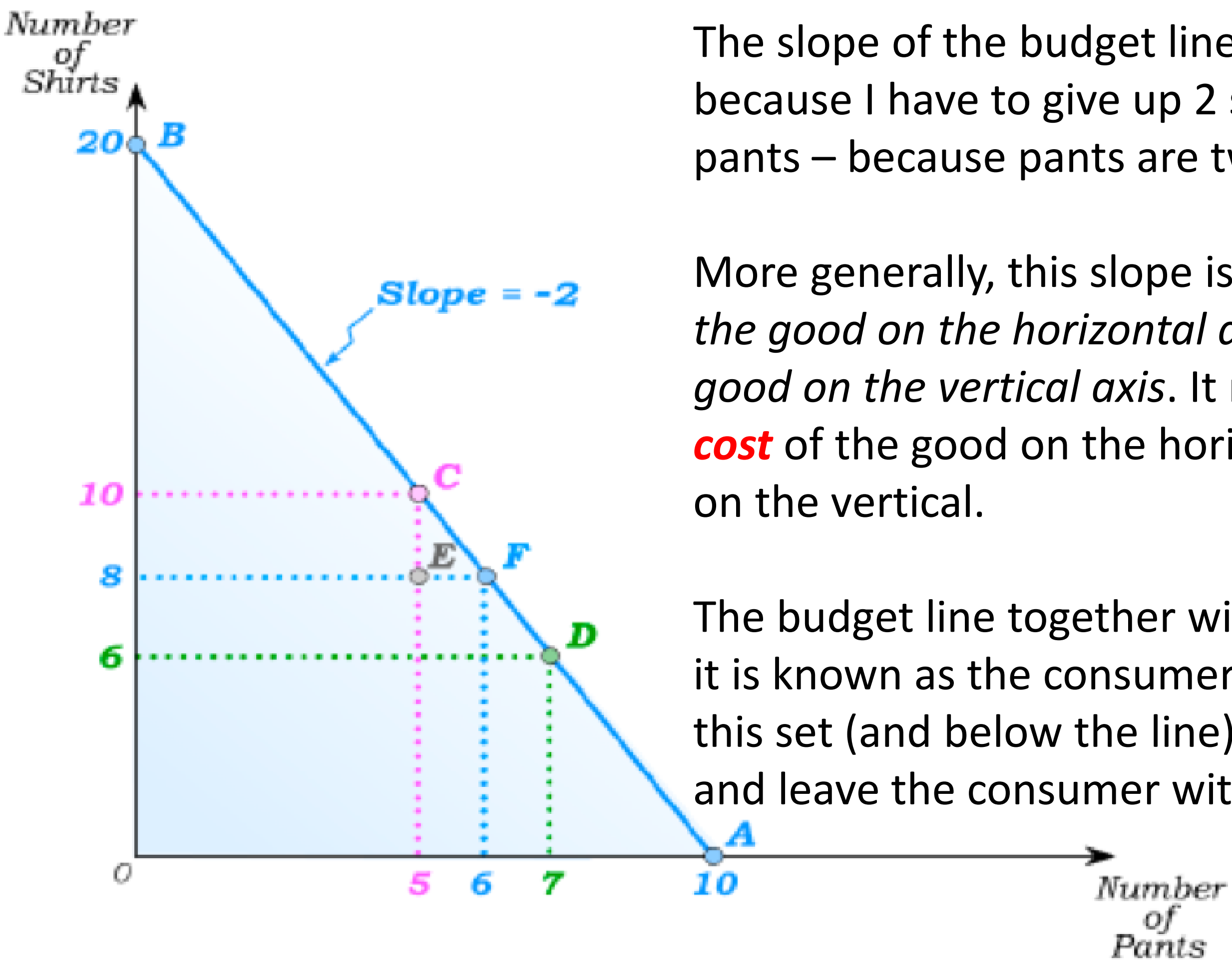


In fact, I can buy any combination of pants and shirts that lies on the line connecting ***A*** and ***B***.

This is called my ***"budget line"*** or ***"budget constraint"*** -- the set of bundles that is affordable and that leaves me with no unspent income.

Go to Math

Consumer Choice Sets… - Shopping on a Fixed (Exogenous) Income



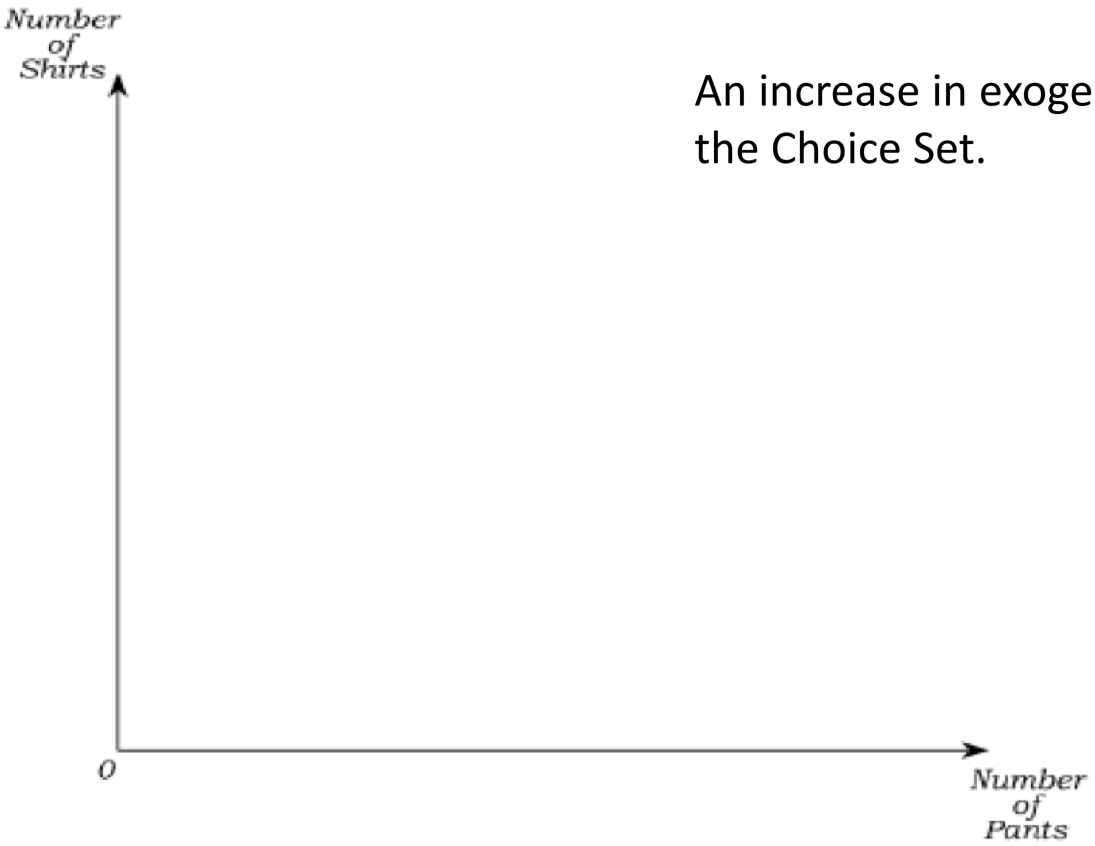
The slope of the budget line is –2 in our case. This is because I have to give up 2 shirts for every 1 pair of pants – because pants are twice as expensive as shirts.

More generally, this slope is equal to *minus the price of the good on the horizontal divided by the price of the good on the vertical axis*. It represents the ***opportunity cost*** of the good on the horizontal in terms of the good on the vertical.

The budget line together with the shaded area beneath it is known as the consumer's ***choice set***. Bundles within this set (and below the line) (such as ***E***) are affordable and leave the consumer with some money.

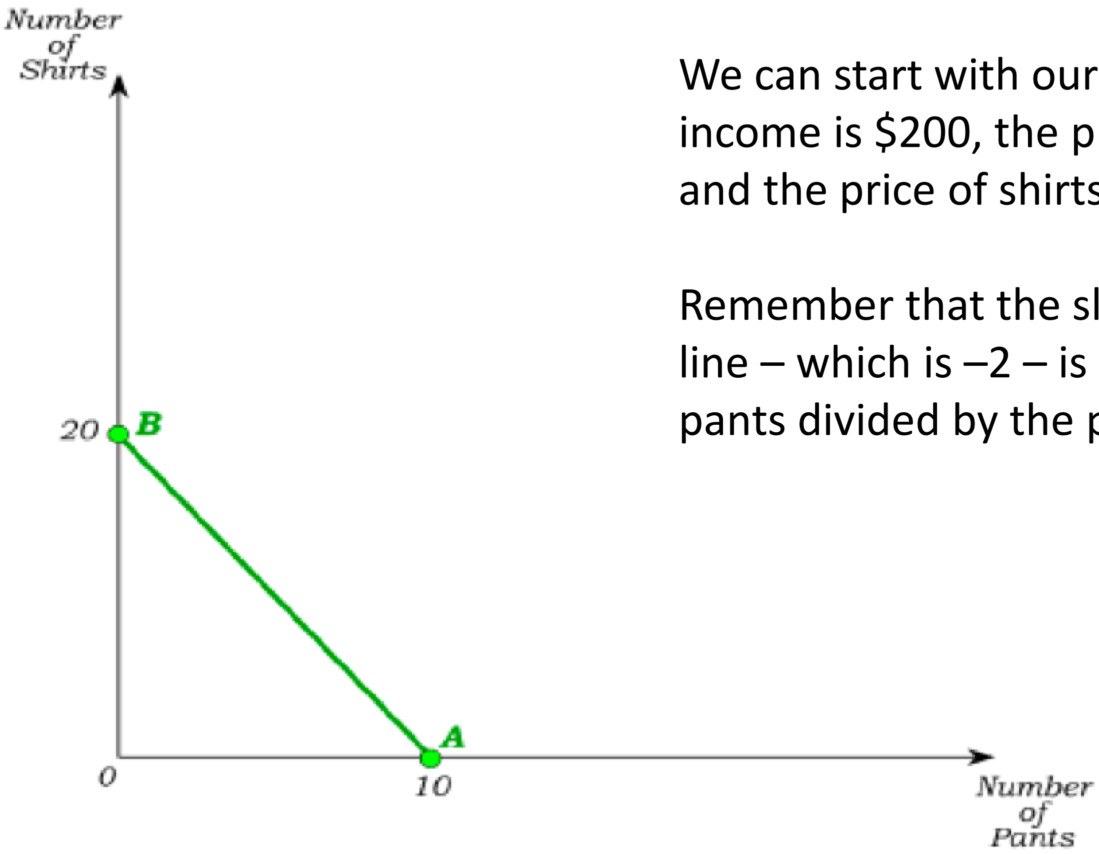
Go to Math

Consumer Choice Sets… -
An Increase or Decrease in Fixed Incomes



An increase in exogenous income will expand the Choice Set.

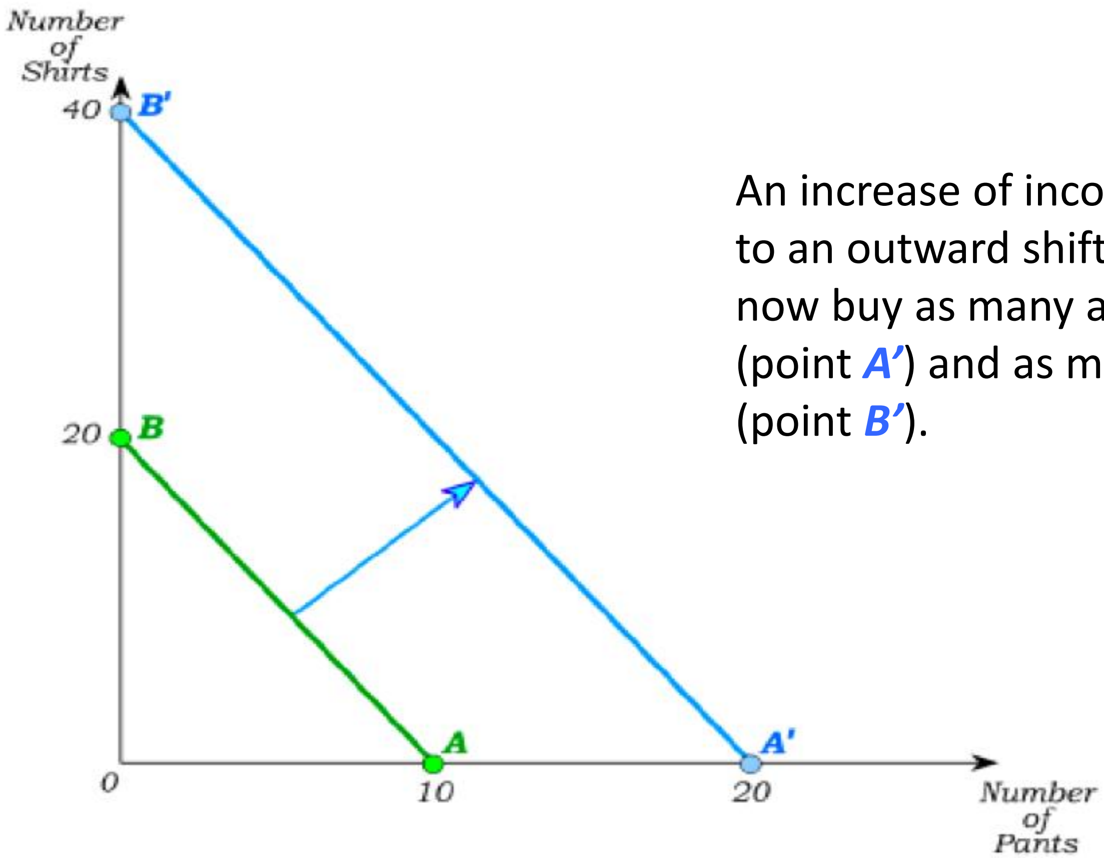
Consumer Choice Sets… - An Increase or Decrease in Fixed Incomes



We can start with our initial set-up where income is $200, the price of pants is $20 and the price of shirts is $10.

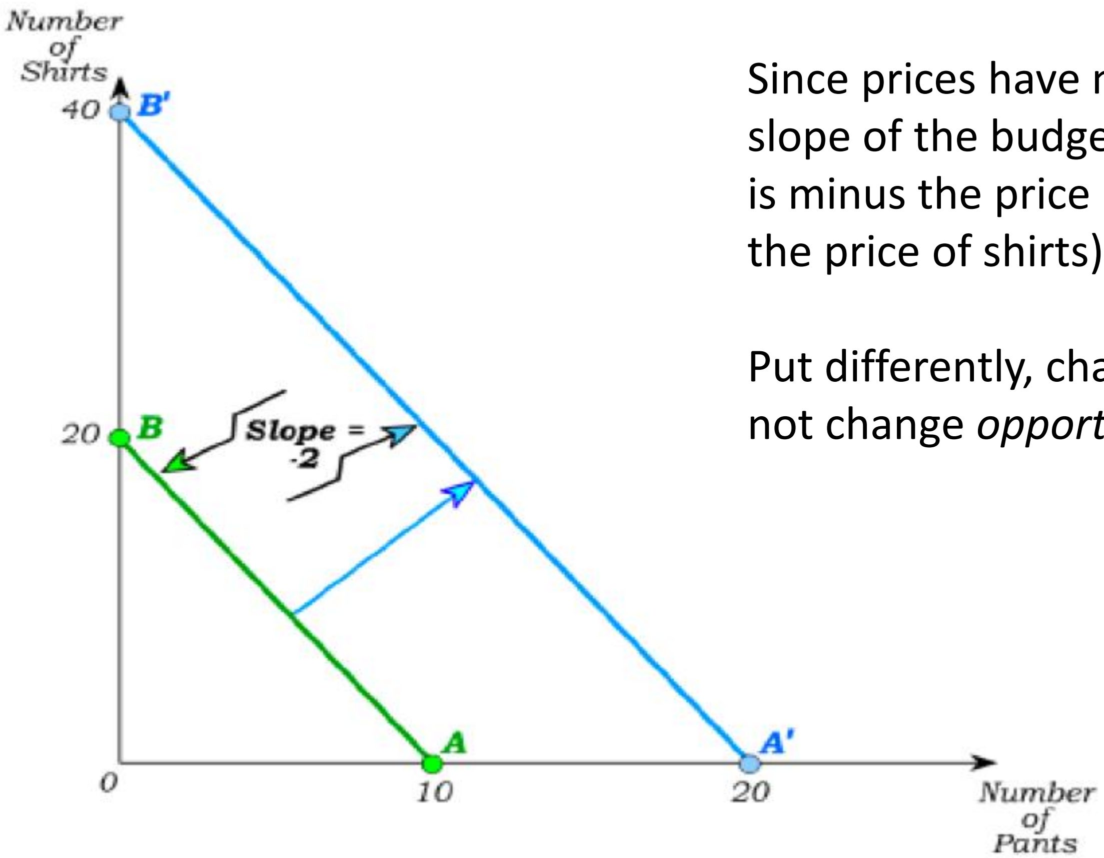
Remember that the slope of this budget line – which is –2 – is (minus) the price of pants divided by the price of shirts.

Consumer Choice Sets… -
An Increase or Decrease in Fixed Incomes



An increase of income to $400 then leads to an outward shift of the budget – I can now buy as many as 20 pants at $20 (point ***A’***) and as many as 40 shirts at $10 (point ***B’***).

Consumer Choice Sets… - An Increase or Decrease in Fixed Incomes

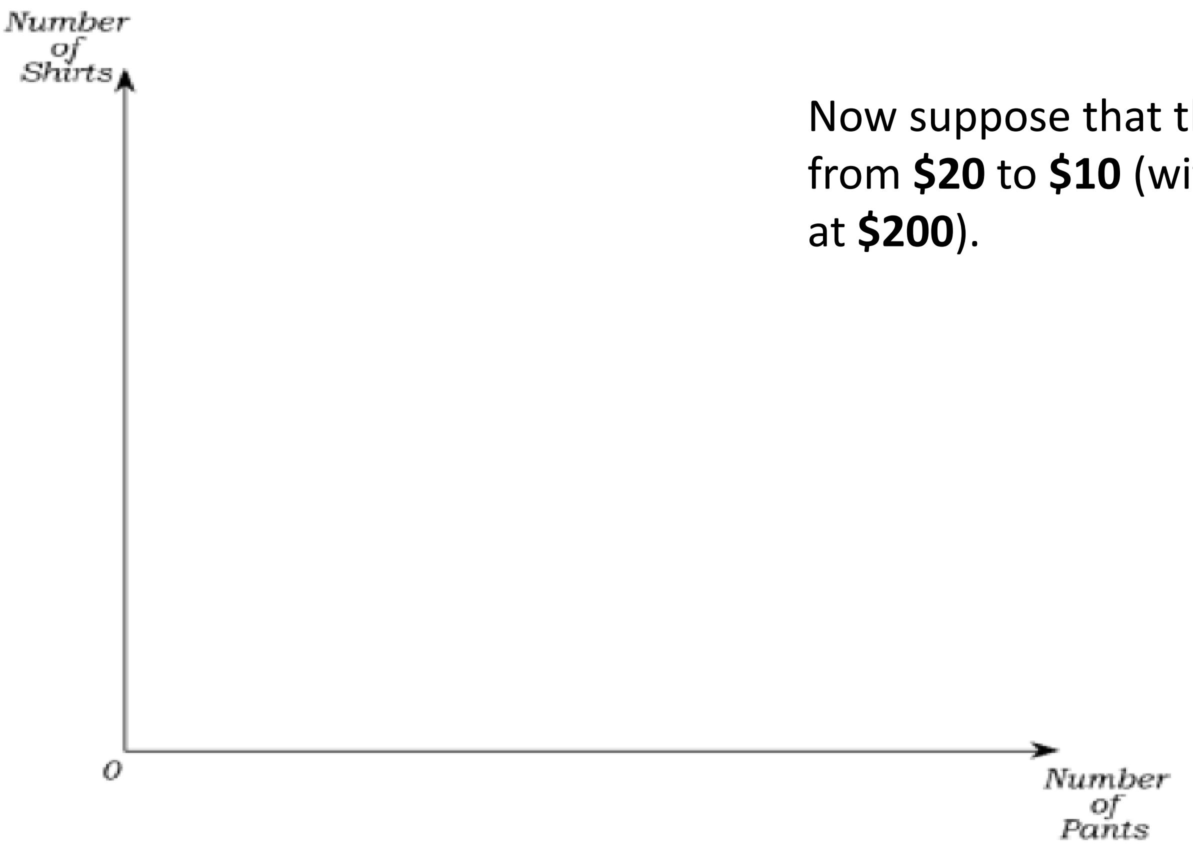


Since prices have not changed, the slope of the budget constraint (which is minus the price of pants divided by the price of shirts) does not change.

Put differently, changes in income do not change *opportunity costs.*

Go to Math

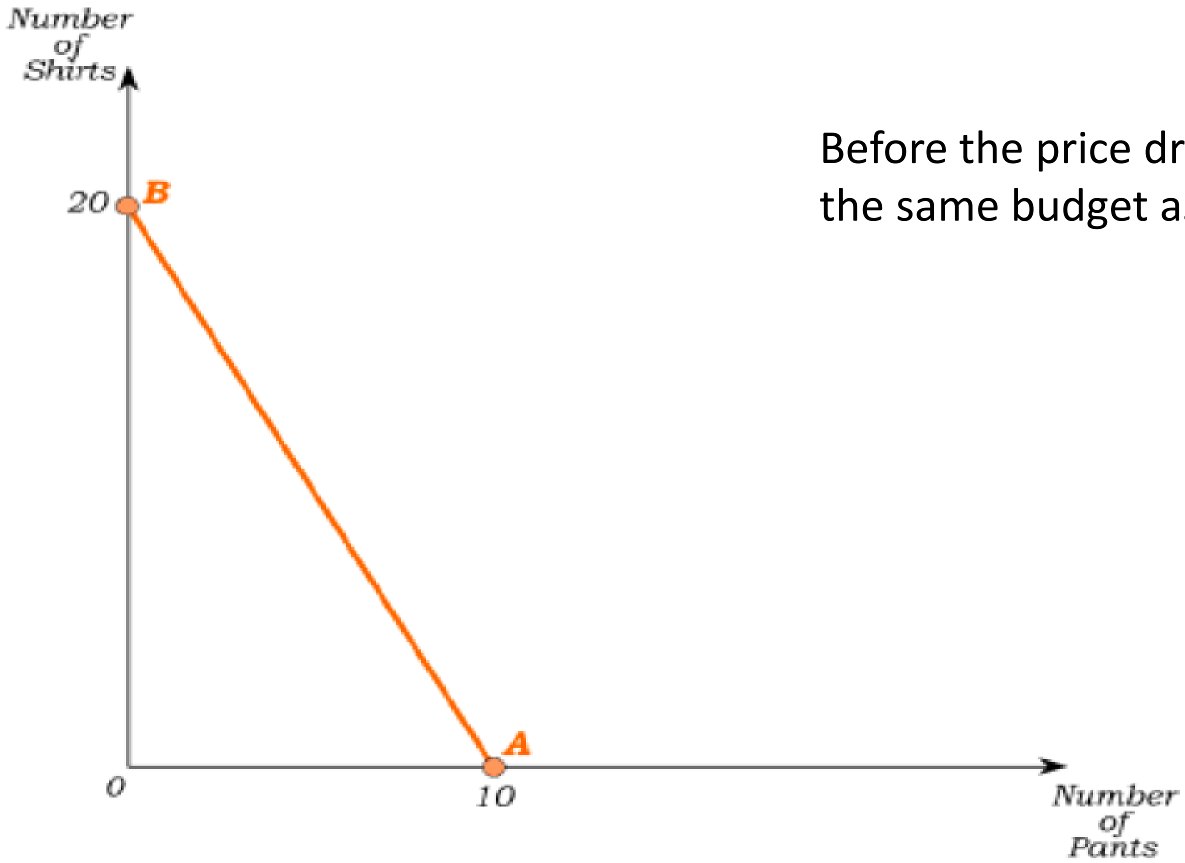
Consumer Choice Sets… -
A Change in Price



Now suppose that the price of pants falls from **$20** to **$10** (with income remaining at **$200**).

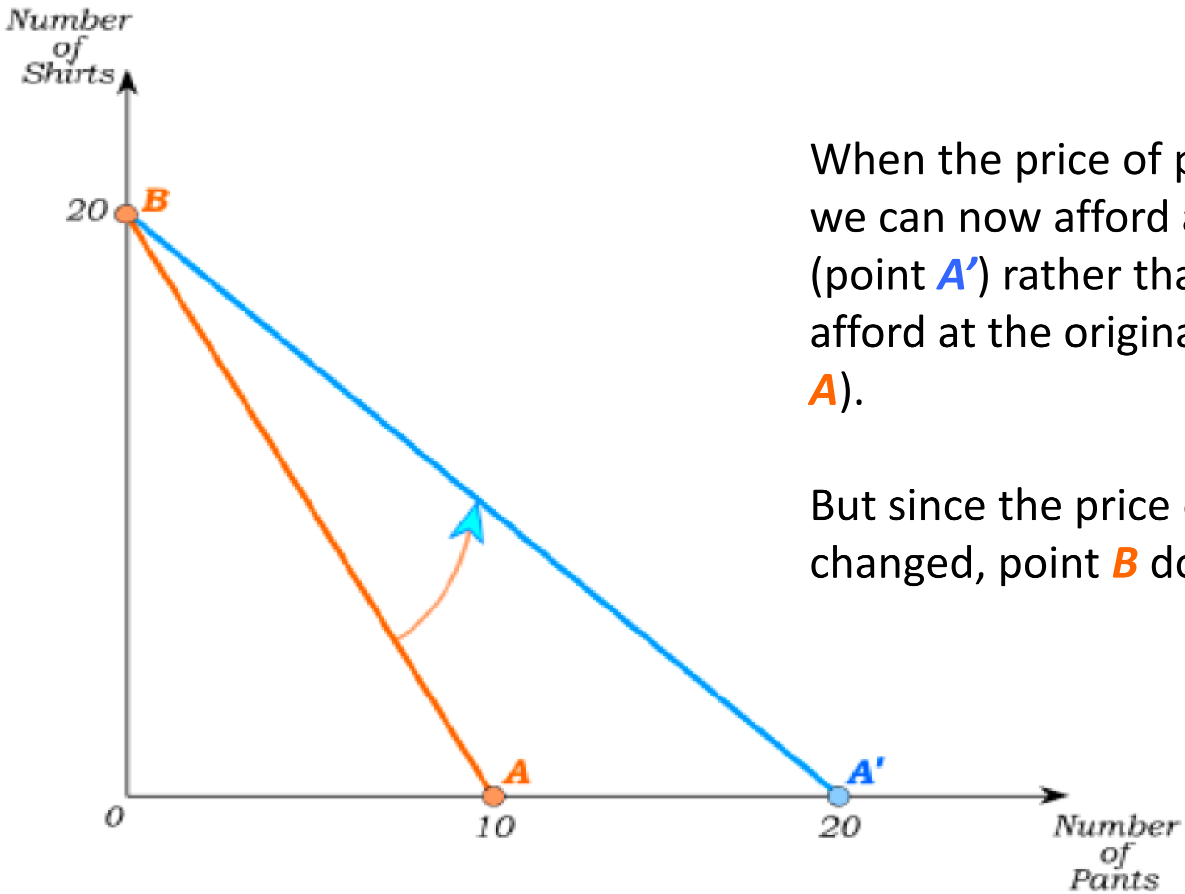
Consumer Choice Sets…-

A Change in Price



Before the price drop, we start with the same budget as before.

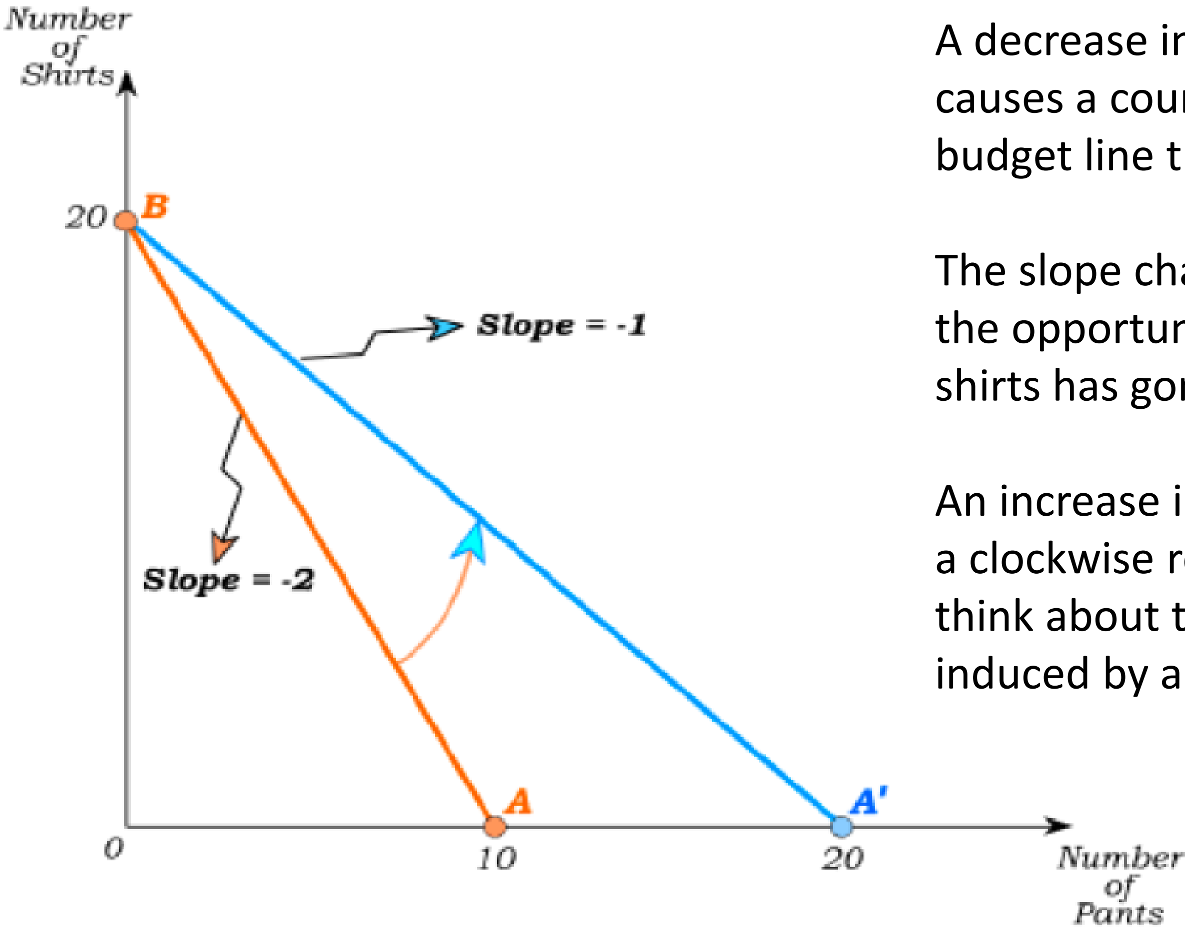
Consumer Choice Sets…-
A Change in Price



When the price of pants drops to $10, we can now afford as many as 20 pants (point ***A'***) rather than the 10 we could afford at the original price of $20 (point ***A***).

But since the price of shirts has not changed, point ***B*** does not change.

Consumer Choice Sets… - A Change in Price



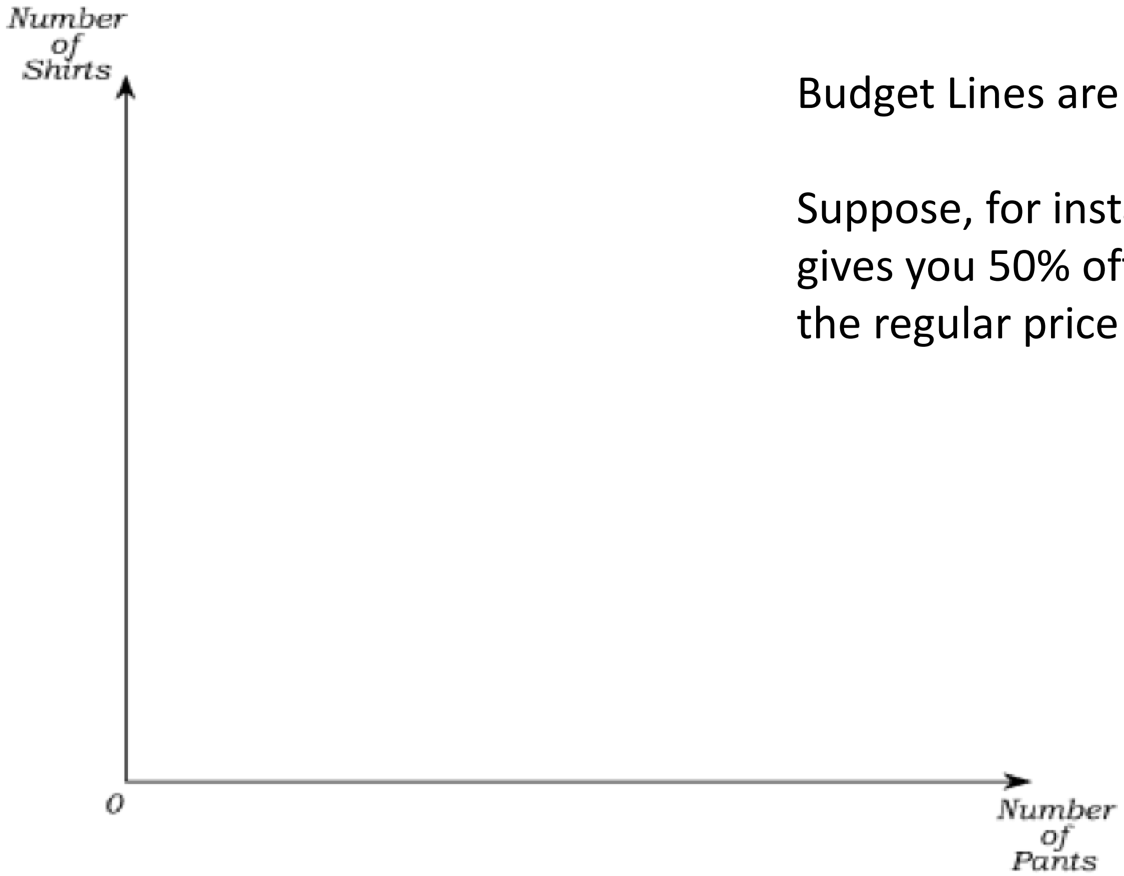
A decrease in the price of pants therefore causes a counter-clockwise rotation of the budget line through the vertical intercept.

The slope changes from –2 to –1 – because the opportunity cost of pants in terms of shirts has gone from 2 to 1.

An increase in the price of pants would cause a clockwise rotation inward. You can also think about the rotation of the budget induced by a change in the price of shirts.

Go to Math

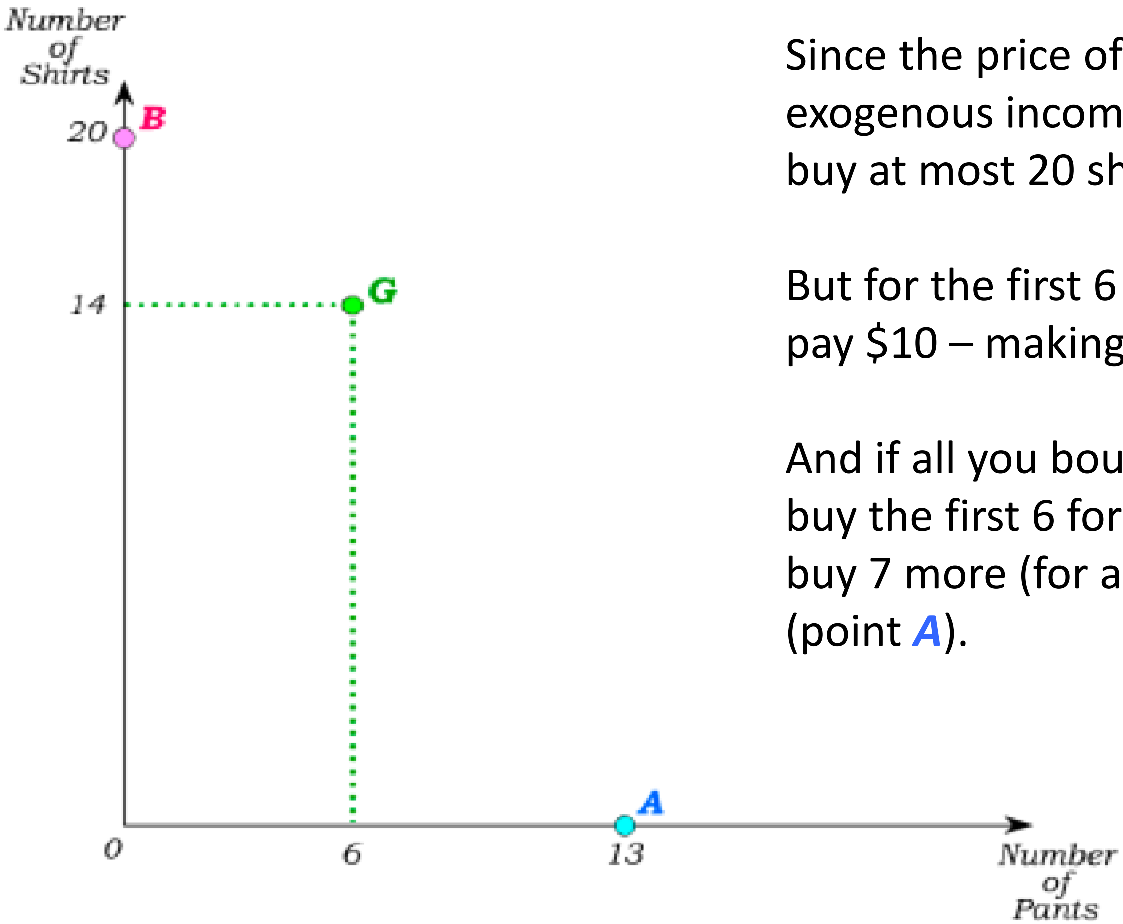
Consumer Choice Sets… - Kinky Budgets



Budget Lines are not always straight lines.

Suppose, for instance, you held a coupon that gives you 50% off the first 6 pair of pants (when the regular price of pants is $20.)

Consumer Choice Sets…-
Kinky Budgets

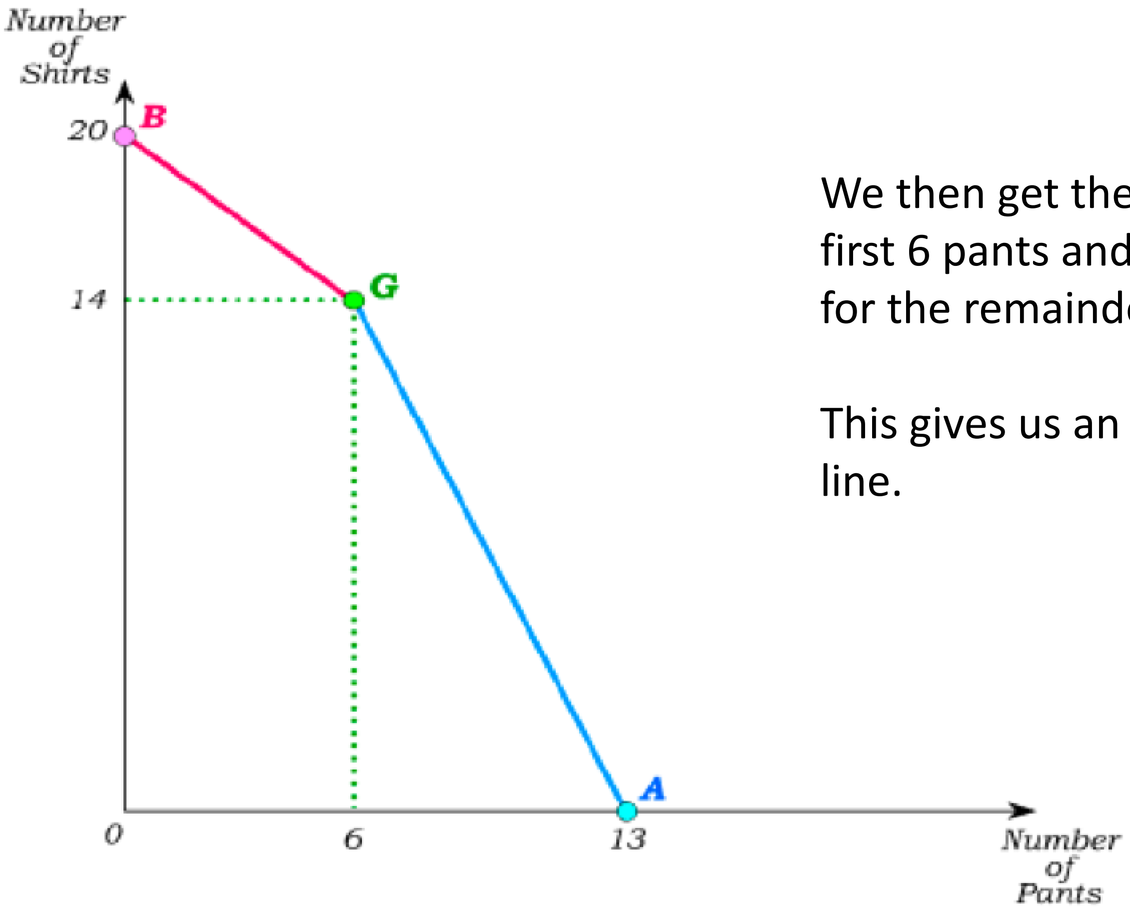


Since the price of shirts is $10 and your exogenous income is $200, you'd still be able to buy at most 20 shirts (point ***B***).

But for the first 6 pants you buy, you now only pay $10 – making possible the bundle ***G***.

And if all you bought was pants, you'd be able to buy the first 6 for $60, leaving you with $140 to buy 7 more (for a total of 13) at regular price (point ***A***).

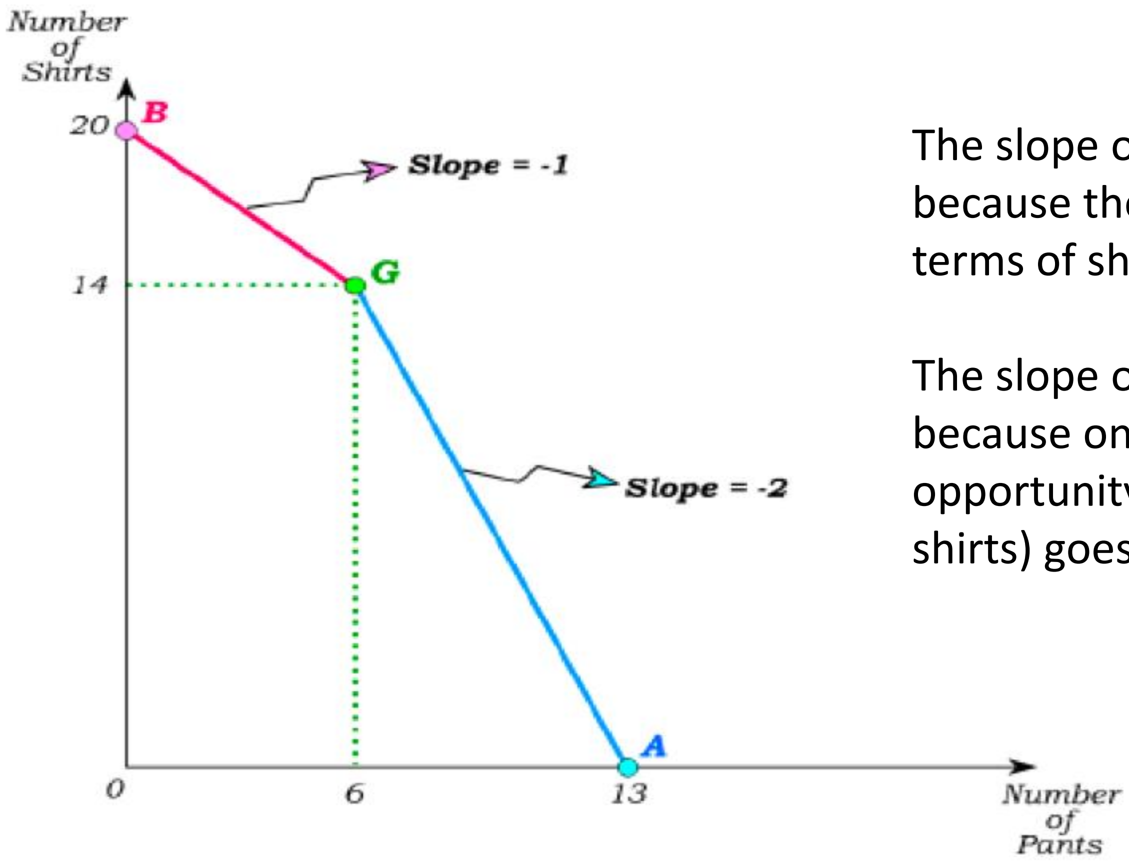
Consumer Choice Sets... - Kinky Budgets



We then get the **red** line segment for the first 6 pants and the **blue** line segment for the remainder.

This gives us an ***outwardly kinked*** budget line.

Consumer Choice Sets… - Kinky Budgets

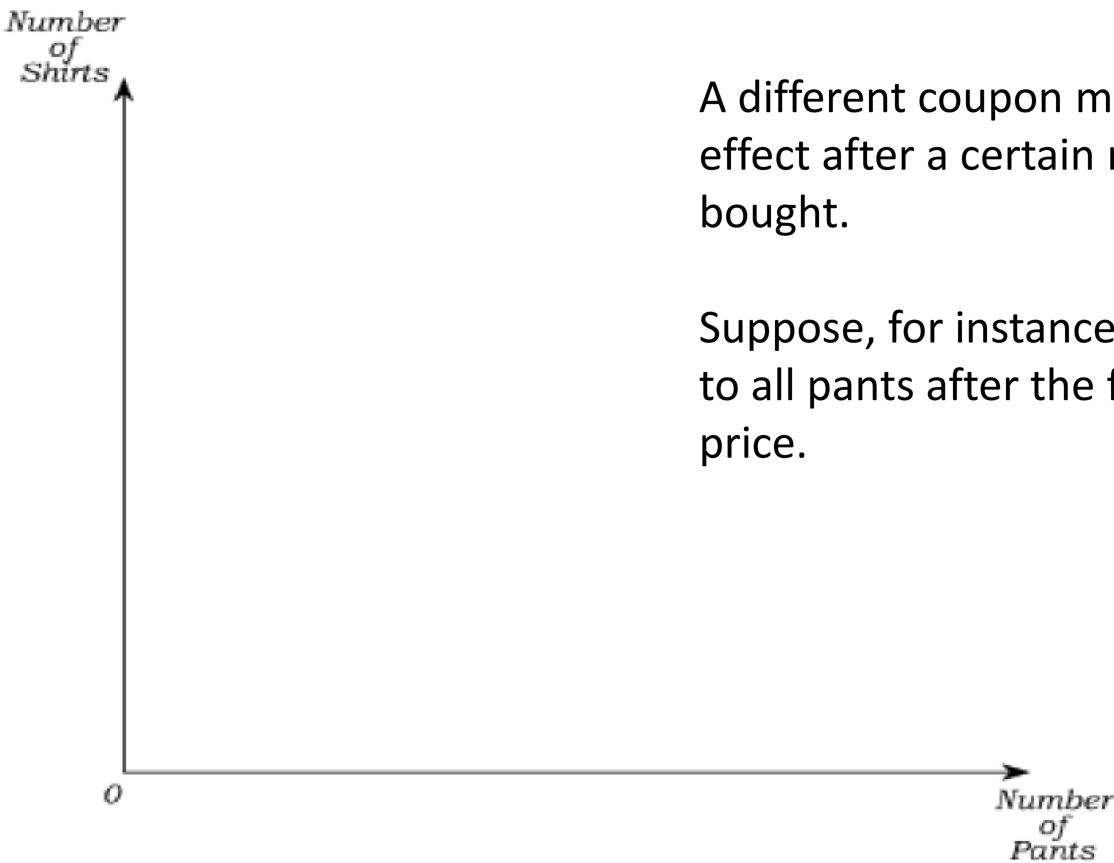


The slope of the **red** portion is –1, because the opportunity cost of pants in terms of shirts is 1 with the coupon.

The slope of the **blue** portion remains –2, because once the coupon is used, the opportunity cost of pants (in terms of shirts) goes back to 2.

Go to Math

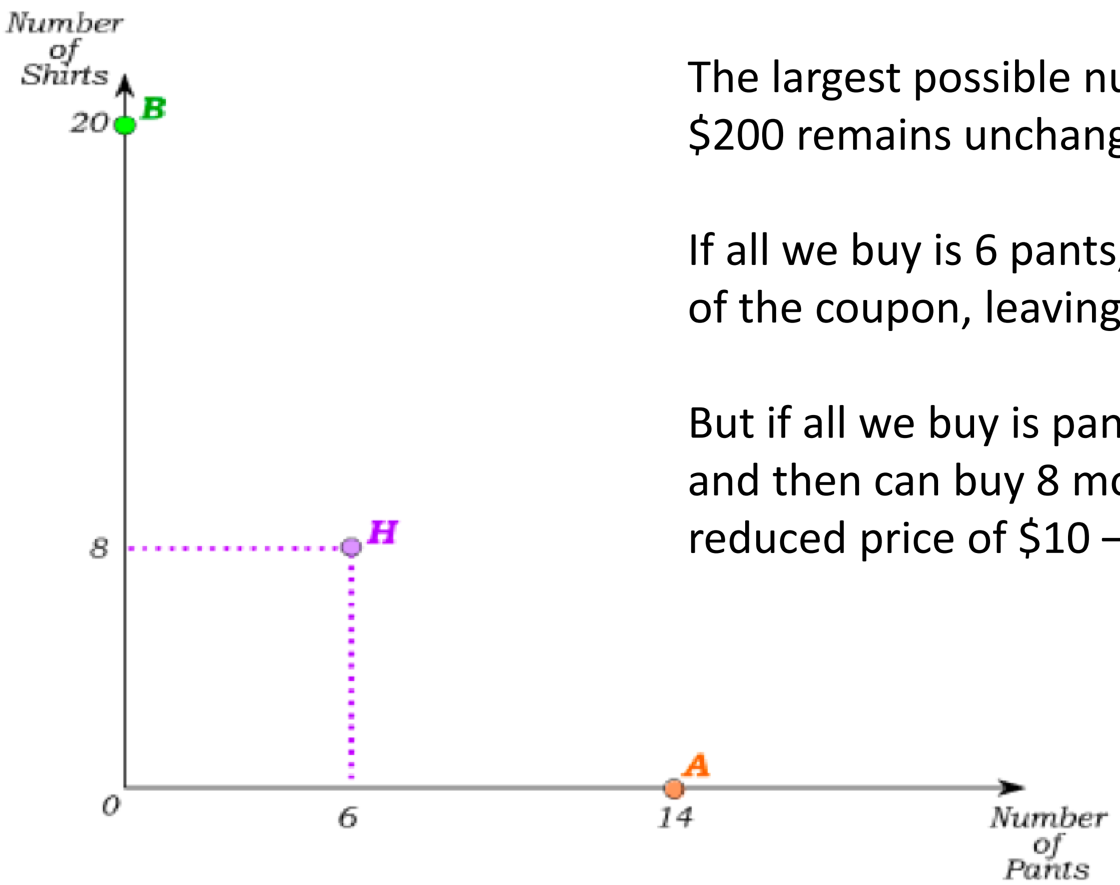
Consumer Choice Sets… - Kinky Budgets



A different coupon might be structured to go into effect after a certain number of pants have been bought.

Suppose, for instance, that a 50% discount applies to all pants after the first 6 are bought at full price.

Consumer Choice Sets…- Kinky Budgets

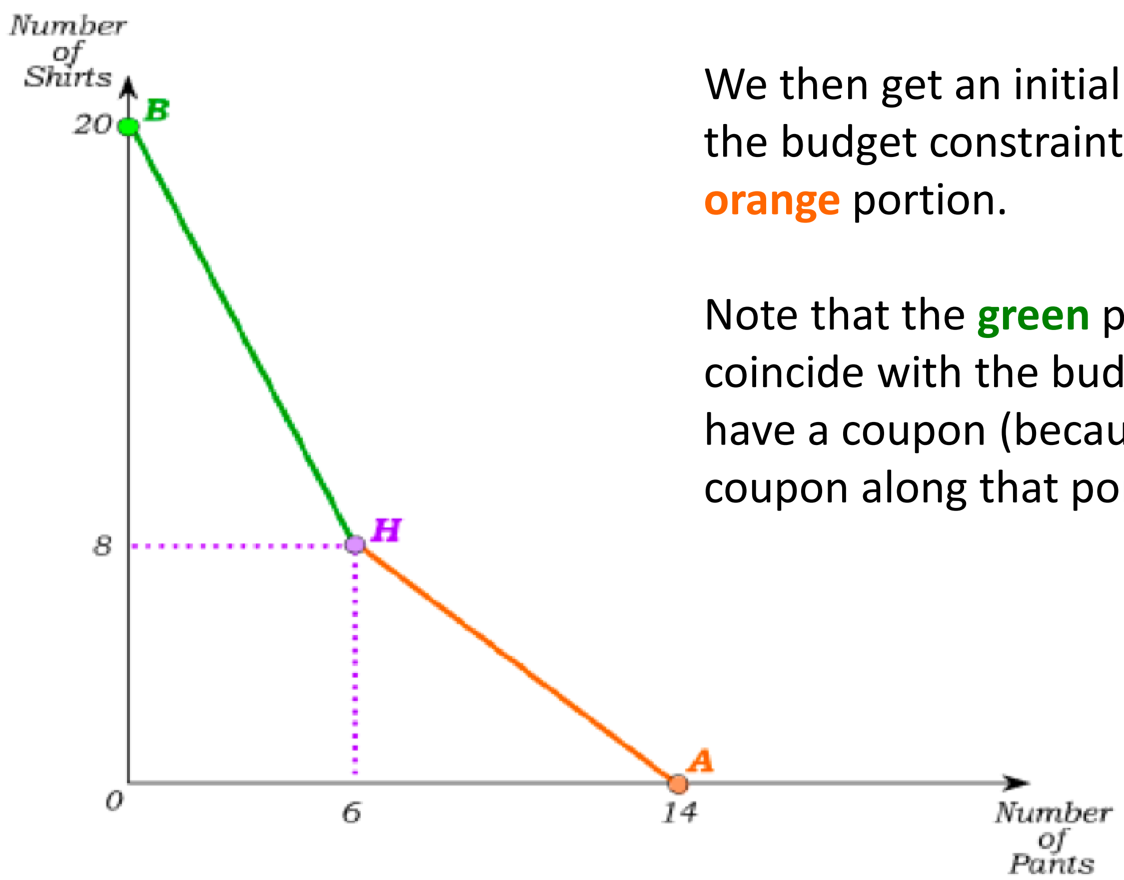


The largest possible number of shirts we can buy with $200 remains unchanged, leaving us with point ***B***.

If all we buy is 6 pants, we don't get to take advantage of the coupon, leaving us with point ***H***.

But if all we buy is pants, we spend $120 on the first 6 and then can buy 8 more (for a total of 14) at the reduced price of $10 – giving us point ***A***.

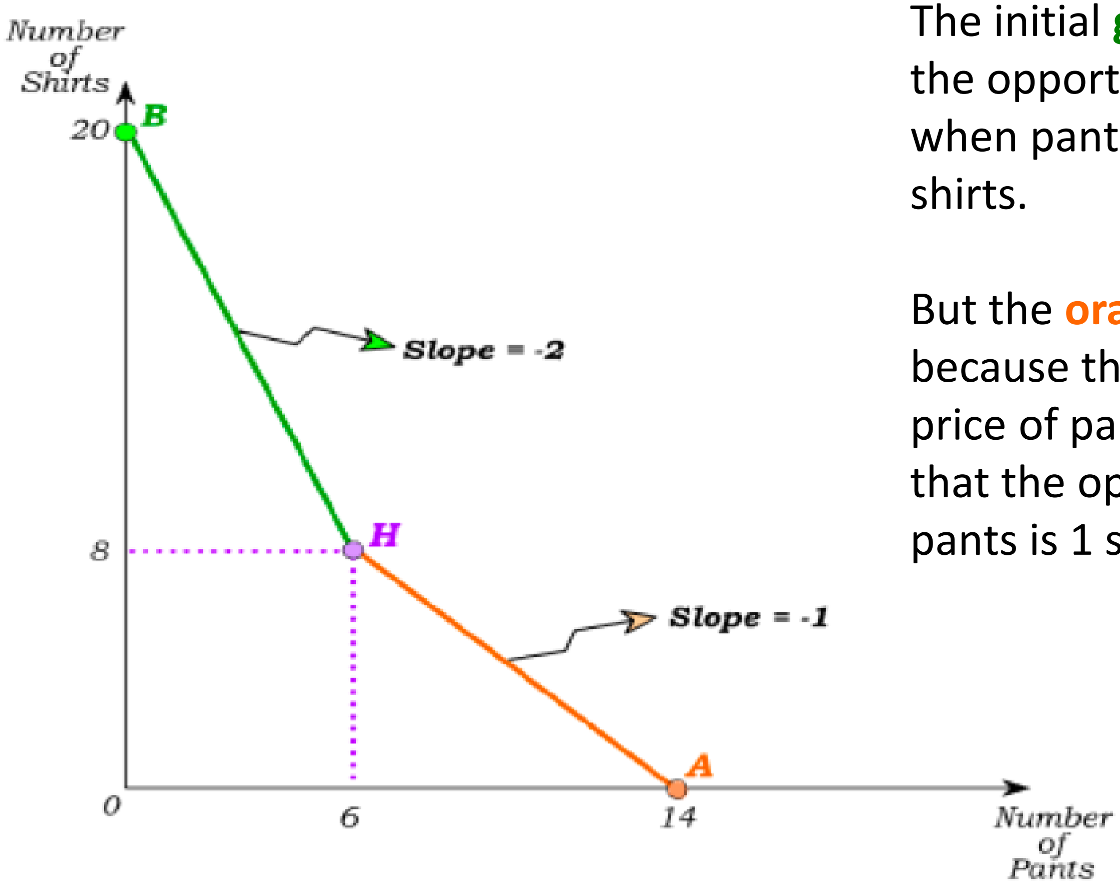
Consumer Choice Sets... -
Kinky Budgets



We then get an initially steep **green** portion of the budget constraint followed by a shallower **orange** portion.

Note that the **green** portion must exactly coincide with the budget line when we don't have a coupon (because we don't use the coupon along that portion.)

Consumer Choice Sets… - Kinky Budgets

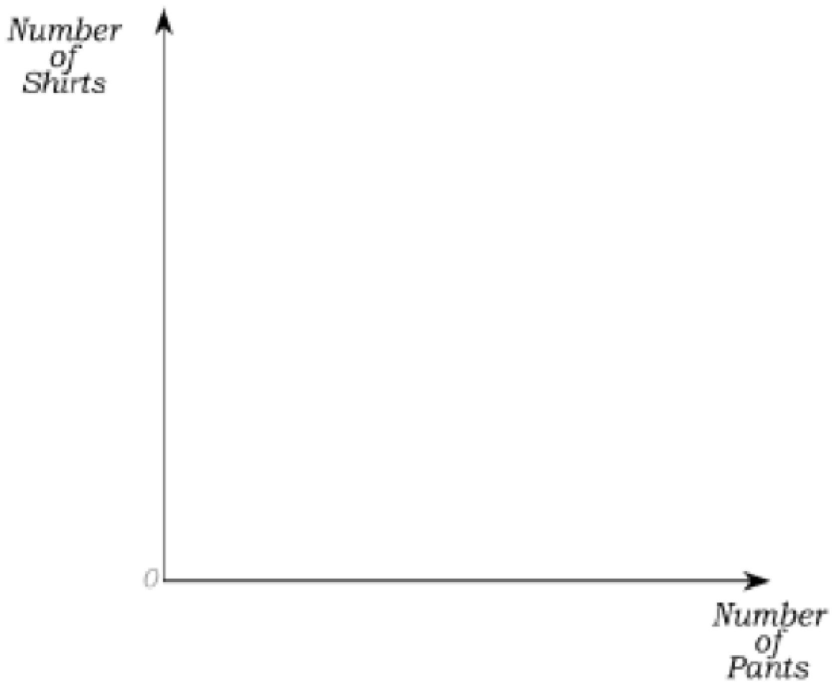


The initial **green** slope is then –2 because the opportunity cost of pants is 2 shirts when pants are twice as expensive as shirts.

But the **orange** portion has a slope of –1, because the coupon now reduces the price of pants to that of shirts, implying that the opportunity cost of a pair of pants is 1 shirt.

Discuss Convexity

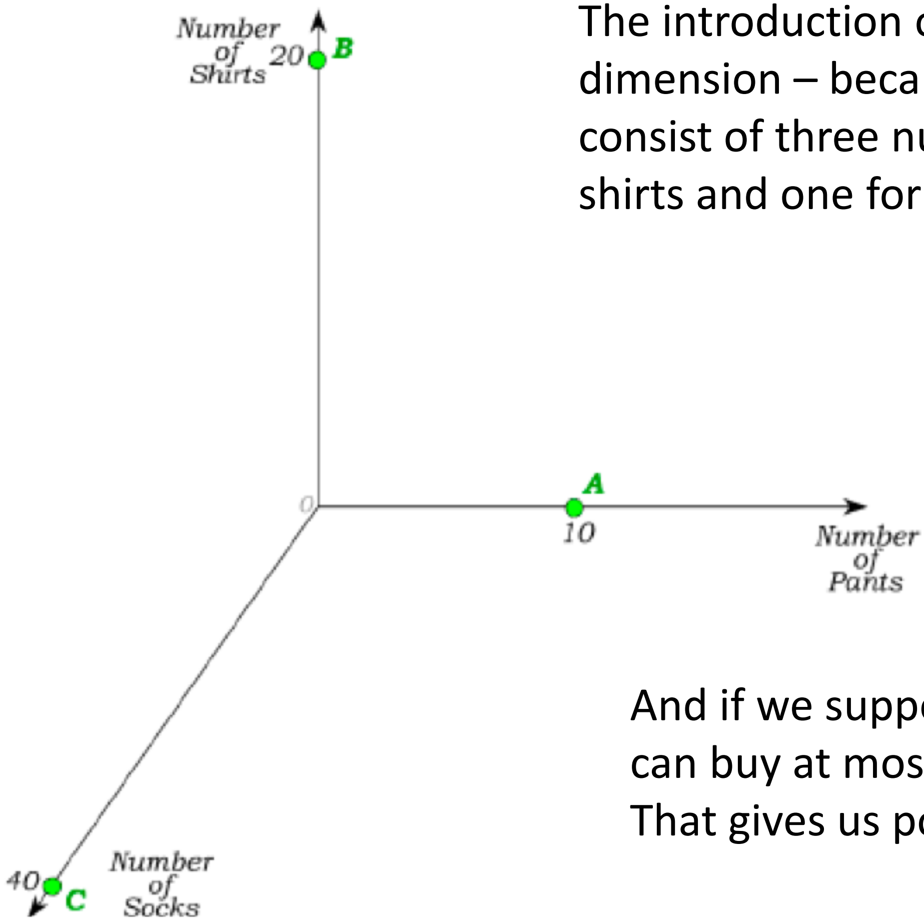
Modeling More General Choices–
Choice Sets with Three Goods



Suppose now that, in addition to shirts and pants, we have a third good we can choose: Socks.

We begin with our previous space of bundles consisting of two goods (shirts and pants.)

Modeling More General Choices–
Choice Sets with Three Goods

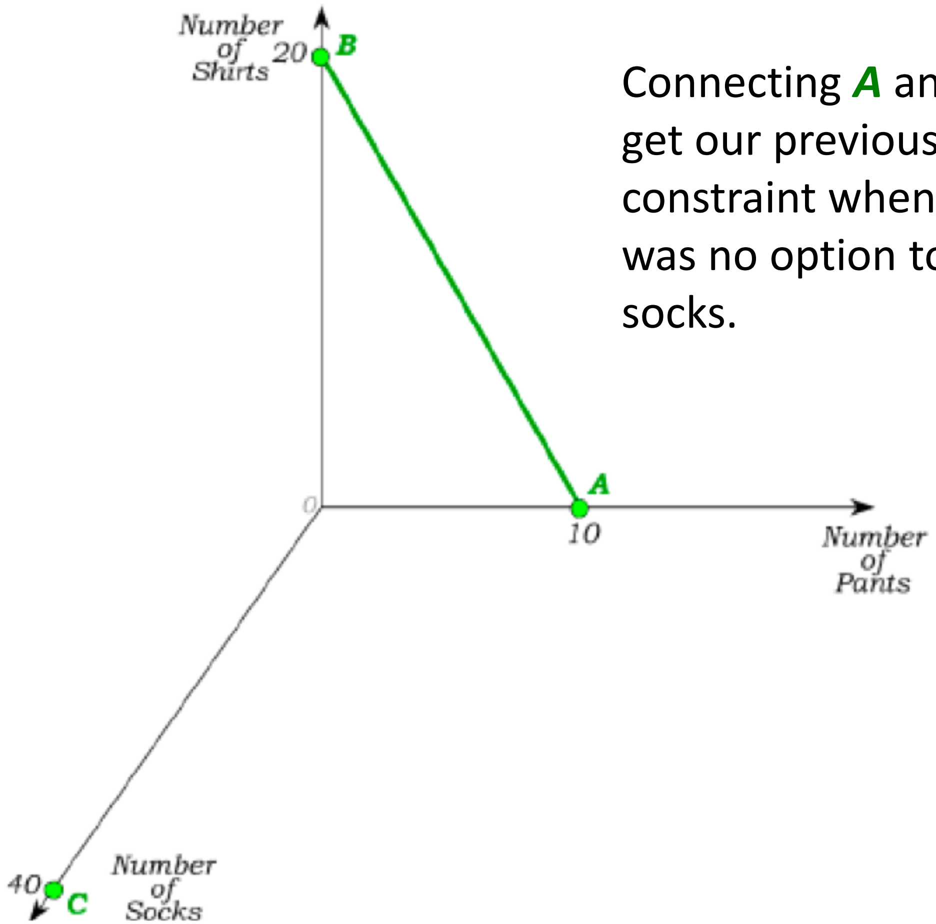


The introduction of socks then adds a third dimension – because bundles of goods now consist of three numbers: one for pants, one for shirts and one for socks.

With our income of $200 a $20 price for pants, and a $10 price for shirts, we can buy at most 10 pants (point ***A***) and at most 20 shirts (point ***B***).

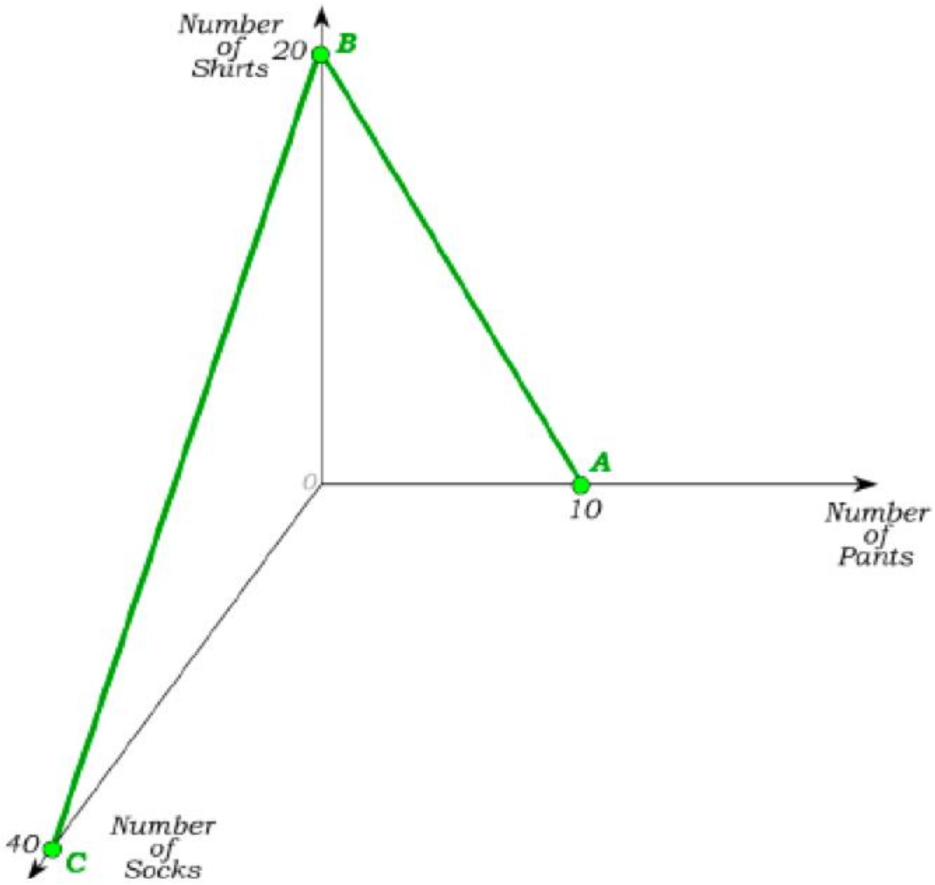
And if we suppose that the price of socks is $5, we can buy at most 40 socks (if we buy nothing else). That gives us point ***C*** on the socks axis.

Modeling More General Choices–
Choice Sets with Three Goods



Connecting ***A*** and ***B***, we get our previous budget constraint when there was no option to buy socks.

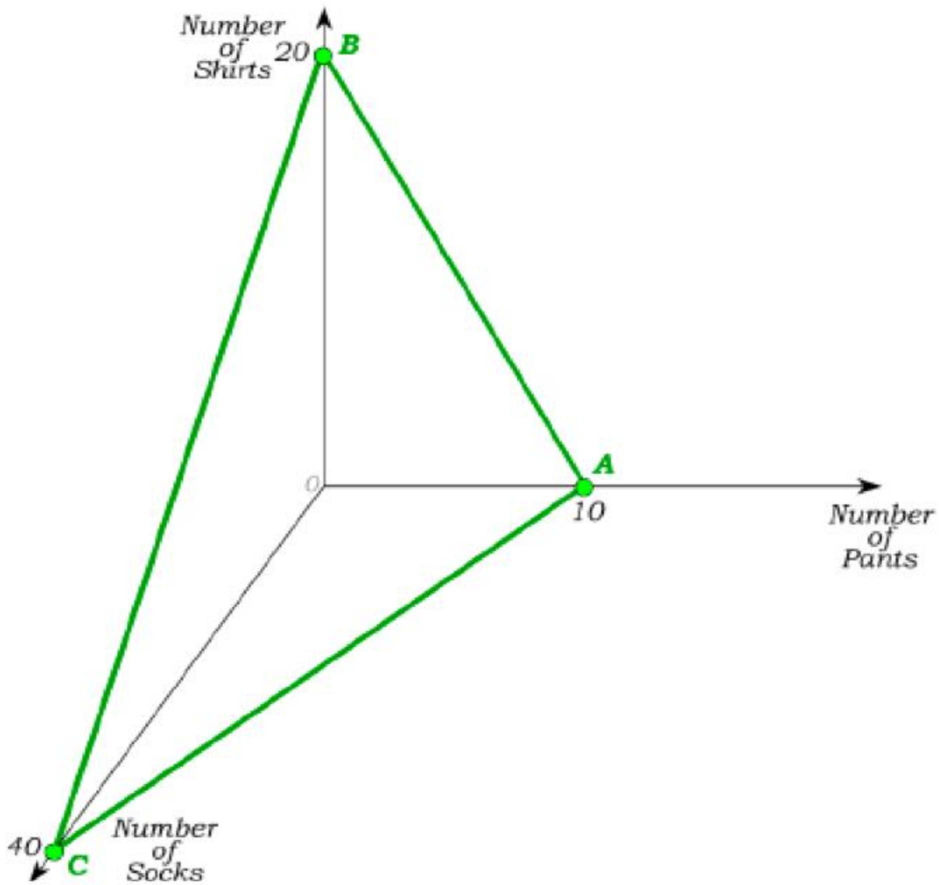
Modeling More General Choices–
Choice Sets with Three Goods



We similarly get the 2-dimensional budget constraint assuming there are no pants when we connect ***B*** and ***C***.

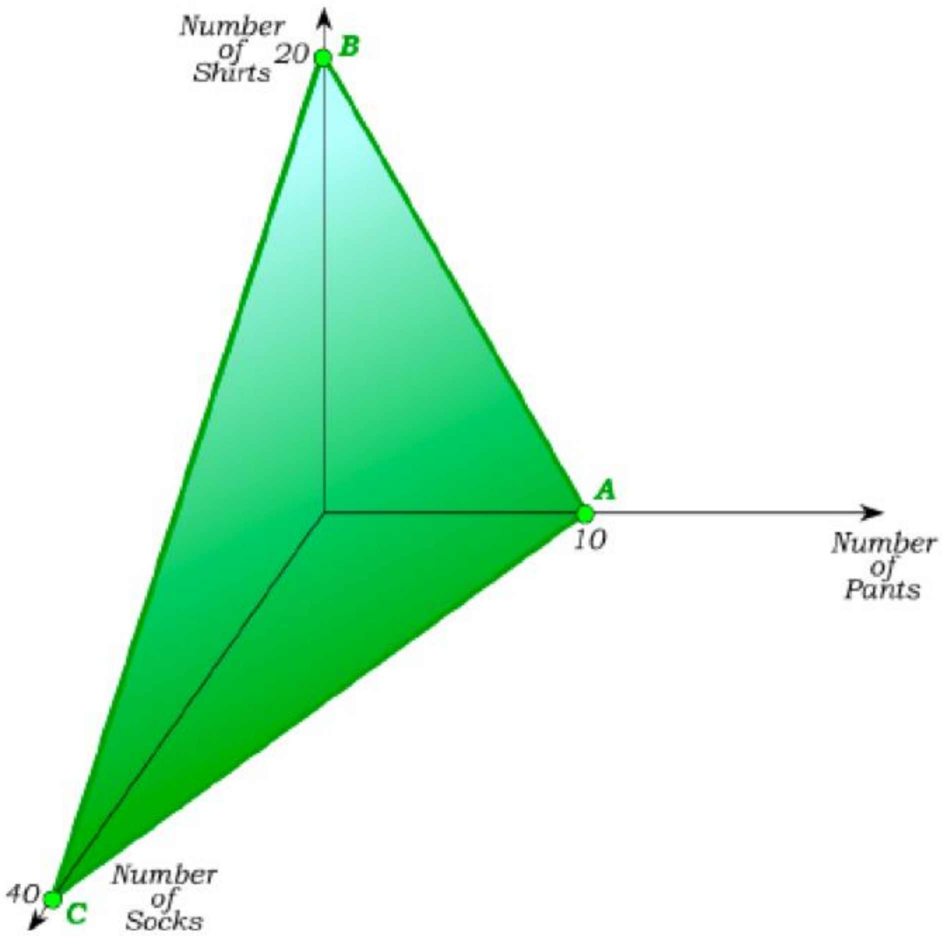
This gives all the bundles of shirts and socks that exhaust our $200 exogenous income.

Modeling More General Choices – Choice Sets with Three Goods



And, when we connect ***A*** and ***C***, we get the 2-dimensional budget constraint assuming there are no shirts and all we are choosing is bundles of pants and socks.

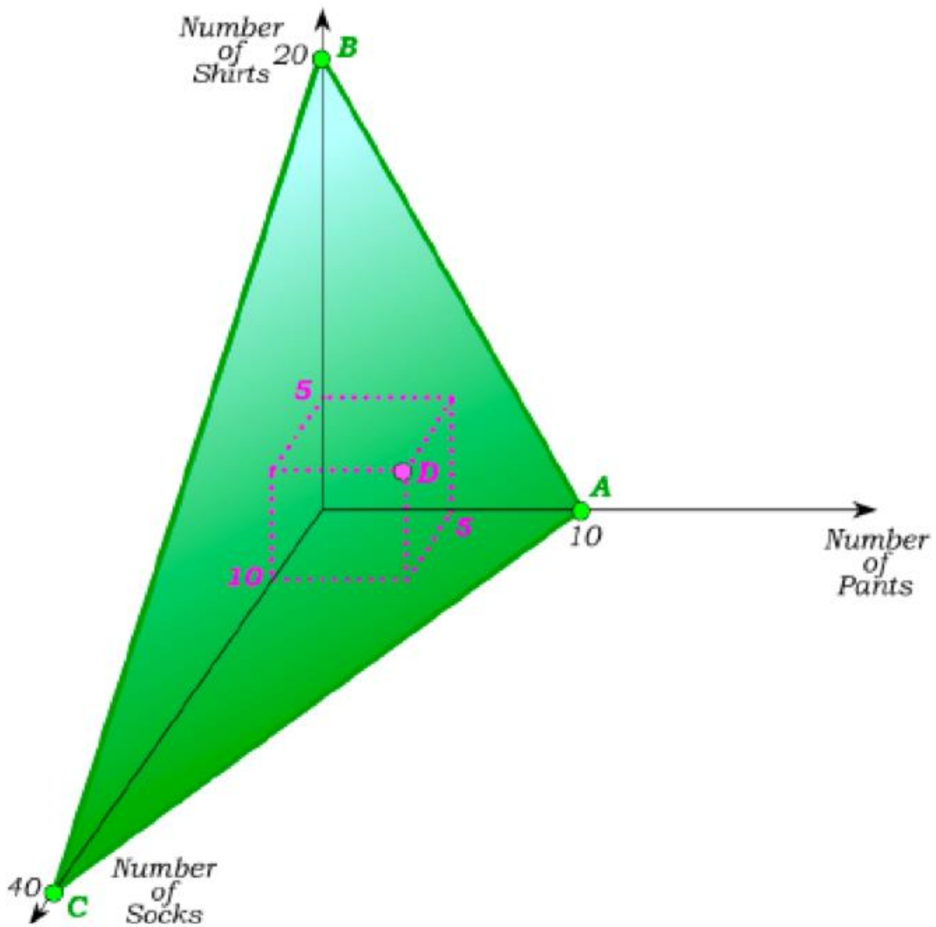
Modeling More General Choices- Choice Sets with Three Goods



When we then fill in the plane that connects the three 2-dimensional budget lines we have drawn, we get the 3-dimensional budget constraints over pants, shirts and socks.

The bundles that lie on or below this plane then constitute the ***choice set*** for this consumer.

Modeling More General Choices–
Choice Sets with Three Goods



One bundle that lies on the budget constraint, for instance, is bundle ***D***. This bundle contains 5 pants, 5 shirts and 10 socks.

The 5 pants cost $100 (at $20 each), the 5 shirts cost $50 (at $10 each), and the 10 socks cost $50 (at $5 each). Thus, bundle ***D*** costs $200.

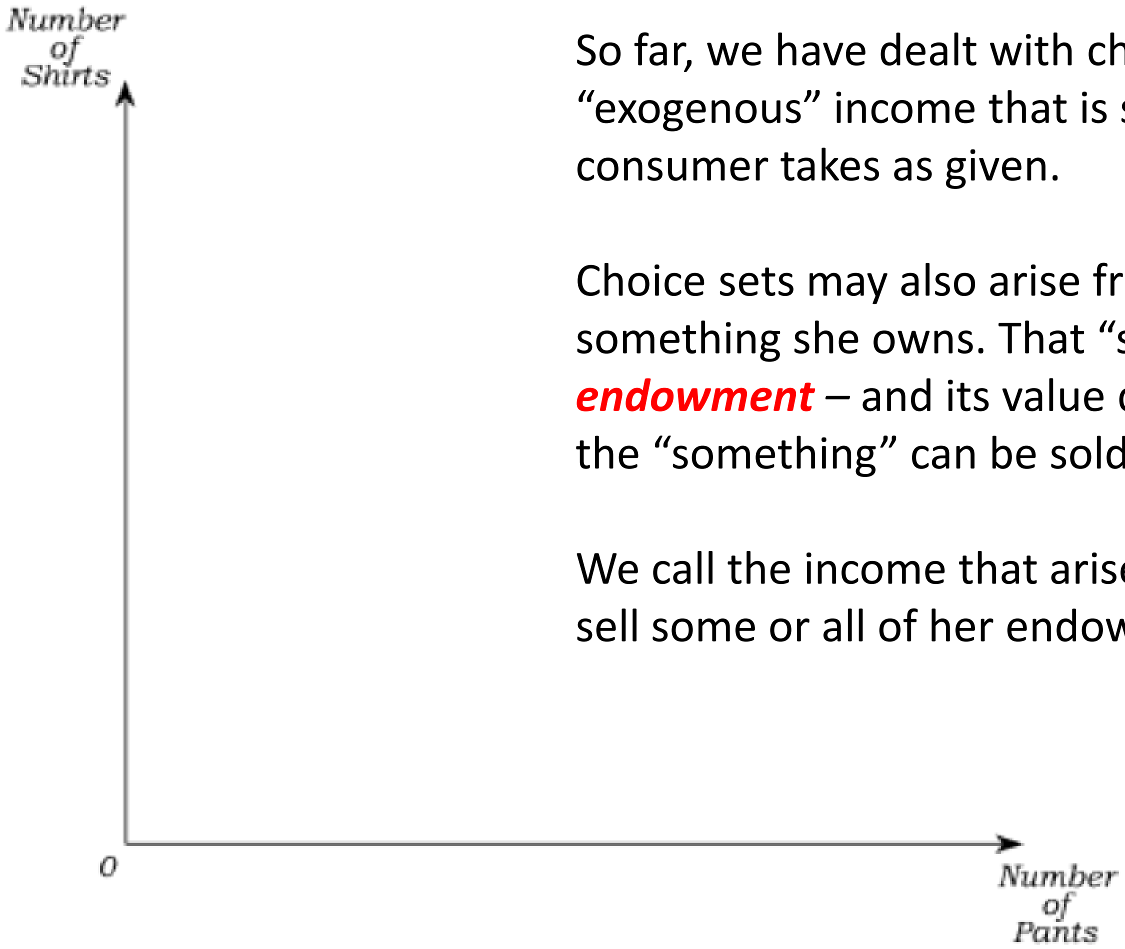
Go to Math

Modeling More General Choices–
Modeling Composite Goods

- But if we have more than 3 goods, it becomes impossible to graph the budget constraint.
- We thus often use the trick of aggregating all goods except for one into a ***"composite good"***.
- A ***composite good*** is an index of "dollars worth of all other goods", with the price of a composite good therefore equal to 1.
- When a composite good is put on the vertical axis, the slope of the budget is then simply (minus) the price of the good on the horizontal.

Go to Math

Consumer Choice Sets… –
… Incomes Arising from Endowments

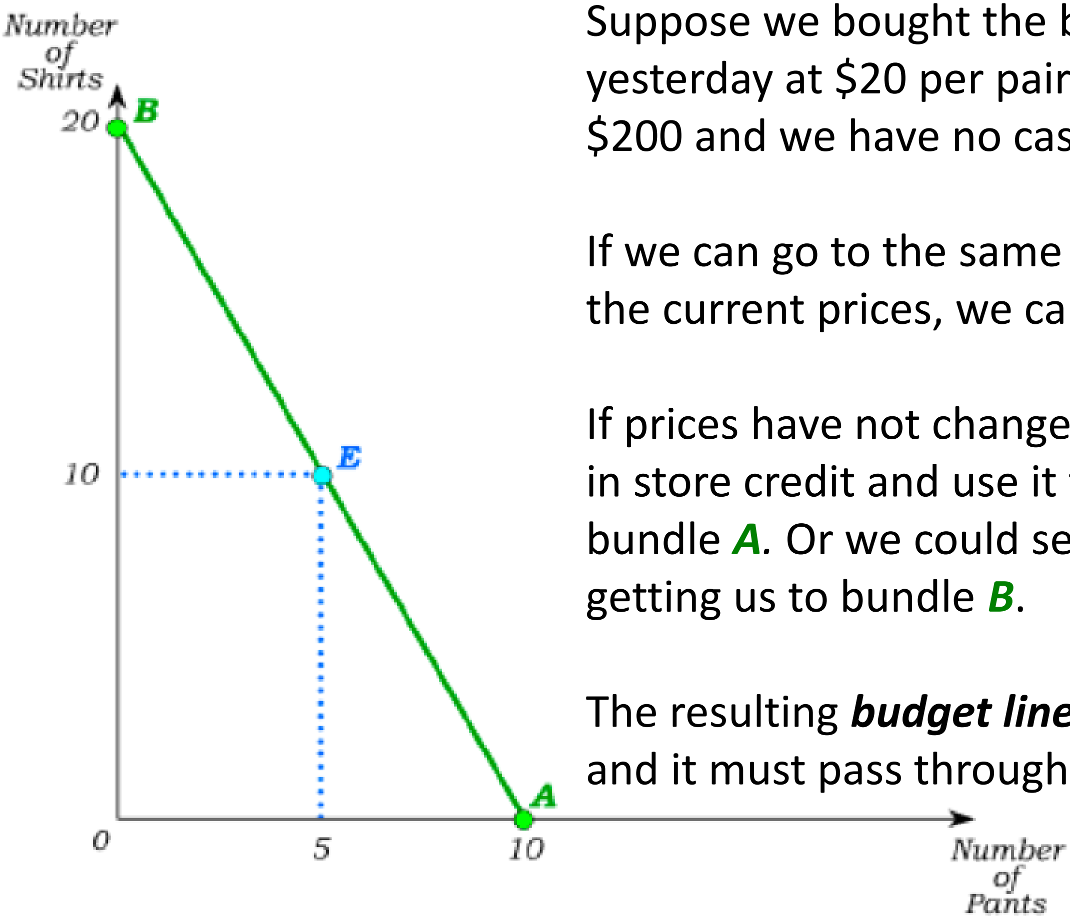


So far, we have dealt with choice sets that arise from "exogenous" income that is simply a dollar amount the consumer takes as given.

Choice sets may also arise from a consumer's ability to sell something she owns. That "something" is called an ***endowment*** – and its value depends on the prices at which the "something" can be sold.

We call the income that arises from a consumer's decision to sell some or all of her endowment ***endogenous***.

Consumer Choice Sets… –
… Incomes Arising from Endowments



Suppose we bought the bundle ***E*** – 5 pants an 10 shirts – yesterday at $20 per pair of pants and $10 per shirt. This cost us $200 and we have no cash left. But we now ***own*** the bundle ***E***.

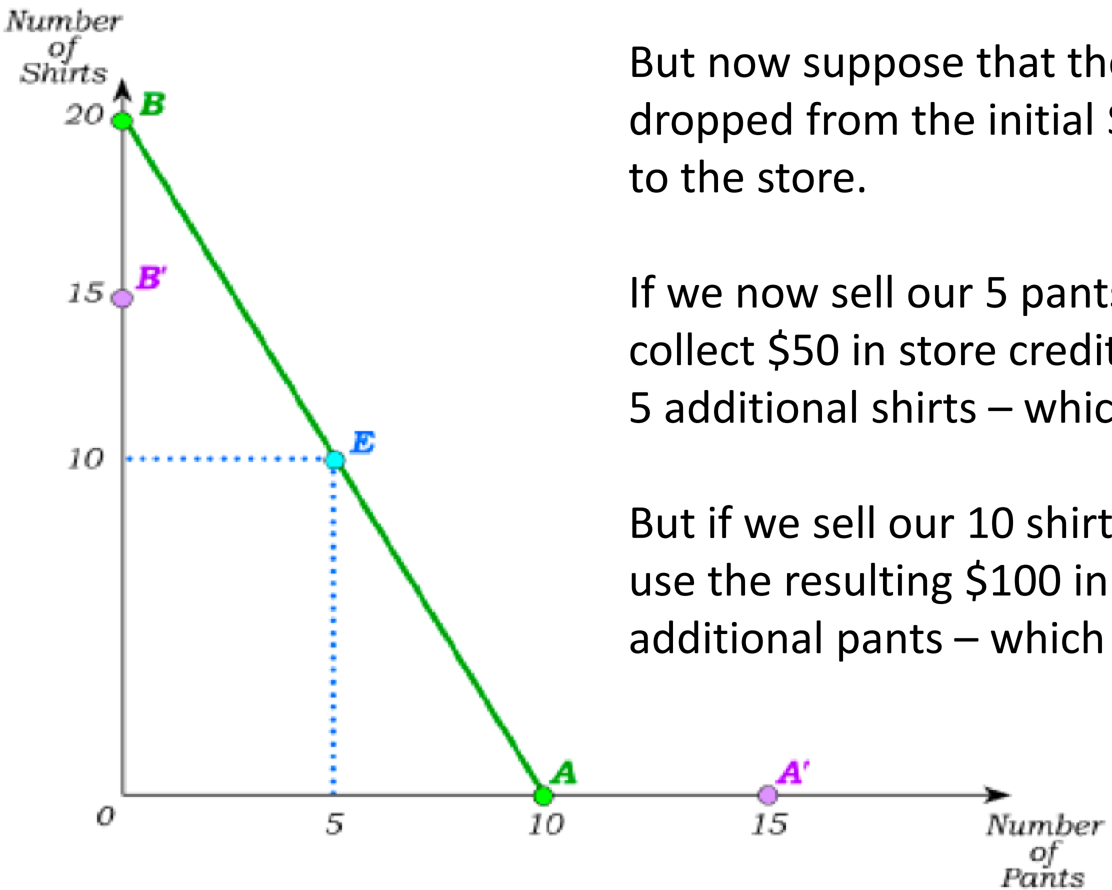
If we can go to the same store and sell pants and shirts back at the current prices, we can create a new choice set.

If prices have not changed, we could sell the 10 shirts, get $100 in store credit and use it to buy 5 more pants – getting us to bundle ***A***. Or we could sell the 5 pants and buy 10 more shirts – getting us to bundle ***B***.

The resulting ***budget line*** is as if we had $200 – the value of ***E***, and it must pass through ***E*** since we could always just consume ***E***.

Go to Math

Consumer Choice Sets… –
… Incomes Arising from Endowments

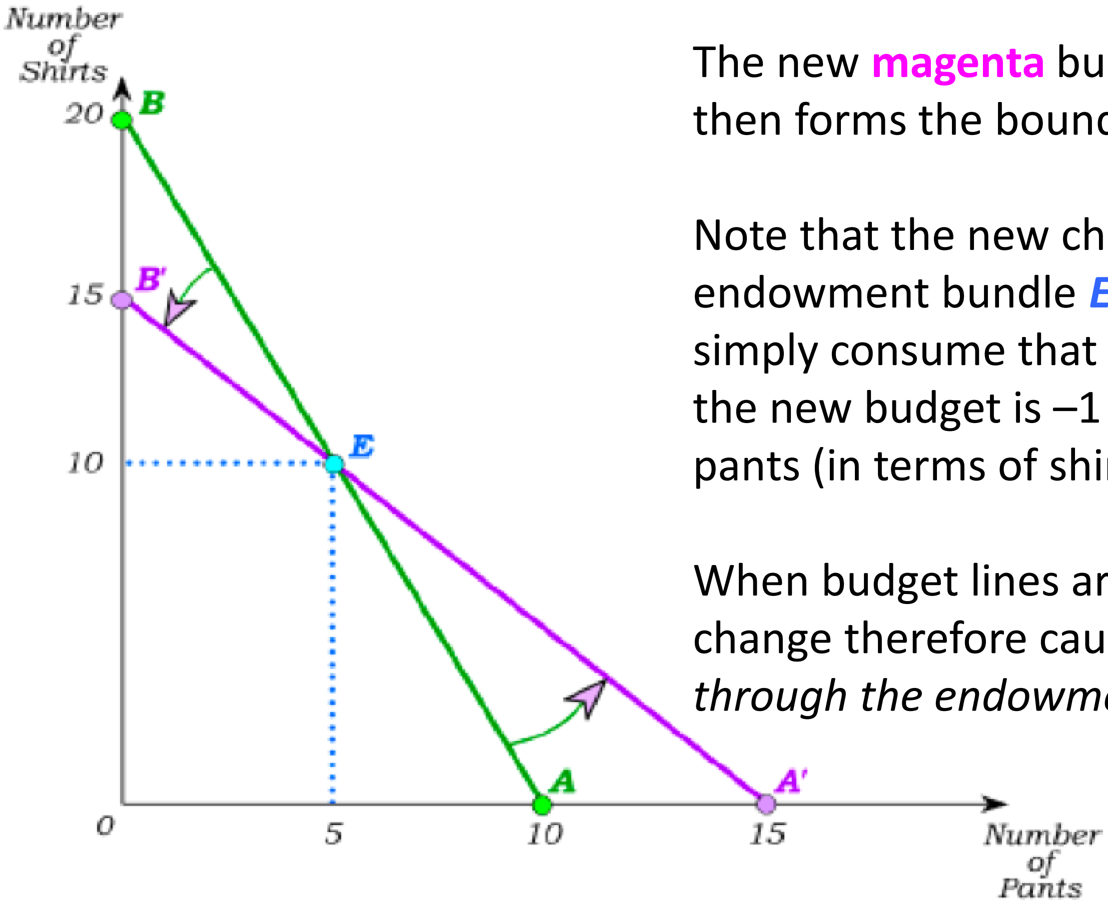


But now suppose that the price of pants has dropped from the initial $20 to $10 when we return to the store.

If we now sell our 5 pants in our bundle ***E***, we only collect $50 in store credit and can therefore only buy 5 additional shirts – which gets us bundle ***B'***.

But if we sell our 10 shirts from ***E*** instead, we can use the resulting $100 in store credit to get 10 additional pants – which gets us to bundle ***A'***.

Consumer Choice Sets…-
…Incomes Arising from Endowments



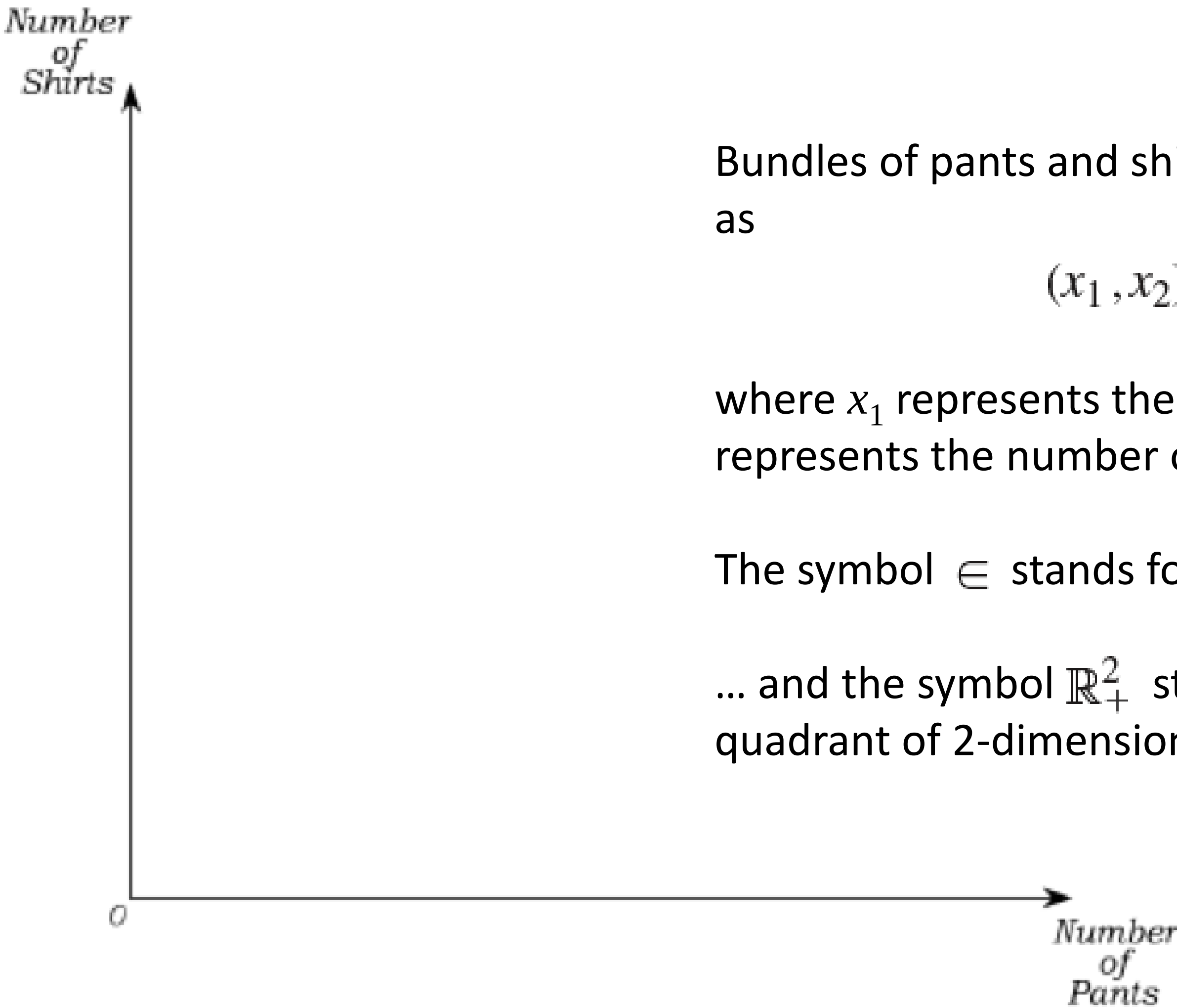
The new magenta budget line that connects ***A'*** and ***B'*** then forms the boundary of our new choice set.

Note that the new choice set must pass through our endowment bundle ***E*** because it is always possible to simply consume that bundle. Note also that the slope of the new budget is –1 – the *new* opportunity cost of pants (in terms of shirts).

When budget lines arise from ***endowments***, a price change therefore causes a rotation of the budget line *through the endowment bundle*.

Go to Math

Budget Math



Bundles of pants and shirts can be represented as

$$(x_1, x_2) \in \mathbb{R}^2_+$$

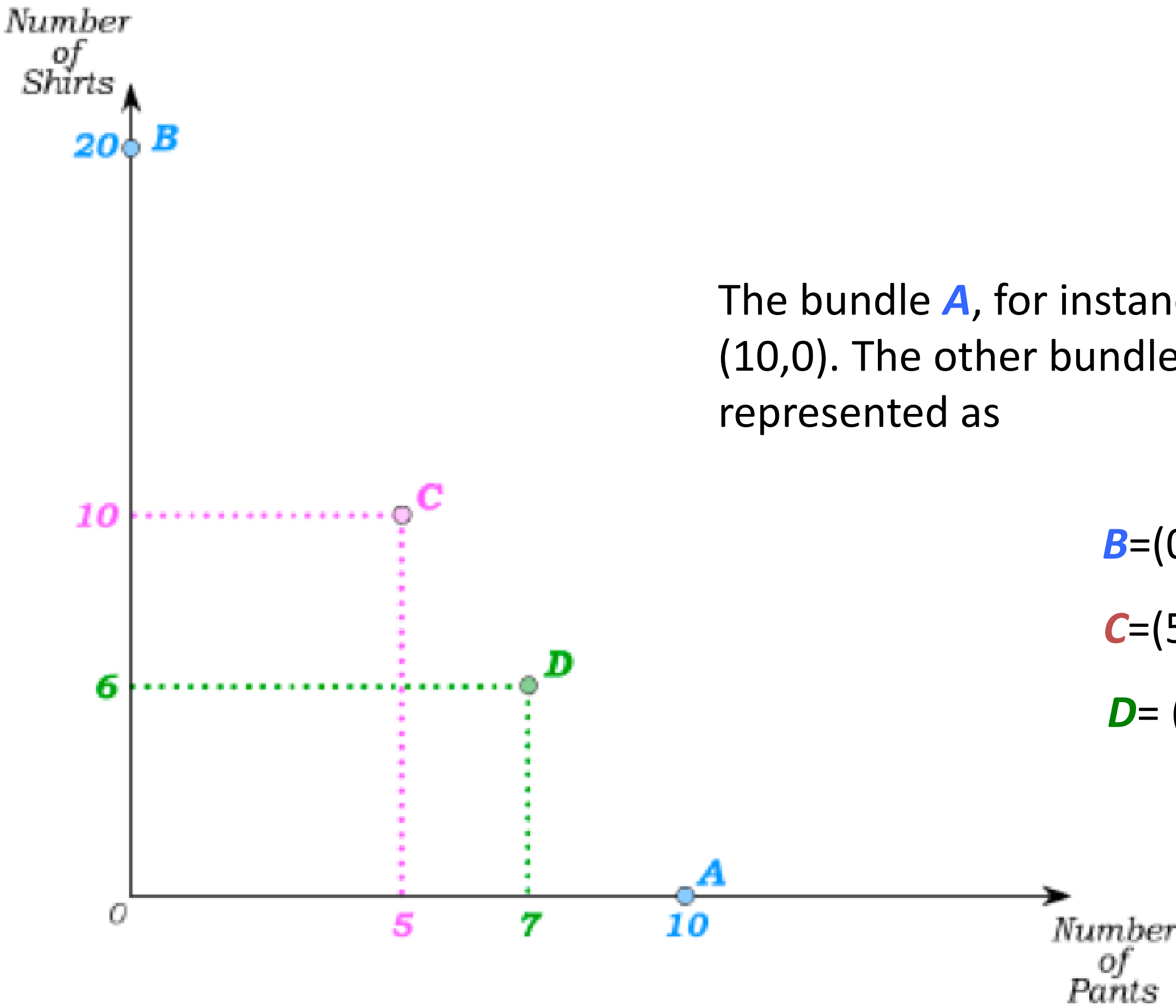
where x_1 represents the number of pants and x_2 represents the number of shirts.

The symbol $\in$ stands for “is an element of” …

… and the symbol $\mathbb{R}^2_+$ stands for the positive quadrant of 2-dimensional space

Back to Graphs

Budget Math



The bundle *A*, for instance, is represented as (10,0). The other bundles that are indicated are represented as

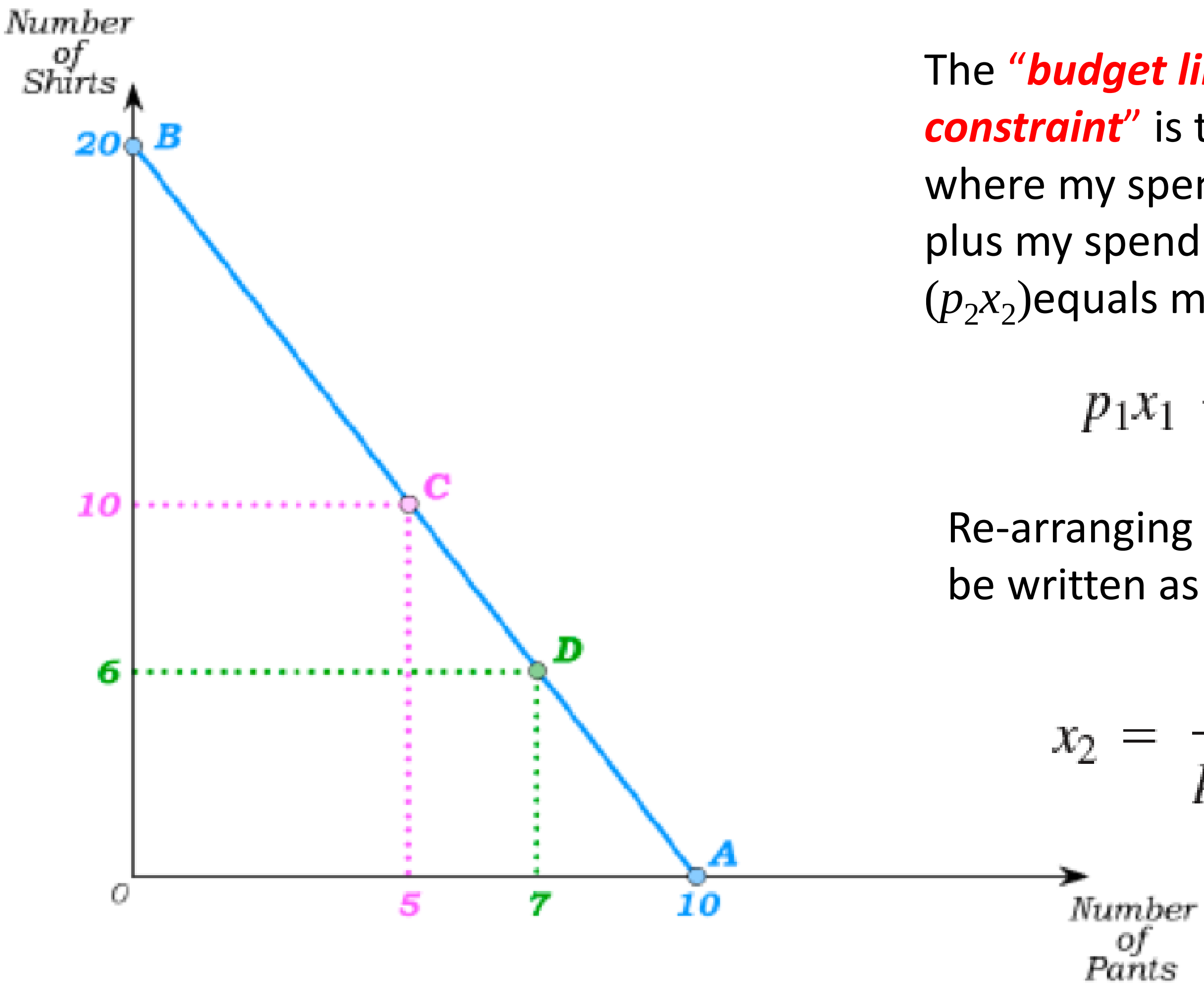
B=(0,20)

C=(5,10)

D= (7,6)

Back to Graphs

Budget Math



The ***"budget line"*** or ***"budget constraint"*** is then the set of bundles where my spending on good 1 (p_1x_1) plus my spending on good 2 (p_2x_2)equals my income I:

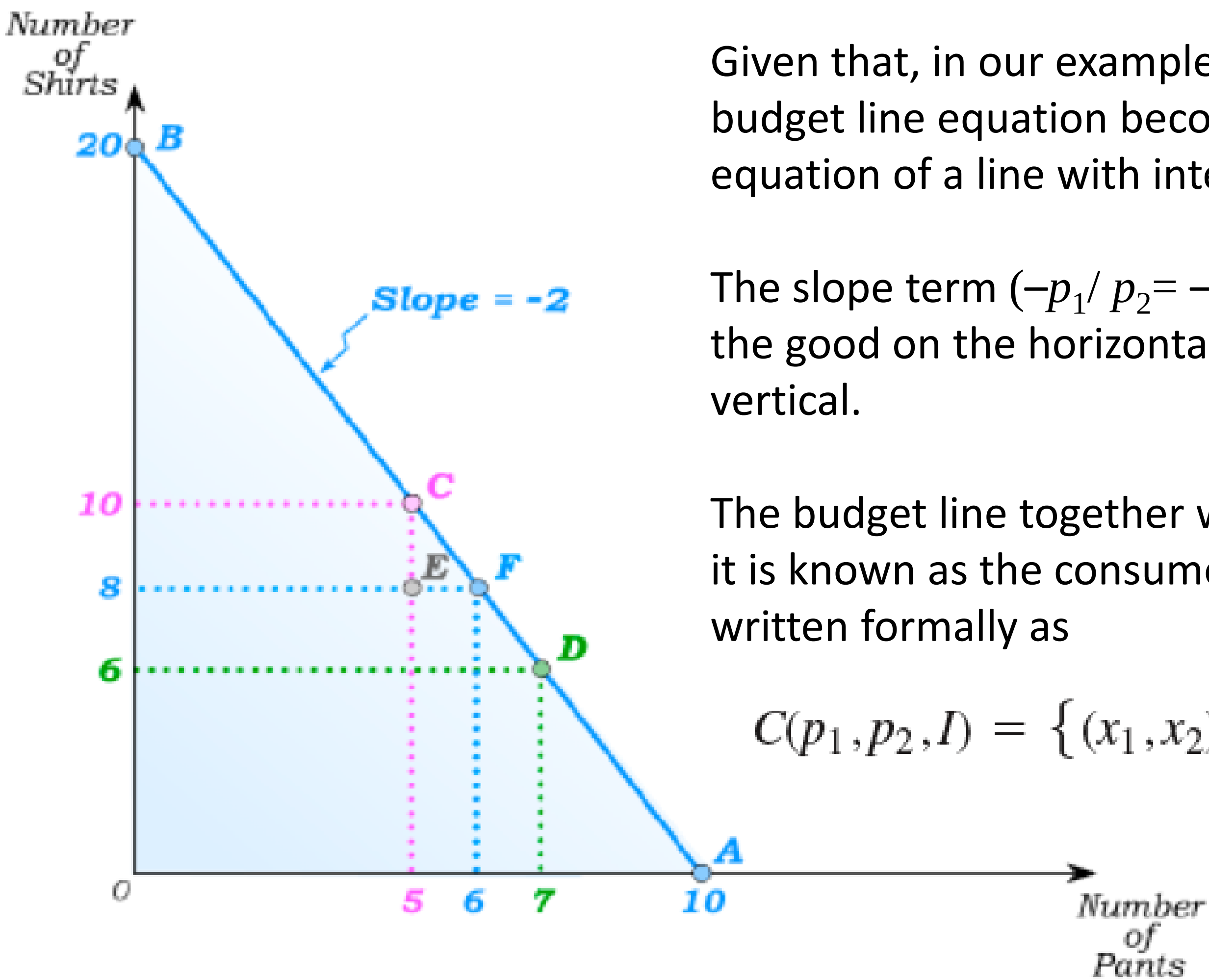
$$p_1x_1 + p_2x_2 = I$$

Re-arranging terms, this can also be written as

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1$$

Back to Graphs

Budget Math



Given that, in our example, I=200, p_1=20 and p_2=20, our budget line equation becomes $x_2 = 20 - 2x_1$ – i.e. an equation of a line with intercept of 20 and slope of –2.

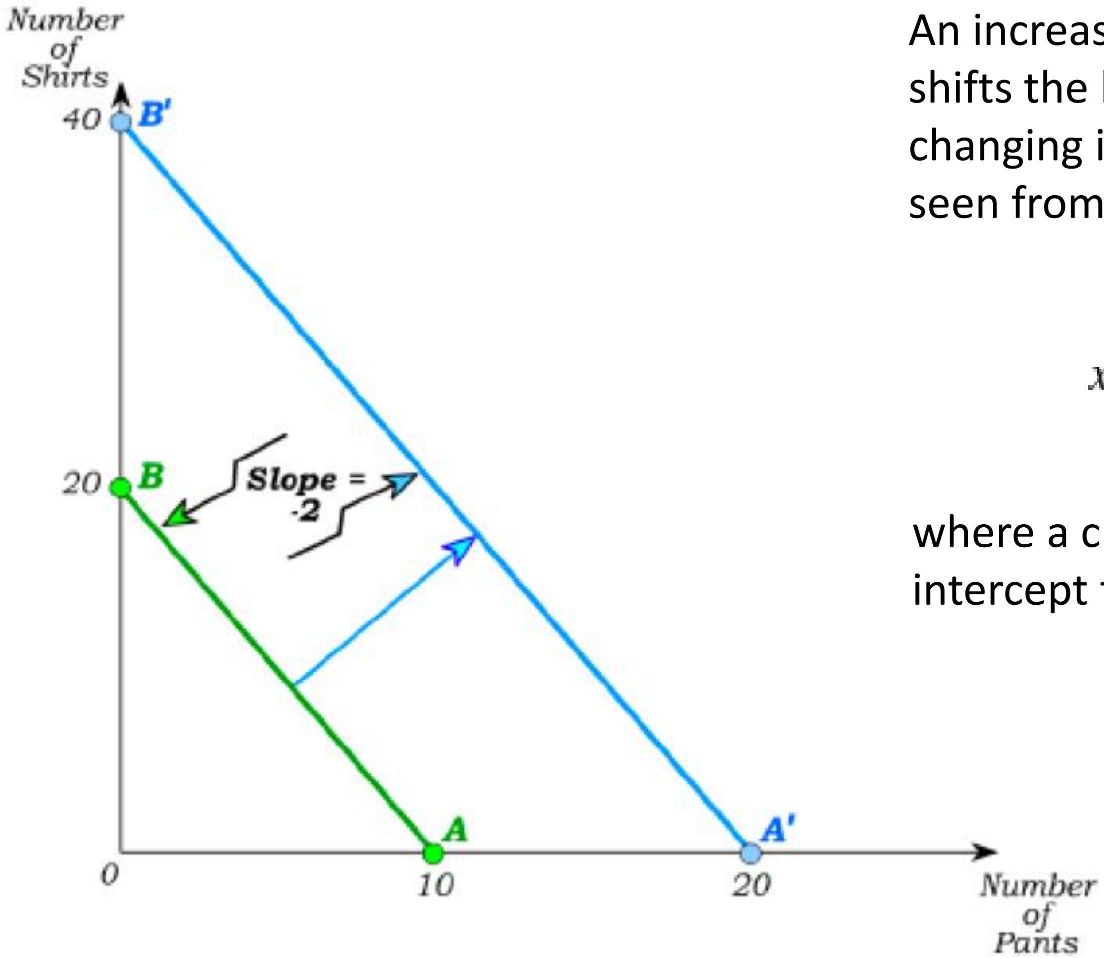
The slope term ($-p_1/\ p_2 = -2$) is the ***opportunity cost*** of the good on the horizontal in terms of the good on the vertical.

The budget line together with the shaded area beneath it is known as the consumer's ***choice set***. It can be written formally as

$$C(p_1, p_2, I) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid p_1 x_1 + p_2 x_2 \leq I\}.$$

Back to Graphs

An Increase in Exogenous Income



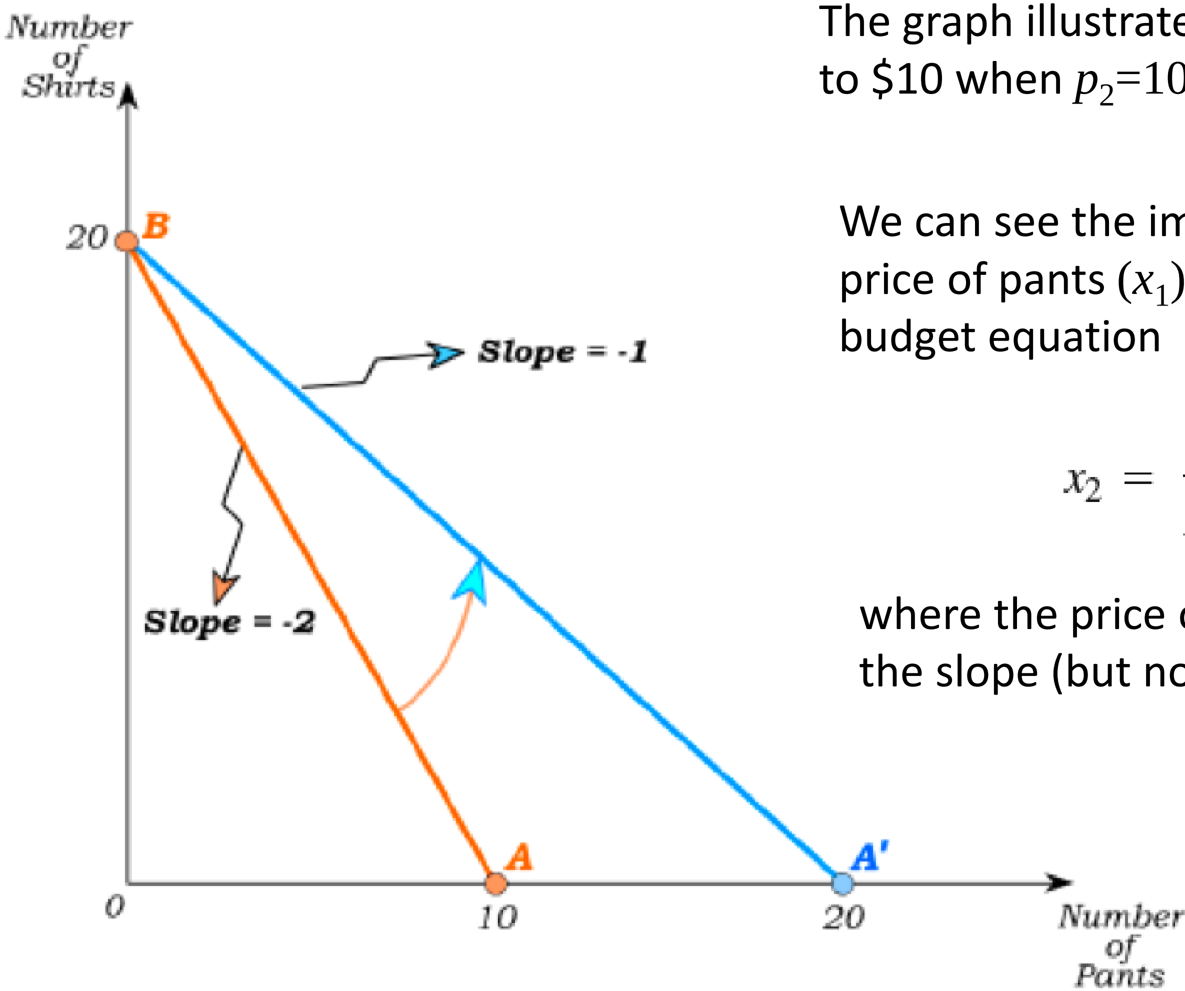
An increase in (exogenous) income shifts the budget line outward without changing its slope. This can easily be seen from the budget line equation

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1$$

where a change in *I* changes the intercept term but not the slope.

Back to Graphs

A Decrease in Price



The graph illustrates a change in p_1 from \$20 to \$10 when p_2=10 and I=200.

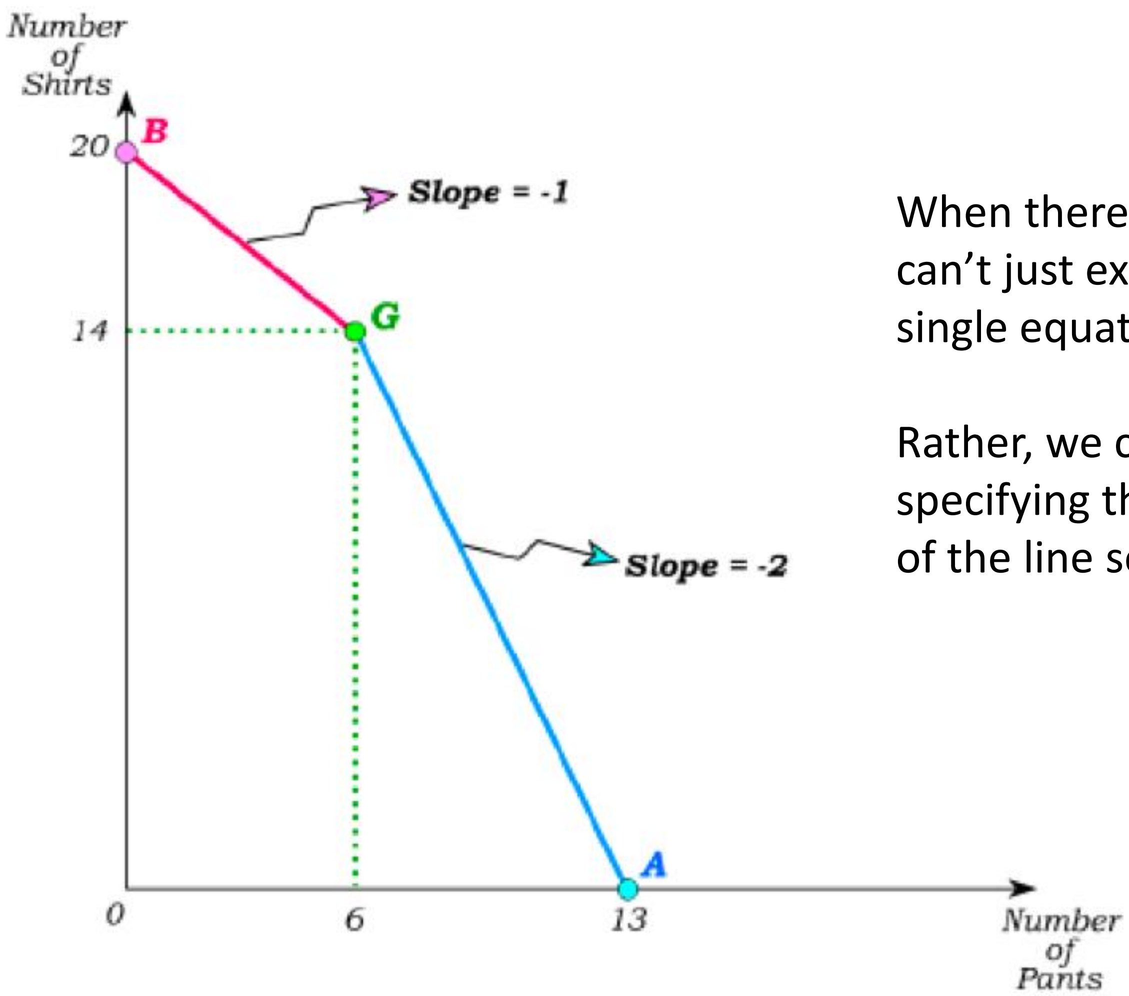
We can see the impact of a decrease in the price of pants (x_1) by again looking at our budget equation

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2} x_1$$

where the price of pants (p_1) only appears in the slope (but not the intercept) term.

Back to Graphs

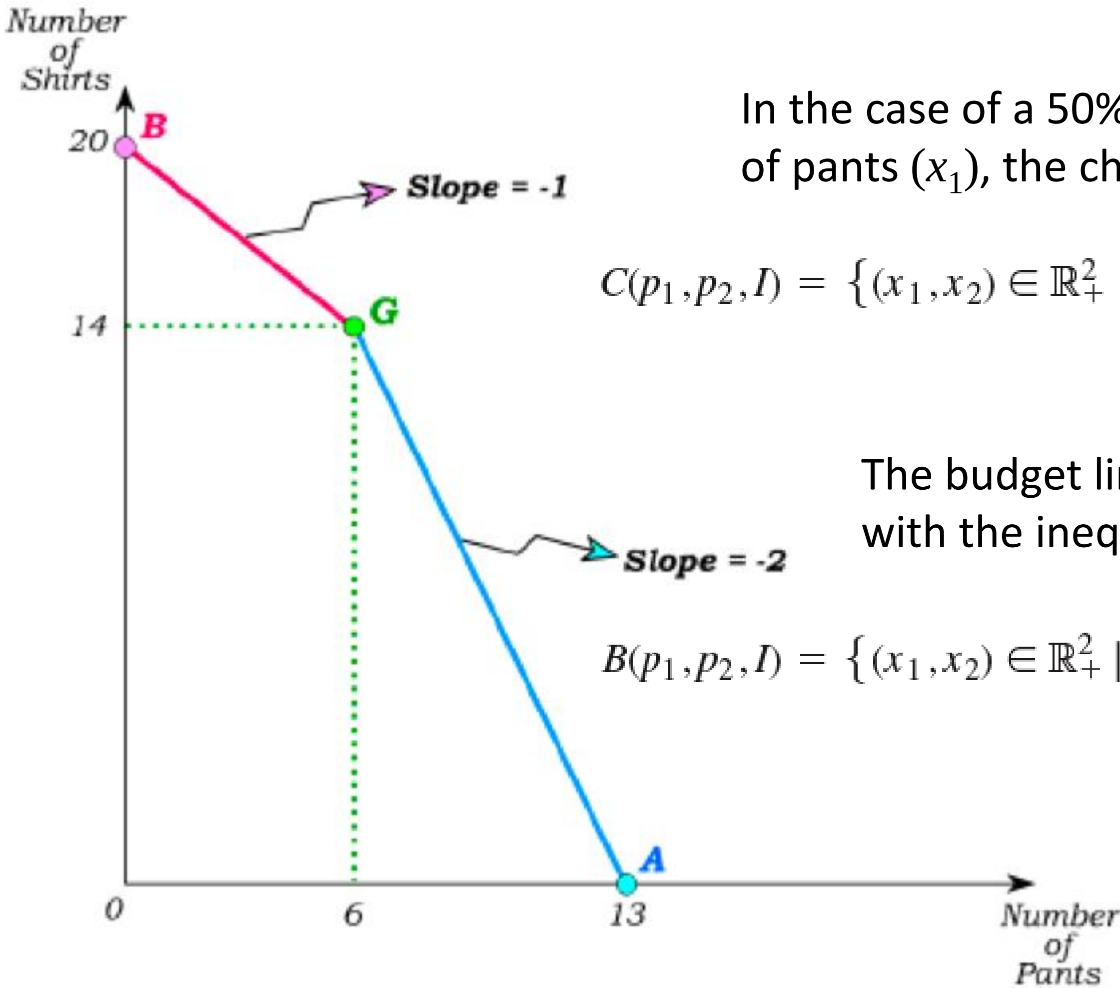
Kinky Budgets



When there are kinks in budgets, we can't just express the budget line in a single equation.

Rather, we can express the choice set by specifying the relevant equation for each of the line segments.

Kinky Budgets



In the case of a 50% coupon for the first 6 pair of pants (x_1), the choice set then becomes

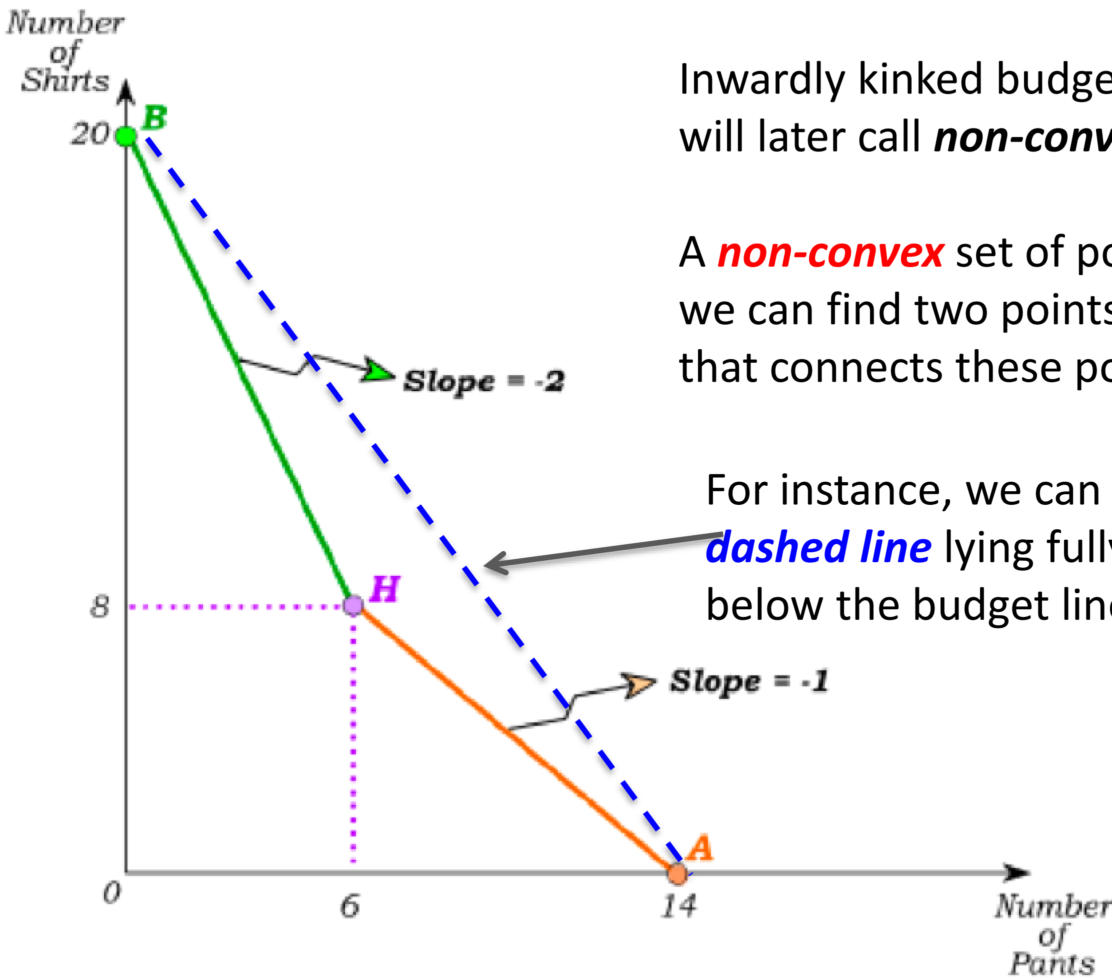
$$C(p_1, p_2, I) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid 0.5\, p_1 x_1 + p_2 x_2 \leq I \text{ for } x_1 \leq 6 \text{ and } p_1 x_1 + p_2 x_2 \leq I + 3p_1 \text{ for } x_1 > 6\}$$

The budget line can similarly be expressed – with the inequalities replaced by equalities.

$$B(p_1, p_2, I) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid 0.5\, p_1 x_1 + p_2 x_2 = I \text{ for } x_1 \leq 6 \text{ and } p_1 x_1 + p_2 x_2 = I + 3p_1 \text{ for } x_1 > 6\}.$$

Back to Graphs

Non-Convex and Convex Sets

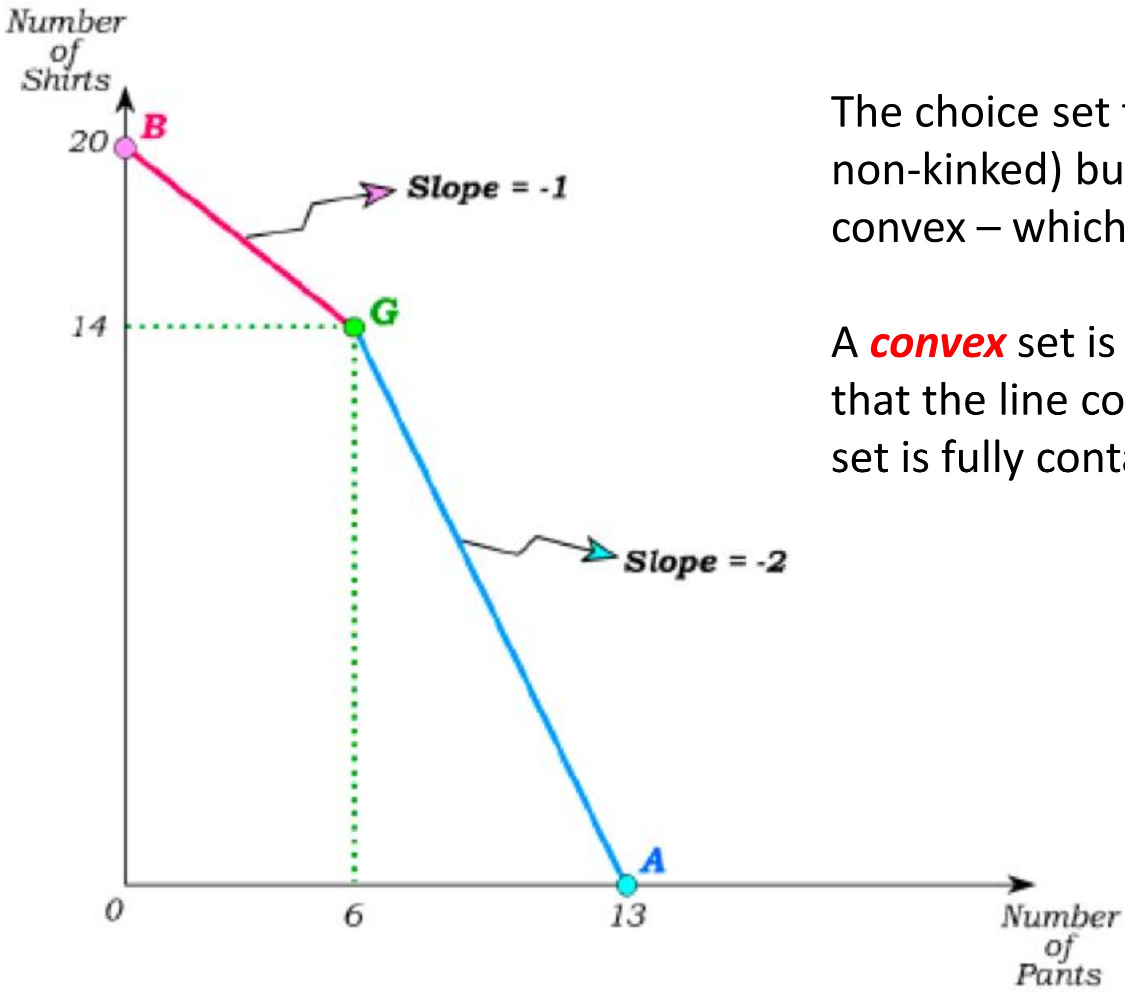


Inwardly kinked budget constraints give rise to what we will later call ***non-convex*** choice sets.

A ***non-convex*** set of points is defined as a set in which we can find two points such that a portion of the line that connects these points lies outside the set.

For instance, we can connect ***A*** and ***B*** and the resulting ***dashed line*** lying fully outside the choice set (that lies below the budget line).

Non-Convex and Convex Sets



The choice set for an outwardly-kinked (or a non-kinked) budget constraint is *NOT* non-convex – which we then defined as a ***convex*** set.

A ***convex*** set is therefore a set of points such that the line connecting any two points in the set is fully contained in the same set.

Back to Graphs

Choice Sets with Three Goods

The budget line equation $p_1x_1+p_2x_2=I$ extends straightforwardly to 3 goods, with expenditure on the third good (p_3x_3) simply added to the right hand side to give us

$$p_1x_1 + p_2x_2 + p_3x_3 = I$$

with the corresponding choice set defined as

$$C(p_1,p_2,p_3,I) = \{(x_1,x_2,x_3) \in \mathbb{R}^3_+ \mid p_1x_1 + p_2x_2 + p_3x_3 \leq I\}$$

And if furthermore extends to the *n*-good case, with a budget line of

$$p_1x_1 + p_2x_2 + \ldots + p_nx_n = I$$

and corresponding choice set

$$C(p_1,p_2,\ldots,p_n,I) = \{(x_1,x_2,\ldots,x_n) \in \mathbb{R}^n_+ \mid p_1x_1 + p_2x_2 + \ldots + p_nx_n \leq I\}$$

Back to Graphs

Composite Goods

When we are primarily interested in one of *n* goods that a person consumes, we often model that good as x_1 and *aggregate* the remaining (*n*-1) goods into a ***composite good*** (x_2) defined as "dollars worth of other consumption".

Since a "dollar's worth of other consumption" costs by definition $1, we can then simply set the price of the composite good to $p_2=1$ and write the budget equation as

$$p_1x_1 + x_2 = I$$

.

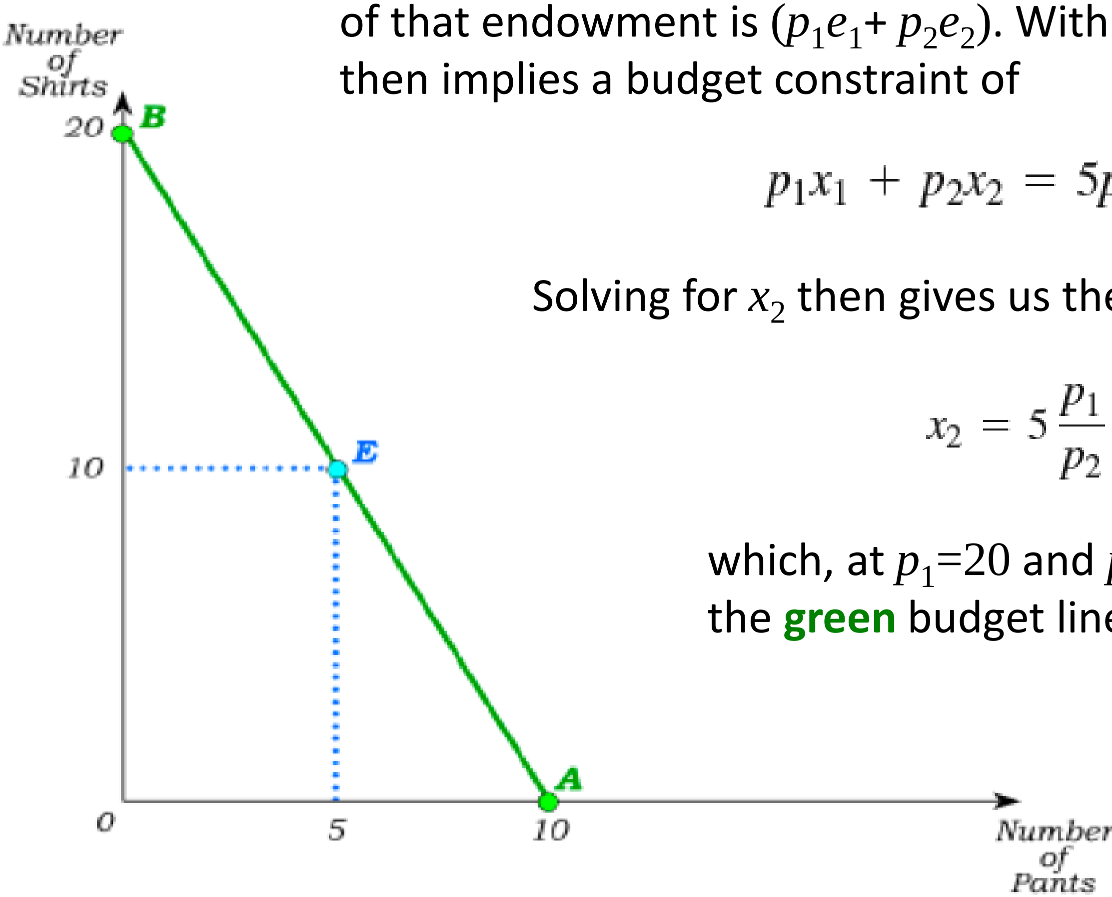
Solving this for x_2, we get the budget line

$$x_2 = I - p_1x_1$$

with intercept I and slope $-p_1$.

Back to Graphs

Endowments



If we have an endowment e_1 of good 1 and e_2 of good 2, then the value of that endowment is $(p_1e_1 + p_2e_2)$. With the endowment ***E***=(5,10), this then implies a budget constraint of

$$p_1x_1 + p_2x_2 = 5p_1 + 10p_2.$$

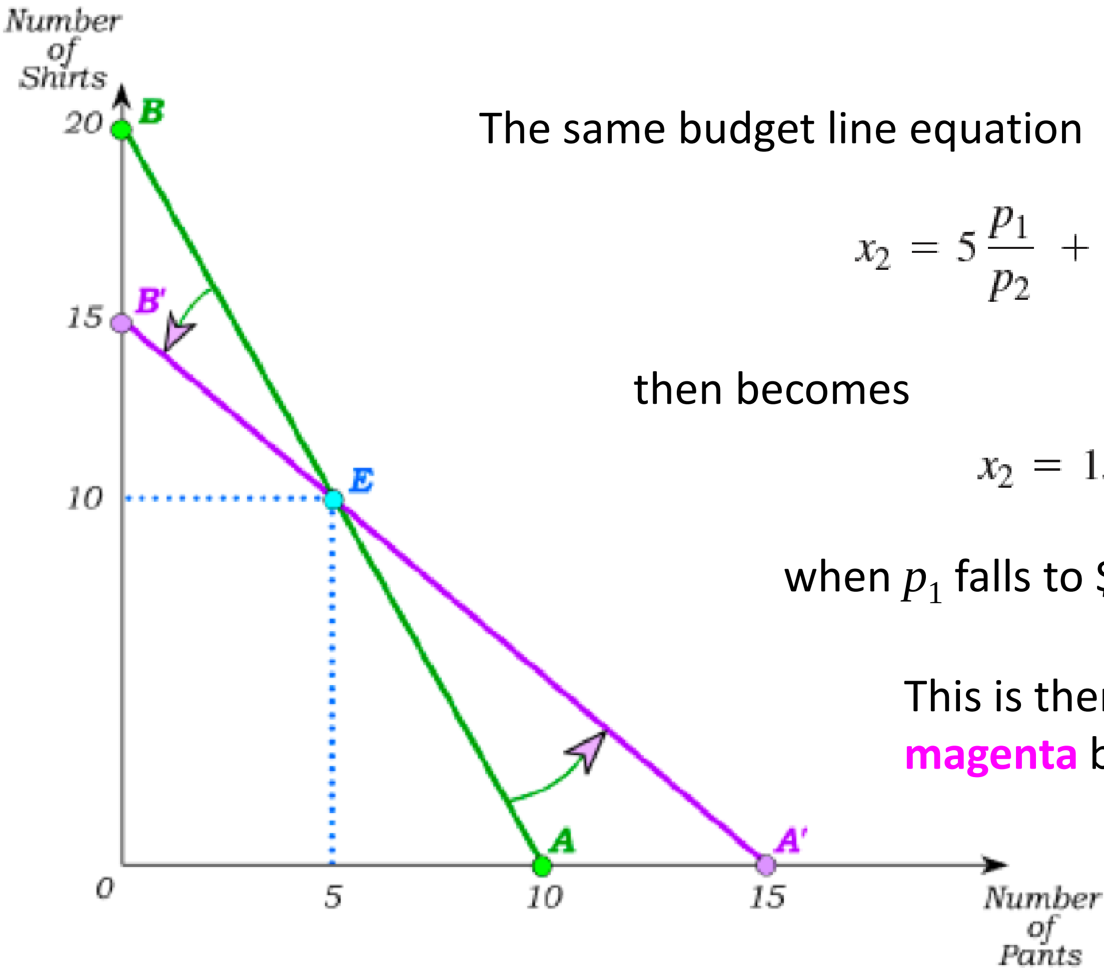
Solving for x_2 then gives us the budget line

$$x_2 = 5\frac{p_1}{p_2} + 10 - \frac{p_1}{p_2}x_1$$

which, at p_1=20 and p_2=10, reduces to $x_2 = 20 - 2x_1$, the **green** budget line drawn here.

Back to Graphs

Endowments



The same budget line equation

$$x_2 = 5\frac{p_1}{p_2} + 10 - \frac{p_1}{p_2}x_1$$

then becomes

$$x_2 = 15 - x_1$$

when p_1 falls to \$10 (with p_2 remaining at \$10).

This is then the equation for the new **magenta** budget line.

2B
Consumer Choice Sets & Budget Equations

- For ease of understanding, refer to the A section of the text for each topic
- **2B.1 Shopping on a Fixed Income**
  - Choice set: pants & shirts at Walmart
  - **2B.1.1 Defining Choice Sets & Budget Lines Mathematically**
    - If pants are denoted by variable x_1, and shorts by x_2, we can define the choice set formally as:

$$\{(x_1, x_2) \in \mathbb{R}^2_+ \mid 20x_1 + 10x_2 \leq 200\}.$$

2B.1 Shopping on a Fixed Income

2B.1.1 Defining Choice Sets… Mathematically

- We can define the *budget line* as the set of bundles that lie on the boundary of the choice set:

$$\{(x_1, x_2) \in \mathbb{R}^2_+ \mid 20x_1 + 10x_2 = 200\}.$$

- If the price of pants is p_1, the price of shirts p_2, and income as I, we define a consumer's choice set *C* as:

$$C(p_1, p_2, I) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid p_1x_1 + p_2x_2 \leq I\}.$$

- We can define budget line *B* as:

$$B(p_1, p_2, I) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid p_1x_1 + p_2x_2 = I\},$$

- Subtract p_1x_1 from both sides and divide both by p_2 :

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1.$$

2B.1 Shopping on a Fixed Income

2B.1.2 An Increase (or Decrease) in Fixed Income

– When fixed income changes, only the first term (I/p_2) in equation 2.5 changes; the second term remains the same

– The choice set has become larger, but the trade-off between goods remains the same

2B.1.3 A Change in Price

- A 50% coupon effectively lowers the price of pants from $20 to $10
- In equation 2.5, p_1 does not appear in the intercept term, but does appear in the slope term
- The x_2-axis intercept remains unchanged (I/p_2), but the slope becomes shallower as p_1/p_2 becomes smaller

2B.2 Kinky Budgets

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- Kinked budget lines are more difficult to describe mathematically
- Consider the 50% off coupon for the first 6 pairs of pants
- We could define the choice set as:

$$C(p_1,p_2,I) = \{(x_1,x_2) \in \mathbb{R}^2_+ \mid 0.5\, p_1x_1 + p_2x_2 \leq I \text{ for } x_1 \leq 6 \text{ and } p_1x_1 + p_2x_2 \leq I + 3p_1 \text{ for } x_1 > 6\}.$$

- Graph 2.4a:

$$B(p_1,p_2,I) = \{(x_1,x_2) \in \mathbb{R}^2_+ \mid 0.5\, p_1x_1 + p_2x_2 = I \text{ for } x_1 \leq 6 \text{ and } p_1x_1 + p_2x_2 = I + 3p_1 \text{ for } x_1 > 6\}.$$

2B.3
Choice Sets with More Than Two Goods

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– We can mathematically formulate the choice sets, or treat the goods as a composite good

2B.3.1 Choice Sets with 3 or More Goods

- The choice set is $C(p_1,p_2,p_3,I) = \{(x_1,x_2,x_3) \in \mathbb{R}^3_+ \mid p_1x_1 + p_2x_2 + p_3x_3 \leq I\}$, with the corresponding budget constraint defined by:

$$B(p_1,p_2,p_3,I) = \{(x_1,x_2,x_3) \in \mathbb{R}^3_+ \mid p_1x_1 + p_2x_2 + p_3x_3 = I\}.$$

- For the general case of *n* different goods with *n* different prices, we would extend 2.8 and 2.9 to

$$C(p_1,p_2, \ldots ,p_n,I) = \{(x_1,x_2, \ldots ,x_n) \in \mathbb{R}^n_+ \mid p_1x_1 + p_2x_2 + \ldots + p_nx_n \leq I\},$$

and

$$B(p_1,p_2, \ldots ,p_n,I) = \{(x_1,x_2, \ldots ,x_n) \in \mathbb{R}^n_+ \mid p_1x_1 + p_2x_2 + \ldots + p_nx_n = I\}.$$

2B.4 Choice Sets That Arise from Endowments

- Sometimes, money that can be devoted to consumption is not *exogenous*, but arises *endogenously* from the decisions a consumer makes and the prices she faces in the market
- In Section 2A.4, I returned to Walmart with 10 shirts and 5 pants to get a store credit at the current price
- My income can be expressed as $I = 5p_1 + 10p_2.$
- My choice set is then composed of all combinations of pants and shirts such that my total spending is no more than this income level $C(p_1,p_2) = \{(x_1,x_2) \in \mathbb{R}^2_+ \mid p_1x_1 + p_2x_2 \leq 5p_1 + 10p_2\}.$
- When the inequality in (2.15) is replaced with an equality to get the equation for the budget line, we get

$$p_1x_1 + p_2x_2 = 5p_1 + 10p_2.$$

2B.4
Choice Sets That Arise from Endowments

- Subtracting p_1x_1 from both sides and dividing both by p_2, this turns into
$$x_2 = 5\frac{p_1}{p_2} + 10 - \frac{p_1}{p_2}x_1.$$
- Graph 2.6, we plotted this budget set for the case where Walmart was charging $10 for both shirts and pants. When these prices are plugged into equation (2.17), we get
$$x_2 = 15 - x_1,$$
- We can denote someone's endowment as the number of goods of each kind a consumer has as he enters Walmart
- If my endowment of good 1 is e1, and my endowment of good 2 is e2, we can define my choice set as a function of my endowment and the prices of the two goods
$$C(p_1, p_2, e_1, e_2) = \{(x_1, x_2) \mid p_1x_1 + p_2x_2 \leq p_1e_1 + p_2e_2\},$$