

Answers to Discussion Questions

1. After terrorists destroyed the World Trade Center and surrounding office buildings on September 11, 2001, some businesspeople worried about the risks of remaining in Manhattan. What effect would you expect their concern to have on the price of office space in Manhattan? Over time, those fears eased and the area around the World Trade Center site was made into a park, so the destroyed office buildings were never rebuilt. Who would be likely to gain economically from the creation of this park? Who would be likely to lose?

Answer:

The destruction of the World Trade Center caused a change in both the supply of and the demand for office space in Manhattan. The change in supply was a physical reduction in the available space; the change in demand came from worry from businesspeople about issues of safety. When both supply and demand decrease in this way, the effect on quantity is obvious: there will be less office space rented in Manhattan. What is not clear is the effect on price. If the decrease in supply were greater than the decrease in demand, price would rise; if the other way around, price would fall.

In the long run, after office space is rebuilt, the supply curve would shift back out to (or closer to) its original position. If demand never rebounded, then this would cause an overall reduction in the price of office space.

However, now that the area around the World Trade Center has been turned into a park, the decrease in supply is permanent. Further, if demand were to rebound (say because the park causes a lower density of office space, making it a less likely target for another attack, thereby alleviating concerns), this would mean a higher price in the long run. Those who owned the office space not destroyed by the attacks would be the “winners,” as they would see their prices rise. The “losers” would be those who pay higher rents for their office space.

2. If the U.S. government were to ban imports of Canadian beef for reasons unrelated to health concerns, what would be the effect on the price of beef in the United States? How would the typical American’s diet change? What about the typical Canadians? What if the ban suggested to consumers that there might be health risks associated with beef?

Answer:

A ban on Canadian beef would lower the supply of beef available to U.S. consumers, which would cause an increase in the price of beef. Initially, before the price changes, there will not be enough beef to satisfy demand. This will cause upward pressure on prices, driving some consumers out of the market, and leading some suppliers into the market. Depending on how much of the beef being sold in the U.S. was Canadian beef, the increase could be great. Americans will reduce their beef consumption (though more Americans are producing beef than before).

In Canada, beef producers would be supplying too much beef to the market, with no one to buy it. This will cause downward pressure on prices. Because of the decrease in price, some Canadian consumers will enter the market and some producers will leave the market. Canadians will consume more beef (and produce less).

Unless U.S. consumers believed that the health risk associated with Canadian beef also implied health risks associated with U.S.-produced beef, the analysis for the U.S. would not be any different. If U.S. consumers did believe that all beef was unsafe, this would cause a decrease in the demand for beef, which would reverse the price-increasing trend of the ban (making the final effect on price ambiguous) while further reducing beef consumption.

If Canadian consumers believed that their beef was unsafe, there would also be a decrease in demand. This would counter the trend toward consuming more beef (so that the final effect on beef consumption would be ambiguous, but it would further reduce the price).

3. Published reports indicate that Economics professors have higher salaries than English professors. Discuss the factors that might be responsible for this.

Answer:

Salaries are determined by demand and supply. Relative to English, fewer Economics Ph.D.'s are granted every year. Additionally, economists have relatively more nonacademic job options, further reducing the supply of Economics professors, relative to English. There may be more demand for English professors, but the difference in demand is not large enough to make up for the difference in supply.

4. In the last 30 years, the wage difference between high school and college graduates has grown dramatically. At the same time, the fraction of adults over age 25 that have college degrees has risen from around 16 percent to over 27 percent. How might the widespread adoption of computers explain these trends?

Answer:

Widespread adoption of computers has increased the demand for computer literate workers while decreasing the demand for computer illiterate workers. College graduates are more likely to have the necessary computer literacy. Also, the increase in demand for computer literate workers has been faster than the increase in supply of college graduates.

Answers to Problems

- 2.1 Consider again the demand function for corn in formula (1). Graph the corresponding demand curve when potatoes and butter cost \$0.75 and \$4 per pound, respectively, and average income is \$40,000 per year. At what price does the amount demanded equal 15 billion bushels per year? Show your answer using algebra.

Answer:

Formula 1 - Consider the demand function for corn:

$$Q_{corn}^d = 5 - 2P_{corn} + 4P_{potatoes} - 0.25P_{butter} + 0.0003M,$$

where Q_{corn}^d is the amount of corn demanded per year in billions of bushels; P_{corn} is the price of corn per bushel; $P_{potatoes}$ and P_{butter} are the price of potatoes and butter per pound, respectively; and M is consumers' average annual income.

The corresponding demand curve when potatoes and butter cost \$0.75 and \$4 per pound, respectively, and average income is \$40,000 per year---see the graph below:



At the price of \$2.00 the amount demanded is equal to 15 billion bushels per year.

Explanation:

Graphing a linear demand curve is most easily done by finding the x - and y -intercepts.

Y -intercept = 9.5: Substitute the values of $P_{potatoes}$, P_{butter} , and M into the demand function and find the price of corn, P_{corn} , at which Q^d equals zero.

$$\begin{aligned} Q^d &= 5 - 2P_{corn} + 4P_{potatoes} - 0.25P_{butter} + 0.0003M \\ 0 &= 5 - 2P_{corn} + 4(0.75) - 0.25(4.00) + 0.0003(40,000) \\ 0 &= 5 - 2P_{corn} + 3 - 1 + 12 \\ -19 &= -2P_{corn} \\ P_{corn} &= 9.5 = y \end{aligned}$$

X -intercept = 19: Substitute the values of $P_{potatoes}$, P_{butter} , and M into the demand function. Set the price of corn, P_{corn} , equal to zero and solve for Q^d .

$$\begin{aligned} Q^d &= 5 - 2P_{corn} + 4P_{potatoes} - 0.25P_{butter} + 0.0003M \\ Q^d &= 5 - 2P_{corn} + 4(0.75) - 0.25(4.00) + 0.0003(40,000) \\ Q^d &= 5 - 2(0) + 3 - 1 + 12 \\ Q^d &= 19 = x\text{-intercept.} \end{aligned}$$

To find the price, set the demand function equal to 15. Substitute the values of $P_{potatoes}$, P_{butter} , and M into the demand function and find the price of corn, P_{corn} , at which Q^d equals 15.

$$\begin{aligned} Q^d &= 5 - 2P_{corn} + 4P_{potatoes} - 0.25P_{butter} + 0.0003M \\ 15 &= 5 - 2P_{corn} + 4(0.75) - 0.25(4.00) + 0.0003(40,000) \\ 15 &= 5 - 2P_{corn} + 3 - 1 + 12 \\ 15 &= 19 - 2P_{corn} \\ -4 &= -2P_{corn} \\ P_{corn} &= \$2.00. \end{aligned}$$

- 2.2 Consider again the supply function for corn in formula (2). Graph the corresponding supply curve when diesel fuel costs \$2.75 per gallon and the price of soybeans is \$10 per bushel. At what price does the amount supplied equal 21 billion bushels per year? Show your answer using algebra.

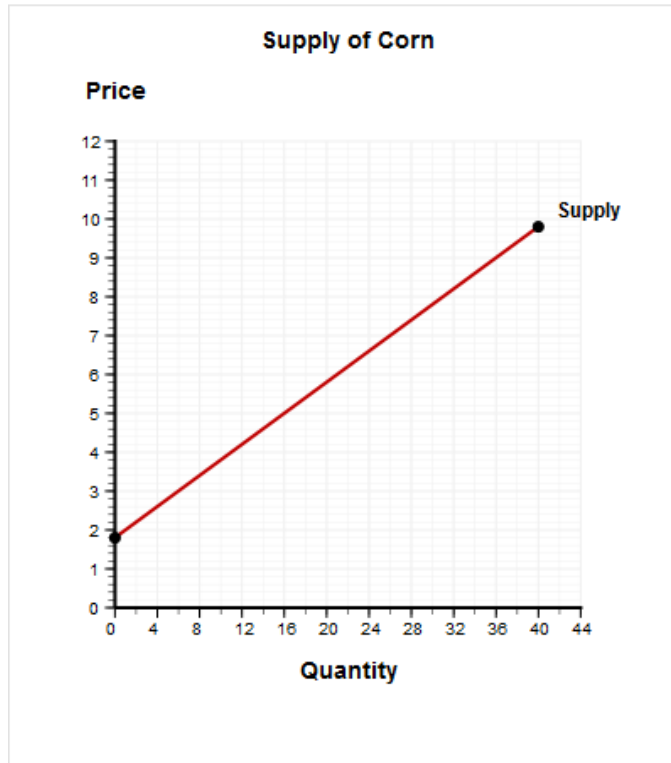
Answer:

Consider the supply function for corn:

$$Q_{corn}^S = 9 + 5P_{corn} - 2P_{fuel} - 1.25P_{soybeans}$$

where Q_{corn}^S is the amount of corn supplied per year in billions of bushels; P_{corn} is the price of corn per bushel; P_{fuel} is the price of diesel fuel per gallon; and $P_{soybeans}$ is the price of soybeans per bushel.

The corresponding supply curve is presented below:



At a price of \$6.00 the amount supplied is equal to 21 billion bushels per year.

Explanation:

Graphing a linear supply curve is not quite as simple as graphing a linear demand curve (because it is upward-sloping, it only has one positive intercept).

Determine the y-intercept: Substitute the price of fuel and the price of soybeans into the supply function. Set Q^s at zero and solve for the price at which no bushels of corn will be supplied to the market. The y-intercept = \$1.80.

Determine the supply curve: Substitute the price of fuel and the price of soybeans into the supply function. Set the quantity of corn supplied equal to the upper end of the relevant output range (40). Solve for $Q^S = 40 = -9 + 5P$, so that $P = \$9.80$.

First, set the supply function equal to 21 billion bushels and substitute the price of fuel and the price of soybeans. Next, solve for the price of corn, P_{corn} .

$$Q_{\text{corn}}^2 = 9 + 5P_{\text{corn}} - 2P_{\text{fuel}} - 1.25P_{\text{soybeans}}$$

$$21 = 9 + 5P_{\text{corn}} - 2(\$2.75) - 1.25(\$10)$$

$$21 = 9 + 5P_{\text{corn}} - 5.50 - 12.50$$

$$21 = -9 + 5P_{\text{corn}}$$

$$30 = 5P_{\text{corn}}$$

$$P_{\text{corn}} = 6$$

When the quantity supplied is 21 billion bushels, the price of corn is \$6.00.

- 2.3 What is the equilibrium price for the demand and supply conditions described in Problems 1 and 2? How much corn is bought and sold? What if the price of diesel fuel increases to \$4.50 per gallon? Show the equilibrium price before and after the change in a graph.

Answer:

The demand for corn (measured in billions of bushels) is given by:

$$Q_{corn}^d = 5 - 2P_{corn} + 4P_{potatoes} - 0.25P_{butter} + 0.0003M.$$

The supply of corn is given by:

$$Q_{corn}^s = 9 + 5P_{corn} - 2P_{fuel} - 1.25P_{soybeans}$$

The equilibrium price for the demand and supply conditions described in Problems 1 and 2 is \$4.00.

11 billion bushels of corn is bought and sold.

If the price of diesel fuel increases to \$4.50 per gallon, the price of corn will increase to \$4.50.

Explanation:

At equilibrium $Q^s = Q^d$.

$$9 + 5P_{corn} - 2P_{fuel} - 1.25P_{soybeans} = 5 - 2P_{corn} + 4P_{potatoes} - 0.25P_{butter} + 0.0003M$$

$$9 + 5P_{corn} - 5.5 - 12.50 = 5 - 2P_{corn} + 3 - 1 + 12$$

$$-28 = -7P_{corn}$$

$$P_{corn} = \$4.00$$

By inserting the new price into either the quantity demanded or quantity supplied equation, we can see how much corn is bought and sold. When the price is set by the market, the quantity of corn bought and sold is 11 billion bushels.

$$Q_{corn}^s = 9 + 5(\$4.00) - 2(\$2.75) - 1.25(\$10)$$

$$Q_{corn}^s = 11$$

If fuel prices increase to \$4.50 per gallon, the price of corn will rise to \$4.50 and less will be produced.

$$9 + 5P_{corn} - 9.00 - 12.50 = 5 - 2P_{corn} + 3 - 1 + 12$$

$$-31.50 = -7P_{corn}$$

$$P_{corn} = \$4.50$$

$$Q_{corn}^s = 9 + 5(\$4.50) - 2(\$4.50) - 1.25(\$10)$$

$$Q_{corn}^s = 10$$

- 2.4 Suppose that the demand function for a product is $Q^d = 200/P$ and the supply function is $Q^s = 2P$. What is the equilibrium price and the amount bought and sold?

Answer:

The equilibrium price is \$10. The equilibrium quantity that will be bought and sold is 20 units.

Explanation:

Solve $Q^d = Q^s$.

$$200/P = 2P$$

$$P^2 = 100$$

$$P = 10.$$

Therefore:

$$Q^d = 200/10 = 20.$$

$$Q^s = 2(10) = 20.$$

- 2.5 The daily worldwide demand function for crude oil (in millions of gallons) is $Q^d = 17 - \sqrt{P}$ and the daily supply function is $Q^s = \sqrt{P} - 1$, where P is the price per gallon. What is the equilibrium price of crude oil? What is the quantity bought and sold each day?

Answer:

The equilibrium price of crude oil is \$81. The equilibrium quantity bought and sold each day is 8 million gallons.

Explanation:

Equate Q^d and Q^s .

$$\Rightarrow 17 - \sqrt{P} = \sqrt{P} - 1$$

$$\Rightarrow 2\sqrt{P} = 18$$

$$\Rightarrow \sqrt{P} = 9$$

$$\Rightarrow P = 81.$$

When $P = 81$,

$$Q^d = 17 - \sqrt{81} = 17 - 9 = 8.$$

$$Q^S = \sqrt{81} - 1 = 9 - 1 = 8.$$

- 2.6 Consider again the demand and supply functions in Worked-Out Problem 2.2 (page 31). Suppose the government needs to buy 3.5 billion bushels of corn for a third-world famine relief program. What effect will the purchase have on the equilibrium price of corn? How will it change the amount of corn that is bought and sold?

Answer:

Consider the following demand and supply functions for corn:

Demand is given by:

$$Q_{corn}^d = 15 - 2P_{corn}$$

Supply is given by:

$$Q_{corn}^S = 5P_{corn} - 6$$

where P is the price in dollars and Q is the quantity in billions of bushels. Suppose the government needs to buy 3.5 billion bushels of corn for a third-world famine relief program.

The equilibrium price of corn *before* the government purchase is \$3. The equilibrium price of corn *after* the government purchase is \$3.50.

Explanation:

To find the equilibrium price before the government purchase we set supply equal to demand:

$$\begin{aligned} Q_{corn}^S &= Q_{corn}^d \\ 5P_{corn} - 6 &= 15 - 2P_{corn} \\ 7P_{corn} &= 21 \\ P_{corn} &= 3 \end{aligned}$$

The effect of the government purchase will be to reduce the available quantity supplied by 3.5 billion bushels.

Therefore, the new supply function is given by:

$$Q_{corn}^S = 5P_{corn} - 6 - 3.5$$

Demand is given by:

$$Q_{corn}^d = 15 - 2P_{corn}$$

At equilibrium $Q^S = Q^d$.

$$5P_{corn} - 6 - 3.5 = 15 - 2P_{corn}$$

Equilibrium price equals \$3.50, and equilibrium quantity equals 8.0 billion bushels plus 3.5 billion bushels purchased by the government.

Alternatively, the government purchase of 3.5 billion bushels of corn can be seen as an outward shift in demand by 3.5. This will result in a new demand function: $Q^d = 18.5 - 2P$. Note that the new equilibrium price and quantity will still be \$3.50 per bushel and 11.5 billion bushels of corn including the 3.5 billion bushels purchased by the government.

- 2.7 Suppose that the U.S. demand for maple syrup, in thousands of gallons per year, is $Q^d = 6000 - 30P$, where P is the price per gallon. What is the elasticity of demand at a price of \$75 per gallon?

Answer:

The elasticity of demand is -0.60.

Explanation:

At price P , denoted E^d equals the percentage change in the amount demanded for each 1 percent increase in the price. Demand for maple syrup is given by:

$$Q_d = 6,000 - 30P$$

The elasticity of demand for a linear demand curve starting at price P and quantity Q is:

$$E^d = \left(\frac{\Delta Q}{\Delta P} \right) \left(\frac{P}{Q} \right).$$

For a linear demand curve of the form $Q^D = A - BP$, the first term in parentheses $\left(\frac{\Delta Q}{\Delta P} \right)$ equals $-B$. Therefore:

$$E^d = -B \times \left(P / Q \right).$$

Specifically:

$$\begin{aligned} E^d &= -30 \times 75 / (6,000 - (30 \times 75)) \\ E^d &= -0.6 \end{aligned}$$

- 2.8 Consider again Problem 7. At what price is the expenditure on maple syrup by U.S. consumers highest?

Answer:

The expenditure on maple syrup by U.S. consumers is highest at \$100.

Explanation:

The largest total expenditure and maximum revenue occur when price elasticity of demand equals -1 . A small increase in price causes total expenditure to increase if demand is inelastic and decrease if demand is elastic. Total expenditure is largest at a price for which the elasticity equals -1 .

$$E^d = -B\left(\frac{P}{Q}\right) = -1$$

Specifically,

$$\begin{aligned} -1 &= -30 \times (P / (6,000 - (30P))) \\ 6,000 - 30P &= 30P \\ 6,000 &= 60P \\ P &= 100 \end{aligned}$$

- 2.9 Suppose the demand function for jelly beans in Cincinnati is linear. Two years ago, the price of jelly beans was \$1 per pound, and consumers purchased 100,000 pounds of jelly beans. Last year the price was \$2, and consumers purchased 50,000 pounds of jelly beans. No other factors that might affect the demand for jelly beans changed. What was the elasticity of demand at last year's price of \$2? At what price would the total expenditure on jelly beans have been largest?

Answer:

The elasticity of demand at last year's price is -2 . The total expenditure on jelly beans would have been largest at a price of \$1.50.

Explanation:

First find the elasticity of demand.

$$E^d = \left(\frac{\Delta Q}{Q}\right)\left(\frac{P}{\Delta P}\right) = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{P}{Q}\right)$$

$$\begin{aligned} E^d &= -50,000 \times 2 / 50,000 \\ E^d &= -2 \end{aligned}$$

Use E^d to derive the linear demand curve of the form: $Q^d = A - BP$.

$$E^d = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{P}{Q}\right) = (-B)\left(\frac{P}{Q}\right)$$

Use known values for Q^d , B , and P to solve for A .

$$\begin{aligned} 50,000 &= A - 50,000 \times 2 \\ A &= 150,000 \end{aligned}$$

$$Q^d = 150,000 - 50,000P$$

Use this result to answer the next question. The largest total expenditure and maximum revenue occur when price elasticity of demand equals -1 .

$$\begin{aligned}-1 &= -50,000P / (150,000 - (50,000P)) \\ P &= \$1.50\end{aligned}$$

2.10 Consider again the demand and supply functions in In-Text Exercise 2.2 (page 31). At the equilibrium price, what are the elasticities of demand and supply?

Answer:

Consider the following demand and supply functions for corn:

Demand is given by:

$$Q^d_{\text{corn}} = 15 - 2P$$

Supply is given by:

$$Q^s_{\text{corn}} = 5P - 6$$

At the equilibrium price, the elasticities of demand (E^d) is equal to -0.67 and supply (E^s) is equal to 1.67 .

Explanation:

Find equilibrium price by setting $Q^d = Q^s$.

$$\begin{aligned}15 - 2P &= 5P - 6 \\ 21 &= 7P \\ P &= \$3.00\end{aligned}$$

Price elasticity of demand:

$$\begin{aligned}E^d &= (\Delta Q / \Delta P) \times (P / Q) \\ E^d &= -B(P / Q) \\ E^d &= -2 \times 3.00 / (15 - (2 \times 3.00)) \\ E^d &= -2 \times (3.00 / 9.0) \\ E^d &= -0.67\end{aligned}$$

Price elasticity of supply:

$$\begin{aligned}E^s &= (\Delta Q / \Delta P) \times (P / Q) \\ E^s &= -d(P / Q) \\ E^s &= 5 \times 3.00 / (5 \times 3.00 - 6)\end{aligned}$$

$$E^s = 5 \times (3.00/9.00)$$

$$E^s = 1.67$$

- 2.11 Last September, the price of gasoline in Chattanooga was \$2 a gallon, and consumers bought 1 million gallons. Suppose the elasticity of demand in September at a price of \$2 was -0.5, and that the demand function for gasoline that month was linear. What was that demand function? At what price does consumers' total expenditure on gasoline reach its largest level?

Answer:

The demand function is $Q^d = 1,500,000 - 250,000P$. Consumer's total expenditure on gasoline reaches its largest level at a price of \$3.

Explanation:

Derive the linear demand curve of the form $Q^d = A - BP$.

Step1: Find the slope ($-B$) using price elasticity of demand.

$$E^d = (\Delta Q / \Delta P) \times (P/Q)$$

$$E^d = (B) \times (P/Q)$$

$$-0.50 = (B) \times (2.00/1,000,000)$$

$$B = -250,000$$

Step 2: Determine the constant (intercept).

$$Q^d = A - BP$$

$$1,000,000 = A - (250,000 \times 2.00)$$

$$A = 1,500,000$$

The equation is $Q^d = 1,500,000 - 250,000P$

The largest total expenditure occurs when $E^d = -1$.

$$E^d = (\Delta Q / \Delta P) \times (P/Q)$$

$$-1 = -250,000 \times P / (1,500,000 - (250,000P))$$

$$-1,500,000 + 250,000P = -250,000P$$

$$500,000P = 1,500,000$$

$$P = \$3$$

- 2.12 Suppose the annual demand function for the Honda Accord is $Q^d = 430 - 10 P_A + 10 P_C - 10 P_G$, where P_A and P_C are the prices of the Accord and the Toyota Camry respectively (in thousands), and P_G is the price of gasoline (per gallon). What is the elasticity of demand for the Accord with respect to the price of a Camry when both cars sell for \$20,000 and fuel costs \$3 per gallon? What is the elasticity with respect to the price of gasoline?

Answer:

The elasticity of demand for the Accord with respect to the price of a Camry when both cars sell for \$20,000 and fuel costs \$3 per gallon is 500. The elasticity with respect to the price of gasoline is -0.075.

Explanation:

$$Q^d = 430 - 10 P_A + 10 P_C - 10 P_G$$

$$Q^d = 430 - (10 \times 20,000) + (10 \times 20,000) - (10 \times 3.00) = 400$$

Cross-price elasticity of demand for Honda Accords with respect to the price of a Toyota Camry:

$$E_{pc} = (\Delta Q / \Delta P) \times (P / Q)$$

$$E_{pc} = 10 \times (20,000 / 400)$$

$$E_{pc} = 500.0$$

Cross-price elasticity of demand for Honda Accords with respect to the price of gasoline:

$$E_{pg} = (\Delta Q / \Delta P) \times (P / Q)$$

$$E_{pg} = -10 \times (3 / 400)$$

$$E_{pg} = -0.075$$

- 2.13 The demand for a product is $Q^d = A - BP$, where P is its price and A and B are positive numbers. Suppose that when the price is \$1 the amount demanded is 60 and the elasticity of demand is -1. What are the values of A and B ?

Answers:

$A = 120$.

$B = 60$.

Explanation:

Derive the linear demand curve

Step 1: Solve for $(-B)$ using elasticity of demand along with the given values for P and Q .

$$E^d = (\Delta Q / \Delta P) \times (P / Q)$$

$$-1 = (\Delta Q / \Delta P) \times (1.00 / 60)$$

$$-B = (\Delta Q / \Delta P)$$

$$B = 60$$

Step 2: Solve for A using the known values for Q^d , B , and P .

$$Q^d = A - BP$$

$$60 = A - (60 \times 1.00)$$

$$A = 120$$

$$Q^d = 120 - 60P$$

Answers to Calculus Problems

- 2.1 The demand function for a product is $Q^d = 100 - B_d P$. Suppose that there is a tax of t dollars per unit that producers must pay and that the supply function for the product when the tax is t and the price is P is $Q^s = B_s (P - t) - 5$. What is the equilibrium price as a function of the tax t ? Define the “pass-through rate” of a small increase in the tax as the derivative of the market price consumers pay with respect to the tax: dP/dt . What is the pass-through rate of a small tax increase in this market? How does it depend on B_d and B_s ?

Answer:

The equilibrium price as a function of the tax t is

$$\frac{105}{B_d + B_s} + \frac{tB_s}{B_d + B_s}$$

Define the "pass-through rate" of a small increase in the tax as the derivative of the market price consumers pay with respect to the tax: dP/dt . The pass-through rate of a small tax increase in this market is

$$\frac{B_s}{B_d + B_s}$$

The pass-through rate increases as B_s increases and decreases as B_d increases.

Explanation:

Equate Q^d and Q^s .

$$100 - B_d P = B_s (P - t) - 5$$

$$\Rightarrow 100 - B_d P = B_s P - tB_s - 5$$

$$\Rightarrow (B_s + B_d)P = 105 + tB_s$$

$$\Rightarrow P = \frac{105}{B_s + B_d} + \frac{tB_s}{B_s + B_d}$$

Therefore,

$$\frac{dP}{dt} = \frac{B_S}{B_S + B_D}.$$

To see how the pass-through rate changes with B_S and B_D note that:

$$\frac{d}{dB_S} \left(\frac{B_S}{B_S + B_D} \right) > 0$$

$$\frac{d}{dB_D} \left(\frac{B_S}{B_S + B_D} \right) < 0$$

- 2.2 Suppose the daily demand for coffee in Seattle is $Q^d = 100,000 (3 - P)^2$. What is the elasticity of demand at a price of \$2? At what price would the total expenditure on coffee be largest?

Answers:

The elasticity of demand is -4. Total expenditure on coffee will be largest at a price of \$1.

Explanation:

$$\frac{dQ^d}{dP} = \frac{d}{dP} \left(100,000 (3 - P)^2 \right) = -200,000 (3 - P)$$

Hence, the elasticity is:

$$\frac{dQ^d}{dP} \left(\frac{P}{Q^d} \right) = \frac{-200,000 (3 - P) P}{100,000 (3 - P)^2} = -\frac{2P}{3 - P}.$$

When $P = 2$, the elasticity is -4.

The price at which the elasticity is -1 (or 1 in absolute value), maximizes the total expenditure.

Note that at $P = 1$,

$$\frac{2P}{3 - P} = \frac{2}{2} = 1$$

- 2.3 Suppose that the demand function for a product takes the form $Q^d = (A - BP)^D$ where A , B , and D are positive numbers. Derive a formula for the elasticity of demand as a function of the price. (Note: Your formula can have A , B , and D in it.) At what price is consumers' total expenditure the largest? (Note: Your answer should be in terms of A , B , and D .)

Answer:

The formula for the elasticity of demand is:

$$-\frac{BDP}{A-BP}$$

Consumers' total expenditure the largest at a price of:

$$\frac{A}{B(D+1)}$$

Explanation:

$$\frac{dQ^d}{dP} = \frac{d}{dP} \left((A - BP)^D \right) = -BD(A - BP)^{D-1}$$

Therefore, the formula for elasticity is:

$$\frac{dQ^d}{dP} \frac{P}{Q^d} = -BD(A - BP)^{D-1} \left(\frac{P}{(A - BP)^D} \right) = -\frac{BDP}{A - BP}.$$

Total expenditure is maximized at the price where the absolute value of the elasticity is 1.

Hence, look for P such that:

$$\frac{BDP}{A - BP} = 1$$

$$\Rightarrow BDP = A - BP$$

$$\Rightarrow BDP + BP = A$$

$$\Rightarrow BP(D + 1) = A$$

$$\Rightarrow P = \frac{A}{B(D+1)}$$

- 2.4 Let P denote the price of a product that is produced using a single input whose price is W . The demand function for the product is $Q^d = 10 - \sqrt{P}$, and the supply function is $Q^s = \sqrt{P/W}$ if $P > W$ and $Q^s = 0$ if $P \leq W$. How does the equilibrium price depend on W ? What is the elasticity of the equilibrium price P with respect to W when $W = 1$?

Answer:

When $P \leq W$, quantity supplied is zero. Hence, no exchange takes place between buyers and sellers. When $P > W$, the equilibrium price is expressed as:

$$\frac{100W}{(1 + \sqrt{W})^2}$$

The elasticity of the equilibrium price P with respect to W when $W = 1$ is 0.5.

Explanation:

Let $P > W$.

$$\begin{aligned} Q^d &= Q^s \\ \Rightarrow 10 - \sqrt{P} &= \sqrt{\frac{P}{W}} \\ \Rightarrow 10 &= \sqrt{P} + \sqrt{\frac{P}{W}} \\ \Rightarrow 10 &= \sqrt{P} \left(1 + \sqrt{\frac{1}{W}} \right) \\ \Rightarrow 10 &= \sqrt{P} \left(\frac{1 + \sqrt{W}}{\sqrt{W}} \right) \\ \Rightarrow 10\sqrt{W} &= \sqrt{P} (1 + \sqrt{W}) \\ \Rightarrow \frac{10\sqrt{W}}{1 + \sqrt{W}} &= \sqrt{P} \\ \Rightarrow P &= \frac{100W}{(1 + \sqrt{W})^2} \end{aligned}$$

The elasticity of the equilibrium price with respect to W is:

$$\frac{dP}{dW} \left(\frac{W}{P} \right)$$

Note that:

$$\frac{dP}{dW} = \frac{d}{dW} \left(\frac{100W}{(1 + \sqrt{W})^2} \right) = \frac{100}{(1 + \sqrt{W})^2} - \frac{100\sqrt{W}}{(1 + \sqrt{W})^3}$$

Also:

$$\frac{W}{P} = \frac{(1 + \sqrt{W})^2}{100}$$

Hence, the elasticity of the equilibrium price with respect to W is:

$$\frac{dP}{dW} \left(\frac{W}{P} \right) = 1 - \frac{\sqrt{W}}{1 + \sqrt{W}}$$

Note that when $W = 1$, the elasticity of the equilibrium price with respect to W is 0.5.

- 2.5 Suppose that the demand function for jelly beans is $Q^d = AP^{-B}$ and the supply function is $Q^s = CP^D$, where A , B , C , and D are all positive numbers.
- What is the elasticity of demand with respect to changes in A ?
 - What is the elasticity of supply with respect to the price?
 - Solve for the equilibrium price as a function of A , B , C , and D .
 - What is the elasticity of the equilibrium price with respect to changes in A ?

Answer:

- The elasticity of demand is 1.
- The elasticity of supply is D .
- The equilibrium price is:

$$\Rightarrow P = \left[\frac{A}{C} \right]^{\frac{1}{B+D}}$$

- The elasticity of the equilibrium price with respect to changes in A is:
 $1/(B + D)$

Explanation:

- Elasticity of demand with respect to A :

$$Q^d = AP^{-B}$$

$$\Rightarrow \frac{dQ^d}{dA} = P^{-B}$$

Elasticity of demand with respect to A :

$$\left(\frac{dQ^d}{dA}\right)\left(\frac{A}{Q^d}\right) = P^{-B}\left(\frac{A}{AP^{-B}}\right) = 1$$

b. Elasticity of supply with respect to P :

$$Q^S = CP^D$$

$$\frac{dQ^S}{dP} = CD P^{D-1}$$

Elasticity of supply with respect to the price:

$$\left(\frac{dQ^S}{dP}\right)\left(\frac{P}{Q^S}\right) = CD P^{D-1}\left(\frac{P}{CP^D}\right) = D$$

c. Equilibrium price:

$$Q^d = Q^S \Rightarrow AP^{-B} = CP^D$$

$$\Rightarrow P^{B+D} = \frac{A}{C}$$

$$\Rightarrow \frac{A}{P^B} = CP^D$$

$$\Rightarrow P = \left(\frac{A}{C}\right)^{\frac{1}{B+D}}$$

d. Elasticity of the equilibrium price with respect to A :

$$\frac{dP}{dA} = \frac{d}{dA} \left(\left(\frac{A}{C}\right)^{\frac{1}{B+D}} \right) = \left(\frac{1}{B+D}\right) \left(\frac{A}{C}\right)^{\frac{1}{B+D}-1} \left(\frac{1}{C}\right)$$

Elasticity of the equilibrium price with respect to A is:

$$\left(\frac{dP}{dA}\right)\left(\frac{A}{P}\right) = \left(\frac{1}{B+D}\right) \left(\frac{1}{C}\right) \left(\frac{A}{C}\right)^{\frac{1}{B+D}-1} \left(\frac{A}{\left(\frac{A}{C}\right)^{\frac{1}{B+D}}}\right) = \frac{1}{B+D}$$