**2–1.**

An air-filled rubber ball has a diameter of 6 in. If the air pressure within the ball is increased until the diameter becomes 7 in., determine the average normal strain in the rubber.

## SOLUTION

$d_0 = 6$ in.

$d = 7$ in.

$$\epsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.}$$ **Ans.**



**Ans:**
$\epsilon = 0.167$ in./in.

**2–2.**

A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

## SOLUTION

$L_0 = 15 \text{ in.}$

$L = \pi(5 \text{ in.})$

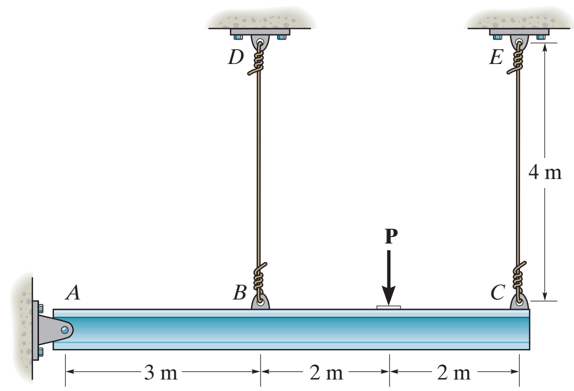
$$\epsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.}$$ **Ans.**



**Ans:**
$\epsilon = 0.0472 \text{ in./in.}$

**2–3.**

If the load **P** on the beam causes the end $C$ to be displaced 10 mm downward, determine the normal strain in wires $CE$ and $BD$.



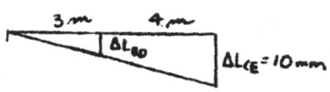

## SOLUTION

$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**

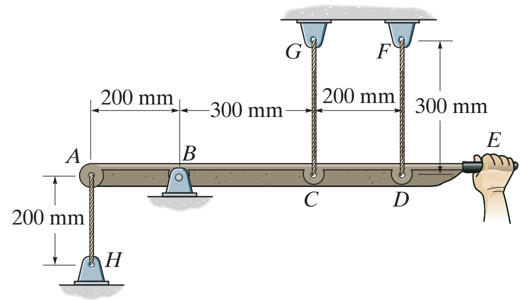





**Ans:**
$\epsilon_{CE} = 0.00250$ mm/mm, $\epsilon_{BD} = 0.00107$ mm/mm

***2–4.**

The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin $B$ through an angle of 2°. Determine the average normal strain in each wire. The wires are unstretched when the lever is in the horizontal position.




## SOLUTION

**Geometry:** The lever arm rotates through an angle of $\theta = \left(\frac{2°}{180}\right)\pi \text{ rad} = 0.03491 \text{ rad}$.

Since $\theta$ is small, the displacements of points $A$, $C$, and $D$ can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

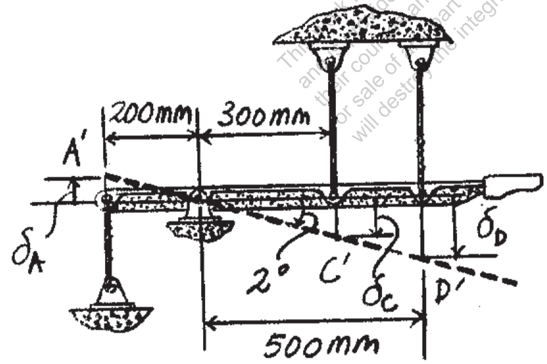
**Average Normal Strain:** The unstretched length of wires $AH$, $CG$, and $DF$ are $L_{AH} = 200$ mm, $L_{CG} = 300$ mm, and $L_{DF} = 300$ mm. We obtain

$$(\epsilon_{\text{avg}})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm} \qquad \textbf{Ans.}$$



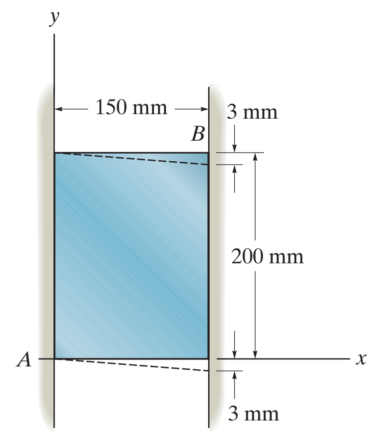



(a)

**Ans:**
$(\epsilon_{\text{avg}})_{AH} = 0.0349$ mm/mm
$(\epsilon_{\text{avg}})_{CG} = 0.0349$ mm/mm
$(\epsilon_{\text{avg}})_{DF} = 0.0582$ mm/mm

**2–5.**

The rectangular plate is subjected to the deformation shown by the dashed line. Determine the average shear strain $\gamma_{xy}$ in the plate.




## SOLUTION

**Geometry:**

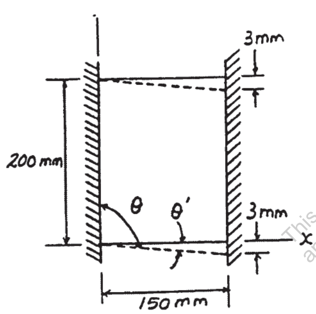
$$\theta' = \tan^{-1}\frac{3}{150} = 0.0200 \text{ rad}$$

$$\theta = \left(\frac{\pi}{2} + 0.0200\right) \text{rad}$$

**Shear Strain:**

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \left(\frac{\pi}{2} + 0.0200\right)$$

$$= -0.0200 \text{ rad} \qquad \textbf{Ans.}$$

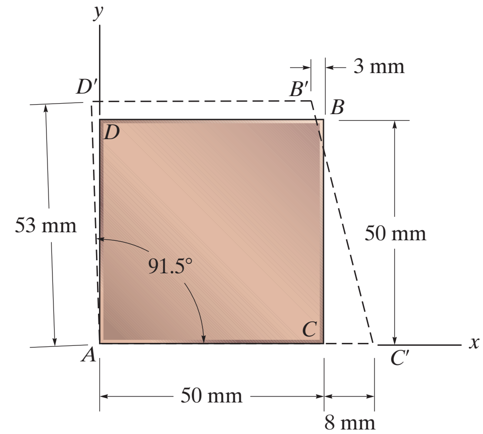





**Ans:**
$\gamma_{xy} = -0.0200$ rad

**2–6.**

The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners, $A$, $B$, $C$, and $D$, relative to the $x$, $y$ axes. Side $D'B'$ remains horizontal.




## SOLUTION

**Geometry:**

$$B'C' = \sqrt{(8 + 3)^2 + (53 \sin 88.5°)^2} = 54.1117 \text{ mm}$$

$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58) \cos 91.5°}$$

$$= 79.5860 \text{ mm}$$

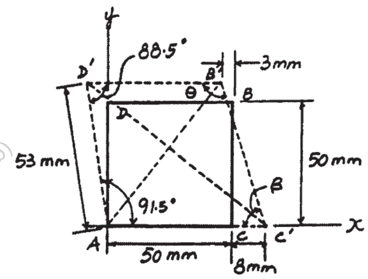
$$B'D' = 50 + 53 \sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$\cos \theta = \frac{(B'D')^2 + (B'C')^2 - (C'D')^2}{2(B'D')(B'C')}$$

$$= \frac{48.3874^2 + 54.1117^2 - 79.5860^2}{2(48.3874)(54.1117)} = -0.20328$$

$$\theta = 101.73°$$

$$\beta = 180° - \theta = 78.27°$$




**Shear Strain:**

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \pi\left(\frac{91.5°}{180°}\right) = -0.0262 \text{ rad}$$ **Ans.**

$$(\gamma_B)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \pi\left(\frac{101.73°}{180°}\right) = -0.205 \text{ rad}$$ **Ans.**

$$(\gamma_C)_{xy} = \beta - \frac{\pi}{2} = \pi\left(\frac{78.27°}{180°}\right) - \frac{\pi}{2} = -0.205 \text{ rad}$$ **Ans.**

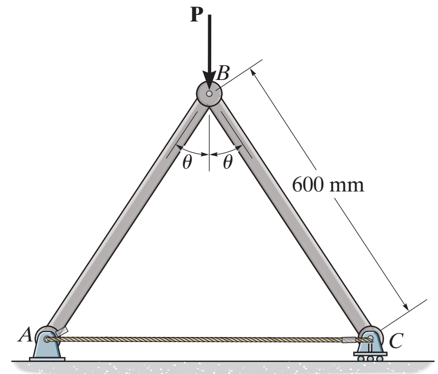
$$(\gamma_D)_{xy} = \pi\left(\frac{88.5°}{180°}\right) - \frac{\pi}{2} = -0.0262 \text{ rad}$$ **Ans.**



**Ans:**
$(\gamma_A)_{xy} = -0.0262$ rad
$(\gamma_B)_{xy} = -0.205$ rad
$(\gamma_C)_{xy} = -0.205$ rad
$(\gamma_D)_{xy} = -0.0262$ rad

**2–7.**

The pin-connected rigid rods $AB$ and $BC$ are inclined at $\theta = 30°$ when they are unloaded. When the force **P** is applied $\theta$ becomes 30.2°. Determine the average normal strain in wire $AC$.




## SOLUTION

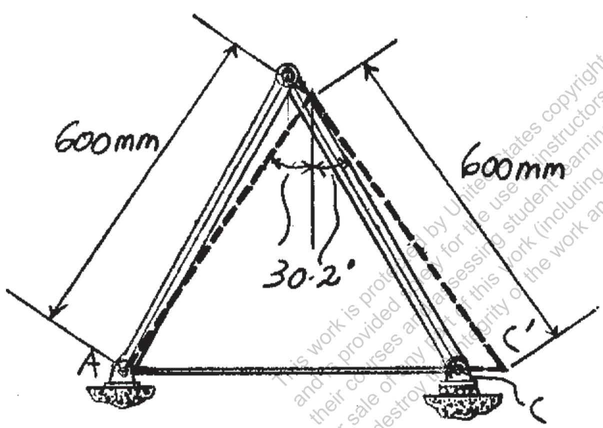
**Geometry:** Referring to Fig. $a$, the unstretched and stretched lengths of wire $AD$ are

$$L_{AC} = 2(600 \sin 30°) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2°) = 603.6239 \text{ mm}$$

**Average Normal Strain:**

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3}) \text{ mm/mm}$$ **Ans.**



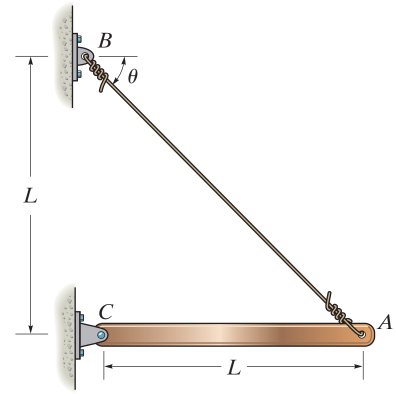

(a)



**Ans:**
$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3})$ mm/mm

***2–8.**

The wire $AB$ is unstretched when $\theta = 45°$. If a load is applied to the bar $AC$, which causes $\theta$ to become 47°, determine the normal strain in the wire.




## SOLUTION

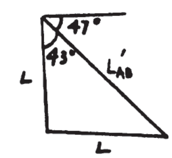
$$L^2 = L^2 + L'^2_{AB} - 2LL'_{AB}\cos 43°$$

$$L'_{AB} = 2L\cos 43°$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{2L\cos 43° - \sqrt{2}L}{\sqrt{2}L}$$

$$= 0.0343$$ **Ans.**

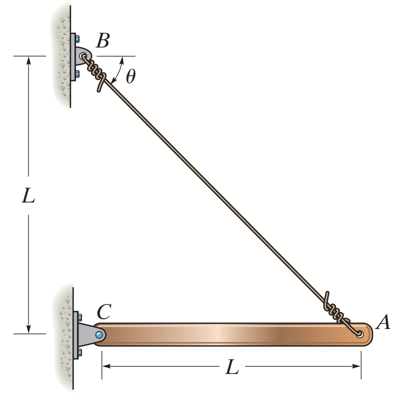





**Ans:**
$\epsilon_{AB} = 0.0343$

**2–9.**

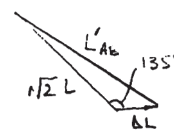
If a horizontal load applied to the bar $AC$ causes point $A$ to be displaced to the right by an amount $\Delta L$, determine the normal strain in the wire $AB$. Originally, $\theta = 45°$.




## SOLUTION

$$L'_{AB} = \sqrt{(\sqrt{2}L)^2 + \Delta L^2 - 2(\sqrt{2}L)(\Delta L)\cos 135°}$$

$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$



$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}L}{\sqrt{2}L}$$

$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$

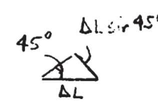
Neglecting the higher-order terms,

$$\epsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots\ldots - 1 \qquad \text{(binomial theorem)}$$

$$= \frac{0.5\Delta L}{L} \qquad \textbf{Ans.}$$

Also,



$$\epsilon_{AB} = \frac{\Delta L \sin 45°}{\sqrt{2}L} = \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans.}$$

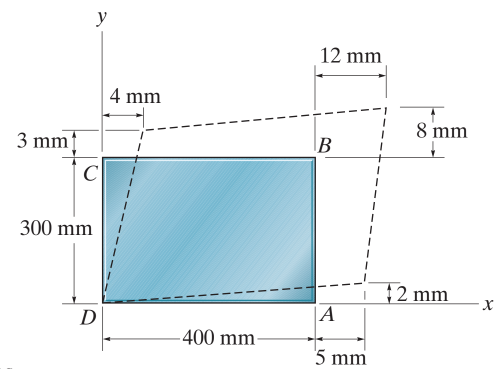


**Ans:**

$$\epsilon_{AB} = \frac{0.5\Delta L}{L}$$

**2–10.**

Determine the shear strain $\gamma_{xy}$ at corners $A$ and $B$ if the plastic distorts as shown by the dashed lines.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{7}{306} = 0.022876 \text{ rad}$$

$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

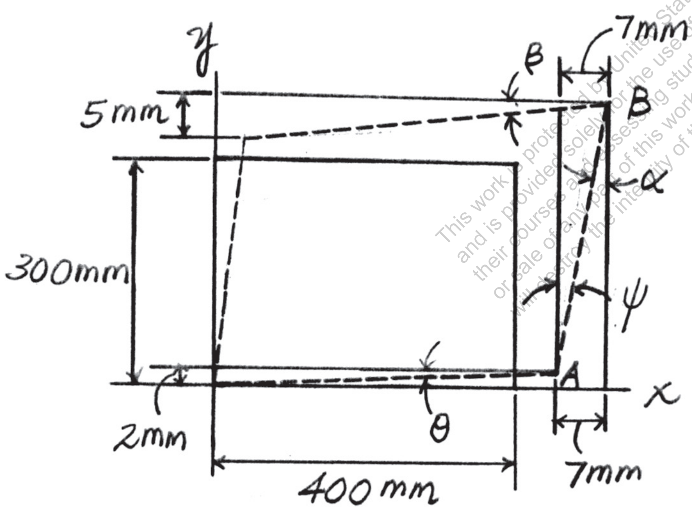
$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

**Shear Strain:** By definition,

$$(\gamma_A)_{xy} = \theta + \psi = 0.02781 \text{ rad} = 27.8(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$

$$(\gamma_B)_{xy} = \alpha + \beta = 0.03513 \text{ rad} = 35.1(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$



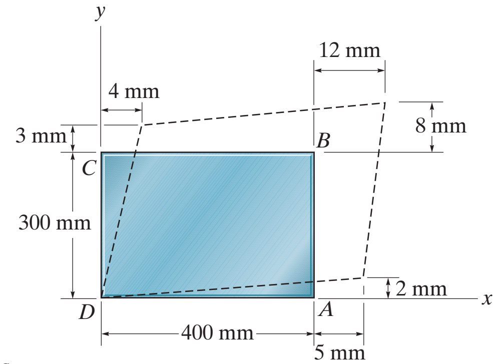



(a)

**Ans:**
$(\gamma_A)_{xy} = 27.8(10^{-3})$ rad
$(\gamma_B)_{xy} = 35.1(10^{-3})$ rad

**2–11.**

Determine the shear strain $\gamma_{xy}$ at corners $D$ and $C$ if the plastic distorts as shown by the dashed lines.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{4}{303} = 0.013201 \text{ rad}$$

$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

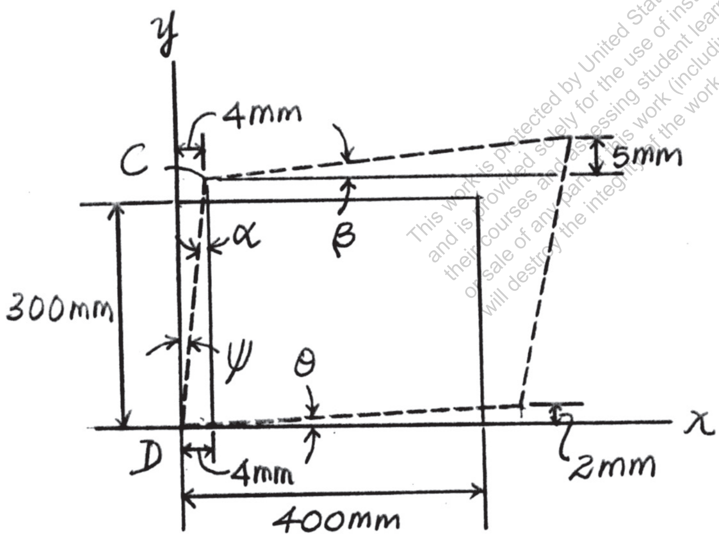
$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

**Shear Strain:** By definition,

$$(\gamma_{xy})_C = \alpha + \beta = 0.02546 \text{ rad} = 25.5(10^{-3}) \text{ rad}$$ **Ans.**

$$(\gamma_{xy})_D = \theta + \psi = 0.01814 \text{ rad} = 18.1(10^{-3}) \text{ rad}$$ **Ans.**



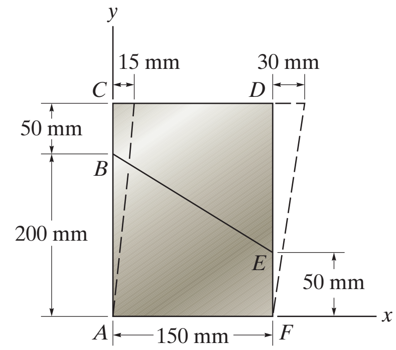



(a)

**Ans:**
$(\gamma_{xy})_C = 25.5(10^{-3})$ rad
$(\gamma_{xy})_D = 18.1(10^{-3})$ rad

***2–12.**

The material distorts into the dashed position shown. Determine the average normal strains $\epsilon_x$, $\epsilon_y$ and the shear strain $\gamma_{xy}$ at $A$, and the average normal strain along line $BE$.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. *a*,

$$\tan\theta = \frac{15}{250}; \qquad \theta = (3.4336°)\left(\frac{\pi}{180°}\text{ rad}\right) = 0.05993 \text{ rad}$$

$$L_{AC}' = \sqrt{15^2 + 150^2} = \sqrt{62725} \text{ mm}$$

$$\frac{BB'}{15} = \frac{200}{250}; \qquad BB' = 12 \text{ mm} \qquad \frac{EE'}{30} = \frac{50}{250}; \qquad EE' = 6 \text{ mm}$$

$$x' = 150 + EE' - BB' = 150 + 6 - 12 = 144 \text{ mm}$$

$$L_{BE} = \sqrt{150^2 + 150^2} = 150\sqrt{2} \text{ mm} \quad L_{B'E'} = \sqrt{144^2 + 150^2} = \sqrt{43236} \text{ mm}$$

**Average Normal and Shear Strain:** Since no deformation occurs along $x$ axis,

$$(\epsilon_x)_A = 0 \qquad \textbf{Ans.}$$

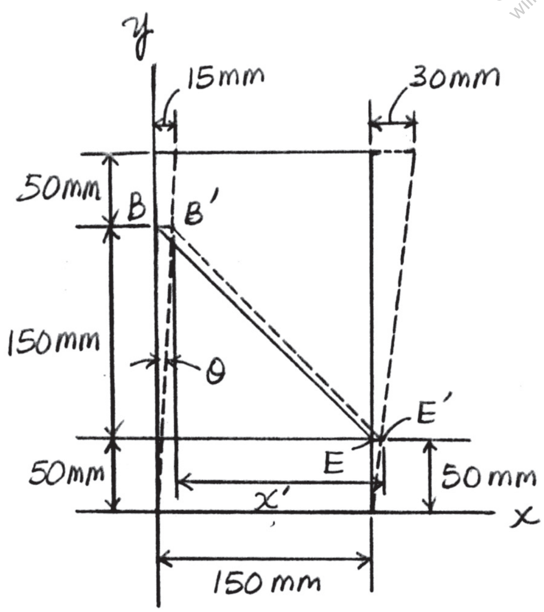
$$(\epsilon_y)_A = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{62725} - 250}{250} = 1.80(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$

By definition,

$$(\gamma_{xy})_A = \theta = 0.0599 \text{ rad} \qquad \textbf{Ans.}$$

$$\epsilon_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{43236} - 150\sqrt{2}}{150\sqrt{2}} = -0.0198 \text{ mm/mm} \qquad \textbf{Ans.}$$



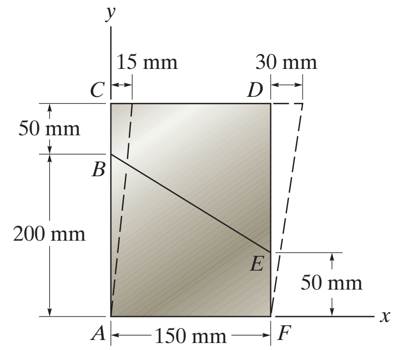



(a)

**Ans:**
$(\epsilon_x)_A = 0$
$(\epsilon_y)_A = 1.80(10^{-3})$ mm/mm
$(\gamma_{xy})_A = 0.0599$ rad
$\epsilon_{BE} = -0.0198$ mm/mm

**2–13.**

The material distorts into the dashed position shown. Determine the average normal strains along the diagonals *AD* and *CF*.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. *a*,

$$L_{AD} = L_{CF} = \sqrt{150^2 + 250^2} = \sqrt{85000} \text{ mm}$$

$$L_{AD'} = \sqrt{(150 + 30)^2 + 250^2} = \sqrt{94900} \text{ mm}$$

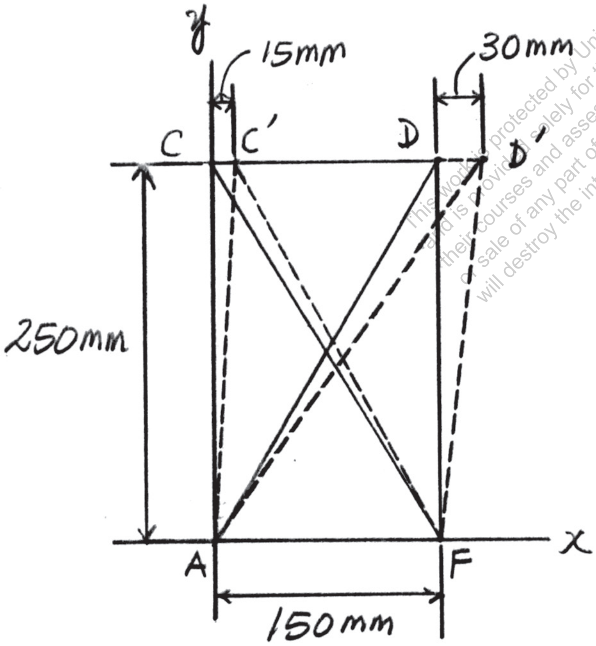
$$L_{C'F} = \sqrt{(150 - 15)^2 + 250^2} = \sqrt{80725} \text{ mm}$$

**Average Normal Strain:**

$$\epsilon_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{94900} - \sqrt{85000}}{\sqrt{85000}} = 0.0566 \text{ mm/mm}$$ **Ans.**

$$\epsilon_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{80725} - \sqrt{85000}}{\sqrt{85000}} = -0.0255 \text{ mm/mm}$$ **Ans.**



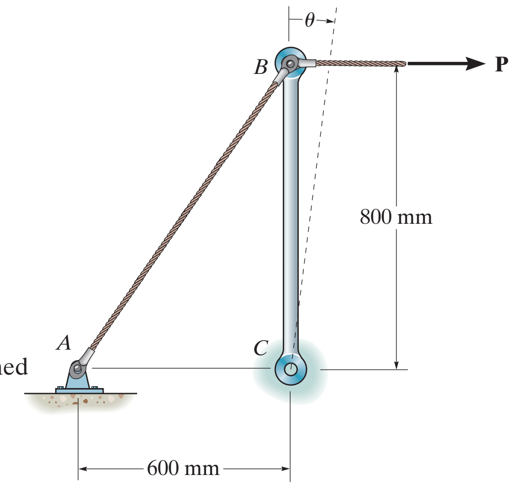



(a)

**Ans:**
$\epsilon_{AD} = 0.0566 \text{ mm/mm}$
$\epsilon_{CF} = -0.0255 \text{ mm/mm}$

**2–14.**

Part of a control linkage for an airplane consists of a rigid member $CB$ and a flexible cable $AB$. If a force is applied to the end $B$ of the member and causes it to rotate by $\theta = 0.5°$, determine the normal strain in the cable. Originally the cable is unstretched.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. $a$, the unstretched and stretched lengths of cable $AB$ are

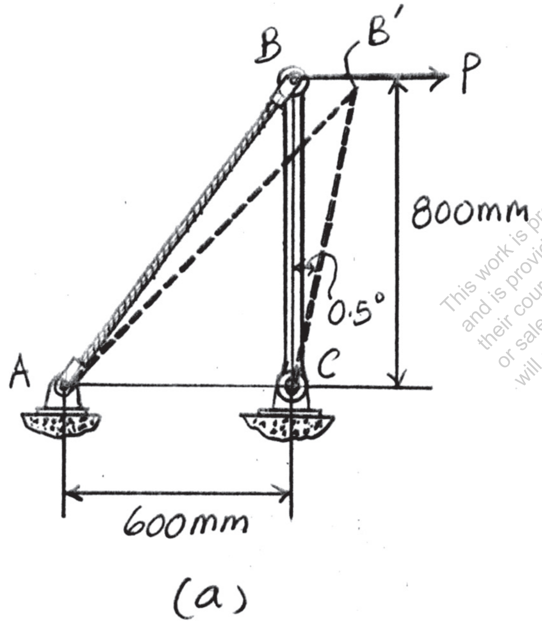
$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos 90.5°} = 1004.18 \text{ mm}$$

**Average Normal Strain:**

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{1004.18 - 1000}{1000} = 0.00418 \text{ mm/mm}$$ **Ans.**

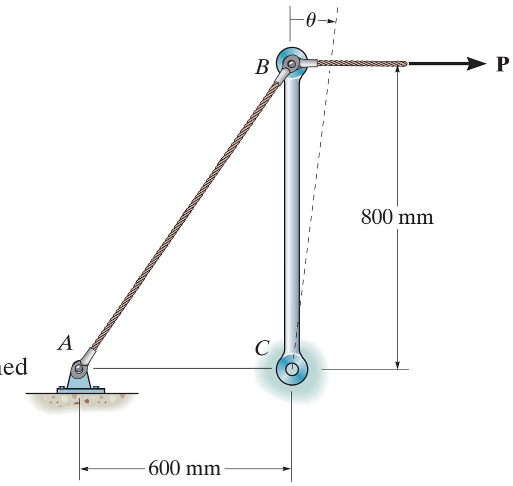





**Ans:**
$\epsilon_{AB} = 0.00418 \text{ mm/mm}$

**2–15.**

Part of a control linkage for an airplane consists of a rigid member $CB$ and a flexible cable $AB$. If a force is applied to the end $B$ of the member and causes a normal strain in the cable of 0.004 mm/mm, determine the displacement of point $B$. Originally the cable is unstretched.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. $a$, the unstretched and stretched lengths of cable $AB$ are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos(90° + \theta)}$$

$$L_{AB'} = \sqrt{1(10^6) - 0.960(10^6)\cos(90° + \theta)}$$

**Average Normal Strain:**

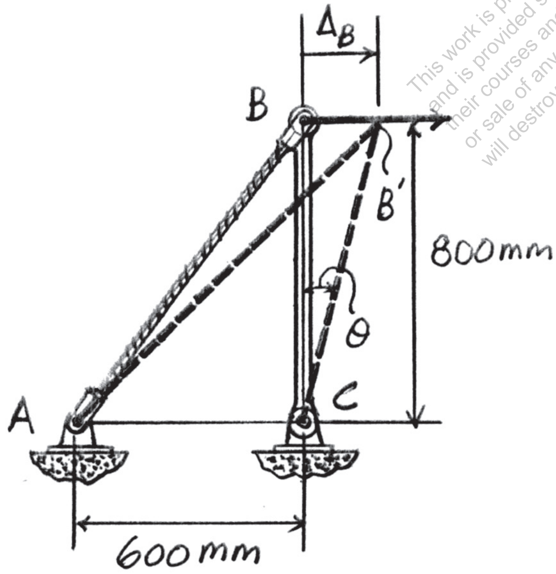
$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}}; \quad 0.004 = \frac{\sqrt{1(10^6) - 0.960(10^6)\cos(90° + \theta)} - 1000}{1000}$$

$$\theta = 0.4784°\left(\frac{\pi}{180°}\right) = 0.008350 \text{ rad}$$

Thus,

$$\Delta_B = \theta L_{BC} = 0.008350(800) = 6.68 \text{ mm} \qquad \textbf{Ans.}$$



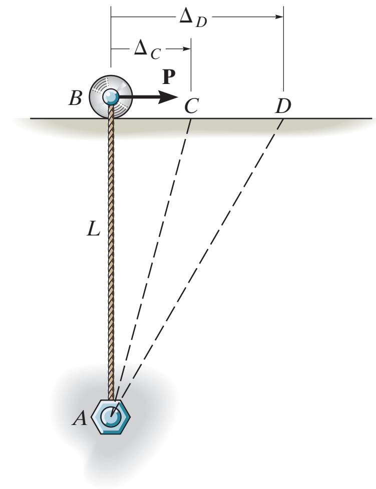



(a)

**Ans:**
$\Delta_B = 6.68$ mm

***2–16.**

The nylon cord has an original length $L$ and is tied to a bolt at $A$ and a roller at $B$. If a force $\mathbf{P}$ is applied to the roller, determine the normal strain in the cord when the roller is at $C$, and at $D$. If the cord is originally unstrained when it is at $C$, determine the normal strain $\epsilon'_D$ when the roller moves to $D$. Show that if the displacements $\Delta_C$ and $\Delta_D$ are small, then $\epsilon'_D = \epsilon_D - \epsilon_C$.




## SOLUTION

$$L_C = \sqrt{L^2 + \Delta_C^2}$$

$$\epsilon_C = \frac{\sqrt{L^2 + \Delta_C^2} - L}{L}$$

$$= \frac{L\sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - L}{L} = \sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - 1$$

For small $\Delta_C$,

$$\epsilon_C = 1 + \frac{1}{2}\left(\frac{\Delta_C^2}{L^2}\right) - 1 = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$ **Ans.**

In the same manner,

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$ **Ans.**

$$\epsilon_D{}' = \frac{\sqrt{L^2 + \Delta_D^2} - \sqrt{L^2 + \Delta_C^2}}{\sqrt{L^2 + \Delta_C^2}} = \frac{\sqrt{1 + \frac{\Delta_D^2}{L^2}} - \sqrt{1 + \frac{\Delta_C^2}{L^2}}}{\sqrt{1 + \frac{\Delta_C^2}{L^2}}}$$

For small $\Delta_C$ and $\Delta_D$,

$$\epsilon_D{}' = \frac{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right) - \left(1 + \frac{1}{2}\frac{\Delta_D^2}{L^2}\right)}{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right)} = \frac{\frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right)}{\frac{1}{2L^2}\left(2L^2 + \Delta_C^2\right)}$$

$$\epsilon_D{}' = \frac{\Delta_C^2 - \Delta_D^2}{2L^2 - \Delta_C^2} = \frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right) = \epsilon_C - \epsilon_D$$ **QED**



Also this problem can be solved as follows:

$$A_C = L \sec \theta_C\,; \qquad A_D = L \sec \theta_D$$

$$\epsilon_C = \frac{L \sec \theta_C - L}{L} = \sec \theta_C - 1$$

$$\epsilon_D = \frac{L \sec \theta_D - L}{L} = \sec \theta_D - 1$$

Expanding $\sec \theta$

$$\sec \theta = 1 + \frac{\theta^2}{2\,!} + \frac{5\,\theta^4}{4\,!} \ldots\ldots$$

***2–16. Continued**

For small $\theta$ neglect the higher order terms

$$\sec\theta = 1 + \frac{\theta^2}{2}$$

Hence,

$$\epsilon_C = 1 + \frac{{\theta_C}^2}{2} - 1 = \frac{{\theta_C}^2}{2}$$

$$\epsilon_D = 1 + \frac{{\theta_D}^2}{2} - 1 = \frac{{\theta_D}^2}{2}$$

$$\epsilon_D' = \frac{L\sec\theta_D - L\sec\theta_C}{L\sec\theta_C} = \frac{\sec\theta_D}{\sec\theta_C} - 1 = \sec\theta_D\cos\theta_C - 1$$

$$\text{Since } \cos\theta = 1 - \frac{\theta^2}{2\,!} + \frac{\theta^4}{4\,!}\ldots\ldots$$

$$\sec\theta_D\cos\theta_C = \left(1 + \frac{{\theta_D}^2}{2}\ldots\ldots\right)\left(1 - \frac{{\theta_C}^2}{2}\ldots\ldots\right)$$

$$= 1 - \frac{\theta_C^2}{2} + \frac{\theta_D^2}{2} - \frac{\theta_C^2\,\theta_D^2}{4}$$

Neglecting the higher order terms

$$\sec\theta_D\cos\theta_C = 1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$\epsilon_D' = \left[1 + \frac{{\theta_2}^2}{2} - \frac{{\theta_1}^2}{2}\right] - 1 = \frac{{\theta_D}^2}{2} - \frac{{\theta_C}^2}{2}$$

$$= \epsilon_D - \epsilon_C$$

**QED**



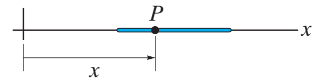
**Ans:**

$$\epsilon_C = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$

**2–17.**

A thin wire, lying along the $x$ axis, is strained such that each point on the wire is displaced $\Delta x = kx^2$ along the $x$ axis. If $k$ is constant, what is the normal strain at any point $P$ along the wire?




## SOLUTION

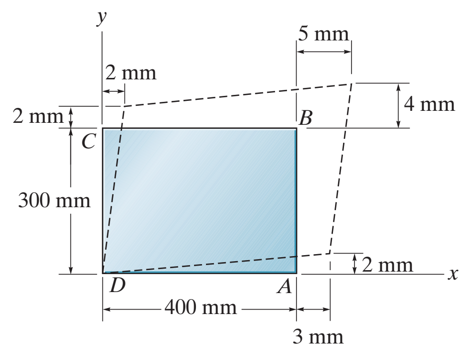
$$\epsilon = \frac{d(\Delta x)}{dx} = 2\,k\,x$$ **Ans.**



**Ans:**
$\epsilon = 2kx$

**2–18.**

Determine the shear strain $\gamma_{xy}$ at corners $A$ and $B$ if the plate distorts as shown by the dashed lines.

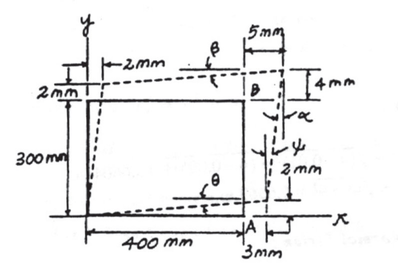



## SOLUTION

**Geometry:** For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$




**Shear Strain:**

$$(\gamma_B)_{xy} = \alpha + \beta$$
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad}$$ **Ans.**

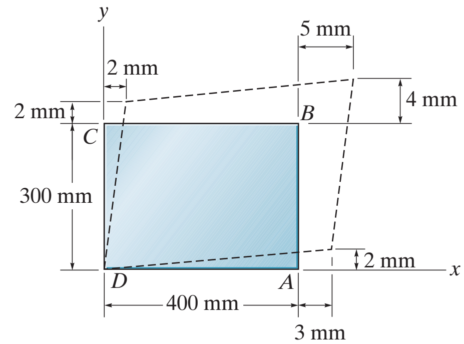
$$(\gamma_A)_{xy} = \theta + \psi$$
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad}$$ **Ans.**



**Ans:**
$(\gamma_B)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_A)_{xy} = 11.6(10^{-3})$ rad

**2–19.**

Determine the shear strain $\gamma_{xy}$ at corners $D$ and $C$ if the plate distorts as shown by the dashed lines.

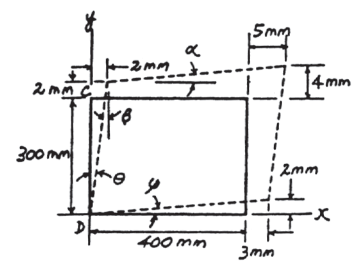



## SOLUTION

**Geometry:** For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$




**Shear Strain:**

$$(\gamma_C)_{xy} = \alpha + \beta$$
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad}$$ **Ans.**

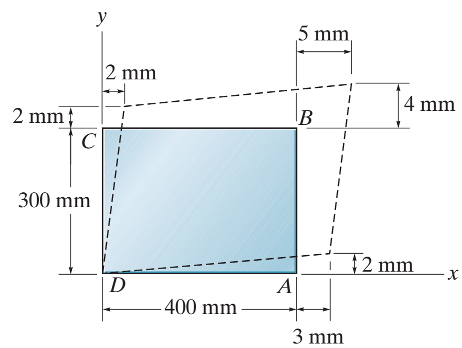
$$(\gamma_D)_{xy} = \theta + \psi$$
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad}$$ **Ans.**



**Ans:**
$(\gamma_C)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_D)_{xy} = 11.6(10^{-3})$ rad

***2–20.**

Determine the average normal strain that occurs along the diagonals $AC$ and $DB$.



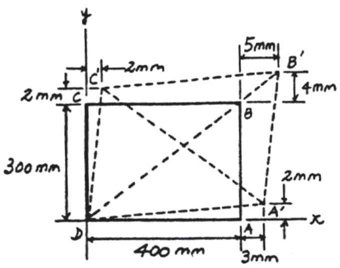

## SOLUTION

**Geometry:**

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$




**Average Normal Strain:**

$$\epsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

$$\epsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

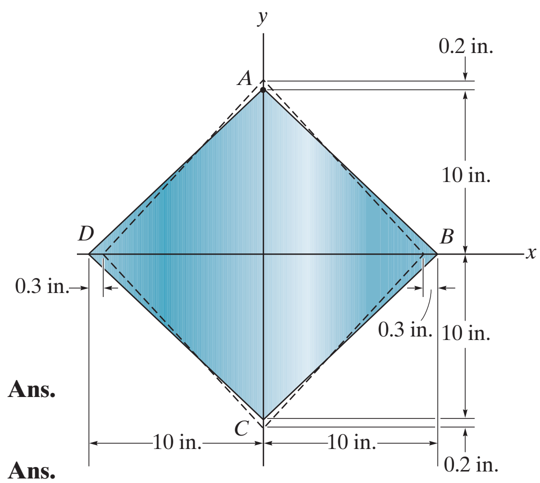
$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**



**Ans:**
$\epsilon_{AC} = 1.60\left(10^{-3}\right)$ mm/mm
$\epsilon_{DB} = 12.8\left(10^{-3}\right)$ mm/mm

**2–21.**

The corners of the square plate are given the displacements indicated. Determine the average normal strains $\epsilon_x$ and $\epsilon_y$ along the $x$ and $y$ axes.




## SOLUTION

$$\epsilon_x = \frac{-0.3}{10} = -0.03 \text{ in./in.}$$ **Ans.**

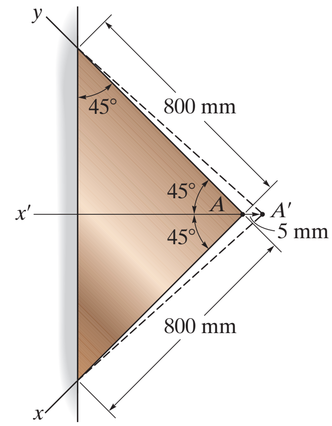
$$\epsilon_y = \frac{0.2}{10} = 0.02 \text{ in./in.}$$ **Ans.**



**Ans:**
$\epsilon_x = -0.03$ in./in.
$\epsilon_y = 0.02$ in./in.

**2–22.**

The triangular plate is fixed at its base, and its apex $A$ is given a horizontal displacement of 5 mm. Determine the shear strain, $\gamma_{xy}$, at $A$.



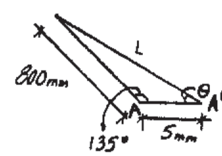

## SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\frac{\sin 135^\circ}{803.54} = \frac{\sin\theta}{800}; \qquad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

$$= 0.00880 \text{ rad}$$ **Ans.**

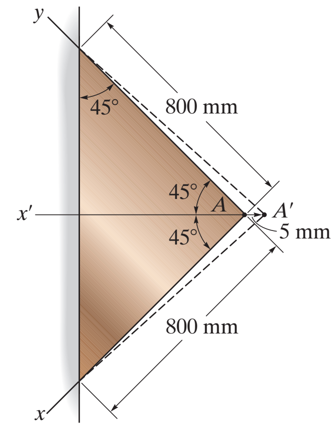





**Ans:**
$\gamma_{xy} = 0.00880$ rad

**2–23.**

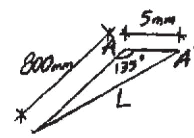
The triangular plate is fixed at its base, and its apex $A$ is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_x$ along the $x$ axis.




## SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm}$$ **Ans.**

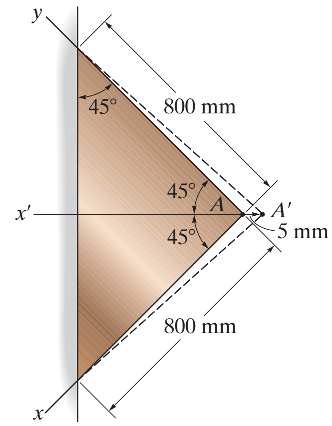





**Ans:**
$\epsilon_x = 0.00443 \text{ mm/mm}$

***2–24.**

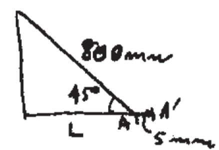
The triangular plate is fixed at its base, and its apex $A$ is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the $x'$ axis.




## SOLUTION

$L = 800 \cos 45° = 565.69$ mm

$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm}$$ **Ans.**

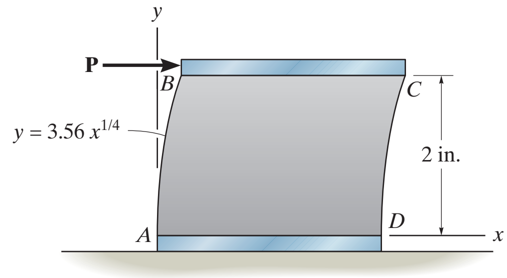





**Ans:**
$\epsilon_{x'} = 0.00884$ mm/mm

**2–25.**

The polysulfone block is glued at its top and bottom to the rigid plates. If a tangential force, applied to the top plate, causes the material to deform so that its sides are described by the equation $y = 3.56\,x^{1/4}$, determine the shear strain at the corners $A$ and $B$.




## SOLUTION

$$y = 3.56\,x^{1/4}$$

$$\frac{dy}{dx} = 0.890\,x^{-3/4}$$

$$\frac{dx}{dy} = 1.123\,x^{3/4}$$

At $A$, $x = 0$

$$\gamma_A = \frac{dx}{dy} = 0$$ **Ans.**

At $B$,

$$2 = 3.56\,x^{1/4}$$

$$x = 0.0996 \text{ in.}$$

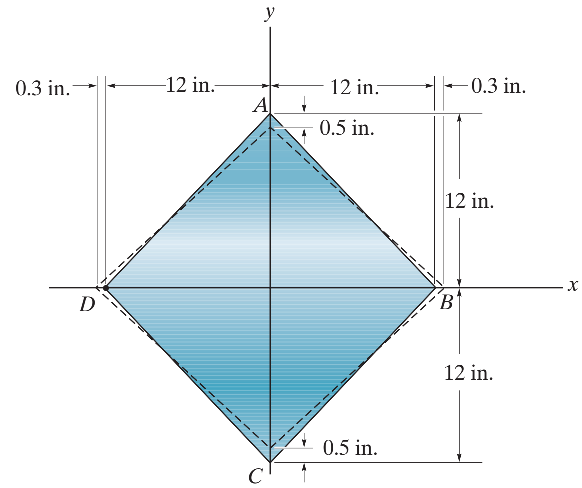
$$\gamma_B = \frac{dx}{dy} = 1.123(0.0996)^{3/4} = 0.199 \text{ rad}$$ **Ans.**



**Ans:**
$\gamma_A = 0$
$\gamma_B = 0.199$ rad

**2–26.**

The corners of the square plate are given the displacements indicated. Determine the shear strain at $A$ relative to axes that are directed along $AB$ and $AD$, and the shear strain at $B$ relative to axes that are directed along $BC$ and $BA$.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. *a*,

$$\tan\frac{\theta}{2} = \frac{12.3}{11.5} \qquad \theta = (93.85°)\left(\frac{\pi}{180°}\text{ rad}\right) = 1.6380 \text{ rad}$$

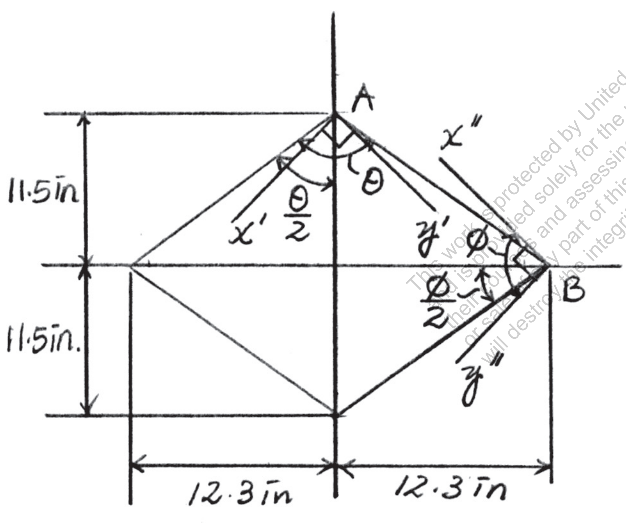
$$\tan\frac{\phi}{2} = \frac{11.5}{12.3} \qquad \phi = (86.15°)\left(\frac{\pi}{180°}\text{ rad}\right) = 1.5036 \text{ rad}$$

**Shear Strain:** By definition,

$$(\gamma_{x'y'})_A = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.6380 = -0.0672 \text{ rad}$$ **Ans.**

$$(\gamma_{x''y''})_B = \frac{\pi}{2} - \phi = \frac{\pi}{2} - 1.5036 = 0.0672 \text{ rad}$$ **Ans.**



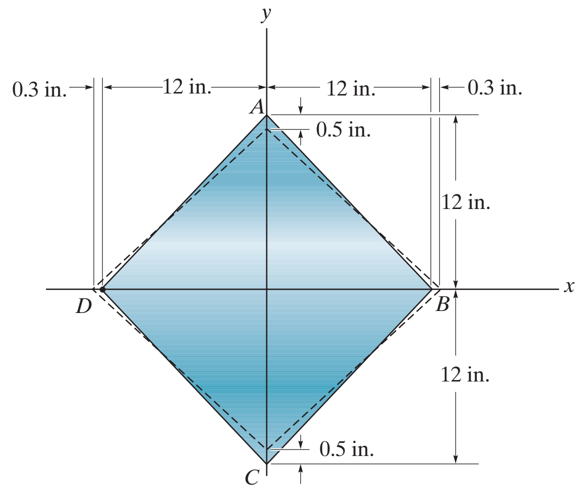



(a)

**Ans:**
$(\gamma_{x'y'})_A = -0.0672$ rad
$(\gamma_{x''y''})_B = 0.0672$ rad

**2–27.**

The corners of the square plate are given the displacements indicated. Determine the average normal strains along side $AB$ and diagonals $AC$ and $BD$.




## SOLUTION

**Geometry:** Referring to the geometry shown in Fig. *a*,

$$L_{AB} = \sqrt{12^2 + 12^2} = 12\sqrt{2}\ \text{in.}$$

$$L_{A'B'} = \sqrt{12.3^2 + 11.5^2} = \sqrt{283.54}\ \text{in.}$$

$$L_{BD} = 2(12) = 24\ \text{in.}$$

$$L_{B'D'} = 2(12 + 0.3) = 24.6\ \text{in.}$$

$$L_{AC} = 2(12) = 24\ \text{in.}$$

$$L_{A'C'} = 2(12 - 0.5) = 23\ \text{in.}$$

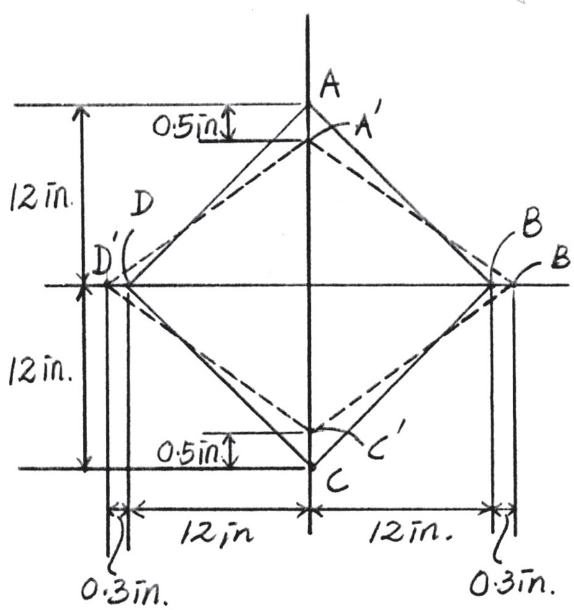
**Average Normal Strain:**

$$\epsilon_{AB} = \frac{L_{A'B'} - L_{AB}}{L_{AB}} = \frac{\sqrt{283.54} - 12\sqrt{2}}{12\sqrt{2}} = -7.77(10^{-3})\ \text{in./in.} \quad \textbf{Ans.}$$

$$\epsilon_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{24.6 - 24}{24} = 0.025\ \text{in./in.} \quad \textbf{Ans.}$$

$$\epsilon_{AC} = \frac{L_{A'C'} - L_{AC}}{L_{AC}} = \frac{23 - 24}{24} = -0.0417\ \text{in./in.} \quad \textbf{Ans.}$$



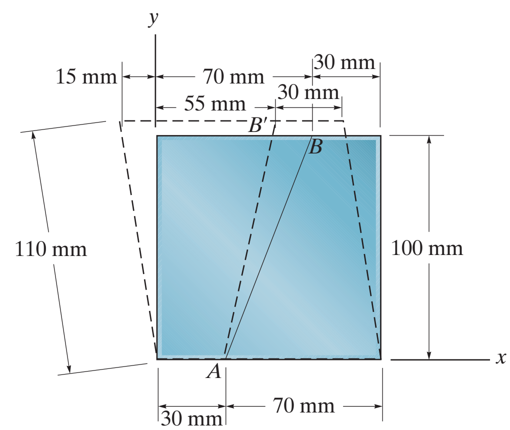



(a)

**Ans:**
$\epsilon_{AB} = -7.77(10^{-3})$ in./in.
$\epsilon_{BD} = 0.025$ in./in.
$\epsilon_{AC} = -0.0417$ in./in.

***2–28.**

The block is deformed into the position shown by the dashed lines. Determine the average normal strain along line $AB$.



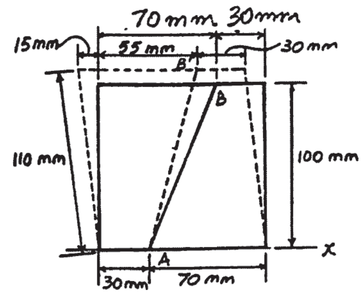

## SOLUTION

**Geometry:**

$$AB = \sqrt{100^2 + (70 - 30)^2} = 107.7033 \text{ mm}$$

$$AB' = \sqrt{(70 - 30 - 15)^2 + \left(110^2 - 15^2\right)} = 111.8034 \text{ mm}$$

**Average Normal Strain:**




$$\epsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{111.8034 - 107.7033}{107.7033}$$

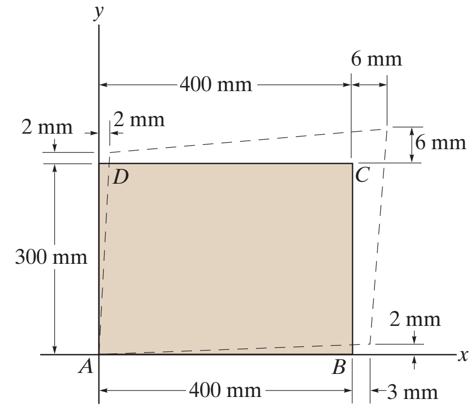
$$= 0.0381 \text{ mm/mm} = 38.1\left(10^{-3}\right) \text{ mm}$$ **Ans.**



**Ans:**
$\epsilon_{AB} = 38.1\left(10^{-3}\right)$ mm

**2–29.**

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal $AC$, and the average shear strain at corner $A$ relative to the $x$, $y$ axes.




## SOLUTION

**Geometry:** The unstretched length of diagonal $AC$ is

$$L_{AC} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. $a$, the stretched length of diagonal $AC$ is

$$L_{AC'} = \sqrt{(400 + 6)^2 + (300 + 6)^2} = 508.4014 \text{ mm}$$

Referring to Fig. $a$ and using small angle analysis,

$$\phi = \frac{2}{300 + 2} = 0.006623 \text{ rad}$$

$$\alpha = \frac{2}{400 + 3} = 0.004963 \text{ rad}$$

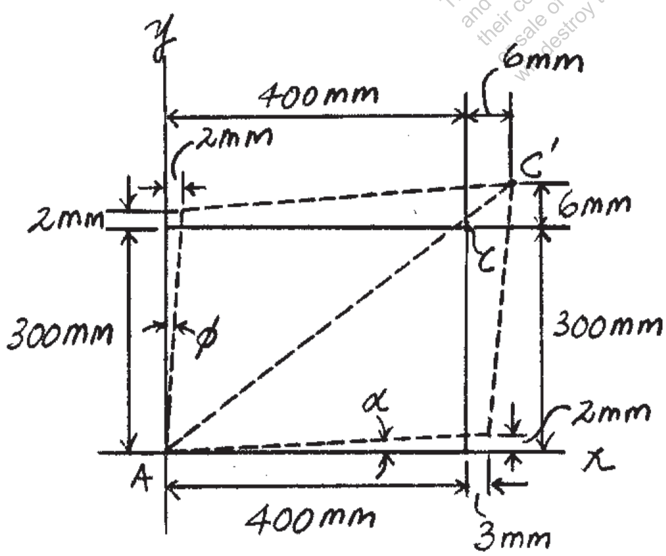
**Average Normal Strain:** Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{508.4014 - 500}{500} = 0.0168 \text{ mm/mm} \quad \textbf{Ans.}$$

**Shear Strain:** Referring to Fig. $a$,

$$(\gamma_A)_{xy} = \phi + \alpha = 0.006623 + 0.004963 = 0.0116 \text{ rad} \quad \textbf{Ans.}$$



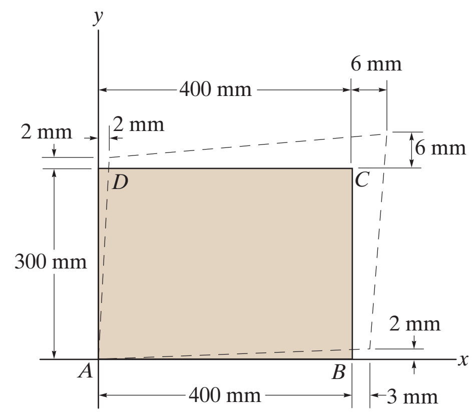



(a)

**Ans:**
$(\epsilon_{\text{avg}})_{AC} = 0.0168 \text{ mm/mm}$, $(\gamma_A)_{xy} = 0.0116$ rad

**2–30.**

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal $BD$, and the average shear strain at corner $B$ relative to the $x$, $y$ axes.




## SOLUTION

**Geometry:** The unstretched length of diagonal $BD$ is

$$L_{BD} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. $a$, the stretched length of diagonal $BD$ is

$$L_{B'D'} = \sqrt{(300 + 2 - 2)^2 + (400 + 3 - 2)^2} = 500.8004 \text{ mm}$$

Referring to Fig. $a$ and using small angle analysis,

$$\phi = \frac{2}{403} = 0.004963 \text{ rad}$$

$$\alpha = \frac{3}{300 + 6 - 2} = 0.009868 \text{ rad}$$

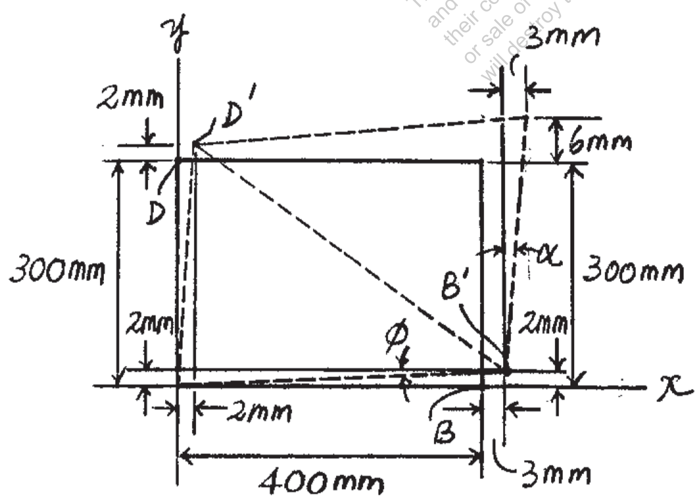
**Average Normal Strain:** Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{500.8004 - 500}{500} = 1.60(10^{-3}) \text{ mm/mm} \quad \textbf{Ans.}$$

**Shear Strain:** Referring to Fig. $a$,

$$(\gamma_B)_{xy} = \phi + \alpha = 0.004963 + 0.009868 = 0.0148 \text{ rad} \quad \textbf{Ans.}$$



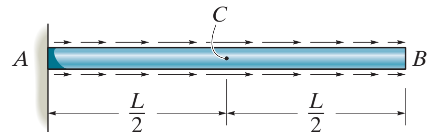



(a)

**Ans:**
$(\epsilon_{\text{avg}})_{BD} = 1.60(10^{-3})$ mm/mm,
$(\gamma_B)_{xy} = 0.0148$ rad

**2–31.**

The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$, where $k$ is a constant. Determine the displacement of the center $C$ and the average normal strain in the entire rod.




## SOLUTION

$$\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$$

$$(\Delta x)_C = \int_0^{L/2} \epsilon_x \, dx = \int_0^{L/2} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right)\Big|_0^{L/2} = -k\left(\frac{L}{\pi}\right)\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= \frac{kL}{\pi}$$ **Ans.**

$$(\Delta x)_B = \int_0^{L} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right)\Big|_0^{L} = -k\left(\frac{L}{\pi}\right)(\cos \pi - \cos 0) = \frac{2kL}{\pi}$$

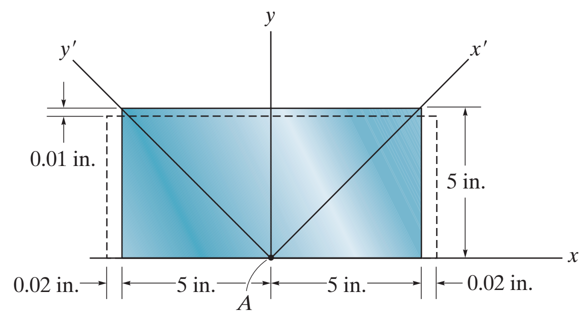
$$\epsilon_{avg} = \frac{(\Delta x)_B}{L} = \frac{2k}{\pi}$$ **Ans.**



**Ans:**
$(\Delta x)_C = \frac{kL}{\pi}$
$\epsilon_{avg} = \frac{2k}{\pi}$

***2–32.**

The rectangular plate undergoes a deformation shown by the dashed lines. Determine the shear strain $\gamma_{xy}$ and $\gamma_{x'y'}$ at point $A$.

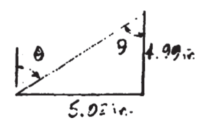



## SOLUTION

Since the right angle of an element along the $x$,$y$ axes does not distort, then

$\gamma_{xy} = 0$ **Ans.**

$$\tan\theta = \frac{5.02}{4.99}$$




$$\theta = 45.17^\circ = 0.7884 \text{ rad}$$

$$\gamma_{x'y'} = \frac{\pi}{2} - 2\theta$$

$$= \frac{\pi}{2} - 2(0.7884)$$

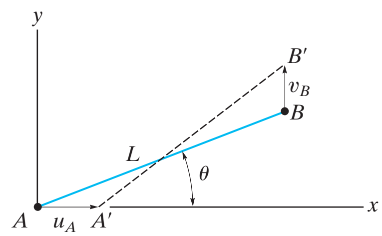
$$= -0.00599 \text{ rad}$$ **Ans.**



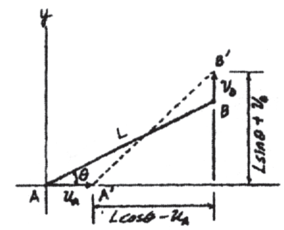
**Ans:**
$\gamma_{xy} = 0$
$\gamma_{x'y'} = -0.00599$ rad

**2–33.**

The fiber $AB$ has a length $L$ and orientation $\theta$. If its ends $A$ and $B$ undergo very small displacements $u_A$ and $v_B$ respectively, determine the normal strain in the fiber when it is in position $A'B'$.




## SOLUTION



**Geometry:**

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

**Average Normal Strain:**

$$\epsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

Neglecting higher terms $u_A^2$ and $v_B^2$

$$\epsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\epsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots -1$$

$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L} \qquad \textbf{Ans.}$$



**Ans:**

$$\epsilon_{AB} = \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$

**2–34.**

If the normal strain is defined in reference to the final length $\Delta s'$, that is,

$$\epsilon' = \lim_{\Delta s' \to 0} \left( \frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon - \epsilon' = \epsilon \epsilon'$.

## SOLUTION

$$\epsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon - \epsilon' = \frac{\Delta s' - \Delta s}{\Delta s} - \frac{\Delta s' - \Delta s}{\Delta s'}$$

$$= \frac{\Delta s'^2 - \Delta s \Delta s' - \Delta s' \Delta s + \Delta s^2}{\Delta s \Delta s'}$$

$$= \frac{\Delta s'^2 + \Delta s^2 - 2\Delta s' \Delta s}{\Delta s \Delta s'}$$

$$= \frac{(\Delta s' - \Delta s)^2}{\Delta s \Delta s'} = \left( \frac{\Delta s' - \Delta s}{\Delta s} \right)\left( \frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

$$= \epsilon \epsilon'$$ **(Q.E.D)**



**Ans:**
N/A