

CHAPTER 2

EXERCISE 2.1

Part A

1. $S = 100(1.055)^5 = \$130.70$, Int. = \$30.70
2. $S = 500\left(1 + \frac{0.03}{12}\right)^{24} = \530.88 , Int. = \$30.88
3. $S = 220\left(1 + \frac{0.088}{4}\right)^{12} = \285.65 , Int. = \$65.65
4. $S = 1000(1.045)^{12} = \$1695.88$, Int. = \$695.88
5. $S = 50(1.005)^{48} = \$63.52$, Int. = \$13.52
6. $S = 800(1.0775)^{10} = \$1687.57$, Int. = \$887.57
7. $S = 300\left(1 + \frac{0.08}{52}\right)^{156} = \381.30 , Int. = \$81.30
8. $S = 1000\left(1 + \frac{0.045}{365}\right)^{730} = \1094.17 , Int. = \$94.17
9. a) $S = 500\left(1 + \frac{0.04}{12}\right)^{12} = \520.37
 b) $S = 500\left(1 + \frac{0.08}{12}\right)^{12} = \541.50
 c) $S = 500\left(1 + \frac{0.10}{12}\right)^{12} = \563.41
10. $S = 2000(1.0175)^{12} = \2462.88
11. a) $S = 100(1.08)^5 = \$146.93$
 b) $S = 100(1.04)^{10} = \$148.02$
 c) $S = 100(1.02)^{20} = \$148.59$
 d) $S = 100\left(1 + \frac{0.08}{12}\right)^{60} = \148.98
 e) $S = 100\left(1 + \frac{0.12}{365}\right)^{1825} = \149.18
12. $S = 1000(1.005)^{216} = \2936.77
13. a) $S = 10\,000(1.03)^{522} = 5.02379 \times 10^{10} = \50.2379 billion
 b) $S = 10\,000[1 + (0.03)(522)] = \$166\,600$

14. a) $S = 1000(1.06136)^1 = \$1061.36$

b) $S = 1000(1.030225)^2 = \$1061.36$

c) $S = 1000(1.015)^4 = \$1061.36$

d) $S = 1000(1.004975)^{12} = \1061.36

15. At the end of 5 years $= 8000 \left(1 + \frac{0.035}{2}\right)^{10} = \9515.56

At the end of 6 years $= 9515.56 \left(1 + \frac{0.04}{2}\right)^2 = \9899.99

Interest earned $= \$384.43$

EXERCISE 2.1

Part B

1. a) $S = 100 \left(1 + \frac{0.06}{365}\right)^{365} = \1061.83 , Interest $= \$61.83$

b)	Period	Interest
	January 1-June 30	$1000 \times 0.06 \times \frac{181}{365} = \29.75
	July 1 - December 31	$1029.75 \times 0.06 \times \frac{184}{365} = \31.15
	Total interest earned	$= \$60.90$

c)	Period	Interest
	January	$1000 \times 0.06 \times \frac{31}{365} = \5.10
	February	$1005.10 \times 0.06 \times \frac{28}{365} = \4.63
	March	$1009.73 \times 0.06 \times \frac{31}{365} = \5.15
	April	$1014.88 \times 0.06 \times \frac{30}{365} = \5.00
	May	$1019.88 \times 0.06 \times \frac{31}{365} = \5.20
	June	$1025.08 \times 0.06 \times \frac{30}{365} = \5.06
	July	$1030.14 \times 0.06 \times \frac{31}{365} = \5.25
	August	$1035.39 \times 0.06 \times \frac{31}{365} = \5.28
	September	$1040.67 \times 0.06 \times \frac{30}{365} = \5.13
	October	$1045.80 \times 0.06 \times \frac{31}{365} = \5.33
	November	$1051.13 \times 0.06 \times \frac{30}{365} = \5.18
	December	$1056.31 \times 0.06 \times \frac{31}{365} = \5.38
	Total interest earned	$= \$61.69$

2.

Growth of \$1000

Years	n	$j_{365} = 4\%$	$j_{365} = 7\%$	$j_{365} = 10\%$
5	1825	1221.39	1419.02	1648.61
10	3650	1491.79	2013.62	2717.91
15	5475	1822.06	2857.36	4480.77
20	7300	2225.44	4054.66	7387.03
25	9125	2718.13	5753.63	12 178.32

3.

m	i	n	S	Interest
1	0.054	10	16 920.22	6920.22
2	0.027	20	17 037.62	7037.62
4	0.0135	40	17 098.19	7098.19
12	0.0045	120	17 139.29	7139.29
52	$\frac{0.054}{52}$	520	17 155.26	7155.26
365	$\frac{0.054}{365}$	3650	17 159.38	7159.38

EXERCISE 2.2

Part A

1. a) $j = (1.035)^2 - 1 = 0.071225 = 7.12\%$
b) $j = \left(1 + \frac{0.03}{4}\right)^4 - 1 = 0.030339191 = 3.03\%$
c) $j = (1.02)^4 - 1 = 0.08243216 = 8.24\%$
d) $j = \left(1 + \frac{0.12}{365}\right)^{365} - 1 = 0.127474614 = 12.75\%$
e) $j = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 0.093806897 = 9.38\%$

2. a) $(1 + i)^2 = 1.06 \rightarrow i = (1.06)^{1/2} - 1$
 $j_2 = 2[(1.06)^{1/2} - 1] = 5.91\%$
b) $(1 + i)^4 = 1.09 \rightarrow i = (1.09)^{1/4} - 1$
 $j_4 = 4[(1.09)^{1/4} - 1] = 8.71\%$
c) $(1 + i)^{12} = 1.10 \rightarrow i = (1.10)^{1/12} - 1$
 $j_{12} = 12[(1.10)^{1/12} - 1] = 9.57\%$
d) $(1 + i)^{365} = 1.17 \rightarrow i = (1.17)^{1/365} - 1$
 $j_{365} = 365[(1.17)^{1/365} - 1] = 15.70\%$
e) $(1 + i)^{52} = 1.045 \rightarrow i = (1.045)^{1/52} - 1$
 $j_{52} = 52[(1.045)^{1/52} - 1] = 4.40\%$

3. a) $(1 + i)^4 = (1.04)^2 \rightarrow i = (1.04)^{1/2} - 1$
 $j_4 = 4[(1.04)^{1/2} - 1] = 7.92\%$
b) $(1 + i)^2 = (1.05)^4 \rightarrow i = (1.05)^2 - 1$
 $j_2 = 2[(1.05)^2 - 1] = 6.05\%$
c) $[(1 + i)^4 = \left(1 + \frac{0.18}{12}\right)^2 \rightarrow i = (1.015)^3 - 1$
 $j_4 = 4[(1.015)^3 - 1] = 18.27\%$
d) $(1 + i)^{12} = \left(1 + \frac{0.1}{6}\right)^6 \rightarrow i = \left(1 + \frac{0.10}{6}\right)^{1/2} - 1$
 $j_{12} = 12\left[\left(1 + \frac{0.10}{6}\right)^{1/2} - 1\right] = 9.96\%$
e) $(1 + i)^2 = (1.02)^4 \rightarrow i = (1.02)^2 - 1$
 $j_2 = 2[(1.02)^2 - 1] = 8.08\%$
f) $(1 + i)^2 = \left(1 + \frac{0.04}{52}\right)^{52} \rightarrow i = \left(1 + \frac{0.04}{52}\right)^{26} - 1$
 $j_2 = 2\left[\left(1 + \frac{0.04}{52}\right)^{26} - 1\right] = 4.04\%$

$$\text{g) } (1+i)^{12} = \left(1 + \frac{0.0525}{2}\right)^2 \rightarrow i = \left(1 + \frac{0.0525}{2}\right)^{1/6} - 1$$

$$j_{12} = 12\left[\left(1 + \frac{0.0525}{2}\right)^{1/6} - 1\right] = 5.19\%$$

$$\text{h) } (1+i)^{365} = \left(1 + \frac{0.1279}{4}\right)^4 \rightarrow i = \left(1 + \frac{0.1279}{4}\right)^{4/365} - 1$$

$$j_{365} = 365\left[\left(1 + \frac{0.1279}{4}\right)^{4/365} - 1\right] = 12.59\%$$

$$4. \quad 1 + 2r = \left(1 + \frac{0.057}{12}\right)^{24} \rightarrow r = \frac{1}{2}\left[\left(1 + \frac{0.057}{12}\right)^{24} - 1\right] = 6.02\%$$

$$5. \quad 1 + 3r = \left(1 + \frac{0.08}{365}\right)^{1095} \rightarrow r = \frac{1}{3}\left[\left(1 + \frac{0.08}{365}\right)^{1095} - 1\right] = 9.04\%$$

$$6. \quad j = (1.0175)^{12} - 1 = 23.14\%$$

$$7. \quad j_2 = 4.9\% \rightarrow j = \left(1 + \frac{0.049}{2}\right)^2 - 1 = 4.96\%$$

$$j_1 = 5\% \rightarrow j = 5\%$$

Thus $j_1 = 5\%$ yields the higher annual effective rate of interest.

$$8. \quad \text{a) } j_{12} = 15\% \rightarrow j = \left(1 + \frac{0.15}{12}\right)^{12} - 1 = 16.08\%$$

$$j_2 = 15\frac{1}{2}\% \rightarrow j = \left(1 + \frac{0.155}{2}\right)^2 - 1 = 16.10\% \quad \text{BEST}$$

$$j_{365} = 14.9\% \rightarrow j = \left(1 + \frac{0.149}{365}\right)^{365} - 1 = 16.06\% \quad \text{WORST}$$

$$\text{b) } j_{12} = 6\% \rightarrow j = (1.005)^{12} - 1 = 6.17\%$$

$$j_2 = 6\frac{1}{2}\% \rightarrow j = (1.0325)^2 - 1 = 6.61\% \quad \text{BEST}$$

$$j_{365} = 5.9\% \rightarrow j = \left(1 + \frac{0.059}{365}\right)^{365} - 1 = 6.08\% \quad \text{WORST}$$

$$9. \quad \text{Bank A : } j_1 = 0.10 \rightarrow \text{annual effective rate} = 0.10$$

$$\text{Bank B : } j_m = 0.0975 \rightarrow \text{annual effective rate} = j$$

$$\text{Calculate } m, \text{ such that } j = \left(1 + \frac{0.0975}{m}\right)^m - 1 \geq 0.10$$

$$\text{for } m = 2, j = 0.0998766$$

$$\text{for } m = 4, j = 0.1011231$$

The minimum frequency of compounding for Bank B is $m = 4$.

However, if Bank A offered 5% and Bank B offered 4.75%, $j_m = 4.75\%$

will never be equivalent to $j_1 = 5\%$, no matter what value of m is chosen.

EXERCISE 2.2

Part B

3. a) $m = 1: S = 20\,000(1.06)^5 = \$26\,764.51$
 $m = 2: S = 20\,000(1.03)^{10} = \$26\,878.33$
 $m = 4: S = 20\,000(1.015)^{20} = \$26\,937.10$
 $m = 12: S = 20\,000(1.005)^{60} = \$26\,977.00$
 $m = 365: S = 20\,000 \left(1 + \frac{0.06}{365}\right)^{1825} = \$26\,996.51$

b)

$j_m = 6\%$	$j = (1+i)^m$	$S = 20\,000(1+j)^5$
j_1	$j = (1.06)^1 - 1$	26 764.51
j_2	$j = (1.03)^2 - 1$	26 878.33
j_4	$j = (1.015)^4 - 1$	26 937.10
j_{12}	$j = (1.005)^{12} - 1$	26 977.00
j_{365}	$j = \left(1 + \frac{0.06}{365}\right)^{365} - 1$	26 996.51

c)

$j_m = 6\%$	$j_{12} = 12 \left[\left(1 + \frac{j_m}{m}\right)^{m/12} - 1 \right]$	$S = 20\,000 \left(1 + \frac{j_{12}}{12}\right)^{60}$
j_1	0.058410607	26 764.51
j_2	0.059263464	26 878.33
j_4	0.059702475	26 937.10
j_{12}	0.06	26 977.00
j_{365}	0.060145294	26 996.51

4. a) $(1+j)^2 = (1.06)^2 \rightarrow j = 6\%$
b) $(1+j)^3 = (1.06)^3(1.02) \rightarrow j = [(1.06)^3(1.02)]^{1/3} - 1 = 6.70\%$
c) $(1+j)^4 = (1.06)^4(1.02) \rightarrow j = [(1.06)^4(1.02)]^{1/4} - 1 = 6.53\%$
5. Annual effective yield = $[(1 + 0.0201)(0.995)]^4 - 1 = 6.14\%$
6. Let $j_2 = 2i$; Then at the present time:
 $100 = 51.50 + 51.50(1+i)^{-1}$
 $(1+i) = \frac{51.50}{48.50} \rightarrow i = 0.06185567 \rightarrow j_2 = 2i = 0.12371134 = 12.37\%$
7. a) Amount of interest during the n -th year = $P[1 + rn] - P[1 + r(n-1)]$
 $= P + Prn - P - Pr(n-1) = Pr$
Amount of principal at the beginning of the n -th year = $P[1 + r(n-1)]$
Annual effective rate interest = $\frac{Pr}{P[1 + r(n-1)]} = \frac{r}{1 + r(n-1)}$
- b) Amount of interest during the n -th year = $P(1+i)^n - P(1+i)^{n-1}$
 $= P(1+i)^{n-1}[(1+i) - 1]$
 $= P(1+i)^{n-1} i$
Amount of principal at the beginning of n -th year = $P(1+i)^{n-1}$
Annual effective rate of interest = $\frac{P(1+i)^{n-1} i}{P(1+i)^{n-1}} = i$

EXERCISE 2.3

Part A

1. $P = 100(1.015)^{-12} = \$83.64$

2. $P = 50(1 + \frac{0.085}{12})^{-24} = \42.21

3. $P = 2000(1.118)^{-10} = \655.56

4. $P = 500(1.05)^{-10} = \$306.96$

5. $P = 800(1 + \frac{0.05}{365})^{-1095} = \688.57

6. $P = 1000(1 + \frac{0.08}{4})^{-20} = \672.97

7. $P = 2000(1 + \frac{0.048}{12})^{-36} = \1732.27

8. $P = 250\,000(1 + \frac{0.065}{2})^{-20} = \$131\,867.81$

9. $P = 10\,000(1.03)^{-40} = \3065.57

10. $P = 2000(1 + \frac{0.055}{4})^{-18} = \1564.14

11. $P = 800(1.03)^{-24} = \$393.55$

12. Maturity value $S = 250(1 + \frac{0.09}{12})^{48} = \357.85

Proceeds $P = 357.85(1 + \frac{0.075}{4})^{-11} = \291.71

13. Maturity value $S = 1000(1.03)^{10} = \$1343.92$

Proceeds $P = 1343.92(1 + \frac{0.07}{4})^{-14} = \1054.12

14. Discounted value of the payment plan : $230\,000 + 200\,000(1 + \frac{0.04}{2})^{-10}$
 $= 230\,000 + 164\,069.66 = \$394\,069.66$

The payment scheme is cheaper by $400\,000 - 394\,069.66 = \$5\,930.34$

15. Total current value = $1000(1.045)^{20} + 600(1.045)^{-14}$
 $= 2411.71 + 323.98 = \$2735.69$

16. $P = 3000(1 + \frac{0.0575}{2})^{10}(1.05)^{-5} = \3120.85

EXERCISE 2.3

Part B

1. Maturity value $S = 2500 \left(1 + \frac{0.12}{12}\right)^{40} = \3722.16
 Financial Consultants pay: $3722.16 \left(1 + \frac{0.1325}{4}\right)^{-12} = \2517.45
 Financial Consultants receive: $3722.16(1.13)^{-3} = \$2579.64$
 Financial Consultants profit: $2579.64 - 2517.45 = \$62.19$

2. $S = 1000(1.075)^5 = \$1435.63$

m	i	n	P	Discount
1	0.06	5	1072.79	362.84
2	0.03	10	1068.24	367.39
4	0.015	20	1065.91	369.72
12	$\frac{0.06}{12}$	60	1064.34	371.29
52	$\frac{0.06}{52}$	260	1063.72	371.91
365	$\frac{0.06}{365}$	1825	1063.57	372.06

3. Net present value of proposal A :
 $95\,400(1.14)^{-1} + 39\,000(1.14)^{-2} + 12\,000(1.14)^{-3} - 80\,000$
 $= 83\,684.21 + 30\,009.23 + 8099.66 - 80\,000 = \$41\,793.10$
 Net present value of proposal B:
 $35\,000(1.14)^{-1} + 58\,000(1.14)^{-2} + 80\,000(1.14)^{-3} - 100\,000$
 $= 30\,701.75 + 44\,629.12 + 53\,997.72 - 100\,000 = \$29\,328.59$
 Select proposal A with higher net present value.

EXERCISE 2.4

Part A

1. a) $S = 100 \left(1 + \frac{0.065}{2}\right)^{11\frac{1}{6}} = \142.92
b) $S = 100 \left(1 + \frac{0.065}{2}\right)^{11} \left[1 + (0.065) \left(\frac{1}{12}\right)\right] = \142.93
2. a) $S = 800(1.01)^{18\frac{1}{3}} = \960.10
b) $S = 800(1.01)^{18} \left[1 + (0.04) \left(\frac{1}{12}\right)\right] = \960.11
3. a) $S = 5000 \left(1 + \frac{0.074}{2}\right)^{-17\frac{2}{3}} = \2631.55
b) $S = 5000 \left(1 + \frac{0.074}{2}\right)^{-18} \left[1 + (0.074) \left(\frac{2}{12}\right)\right] = \2631.94
4. a) $S = 280(1.0175)^{-3\frac{7}{12}} = \263.12
b) $S = 280(1.0175)^{-4} \left[1 + (0.0175) \left(\frac{5}{12}\right)\right] = \263.13
5. Maturity date is October 20, 2019.
Time = 22 interest periods less 8 days
 $P = 2000(1.03)^{-22} \left[1 + (0.12) \left(\frac{8}{365}\right)\right] = 1046.53$
6. $S = 1200(1.00525)^{38} \left[1 + (0.063) \left(\frac{11}{365}\right)\right] = \1466.97
7. $S = 4000(1.05)^{10} \left[1 + (0.10) \left(\frac{165}{365}\right)\right] = \6810.12
8. Maturity date is December 8, 2017.
Time = 7 interest periods less 60 days
 $P = 850 \left(1 + \frac{0.0525}{2}\right)^{-7} \left[1 + (0.0525) \left(\frac{60}{365}\right)\right] = \715.12
9. Maturity date is August 24, 2015:
 $S = 1200 \left(1 + \frac{0.0875}{12}\right)^{24} = \1428.59
Proceeds: $P = 1428.59 \left(1 + \frac{0.095}{4}\right)^{-5} \left[1 + (0.095) \left(\frac{25}{365}\right)\right] = \1278.66
Compound discount: $S - P = \$149.93$

EXERCISE 2.4

Part B

1. a) From the binomial theorem

$$(1 + i)^t = 1 + it + \binom{t}{2} i^2 + \dots$$

The 3rd term in the series will overshadow all the remaining terms.

If $0 < t < 1$ then $\binom{t}{2} i^2$ is negative

And $(1 + i)^t < 1 + it$

If $t > 1$ then $\binom{t}{2} i^2$ is positive

and $(1 + i)^t > 1 + it$

- c) For $0 < t < 1$: $P(1 + i)^k(1 + i)^t < P(1 + i)^k[1 + it]$

$$S(1 + i)^{-k}(1 + i)^t < S(1 + i)^{-k}[1 + it]$$

2. $(1 - k)(1 + i)^n + k(1 + i)^{n+1} = (1 - k)(1 + i)^n + k(1 + i)(1 + i)^n$
 $= (1 + i)^n[(1 - k) + k(1 + i)]$
 $= (1 + i)^n(1 + ki)$

3. Maturity value on October 4, 2018:

$$S = 2000(1.03)^4 \left[1 + (0.03) \left(\frac{182}{365} \right) \right] = \$2284.69$$

$$\text{Proceeds: } P = 2283.69 \left(1 + \frac{0.035}{4} \right)^{-14} \left[1 + (0.035) \left(\frac{64}{365} \right) \right] = \$2034.77$$

$$\text{Compound discount: } S - P = \$1750.08$$

EXERCISE 2.5

Part A

1. $2000(1+i)^{15} = 3000$
 $(1+i)^{15} = 1.5$
 $1+i = (1.5)^{1/15}$
 $i = (1.5)^{1/15} - 1$
 $i = 0.027399659$
 $j_4 = 0.109598636$
 $j_4 = 10.96\%$
2. $100(1+i)^{55} = 150$
 $(1+i)^{55} = 1.5$
 $1+i = (1.5)^{1/55}$
 $i = (1.5)^{1/55} - 1$
 $i = 0.007399334$
 $j_{12} = 0.088792004$
 $j_{12} = 8.88\%$
3. $200(1+i)^{15} = 600$
 $(1+i)^{15} = 3$
 $1+i = 3^{1/15}$
 $i = 3^{1/15} - 1$
 $i = 0.075989625$
 $j_1 = 7.60\%$
4. $1000(1+i)^7 = 1181.72$
 $(1+i)^7 = 1.18172$
 $1+i = (1.18172)^{1/7}$
 $i = (1.18172)^{1/7} - 1$
 $i = 0.024139759$
 $j_2 = 0.048279518$
 $j_2 = 4.83\%$
5. $2000(1.01)^n = 2800$
 $(1.01)^n = 1.4$
 $n \log(1.01) = \log 1.4$
 $n = 33.81518078$ quarters
 $n = 8$ years, 5 months, 14 days
6. $1000(1.045)^n = 130$
 $(1.045)^n = 1.3$
 $n \log(1.045) = \log 1.3$
 $n = 5.96053678$ half years
 $n = 2$ years, 11 months, 23 days

7. $500(1.005)^n = 800$
 $(1.005)^n = 1.6$
 $n \log 1.005 = \log 1.6$
 $n = 94.23553231 \text{ months}$
 $n = 7 \text{ years, 10 months, 8 days}$
8. $1800(1.02)^n = 2200$
 $(1.02)^n = \frac{22}{18}$
 $n \log 1.02 = \log \frac{22}{18}$
 $n = 10.13353897 \text{ quarters}$
 $n = 2 \text{ years, 6 months, 12 days}$
9. $(1 + i)^{10} = 2$
 $i = 2^{1/10} - 1$
 $j_1 = 2^{1/10} - 1 = 7.18\%$
10. $(1 + i)^{16} = 1.5$
 $i = (1.5)^{1/16} - 1$
 $j_4 = 4[(1.5)^{1/16} - 1] = 10.27\%$
11. $4.71(1 + i)^5 = 9.38$
 $(1 + i)^5 = \frac{9.38}{4.71}$
 $i = \left(\frac{9.38}{4.71}\right)^{1/5} - 1 = 14.77\%$
12. $4000(1 + i)^{1095} = 5000$
 $(1 + i)^{1095} = 1.25$
 $i = (1.25)^{1/1095} - 1$
 $j_{365} = 365[(1.25)^{\frac{1}{1095}} - 1] = 7.44\%$
13. a) $(1.0456)^n = 2$
 $n \log 1.10456 = \log 2$
 $n = 15.54459407 \text{ years}$
 $n = 15 \text{ years, 199 days OR } 15 \text{ years, 6 months, 17 days}$
- Rule of 70
 $n = \frac{70}{4.56}$
 $n = 15.35087719 \text{ years}$
 $n = 15 \text{ years, 129 days OR } 15 \text{ years, 4 months, 7 days}$

$$\begin{aligned}
 \text{b)} \quad & \left(1 + \frac{0.07}{365}\right)^n = 2 \\
 & n \log\left(1 + \frac{0.07}{365}\right) = \log 2 \\
 & n = 3614.614035 \text{ days} \\
 & n = 9 \text{ years, 330 days OR } 9 \text{ years, 10 months, 26 days}
 \end{aligned}$$

Rule of 70

$$\begin{aligned}
 n &= \frac{70}{\frac{7}{365}} \\
 n &= \frac{70}{0.019178082} \\
 n &= 3650 \text{ days} = 10 \text{ years}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & 800 \left(1 + \frac{0.098}{2}\right)^n = 1500 \\
 & (1.049)^n = 1.875 \\
 & n \log(1.049) = \log 1.875 \\
 & n = 13.14054666 \text{ half – years} \\
 & n = 6 \text{ years, 208 days OR } 6 \text{ years, 6 months, 26 days}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \left(1 + \frac{0.05}{365}\right)^n = 1.5 \\
 & n \log\left(1 + \frac{0.05}{365}\right) = \log 1.5 \\
 & n = 2960.098047 \text{ days} \\
 & n = 8 \text{ years, 41 days OR } 8 \text{ years, 1 month, 10 days}
 \end{aligned}$$

EXERCISE 2.5

Part B

1. $(1 + i)^{16} = 2$

$$(1 + i) = 2^{1/16}$$

$$1 + i = 1.044273782$$

a) $S = 1000(1 + i)^{10} = \$1542.21$

b) $S = 1000(1 + i)^{20} = \$2378.41$

2. $(1 + i)^{2190} = 2$

$$(1 + i)^n = 3$$

$$1 + i = 2^{1/2190}$$

$$n \log(1 + i) = \log 3$$

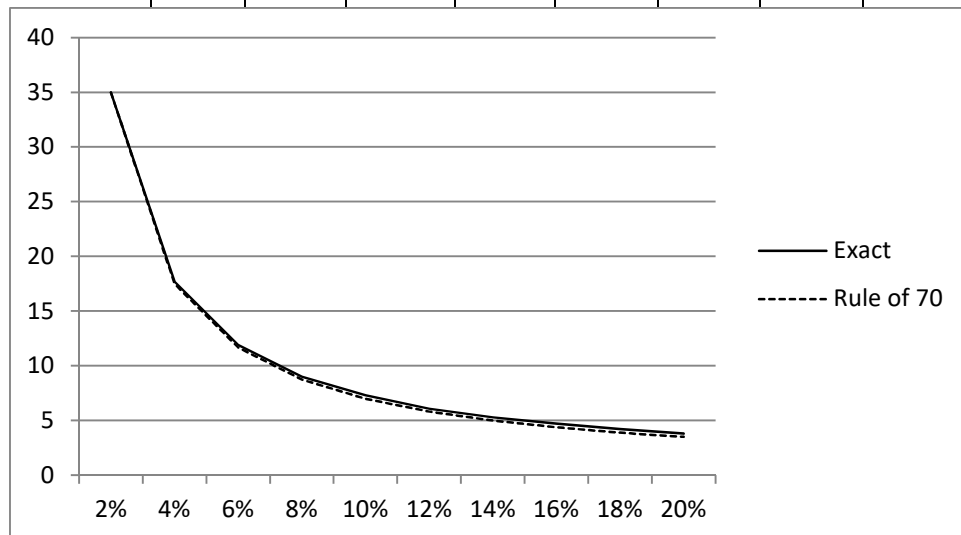
$$1 + i = 1.000316556$$

$$n = 3471.06782 \text{ days}$$

$$n = 9 \text{ yrs, 186 days OR } 9 \text{ yrs, 6 mths, 4 days}$$

3.

Rate j_1	2%	4%	6%	8%	10%	12%	14%	16%	18%	20%
Years	35	17.7	11.9	9	7.3	6.1	5.3	4.7	4.2	3.8
Rule of 70	35	17.5	11.67	8.75	7	5.83	5	4.375	3.89	3.5



4. $\left(1 + \frac{0.045}{12}\right)^{12n} = 2 \left(1 + \frac{0.025}{2}\right)^{2n}$

$$\left[\frac{(1.00375)^{12}}{(1.0125)^2}\right]^n = 2$$

$$n \log \left[\frac{(1.00375)^{12}}{(1.0125)^2}\right] = \log 2$$

$$n = 34.535 \text{ years}$$

$$5. (1 + j_1^*)^{t/2} = (1 + j_1)^t \rightarrow 1 + j_1^* = (1 + j_1)^2 \rightarrow j_1^* = (1 + j_1)^2 - 1$$

$$6. 800(1.045)^n = 2(600)(1.035)^n$$

$$\left(\frac{1.045}{1.035}\right)^n = \frac{1200}{800}$$

$$n = \frac{\log(1200/800)}{\log(1.045/1.035)}$$

$$n = 42.16804634 \text{ half-years}$$

$$n = 21 \text{ years, 31 days OR } 21 \text{ years, 1 month, 1 day}$$

7. In n years:

$$1.5[100(1.04)^n + 25(1.04)^{n-2}] = 95(1.08)^{n-1}$$

$$1.5(1.04)^n[100 + 25(1.04)^{-2}] = 95(1.08)^n(1.08)^{-1}$$

$$\left(\frac{1.04}{1.08}\right)^n = \frac{95(1.08)^{-1}}{1.5[100 + 25(1.04)^{-2}]}$$

$$\left(\frac{1.04}{1.08}\right)^n = 0.476322924$$

$$n = \frac{\log 0.476322924}{\log \frac{1.04}{1.08}}$$

$$n = 19.65163748 \text{ years} = 19 \text{ years, 238 days}$$

Using simple interest for the last X days we obtain:

$$1.5(1.04)^{19}[100 + 25(1.04)^{-2}]\left[1 + (0.04)\left(\frac{X}{365}\right)\right] = 95(1.08)^{18}\left[1 + (0.08)\left(\frac{X}{365}\right)\right]$$

This solves for $X = 233$; It takes 19 years and 233 days

$$8. 500(1.08)^n + 800(1.08)^{n-3} = 2000$$

$$(1.08)^n[500 + 800(1.08)^{-3}] = 2000$$

$$1135.065793(1.08)^n = 2000$$

$$(1.08)^n = 1.762012398$$

$$n = \frac{\log 1.762012398}{\log 1.08}$$

$$n = 7.360302768 \text{ years} = 7 \text{ years, 132 days}$$

Using compound interest for 7 years and simple interest for X days we have

$$500(1.08)^7\left[1 + (0.08)\left(\frac{X}{365}\right)\right] + 800(1.08)^4\left[1 + (0.08)\left(\frac{X}{365}\right)\right] = 2000$$

Solving for X we obtain $X = 128$ days; It takes 7 years, 128 days

$$\begin{aligned} \text{Check: } & 500(1.08)^7\left[1 + (0.08)\left(\frac{128}{365}\right)\right] + 800(1.08)^4\left[1 + (0.08)\left(\frac{128}{365}\right)\right] \\ & = 880.95 + 1118.93 = 1999.88 \end{aligned}$$

EXERCISE 2.6

Part A

1. $X = 1000(1.01)^{36} = \$1709.14$

2. $X = 1800 \left(1 + \frac{0.1175}{2}\right)^{-14} = \809.40

3. a) $X = 2500 \left(1 + \frac{0.09}{12}\right)^{-48} = \1746.54

b) $Y = 2500 \left(1 + \frac{0.09}{12}\right)^{36} = \3271.61

Note: $1746.54 \left(1 + \frac{0.09}{12}\right)^{84} = \3271.61

4. $X = 1000(1.02)^4 + 1500(1.02)^{-4} = 1082.43 + 1385.77 = \2468.20

5. a) $X = 800(1.0025)^{-24} + 700(1.0025)^{-72} = \1338.29

b) $Y = 800(1.0025)^{24} + 700(1.0025)^{72} = \1508.69

c) $Z = 800(1.0025)^{72} + 700(1.0025)^{24} = \1700.79

$$X(1.0025)^{48} = Y \quad 1338.29(1.0025)^{48} = \$1508.69$$

$$Y(1.0025)^{48} = Z \quad 1508.69(1.0025)^{48} = \$1700.79$$

6. At the end of 7 years: $X + 1000(1.035)^8 = 2000(1.035)^{-2}$

$$X + 1316.81 = 1867.02$$

$$X = \$550.21$$

7. $X = 4000(1.015)^{12} - 1000(1.015)^8 - 2000(1.015)^4$
 $= 4782.47 - 1126.49 - 2122.73 = \1533.25

8. $X = 1200(1.015)^{12} - 500(1.015)^6 = 1434.74 - 546.72 = \888.02

9. At the end of 4 years:

$$375 \left(1 + \frac{0.08}{12}\right)^{36} + X \left(1 + \frac{0.08}{12}\right)^{24} + X \left(1 + \frac{0.08}{12}\right)^{12} = 1000$$

$$476.34 + 1.172887932X + 1.082999507X = 1000$$

$$2.255887439X = 523.66$$

$$X = \$232.13$$

10. a) On December 1, 2015:

$$X + X(1.03)^2 + 1200(1.03)^4 + 900(1.03)^7 = 3000(1.03)^9$$

$$X + 1.0609X + 1350.61 + 1106.89 = 3914.32$$

$$2.0609X = 1456.82$$

$$X = \$706.89$$

b) Balance on September 1, 2015:

$$= 3000(1.03)^8 - 900(1.03)^6 - 1200(1.03)^3 - 900(1.03)^2$$

$$= 3800.31 - 1074.65 - 1311.27 - 954.81 = \$459.58$$

$$11. X = 200(1.03)^4 + 150(1.03)^3 - 250(1.03)^2 + 100(1.03) \\ = 225.10 + 163.91 - 265.23 + 103 = \$226.78$$

12. At the end of 3 years:

$$X + 2X(1.1)^{-3} = 400(1.1)^{-2} + 300(1.1)^{-7} \\ X + 1.502629602X = 330.58 + 153.95 \\ 2.502629602X = 484.53 \\ X = \$193.61$$

13. At the time of the man's death:

$$X(1.03)^{-4} + X(1.03)^{-12} + X(1.03)^{-16} = 50\,000 \\ 2.17033867X = 50\,000 \\ X = \$22\,593.42$$

$$14. X(1.006)^9 + 2X(1.006)^5 + 2X = 4000(1.006)^{12} \\ 5.11603864X = 4297.70 \\ X = \$840.04$$

15. Maturity value of original debt:

$$S = 3000(1.0075)^{24} = \$3589.24$$

Equation of value at time 5:

$$1000(1.03)^4 + 1500(1.03)^3 + X = 3589.24(1.03) \\ 1125.50881 + 1639.0905 + X = 3696.9172 \\ X = \$932.32$$

$$16. 500 + 800(1.08)^{-3} = 2000(1.08)^{-t} \\ (1.08)^{-t} = \frac{1135.065793}{2000} = 0.567532896 \\ t = 7.36 \text{ years}$$

$$17. 1000 = 700(1+i)^{-6} + 400(1+i)^{-10}$$

By trial and error,

$i = j_4/4$	Right hand side
0.02	949.72
0.015	984.85
0.013	999.33
0.0128	1000.80
0.0129	1000.06

$$\text{Thus, } j_4 \doteq 4(0.0129) = 0.0516 = 5.16\%$$

EXERCISE 2.6

Part B

1. a) Let X and Y be the two dated values due n_1 and n_2 periods from now.
Let D_1 and D_2 be two equivalent dated values of the set at t_1 and t_2 interest periods from now.

	X	D_1	Y	D_2
0	n_1	t_1	n_2	t_2

$$D_1 = X(1+i)^{t_1-n_1} + Y(1+i)^{t_1-n_2}$$

$$D_2 = X(1+i)^{t_2-n_1} + Y(1+i)^{t_2-n_2}$$

Multiplying the first equation by $(1+i)^{t_2-t_1}$ and simplifying we obtain

$$D_1(1+i)^{t_2-t_1} = X(1+i)^{t_2-n_1} + Y(1+i)^{t_2-n_2} = D_2$$

which is the condition that D_1 and D_2 are equivalent

- b) Assuming that the times are in years

$$D_1 = X[1+r(t_1-n_1)] + Y[1+r(n_2-t_1)]^{-1}$$

$$D_2 = X[1+r(t_2-n_1)] + Y[1+r(t_2-n_2)]$$

Multiplying the first equation by $[1+r(t_2-t_1)]$ we obtain

$$\begin{aligned} D_1[1+r(t_2-t_1)] &= X[1+r(t_1-n_1)][1+r(t_2-t_1)] \\ &\quad + Y[1+r(n_2-t_1)]^{-1}[1+r(t_2-t_1)] \\ &\neq X[1+r(t_2-n_1)] + Y[1+r(t_2-n_2)] = D_2 \end{aligned}$$

$$\begin{aligned} 2. \quad X &= 1000(1.08)^2 + 2000\left(1 + \frac{0.125}{2}\right)^8 (1.08)^{-2} \\ &= 1166.40 + 2784.93 = \$3951.33 \end{aligned}$$

3. On January 1, 2018:

$$\begin{aligned} X + X(1.02)^8 + 500(1.02)^{16} &= 5000(1.0225)^{24} \\ X + 1.171659381X + 686.39 &= 8528.83 \\ 2.171659281X &= 7842.44 \\ X &= \$3611.27 \end{aligned}$$

4. At the present time:

$$\begin{aligned} X + X(1.06)^{-2} &= 3000(1.05)^{-8} + 4000(1.04)^{-10} \\ 1.88999644X &= 2030.52 + 2702.26 \\ 1.88999644X &= 4732.78 \\ X &= \$2504.12 \end{aligned}$$

5. Let i be the interest rate per year.

At the end of year 18:

$$\begin{aligned} (1) \quad & 240(1+i)^{12} + 200(1+i)^6 + 300 = X \\ (2) \quad & 360(1+i)^6 + 700 = X + 100 \\ (3) \quad & Y(1+i)^{12} + 600(1+i)^6 = X \end{aligned}$$

Let $(1+i)^6 = Z$. Then,

$$\begin{aligned} (1) \quad & 240Z^2 + 200Z + 300 = X \\ (2) \quad & 360Z + 600 = X \\ (3) \quad & YZ^2 + 600Z = X \end{aligned}$$

From the first two equations:

$$\begin{aligned} 240Z^2 + 200Z + 300 &= 360Z + 600 \\ 240Z^2 - 160Z - 300 &= 0 \\ 12Z^2 - 8Z - 15 &= 0 \end{aligned}$$

$$\begin{aligned} Z &= \frac{8 \pm \sqrt{64 + 720}}{24} = \frac{36}{24} = 1.5 \\ \text{or } -\frac{20}{24} & \text{ (not applicable)} \end{aligned}$$

Substituting $Z = 1.5$ into (1) we obtain

$$\begin{aligned} 240(1.5)^2 + 200(1.5) + 300 &= X \\ X &= \$1140 \end{aligned}$$

Substituting $Z = 1.5, X = 1140$ into (3) we obtain

$$\begin{aligned} Y(1.5)^2 + 600(1.5) &= 1140 \\ 2.25Y &= 1140 - 900 \\ Y &= \frac{240}{2.25} \\ Y &= \$106.67 \end{aligned}$$

EXERCISE 2.7

Part A

1. $S = 2000 \left(1 + \frac{0.04}{12}\right)^{72} \left(1 + \frac{0.09}{12}\right)^{72} = \3042.03
2. $P = 1000(1.07)^{-4}(1.08)^{-2} = 654.06$
3. $S = 500(1.025)^2(1.03)^4(1.0225)^4 = 646.28$
 $500(1 + j_1)^5 = 646.28$
 $j_1 = 5.27\%$
4. $S = 2000(1.025)^6(1.02)^{16}(1.005)^{36} = \3810.26
Compound interest = $3810.26 - 2000 = \$1810.26$
 $200 \left(1 + \frac{j_2}{2}\right)^{20} = 3810.26$
 $j_2 = 6.55\%$
5. $X = 2000(1.05)^4(1.045)^9 = \3612.72
6. At the present time:
 $X + X(1.048)^{-4}(1.061)^{-6} = 5000(1.061)^{-5}$
 $1.581115643X = 3718.72$
 $X = \$2351.96$
7. $Y = 20\,000(1.06)^5 + 30\,000 + 35\,000(1.05)^{-7}$
 $= 26\,764.51 + 30\,000 + 24\,873.85$
 $= \$81\,638.36$
8. Present value of the offer = $65\,000 + 150\,000(1.02)^{-4} + 150\,000(1.02)^{-4}(1.015)^{-8}$
 $= 65\,000 + 138\,576.81 + 123\,016.18 = \$326\,592.99$
They should accept the offer.
9. a) Discounted value of the payments option:
 $60\,000 + 60\,000 \left(1 + \frac{0.072}{12}\right)^{-24} + 60\,000 \left(1 + \frac{0.072}{12}\right)^{-60}$
 $= 60\,000 + 51\,975.62 + 41\,905.63 = \$153\,881.26$
The payment option is better.

b) Discounted value of the payments option:
 $60\,000 + 60\,000 \left(1 + \frac{0.075}{4}\right)^{-8} + 60\,000 \left(1 + \frac{0.075}{4}\right)^{-12} \left(1 + \frac{0.04}{4}\right)^{-8}$
 $= 60\,000 + 51\,714.26 + 44\,337.25 = \$156\,051.51$
The cash option is better.
10. $(1 + j)^6 = (1.015)^8 \left(1 + \frac{0.08}{12}\right)^{48}$
 $(1 + j)^6 = 1.549677664$
 $1 + j = 1.075738955$
 $j = 7.57\%$

EXERCISE 2.7

Part B

$$1. (1+i)^n \times (1+j)^n = [(1+i)(1+j)]^n = (1+i+j+ij)^n$$

$$\left(1 + \frac{i+j}{2}\right)^{2n} - \left[\left(1 + \frac{i+j}{2}\right)^2\right]^n = \left[1 + \frac{2(i+j)}{2} + \frac{i^2+2ij+j^2}{4}\right]^n$$

$$= \left(1 + i + j + \frac{i^2+2ij+j^2}{4}\right)^n$$

Since $ij \neq \frac{i^2+2ij+j^2}{4}$ then $(1+i)^n \times (1+j)^n \neq \left(1 + \frac{i+j}{2}\right)^{2n}$

$$2. S = 500(1.04)^2(1.02)^4 \left(1 + \frac{0.08}{12}\right)^{12} \left(1 + \frac{0.08}{365}\right)^{365} = \$686.76$$

$$\text{Difference} = 686.76 - 500(1.04)^8 = 686.76 - 684.28 = \$2.48$$

$$3. X = 1000(1.02)^{14} + 2000(1.01)^{-20}$$

$$= 1319.48 + 1639.09 = \$2958.57$$

$$4. P = 20\,000(1.12)^{-3}(1.05)^{-10} + 30\,000(1.12)^{-3}(1.05)^{-12}(1.02)^{-12}(1.0075)^{-36}$$

$$= 8739.43 + 7164.26 = \$15\,903.69$$

5. Amount in the account on April 21, 2014 :

$$X = 1000(1.0175)^{11}(1.025)^3 + 2000(1.0175)(1.025)^3$$

$$= 1303.32 + 2191.47 = \$3494.79$$

Calculate $i = \frac{j_{12}}{12}$ such that $1000(1+i)^{51} + 2000(1+i)^{21} = 3494.79$

By trial and error we determine:

at $j_{12} = 5\%$: $1000(1+i)^{51} + 2000(1+i)^{21} = 3418.71$

at $j_{12} = 6\%$: $1000(1+i)^{51} + 2000(1+i)^{21} = 3510.48$

91.77	{	76.08	{	amount	j_{12}	3418.71	5%	3494.79	j_{12}	3510.48	6%	}	d	}	1%	$\frac{d}{1\%} = \frac{76.08}{91.77}$ $d \doteq 0.83\%$ $j_{12} = 5.83\%$
amount	j_{12}															
3418.71	5%															
3494.79	j_{12}															
3510.48	6%															

Check at $j_{12} = 5.83\%$: $1000(1+i)^{51} + 2000(1+i)^{21} = \3494.68

$$6. (1+j_1)^3 = \left(1 + \frac{0.04}{12}\right)^{12} \left(1 + \frac{0.08}{4}\right)^4 \left(1 + \frac{0.055}{365}\right)^{365}$$

$$(1+j_1)^3 = 1.190222002$$

$$j_1 = (1.190222002)^{1/3} - 1 = 0.059764396 = 5.98\%$$

7. Let $j_4 = 4i$

$$(1+i)^{12} = [1 + (0.06)(1)][1 - (0.08)(2)]^{-1}$$

$$(1+i)^{12} = 1.261904762$$

$$1+i = 1.019574304$$

$$i = 0.019574304$$

and $j_4 = 4i = 0.078297216 = 7.83\%$

EXERCISE 2.8

Part A

- $0.6(1.08)^n = 1$
 $(1.08)^n = \frac{1}{0.6}$
 $n \log 1.08 = \log \frac{1}{0.6}$
 $n = 6.637457293 \text{ years}$
- $S = 320\,000(1.021)^5 = \$355\,041.15$
Increase = \$35 041.15
- $i_{real} = \frac{0.06-0.02}{1+0.02} = 3.92\% \quad i_{realAT} = \frac{0.06(1-0.26)-0.02}{1+0.02} = 2.39\%$
 - $i_{real} = \frac{0.08-0.04}{1+0.04} = 3.85\% \quad i_{realAT} = \frac{0.08(1-0.26)-0.04}{1+0.04} = 1.85\%$
 - $i_{real} = \frac{0.10-0.06}{1+0.06} = 3.77\% \quad i_{realAT} = \frac{0.10(1-0.26)-0.06}{1+0.06} = 1.32\%$

EXERCISE 2.8

Part B

- Let $X = \$1000$.
You need $1000(1.03)^{-1}$ U.S. dollars now in U.S. dollars account,
which is equivalent to $1000(1.03)^{-1} \left(\frac{1}{0.9717} \right) = \999.15 Cdn.
This amount invested in a Canadian dollar account will accumulate to

$$\$999.15(1.04) = \$1039.12$$

The implied exchange rate one year from now is

$$\$1000 \text{ U.S.} = \$1039.12 \text{ Cdn. OR } \$0.9624 \text{ U.S.} = \$1 \text{ Cdn.}$$

- Present value of $(1+r)^n$ due in n -years at annual effective rate i is:

$$(1+r)^n(1+i)^{-n} = \left(\frac{1+r}{1+i} \right)^n$$

Present value of 1 due in n years at annual effective rate $\frac{i-r}{1+r}$ is:

$$\left(1 + \frac{i-r}{1+r} \right)^{-n} = \left(\frac{1+r+i-r}{1+r} \right)^{-n} = \left(\frac{1+i}{1+r} \right)^{-n} = \left(\frac{1+r}{1+i} \right)^n$$

EXERCISE 2.9

Part A

1. $S = 40\,000(1.04)^{20} \doteq 87\,645$
2. Increase = 2% of $15\,000(1.02)^7 \doteq 345$
3. $(1+j)^{11} = 2$
 $j = 2^{1/11} - 1$
 $j = 0.065041089$
 $j = 6.50\%$
4. $S = 48\,000(1.05)^{42} = \$372\,556.20$
5. $P = 0.25$ $S = 10$ $i = 0.10$
 $(0.25)(1.10)^n = 10$
 $(1.10)^n = 40$
 $n = \frac{\log 40}{\log 1.10}$
 $n = 38.70393972$ hours
 $n = 1.61$ days

EXERCISE 2.9

Part B

1. a) Number of flies at 7 a.m. = $100\,000(1.04)^{27} \doteq 288\,337$
Number of flies at 11 a.m. = $100\,000(1.04)^{33} \doteq 364\,838$
Increase between 7 a.m. and 10 a.m. = 76 501
b) $(1.04)^n = 2$
 $n = \frac{\log 2}{\log 1.04} = 17.67298769$ periods $\doteq 707$ minutes
At 0:47 a.m. there will be 20 000 flies in the lab.
2. $200\,000(1+i)^{10} = 250\,000$
 $(1+i)^{10} = 1.25$
 $i = (1.25)^{1/10} - 1$
 $i = 0.022565183$
Population in 2014 = $200\,000(1+i)^{20} = 312\,500$
Population in 2019 = $200\,000(1+i)^{25} = 349\,386$
Increase in population = 36 886

EXERCISE 2.10

Part A

1. a) $S = 1500(1.09)^{1.5} = \$1706.99$

b) $S = 1500 \left(1 + \frac{0.09}{12}\right)^{18} = \1715.94

c) $S = 1500e^{(0.09)(1.5)} = \1716.81

2. a) $P = 8000(1.02)^{-20} = \$5383.77$

b) $P = 8000 \left(1 + \frac{0.08}{365}\right)^{-1825} = \5362.80

c) $P = 8000e^{-(0.08)(5)} = \5362.56

3. $e^{j_{\infty}(5)} = 1.5$

$$5 j_{\infty} = \ln 1.5$$

$$j_{\infty} = \frac{\ln 1.5}{5} = 0.081093022 = 8.11\%$$

$$j = e^{j_{\infty}} - 1 = 0.084471771 = 8.45\%$$

4. a) $800 \left(1 + \frac{0.06}{365}\right)^n = 1200$

$$\left(1 + \frac{0.06}{365}\right)^n = 1.5$$

$$n = \frac{\log 1.5}{\log \left(1 + \frac{0.06}{365}\right)} = 2466.782157 \rightarrow 2467 \text{ days} = 6 \text{ years, } 277 \text{ days}$$

On November 8, 2020 the deposit will be worth at least \$1200.

b) $800e^{0.06t} = 1200$

$$e^{0.06t} = 1.5$$

$$0.06t = \ln 1.5$$

$$t = \frac{\ln 1.5}{0.06} = 6.757751802 \text{ years} \doteq 6 \text{ years } 277 \text{ days}$$

On November 8, 2016 the deposit will be worth at least \$1200.

5. $e^{5j_{\infty}} = 2$

$$5 j_{\infty} = \ln 2$$

$$j_{\infty} = \frac{\ln 2}{5}$$

$$e^{j_{\infty}t} = 3$$

$$j_{\infty} t = \ln 3$$

$$t = \frac{\ln 3}{j_{\infty}} = \frac{\ln 3}{\frac{\ln 2}{5}} = 7.924812504 \text{ years}$$

6. a) $S = 1000e^{0.08(2)} = \$1173.51$

b) $S = 1000 \left(1 + \frac{0.0825}{2}\right)^4 = \1175.49

c) $S = 1000[1 + (0.085)(2)] = \1170

She should accept offer c) as it has the lowest interest charges.

EXERCISE 2.10

Part B

$$\begin{aligned}1. \quad (1 + 5r) &= e^{0.07(5)} \\ 5r &= e^{0.35} - 1 \\ r &= \frac{e^{0.35} - 1}{5} \\ r &= 0.08381351 \\ r &= 8.38\%\end{aligned}$$

$$\begin{aligned}2. \quad e^{j_{\infty}(25)} &= 2 & e^{\frac{\ln 2}{25}t} &= 1.5 \\ j_{\infty} &= \frac{\ln 2}{25} & t \frac{(\ln 2)}{25} &= \ln 1.5 \\ & & t &= 25\left(\frac{\ln 1.5}{\ln 2}\right) \\ & & t &= 14.62406252 \text{ years}\end{aligned}$$

$$\begin{aligned}3. \quad \text{At the end of } t\text{-years :} \\ 1000e^{0.10(t-1.25)} + 1500e^{-0.10(6.5-t)} &= 2500 \\ e^{0.10t}(1000e^{-0.125} + 1500e^{-0.65}) &= 2500 \\ e^{0.10t} &= 1.500991644 \\ 0.10t &= \ln 1.500991644 \\ t &= 4.061259858 \text{ years}\end{aligned}$$

$$\begin{aligned}4. \quad 250e^{0.07(2)}e^{0.08(n-2)} &= 400 \\ e^{0.08(n-2)} &= \frac{400}{250}e^{-0.14} \\ 0.08(n-2) &= \ln\left(\frac{400}{250}\right) - 0.14 \\ n-2 &= \frac{1}{0.08}\left(\ln\frac{400}{250} - 0.14\right) \\ n &= 6.125045366 \text{ years}\end{aligned}$$

$$\begin{aligned}5. \quad \text{At the end of 12 months:} \\ 400e^{0.04(0.75)} + Xe^{0.04(0.5)} + X &= 1000e^{0.04} \\ 412.187 + 1.02020134X + X &= 1040.81 \\ 2.02020134X &= 628.63 \\ X &= \$311.17\end{aligned}$$

$$\begin{aligned}6. \quad 1 - 4d &= e^{-0.08(4)} \\ d &= \frac{1 - e^{-0.32}}{4} \\ d &= 0.068462741 \\ d &= 6.85\%\end{aligned}$$

REVIEW EXERCISE 2.11

1. $X = 1000 \left(1 + \frac{0.0638}{2}\right)^9 + 800 \left(1 + \frac{0.0638}{2}\right)^{-11} = 1326.60 + 566.34 = \1892.94
2. $S = 1500 \left(1 + \frac{0.098}{365}\right)^{3650} = \3996.16
3. $S = 1000(1.045)^{20} = \$2411.71$
4. a) Theoretical method : $S = 2000(1.04)^{2\frac{1}{3}} = \2191.67
 Practical method : $S = 2000(1.04)^2 \left[1 + (0.08)\left(\frac{2}{12}\right)\right] = \2192.04
 b) Theoretical method : $S = 2000(1.04)^{-2\frac{1}{3}} = \1825.10
 Practical method : $S = 2000(1.04)^{-3} \left[1 + (0.08)\left(\frac{4}{12}\right)\right] = \1825.41
5. $S = 680\,000(1.04)^5 = \$827\,323.97$
6. $\text{Interest} = 100(1.035)^{20} - 100(1.035)^{10} = 198.98 - 141.06 = \57.92
7. $D = 1500 - 500 \left(1 + \frac{0.21}{12}\right)^{-3} - 600 \left(1 + \frac{0.21}{12}\right)^{-6} - 300 \left(1 + \frac{0.21}{12}\right)^{-9}$
 $= 1500 - 474.64 - 540.69 - 256.63 = \228.04
8. $j_2 = 6.75\% \rightarrow j = \left(1 + \frac{0.0675}{2}\right)^2 - 1 \doteq 6.86\% \text{ BEST}$
 $j_4 = 6.25\% \rightarrow j = \left(1 + \frac{0.0625}{4}\right)^4 - 1 \doteq 6.40\% \text{ MIDDLE}$
 $j_{12} = 6.125\% \rightarrow j = \left(1 + \frac{0.06125}{12}\right)^{12} - 1 \doteq 6.30\% \text{ WORST}$
9. Maturity date is November 21, 2018
 Proceeds $P = 3000(1.015)^{-19} \left[1 + (0.06)\left(\frac{41}{365}\right)\right] = \2276.06
10. $1000 \left(1 + \frac{0.06}{365}\right)^n = 2500$
 $\left(1 + \frac{0.06}{365}\right)^n = 2.5$
 $n \log \left(1 + \frac{0.06}{365}\right) = \log 2.5$
 $n = 5547.560135 \text{ days}$
 $n = 15 \text{ years, } 100 \text{ days OR } 15 \text{ years, } 2 \text{ months, } 12 \text{ days}$
11. $(1 + i)^{60} = 3$
 $i = 3^{1/60} - 1$
 $j_4 = 4 \left[3^{1/60} - 1\right] = 7.39\%$

$$12. \text{Maturity Value of Loan} = 10\,000 \left(1 + \frac{0.12}{2}\right)^{12} = \$20\,121.96$$

On January 1, 2019:

$$2000(1.02)^{12} + X(1.02)^4 + X = 20\,121.96$$

$$2536.48 + 2.08243216X = 20\,121.96$$

$$X = \$8\,444.68$$

$$13. \left(1 + \frac{0.045}{365}\right)^n = 1.25$$

$$n = \frac{\log 1.25}{\log \left(1 + \frac{0.045}{365}\right)}$$

$$n \doteq 1810 \text{ days}$$

4 years, 350 days from November 20, 2013 is November 5, 2017.

14. At the present time:

$$X(1.0075)^{-2} + 2X(1.0075)^{-5} + 3X(1.0075)^{-10} = 5000$$

$$5.695834944X = 5000$$

$$X = \$877.83$$

$$15. \text{a) at } j_{12}: \quad \left(1 + \frac{j_{12}}{12}\right)^{120} = 3$$

$$j_{12} = 12 \left[3^{\frac{1}{120}} - 1\right] \doteq 11.04\%$$

$$\text{b) at } j_{365}: \quad \left(1 + \frac{j_{365}}{365}\right)^{3650} = 3$$

$$j_{365} = 365 \left[3^{\frac{1}{3650}} - 1\right] \doteq 10.99\%$$

$$\text{c) at } j_{\infty} \equiv \sigma: \quad e^{10\sigma} = 3$$

$$10\sigma = \ln 3$$

$$\sigma = \frac{\ln 3}{10} \doteq 10.99\%$$

$$16. 1000(1.045)^n = 1246.18$$

$$(1.045)^n = 1.24618$$

$$n = \frac{\log 1.24618}{\log 1.045}$$

$$n = 5$$

$$S = 1000(1.06)^5 = \$1338.23$$

17. Discounted value of the payments option:

$$P = 20\,000 + 20\,000(1.04)^{-4} + 20\,000(1.04)^{-8}$$

$$= 20\,000 + 17\,096.08 + 14\,613.80 = \$51\,709.88$$

Cash option is better by \$1709.88

18. Maturity value on October 6, 2012:

$$S = 2000 \left(1 + \frac{0.08}{12}\right)^{24} = \$2345.78$$

Proceeds on January 16, 2014:

$$P = 2345.78 \left(1 + \frac{0.09}{4}\right)^{-9} \left[1 + (0.09) \left(\frac{10}{365}\right)\right] = \$2199.72$$

$$\text{Compound discount} = 2345.78 - 2199.72 = \$146.06$$

$$19. S = 500 \left(1 + \frac{0.053}{2}\right)^4 \left(1 + \frac{0.07}{12}\right)^{36} \left(1 + \frac{0.045}{365}\right)^{365} = \$715.95$$

$$500(1 + j_1)^6 = 715.95$$

$$j_1 = 6.17\%$$

$$20. P = 2000(1.02)^{-8}(1.05)^{-7} = \$1213.12$$

$$21. a) 1000(1.06)^5 = \$1338.23$$

$$b) 1000 \left(1 + \frac{0.06}{12}\right)^{60} = \$1348.85$$

$$c) 1000e^{0.06(5)} = \$1349.86$$

$$22. \text{She will receive } 2000(1.05)^5 \left[1 + (0.05) \left(\frac{3}{12}\right)\right] = \$2584.47$$

$$23. a) S = 5000 \left(1 + \frac{0.036}{12}\right)^{22} \left[1 + (0.036) \left(\frac{26}{365}\right)\right] = \$5354.30$$

$$b) S = 5000 \left(1 + \frac{0.036}{12}\right)^{22 + \frac{26}{365}(12)} = \$5354.30$$

24. Value on December 13, 2013:

$$2000(1.025)^{-9} \left[1 + (0.1) \left(\frac{39}{365}\right)\right] = \$1618.57$$

25. a) Equation of value at 12 months:

$$X(1.0075)^9 + 2X(1.0075)^5 + 2X = 4000(1.0075)^{12}$$

$$1.069560839X + 2.076133469X + 2X = 4375.23$$

$$5.145694308X = 4375.23$$

$$X = \$850.27$$

b) Equation of value at 12 months:

$$Xe^{0.09\left(\frac{9}{12}\right)} + 2Xe^{0.09\left(\frac{5}{12}\right)} + 2X = 4000e^{0.09\left(\frac{12}{12}\right)}$$

$$1.0698026X + 2.076423994X + 2X = 4376.70$$

$$5.146254254X = 4376.70$$

$$X = \$850.46$$

26. a) At $j_{365} = 10\%$

$$\left(1 + \frac{0.10}{365}\right)^n = 2$$

$$n = \frac{\log 2}{\log\left(1 + \frac{0.10}{365}\right)}$$

$$n \doteq 2530.33 = 2531 \text{ days}$$

$$n \doteq 6 \text{ years, } 341 \text{ days OR } 6 \text{ years, } 11 \text{ months, } 7 \text{ days}$$

b) At $j_{\infty} = 10\%$

$$e^{0.1t} = 2$$

$$0.1t = \ln 2$$

$$t = \frac{\ln 2}{0.1}$$

$$t = 6.931471806 \text{ years}$$

$$t \doteq 6 \text{ years, } 340 \text{ days OR } 6 \text{ years, } 11 \text{ months, } 6 \text{ days}$$

c) At $j_4 = 10\%$

$$(1.05)^n = 2$$

$$n = 28.07103453 \text{ quarters}$$

$$n = 7 \text{ years, } 0 \text{ months, } 7 \text{ days}$$

d) At $j_2 = 10\%$

$$(1.05)^n = 2$$

$$n = 14.20669908 \text{ half years}$$

$$n = 7 \text{ years, } 38 \text{ days OR } 7 \text{ years, } 1 \text{ months, } 8 \text{ days}$$

Rule of 70

$$\text{a) } \frac{70}{\frac{10}{365}} = 2555 \text{ days} = 7 \text{ years}$$

$$\text{c) } \frac{70}{\frac{10}{4}} = 28 \text{ quarters} = 7 \text{ years}$$

$$\text{d) } \frac{70}{\frac{10}{2}} = 14 \text{ half years} = 7 \text{ years}$$

Case Study I – Payday Loans

- a) Calculate
- j
- such that:

$$(1 + j) = (1.25)^{\frac{365}{14}}$$

$$1 + j = 336.188$$

$$j = 335.2\%$$

- b) If you are one week late, the penalty is 10% of 1000 or another \$100. Thus you borrow \$800 and pay back \$1100 in 21 days. Thus:

$$(1 + j) = \left(\frac{1100}{800}\right)^{\frac{365}{21}}$$

$$1 + j = 253.415$$

$$j = 252.4\%$$

If you are two weeks late, you owe \$1200 in 28 days. Thus:

$$(1 + j) = \left(\frac{1200}{800}\right)^{\frac{365}{28}}$$

$$1 + j = 197.458$$

$$j = 196.5\%$$

- c) When the fee is 15%:

$$(1 + j) = (1.15)^{\frac{365}{14}}$$

$$1 + j = 38.2366$$

$$j = 37.2\%$$

At 20%:

$$(1 + j) = (1.20)^{\frac{365}{14}}$$

$$1 + j = 115.976$$

$$j = 114.98\%$$

At 30%

$$(1 + j) = (1.30)^{\frac{365}{14}}$$

$$1 + j = 934.687$$

$$j = 933.7\%$$

Case Study II – Overnight Rates

- a) $I = 20,000,000 \left(\frac{0.04}{365}\right) = \2191.78
- b) $I = 20,000,000 \left(e^{\frac{0.04}{365}} - 1\right) = \2191.90
- c) $j = \left(\frac{25,002,568}{25,000,000}\right)^{\frac{365}{1}} - 1 = 0.0328 = 3.28\%$