Chapter 2 Solutions for Introduction to Robotics

1. a) Use (2.3) to obtain

$$
{}^{A}_{B}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}
$$

   b) Use (2.74) to get

$$
\begin{aligned}
\alpha &= 90 \text{ degrees} \\
\beta &= 90 \text{ degrees} \\
\gamma &= -90 \text{ degrees}
\end{aligned}
$$

2. a) Use (2.64) to obtain

$$
{}^{A}_{B}R = \begin{bmatrix} .330 & -.770 & .547 \\ .908 & .418 & .0396 \\ -.259 & .483 & .837 \end{bmatrix}
$$

   b) Answer is the same as in (a) according to (2.71)

3. Use (2.19) to obtain the transformation matrices. The rotation is X-Y-Z fixed angles, so use (2.64) for that 3×3 submatrix, with angles

$$
\begin{aligned}
\gamma &= 0 \text{ degrees} \\
\beta &= -\sin^{-1}\left(\frac{\text{tripod\_height}}{\text{distance\_along\_optical\_axis}}\right) = -\sin^{-1}\left(\frac{1.5}{5}\right) = -107 \text{ degrees} \\
\alpha_C &= 0 \text{ degrees} \\
\alpha_D &= 120 \text{ degrees} \\
\alpha_E &= 240 \text{ degrees}
\end{aligned}
$$

   The position vectors to the camera-frame origins are

$$
{}^{B}P_{CORG} = \begin{bmatrix} \text{horizontal\_distance} \\ 0 \\ \text{tripod\_height} \end{bmatrix} = \begin{bmatrix} 4.77 \\ 0 \\ 1.50 \end{bmatrix}
$$

$$
{}^{B}P_{DORG} = \begin{bmatrix} \text{horizontal\_distance} \times \cos\alpha_D \\ \text{horizontal\_distance} \times \sin\alpha_D \\ \text{tripod\_height} \end{bmatrix} = \begin{bmatrix} -2.39 \\ 4.13 \\ 1.5 \end{bmatrix}
$$

$$
{}^{B}P_{EORG} = \begin{bmatrix} \text{horizontal\_distance} \times \cos\alpha_E \\ \text{horizontal\_distance} \times \sin\alpha_E \\ \text{tripod\_height} \end{bmatrix} = \begin{bmatrix} -2.38 \\ -4.13 \\ 1.50 \end{bmatrix},
$$

where horizontal_distance $= \sqrt{(\text{distance\_along\_optical\_axis})^2 - (\text{tripod\_height})^2}$.
Combining the rotation and translation yields the transformation matrices via (2.19) as

$$
{}^{B}_{C}T = \begin{bmatrix} -.300 & 0 & -.954 & 4.77 \\ 0 & 1.00 & 0 & 0 \\ .954 & 0 & -.300 & 1.50 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{B}_{D}T = \begin{bmatrix} .150 & -.866 & .477 & -2.39 \\ -.260 & -.500 & -.826 & 4.13 \\ .954 & 0 & -.300 & 1.50 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{B}_{E}T = \begin{bmatrix} .150 & .866 & .477 & -2.39 \\ .260 & -.500 & .826 & -4.13 \\ .954 & 0 & -.300 & 1.50 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

4. The camera-frame origin is located at ${}^{B}P_{CORG} = [7 \quad -2 \quad 5]^{\mathsf{T}}$. Use (2.19) to get the transformation, ${}^{B}_{C}T$. The rotation is Z-Y-X Euler angles, so use (2.71) with

$$
\alpha = 0 \text{ degrees}
$$
$$
\beta = -110 \text{ degrees}
$$
$$
\gamma = -20 \text{ degrees}
$$

to get

$$
{}^{B}_{C}T = \begin{bmatrix} -.342 & .321 & -.883 & 7.00 \\ 0 & .940 & .342 & -2.00 \\ .940 & .117 & -.321 & 5.00 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

5. Let

$$^{B}P_1 = {}^{B}P_0 + 5\,{}^{B}V_0 = [9.5 \quad 1.00 \quad -1.50]^{\mathsf{T}}$$

The object's position in $\{A\}$ is

$$^{A}P_1 = {}^{A}_{B}T\,{}^{B}P_1 = [-4.89 \quad 2.11 \quad 3.60]^{\mathsf{T}}$$

6. *(2.1)*

$$\begin{aligned} R &= \mathrm{rot}(\hat{Y},\,\phi)\,\mathrm{rot}(\hat{Z},\,\theta) \\ &= \begin{bmatrix} \mathrm{c}\phi & 0 & \mathrm{s}\phi \\ 0 & 1 & 0 \\ -\mathrm{s}\phi & 0 & \mathrm{c}\phi \end{bmatrix} \begin{bmatrix} \mathrm{c}\theta & -\mathrm{s}\theta & 0 \\ \mathrm{s}\theta & \mathrm{c}\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathrm{c}\phi\,\mathrm{c}\theta & -\mathrm{c}\phi\,\mathrm{s}\theta & \mathrm{s}\phi \\ \mathrm{s}\theta & \mathrm{c}\theta & 0 \\ -\mathrm{s}\phi\,\mathrm{c}\theta & \mathrm{s}\phi\,\mathrm{s}\theta & \mathrm{c}\phi \end{bmatrix} \end{aligned}$$

7. *(2.2)*

$$\begin{aligned} R &= \mathrm{rot}(\hat{X},\,60)\,\mathrm{rot}(\hat{Y},\,-45) \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & .500 & -.866 \\ 0 & .866 & .500 \end{bmatrix} \begin{bmatrix} .707 & 0 & -.707 \\ 0 & 1 & 0 \\ .707 & 0 & .707 \end{bmatrix} \\ &= \begin{bmatrix} .707 & 0 & -.707 \\ -.612 & .500 & -.612 \\ .353 & .866 & .353 \end{bmatrix} \end{aligned}$$

8. *(2.12)* Velocity is a "free vector" and only will be affected by rotation, and not by translation:

$$\begin{aligned} {}^{A}V = {}^{A}_{B}R\,{}^{B}V &= \begin{bmatrix} .707 & 0 & -.707 \\ -.612 & .500 & -.612 \\ .353 & .866 & .353 \end{bmatrix} \begin{bmatrix} 30.0 \\ 40.0 \\ 50.0 \end{bmatrix} \\ &= [-14.1 \quad -29.0 \quad 62.9]^{\mathsf{T}} \end{aligned}$$

9. *(2.31)*

$$^{C}_{B}T = \begin{bmatrix} 0 & 0 & -1 & 2 \\ .500 & -.866 & 0 & 0 \\ -.866 & -.500 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10. *(2.37)* Using (2.45) we get that

$$^{B}P_{AORG} = -{}^{A}_{B}R^{\mathsf{T}}\,{}^{A}P_{AORG} = -\begin{bmatrix} .25 & .87 & .43 \\ .43 & -.50 & .75 \\ .86 & .00 & -.50 \end{bmatrix} \begin{bmatrix} 5.0 \\ -4.0 \\ 3.0 \end{bmatrix} = \begin{bmatrix} .94 \\ -6.4 \\ -2.8 \end{bmatrix}$$