2 Review and Applications of Algebra

Exercise 2.1

Basic Problems

1. $(-p) + (-3p) + 4p = -p - 3p + 4p = \underline{\underline{0}}$

2. $(5s - 2t) - (2s - 4t) = 5s - 2t - 2s + 4t = \underline{\underline{3s + 2t}}$

3. $4x^2y + (-3x^2y) - (-5x^2y) = 4x^2y - 3x^2y + 5x^2y = \underline{\underline{6x^2y}}$

4. $1 - (7e^2 - 5 + 3e - e^3) = 1 - 7e^2 + 5 - 3e + e^3 = \underline{\underline{e^3 - 7e^2 - 3e + 6}}$

5. $(6x^2 - 3xy + 4y^2) - (8y^2 - 10xy - x^2) = 6x^2 - 3xy + 4y^2 - 8y^2 + 10xy + x^2$
   $= \underline{\underline{7x^2 - 4y^2 + 7xy}}$

6. $6a - 3a - 2(2b - a) = 6a - 3a - 4b + 2a = \underline{\underline{5a - 4b}}$

7. $\frac{3y}{1.2} + 6.42y - 4y + 7 = 2.5y + 6.42y - 4y + 7 = \underline{\underline{4.92y + 7}}$

8. $13.2 + 7.4t - 3.6 + \frac{2.8t}{0.4} = 13.2 + 7.4t - 3.6 + 7t = \underline{\underline{14.4t + 9.6}}$

Intermediate Problems

9. $4a(3ab - 5a + 6b) = \underline{\underline{12a^2b - 20a^2 + 24ab}}$

10. $9k(4 - 8k + 7k^2) = \underline{\underline{36k - 72k^2 + 63k^3}}$

11. $-5xy(2x^2 - xy - 3y^2) = \underline{\underline{-10x^3y + 5x^2y^2 + 15xy^3}}$

12. $(3p^2 - 5p)(-4p + 2) = -12p^3 + 6p^2 + 20p^2 - 10p = \underline{\underline{-12p^3 + 26p^2 - 10p}}$

13. $3(a - 2)(4a + 1) - 5(2a + 3)(a - 7) = 3(4a^2 + a - 8a - 2) - 5(2a^2 - 14a + 3a - 21)$
    $= 12a^2 - 21a - 6 - 10a^2 + 55a + 105$
    $= \underline{\underline{2a^2 + 34a + 99}}$

14. $5(2x - y)(y + 3x) - 6x(x - 5y) = 5(2xy + 6x^2 - y^2 - 3xy) - 6x^2 + 30xy$
    $= -5xy + 30x^2 - 5y^2 - 6x^2 + 30xy$
    $= \underline{\underline{24x^2 + 25xy - 5y^2}}$

15. $\frac{18x^2}{3x} = \underline{\underline{6x}}$

16. $\frac{6a^2b}{-2ab^2} = \underline{\underline{-3\frac{a}{b}}}$

17. $\frac{x^2y - xy^2}{xy} = \underline{\underline{x - y}}$

18. $\frac{-4x + 10x^2 - 6x^3}{-0.5x} = \underline{\underline{8 - 20x + 12x^2}}$

19. $\frac{12x^3 - 24x^2 + 36x}{48x} = \underline{\underline{\frac{x^2 - 2x + 3}{4}}}$

Exercise 2.1 *(continued)*

20. $\dfrac{120(1+i)^2+180(1+i)^3}{360(1+i)} = \underline{\underline{\dfrac{2(1+i)+3(1+i)^2}{6}}}$

21. $3d^2 - 4d + 15 = 3(2.5)^2 - 4(2.5) + 15$
$= 18.75 - 10 + 15$
$= \underline{\underline{23.75}}$

22. $15g - 9h + 3 = 15(14) - 9(15) + 3 = \underline{\underline{78}}$

23. $7x(4y - 8) = 7(3.2)(4 \times 1.5 - 8) = 22.4(6 - 8) = \underline{\underline{-44.8}}$

24. $(1 + i)^m - 1 = (1 + 0.0225)^4 - 1 = \underline{\underline{0.093083}}$

25. $I \div Pr = \dfrac{\$13.75}{\$500 \times 0.11} = \underline{\underline{0.250}}$

26. $\dfrac{N}{1-d} = \dfrac{\$89.10}{1-0.10} = \underline{\underline{\$99.00}}$

27. $P(1 + rt) = \$770\left(1+0.013\times\dfrac{223}{365}\right) = \$770(1.0079425) = \underline{\underline{\$776.12}}$

28. $\dfrac{S}{1+rt} = \dfrac{\$2500}{1+0.085\times\frac{123}{365}} = \dfrac{\$2500}{1.028644} = \underline{\underline{\$2430.38}}$

29. $P(1 + i)^n = \$1280(1 + 0.025)^3 = \underline{\underline{\$1378.42}}$

30. $\dfrac{S}{(1+i)^n} = \dfrac{\$850}{(1+0.0075)^6} = \dfrac{\$850}{1.045852} = \underline{\underline{\$812.73}}$

Advanced Problems

31. $\dfrac{2x+9}{4} - 1.2(x-1) = 0.5x + 2.25 - 1.2x + 1.2 = \underline{\underline{-0.7x + 3.45}}$

32. $\dfrac{x}{2} - x^2 + \dfrac{4}{5} - 0.2x^2 - \dfrac{4}{5}x + \dfrac{1}{2} = 0.5x - x^2 + 0.8 - 0.2x^2 - 0.8x + 0.5$
$= \underline{\underline{-1.2x^2 - 0.3x + 1.3}}$

33. $\dfrac{8x}{0.5} + \dfrac{5.5x}{11} + 0.5(4.6x - 17) = 16x + 0.5x + 2.3x - 8.5 = \underline{\underline{18.8x - 8.5}}$

34. $\dfrac{2x}{1.045} - \dfrac{2.016x}{3} + \dfrac{x}{2} = 1.9139x - 0.6720x + 0.5x = \underline{\underline{1.7419x}}$

35. $R\left[\dfrac{(1+i)^n - 1}{i}\right] = \$550\left(\dfrac{1.085^3 - 1}{0.085}\right) = \$550\left(\dfrac{0.2772891}{0.085}\right) = \underline{\underline{\$1794.22}}$

Exercise 2.1 *(continued)*

36. $$R\left[\frac{(1+i)^n-1}{i}\right](1+i)=\$910\left(\frac{1.1038129^4-1}{0.1038129}\right)(1.1038129)$$
$$=\$910\left(\frac{0.4845057}{0.1038129}\right)(1.1038129)$$
$$=\underline{\underline{\$4687.97}}$$

37. $$\frac{R}{i}\left[1-\frac{1}{(1+i)^n}\right]=\frac{\$630}{0.115}\left(1-\frac{1}{1.115^2}\right)=\underline{\underline{\$1071.77}}$$

Exercise 2.2

Basic Problems

1. $$I = Prt$$
$$\$6.25 = P(0.05)0.25$$
$$\$6.25 = 0.0125P$$
$$P = \frac{\$6.25}{0.0125} = \underline{\underline{\$500.00}}$$

2. $$PV = \frac{PMT}{i}$$
$$\$150{,}000 = \frac{\$900}{i}$$
$$\$150{,}000i = \$900$$
$$i = \frac{\$900}{\$150{,}000} = \underline{\underline{0.00600}}$$

3. $$S = P(1 + rt)$$
$$\$3626 = P(1 + 0.004\times 9)$$
$$\$3626 = 1.036P$$
$$P = \frac{\$3626}{1.036} = \underline{\underline{\$3500.00}}$$

4. $$N = L(1 - d)$$
$$\$891 = L(1 - 0.10)$$
$$\$891 = 0.90L$$
$$L = \frac{\$891}{0.90} = \underline{\underline{\$990.00}}$$

5. $$N = L(1 - d)$$
$$\$410.85 = \$498(1 - d)$$
$$\frac{\$410.85}{\$498} = 1 - d$$
$$0.825 = 1 - d$$
$$d = 1 - 0.825 = \underline{\underline{0.175}}$$

Exercise 2.2 *(continued)*

6.
$$S = P(1 + rt)$$
$$\$5100 = \$5000(1 + 0.0025t)$$
$$\$5100 = \$5000 + \$12.5t$$
$$\$5100 - \$5000 = \$12.5t$$
$$t = \frac{\$100}{\$12.5} = \underline{\underline{8.00}}$$

7.
$$NI = (CM)X - FC$$
$$\$15{,}000 = CM(5000) - \$60{,}000$$
$$\$15{,}000 + \$60{,}000 = 5000CM$$
$$CM = \frac{\$75{,}000}{5000} = \underline{\underline{\$15.00}}$$

8.
$$NI = (CM)X - FC$$
$$-\$542.50 = (\$13.50)X - \$18{,}970$$
$$\$18{,}970 - \$542.50 = (\$13.50)X$$
$$X = \frac{\$18{,}427.50}{\$13.50} = \underline{\underline{1365}}$$

9.
$$N = L(1 - d_1)(1 - d_2)(1 - d_3)$$
$$\$1468.80 = L(1 - 0.20)(1 - 0.15)(1 - 0.10)$$
$$\$1468.80 = L(0.80)(0.85)(0.90)$$
$$L = \frac{\$1468.80}{0.6120} = \underline{\underline{\$2400.00}}$$

10.
$$c = \frac{V_f - V_i}{V_i}$$
$$0.12 = \frac{V_f - \$6700}{\$6700}$$
$$0.12(\$6700) = V_f - \$6700$$
$$\$804 + \$6700 = V_f$$
$$V_f = \underline{\underline{\$7504.00}}$$

11.
$$c = \frac{V_f - V_i}{V_i}$$
$$0.07 = \frac{\$1850 - V_i}{V_i}$$
$$0.07V_i = \$1850 - V_i$$
$$0.07V_i + V_i = \$1850$$
$$1.07V_i = \$1850$$
$$V_i = \frac{\$1850}{1.07}$$
$$V_i = \underline{\underline{\$1728.97}}$$

Exercise 2.2 *(continued)*

Intermediate Problems

12. $a^2 \times a^3 = \underline{\underline{a^5}}$
13. $(x^6)(x^{-4}) = \underline{\underline{x^2}}$
14. $b^{10} \div b^6 = b^{10-6} = \underline{\underline{b^4}}$
15. $h^7 \div h^{-4} = h^{7-(-4)} = \underline{\underline{h^{11}}}$
16. $(1 + i)^4 \times (1 + i)^9 = \underline{\underline{(1 + i)^{13}}}$
17. $(1 + i) \times (1 + i)^n = \underline{\underline{(1 + i)^{n+1}}}$
18. $(x^4)^7 = x^{4x7} = \underline{\underline{x^{28}}}$
19. $(y^3)^3 = \underline{\underline{y^9}}$
20. $\left(t^6\right)^{\frac{1}{3}} = \underline{\underline{t^2}}$
21. $(n^{0.5})^8 = \underline{\underline{n^4}}$
22. $\dfrac{\left(x^5\right)\left(x^6\right)}{x^9} = x^{5+6-9} = \underline{\underline{x^2}}$
23. $\dfrac{\left(x^5\right)^6}{x^9} = x^{5\times6-9} = \underline{\underline{x^{21}}}$
24. $\left[2(1+i)\right]^2 = \underline{\underline{4(1+i)^2}}$
25. $\left(\dfrac{1+i}{3i}\right)^3 = \underline{\underline{\dfrac{(1+i)^3}{27i^3}}}$
26. $8^{4/3} = \left(8^{1/3}\right)^4 = 2^4 = \underline{\underline{16}}$
27. $-27^{2/3} = -\left(27^{1/3}\right)^2 = \underline{\underline{-9}}$
28. $\left(\dfrac{2}{5}\right)^3 = 0.4^3 = \underline{\underline{0.064}}$
29. $5^{-3/4} = 5^{-0.75} = \underline{\underline{0.299070}}$
30. $(0.001)^{-2} = \underline{\underline{1,000,000}}$
31. $0.893^{-1/2} = 0.893^{-0.5} = \underline{\underline{1.05822}}$
32. $(1.0085)^5(1.0085)^3 = 1.0085^8 = \underline{\underline{1.07006}}$
33. $(1.005)^3(1.005)^{-6} = 1.005^{-3} = \underline{\underline{0.985149}}$

Exercise 2.2 *(continued)*

34. $\sqrt[3]{1.03} = 1.03^{0.\overline{3}} = \underline{\underline{1.00990}}$

35. $\sqrt[6]{1.05} = \underline{\underline{1.00816}}$

Advanced Problems

36. $$\frac{4r^5t^6}{(2r^2t)^3} = \frac{4r^5t^6}{8r^6t^3} = \frac{r^{5-6}t^{6-3}}{2} = \frac{t^3}{\underline{\underline{2r}}}$$

37. $$\frac{(-r^3)(2r)^4}{(2r^{-2})^2} = \frac{-r^3(16r^4)}{4r^{-4}} = -4r^{3+4-(-4)} = \underline{\underline{-4r^{11}}}$$

38. $$\frac{(3x^2y^3)^5}{(xy^2)^3} = \frac{243x^{10}y^{15}}{x^3y^6} = 243x^{10-3}y^{15-6} = \underline{\underline{243x^7y^9}}$$

39. $$\frac{6(-3xy)^4}{(-3x^{-3})^2} = \frac{6(81x^4y^4)}{9x^{-6}} = \frac{486x^4y^4}{9x^{-6}} = 54x^{4-(-6)}y^4 = \underline{\underline{54x^{10}y^4}}$$

40. $$(4^4)(3^{-3})\left(-\frac{3}{4}\right)^3 = \frac{4^4}{3^3}\left(-\frac{3^3}{4^3}\right) = \underline{\underline{-4}}$$

41. $$\left[\left(-\frac{3}{4}\right)^2\right]^{-2} = \left(-\frac{3}{4}\right)^{-4} = \left(-\frac{4}{3}\right)^4 = \frac{256}{81} = \underline{\underline{3.16049}}$$

42. $$\left(\frac{2}{3}\right)^3\left(-\frac{3}{2}\right)^2\left(-\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3\left(\frac{3}{2}\right)^2\left(-\frac{2}{3}\right)^3 = \frac{2}{3}\left(-\frac{2}{3}\right)^3 = -\frac{16}{81} = \underline{\underline{-0.197531}}$$

43. $$\frac{1.03^{16} - 1}{0.03} = \underline{\underline{20.1569}}$$

44. $$\frac{1 - 1.0225^{-20}}{0.0225} = \frac{0.3591835}{0.0225} = \underline{\underline{15.9637}}$$

45. $(1 + 0.055)^{1/6} - 1 = \underline{\underline{0.00896339}}$

Exercise 2.3

Basic Problems

1. $10a + 10 = 12 + 9a$
   $10a - 9a = 12 - 10$
   $a = \underline{\underline{2}}$

Exercise 2.3 *(continued)*

2. $29 - 4y = 2y - 7$
   $36 = 6y$
   $y = \underline{\underline{6}}$

3. $0.5(x - 3) = 20$
   $x - 3 = 40$
   $x = \underline{\underline{43}}$

4. $\frac{1}{3}(x - 2) = 4$
   $x - 2 = 12$
   $x = \underline{\underline{14}}$

5. $y = 192 + 0.04y$
   $y - 0.04y = 192$
   $y = \frac{192}{0.96} = \underline{\underline{200}}$

6. $x - 0.025x = 341.25$
   $0.975x = 341.25$
   $x = \frac{341.25}{0.975} = \underline{\underline{350}}$

7. $12x - 4(2x - 1) = 6(x + 1) - 3$
   $12x - 8x + 4 = 6x + 6 - 3$
   $-2x = -1$
   $x = \underline{\underline{0.5}}$

8. $3y - 4 = 3(y + 6) - 2(y + 3)$
   $= 3y + 18 - 2y - 6$
   $2y = 16$
   $y = \underline{\underline{8}}$

9. $8 - 0.5(x + 3) = 0.25(x - 1)$
   $8 - 0.5x - 1.5 = 0.25x - 0.25$
   $-0.75x = -6.75$
   $x = \underline{\underline{9}}$

10. $5(2 - c) = 10(2c - 4) - 6(3c + 1)$
    $10 - 5c = 20c - 40 - 18c - 6$
    $-7c = -56$
    $c = 8$

Exercise 2.3 *(continued)*

Intermediate Problems

11.

$$\begin{aligned} x - y &= 2 \quad ① \\ 3x + 4y &= 20 \quad ② \\ ① \times 3: \quad \underline{3x - 3y} &= \underline{\;\;6} \\ \text{Subtract:} \quad 7y &= 14 \\ y &= 2 \end{aligned}$$

Substitute into equation ①:

$$\begin{aligned} x - 2 &= 2 \\ x &= 4 \\ (x, y) &= \underline{(4, 2)} \end{aligned}$$

Check: LHS of ② = 3(4) + 4(2) = 20 = RHS of ②

12.

$$\begin{aligned} y - 3x &= 11 \quad ① \\ -4y + 5x &= -30 \quad ② \\ ① \times 4: \quad \underline{4y - 12x} &= \underline{\;\;44} \\ \text{Add:} \quad -7x &= 14 \\ x &= -2 \end{aligned}$$

Substitute into equation ①:

$$\begin{aligned} y - 3(-2) &= 11 \\ y &= 11 - 6 = 5 \\ (x, y) &= \underline{(-2, 5)} \end{aligned}$$

Check: LHS of ② = – 4(5) + 5(–2) = –30= RHS of ②

13.

$$\begin{aligned} 7p - 3q &= 23 \quad ① \\ \underline{-2p - 3q} &= \underline{\;\;5} \quad ② \\ \text{Subtract:} \quad 9p \quad &= 18 \\ p &= 2 \end{aligned}$$

Substitute into equation ①:

$$\begin{aligned} 7(2) - 3q &= 23 \\ 3q &= -23 + 14 \\ q &= -3 \\ (p, q) &= \underline{(2, -3)} \end{aligned}$$

Check: LHS of ② = –2(2) –3(–3) = 5 = RHS of ②

14.

$$\begin{aligned} y &= 2x \quad ① \\ \underline{7x - y} &= \underline{\quad\;\; 35} \quad ② \\ \text{Add:} \quad 7x \quad &= 2x + 35 \\ 5x &= 35 \\ x &= 7 \end{aligned}$$

Substitute into ①:

$$\begin{aligned} y &= 2(7) = 14 \\ (x, y) &= \underline{(7, 14)} \end{aligned}$$

Check: LHS of ② = 7(7) – 14 = 49 – 14 = 35 = RHS of ②

Exercise 2.3 *(continued)*

15.

$$-3c + d = -500 \quad ①$$
$$0.7c + 0.2d = 550 \quad ②$$

To eliminate d,

$$\begin{aligned} ① \times 0.2: &\quad -0.6c + 0.2d = -100 \\ ②: &\quad \underline{0.7c + 0.2d} = \underline{\;550} \\ \text{Subtract:} &\quad -1.3c + 0 = -650 \\ &\quad c = 500 \end{aligned}$$

Substitute into ①:

$$d = 3(500) - 500 = 1000$$
$$(c, d) = \underline{(500, 1000)}$$

Check: LHS of ② = 0.7(500) + 0.2(1000) = 550 = RHS of ②

16.

$$0.03x + 0.05y = 51 \quad ①$$
$$0.8x - 0.7y = 140 \quad ②$$

To eliminate y,

$$\begin{aligned} ① \times 0.7: &\quad 0.021x + 0.035y = 35.7 \\ ② \times 0.05: &\quad \underline{0.04x - 0.035y} = \underline{\;7\;} \\ \text{Add:} &\quad 0.061x + 0 = 42.7 \\ &\quad x = 700 \end{aligned}$$

Substitute into ②:

$$0.8(700) - 0.7y = 140$$
$$-0.7y = -420$$
$$y = 600$$
$$(x, y) = \underline{(700, 600)}$$

Check: LHS of ① = 0.03(700) + 0.05(600) = 51 = RHS of ①

17.

$$2v + 6w = 1 \quad ①$$
$$10v - 9w = 18 \quad ②$$

To eliminate v,

$$\begin{aligned} ① \times 10: &\quad 20v + 60w = 10 \\ ② \times 2: &\quad \underline{20v - 18w} = \underline{36} \\ \text{Subtract:} &\quad 0 + 78w = -26 \\ &\quad w = -\tfrac{1}{3} \end{aligned}$$

Substitute into ①:

$$2v + 6\left(-\tfrac{1}{3}\right) = 1$$
$$2v = 1 + 2$$
$$v = \tfrac{3}{2}$$
$$(v, w) = \underline{\underline{\left(\tfrac{3}{2}, -\tfrac{1}{3}\right)}}$$

Check: LHS of ② = $10\left(\tfrac{3}{2}\right) - 9\left(-\tfrac{1}{3}\right)$ = 18 = RHS of ②

Exercise 2.3 *(continued)*

18.

$$2.5a + 2b = 11 \quad ①$$
$$8a + 3.5b = 13 \quad ②$$

To eliminate b,

$$① \times 3.5: \quad 8.75a + 7b = 38.5$$
$$② \times 2: \quad \underline{16a + 7b} = \underline{26}$$
$$\text{Subtract:} \quad -7.25a + 0 = 12.5$$
$$a = -1.724$$

Substitute into ①:

$$2.5(-1.724) + 2b = 11$$
$$2b = 11 + 4.31$$
$$b = 7.655$$
$$(a, b) = \underline{(-1.72, 7.66)}$$

Check: LHS of ② = 8(–1.724) + 3.5(7.655) = 13.00 = RHS of ②

19.

$$37x - 63y = 235 \quad ①$$
$$18x + 26y = 468 \quad ②$$

To eliminate x,

$$① \times 18: \quad 666x - 1134y = 4230$$
$$② \times 37: \quad \underline{666x + 962y} = \underline{17{,}316}$$
$$\text{Subtract:} \quad 0 - 2096y = -13{,}086$$
$$y = 6.243$$

Substitute into ①:

$$37x - 63(6.243) = 235$$
$$37x = 628.3$$
$$x = 16.98$$
$$(x, y) = \underline{(17.0, 6.24)}$$

Check: LHS of ② = 18(16.98) + 26(6.243) = 468.0 = RHS of ②

20.

$$68.9n - 38.5m = 57 \quad ①$$
$$45.1n - 79.4m = -658 \quad ②$$

To eliminate n,

$$① \times 45.1: \quad 3107n - 1736.4m = 2571$$
$$② \times 68.9: \quad \underline{3107n - 5470.7m} = \underline{-45{,}336}$$
$$\text{Subtract:} \quad 0 + 3734.3m = 47{,}907$$
$$m = 12.83$$

Substitute into ①:

$$68.9n - 38.5(12.83) = 57$$
$$68.9n = 551.0$$
$$n = 7.996$$
$$(m, n) = \underline{(12.8, 8.00)}$$

Check: LHS of ② = 45.1(7.996) – 79.4(12.83) = –658.1 = RHS of ②

Advanced Problems

21.

$$\frac{x}{1.1^2} + 2x(1.1)^3 = \$1000$$
$$0.8264463x + 2.622x = \$1000$$
$$3.488446x = \$1000$$
$$x = \underline{\underline{\$286.66}}$$

Exercise 2.3 *(continued)*

22. $$\frac{3x}{1.025^6} + x(1.025)^8 = \$2641.35$$

$$2.586891x + 1.218403x = \$2641.35$$

$$x = \underline{\underline{\$694.13}}$$

23. $$\frac{2x}{1.03^7} + x + x\left(1.03^{10}\right) = \$1000 + \frac{\$2000}{1.03^4}$$

$$1.626183x + x + 1.343916x = \$1000 + \$1776.974$$

$$3.970099x = \$2776.974$$

$$x = \underline{\underline{\$699.47}}$$

24. $$x(1.05)^3 + \$1000 + \frac{x}{1.05^7} = \frac{\$5000}{1.05^2}$$

$$1.157625x + 0.7106813x = \$4535.147 - \$1000$$

$$x = \underline{\underline{\$1892.17}}$$

25. $$x\left(1 + 0.095 \times \frac{84}{365}\right) + \frac{2x}{1 + 0.095 \times \frac{108}{365}} = \$1160.20$$

$$1.021863x + 1.945318x = \$1160.20$$

$$2.967181x = \$1160.20$$

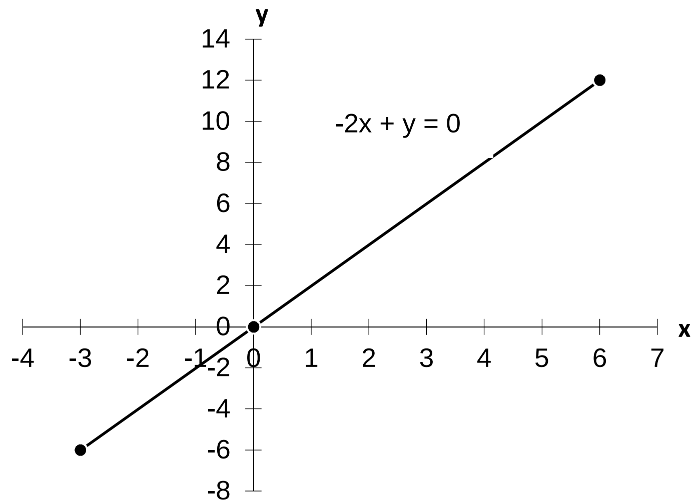
$$x = \underline{\underline{\$391.01}}$$

Exercise 2.4

Basic Problems

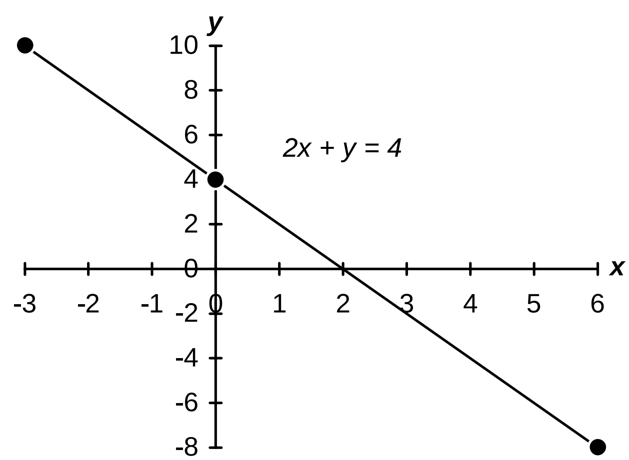
1.

x:	–3	0	6
y:	–6	0	12



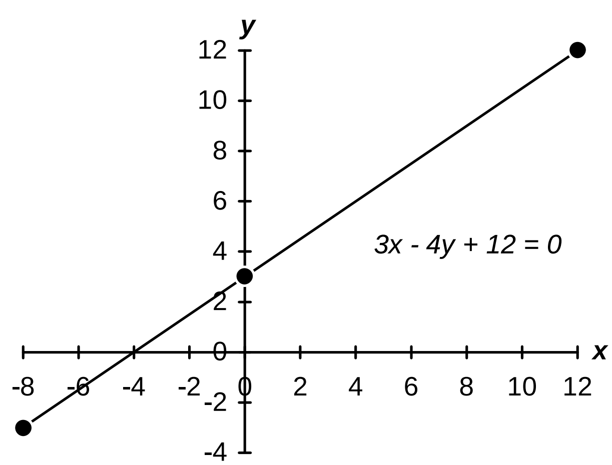
2.

x:	–3	0	6
y:	10	4	–8



3.

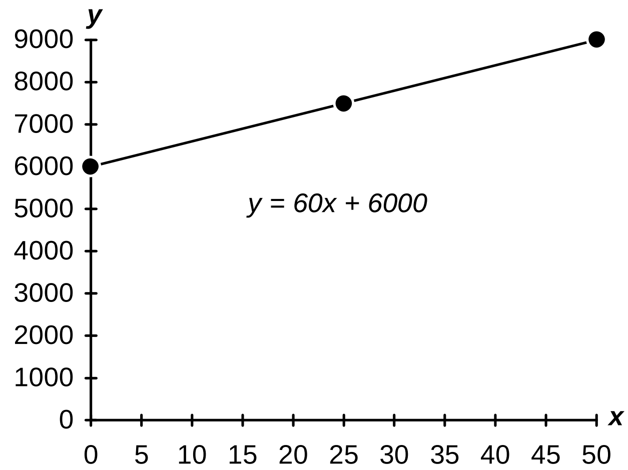
x:	–8	0	12
y:	–3	3	12



Exercise 2.4 *(continued)*

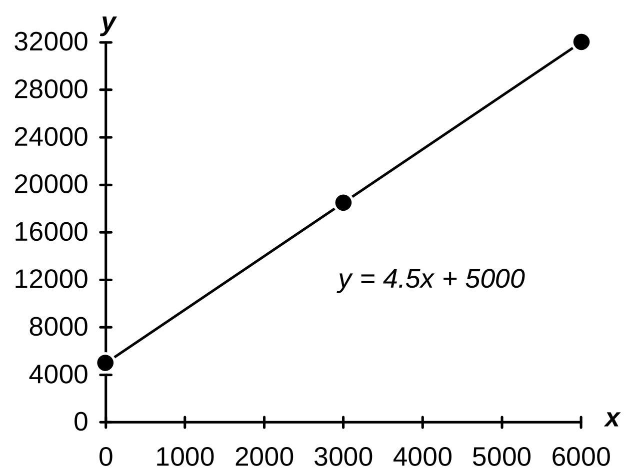
4.

x:	0	25	50
y:	6000	7500	9000



5.

x:	0	3000	6000
y:	5000	18,500	32,000



6. In each part, rearrange the equation to render it in the form $y = mx + b$

a. $2x = 3y + 4$

$3y = 2x - 4$

$y = \frac{2}{3}x - \frac{4}{3}$

The slope is $m = \underline{\underline{\frac{2}{3}}}$ and the y-intercept is $b = \underline{\underline{-\frac{4}{3}}}$.

b. $8 - 3x = 2y$

$2y = -3x + 8$

$y = -\frac{3}{2}x + 4$

The slope is $m = \underline{\underline{-\frac{3}{2}}}$ and the y-intercept is $b = \underline{\underline{4}}$

(continued)

Exercise 2.4 *(continued)*

6. *c.* $8x - 2y - 3 = 0$

$$-2y = -8x + 3$$

$$y = 4x - \tfrac{3}{2}$$

The slope is $m = \underline{\underline{4}}$ and the y-intercept is $b = \underline{\underline{-\tfrac{3}{2}}}$.

d. $6x = 9y$

$$y = \tfrac{6}{9}x = \tfrac{2}{3}x$$

$$y = \tfrac{2}{3}x$$

The slope is $m = \underline{\underline{\tfrac{2}{3}}}$ and the y-intercept is $b = \underline{\underline{0}}$.

Intermediate Problems

7. The plumber charges a \$100 service charge plus 4(\$20) = \$80 per hour

Then $C = \$100 + \$80H$

Expressing this equation in the form $y = mx + b$

$$C = \$80H + \$100$$

On a plot of C vs. H, slope = \$80 and C-intercept = \$100.

8. Ehud earns \$1500 per month plus 5% of sales. Then gross earnings

$$E = \$1500 + 0.05R$$

Expressing this equation in the form $y = mx + b$

$$E = 0.05R + \$1500$$

On a plot of E vs. R, slope = 0.05 and E-intercept = \$1500.

9. *a.* Comparing the equation $F = \tfrac{9}{5}C + 32$ to $y = mx + b$,

we can conclude that a plot of F vs. C will have

slope = $\underline{\underline{\tfrac{9}{5}}}$ and F-intercept = 32 .

b. $\text{Slope} = \dfrac{\text{Change in F}}{\text{Change in C}}$

Therefore, (Change in F) = Slope(Change in C) = $\tfrac{9}{5}$(10 Celsius) = 18 Fahrenheit

c. $F = \tfrac{9}{5}C + 32$

$$\tfrac{9}{5}C = F - 32$$

$$C = \tfrac{5}{9}F - \tfrac{5}{9}(32) = \tfrac{5}{9}F - 17\tfrac{7}{9}$$

On a plot of C vs. F, slope = $\underline{\underline{\tfrac{5}{9}}}$ and C-intercept = $\underline{\underline{-17\tfrac{7}{9}}}$.

Exercise 2.4 *(continued)*

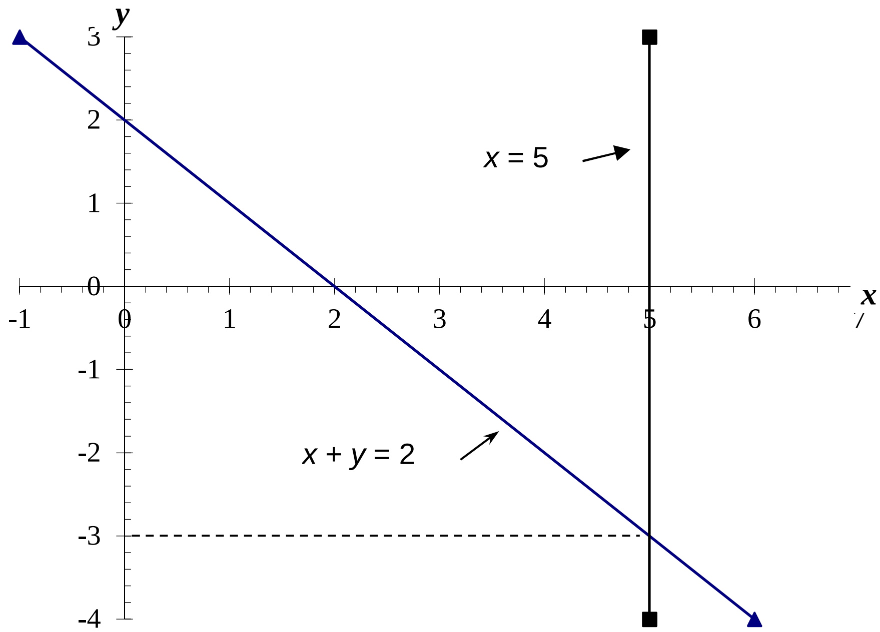
10. $x + y = 2$

x:	–1	6
y:	3	– 4

$x = 5$

x:	5	5
y:	3	– 4

The solution is
$(x, y) = (5, –3)$.



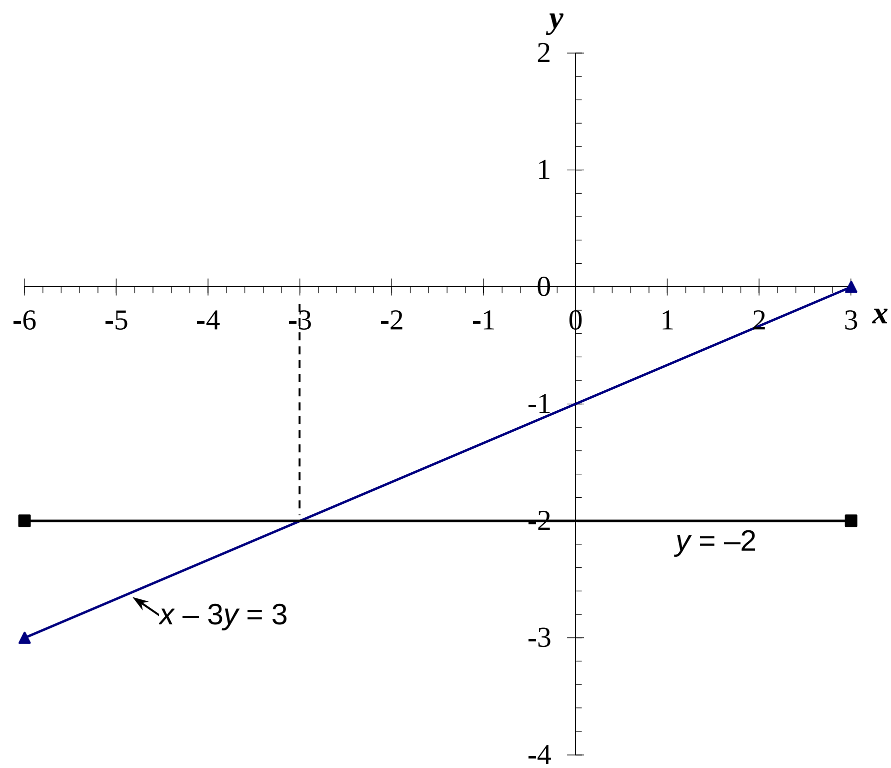
11. $x – 3y = 3$

x:	–6	3
y:	–3	0

$y = –2$

x:	–6	3
y:	–2	–2

The solution is
$(x, y) = (–3, –2)$.



Exercise 2.4 *(continued)*

12.

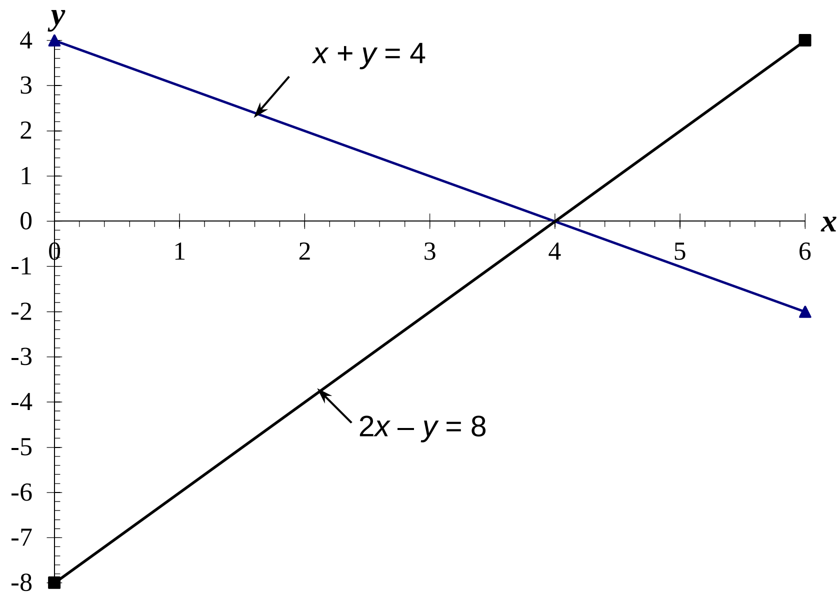
$x + y = 4$

x:	0	6
y:	4	–2

$2x - y = 8$

x:	0	6
y:	–8	4

The solution is
$(x, y) = (4, 0)$.



13.

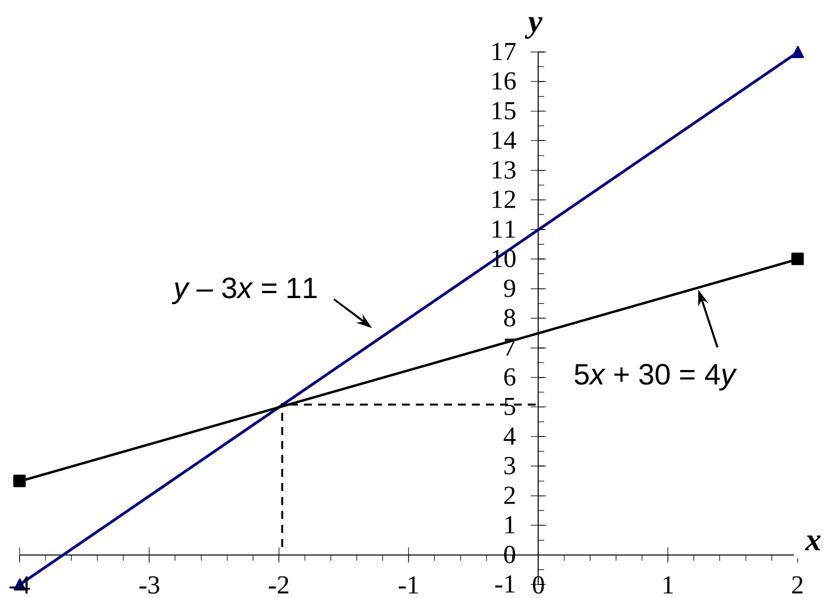
$y - 3x = 11$

x:	– 4	2
y:	–1	17

$5x + 30 = 4y$

x:	– 4	2
y:	2.5	10

The solution is
$(x, y) = (–2, 5)$.



Exercise 2.4 *(continued)*

Advanced Problem

14. *a.* Given: $TR = \$6X$

    On a plot of TR vs. X, slope = \$6 and TR-intercept = \$0.

    b. $TC = \$2X + \$80,000$

    On a plot of TC vs. X, slope = \$2 and TC-intercept = \$80,000.

    c. $NI = \$4X – \$80,000$

    On a plot of NI vs. X, slope = \$4 and NI-intercept = – \$80,000.

    d. The steepest line is the one with the largest slope.

    Therefore, the *TR* line is steepest.

    e. The increase in *NI* per pair of sunglasses sold is the "change in *NI*" divided by the "change in *X*". This is just the slope of the *NI* vs. *X* line. Therefore, *NI* increases by \$4 for each pair of sunglasses sold.

    f. The coefficient of *X* in the *TR* equation is the unit selling price, which is unchanged.

    Therefore, the slope remains unchanged.

    The coefficient of *X* in the *TC* equation is the unit cost.

    Therefore, the slope decreases (from \$2 to \$1.75).

    The coefficient of *X* in the *NI* equation equals

    (Unit selling price) – (Unit cost)

    Therefore, the slope increases (from \$4 to \$4.25).

Exercise 2.5

Basic Problems

1. Step 2: Hits last month = 2655 after the $\frac{2}{7}$ increase.

   Let the number of hits 1 year ago be n.

   Step 3: Hits last month = Hits 1 year ago + $\frac{2}{7}$(Hits 1 year ago)

   Step 4: $2655 = n + \frac{2}{7}n$

   Step 5: $2655 = \frac{9}{7}n$

   Multiply both sides by $\frac{7}{9}$.

   $n = 2655 \times \frac{7}{9} = 2065$

   The Web site had 2065 hits in the same month 1 year ago.

2. Step 2: Retail price = \$712; Markup = 60% of wholesale of cost.
   Let the wholesale cost be C.
   Step 3: Retail price = Cost + 0.60(Cost)
   Step 4: \$712 = C + 0.6C
   Step 5: \$712 = 1.6C

   $C = \frac{\$712}{1.6} = \underline{\underline{\$445.00}}$. The wholesale cost is \$445.00.

Exercise 2.5 *(continued)*

3. Step 2: Tag price = \$39.95 (including 13% HST). Let the plant's pretax price be P.
   Step 3: Tag price = Pretax price + HST

   Step 4: \$39.95 = P + 0.13P

   Step 5: \$39.95 = 1.13P

   $$P = \frac{\$39.95}{1.13} = \$35.35$$

   The amount of HST is \$39.95 – \$35.35 = \$4.60

4. Step 2: Commission rate = 2.5% on the first \$5000 and 1.5% on the remainder
   Commission amount = \$227. Let the transaction amount be x.

   Step 3: Commission amount = 0.025(\$5000) + 0.015(Remainder)

   Step 4: \$227 = \$125.00 + 0.015(x – \$5000)

   Step 5: \$102 = 0.015x – \$75.00
   \$102 + \$75 = 0.015x

   $$x = \frac{\$177}{0.015} = \$11,800.00$$

   The amount of the transaction was \$11,800.00.

5. Step 2: Let the basic price be P. First 20 meals at P.
   Next 20 meals at P – \$2. Additional meals at P – \$3.

   Step 3: Total price for 73 meals = \$1686

   Step 4: 20P + 20(P – \$2) + (73 – 40)(P – \$3) = \$1686

   Step 5: 20P + 20P – \$40 + 33P – \$99 = \$1686
   73P = \$1686 + \$99 + \$40

   $$P = \frac{\$1825}{73} = \$25.00$$

   The basic price per meal is \$25.00.

6. Step 2: Rental Plan 1: \$295 per week + \$0.15 × (Distance in excess of 1000 km)
   Rental Plan 2: \$389 per week
   Let d represent the distance at which the costs of both plans are equal.

   Step 3: Cost of Plan 1 = Cost of Plan 2

   Step 4: \$295 + \$0.15(d – 1000) = \$389

   Step 5: \$295 + \$0.15d – \$150 = \$389
   \$0.15d = \$244
   d = 1627 km

   To the nearest kilometre, the unlimited driving plan will be cheaper if you drive more than 1627 km in the one-week interval.

Exercise 2.5 *(continued)*

7. Step 2: Tax rate = 38%; Overtime hourly rate = 1.5($23.50) = $35.25
   Cost of canoe = $2750
   Let h represent the hours of overtime Alicia must work.

   Step 3: Gross overtime earnings – Income tax = Cost of the canoe

   Step 4: 35.25h$ – 0.38(35.25h$) = $2750

   Step 5: 21.855h$ = $2750
   h = 125.83 hours

   Alicia must work 125¾ hours of overtime to earn enough money to buy the canoe.

8. Let x represent the number of units of product X and y represent the number of units of product Y. Then

   $x + y = 93$ ①
   $0.5x + 0.75y = 60.5$ ②
   ① × 0.5: $0.5x + 0.5y = 46.5$
   Subtract: $0 + 0.25y = 14$
   $y = 56$
   Substitute into ①: $x + 56 = 93$
   $x = 37$

   Therefore, 37 units of X and 56 units of Y were produced last week.

9. Let the price per litre of milk be m and the price per dozen eggs be e. Then

   5m + 4e = $19.51 ①
   9m + 3e = $22.98 ②

   To eliminate e,
   ① × 3: 15m + 12e = $58.53
   ② × 4: 36m + 12e = $91.92
   Subtract: –21 m + 0 = –$33.39
   m = $1.59

   Substitute into ①: 5($1.59) + 4e = $19.51
   e = $2.89

   Milk costs $1.59 per litre and eggs cost $2.89 per dozen.

10. Let M be the number of litres of milk and J be the number of cans of orange juice per week.

    $1.50M + $1.30J = $57.00 ①
    $1.60M + $1.39J = $60.85 ②

    To eliminate M,
    ① × 1.60: $2.40M + $2.08J = $91.20
    ② × 1.50: $2.40M + $2.085J = $91.275
    Subtract: 0 – 0.005J = –$0.075
    J = 15

    Substitution of J = 15 into either equation will give M = 25. Hence 25 litres of milk and 15 cans of orange juice are purchased each week.

Exercise 2.5 *(continued)*

Intermediate Problems

11. Step 2: Number of two-bedroom homes = 0.4(Number of three-bedroom homes)
Number of two-bedroom homes = 2(Number of four-bedroom homes)
Total number of homes = 96
Let h represent the number of two-bedroom homes

Step 3: # 2-bedroom homes + # 3-bedroom homes + # 4-bedroom homes = 96

Step 4: $h+\frac{h}{0.4}+\frac{h}{2}=96$

Step 5: $h + 2.5h + 0.5h = 96$
$4h = 96$
$h = 24$

There should be <u>24 two-bedroom homes</u>, 2.5(24) = <u>60 three-bedroom homes</u>, and 0.5(24) = <u>12 four-bedroom homes</u>.

12. Step 2: Cost of radio advertising = 0.5(Cost of internet advertising)
Cost of TV advertising = 0.6(Cost of radio advertising)
Total advertising budget = \$160,000
Let r represent the amount allocated to radio advertising

Step 3: Radio advertising + TV advertising + Internet advertising = \$160,000

Step 4: $r+0.6r+\frac{r}{0.5}=\$160{,}000$

Step 5: $3.6r = \$160{,}000$
$r = \$44{,}444.44$

The advertising budget allocations should be:
<u>\$44,444 to radio advertising</u>,
0.6(\$44,444.44) = <u>\$26,667 to TV advertising</u>, and
2(\$44,444.44) = <u>\$88,889 to internet advertising</u>.

13. Step 2: By-laws require: 5 parking spaces per 100 square meters,
4% of spaces for customers with physical disabilities
In remaining 96%, # regular spaces = 1.4(# small car spaces)
Total area = 27,500 square meters
Let s represent the number of small car spaces.

Step 3: Total # spaces = # spaces for customers with physical disabilities + # regular spaces + # small spaces

Step 4: $\frac{27{,}500}{100}\times 5 = 0.04\times\frac{27{,}500}{100}\times 5 + s + 1.4s$

Step 5: $1375 = 55 + 2.4s$
$s = 550$

The shopping centre must have <u>55 parking spaces for customers with physical disabilities</u>, <u>550 small-car spaces</u>, and <u>770 regular parking spaces</u>.

Exercise 2.5 *(continued)*

14. Step 2: Overall portfolio's rate of return = 1.1%, equity fund's rate of return = –3.3%,
    bond fund's rate of return = 7.7%.
    Let e represent the fraction of the portfolio initially invested in the equity fund.

    Step 3: Overall rate of return = Weighted average rate of return
    = (Equity fraction)(Equity return) + (Bond fraction)(Bond return)

    Step 4: $1.1\% = e(-3.3\%) + (1 - e)(7.7\%)$

    Step 5:
    $$1.1 = -3.3e + 7.7 - 7.7e$$
    $$-6.6 = -11.0e$$
    $$e = 0.600$$

    Therefore, 60.0% of Erin's original portfolio was invested in the equity fund.

15. Step 2: Pile A steel is 5.25% nickel; pile B steel is 2.84% nickel.
    We want a 32.5-tonne mixture from A and B averaging 4.15% nickel.
    Let A represent the tonnes of steel required from pile A.

    Step 3: Wt. of nickel in 32.5 tonnes of mixture
    = Wt. of nickel in steel from pile A + Wt. of nickel in steel from pile B
    = (% nickel in pile A)(Amount from A) + (% nickel in pile B)(Amount from B)

    Step 4: $0.0415(32.5) = 0.0525A + 0.0284(32.5 - A)$

    Step 5:
    $$1.34875 = 0.0525A + 0.9230 - 0.0284A$$
    $$0.42575 = 0.0241A$$
    $$A = 17.67 \text{ tonnes}$$

    The recycling company should mix 17.67 tonnes from pile A with 14.83 tonnes from pile B.

16. Step 2: Total options = 100,000
    \# of options to an executive = 2000 + # of options to an engineer
    \# of options to an engineer = 1.5(# of options to a technician)
    There are 3 executives, 8 engineers, and 14 technicians.
    Let t represent the number of options to each technician.

    Step 3: Total options = Total options to engineers
    + Total options to technicians + Total options to executives

    Step 4: $100{,}000 = 8(1.5t) + 14t + 3(2000 + 1.5t)$

    Step 5:
    $$= 12t + 14t + 6000 + 4.5t$$
    $$94{,}000 = 30.5t$$
    $$t = 3082 \text{ options}$$

    Each technician will receive 3082 options,
    each engineer will receive 1.5(3082) = 4623 options,
    and each executive will receive 2000 + 4623 = 6623 options.

17. Step 2: Plan A: 20 cents/minute for local calls and 40 cents/minute for long distance calls
Plan B: 35 cents/minute any time
Let d represent the fraction of long-distance usage at which costs are equal.

Step 3: Cost of Plan A = Cost of plan B

Step 4: Pick any amount of usage in a month—say 1000 minutes.
$d(1000)\$0.40 + (1 - d)(1000)\$0.20 = 1000(\$0.35)$

Step 5:
$$400d + 200 - 200d = 350$$
$$200d = 150$$
$$d = 0.75$$

If long distance usage exceeds 75% of overall usage, plan B will be cheaper.

18. Step 2: Raisins cost \$3.75 per kg; peanuts cost \$2.89 per kg.
Cost per kg of ingredients in 50 kg of "trail mix" is to be \$3.20.
Let p represent the weight of peanuts in the mixture.

Step 3: Cost of 50 kg of trail mix = Cost of p kg peanuts + Cost of $(50 - p)$ kg of raisins

Step 4: $50(\$3.20) = p(\$2.89) + (50 - p)(\$3.75)$

Step 5: $\$160.00 = \$2.89p + \$187.50 - \$3.75p$
$-\$27.50 = -\$0.86p$
$p = 31.98$ kg

32.0 kg of peanuts should be mixed with 18.0 kg of raisins.

19. Step 2: Total bill = \$3310. Total hours = 41.
Hourly rate = \$120 for CGA
= \$50 for clerk.
Let x represent the CGA's hours.

Step 3: Total bill = (CGA hours x CGA rate) + (Clerk hours x Clerk rate)

Step 4: $\$3310 = x(\$120) + (41 - x)\$50$

Step 5: $\$3310 = \$120x + \$2050 - \$50x$
$1260 = 70x$
$x = 18$

The CGA worked 18 hours and the clerk worked 41 – 18 = 23 hours.

20. Step 2: Total investment = \$32,760
Sue's investment = 1.2(Joan's investment)
Joan's investment = 1.2(Stella's investment)
Let L represent Stella's investment.

Step 3: Sue's investment + Joan's investment + Stella's investment = Total investment

Step 4: Joan's investment = 1.2L
Sue's investment = 1.2(1.2L) = 1.44L
1.44L + 1.2L + L = \$32,760

Step 5:
$$3.64L = \$32{,}760$$
$$L = \frac{\$32{,}760}{3.64} = \$9000$$

(continued)

Exercise 2.5 *(continued)*

Stella will contribute $9000, Joan will contribute 1.2($9000) = $10,800, and Sue will contribute 1.2($10,800) = $12,960

21. Step 2: Sven receives 30% less than George (or 70% of George's share).
    Robert receives 25% more than George (or 1.25 times George's share).
    Net income = $88,880
    Let G represent George's share.

    Step 3: George's share + Robert's share + Sven's share = Net income

    Step 4: G + 1.25G + 0.7G = $88,880

    Step 5: 2.95G = $88,880
    G = $30,128.81
    George's share is $30,128.81, Robert's share is 1.25($30,128.81) = $37,661.02, and Sven's share is 0.7($30,128.81) = $21,090.17.

22. Step 2: Time to make X is 20 minutes.
    Time to make Y is 30 minutes.
    Total time is 47 hours. Total units = 120. Let Y represent the number of units of Y.

    Step 3: Total time = (Number of X) × (Time for X) + (Number of Y) × (Time for Y)

    Step 4: 47 × 60 = (120 – Y)20 + Y(30)

    Step 5: 2820 = 2400 – 20Y + 30Y
    420 = 10Y
    Y = 42
    Forty-two units of product Y were manufactured.

23. Step 2: Price of blue ticket = $19.00. Price of red ticket = $25.50.
    Total tickets = 4460. Total revenue = $93,450.
    Let the number of tickets in the red section be R.

    Step 3: Total revenue = (Number of red × Price of red) + (Number of blue × Price of blue)

    Step 4: $93,450 = R($25.50) + (4460 – R)$19.00

    Step 5: 93,450 = 25.5R + 84,740 – 19R
    6.5R = 8710
    R = 1340
    1340 seats were sold in the red section and 4460 – 1340 = 3120 seats were sold in the blue section.

24. Step 2: Regal owns a 58% interest in a mineral claim. Yukon owns the remainder (42%).
    Regal sells one fifth of its interest for $1.2 million.
    Let the V represent the implied value of the entire mineral claim.

    Step 3: $\frac{1}{5}$ (or 20%) of a 58% interest is worth $1.2 million

    Step 4: 0.20(0.58)V = $1,200,000

    Step 5: $V = \frac{\$1,200,000}{0.20 \times 0.58} = \$10,344,828$

    The implied value of Yukon's interest is
    0.42V = 0.42 × $10,344,828 = $4,344,828

Exercise 2.5 *(continued)*

25. Step 2: $\frac{5}{7}$ of entrants complete Level 1. $\frac{2}{9}$ of Level 1 completers fail Level 2.
587 students completed Level 2 last year.
Let the N represent the original number who began Level 1.

Step 3: $\frac{7}{9}$ of $\frac{5}{7}$ of entrants will complete Level 2.

Step 4: $\frac{7}{9}\times\frac{5}{7}\text{N} = 587$

Step 5: $\text{N} = \frac{9\times 7}{7\times 5}$ x 587 = 1056.6

<u>1057</u> students began Level 1.

26. Step 2: $\frac{4}{7}$ of inventory was sold at cost.
$\frac{3}{7}$ inventory was sold to liquidators at 45% of cost, yielding \$6700.
Let C represent the original cost of the entire inventory.

Step 3: $\frac{3}{7}$ of inventory was sold to liquidators at 45% of cost, yielding \$6700.

Step 4: $\frac{3}{7}(0.45\text{C}) = \$6700$

Step 5: $\text{C} = \frac{7\times \$6700}{3\times 0.45} = \$34{,}740.74$

a. The cost of inventory sold to liquidators was
$\frac{3}{7}$(\$34,740.74) = <u>\$14,888.89</u>

b. The cost of the remaining inventory sold in the bankruptcy sale was
\$34,740.74 – \$14,888.89 = <u>\$19,851.85</u>

27. Let r represent the number of regular members and s the number of student members.

Then	$r + s =$	583	①
Total revenue:	$\$2140r + \$856s =$	\$942,028	②
① × \$856:	$\$856r + \$856s =$	\$499,048	
Subtract:	$\$1284r + 0 =$	\$442,980	
	$r =$	345	
Substitute into ①:	$345 + s =$	583	
	$s =$	238	

The club had <u>238 student members</u> and <u>345 regular members</u>.

28. Let a represent the adult airfare and c represent the child airfare.

Mrs. Ramsey's cost:	$a + 2c =$	\$610	①
Chudnowskis' cost:	$2a + 3c =$	\$1050	②
① × 2:	$2a + 4c =$	\$1220	
Subtract:	$0 + -c =$	–\$170	
Substitute c = \$170 into ①:	a +2(\$170) =	\$610	
	$a =$	\$610 – \$340 = \$270	

The airfare is <u>\$270 per adult</u> and <u>\$170 per child</u>.

Exercise 2.5 *(continued)*

29. Let h represent the rate per hour and k represent the rate per km.

Vratislav's cost: $2h + 47k = \$\ 54.45$ ①

Bryn's cost: $5h + 93k = \$127.55$ ②

To eliminate h,

① × 5: $10h + 235k = \$272.25$ ①

② × 2: $\underline{10h + 186k} = \underline{\$255.10}$ ②

Subtract: $0 + 49k = \$\ 17.15$

$k = \$0.35$ per km

Substitute into ①:

$2h + 47(\$0.35) = \$54.45$

$2h = \$54.45 - \$16.45$

$h = \$19.00$ per hour

Budget Truck Rentals charged $\underline{\$19.00 \text{ per hour}}$ plus $\underline{\$0.35 \text{ per km}}$.

Advanced Problems

30. Step 2: Each of 4 children receive 0.5(Wife's share).

Each of 13 grandchildren receive $0.\overline{3}$ (Child's share).

Total distribution = \$759,000. Let w represent the wife's share.

Step 3: Total amount = Wife's share + 4(Child's share) + 13(Grandchild's share)

Step 4: $\$759,000 = w + 4(0.5w) + 13\left(0.\overline{3}\right)(0.5w)$

Step 5: $\$759,000 = w + 2w + 2.1\overline{6}w$

$= 5.1\overline{6}w$

$w = \$146,903.226$

Each child will receive 0.5(\$146,903.226) = $\underline{\$73,451.61}$

and each grandchild will receive $0.\overline{3}$ (\$73,451.61) = $\underline{\$24,483.87}$.

31. Step 2: Stage B workers = 1.6(Stage A workers)

Stage C workers = 0.75(Stage B workers)

Total workers = 114. Let A represent the number of Stage A workers.

Step 3: Total workers = A workers + B workers + C workers

Step 4: 114 = A + 1.6A + 0.75(1.6A)

Step 5: 114 = 3.8A

A = 30

$\underline{30}$ workers should be allocated to Stage A, 1.6(30) = $\underline{48}$ workers to Stage B, and 114 – 30 – 48 = $\underline{36}$ workers to Stage C.

32. Step 2: Hillside charge = 2(Barnett charge) – \$1000

Westside charge = Hillside charge + \$2000

Total charges = \$27,600. Let B represent the Barnett charge.

Step 3: Total charges = Barnett charge + Hillside charge + Westside charge

Step 4: \$27,600 = B + 2B – \$1000 + 2B – \$1000 + \$2000

Step 5: \$27,600 = 5B

B = \$5520

Hence, the Westside charge is 2(\$5520) – \$1000 + \$2000 = $\underline{\$12,040}$

Exercise 2.6

Basic Problems

1. $c = \frac{V_f - V_i}{V_i} \times 100 = \frac{\$100 - \$95}{\$95} \times 100 = \underline{\underline{5.26\%}}$

2. $c = \frac{V_f - V_i}{V_i} \times 100 = \frac{35\text{ kg} - 135\text{ kg}}{135\text{ kg}} \times 100 = \underline{\underline{-74.07\%}}$

3. $c = \frac{V_f - V_i}{V_i} \times 100 = \frac{0.13 - 0.11}{0.11} \times 100 = \underline{\underline{18.18\%}}$

4. $c = \frac{V_f - V_i}{V_i} \times 100 = \frac{0.085 - 0.095}{0.095} \times 100 = \underline{\underline{-10.53\%}}$

5. $V_f = V_i(1 + c) = \$134.39[1 + (-0.12)] = \$134.39(0.88) = \underline{\underline{\$118.26}}$

6. $V_f = V_i(1 + c) = 112\text{g}(1 + 1.12) = \underline{\underline{237.44\text{g}}}$

7. $V_i = \frac{V_f}{1 + c} = \frac{\$75}{1 + 2.00} = \underline{\underline{\$25.00}}$

8. $V_i = \frac{V_f}{1 + c} = \frac{\$75}{1 + (-0.50)} = \underline{\underline{\$150.00}}$

9. Given: V_i = \$90, V_f = \$100

   $$c = \frac{\$100 - \$90}{\$90} \times 100 = \underline{\underline{11.11\%}}$$

   \$100 is 11.11% more than \$90.

10. Given: V_i = \$110, V_f = \$100

    $$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{\$100 - \$110}{\$110} \times 100 = \underline{\underline{-9.09\%}}$$

    \$100 is 9.09% less than \$110.

11. Given: c = 25%, V_f = \$100

    $$V_i = \frac{V_f}{1 + c} = \frac{\$100}{1 + 0.25} = \underline{\underline{\$80.00}}$$

    \$80.00 increased by 25% equals \$100.00.

12. Given: V_f = \$75, c = 75%

    $$V_i = \frac{V_f}{1 + c} = \frac{\$75}{1 + 0.75} = \underline{\underline{\$42.86}}$$

    \$75 is 75% more than \$42.86.

13. Given: V_i = \$759.00, V_f = \$754.30

    $$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{\$754.30 - \$759.00}{\$759.00} \times 100 = \underline{\underline{-0.62\%}}$$

    \$754.30 is 0.62% less than \$759.00.

Exercise 2.6 *(continued)*

14. Given: V_i = \$75, c = 75%

$$V_f = V_i(1 + c) = \$75(1 + 0.75) = \underline{\underline{\$131.25}}$$

\$75.00 becomes \$131.25 after an increase of 75%.

15. Given: V_f = \$100, c = – 10%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.10)} = \underline{\underline{\$111.11}}$$

\$100.00 is 10% less than \$111.11.

16. Given: V_f = \$100, c = – 20%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.20)} = \underline{\underline{\$125.00}}$$

\$125 after a reduction of 20% equals \$100.

17. Given: V_i = \$900, c = –90%

$$V_f = V_i(1 + c) = \$900[1 + (-0.9)] = \underline{\underline{\$90.00}}$$

\$900 after a decrease of 90% is \$90.00.

18. Given: c = 0.75%, V_i = \$10,000

$$V_f = V_i(1 + c) = \$10,000(1 + 0.0075) = \underline{\underline{\$10,075.00}}$$

\$10,000 after an increase of $\frac{3}{4}$% is \$10,075.00.

19. Given: c = 210%, V_f = \$465

$$V_i = \frac{V_f}{1+c} = \frac{\$465}{1+2.1} = \underline{\underline{\$150.00}}$$

\$150.00 after being increased by 210% equals \$465.

Intermediate Problems

20. Let the retail price be p. Then

$$p + 0.13p = \$281.37$$

$$p = \frac{\$281.37}{1.13} = \underline{\underline{\$249.00}}$$

The jacket's retail price was \$249.00.

21. Let the number of students enrolled in September, 2014 be s. Then

$$s + 0.0526s = 1200$$

$$1.0526s = 1200$$

$$s = \frac{1200}{1.0526} \approx \underline{\underline{1140}}$$

Rounded to the nearest person, the number of students enrolled in September, 2014 was 1140.

22. Let next year's sales be n. Then

$$n = \$18,400(1+ 0.12)$$

$$n = \underline{\underline{\$20,608}}$$

Nykita is expecting next year's sales to be \$20,608.

Exercise 2.6 *(continued)*

23. Given: V_i = \$285,000, V_f = \$334,000

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$334{,}000 - \$285{,}000}{\$285{,}000} \times 100\% = \underline{\underline{17.19\%}}$$

The value of Amir's real estate investment grew by 17.19%.

24. Let Jamal's earnings this year be e. Then

$$e = \$87{,}650(1 - 0.065)$$
$$e = \$81{,}952.75$$

Rounded to the nearest dollar, Jamal's earnings this year were $\underline{\underline{\$81{,}953}}$.

25. Let the population figure on July 1, 2011 be p. Then

$$p + 0.0439p = 35{,}851{,}800$$
$$p = \frac{35{,}851{,}800}{1.0439} \approx 34{,}344{,}094$$

Rounded to the nearest 1000, the population on July 1, 2011 was $\underline{\underline{34{,}344{,}000}}$.

26. *a.* Given: V_i = 32,400, V_f = 27,450

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{27{,}450 - 32{,}400}{32{,}400} \times 100\% = \underline{\underline{-15.28\%}}$$

The number of hammers sold declined by 15.28%.

b. Given: V_i = \$15.10, V_f = \$15.50

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15.50 - \$15.10}{\$15.10} \times 100\% = \underline{\underline{2.65\%}}$$

The average selling price increased by 2.65%.

c. Year 1 revenue = 32,400(\$15.10) = \$489,240
Year 2 revenue = 27,450(\$15.50) = \$425,475

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$425{,}475 - \$489{,}240}{\$489{,}240} \times 100\% = \underline{\underline{-13.03\%}}$$

The revenue decreased by 13.03%.

27. *a.* Given: V_i = \$0.55, V_f = \$1.55

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$1.55 - \$0.55}{\$0.55} \times 100\% = \underline{\underline{181.82\%}}$$

The share price rose by 181.82% in the first year.

b. Given: V_i = \$1.55, V_f = \$0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$1.55}{\$1.55} \times 100\% = \underline{\underline{-51.61\%}}$$

The share price declined by 51.61% in the second year.

c. Given: V_i = \$0.55, V_f = \$0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$0.55}{\$0.55} \times 100\% = \underline{\underline{36.36\%}}$$

The share price rose by 36.36% over 2 years.

Exercise 2.6 *(continued)*

28. Initial unit price = $\frac{\$5.49}{1.65\ l}$ = \$3.327 per litre

Final unit price = $\frac{\$7.98}{2.2\ l}$ = \$3.627 per litre

The percent increase in the unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$3.627 - \$3.327}{\$3.327} \times 100\% = \underline{\underline{9.02\%}}$$

29. Initial unit price = $\frac{1098\ \text{cents}}{700\ \text{g}}$ = 1.5686 cents per g

Final unit price = $\frac{998\ \text{cents}}{600\ \text{g}}$ = 1.6633 cents per g

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{1.6633 - 1.5686}{1.5686} \times 100 = \underline{\underline{6.04\%}}$$

30. Given: V_f = \$348,535, c = −1.8%

$$V_i = \frac{V_f}{1+c} = \frac{\$348{,}535}{0.982} \approx \underline{\underline{\$354{,}900}}$$

Rounded to the nearest \$100, the average price one month ago was \$354,900.

31. Given: V_f = \$348.60, c = –0.30

$$V_i = \frac{V_f}{1+c} = \frac{\$348.60}{1+(-0.30)} = \frac{\$348.60}{0.70} = \underline{\underline{\$498.00}}$$

The regular price of the boots is \$498.00.

32. Given: V_f = 231,200,000, c = 3.66%

$$V_i = \frac{V_f}{1+c} = \frac{231{,}200{,}000}{1+0.0366} = \frac{231{,}200{,}000}{1.0366} \approx \underline{\underline{223{,}037{,}000}}$$

Rounded to the nearest 1000 units, Apple sold 223,037,000 iPhones in 2014.

33. Given: V_f = \$2,030,000,000, c = 241%

$$c = \frac{V_f - V_i}{V_i} \times 100$$

$$241 = \frac{\$2{,}030{,}000{,}000 - V_i}{V_i} \times 100$$

$$2.41 = \frac{\$2{,}030{,}000{,}000 - V_i}{V_i}$$

$$2.41V_i = \$2{,}030{,}000{,}000 - V_i$$

$$3.41V_i = \$2{,}030{,}000{,}000$$

$$V_i = \frac{\$2{,}030{,}000{,}000}{3.41} \approx \$595{,}308{,}000$$

Rounded to the nearest \$1000, Twitter's 2013 advertising revenues were $\underline{\underline{\$595{,}308{,}000}}$.

Exercise 2.6 *(continued)*

34. The fees to Fund A will be

$$\frac{(\text{Fees to Fund A})-(\text{Fees to Fund B})}{(\text{Fees to Fund B})}\times 100\% = \frac{2.38\%-1.65\%}{1.65\%}\times 100\% = \underline{\underline{44.24\%}}$$

more than the fees to Fund B.

35. Percent change in the GST rate

$$= \frac{(\textit{Final GST rate})-(\textit{Initial GST rate})}{(\textit{Initial GST rate})}\times 100\% = \frac{5\%-6\%}{6\%}\times 100\% = \underline{\underline{-16.67\%}}$$

The GST paid by consumers was reduced by 16.67%.

36. Given: V_f = \$0.45, c = 76%

$$V_i = \frac{V_f}{1+c} = \frac{\$0.45}{1+(-0.76)} = \$1.88$$

Price decline = $V_i - V_f$ = \$1.88 – \$0.45 = $\underline{\underline{\$1.43}}$
The share price dropped by \$1.43.

37. If the Canadian dollar is worth 1.5% less than the US dollar,
Canadian dollar = (1 – 0.015)(US dollar) = 0.985(US dollar)

Hence, US dollar = $\frac{\text{Canadian dollar}}{0.985}$ = 1.0152(Canadian dollar)

Therefore, the US dollar is worth $\underline{\underline{1.52\%}}$ more than the Canadian dollar.

38. Current unit price = $\frac{115 \text{ cents}}{100 \text{ g}}$ = 1.15 cents per g

New unit price = 1.075(1.15 cents per g) = 1.23625 cents per g
Price of an 80-g bar = (80 g) × (1.23625 cents per g) = 98.9 cents = $\underline{\underline{\$0.99}}$

39. Canada's exports to US exceeded imports from the US by 14.1%.
That is, Exports = 1.141(Imports)

Therefore, Imports = $\frac{\text{Exports}}{1141}$ = 0.8764(Exports)

That is, Canada's imports from US (= US exports to Canada) were
1 – 0.8764 = 0.1236 = $\underline{\underline{12.36\%}}$
less than Canada's exports to US (= US imports from Canada.)

40. Given: 2014 sales revenues were 14.2% less than 2013 sales revenues
Hence, (Sales for 2014) = (1 – 0.142)(Sales for 2013) = 0.858(Sales for 2013)

Therefore, (Sales for 2013) = $\frac{\text{(Sales for 2014)}}{0.858}$ = 1.1655(Sales for 2014)

That is, sales revenues for 2013 were $\underline{\underline{116.55\%}}$ of sales revenues for 2014.

Exercise 2.6 *(continued)*

Advanced Problems

41. Given: For the appreciation, V_i = Purchase price, c = 140%, V_f = List price
For the price reduction, V_i = List price, c = –10%, V_f = \$172,800

List price = $\frac{V_f}{1+c} = \frac{\$172{,}800}{1+(-0.1)} = \$192{,}000$

Original purchase price = $\frac{V_f}{1+c} = \frac{\$192{,}000}{1+1.4} = \underline{\underline{\$80{,}000}}$

The owner originally paid \$80,000 for the property.

42. Given: For the markup, V_i = Cost, c = 22%, V_f = List price
For the markdown, V_i = List price, c = –10%, V_f = \$17,568

List price = $\frac{V_f}{1+c} = \frac{\$17{,}568}{1+(-0.10)} = \$19{,}520$

Cost (to dealer) = $\frac{V_f}{1+c} = \frac{\$19{,}520}{1+0.22} = \underline{\underline{\$16{,}000}}$

The dealer paid \$16,000 for the car.

43. Next year there must be 15% fewer students per teacher.
With the same number of students,

$$\frac{\text{Students}}{\text{Teachers next year}} = 0.85\left(\frac{\text{Students}}{\text{Teachers now}}\right)$$

Therefore, Teachers next year = $\frac{\text{Teachers now}}{0.85}$ = 1.1765(Teachers now)

That is, if the number of students does not change, the number of teachers must be <u>increased by 17.65%</u>.

44. Use ppm as the abbreviation for "pages per minute".
Given: Lightning printer prints 30% more ppm than the Reliable printer.
That is, the Lightning's printing speed is 1.30 times the Reliable's printing speed.
Therefore, the Reliable's printing speed is

$\frac{1}{1.3}$ = 0.7692 = 76.92% of the Lightning's printing speed

Therefore, the Reliable's printing speed is
100% − 76.92% = 23.08% less than the Lighting's speed.
The Lightning printer will require <u>23.08% less time</u> than the Reliable for a long printing job.

45. Given: Euro is worth 32% more than the Canadian dollar.
That is, Euro = 1.32(Canadian dollar)

Therefore, Canadian dollar = $\frac{\text{Euro}}{1.32}$ = 0.7576(Euro) = 75.76% of a Euro.

That is, the Canadian dollar is worth 100% − 75.76% = <u>24.24% less</u> than the Euro.

Exercise 2.6 *(continued)*

46. Let us use OT as an abbreviation for "overtime".
The number of OT hours permitted by this year's budget is

$$\text{OT hours (this year)} = \frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

The number of overtime hours permitted by next year's budget is

$$\text{OT hours (next year)} = \frac{\text{OT budget (next year)}}{\text{OT hourly rate (next year)}} = \frac{103[\text{OT budget (this year)}]}{105[\text{OT hourly rate (this year)}]}$$

$$= 0.98095\frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

= 98.10% of this year's OT hours

The number of OT hours must be reduced by 100% − 98.10% = <u>1.90%</u>.

Review Problems

Basic Problems

1. *a.* 2(7x – 3y) – 3(2x – 3y) = 14x – 6y – 6x + 9y = <u>8x + 3y</u>

   b. 15x – (4 – 10x + 12) = 15x – 4 + 10x – 12 = <u>25x – 16</u>

2. Given: *NI* = \$200,000, *CM* = \$8, *X* = 40,000

   NI = (CM)X – FC
   \$200,000 = \$8(40,000) – *FC*
   \$200,000 – \$320,000 = –*FC*
   –\$120,000 = –*FC*
   FC = <u>\$120,000</u>

3. Given: *S* = \$1243.75, *P* = \$1200, $t = \frac{7}{12}$

   S = P(1 + rt)
   $\$1243.75 = \$1200\left[1 + r\left(\frac{7}{12}\right)\right]$
   $\frac{\$1243.75}{\$1200} = 1 + r\left(\frac{7}{12}\right)$
   $1.0365 - 1 = r\left(\frac{7}{12}\right)$
   $0.0365 = 0.58\overline{3}\, r$
   $r = \frac{0.0365}{0.58\overline{3}}$
   r = 0.0626 x 100% = <u>6.26%</u>

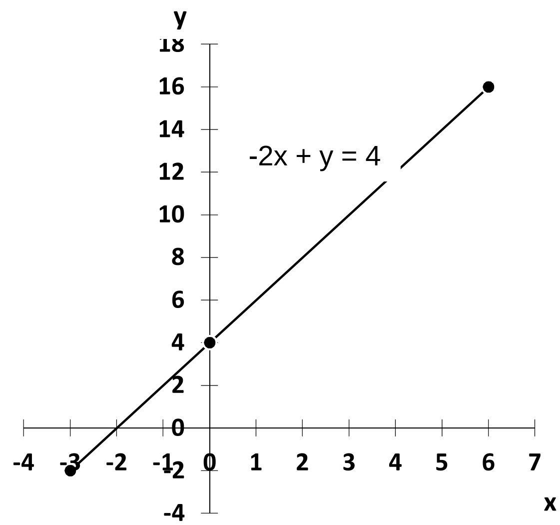
4. *a.* 3.1t + 145 = 10 + 7.6t
   3.1t – 7.6t = 10 – 145
   -4.5t = -135
   <u>t = 30</u>

   b. 1.25y – 20.5 = 0.5y – 11.5
   1.25y – 0.5y = -11.5 + 20.5
   0.75y = 9
   <u>y = 12</u>

Review Problems *(continued)*

5.

x:	–3	0	6
y:	–2	4	16



6. In each part, rearrange the equation to render it in the form y = (slope)x + (intercept)

a. $2b + 3 = 5a$

$2b = 5a - 3$

$b = \frac{5}{2}a - \frac{3}{2}$

The slope is $\frac{5}{2}$ and the b-intercept is $-\frac{3}{2}$.

b. $3a - 4b = 12$

$-4b = -3a + 12$

$b = \frac{3}{4}a - 3$

The slope is $\frac{3}{4}$ and the b-intercept is -3 .

c. $7a = -8b$

$8b = -7a$

$b = -\frac{7}{8}a$

The slope is $-\frac{7}{8}$ and the b-intercept is 0 .

7. Step 2: Total revenue for the afternoon: \$240.75
   Total number of swimmers for the afternoon: 126
   Adult price: \$3.50
   Child price: \$1.25
   Let A represent the number of adults and C represent the number of children.

   Step 3: Total number of swimmers = Number of adults + Number of children
   Total revenue = Revenue from adults + Revenue from children

   Step 4: $126 = A + C$ ①
   $\$240.75 = \$3.50A + \$1.25C$ ②

   Step 5: Rearrange ①: $A = 126 – C$
   Substitute into ②: $\$240.75 = \$3.50(126 – C) + \$1.25C$
   Sove: $\$240.75 = \$441 – \$3.50C + \$1.25C$
   $\$240.75 = \$441 – \$2.25C$
   $\$240.75 - \$441 = –\$2.25C$
   $–\$200.25 = –\$2.25C$
   $C = –\$200.25/–\$2.25 = 89$

   There were 89 children and 126 – 89 = 37 adults who swam during the afternoon.

8. Step 2: Total kilometres paved = 11.5.
   There were 4.25 more kilometres paved on day two than on day one.
   Let the number of kilometres paved on day one be X.
   Then the number of kilometres paved on day two is (X + 4.25)

   Step 3: Total Kms paved = Kms paved on day one + Kms paved on day two

   Step 4: 11.5 = X + (X + 4.25)

   Step 5: 11.5 = 2X + 4.25
   2X = 11.5 – 4.25
   2X = 7.25
   X = 7.25/2 = 3.625

   3.625 kilometres were paved on day one and 3.625 + 4.25 =7.875 kilometres were paved on day two.

9. *a.* Given: $c = 17.5\%$, $V_i = \$29.43$
   $V_f = V_i(1 + c) = \$29.43(1.175) = \$34.58$
   \$34.58 is 17.5% more than \$29.43.

   b. Given: $V_f = \$100$, $c = –80\%$
   $$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.80} = \$500.00$$
   80% off \$500 leaves \$100.

   c. Given: $V_f = \$100$, $c = –15\%$
   $$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.15} = \$117.65$$
   \$117.65 reduced by 15% equals \$100.

(continued)

Review Problems *(continued)*

9. *d.* Given: V_i = \$47.50, c = 320%

   $V_f = V_i(1 + c) = \$47.50(1 + 3.2) = \underline{\$199.50}$

   \$47.50 after an increase of 320% is \$199.50.

   e. Given: c = – 62%, V_f = \$213.56

   $$V_i = \frac{V_f}{1+c} = \frac{\$213.56}{1-0.62} = \underline{\underline{\$562.00}}$$

   \$562 decreased by 62% equals \$213.56.

   f. Given: c = 125%, V_f = \$787.50

   $$V_i = \frac{V_f}{1+c} = \frac{\$787.50}{1+1.25} = \underline{\underline{\$350.00}}$$

   \$350 increased by 125% equals \$787.50.

   g. Given: c = – 30%, V_i = \$300

   $$V_f = V_i(1+c) = \$300(1-0.30) = \underline{\underline{\$210.00}}$$

   \$210 is 30% less than \$300.

Intermediate Problems

10. $$\frac{9y-7}{3} - 2.3(y-2) = 3y - 2.\overline{3} - 2.3y + 4.6 = \underline{\underline{0.7y + 2.2\overline{6}}}$$

11. $$4(3a + 2b)(2b - a) - 5a(2a - b) = 4(6ab - 3a^2 + 4b^2 - 2ab) - 10a^2 + 5ab$$
    $$= \underline{\underline{-22a^2 + 21ab + 16b^2}}$$

12. *a.* $L(1 - d_1)(1 - d_2)(1 - d_3) = \$340(1 - 0.15)(1 - 0.08)(1 - 0.05) = \underline{\underline{\$252.59}}$

    b. $$\frac{R}{i}\left[1-\frac{1}{(1+i)^n}\right] = \frac{\$575}{0.085}\left[1-\frac{1}{(1+0.085)^3}\right] = \$6764.706(1-0.7829081) = \underline{\underline{\$1468.56}}$$

13. $$N = L(1-d_1)(1-d_2)(1-d_3)$$
    $$\$324.30 = \$498(1 - 0.20)(1 - d_2)(1 - 0.075)$$
    $$\$324.30 = \$368.52(1 - d_2)$$
    $$\frac{\$324.30}{\$368.52} = (1 - d_2)$$
    $$d_2 = 1 - 0.8800 = \underline{\underline{0.120}} = \underline{\underline{12.0\%}}$$

14. *a.* $$6(4y - 3)(2 - 3y) - 3(5 - y)(1 + 4y) = 6(8y - 12y^2 - 6 + 9y) - 3(5 + 20y - y - 4y^2)$$
    $$= \underline{\underline{-60y^2 + 45y - 51}}$$

    b. $$\frac{5b-4}{4} - \frac{25-b}{1.25} + \frac{7}{8}b = 1.25b - 1 - 20 + 0.8b + 0.875b = \underline{\underline{2.925b - 21}}$$

    c. $$\frac{96nm^2 - 72n^2m^2}{48n^2m} = \frac{4m-3nm}{2n} = \frac{4m}{2n} - \frac{3nm}{2n} = \underline{\underline{2\frac{m}{n} - 1.5m}}$$

15. $$\frac{(-3x^2)^3(2x^{-2})}{6x^5} = \frac{(-27x^6)(2x^{-2})}{6x^5} = \underline{\underline{-\frac{9}{x}}}$$

Review Problems *(continued)*

16. *a.* $1.0075^{24} = \underline{\underline{1.19641}}$

b. $(1.05)^{1/6} - 1 = \underline{\underline{0.00816485}}$

c. $\dfrac{(1+0.0075)^{36} - 1}{0.0075} = \underline{\underline{41.1527}}$

17. *a.*

$$4a - 5b = 30 \quad ①$$
$$2a - 6b = 22 \quad ②$$

To eliminate a,

① × 1: $4a - 5b = 30$

② × 2: $\underline{4a - 12b} = \underline{44}$

Subtract: $7b = -14$

$b = -2$

Substitute into ①: $4a - 5(-2) = 30$

$4a = 30 - 10$

$a = 5$

Hence, $\underline{\underline{(a, b)}} = \underline{\underline{(5, -2)}}$

b.

$$76x - 29y = 1050 \quad ①$$
$$-13x - 63y = 250 \quad ②$$

To eliminate x,

① × 13: $988x - 377y = 13{,}650$

② × 76: $-\underline{988x - 4788y} = \underline{19{,}000}$

Add: $-5165y = 32{,}650$

$y = -6.321$

Substitute into ①: $76x - 29(-6.321) = 1050$

$76x = 1050 - 183.31$

$x = 11.40$

Hence, $\underline{\underline{(x, y) = (11.40, -6.32)}}$

18.

$$3x + 5y = 11 \quad ①$$
$$2x - y = 16 \quad ②$$

To eliminate y,

①: $3x + 5y = 11$

② × 5: $\underline{10x - 5y} = \underline{80}$

Add: $13x + 0 = 91$

$x = 7$

Substitute into equation ②: $2(7) - y = 16$

$y = -2$

Hence, $\underline{\underline{(x, y)}} = \underline{\underline{(7, -2)}}$

19. The homeowner pays \$28 per month plus \$2.75 per cubic metre of water used.

Then $B = \$28 + \$2.75C$

Expressing this equation in the form $y = mx + b$

$B = \$2.75C + \$28$

On a plot of *B* vs. *C*, slope = \$2.75 and *B*-intercept = \$28.

Review Problems *(continued)*

20.

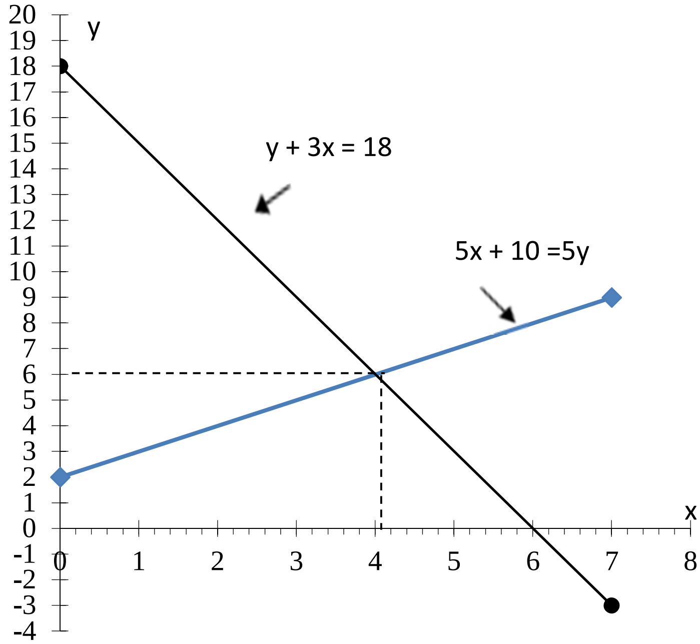
$y + 3x = 18$

x:	0	7
y:	18	-3

$5x + 10 = 5y$

x:	0	7
y:	2	9

The solution is
<u>$(x, y) = (4,6)$</u>.



21. Given: Grace's share = 1.2(Kajsa's share); Mary Anne's share = $\frac{5}{8}$(Grace's share)

Total allocated = \$36,000
Let K represent Kajsa's share.
(Kajsa's share) + (Grace's share) + (Mary Anne's share) = \$36,000

$$K + 1.2K + \tfrac{5}{8}(1.2K) = \$36,000$$
$$2.95\,K = \$36,000$$
$$K = \$12,203.39$$

<u>Kajsa should receive \$12,203.39</u>. <u>Grace should receive</u> 1.2K = <u>\$14,644.07</u>.
<u>Mary Anne should receive</u> $\frac{5}{8}$(\$14,644.07) = <u>\$9152.54</u>.

22. Given: Total initial investment = \$7800; Value 1 year later = \$9310
Percent change in ABC portion = 15%
Percent change in XYZ portion = 25%
Let X represent the amount invested in XYZ Inc.
The solution "idea" is:
(Amount invested in ABC)1.15 + (Amount invested in XYZ)1.25 = \$9310
Hence,

$$(\$7800 - X)1.15 + (X)1.25 = \$9310$$
$$\$8970 - 1.15X + 1.25X = \$9310$$
$$0.10X = \$9310 - \$8970$$
$$X = \$3400$$

Rory invested <u>\$3400 in XYZ</u> Inc. and \$7800 – \$3400 = <u>\$4400 in ABC</u> Ltd.

Review Problems *(continued)*

23. Let R represent the price per kg for red snapper and let L represent the price per kg for lingcod. Then

$370R + 264L = \$2454.20$ ①

$255R + 304L = \$2124.70$ ②

To eliminate R,

① ÷ 370: $R + 0.71351L = \$6.6330$

② ÷ 255: $\underline{R + 1.19216L} = \underline{\$8.3322}$

Subtract: $-0.47865L = -\$1.6992$

$L = \$3.55$

Substitute into ①: $370R + 264(\$3.55) = \$2454.20$

$370R = \$1517.00$

$R = \$4.10$

Nguyen was paid $\underline{\underline{\text{\$3.55 per kg for lingcod}}}$ and $\underline{\underline{\text{\$4.10 per kg for red snapper}}}$.

24. Given:

	Year 1 value (V_i)	Year 2 *value* (V_f)
Gold produced:	34,300 oz.	23,750 oz.
Average price:	$1160	$1280

a. Percent change in gold production = $\frac{23{,}750 - 34{,}300}{34{,}300} \times 100\% = \underline{\underline{-30.76\%}}$

b. Percent change in price = $\frac{\$1280 - \$1160}{\$1160} \times 100\% = \underline{\underline{10.34\%}}$

c. Year 1 revenue, V_i = 34,300($1160) = $39.788 million
Year 2 revenue, V_f, = 23,750($1280) =$30.400 million

Percent change in revenue = $\frac{\$30.400 - \$39.788}{\$39.788} \times 100\% = \underline{\underline{-23.60\%}}$

25. Given: For the first year, $V_i = \$3.40$, $V_f = \$11.50$.
For the second year, $V_i = \$11.50$, $c = -35\%$.

a. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$11.50 - \$3.40}{\$3.40} \times 100\% = \underline{\underline{238.24\%}}$

The share price increased by 238.24% in the first year.

b. Current share price, $V_f = V_i(1 + c) = \$11.50(1 - 0.35) = \underline{\underline{\$7.48}}$.

26. Given: For the first year, $c = 150\%$
For the second year, $c = -40\%$, $V_f = \$24$

The price at the beginning of the second year was

$$V_i = \frac{V_f}{1+c} = \frac{\$24}{1-0.40} = \$40.00 = V_f \text{ for the first year.}$$

The price at the beginning of the first year was

$$V_i = \frac{V_f}{1+c} = \frac{\$40.00}{1+1.50} = \underline{\underline{\$16.00}}$$

Barry bought the stock for $16.00 per share.

Review Problems *(continued)*

27. Given: Last year's revenue = $2,347,000
Last year's expenses = $2,189,000

a. Given: Percent change in revenue = 10%; Percent change in expenses = 5%

Anticipated revenues, $V_f = V_i(1 + c)$ = $2,347,000(1.1) = $2,581,700

Anticipated expenses = $2,189,000(1.05) = $<u>2,298,450</u>

Anticipated profit = $283,250

Last year's profit = $2,347,000 – $2,189,000 = $158,000

$$\text{Percent increase in profit} = \frac{\$283{,}250 - \$158{,}000}{\$158{,}000} \times 100\% = \underline{\underline{79.27\%}}$$

b. Given: c(revenue) = –10%; c(expenses) = – 5%

Anticipated revenues = $2,347,000(1 – 0.10) = $2,112,300

Anticipated expenses = $2,189,000(1 – 0.05) = <u>$2,079,550</u>

Anticipated profit $32,750

$$\text{Percent change in profit} = \frac{\$32{,}750 - \$158{,}000}{\$158{,}000} \times 100\% = \underline{\underline{-79.27\%}}$$

The operating profit will decline by 79.27%.

28. *a.* $$\frac{(1.00\bar{6})^{240} - 1}{0.00\bar{6}} = \frac{4.926802 - 1}{0.00\bar{6}} = \underline{\underline{589.020}}$$

b. $$(1 + 0.025)^{1/3} - 1 = \underline{\underline{0.00826484}}$$

Advanced Problems

29. $$\left(-\frac{2x^2}{3}\right)^{-2}\left(\frac{5^2}{6x^3}\right)\left(-\frac{15}{x^5}\right)^{-1} = \left(\frac{3}{2x^2}\right)^2\left(\frac{25}{6x^3}\right)\left(-\frac{x^5}{15}\right) = -\underline{\underline{\frac{5}{8x^2}}}$$

30. *a.* $$\frac{x}{1.08^3} + \frac{x}{2}(1.08)^4 = \$850$$

$0.793832x + 0.680245x = \$850$

$x = \underline{\$576.63}$

Check: $$\frac{\$576.63}{1.08^3} + \frac{\$576.63}{2}(1.08)^4 = \$457.749 + \$392.250 = \$850.00$$

b. $$2x\left(1 + 0.085 \times \frac{77}{365}\right) + \frac{x}{1 + 0.085 \times \frac{132}{365}} = \$1565.70$$

$2.03586x + 0.97018x = \$1565.70$

$x = \underline{\underline{\$520.85}}$

Check:

$$2(\$520.85)\left(1 + 0.085 \times \frac{77}{365}\right) + \frac{\$520.85}{1 + 0.085 \times \frac{132}{365}} = \$1060.38 + \$505.32 = \$1565.70$$

31. $$P(1 + i)^n + \frac{S}{1 + rt} = \$2500(1.1025)^2 + \frac{\$1500}{1 + 0.09 \times \frac{93}{365}} = \$3038.766 + \$1466.374 = \underline{\underline{\$4505.14}}$$

Review Problems *(continued)*

32. *a.* $\dfrac{2x}{1+0.13\times\frac{92}{365}}+x\left(1+0.13\times\dfrac{59}{365}\right)=\$831$

$1.93655x + 1.02101x = \$831$

$2.95756x = \$831$

$x = \underline{\underline{\$280.97}}$

b. $3x(1.03^5) + \dfrac{x}{1.03^3}+x=\dfrac{\$2500}{1.03^2}$

$3.47782x + 0.915142x + x = \$2356.49$

$x = \underline{\underline{\$436.96}}$

33. 60% of a $\frac{3}{8}$ interest was purchased for $65,000.

Let the V represent the implied value of the entire partnership.

Then $0.60\times\dfrac{3}{8}\text{V} = \$65{,}000$

$\text{V} = \dfrac{8\times\$65{,}000}{0.60\times 3} = \underline{\underline{\$288{,}889}}$

The implied value of the chalet was $288,889.

34. Let b represent the base salary and r represent the commission rate. Then

r($27,000) + b = $2815.00 ①

r($35,500) + b = $3197.50 ②

Subtract: –$8500r = –$382.50

r = 0.045

Substitute into ①: 0.045($27,000) + b = $2815

b = $1600

Deanna's base salary is $\underline{\underline{\text{\$1600 per month}}}$ and her commission rate is $\underline{\underline{4.5\%}}$.

35. Let the regular season ticket prices be R for the red section and B for the blue section. Then

2500R + 4500B = $50,250 ①

2500(1.3R) + 4500(1.2B) = $62,400 ②

① × 1.2: 2500(1.2R) + 4500(1.2B) = $60,300

Subtract: 2500(0.1R) + 0 = $2100

R = $8.40

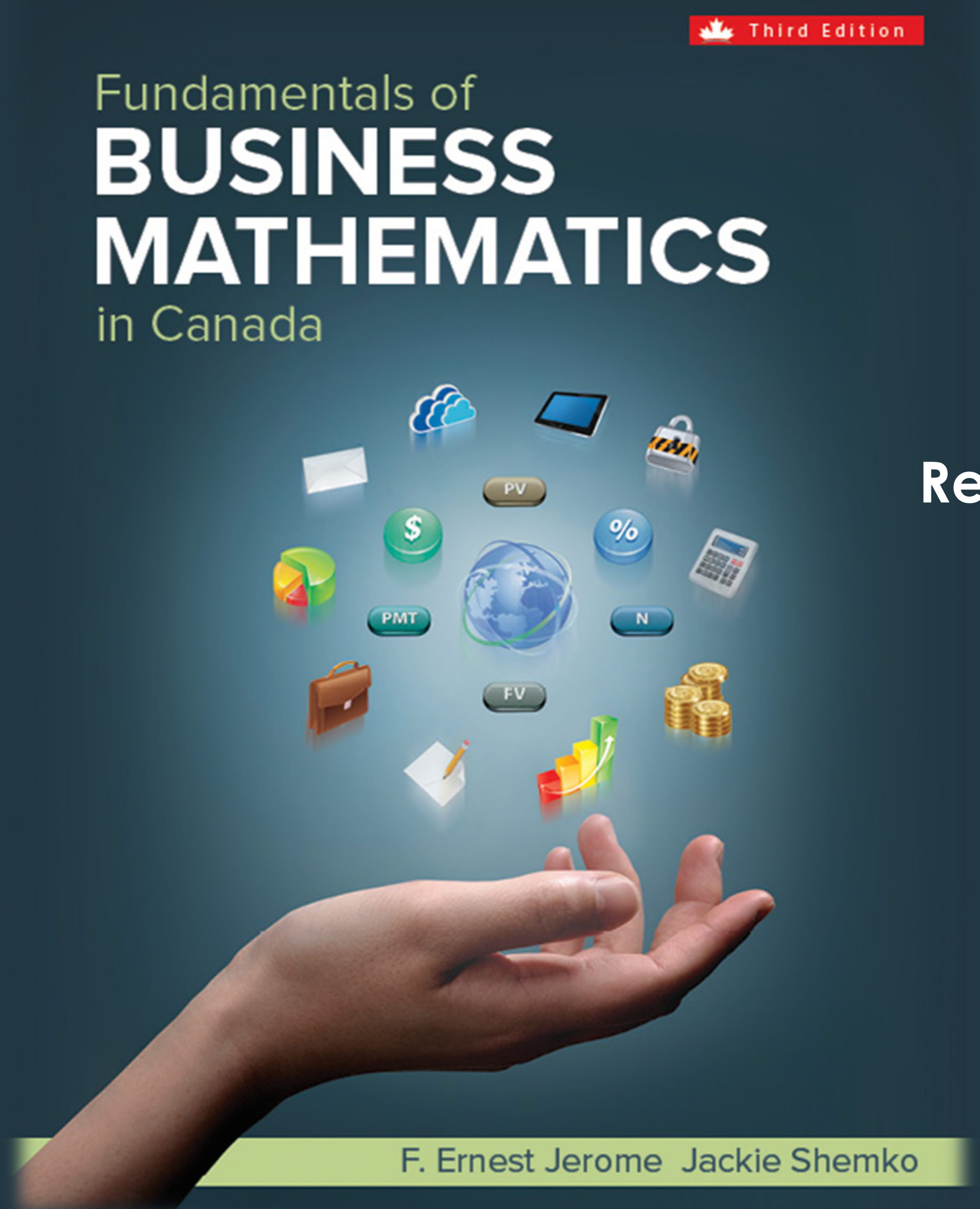
Substitute into ①: 2500($8.40) + 4500B = $50,250

B = $6.50

The ticket prices for the playoffs cost

1.3 × $8.40 = $\underline{\underline{\$10.92}}$ in the "reds"

and 1.2 × $6.50 = $\underline{\underline{\$7.80}}$ in the "blues".



Chapter 2

Review and Applications of Algebra

Prepared By Sarah Chan

Learning Objectives

- **LO1** Simplify algebraic expressions by extracting common factors and applying rules of exponents
- **LO2** Rearrange a formula or equation to isolate a particular variable
- **LO3** Solve a linear equation in one variable, and two linear equations in two variables
- **LO4** Graph a linear equation in two variables
- **LO5** Express a linear equation in slope-intercept form
- **LO6** Solve two equations in two unknowns by a graphical method
- **LO7** Solve "word problems" that lead to a linear equation in one unknown, or two linear equations in two unknowns
- **LO8** Solve problems involving percent change

Definitions

An **algebraic expression** indicates the mathematical operations to be carried out on a combination of numbers and variables.

Example: $2x^2 - 3x + 1$

The components of an algebraic expression that are *separated by addition or subtraction signs* are called **terms.**

Example: $2x^2$ $-3x$ 1

Definitions

A **monomial** is an algebraic expression with only _one_ term.

Example: $2x^2$

A **binomial** is an algebraic expression with _two_ terms.

Example: $2x^2 - 3x$

A **trinomial** is an algebraic expression with _three_ terms.

Example: $2x^2 - 3x + 1$

A **polynomial** is _any_ algebraic expression with _more than one_ term.

Definitions

The components of a term that are *separated by multiplication or division signs* are called **factors.**

Example of a Term: $-5xy$

The Term's Factors: -5 x y

The numerical factor is called the **numerical coefficient.**

-5

The variable factors, *together*, are called the **literal coefficient.**

xy

Combining Like Terms

Terms that have the *same literal coefficient* are called **like terms.**

Like Terms: $-5xy$ $8xy$

Like terms can be **combined** by *adding their numerical coefficients*.

Combining Like Terms:

$$(-5+8)xy = 3xy$$

Example: Simplify by combining like terms: $4x - 10x - 3y + 7y$

$$(4-10)x + (-3+7)y = -6x + 4y$$

Multiplying Polynomials

When multiplying two polynomials, _each term_ in the first polynomial is multiplied by _every term_ in the second polynomial.

Example: $(x-1)(-2x^2+4x-3)$

$$(x)(-2x^2)+(x)(4x)+(x)(-3)+(-1)(-2x^2)+(-1)(4x)+(-1)(-3)$$
$$=-2x^3+4x^2-3x+2x^2-4x+3$$

Then, _combine like terms_:

$$-2x^3+4x^2-3x+2x^2-4x+3$$
$$=-2x^3+(4+2)x^2+(-3-4)x+3$$
$$=-2x^3+6x^2-7x+3$$

Dividing Polynomials

When dividing polynomials, first *identify the factors*.

Example: $\frac{48x^2-32xy}{8x}$

$$\frac{48x^2}{8x}-\frac{32xy}{8x}=\frac{48(x)(x)}{8(x)}-\frac{32(x)(y)}{8(x)}$$

Then, *cancel common factors* from the numerator and denominator:

$$\frac{48(\cancel{x})(x)}{8(\cancel{x})}-\frac{32(\cancel{x})(y)}{8(\cancel{x})}=\frac{48x}{8}-\frac{32y}{8}$$

Then, *divide the numerical coefficients*:

$$\frac{48x}{8}-\frac{32y}{8}=\left(\frac{48}{8}\right)x-\left(\frac{32}{8}\right)y=6x-4y$$

Substitution

Substitution means assigning a numerical value to the variables in an algebraic expression. The expression is then evaluated by carrying out all the indicated operations. Example: $S = P(1 + rt)$

<u>Substitute</u>: $P = \$100, r = 0.09, t = \frac{7}{12}$

$$\boldsymbol{S = \$100\left(1 + 0.09\left(\frac{7}{12}\right)\right)}$$

<u>Evaluate</u>:

$$\boldsymbol{S = \$100\left(1 + 0.09\left(\frac{7}{12}\right)\right)}$$

$$\boldsymbol{S = \$100(1 + 0.0525)}$$

$$\boldsymbol{S = \$100(1.0525)}$$

$$\boldsymbol{S = \$105.25}$$

Manipulating Equations

An **equation** is a statement that one algebraic expression *equals* another algebraic expression. When the equation has broad applications, it is often called a formula.

Equations can be manipulated as long as you adhere to the *all important rule*:

Both sides of an equation must be treated in exactly the same way

This means you are allowed to:

- ✓ **Add the same number or variable to both sides**
- ✓ **Subtract the same number or variable from both sides**
- ✓ **Multiply both sides by the same number or variable**
- ✓ **Divide both sides by the same number or variable**
- ✓ **Raise both sides to the same exponent**

Manipulating Equations

Example: $S = P(1 + rt)$ where $S = \$112, P = \$100, r = 0.04, t = ?$

$$\$112 = \$100(1 + 0.04t)$$

$$\frac{\$112}{\$100} = \frac{\$100(1 + 0.04t)}{\$100}$$

Divide both sides by $100

Manipulating Equations

Example: $S = P(1 + rt) \quad where\ S = \$112, P = \$100, r = 0.04, t = ?$

$$\$112 = \$100(1 + 0.04t)$$

$$\frac{\$112}{\$100} = \frac{\$100(1 + 0.04t)}{\$100}$$

$$1.12 = 1 + 0.04t$$

$$1.12 - 1 = 1 - 1 + 0.04t$$

Subtract 1 from both sides

Manipulating Equations

Example: $S = P(1 + rt) \quad where\ S = \$112, P = \$100, r = 0.04, t = ?$

$$\$112 = \$100(1 + 0.04t)$$

$$\frac{\$112}{\$100} = \frac{\$100(1 + 0.04t)}{\$100}$$

$$1.12 = 1 + 0.04t$$

$$1.12 - 1 = 1 - 1 + 0.04t$$

$$0.12 = 0.04t$$

$$\frac{0.12}{0.04} = \frac{0.04t}{0.04}$$

Divide both sides by 0.04

Manipulating Equations

Example: $S = P(1 + rt) \quad where\ S = \$112, P = \$100, r = 0.04, t = ?$

$$\$112 = \$100(1 + 0.04t)$$

$$\frac{\$112}{\$100} = \frac{\$100(1 + 0.04t)}{\$100}$$

$$1.12 = 1 + 0.04t$$

$$1.12 - 1 = 1 - 1 + 0.04t$$

$$0.12 = 0.04t$$

$$\frac{0.12}{0.04} = \frac{0.04t}{0.04}$$

$$3 = t$$

Exponent Rules

The use of exponents allows us to write algebraic expressions in a more concise form. Example: $4^3 = 4 \times 4 \times 4 = 64$

The **base** is: $\mathbf{4}$

The **exponent** is: $\mathbf{3}$

The **power** is: $\mathbf{64}$

In summary: $\mathbf{base}^{\mathbf{exponent}} = \mathbf{power}$

Exponent Rules

$a^m \times a^n = \mathbf{a^{m+n}}$ $\qquad x^2 \times x^3 = \mathbf{x^{2+3} = x^5}$

$\frac{a^m}{a^n} = \mathbf{a^{m-n}}$ $\qquad \frac{x^7}{x^5} = \mathbf{x^{7-5} = x^2}$

$a^{m}\ {}^{n} = \mathbf{a^{m\times n}}$ $\qquad x^{4}\ {}^{5} = \mathbf{x^{4\times 5} = x^{20}}$

$\mathbf{a^n b^n}$ $\qquad \mathbf{x^3 y^3}$

$\frac{a}{b}\ {}^{n} = \mathbf{\frac{a^n}{b^n}}$ $\qquad \frac{x}{y}\ {}^{2} = \mathbf{\frac{x^2}{y^2}}$

Exponent Rules

$a^0 =$	$\mathbf{1}$	$x^0 =$	$\mathbf{1}$
$a^{-n} =$	$\mathbf{1/a^n}$	$x^{-2} =$	$\mathbf{1/x^2}$
$a^{1}\ n =$	$\sqrt[n]{a}$	$x^{1}\ 2 =$	$\sqrt[2]{x}$
$a^{m}\ n =$	$\sqrt[n]{a^m}$	$x^{3}\ 2 =$	$\sqrt[2]{x^3}$

A solution (or **root**) of an equation is any numerical value of the variable that makes the two sides of the equation equal. A linear equation in one variable has only one root. The process of determining the root (or roots) of an equation is called **solving.**

$$3x - 7 = 5 - 9x$$

Solve

A solution (or **root**) of an equation is any numerical value of the variable that makes the two sides of the equation equal. A linear equation in one variable has only one root. The process of determining the root (or roots) of an equation is called **solving.**

$$3x - 7 = 5 - 9x$$

$$3(1) - 7 = 5 - 9(1)$$

What if $x = 1$?

A solution (or **root**) of an equation is any numerical value of the variable that makes the two sides of the equation equal. A linear equation in one variable has only one root. The process of determining the root (or roots) of an equation is called **solving.**

$$3x - 7 = 5 - 9x$$

$$3(1) - 7 = 5 - 9(1)$$

$$3 - 7 = 5 - 9$$

$$-4 = -4$$

Both sides are equal!

A solution (or **root**) of an equation is any numerical value of the variable that makes the two sides of the equation equal. A linear equation in one variable has only one root. The process of determining the root (or roots) of an equation is called **solving.**

$$3x - 7 = 5 - 9x$$

$$3(1) - 7 = 5 - 9(1)$$

$$3 - 7 = 5 - 9$$

$$-4 = -4$$

$$\therefore x = 1$$

Linear Equations In One Unknown

The steps required to solve a linear equation in one unknown are:

1. **Separate like terms, leaving terms containing the variable on one side and the remaining terms on the other side**
2. **Combine the like terms on each side of the equation**
3. **Obtain the root or solution by dividing both sides of the equation by the numerical coefficient of the variable**

Separate → **Combine** → **Obtain**

Example: Solve $8x - 11 = 5x + 4$

$$\mathbf{8x - 11 = 5x + 4}$$

$$\mathbf{8x - 5x - 11 + 11 = 5x - 5x + 4 + 11}$$

Separate

Example: Solve $8x - 11 = 5x + 4$

$$8x - 11 = 5x + 4$$

$$8x - 5x - 11 + 11 = 5x - 5x + 4 + 11$$

$$(8 - 5)x = (4 + 11)$$

$$3x = 15$$

Combine

Example: Solve $8x - 11 = 5x + 4$

$$8x - 11 = 5x + 4$$

$$8x - 5x - 11 + 11 = 5x - 5x + 4 + 11$$

$$(8 - 5)x = (4 + 11)$$

$$3x = 15$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

Obtain

The solution to a *pair* of linear equations in *two* unknowns is the *pair* of numerical values of the variables that solve both equations simultaneously.

① $2x - 3y = -6$

② $x + y = 2$

Solve

The solution to a *pair* of linear equations in *two* unknowns is the *pair* of numerical values of the variables that solve both equations simultaneously.

① $2x - 3y = -6$

② $x + y = 2$

① $2(0) - 3(2) = -6$

② $0 + 2 = 2$

What if $x = 0$ and $y = 2$?

Linear Equations In Two Unknowns

The solution to a *pair* of linear equations in *two* unknowns is the *pair* of numerical values of the variables that solve both equations simultaneously.

① $2x - 3y = -6$

② $x + y = 2$

① $2(0) - 3(2) = -6$

② $0 + 2 = 2$

① $-6 = -6$

② $2 = 2$

Both equations solved!

The solution to a *pair* of linear equations in *two* unknowns is the *pair* of numerical values of the variables that solve both equations simultaneously.

① $2x - 3y = -6$

② $x + y = 2$

① $2(0) - 3(2) = -6$

② $0 + 2 = 2$

① $-6 = -6$

② $2 = 2$

$\therefore (x, y) = (0, 2)$

The steps required to solve a pair of linear equations are:

1. **Multiply equation ① by the coefficient of x from equation ②**
2. **Multiply equation ② by the coefficient of x from equation ①**
3. **Subtract the two equations (this will eliminate x so you can find y)**
4. **Substitute the value you find for y into the first equation (this will allow you to now find x)**
5. **Check your solution by making sure it solves the second equation**

Multiply → **Subtract** → **Substitute** → **Check**

① $6x - 3y = -9$

② $7x - y = 2$

Solve

①	$6x - 3y = -9$
②	$7x - y = 2$
①$\times 7$	$42x - 21y = -63$
②$\times 6$	$42x - 6y = 12$

Multiply

①	$6x - 3y = -9$
②	$7x - y = 2$
①$\times 7$	$42x - 21y = -63$
②$\times 6$	$42x - 6y = 12$
①−②	$-15y = -75$
	$y = 5$

Subtract to find y

①	$6x - 3y = -9$
②	$7x - y = 2$
①× 7	$42x - 21y = -63$
②× 6	$42x - 6y = 12$
①–②	$-15y = -75$
	$y = 5$
Sub in ①	$6x - 3(5) = -9$
	$6x = 6$
	$x = 1$

Substitute to find x

①	$6x - 3y = -9$
②	$7x - y = 2$
①× 7	$42x - 21y = -63$
②× 6	$42x - 6y = 12$
①–②	$-15y = -75$
	$y = 5$
Sub in ①	$6x - 3(5) = -9$
	$6x = 6$
	$x = 1$
Check in ②	$7(1) - 5 = 2$
	$2 = 2$

Check

①	$6x - 3y = -9$
②	$7x - y = 2$
①× 7	$42x - 21y = -63$
②× 6	$42x - 6y = 12$
①–②	$-15y = -75$
	$y = 5$
Sub in ①	$6x - 3(5) = -9$
	$6x = 6$
	$x = 1$
Check in ②	$7(1) - 5 = 2$
	$2 = 2$

$\therefore (x, y) = (1, 5)$

A *single* linear equation in *two* unknowns has an *infinite* number of solutions. There are many (x, y) pairs that will make both sides of the equation equal.

Example: $\boldsymbol{y = 2x - 1}$

Some Solutions:

$(1, 1)$	$1 = 2(1) - 1$	$1 = 1$
$(2, 3)$	$3 = 2(2) - 1$	$3 = 3$
$(3, 5)$	$5 = 2(3) - 1$	$5 = 5$

Graphing Linear Equations

When all the solutions are plotted on a Cartesian plane, and connected together, a straight line is formed. Therefore, the solution to a linear equation in two unknowns can be represented by a *graph of a line*. The steps required to create this graph are:

1. **Construct a table of values, consisting of pairs of (x, y) values that satisfy the equation**
2. **Construct and label the x-axis and the y-axis**
3. **Plot the (x, y) pairs from the table**
4. **Connect the plotted points with a straight line**

Example: Graph the equation $y = 2x - 1$ over the range $x = -3$ to $x = 3$

Step 1: Construct a table of values

x	
3	
1	
-1	
-3	

Choose values for x that include the endpoints of your range, and some points in the middle

Example: Graph the equation $y = 2x - 1$ over the range $x = -3$ to $x = 3$

Step 1: Construct a table of values

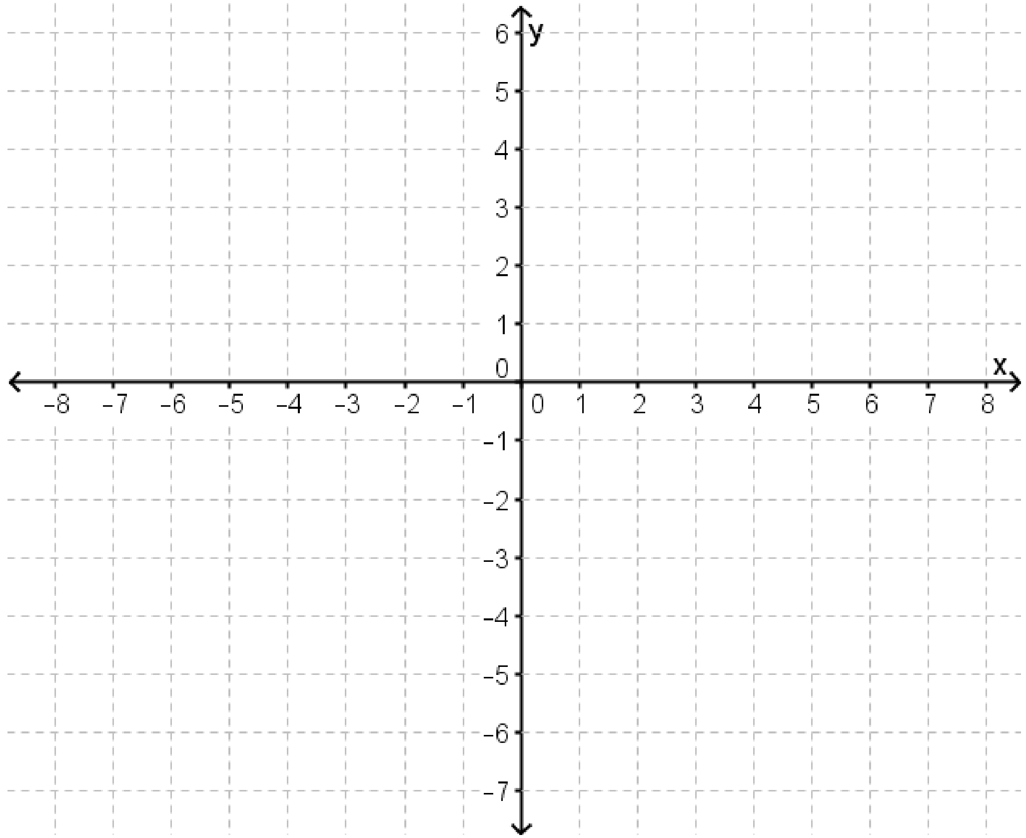
x	
$\mathbf{3}$	$\mathbf{2\ \ 3\ \ - 1 = 5}$
$\mathbf{1}$	$\mathbf{2\ \ 1\ \ - 1 = 1}$
$\mathbf{-1}$	$\mathbf{2\ \ -1\ \ - 1 = -3}$
$\mathbf{-3}$	$\mathbf{2\ \ -3\ \ - 1 = -7}$

Substitute the x values into the equation to determine the y values

Example: Graph the equation $y = 2x - 1$ over the range $x = -3$ to $x = 3$

Step 2: Construct and label the axes

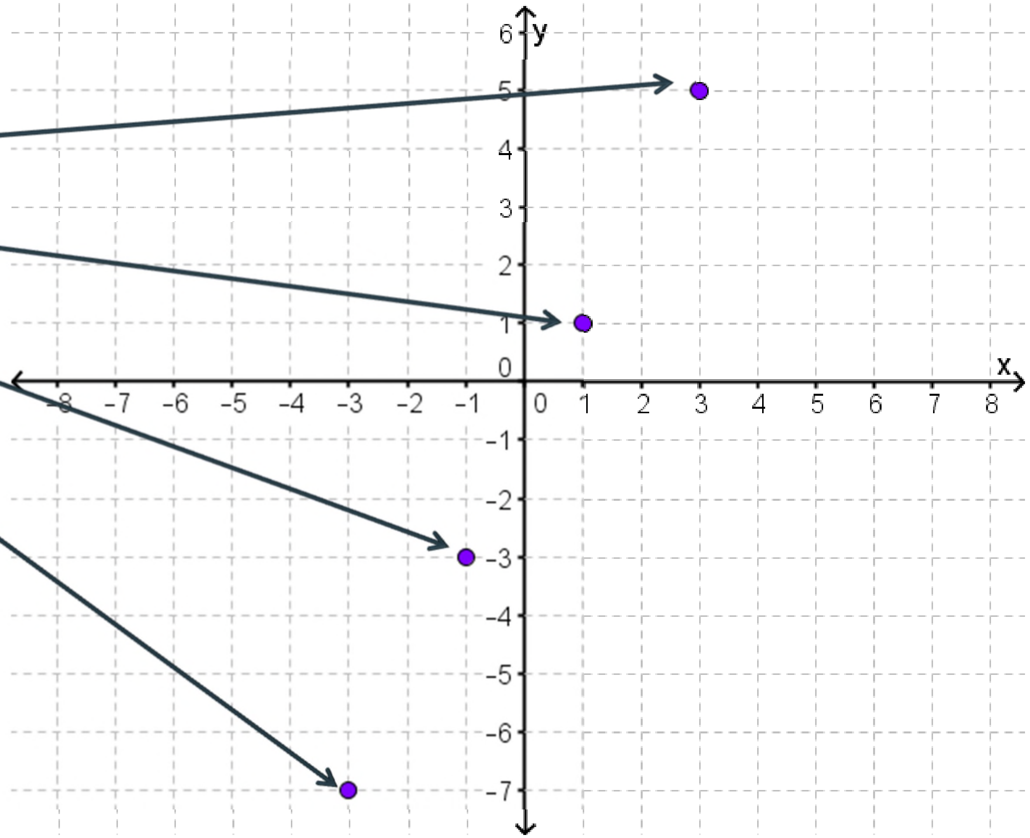
x	y
3	5
1	1
-1	-3
-3	-7



Example: Graph the equation $y = 2x - 1$ over the range $x = -3$ to $x = 3$

Step 3: Plot the pairs

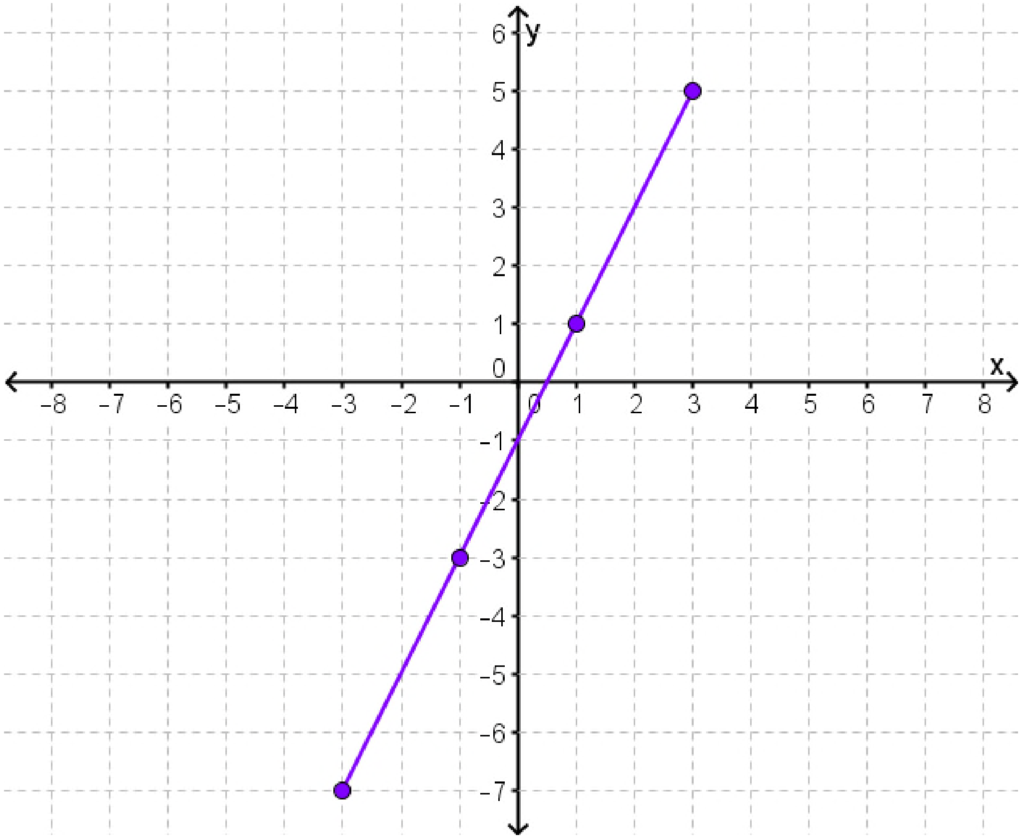
x	y
3	5
1	1
−1	−3
−3	−7



Example: Graph the equation $y = 2x - 1$ over the range $x = -3$ to $x = 3$

Step 4: Connect the points

x	y
3	5
1	1
-1	-3
-3	-7



Slope-Intercept Form

Straight lines have two useful properties. The y-intercept is the value at which the line crosses the y-axis. The slope is a measure of the line's steepness. It is the change in y-coordinate *per unit change* in the x-coordinate.

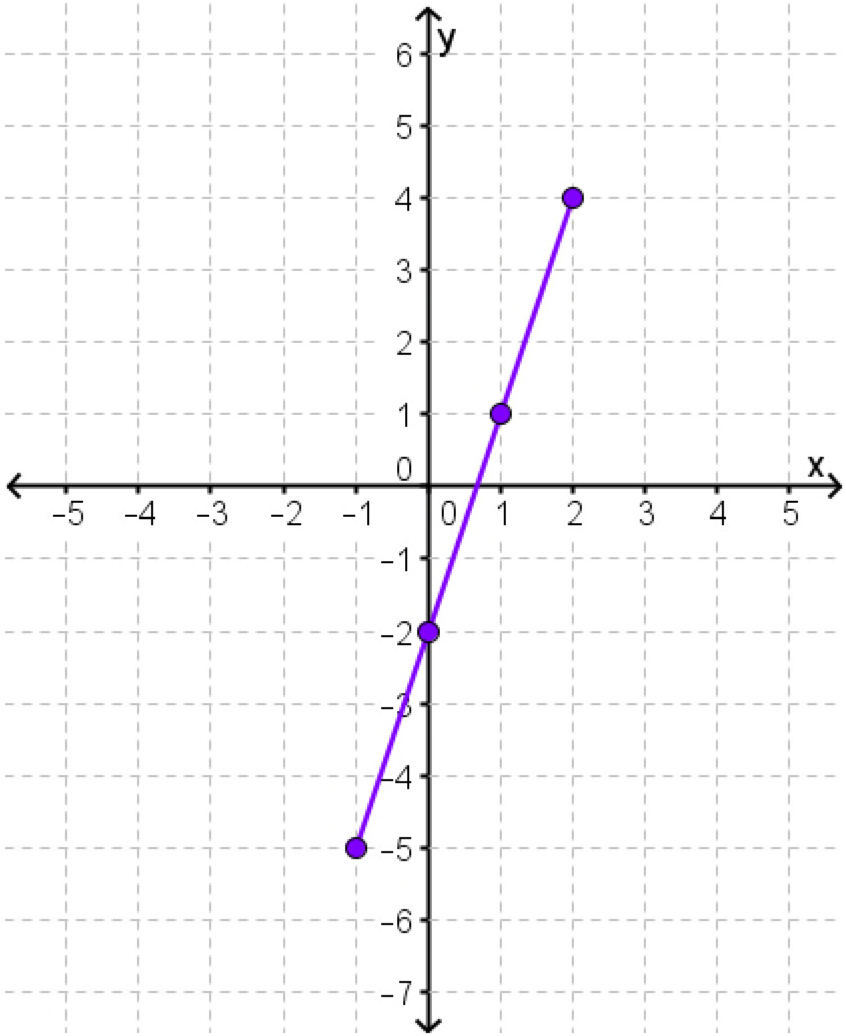
In general, when a linear equation is manipulated into the form

$\boldsymbol{y = mx + b}$ where m and b are constants, then:

The slope will be: $\boldsymbol{m}$

The y-intercept will be: $\boldsymbol{b}$

Slope-Intercept Form Ex. 1



The line crosses the y-axis at -2
Therefore:

$$b = -2$$

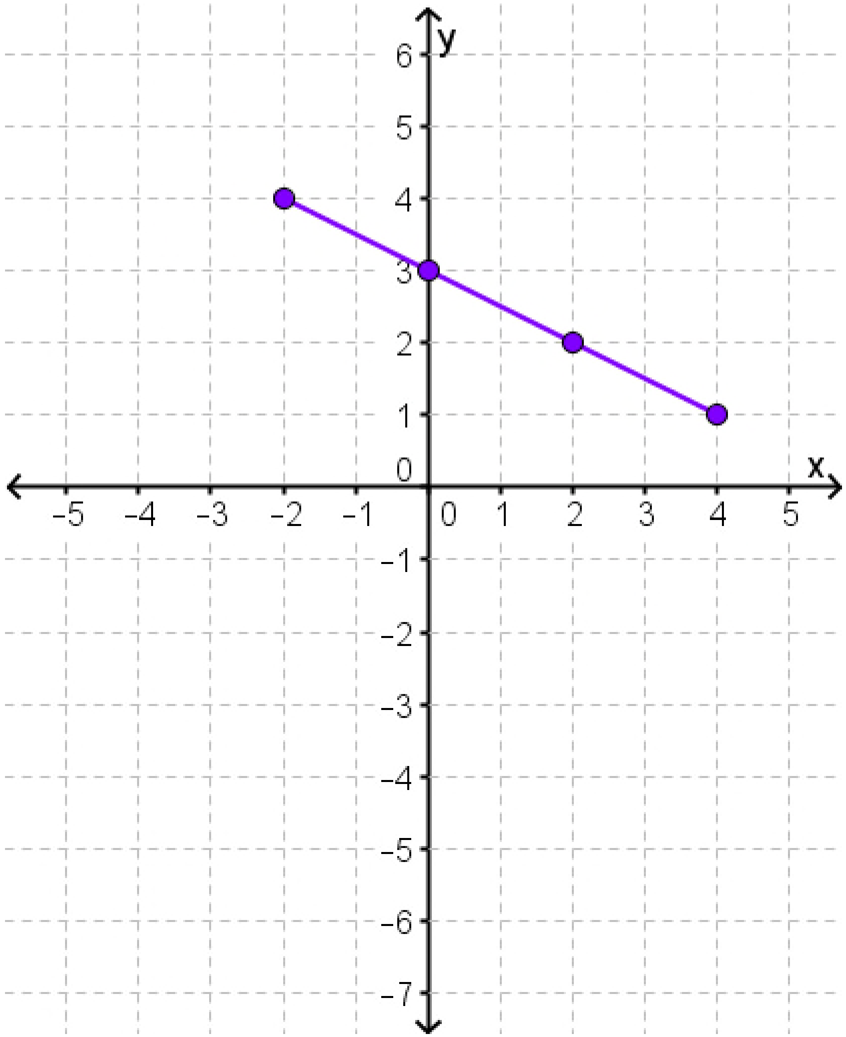
When the x-coordinate increases by 1, the y-coordinate increases by 3
Therefore:

$$m = 3$$

The linear equation is:

$$y = 3x - 2$$

Slope-Intercept Form Ex. 2



The line crosses the y-axis at 3
Therefore:

$$b = 3$$

When the x-coordinate increases by 1, the y-coordinate decreases by $\frac{1}{2}$
Therefore:

$$m = -\frac{1}{2}$$

The linear equation is:

$$y = -\frac{1}{2}x + 3$$

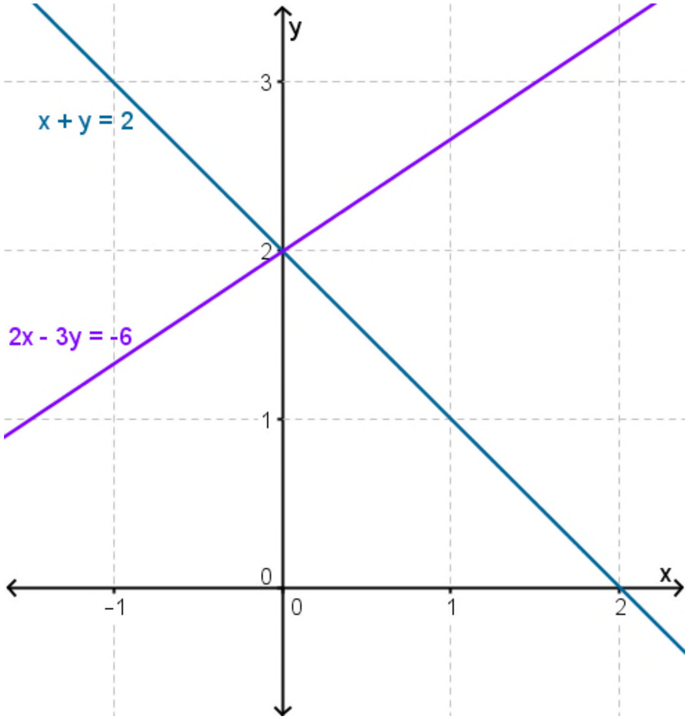
Recall: The solution to a pair of linear equations in two unknowns is the pair of numerical values of the variables that solve both equations simultaneously.

Alternatively: The solution to a pair of linear equations, *graphed as straight lines*, is the point where the two lines *intersect*.

① $x + y = 2$

② $2x - 3y = -6$

Graph the lines

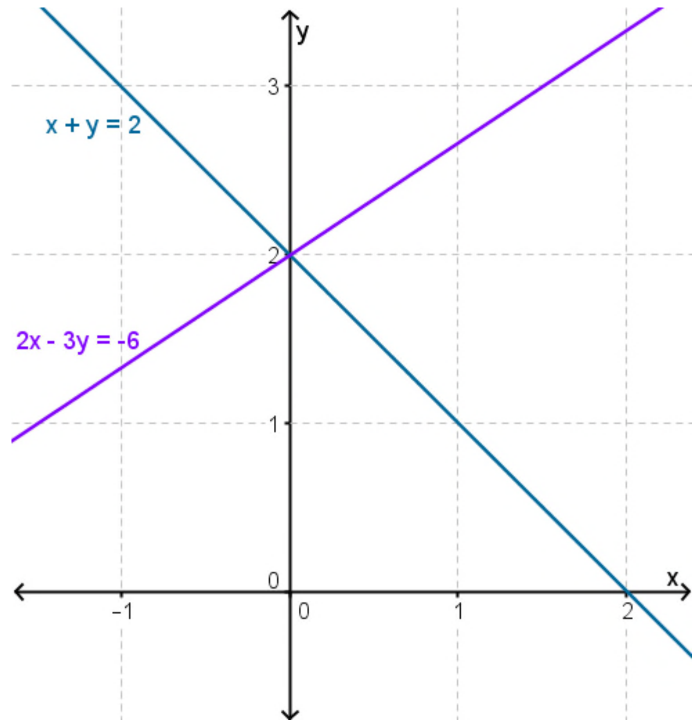


Recall: The solution to a pair of linear equations in two unknowns is the pair of numerical values of the variables that solve both equations simultaneously

Alternatively: The solution to a pair of linear equations, *graphed as straight lines*, is the point where the two lines *intersect*

① $x + y = 2$

② $2x - 3y = -6$



The point of intersection is $(0, 2)$

Therefore, the solution is $(0, 2)$

The steps required to solve two equations in two unknowns graphically are:

1. **Manipulate both equations into $y = mx + b$ form**
2. **Graph both equations (using a table of values or slope-intercept)**
3. **Locate the point of intersection**

Warning: This method is not as precise as finding the solution algebraically. The graphs must be drawn very carefully and accurately to find the correct point of intersection

Manipulate → **Graph** → **Locate**

① $6x - 3y = -9$

② $7x - y = 2$

Original Equations

① $6x - 3y = -9$

② $7x - y = 2$

① $y = \frac{-6x-9}{-3} = 2x + 3$

② $y = \frac{-7x+2}{-1} = 7x - 2$

Manipulate

Solving Equations Graphically

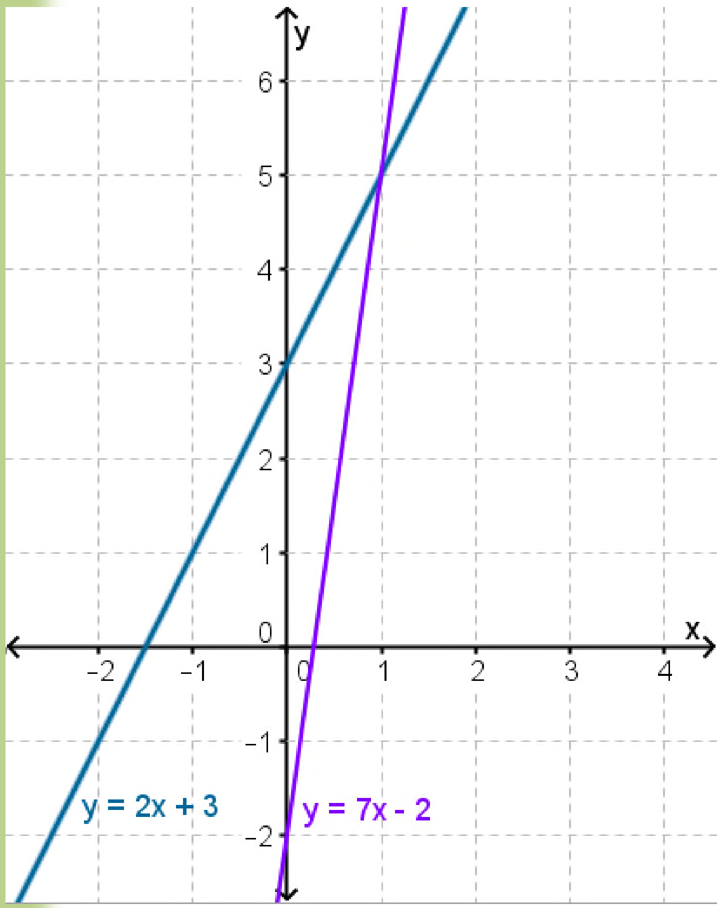
① $6x - 3y = -9$

② $7x - y = 2$

① $y = \frac{-6x-9}{-3} = 2x + 3$

② $y = \frac{-7x+2}{-1} = 7x - 2$

Graph



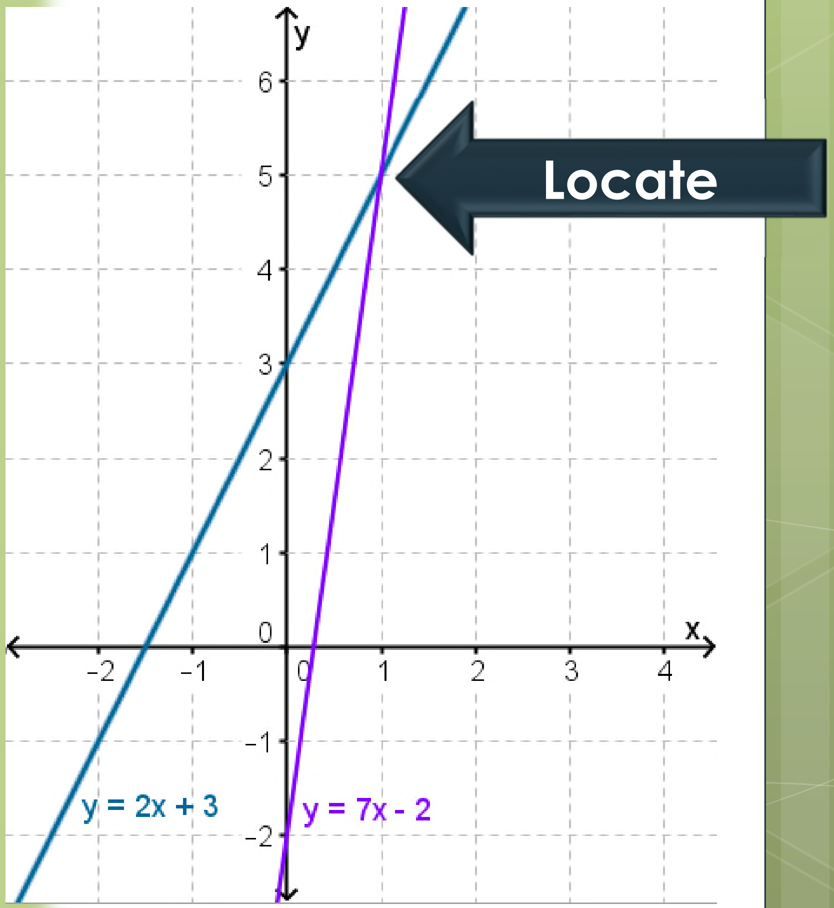
Solving Equations Graphically

① $6x - 3y = -9$

② $7x - y = 2$

① $y = \frac{-6x-9}{-3} = 2x + 3$

② $y = \frac{-7x+2}{-1} = 7x - 2$

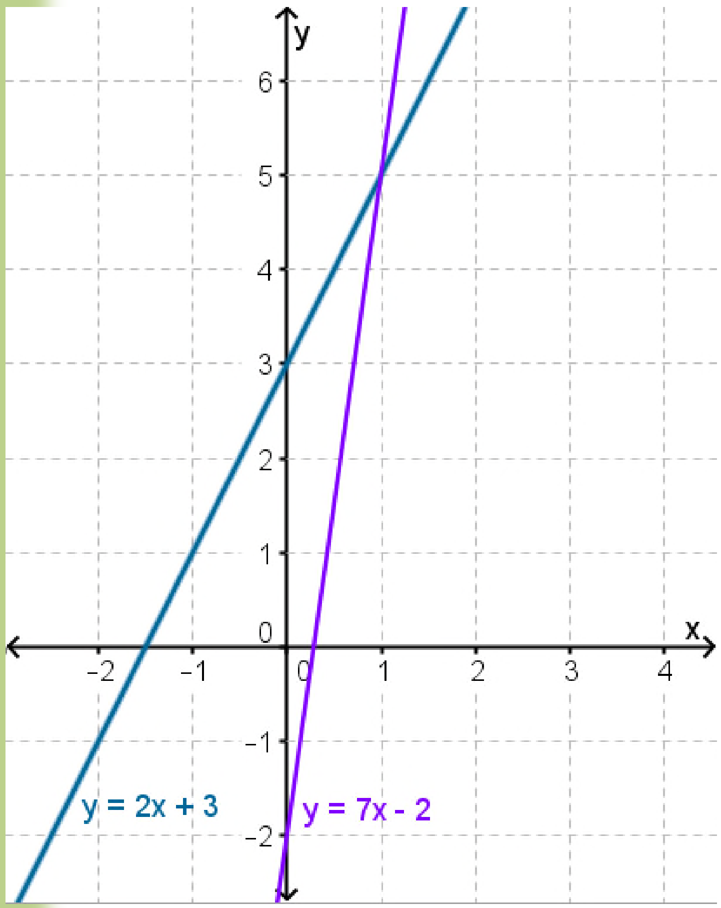


① $6x - 3y = -9$

② $7x - y = 2$

① $y = \frac{-6x-9}{-3} = 2x + 3$

② $y = \frac{-7x+2}{-1} = 7x - 2$



The point of intersection is $(1, 5)$

Therefore, the solution is $(1, 5)$

Word Problems

Word problems require a transition from simply solving *given equations* to *creating equations* yourself based on what you read. The following steps are helpful:

1. **Read the entire problem**
2. **Extract and label the data. Identify the unknown quantity and specify its symbol. Draw and label a diagram if appropriate**
3. **Create a word equation that relates the given data to the unknown quantity**
4. **Convert the word equation to an algebraic equation**
5. **Solve the equation**

Word Problems Ex. 1

Barbie and Ken sell cars at Auto World. In April, Barbie sold twice as many cars as Ken. In total, they sold 15 cars that month. How many cars did Ken sell?

Step 1: Read the entire problem

Step 2: Extract and label the data

Barbie's sales = $2(\boldsymbol{Ken's\ sales})$

Total sales = $\mathbf{15}$

Ken's sales = $\boldsymbol{x}$

Word Problems Ex. 1

Barbie and Ken sell cars at Auto World. In April, Barbie sold twice as many cars as Ken. In total, they sold 15 cars that month. How many cars did Ken sell?

Step 1: Read the entire problem

Step 2: Extract and label the data

Barbie's sales = $2(Ken's\ sales)$ **=** $2x$

Total sales = 15

Ken's sales = x

Word Problems Ex. 1

Barbie and Ken sell cars at Auto World. In April, Barbie sold twice as many cars as Ken. In total, they sold 15 cars that month. How many cars did Ken sell?

Step 3: Create a word equation

Barbie's sales + Ken's sales = Total sales

Step 4: Convert to an algebraic equation

$$2x + x = 15$$

Step 5: Solve the equation

$$\therefore x = 5$$

Ken sold 5 cars (and Barbie sold 10)

Word Problems Ex. 2

A day care purchases the same amount of milk and orange juice each week. The price of milk increased from $1.50 to $1.60 per litre and the price of orange juice increased from $1.30 to $1.39 per can. Before the increase, the weekly bill was $57.00. Now the weekly bill is $60.85. How many litres of milk and cans of orange juice are purchased each week?

Step 1: Read the entire problem

Word Problems Ex. 2

A day care purchases the same amount of milk and orange juice each week. The price of milk increased from $1.50 to $1.60 per litre and the price of orange juice increased from $1.30 to $1.39 per can. Before the increase, the weekly bill was $57.00. Now the weekly bill is $60.85. How many litres of milk and cans of orange juice are purchased each week?

Step 2: Extract and label the data

Price of milk per litre before = $\$1.50$, after = $\$1.60$

Price of orange juice per can before = $\$1.30$, after = $\$1.39$

Weekly bill before = $\$57.00$, after = $\$60.85$

# of milk litres per week = x

# of orange juice cans per week = y

Word Problems Ex. 2

A day care purchases the same amount of milk and orange juice each week. The price of milk increased from \$1.50 to \$1.60 per litre and the price of orange juice increased from \$1.30 to \$1.39 per can. Before the increase, the weekly bill was \$57.00. Now the weekly bill is \$60.85. How many litres of milk and cans of orange juice are purchased each week?

Step 3: Create word equations

① Weekly bill before =

\$ spent on milk before + \$ spent on juice before

② Weekly bill after =

\$ spent on milk after + \$ spent on juice after

Word Problems Ex. 2

A day care purchases the same amount of milk and orange juice each week. The price of milk increased from \$1.50 to \$1.60 per litre and the price of orange juice increased from \$1.30 to \$1.39 per can. Before the increase, the weekly bill was \$57.00. Now the weekly bill is \$60.85. How many litres of milk and cans of orange juice are purchased each week?

Step 4: Convert to algebraic equations

① $\$57.00 = \$1.50x + \$1.30y$

② $\$60.85 = \$1.60x + \$1.39y$

Word Problems Ex. 2

A day care purchases the same amount of milk and orange juice each week. The price of milk increased from $1.50 to $1.60 per litre and the price of orange juice increased from $1.30 to $1.39 per can. Before the increase, the weekly bill was $57.00. Now the weekly bill is $60.85. How many litres of milk and cans of orange juice are purchased each week?

Step 5: Solve the equations

$\therefore (x, y) = (25, 15)$

25 litres of milk and 15 cans of orange juice are purchased each week

Percent Change

When a quantity changes, the amount of the change is often expressed as a percentage of the initial value. That is,

$$\textbf{Percent Change} = \frac{\textbf{Final Value} - \textbf{Initial Value}}{\textbf{Initial Value}} \times 100\%$$

To write this formula more compactly we define the following symbols:

Initial Value: V_i Final Value: V_f Percent Change: c

$$c = \frac{V_f - V_i}{V_i} \times 100\%$$

Percent Change Ex. 1

Monday's sales were $1000 and Tuesday's sales were $2500. Find the percent change.

$$c = \frac{V_f - V_i}{V_i} \times 100\%$$

$$c = \frac{\$2500 - \$1000}{\$1000} \times 100\%$$

$$c = \frac{\$1500}{\$1000} \times 100\%$$

$$c = 1.5 \times 100\%$$

$$c = 150\%$$

$\therefore$ Sales increased by 150%

Percent Change Ex. 2

In the making of dried fruit, 15 kg of fruit shrinks to 3 kg. Find the percent change.

$$c = \frac{V_f - V_i}{V_i} \times 100\%$$

$$c = \frac{3 - 15}{15} \times 100\%$$

$$c = \frac{-12}{15} \times 100\%$$

$$c = -0.80 \times 100\%$$

$$c = -80\%$$

∴ The fruit decreased by 80%

Percent Change Ex. 3

You paid $75 for a coat after a 20% discount. What was the original price of the coat?

$$c = \frac{V_f - V_i}{V_i} \times 100\%$$

$$-20\% = \frac{\$75 - V_i}{V_i} \times 100\%$$

$$\frac{-20\%}{100\%} \times V_i = \$75 - V_i$$

$$-0.20V_i + V_i = \$75$$

$$V_i = \frac{\$75}{0.80}$$

$$V_i = \$93.75$$

∴ The coat originally cost $93.75

In summary, you can now

- Simplify algebraic expressions by extracting common factors and applying rules of exponents
- Rearrange a formula or equation to isolate a particular variable
- Solve a linear equation in one variable, and two linear equations in two variables
- Graph a linear equation in two variables
- Express a linear equation in slope-intercept form
- Solve two equations in two unknowns by a graphical method
- Solve "word problems" that lead to a linear equation in one unknown, or two linear equations in two unknowns
- Solve problems involving percent change