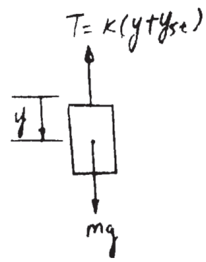
22–1.

A spring is stretched 175 mm by an 8-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 1.50 m/s, determine the differential equation which describes the motion. Assume that positive displacement is downward. Also, determine the position of the block when $t = 0.22$ s.

SOLUTION

$+\downarrow \Sigma F_y = ma_y; \qquad mg - k(y + y_{st}) = m\ddot{y} \qquad \text{where } ky_{st} = mg$



$$\ddot{y} + \frac{k}{m}y = 0$$

Hence $\qquad p = \sqrt{\frac{k}{m}} \qquad \text{Where } k = \frac{8(9.81)}{0.175} = 448.46 \text{ N/m}$

$$= \sqrt{\frac{448.46}{8}} = 7.487$$

$\therefore \qquad \ddot{y} + (7.487)^2 y = 0 \qquad \ddot{y} + 56.1y = 0$ **Ans.**

The solution of the above differential equation is of the form:

$y = A \sin pt + B \cos pt$ **(1)**

$v = \dot{y} = Ap \cos pt - Bp \sin pt$ **(2)**

At $t = 0$, $y = 0.1$ m and $v = v_0 = 1.50$ m/s

From Eq. (1) $\qquad 0.1 = A \sin 0 + B \cos 0 \qquad B = 0.1 \text{ m}$

From Eq. (2) $\qquad v_0 = Ap \cos 0 - 0 \qquad A = \frac{v_0}{p} = \frac{1.50}{7.487} = 0.2003 \text{ m}$

Hence $\qquad y = 0.2003 \sin 7.487t + 0.1 \cos 7.487t$

At $t = 0.22$ s, $\qquad y = 0.2003 \sin [7.487(0.22)] + 0.1 \cos [7.487(0.22)]$

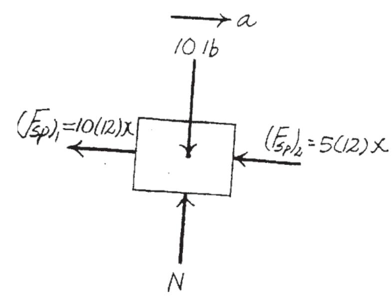
$\qquad = 0.192 \text{ m}$ **Ans.**

Ans:
$\ddot{y} + 56.1\, y = 0$
$y\big|_{t=0.22\text{ s}} = 0.192 \text{ m}$

22–2.

A spring has a stiffness of 800 N/m. If a 2-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation that describes the block's motion. Assume that positive displacement is downward.

SOLUTION



$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{2}} = 20$$

$$x = A \sin pt + B \cos pt$$

$$x = -0.05 \text{ m} \quad \text{when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = 0 \text{ when } t = 0,$$

$$0 = A(20) - 0; \qquad A = 0$$

Thus,

$$x = -0.05 \cos (20t)$$ **Ans.**

Ans:
$x = -0.05 \cos (20t)$

22–3.

A spring is stretched 200 mm by a 15-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 0.75 m/s, determine the equation which describes the motion. What is the phase angle? Assume that positive displacement is downward.

SOLUTION

$$k = \frac{F}{y} = \frac{15(9.81)}{0.2} = 735.75 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{735.75}{15}} = 7.00$$

$$y = A \sin \omega_n t + B \cos \omega_n t$$

$$y = 0.1 \text{ m when } t = 0,$$

$$0.1 = 0 + B; \qquad B = 0.1$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$v = 0.75 \text{ m/s when } t = 0,$$

$$0.75 = A(7.00)$$

$$A = 0.107$$

$$y = 0.107 \sin (7.00t) + 0.100 \cos (7.00t) \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{0.100}{0.107}\right) = 43.0^\circ \qquad \textbf{Ans.}$$

Ans:
$y = 0.107 \sin (7.00t) + 0.100 \cos (7.00t)$
$\phi = 43.0^\circ$

***22–4.**

When a 20-lb weight is suspended from a spring, the spring is stretched a distance of 4 in. Determine the natural frequency and the period of vibration for a 10-lb weight attached to the same spring.

SOLUTION

$$k = \frac{20}{\frac{4}{12}} = 60 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60}{\frac{10}{32.2}}} = 13.90 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\tau = \frac{2\pi}{\omega_n} = 0.452 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 13.90 \text{ rad/s}$
$\tau = 0.452 \text{ s}$

22–5.

When a 3-kg block is suspended from a spring, the spring is stretched a distance of 60 mm. Determine the natural frequency and the period of vibration for a 0.2-kg block attached to the same spring.

SOLUTION

$$k = \frac{F}{\Delta x} = \frac{3(9.81)}{0.060} = 490.5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.2}} = 49.52 = 49.5 \text{ rad/s} \qquad \textbf{Ans.}$$

$$f = \frac{\omega_n}{2\pi} = \frac{49.52}{2\pi} = 7.88 \text{ Hz}$$

$$\tau = \frac{1}{f} = \frac{1}{7.88} = 0.127 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 49.5 \text{ rad/s}$
$\tau = 0.127 \text{ s}$

22–6.

An 8-kg block is suspended from a spring having a stiffness $k = 80$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 90 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is measured downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{8}} = 3.162 \text{ rad/s}$$

$$v = -0.4 \text{ m/s}, \qquad x = -0.09 \text{ m at } t = 0$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.09 = 0 + B$$

$$B = -0.09$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$-0.4 = A(3.162) - 0$$

$$A = -0.126$$

Thus, $x = -0.126 \sin (3.16t) - 0.09 \cos (3.16t)$ m **Ans.**

$C = \sqrt{A^2 + B^2} = \sqrt{(-0.126)^2 + (-0.09)} = 0.155$ m **Ans.**

Ans:
$x = \{-0.126 \sin (3.16t) - 0.09 \cos (3.16t)\}$ m
$C = 0.155$ m

22–7.

A 2-lb weight is suspended from a spring having a stiffness $k = 2$ lb/in. If the weight is pushed 1 in. upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

SOLUTION

$$k = 2(12) = 24 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{\frac{2}{32.2}}} = 19.66 = 19.7 \text{ rad/s} \qquad \textbf{Ans.}$$

$$y = -\frac{1}{12}, \qquad v = 0 \text{ at } t = 0$$

From Eqs. 22–3 and 22–4,

$$-\frac{1}{12} = 0 + B$$

$$B = -0.0833$$

$$0 = A\omega_n + 0$$

$$A = 0$$

$$C = \sqrt{A^2 + B^2} = 0.0833 \text{ ft} = 1 \text{ in.} \qquad \textbf{Ans.}$$

Position equation,

$$y = (0.0833 \cos 19.7t) \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 19.7$ rad/s
$C = 1$ in.
$y = (0.0833 \cos 19.7t)$ ft

***22–8.**

A 6-lb weight is suspended from a spring having a stiffness $k = 3\ \text{lb/in.}$ If the weight is given an upward velocity of 20 ft/s when it is 2 in. above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the weight, measured from the equilibrium position. Assume positive displacement is downward.

SOLUTION

$$k = 3(12) = 36\ \text{lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{36}{\frac{6}{32.2}}} = 13.90\ \text{rad/s}$$

$$t = 0, \qquad v = -20\ \text{ft/s}, \qquad y = -\frac{1}{6}\ \text{ft}$$

From Eq. 22–3,

$$-\frac{1}{6} = 0 + B$$

$$B = -0.167$$

From Eq. 22–4,

$$-20 = A(13.90) + 0$$

$$A = -1.44$$

Thus,

$$y = [-1.44 \sin(13.9t) - 0.167 \cos(13.9t)]\ \text{ft} \qquad \textbf{Ans.}$$

From Eq. 22–10,

$$C = \sqrt{A^2 + B^2} = \sqrt{(1.44)^2 + (-0.167)^2} = 1.45\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$y = [-1.44 \sin(13.9t) - 0.167 \cos(13.9t)]\ \text{ft}$
$C = 1.45\ \text{ft}$

22–9.

A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the frequency of the vibration? Assume that positive displacement is downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{3}} = 8.16 \text{ rad/s} \qquad \textbf{Ans.}$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$x = -0.05 \text{ m when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$v = 0 \text{ when } t = 0,$$

$$0 = A(8.165) - 0; \qquad A = 0$$

Hence,

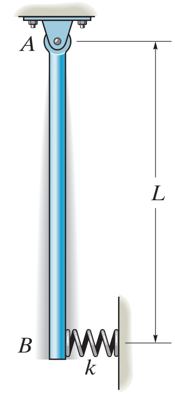
$$x = -0.05 \cos (8.16t) \qquad \textbf{Ans.}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(0)^2 + (-0.05)} = 0.05 \text{ m} = 50 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 8.16$ rad/s
$x = -0.05 \cos (8.16t)$
$C = 50$ mm

22–10.

The uniform rod of mass m is supported by a pin at A and a spring at B. If B is given a small sideward displacement and released, determine the natural period of vibration.



SOLUTION

Equation of Motion. The mass moment of inertia of the rod about A is $I_A = \frac{1}{3}mL^2$. Referring to the FBD. of the rod, Fig. a,

$$\circlearrowleft +\Sigma M_A = I_A\alpha; \qquad -mg\left(\frac{L}{2}\sin\theta\right) - (kx\cos\theta)(L) = \left(\frac{1}{3}mL^2\right)\alpha$$

However; $\quad x = L\sin\theta$. Then

$$\frac{-mgL}{2}\sin\theta - kL^2\sin\theta\cos\theta = \frac{1}{3}mL^2\alpha$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$,

$$\frac{-mgL}{2}\sin\theta - \frac{KL^2}{2}\sin 2\theta = \frac{1}{3}mL^2\alpha$$

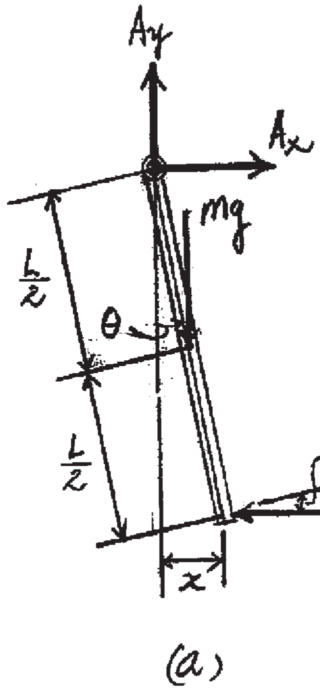
Here since θ is small $\sin\theta \simeq \theta$ and $\sin 2\theta \simeq 2\theta$. Also $\alpha = \ddot{\theta}$. Then the above equation becomes

$$\frac{1}{3}mL^2\ddot{\theta} + \left(\frac{mgL}{2} + kL^2\right)\theta = 0$$

$$\ddot{\theta} + \frac{3mg + 6kL}{2mL}\theta = 0$$

Comparing to that of the Standard form, $\omega_n = \sqrt{\frac{3mg + 6kL}{2mL}}$. Then

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2mL}{3mg + 6kL}} \qquad \textbf{Ans.}$$



(a)

$K L^2 \sin\theta \cos\theta$

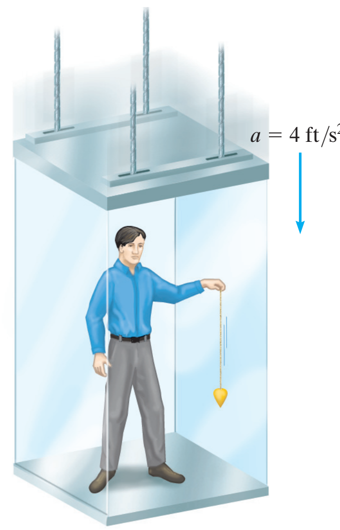
$\frac{kL^2}{2}$

Ans:

$\tau = 2\pi\sqrt{\frac{2mL}{3mg + 6kL}}$

22–11.

While standing in an elevator, the man holds a pendulum which consists of an 18-in. cord and a 0.5-lb bob. If the elevator is descending with an acceleration $a = 4\ \text{ft/s}^2$, determine the natural period of vibration for small amplitudes of swing.



SOLUTION

Since the acceleration of the pendulum is $(32.2 - 4) = 28.2\ \text{ft/s}^2$

Using the result of Example 22–1,

We have

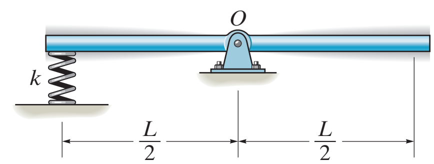
$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{28.2}{18/12}} = 4.336\ \text{rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{4.336} = 1.45\ \text{s} \qquad \textbf{Ans.}$$

Ans:
$\tau = 1.45\ \text{s}$

***22–12.**

Determine the natural period of vibration of the uniform bar of mass m when it is displaced downward slightly and released.



SOLUTION

Equation of Motion. The mass moment of inertia of the bar about O is $I_0 = \frac{1}{12}mL^2$. Referring to the FBD of the rod, Fig. a,

$$\zeta + \Sigma M_0 = I_0\alpha; \quad -ky\cos\theta\left(\frac{L}{2}\right) = \left(\frac{1}{12}mL^2\right)\alpha$$

However, $y = \frac{L}{2}\sin\theta$. Then

$$-k\left(\frac{L}{2}\sin\theta\right)\cos\theta\left(\frac{L}{2}\right) = \frac{1}{12}mL^2\alpha$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$, we obtain

$$\frac{1}{12}mL^2\alpha + \frac{kL^2}{8}\sin 2\theta = 0$$

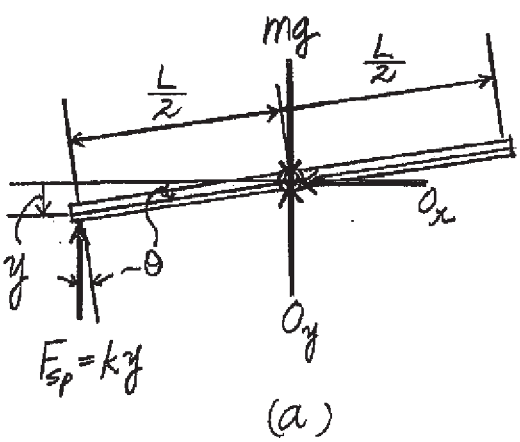
Here since θ is small, $\sin 2\theta \simeq 2\theta$. Also, $\alpha = \ddot{\theta}$. Then the above equation becomes

$$\frac{1}{12}mL^2\ddot{\theta} + \frac{kL^2}{4}\theta = 0$$

$$\ddot{\theta} + \frac{3k}{m}\theta = 0$$

Comparing to that of the Standard form, $\omega_n = \sqrt{\frac{3k}{m}}$. Then

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{3k}} \qquad \textbf{Ans.}$$



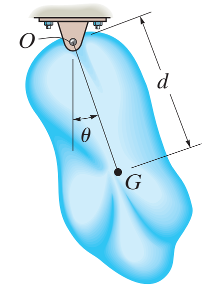
(a)

Ans:

$$\tau = 2\pi\sqrt{\frac{m}{3k}}$$

22–13.

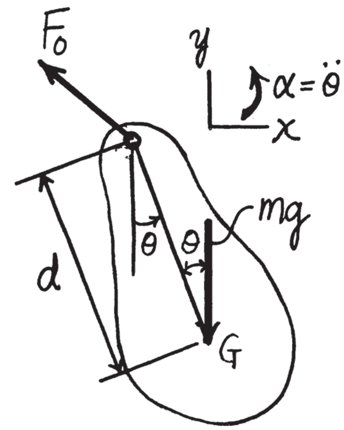
The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.



SOLUTION

$\circlearrowleft +\Sigma M_O = I_O\,\alpha; \qquad -mgd\sin\theta = \left[mk_G^2 + md^2\right]\ddot{\theta}$

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\sin\theta = 0$$



However, for small rotation $\sin\theta \approx \theta$. Hence

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{\dfrac{gd}{k_G^2 + d^2}}$.

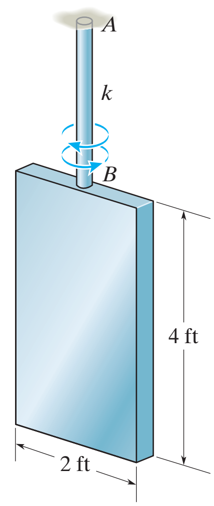
$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{gd}{k_G^2 + d^2}}} = 2\pi\sqrt{\frac{k_G^2 + d^2}{gd}}$$ **Ans.**

Ans:

$\tau = 2\pi\sqrt{\dfrac{k_G^2 + d^2}{gd}}$

22–14.

The 20-lb rectangular plate has a natural period of vibration $\tau = 0.3$ s, as it oscillates around the axis of rod AB. Determine the torsional stiffness k, measured in lb·ft/rad, of the rod. Neglect the mass of the rod.



SOLUTION

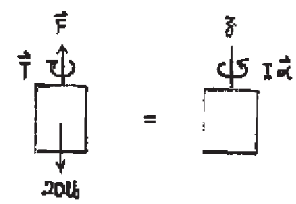
$T = k\theta$

$\Sigma M_z = I_z\alpha; \quad -k\theta = \frac{1}{12}\left(\frac{20}{32.2}\right)(2)^2\ddot{\theta}$

$\ddot{\theta} + k(4.83)\theta = 0$

$\tau = \frac{2\pi}{\sqrt{k(4.83)}} = 0.3$

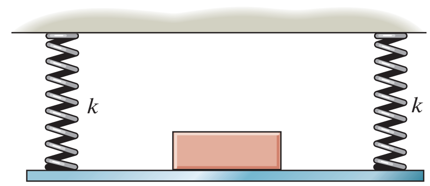
$k = 90.8 \text{ lb} \cdot \text{ft/rad}$ **Ans.**



Ans:
$k = 90.8 \text{ lb} \cdot \text{ft/rad}$

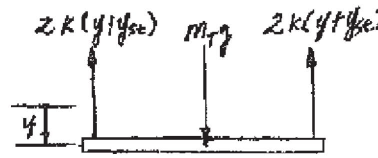
22–15.

A platform, having an unknown mass, is supported by *four* springs, each having the same stiffness k. When nothing is on the platform, the period of vertical vibration is measured as 2.35 s; whereas if a 3-kg block is supported on the platform, the period of vertical vibration is 5.23 s. Determine the mass of a block placed on the (empty) platform which causes the platform to vibrate vertically with a period of 5.62 s. What is the stiffness k of each of the springs?



SOLUTION

$+\downarrow \Sigma F_y = ma_y; \qquad m\tau g - 4k(y + y_{\tau s}) = m\tau \ddot{y} \qquad \text{Where } 4k\, y_{\tau s} = m_\tau g$



$$\ddot{y} + \frac{4k}{m\tau}y = 0$$

Hence $$P = \sqrt{\frac{4k}{m\tau}}$$

$$\tau = \frac{2\pi}{P} = 2\pi\sqrt{\frac{m\tau}{4k}}$$

For empty platform $m\tau = m_P$, where m_P is the mass of the platform.

$$2.35 = 2\pi\sqrt{\frac{m_P}{4k}} \qquad \textbf{(1)}$$

When 3-kg block is on the platform $m_\tau = m_P + 3$.

$$5.23 = 2\pi\sqrt{\frac{m_P + 3}{4k}} \qquad \textbf{(2)}$$

When an unknown mass is on the platform $m_\tau = m_P + m_B$.

$$5.62 = 2\pi\sqrt{\frac{m_P + m_B}{4k}} \qquad \textbf{(3)}$$

Solving Eqs. (1) to (3) yields :

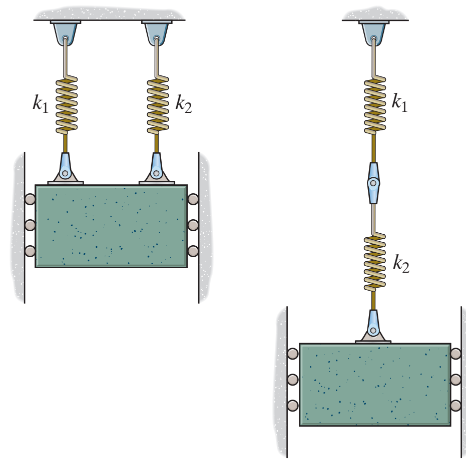
$$k = 1.36\ \text{N/m} \qquad m_B = 3.58\ \text{kg} \qquad \textbf{Ans.}$$

$$m_P = 0.7589\ \text{kg}$$

Ans:
$k = 1.36\ \text{N/m}$
$m_B = 3.58\ \text{kg}$

***22–16.**

A block of mass m is suspended from two springs having a stiffness of k_1 and k_2, arranged a) parallel to each other, and b) as a series. Determine the equivalent stiffness of a single spring with the same oscillation characteristics and the period of oscillation for each case.



(a) (b)

SOLUTION

(a) When the springs are arranged in parallel, the equivalent spring stiffness is

$$k_{eq} = k_1 + k_2 \qquad \textbf{Ans.}$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$$

Thus, the period of oscillation of the system is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{k_1 + k_2}{m}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}} \qquad \textbf{Ans.}$$

(b) When the springs are arranged in a series, the equivalent stiffness of the system can be determined by equating the stretch of both spring systems subjected to the same load F.

$$\frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{k_{eq}}$$

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_{eq}}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{k_{eq}}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} \qquad \textbf{Ans.}$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\left(\dfrac{k_1 k_2}{k_2 + k_1}\right)}{m}}$$

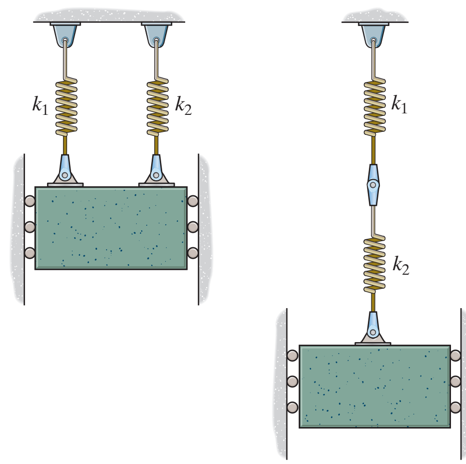
Thus, the period of oscillation of the system is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{\left(\dfrac{k_1 k_2}{k_2 + k_1}\right)}{m}}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}} \qquad \textbf{Ans.}$$

Ans:
$k_{eq} = k_1 + k_2$
$\tau = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$
$k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$
$\tau = 2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$

22–17.

The 15-kg block is suspended from two springs having a different stiffness and arranged a) parallel to each other, and b) as a series. If the natural periods of oscillation of the parallel system and series system are observed to be 0.5 s and 1.5 s, respectively, determine the spring stiffnesses k_1 and k_2.



(a) (b)

SOLUTION

The equivalent spring stiffness of the spring system arranged in parallel is $\left(k_{eq}\right)_P = k_1 + k_2$ and the equivalent stiffness of the spring system arranged in a series can be determined by equating the stretch of the system to a single equivalent spring when they are subjected to the same load.

$$\frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{(k_{eq})_S}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{\left(k_{eq}\right)_S}$$

$$\left(k_{eq}\right)_S = \frac{k_1 k_2}{k_1 + k_2}$$

Thus the natural frequencies of the parallel and series spring system are

$$(\omega_n)_P = \sqrt{\frac{\left(k_{eq}\right)_P}{m}} = \sqrt{\frac{k_1 + k_2}{15}}$$

$$(\omega_n)_S = \sqrt{\frac{\left(k_{eq}\right)_S}{m}} = \sqrt{\frac{\left(\dfrac{k_1 k_2}{k_1 + k_2}\right)}{15}} = \sqrt{\frac{k_1 k_2}{15\left(k_1 + k_2\right)}}$$

Thus, the natural periods of oscillation are

$$\tau_P = \frac{2\pi}{(\omega_n)_P} = 2\pi\sqrt{\frac{15}{k_1 + k_2}} = 0.5 \qquad \textbf{(1)}$$

$$\tau_S = \frac{2\pi}{(\omega_n)_S} = 2\pi\sqrt{\frac{15\left(k_1 + k_2\right)}{k_1 k_2}} = 1.5 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

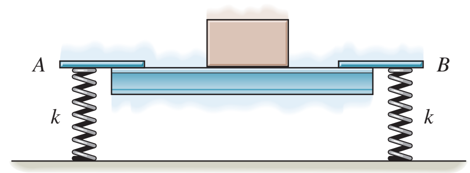
$k_1 = 2067\ \text{N/m}$ or $302\ \text{N/m}$ **Ans.**

$k_2 = 302\ \text{N/m}$ or $2067\ \text{N/m}$ **Ans.**

Ans:
$k_1 = 2067\ \text{N/m}$
$k_2 = 302\ \text{N/m}$
or vice versa

22–18.

The uniform beam is supported at its ends by two springs A and B, each having the same stiffness k. When nothing is supported on the beam, it has a period of vertical vibration of 0.83 s. If a 50-kg mass is placed at its center, the period of vertical vibration is 1.52 s. Compute the stiffness of each spring and the mass of the beam.



SOLUTION

$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{\tau^2}{(2\pi)^2} = \frac{m}{k}$$

$$\frac{(0.83)^2}{(2\pi)^2} = \frac{m_B}{2k} \qquad \textbf{(1)}$$

$$\frac{(1.52)^2}{(2\pi)^2} = \frac{m_B + 50}{2k} \qquad \textbf{(2)}$$

Eqs. (1) and (2) become

$$m_B = 0.03490k$$

$$m_B + 50 = 0.1170k$$

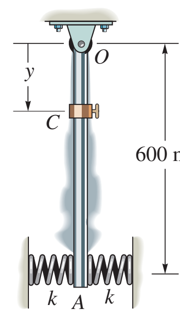
$$m_B = 21.2 \text{ kg} \qquad \textbf{Ans.}$$

$$k = 609 \text{ N/m} \qquad \textbf{Ans.}$$

Ans:
$m_B = 21.2$ kg
$k = 609$ N/m

22–19.

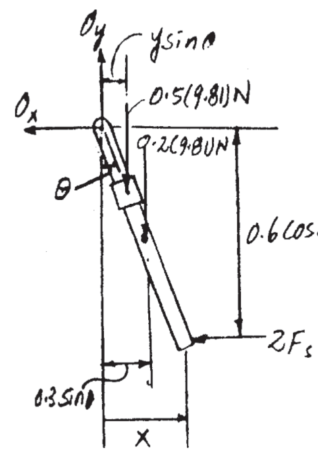
The slender rod has a mass of 0.2 kg and is supported at O by a pin and at its end A by two springs, each having a stiffness $k = 4$ N/m. The period of vibration of the rod can be set by fixing the 0.5-kg collar C to the rod at an appropriate location along its length. If the springs are originally unstretched when the rod is vertical, determine the position y of the collar so that the natural period of vibration becomes $\tau = 1$ s. Neglect the size of the collar.



SOLUTION

Moment of inertia about O:

$$I_O = \frac{1}{3}(0.2)(0.6)^2 + 0.5y^2 = 0.024 + 0.5y^2$$



Each spring force $F_s = kx = 4x$.

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad -2(4x)(0.6\cos\theta) - 0.2(9.81)(0.3\sin\theta)$$

$$-0.5(9.81)(y\sin\theta) = (0.024 + 0.5y^2)\,\ddot{\theta}$$

$$-4.8x\cos\theta - (0.5886 + 4.905y)\sin\theta = (0.024 + 0.5y^2)\ddot{\theta}$$

However, for small displacement $x = 0.6\theta$, $\sin\theta \approx \theta$ and $\cos\theta = 1$. Hence $\ddot{\theta} + \dfrac{3.4686 + 4.905y}{0.024 + 0.5y^2}\theta = 0$

From the above differential equation, $p = \sqrt{\dfrac{3.4686 + 4.905y}{0.024 + 0.5y^2}}$.

$$\tau = \frac{2\pi}{p}$$

$$1 = \frac{2\pi}{\sqrt{\dfrac{3.4686 + 4.905y}{0.024 + 0.5y^2}}}$$

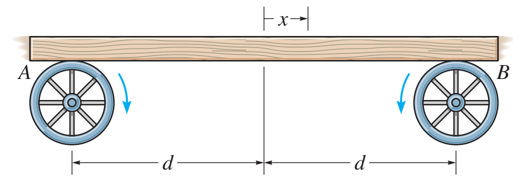
$$19.74y^2 - 4.905y - 2.5211 = 0$$

$$y = 0.503 \text{ m} = 503 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$y = 503$ mm

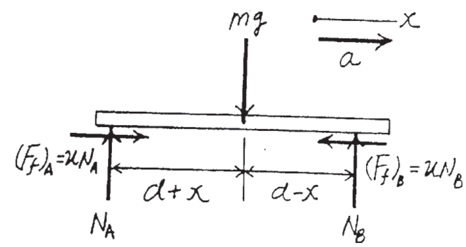
***22–20.**

A uniform board is supported on two wheels which rotate in opposite directions at a constant angular speed. If the coefficient of kinetic friction between the wheels and board is μ, determine the frequency of vibration of the board if it is displaced slightly, a distance x from the midpoint between the wheels, and released.



SOLUTION

Freebody Diagram: When the board is being displaced x to the right, the *restoring force* is due to the unbalance friction force at A and B $\left[(F_f)_B > (F_f)_A\right]$.



Equation of Motion:

$\circlearrowleft + \Sigma M_A = \Sigma (M_A)_k;\qquad N_B(2d) - mg(d + x) = 0$

$$N_B = \frac{mg(d + x)}{2d}$$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + \frac{mg(d + x)}{2d} - mg = 0$

$$N_A = \frac{mg(d - x)}{2d}$$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad \mu\left[\frac{mg(d - x)}{2d}\right] - \mu\left[\frac{mg(d + x)}{2d}\right] = ma$

$$a + \frac{\mu g}{d}x = 0 \qquad \textbf{(1)}$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substitute this value into Eq.(1), we have

$$\ddot{x} + \frac{\mu g}{d}x = 0 \qquad \textbf{(2)}$$

From Eq.(2), $\omega_n{}^2 = \dfrac{\mu g}{d}$, thus, $\omega_n = \sqrt{\dfrac{\mu g}{d}}$. Applying Eq. 22–4, we have

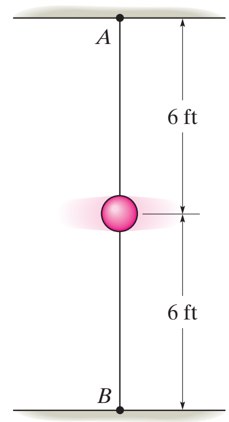
$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\mu g}{d}} \qquad \textbf{Ans.}$$

Ans:

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{\mu g}{d}}$

22–21.

If the wire AB is subjected to a tension of 20 lb, determine the equation which describes the motion when the 5-lb weight is displaced 2 in. horizontally and released from rest.



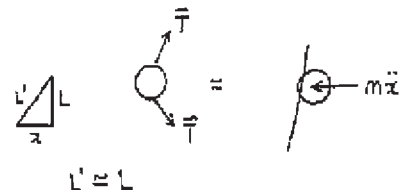
SOLUTION

$L' \equiv L$

$\overset{+}{\leftarrow} \Sigma F_x = m\,a_x; \qquad -2T\dfrac{x}{L} = m\ddot{x}$

$$\ddot{x} + \frac{2T}{Lm}x = 0$$

$$P = \sqrt{\frac{2T}{Lm}} = \sqrt{\frac{2(20)}{6\left(\frac{5}{32.2}\right)}} = 6.55\text{ rad/s}$$



$x = A\sin pt + B\cos pt$

$x = \dfrac{1}{6}$ ft at $t = 0$, $\qquad$ Thus $B = \dfrac{1}{6} = 0.167$

$v = A\,p\cos pt - B\,p\sin pt$

$v = 0$ at $t = 0$, $\qquad$ Thus $A = 0$

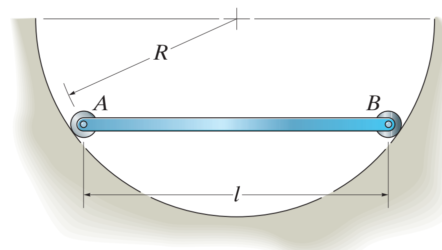
So that

$x = 0.167\cos 6.55t$ **Ans.**

Ans:
$x = 0.167\cos 6.55t$

22–22.

The bar has a length l and mass m. It is supported at its ends by rollers of negligible mass. If it is given a small displacement and released, determine the natural frequency of vibration.



SOLUTION

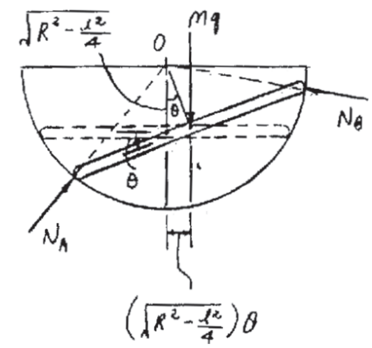
Moment of inertia about point O:

$$I_O = \frac{1}{12}ml^2 + m\left(\sqrt{R^2 - \frac{l^2}{4}}\right)^2 = m\left(R^2 - \frac{1}{6}l^2\right)$$

$$\zeta + \Sigma M_O = I_O\alpha; \qquad mg\left(\sqrt{R^2 - \frac{l^2}{4}}\right)\theta = -m\left(R^2 - \frac{1}{6}l^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{3g(4R^2 - l^2)^{\frac{1}{2}}}{6R^2 - l^2}\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{\dfrac{3g(4R^2 - l^2)^{\frac{1}{2}}}{6R^2 - l^2}}$. **Ans.**

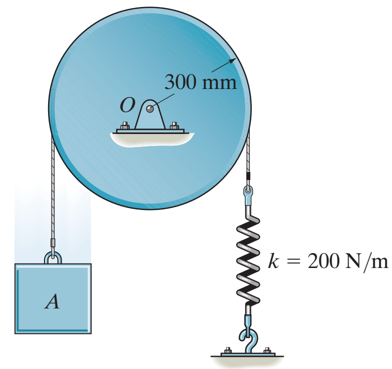


Ans:

$$\omega_n = \sqrt{\frac{3g(4R^2 - l^2)^{1/2}}{6R^2 - l^2}}$$

22–23.

The 20-kg disk, is pinned at its mass center O and supports the 4-kg block A. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.



SOLUTION

Equation of Motion. The mass moment of inertia of the disk about its mass center O is $I_0 = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.3^2) = 0.9\text{ kg}\cdot\text{m}^2$. When the disk undergoes a small angular displacement θ, the spring stretches further by $s = r\theta = 0.3\theta$. Thus, the total stretch is $y = y_{st} + 0.3\theta$. Then $F_{sp} = ky = 200(y_{st} + 0.3\theta)$. Referring to the FBD and kinetic diagram of the system, Fig. a,

$$\circlearrowleft +\Sigma M_0 = \Sigma(\mu_k)_0; \quad 4(9.81)(0.3) - 200(y_{st} + 0.3\theta)(0.3) = 0.90\alpha + 4[\alpha(0.3)](0.3)$$

$$11.772 - 60y_{st} - 18\theta = 1.26\alpha \qquad \textbf{(1)}$$

When the system is in equilibrium, $\theta = 0°$. Then

$$\circlearrowleft +\Sigma M_0 = 0; \qquad 4(9.81)(0.3) - 200(y_{st})(0.3) = 0$$

$$60y_{st} = 11.772$$

Substitute this result into Eq. (1), we obtain

$$-18\theta = 1.26\alpha$$

$$\alpha + 14.28571\theta = 0$$

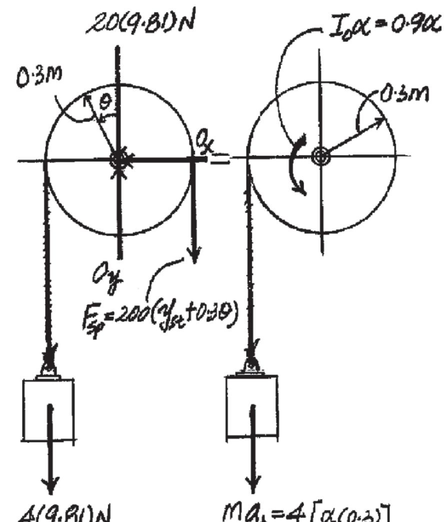
Since $\alpha = \ddot{\theta}$, the above equation becomes

$$\ddot{\theta} + 14.28571\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{14.2857} = 3.7796$ rad/s.

Thus,

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.7796} = 1.6623\text{ s} = 1.66\text{ s} \qquad \textbf{Ans.}$$

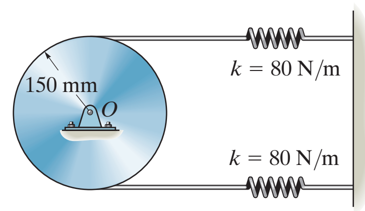


(a)

Ans:
$\tau = 1.66$ s

***22–24.**

The 10-kg disk is pin connected at its mass center. Determine the natural period of vibration of the disk if the springs have sufficient tension in them to prevent the cord from slipping on the disk as it oscillates. *Hint:* Assume that the initial stretch in each spring is δ_O.



SOLUTION

Equation of Motion. The mass moment of inertia of the disk about its mass center O is $I_0 = \frac{1}{2}Mr^2 = \frac{1}{2}(10)(0.15^2) = 0.1125\ \text{kg}\cdot\text{m}^2$. When the disk undergoes a small angular displacement θ, the top spring stretches further but the stretch of the spring is being reduced both by $s = r\theta = 0.15\theta$. Thus, $(F_{sp})_t = Kx_t = 80(\delta_0 - 0.15\theta)$ and $(F_{sp})_b = 80(\delta_0 - 0.15\theta)$. Referring to the FBD of the disk, Fig. a,

$$\zeta + \Sigma M_0 = I_0\alpha; \qquad -80(\delta_0 + 0.15\theta)(0.15) + 80(\delta_0 - 0.15\theta)(0.15) = 0.1125\alpha$$

$$-3.60\theta = 0.1125\alpha$$

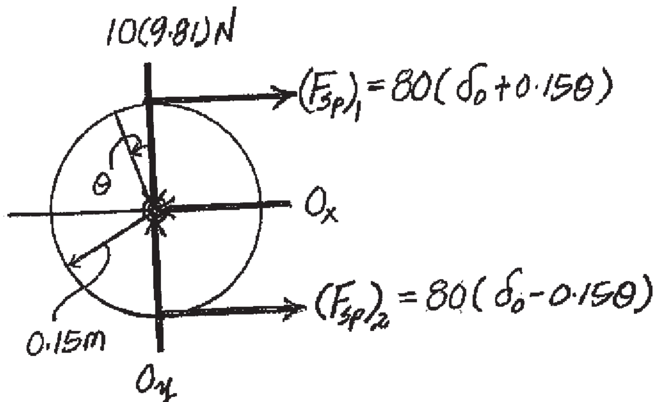
$$\alpha + 32\theta = 0$$

Since $\alpha = \ddot{\theta}$, this equation becomes

$$\ddot{\theta} + 32\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{32}$ rad/s. Then

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{32}} = 1.1107\ \text{s} = 1.11\ \text{s} \qquad \textbf{Ans.}$$

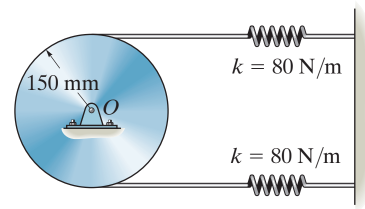


(a)

Ans:
$\tau = 1.11$ s

22–25.

If the disk in Prob. 22–24 has a mass of 10 kg, determine the natural frequency of vibration. *Hint:* Assume that the initial stretch in each spring is δ_O.



SOLUTION

Equation of Motion. The mass moment of inertia of the disk about its mass center O is $I_0 = \frac{1}{2}mr^2 = \frac{1}{2}(10)(0.15^2) = 0.1125\ \text{kg}\cdot\text{m}^2$ when the disk undergoes a small angular displacement θ, the top spring stretches but the bottom spring compresses, both by $s = r\theta = 0.15\theta$. Thus, $(F_{sp})_t = (F_{sp})_b = ks = 80(0.15\theta) = 12\theta$. Referring to the FBD of the disk, Fig. a,

$$\circlearrowleft + \Sigma M_0 = I_0\alpha; \qquad -12\theta(0.3) = 0.1125\alpha$$

$$-3.60\theta = 0.1125\alpha$$

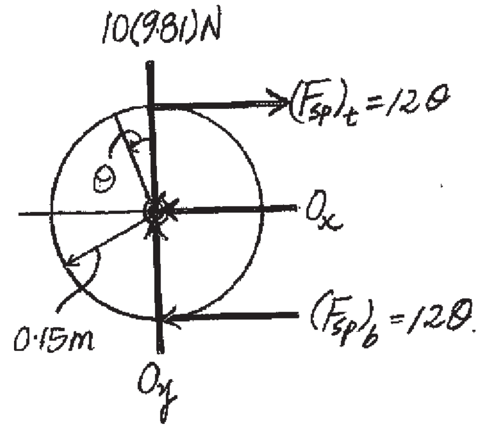
$$\alpha + 32\theta = 0$$

Since $\alpha = \ddot{\theta}$, this equation becomes

$$\ddot{\theta} + 32\theta = 0$$

Comparing to that of Standard form, $\omega_n = \sqrt{32}$ rad/s. Then

$$f = \frac{\omega_n}{2\pi} = \frac{\sqrt{32}}{2\pi} = 0.9003\ \text{Hz} = 0.900\ \text{Hz}$$

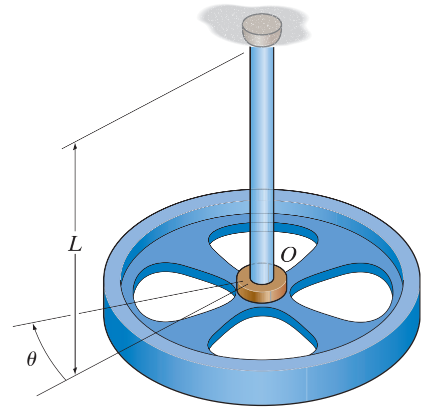


(a)

Ans:
$f = 0.900$ Hz

22–26.

A flywheel of mass m, which has a radius of gyration about its center of mass of k_O, is suspended from a circular shaft that has a torsional resistance of $M = C\theta$. If the flywheel is given a small angular displacement of θ and released, determine the natural period of oscillation.

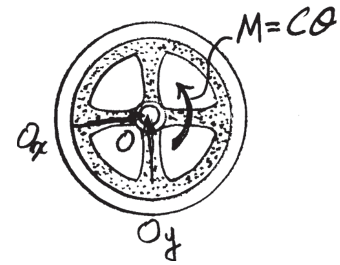


SOLUTION

Equation of Motion: The mass moment of inertia of the wheel about point O is $I_O = mk_O{}^2$. Referring to Fig. a,

$$\circlearrowleft + \qquad \Sigma M_O = I_O\,\alpha; \qquad -C\theta = mk_O{}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{C}{mk_O{}^2}\theta = 0$$



Comparing this equation to the standard equation, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{C}{mk_O{}^2}} = \frac{1}{k_O}\sqrt{\frac{C}{m}}$$

Thus, the natural period of the oscillation is

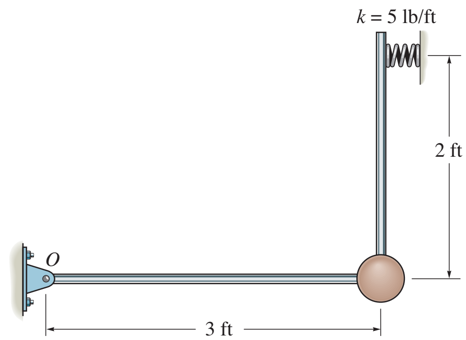
$$\tau = \frac{2\pi}{\omega_n} = 2\pi k_O\sqrt{\frac{m}{C}} \qquad \textbf{Ans.}$$

Ans:

$$\tau = 2\pi k_O\sqrt{\frac{m}{C}}$$

22–27.

The 6-lb weight is attached to the rods of negligible mass. Determine the natural frequency of vibration of the weight when it is displaced slightly from the equilibrium position and released.

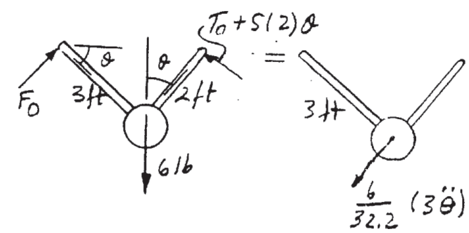


SOLUTION

T_O is the equilibrium force.

$$T_O = \frac{6(3)}{2} = 9 \text{ lb}$$

Thus, for small θ,



$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad 6(3) - \left[9 + 5(2)\theta\right](2) = \left(\frac{6}{32.2}\right)\left(3\ddot{\theta}\right)(3)$$

Thus,

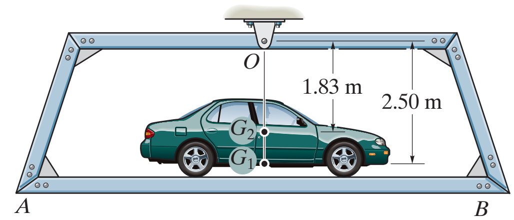
$$\ddot{\theta} + 11.926\theta = 0$$

$$\omega_n = \sqrt{11.926} = 3.45 \text{ rad/s}$$ **Ans.**

Ans:
$\omega_n = 3.45$ rad/s

***22–28.**

The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



SOLUTION

Free-body Diagram: When an object arbitrary shape having a mass m is pinned at O and being displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y.

$$\circlearrowleft + \Sigma M_O = I_O\alpha: \qquad -mg\sin\theta(l) = I_O\alpha \tag{1}$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substituting these values into Eq. (1), we have

$$-mgl\theta = I_O\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \tag{2}$$

From Eq. (2), $\omega_n^2 = \dfrac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\dfrac{mgl}{I_O}}$, Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \tag{3}$$

When the platform is empty, $\tau = \tau_1 = 2.38$ s, $m = 400$ kg and $l = 2.50$ m. Substituting these values into Eq. (3), we have

$$2.38 = 2\pi\sqrt{\frac{(I_O)_p}{400(9.81)(2.50)}} \qquad (I_O)_p = 1407.55\ \text{kg}\cdot\text{m}^2$$

When the car is on the platform, $\tau = \tau_2 = 3.16$ s, $m = 400\ \text{kg} + 1200\ \text{kg} = 1600$ kg. $l = \dfrac{2.50(400) + 1.83(1200)}{1600} = 1.9975$ m and $I_O = (I_O)_C + (I_O)_p = (I_O)_C + 1407.55$. Substituting these values into Eq. (3), we have

$$3.16 = 2\pi\sqrt{\frac{(I_O)_C + 1407.55}{1600(9.81)(1.9975)}} \qquad (I_O)_C = 6522.76\ \text{kg}\cdot\text{m}^2$$

Thus, the mass moment inertia of the car about its mass center is

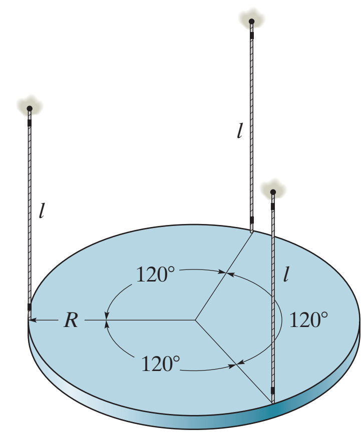
$$(I_G)_C = (I_O)_C - m_C d^2$$

$$= 6522.76 - 1200(1.83^2) = 2.50(10^3)\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$(I_G)_C = 2.50(10^3)\ \text{kg}\cdot\text{m}^2$

22–29.

The plate of mass m is supported by three symmetrically placed cords of length l as shown. If the plate is given a slight rotation about a vertical axis through its center and released, determine the natural period of oscillation.



SOLUTION

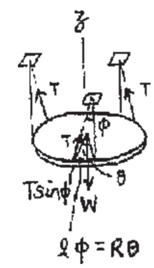
$\Sigma M_z = I_z\alpha \qquad -3(T\sin\phi)R = \frac{1}{2}mR^2\ddot{\theta}$

$\sin\phi \equiv \phi$

$$\ddot{\theta} + \frac{6T}{Rm}\phi = 0$$

$\Sigma F_z = 0 \qquad 3T\cos\phi - mg = 0$

$\phi = 0, \qquad T = \frac{mg}{3}, \qquad \phi = \frac{R}{l}\theta$



$$\ddot{\theta} + \frac{6}{Rm}\left(\frac{mg}{3}\right)\left(\frac{R}{l}\theta\right) = 0$$

$$\ddot{\theta} + \frac{2g}{l}\theta = 0$$

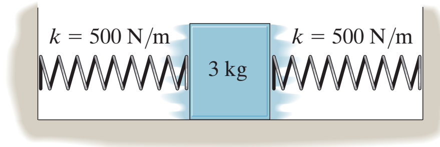
$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l}{2g}}$$ **Ans.**

Ans:

$\tau = 2\pi\sqrt{\frac{l}{2g}}$

22–30.

Determine the differential equation of motion of the 3-kg block when it is displaced slightly and released. The surface is smooth and the springs are originally unstretched.



SOLUTION

$$T + V = \text{const.}$$

$$T = \frac{1}{2}(3)\dot{x}^2$$

$$V = \frac{1}{2}(500)x^2 + \frac{1}{2}(500)x^2$$

$$T + V = 1.5\dot{x}^2 + 500x^2$$

$$1.5(2\dot{x})\,\ddot{x} + 1000x\dot{x} = 0$$

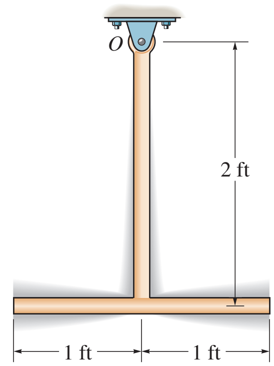
$$3\ddot{x} + 1000x = 0$$

$$\ddot{x} + 333x = 0 \qquad \textbf{Ans.}$$

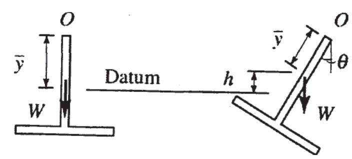
Ans:
$\ddot{x} + 333x = 0$

22–31.

Determine the natural period of vibration of the pendulum. Consider the two rods to be slender, each having a weight of 8 lb/ft.



SOLUTION



$$\bar{y} = \frac{1(8)(2) + 2(8)(2)}{8(2) + 8(2)} = 1.5 \text{ ft}$$

$$I_O = \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(1)^2\right]$$

$$+ \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(2)^2\right] = 2.8157 \text{ slug} \cdot \text{ft}^2$$

$$h = \bar{y}\,(1 - \cos\theta)$$

$$T + V = \text{const}$$

$$T = \frac{1}{2}(2.8157)(\dot{\theta})^2 = 1.4079\,\dot{\theta}^2$$

$$V = 8(4)(1.5)(1 - \cos\theta) = 48(1 - \cos\theta)$$

$$T + V = 1.4079\dot{\theta}^2 + 48(1 - \cos\theta)$$

$$1.4079\,(2\dot{\theta})\ddot{\theta} + 48(\sin\theta)\dot{\theta} = 0$$

For small θ, $\sin\theta = \theta$, then

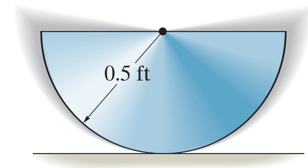
$$\ddot{\theta} + 17.047\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{17.047}} = 1.52 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\tau = 1.52$ s

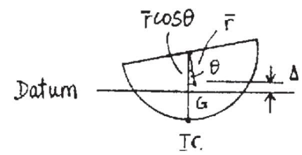
***22–32.**

Determine the natural period of vibration of the 10-lb semicircular disk.



SOLUTION

Datum at initial level of center of gravity of disk.



$$\Delta = \bar{r}(1 - \cos\theta)$$

$$E = T + V$$

$$= \frac{1}{2}I_{IC}(\dot{\theta})^2 + W\bar{r}(1 - \cos\theta)$$

$$\dot{E} = \dot{\theta}(I_{IC}\ddot{\theta} + W\bar{r}\sin\theta) = 0$$

For small θ, $\quad \sin\theta = \theta$

$$\ddot{\theta} + \frac{W\bar{r}}{I_{IC}}\theta = 0$$

$$\bar{r} = \frac{4(0.5)}{3\pi} = 0.212 \text{ ft}$$

$$I_A = I_G + m\bar{r}^2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2 = I_G + \frac{10}{32.2}(0.212)^2$$

$$I_G = 0.02483 \text{ slug}\cdot\text{ft}^2$$

$$I_{IC} = I_G + m(r - \bar{r})^2$$

$$= 0.02483 + \frac{10}{32.2}(0.5 - 0.212)^2$$

$$= 0.05056 \text{ slug}\cdot\text{ft}^2$$

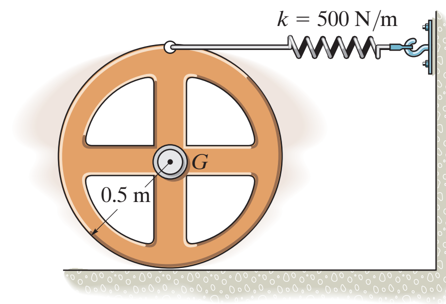
$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_{IC}}{W\bar{r}}} = 2\pi\sqrt{\frac{0.05056}{10(0.212)}}$$

$$\tau = 0.970 \text{ s}$$ **Ans.**

Ans:
$\tau = 0.970$ s

22–33.

If the 20-kg wheel is displaced a small amount and released, determine the natural period of vibration. The radius of gyration of the wheel is $k_G = 0.36$ m. The wheel rolls without slipping.



SOLUTION

Energy Equation. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G = 20(0.361)^2 = 2.592\ \text{kg}\cdot\text{m}^2$. Since the wheel rolls without slipping, $v_G = \omega r = \omega(0.5)$. Thus,

$$T = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$

$$= \frac{1}{2}(2.592)\omega^2 + \frac{1}{2}(20)[\omega(0.5)]^2$$

$$= 3.796\,\omega^2 = 3.796\dot{\theta}^2$$

When the disk undergoes a small angular displacement θ, the spring stretches $s = \theta(1) = \theta$, Fig. a. Thus, the elastic potential energy is

$$V_e = \frac{1}{2}ks^2 = \frac{1}{2}(500)\theta^2 = 250\theta^2$$

Thus, the total energy is

$$E = T + V = 3.796\dot{\theta}^2 + 250\theta^2$$

Time Derivative. Taking the time derivative of the above equation,

$$7.592\dot{\theta}\ddot{\theta} + 500\theta\dot{\theta} = 0$$

$$\dot{\theta}(7.592\ddot{\theta} + 500\theta) = 0$$

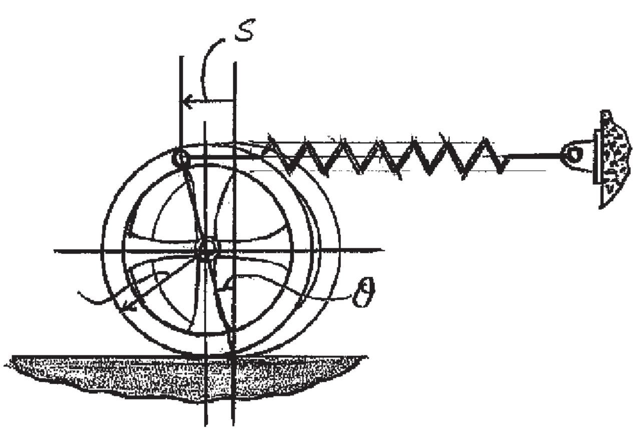
Since $\dot{\theta} \neq 0$, then

$$7.592\ddot{\theta} + 500\theta = 0$$

$$\ddot{\theta} + 65.8588\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{65.8588} = 8.1153$ rad/s. Thus,

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{8.1153} = 0.7742\text{ s} = 0.774\text{ s} \qquad \textbf{Ans.}$$

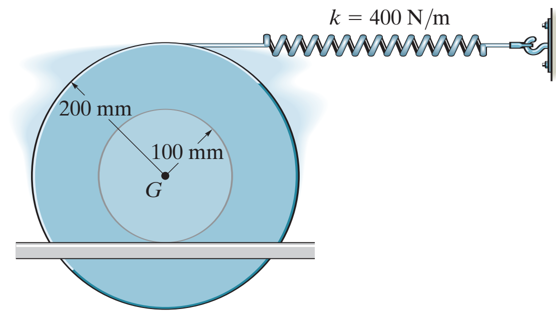


(a)

Ans:
$\tau = 0.774$ s

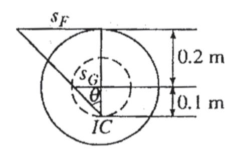
22–34.

Determine the differential equation of motion of the 3-kg spool. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is $k_G = 125$ mm.



SOLUTION

Kinematics: Since no slipping occurs, $s_G = 0.1\theta$ hence $s_F = \frac{0.3}{0.1}S_G = 0.3\theta$. Also,



$$v_G = 0.1\dot{\theta}.$$

$$E = T + V$$

$$E = \frac{1}{2}[(3)(0.125)^2]\dot{\theta}^2 + \frac{1}{2}(3)(0.1\dot{\theta})^2 + \frac{1}{2}(400)(0.3\theta)^2 = \text{const.}$$

$$= 0.038444\dot{\theta}^2 + 18\theta^2$$

$$0.076875\dot{\theta}\ddot{\theta} + 36\theta\dot{\theta} = 0$$

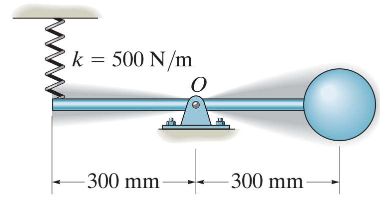
$$0.076875\dot{\theta}(\ddot{\theta} + 468.29\theta) = 0 \text{ Since } 0.076875\dot{\theta} \neq 0$$

$$\ddot{\theta} + 468\theta = 0$$ **Ans.**

Ans:
$\ddot{\theta} + 468\theta = 0$

22–35.

Determine the natural period of vibration of the 3-kg sphere. Neglect the mass of the rod and the size of the sphere.

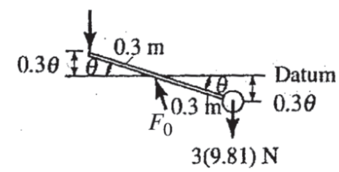


SOLUTION

$$E = T + V$$

$$= \frac{1}{2}(3)(0.3\dot{\theta})^2 + \frac{1}{2}(500)(\delta_{st} + 0.3\theta)^2 - 3(9.81)(0.3\theta)$$

$$\dot{E} = \dot{\theta}[(3(0.3)^2\ddot{\theta} + 500(\delta_{st} + 0.3\theta)(0.3) - 3(9.81)(0.3)] = 0$$



By statics,

$$T(0.3) = 3(9.81)(0.3)$$

$$T = 3(9.81) \text{ N}$$

$$\delta_{st} = \frac{3(9.81)}{500}$$

Thus,

$$3(0.3)^2\ddot{\theta} + 500(0.3)^2\theta = 0$$

$$\ddot{\theta} + 166.67\theta = 0$$

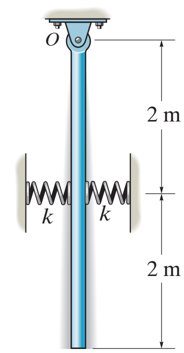
$$\omega_n = \sqrt{166.67} = 12.91 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{12.91} = 0.487 \text{ s}$$ **Ans.**

Ans:
$\tau = 0.487$ s

***22–36.**

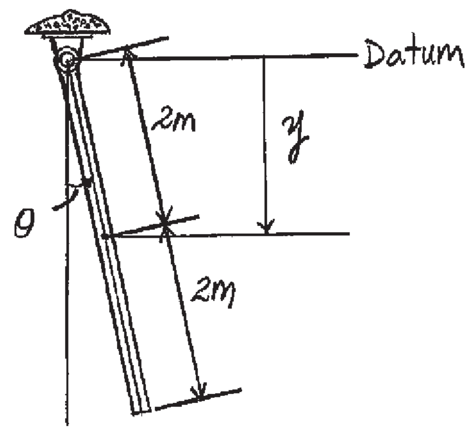
If the lower end of the 6-kg slender rod is displaced a small amount and released from rest, determine the natural frequency of vibration. Each spring has a stiffness of $k = 200\ \text{N/m}$ and is unstretched when the rod is hanging vertically.



SOLUTION

Energy Equation. The mass moment of inertia of the rod about O is $I_0 = \frac{1}{3}ml^2 = \frac{1}{3}(6)(4^2) = 32\ \text{kg}\cdot\text{m}^2$. Thus, the Kinetic energy is

$$T = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(32)\dot{\theta}^2 = 16\dot{\theta}^2$$



(a)

with reference to the datum set in Fig. a, the gravitational potential energy is

$$V_g = mgy = 6(9.81)(-2\cos\theta) = -117.72\cos\theta$$

When the rod undergoes a small angular displacement θ the spring deform $x = 2\sin\Omega$. Thus the elastic potential energy is

$$V_e = 2\left(\frac{1}{2}kx^2\right) = 2\left[\frac{1}{2}(200)(2\sin\theta)^2\right] = 800\sin^2\theta$$

Thus, the total energy is

$$E = T + V = 16\dot{\theta}^2 + 800\sin^2\theta - 117.72\cos\theta$$

Time Derivative. Taking the first time derivative of the above equation

$$32\dot{\theta}\ddot{\theta} + 1600(\sin\theta\cos\theta)\dot{\theta} + 117.72(\sin\theta)\dot{\theta} = 0$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$, we obtain

$$32\dot{\theta}\ddot{\theta} + 800(\sin 2\theta)\dot{\theta} + 117.72(\sin\theta)\dot{\theta} = 0$$

$$\dot{\theta}(32\ddot{\theta} + 800\sin 2\theta + 117.72\sin\theta) = 0$$

Since $\dot{\theta} \neq 0$,

$$32\ddot{\theta} + 800\sin 2\theta + 117.72\sin\theta) = 0$$

Since θ is small, $\sin 2\theta \simeq 2\theta$ and $\sin\theta = \theta$. The above equation becomes

$$32\ddot{\theta} + 1717.72\theta = 0$$

$$\ddot{\theta} + 53.67875\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{53.67875} = 7.3266\ \text{rad/s}$.

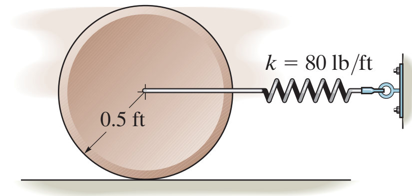
Thus,

$$f = \frac{\omega_n}{2\pi} = \frac{7.3266}{2\pi} = 1.1661\ \text{Hz} = 1.17\ \text{Hz} \qquad \textbf{Ans.}$$

Ans:
$f = 1.17\ \text{Hz}$

22–37.

The disk has a weight of 30 lb and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced, by rolling it counterclockwise 0.2 rad, determine the equation which describes its oscillatory motion and the natural period when it is released.



SOLUTION

Energy Equation. The mass moment of inertia of the disk about its center of gravity is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{30}{32.2}\right)(0.5^2) = 0.11646 \text{ slug} \cdot \text{ft}^2$. Since the disk rolls without slipping, $v_G = \omega r = \omega(0.5)$. Thus

$$T = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$

$$= \frac{1}{2}(0.1146)\,\omega^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)[\omega(0.5)]^2$$

$$= 0.17469\,\omega^2 = 0.17469\dot{\theta}^2$$

When the disk undergoes a small angular displacement θ the spring stretches $s = \theta r = \theta(0.5)$, Fig. *a*. Thus, the elastic potential energy is

$$V_e = \frac{1}{2}ks^2 = \frac{1}{2}(80)[\theta(0.5)]^2 = 10\theta^2$$

Thus, the total energy is

$$E = T + V = 0.17469\dot{\theta}^2 + 10\theta^2$$

$$E = 0.175\dot{\theta}^2 + 10\theta^2 \qquad \textbf{Ans.}$$

Time Derivative. Taking the time derivative of the above equation,

$$0.34938\dot{\theta}\ddot{\theta} + 20\theta\dot{\theta} = 0$$

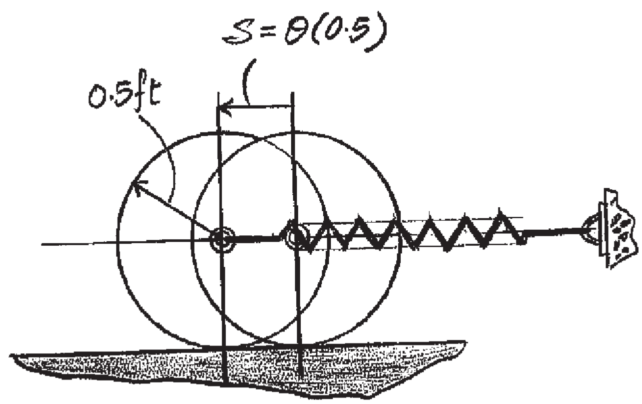
$$\dot{\theta}(0.34938\ddot{\theta} + 20\theta) = 0$$

Since $\dot{\theta} \neq 0$, then

$$0.34938\dot{\theta} + 20\theta = 0$$

$$\ddot{\theta} + 57.244\theta = 0$$

$$\ddot{\theta} = 57.2\theta = 0 \qquad \textbf{Ans.}$$



(a)

Comparing to that of standard form,

$\omega_n = \sqrt{57.2444} = 7.5660 \text{ rad/s}.$

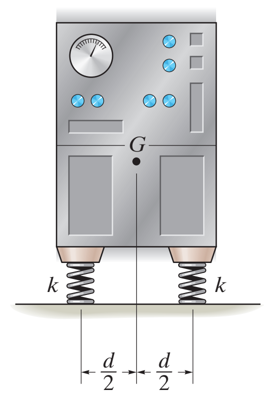
Thus

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{7.5660} = 0.8304 \text{ s} = 0.830 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$E = 0.175\dot{\theta}^2 + 10\,\theta^2$
$\tau = 0.830$ s

22–38.

The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.



SOLUTION

$$T + V = \text{const.}$$

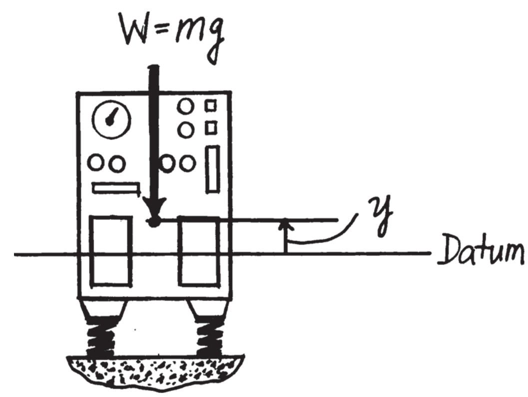
$$T = \frac{1}{2}m(\dot{y})^2$$

$$V = m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$T + V = \frac{1}{2}m(\dot{y})^2 + m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$m\,\dot{y}\,\ddot{y} + m\,g\,\dot{y} - 4k(\Delta s - y)\dot{y} = 0$$

$$m\,\ddot{y} + m\,g + 4ky - 4k\Delta s = 0$$



Since $\Delta s = \dfrac{mg}{4k}$

Then

$$m\ddot{y} + 4ky = 0$$

$$y + \frac{4k}{m}y = 0$$

$$\omega_n = \sqrt{\frac{4k}{m}}$$

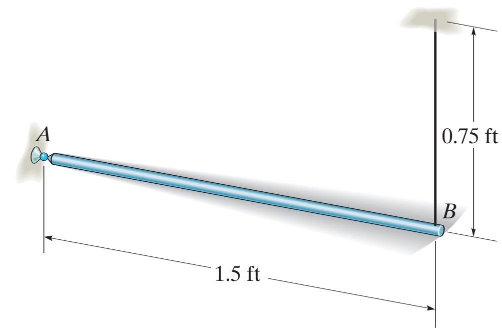
$$\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{m}{k}}$$ **Ans.**

Ans:

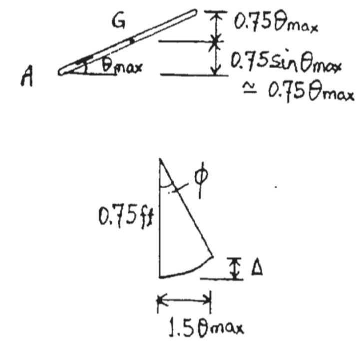
$\tau = \pi\sqrt{\dfrac{m}{k}}$

22–39.

The slender rod has a weight of 4 lb/ft. If it is supported in the horizontal plane by a ball-and-socket joint at A and a cable at B, determine the natural frequency of vibration when the end B is given a small horizontal displacement and then released.



SOLUTION



$$\phi = \frac{1.5\theta_{max}}{0.75}$$

$$\Delta = 0.75(1 - \cos\phi)$$

$$\cong 0.75(1 - 1 + \frac{\phi^2}{2})$$

$$= 0.75(\frac{4\theta_{max}^2}{2})$$

$$\Delta G = \frac{1}{2}\Delta = 0.75\theta_{max}^2$$

$$T_{max} = \frac{1}{2}I_A\,\omega_{max}^2$$

$$= \frac{1}{2}[\frac{1}{3}(\frac{4(1.5)}{32.2})(1.5)^2]\omega_n{}^2\,\theta_{max}^2$$

$$= 0.0699\;\omega_n{}^2\,\theta_{max}^2$$

$$V_{max} = W\Delta_G = 4(1.5)(0.75\theta_{max}^2)$$

$$T_{max} = V_{max}$$

$$0.0699\omega_n{}^2\,\theta_{max}^2 = 4.5\;\theta_{max}^2$$

$$\omega_n{}^2 = 64.40$$

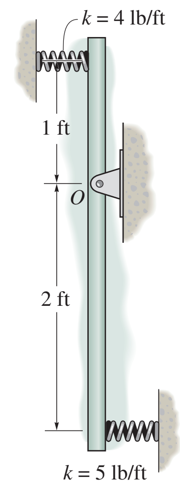
$$\omega_n = 8.025\text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = \frac{8.025}{2\pi} = 1.28\text{ Hz}$$ **Ans.**

Ans:
$f = 1.28$ Hz

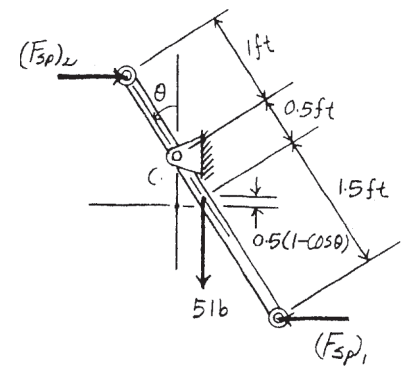
***22–40.**

If the slender rod has a weight of 5 lb, determine the natural frequency of vibration. The springs are originally unstretched.



SOLUTION

***Energy Equation*:** When the rod is being displaced a small angular displacement of θ, the compression of the spring at its ends can be approximated as $x_1 = 2\theta$ and $x_2 = 1\theta$. Thus, the elastic potential energy when the rod is at this position is $V_e = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2 = \frac{1}{2}(5)(2\theta)^2 + \frac{1}{2}(4)(1\theta)^2 = 12\theta^2$. The datum is set at the rod's mass center when the rod is at its original position. When the rod undergoes a small angular displacement θ, its mass center is $0.5(1 - \cos\theta)$ ft *above* the datum hence its gravitational potential energy is $V_g = 5[0.5(1 - \cos\theta)]$. Since θ is small, $\cos\theta$ can be approximated by the first two terms of the power series, that is, $\cos\theta = 1 - \frac{\theta^2}{2}$. Thus, $V_g = 2.5\left[1 - \left(1 - \frac{\theta^2}{2}\right)\right] = 1.25\theta^2$



$$V = V_e + V_g = 12\theta^2 + 1.25\theta^2 = 13.25\theta^2$$

The mass moment inertia of the rod about point O is $I_O = \frac{1}{12}\left(\frac{5}{32.2}\right)(3^2) + \frac{5}{32.2}\left(0.5^2\right) = 0.1553\ \text{slug}\cdot\text{ft}^2$. The kinetic energy is

$$T = \frac{1}{2}I_O\,\omega^2 = \frac{1}{2}(0.1553)\,\dot{\theta}^2 = 0.07764\dot{\theta}^2$$

The total energy of the system is

$$U = T + V = 0.07764\dot{\theta}^2 + 13.25\theta^2 \qquad \textbf{[1]}$$

***Time Derivative*:** Taking the time derivative of Eq.[1], we have

$$0.1553\dot{\theta}\ddot{\theta} + 26.5\theta\dot{\theta} = 0$$

$$\dot{\theta}\left(0.1553\ddot{\theta} + 26.5\theta\right) = 0$$

Since $\dot{\theta} \neq 0$, then

$$0.1553\ddot{\theta} + 26.5\theta = 0$$

$$\ddot{\theta} + 170.66\theta = 0 \qquad \textbf{[2]}$$

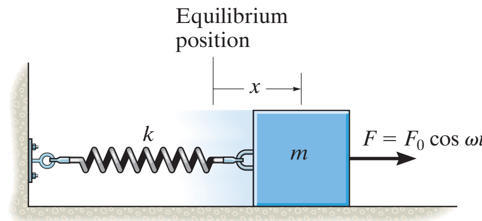
From Eq.[2], $p^2 = 170.66$, thus, $p = 13.06$ rad/s. Applying Eq.22–14, we have

$$f = \frac{p}{2\pi} = \frac{13.06}{2\pi} = 2.08\ \text{Hz} \qquad \textbf{Ans.}$$

Ans:
$f = 2.08$ Hz

22–41.

If the block-and-spring model is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{x} + (k/m)x = (F_0/m)\cos \omega t$, where x is measured from the equilibrium position of the block. What is the general solution of this equation?



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \cos \omega t - kx = m\ddot{x}$

$$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos \omega t \qquad \textbf{(Q.E.D.)}$$

$$\ddot{x} + p^2 x = \frac{F_0}{m}\cos \omega t \qquad \text{Where } p = \sqrt{\frac{k}{m}} \qquad \textbf{(1)}$$

The general solution of the above differential equation is of the form of $x = x_c + x_p$.

The complementary solution:

$$x_c = A \sin pt + B \cos pt$$

The particular solution:

$$s_p = .C \cos \omega t \qquad \textbf{(2)}$$

$$\ddot{x}_P = -C\omega^2 \cos \omega\, t \qquad \textbf{(3)}$$

Substitute Eqs. (2) and (3) into (1) yields:

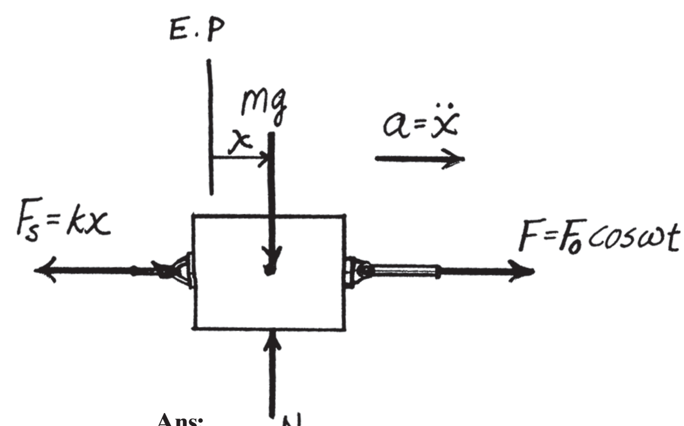
$$-C\omega^2 \cos \omega t + p^2 (C \cos \omega t) = \frac{F_0}{m}\cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{p^2 - \omega^2} = \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2}$$

The general solution is therefore

$$s = A \sin pt + B \cos pt + \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2}\cos \omega t \qquad \textbf{Ans.}$$

The constants A and B can be found from the initial conditions.



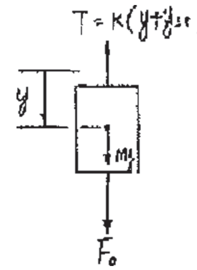
Ans:

$$x = A \sin \omega_n t + B \cos \omega_n t + \frac{F_o/k}{1 - (\omega/\omega_n)^2}\cos \omega t$$

22–42.

A block which has a mass m is suspended from a spring having a stiffness k. If an impressed downward vertical force $F = F_O$ acts on the weight, determine the equation which describes the position of the block as a function of time.

SOLUTION



$+\uparrow \Sigma F_y = ma_y;\qquad k(y_{st} + y) - mg - F_0 = -m\ddot{y}$

$$m\ddot{y} + ky + ky_{st} - mg = F_0$$

However, from equilbrium $ky_{st} - mg = 0$, therefore

$m\ddot{y} + ky = F_0$

$\ddot{y} + \dfrac{k}{m}y = \dfrac{F_0}{m} \qquad \text{where } \omega_n = \sqrt{\dfrac{k}{m}}$

$\ddot{y} + \omega_n^2 y = \dfrac{F_0}{m}$ **[1]**

The general solution of the above differential equation is of the form of $y = y_c + y_p$.

$y_c = A \sin \omega_n t + B \cos \omega_n t$

$y_P = C$ **[2]**

$\ddot{y}_P = 0$ **[3]**

Substitute Eqs. [2] and [3] into [1] yields :

$0 + \omega_n^2 C = \dfrac{F_0}{m} \qquad C = \dfrac{F_0}{mp^2} = \dfrac{F_0}{k}$

The general solution is therefore

$y = A \sin \omega_n t + B \cos \omega_n t + \dfrac{F_0}{k}$ **Ans.**

The constants A and B can be found from the initial conditions.

Ans:

$y = A \sin \omega_n t + B \cos \omega_n t + \dfrac{F_O}{k}$

22–43.

A 4-lb weight is attached to a spring having a stiffness $k = 10\text{ lb/ft}$. The weight is drawn downward a distance of 4 in. and released from rest. If the support moves with a vertical displacement $\delta = (0.5 \sin 4t)$ in., where t is in seconds, determine the equation which describes the position of the weight as a function of time.

SOLUTION

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

$$v = \dot{y} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t + \frac{\delta_0 \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \cos \omega_0 t$$

The initial condition when $t = 0$, $y = y_0$, and $v = v_0$ is

$$y_0 = 0 + B + 0 \qquad B = y_0$$

$$v_0 = A\omega_n - 0 + \frac{\delta_0 \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \qquad A = \frac{v_0}{\omega_n} - \frac{\delta_0 \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}$$

Thus,

$$y = \left(\frac{v_0}{\omega_n} - \frac{\delta_0 \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}\right) \sin \omega_n t + y_0 \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{4/32.2}} = 8.972$$

$$\frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} = \frac{0.5/12}{1 - \left(\frac{4}{8.972}\right)^2} = 0.0520$$

$$\frac{v_0}{\omega_n} - \frac{\delta_0 \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}} = 0 - \frac{(0.5/12)4}{8.972 - \frac{4^2}{8.972}} = -0.0232$$

$$y = (-0.0232 \sin 8.97t + 0.333 \cos 8.97t + 0.0520 \sin 4t) \text{ ft}$$ **Ans.**

Ans:
$y = \{-0.0232 \sin 8.97t + 0.333 \cos 8.97t + 0.0520 \sin 4t\}$ ft

***22–44.**

A 4-kg block is suspended from a spring that has a stiffness of $k = 600$ N/m. The block is drawn downward 50 mm from the equilibrium position and released from rest when $t = 0$. If the support moves with an impressed displacement of $\delta = (10 \sin 4t)$ mm, where t is in seconds, determine the equation that describes the vertical motion of the block. Assume positive displacement is downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25$$

The general solution is defined by Eq. 22–23 with $k\delta_0$ substituted for F_0.

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{\delta_0}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]} \right) \sin \omega t$$

$\delta = (0.01 \sin 4t)$m, hence $\delta_0 = 0.01$, $\omega = 4$, so that

$y = A \sin 12.25t + B \cos 12.25t + 0.0112 \sin 4t$

$y = 0.05$ when $t = 0$

$0.05 = 0 + B + 0; \qquad B = 0.05$ m

$\dot{y} = A(12.25) \cos 12.25t - B(12.25) \sin 12.25t + 0.0112(4) \cos 4t$

$v = y = 0$ when $t = 0$

$0 = A(12.25) - 0 + 0.0112(4); \qquad A = -0.00366$ m

Expressing the result in mm, we have

$y = (-3.66 \sin 12.25t + 50 \cos 12.25t + 11.2 \sin 4t)$ mm **Ans.**

Ans:
$y = (-3.66 \sin 12.25t + 50 \cos 12.25t + 11.2 \sin 4t)$ mm

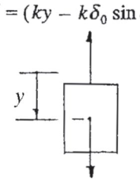
22–45.

Use a block-and-spring model like that shown in Fig. 22–14*a*, but suspended from a vertical position and subjected to a periodic support displacement $\delta = \delta_0 \sin \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

SOLUTION

$+\uparrow \Sigma F_x = ma_x;\qquad k(y - \delta_0 \sin \omega_0 t + y_{st}) - mg = -m\ddot{y}$

$$m\ddot{y} + ky + ky_{st} - mg = k\delta_0 \sin \omega_0 t$$



However, from equilibrium

$$ky_{st} - mg = 0, \text{ therefore}$$

$$\ddot{y} + \frac{k}{m}y = \frac{k\delta_0}{m}\sin \omega\, t \qquad \text{where } \omega_n = \sqrt{\frac{k}{m}}$$

$$\ddot{y} + \omega_n{}^2\, y = \frac{k\delta_0}{m} \sin \omega\, t \qquad \textbf{Ans. (1)}$$

The general solution of the above differential equation is of the form of $y = y_c + y_p$, where

$$y_c = A \sin \omega_n t + B \cos \omega_n t$$

$$y_p = C \sin \omega_0 t \qquad \textbf{(2)}$$

$$\ddot{y}_p = -C\omega_0^2 \sin \omega_0 t \qquad \textbf{(3)}$$

Substitute Eqs. (2) and (3) into (1) yields:

$$-C\omega^2 \sin \omega_0 t + \omega_n^2(C \sin \omega_0 t) = \frac{k\delta_0}{m} \sin \omega_0 t$$

$$C = \frac{\dfrac{k\delta_0}{m}}{\omega_n{}^2 - \omega_0{}^2} = \frac{\delta_0}{1 - \left(\dfrac{\omega_0}{\omega_n}\right)^2}$$

The general solution is therefore

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\dfrac{\omega_0}{\omega_n}\right)^2} \sin \omega\, t \qquad \textbf{Ans.}$$

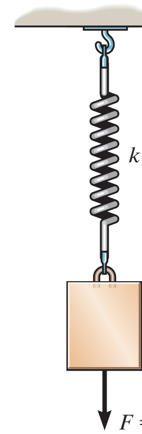
The constants A and B can be found from the initial conditions.

Ans:

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - (\omega/\omega_n)^2} \sin \omega t$$

22–46.

A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical force $F = (7 \sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Assume that positive displacement is downward.



SOLUTION

The general solution is defined by:

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{\frac{F_0}{k}}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \right) \sin \omega_0 t$$

Since

$$F = 7 \sin 8t, \qquad F_0 = 7 \text{ N}, \qquad \omega_0 = 8 \text{ rad/s}, \qquad k = 300 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}} = 7.746 \text{ rad/s}$$

Thus,

$$y = A \sin 7.746t + B \cos 7.746t + \left(\frac{\frac{7}{300}}{1 - \left(\frac{8}{7.746}\right)^2} \right) \sin 8t$$

$y = 0.1$ m when $t = 0$,

$0.1 = 0 + B - 0; \qquad B = 0.1$ m

$\dot{y} = A(7.746) \cos 7.746t - B(7.746) \sin 7.746t - (0.35)(8) \cos 8t$

$y = \dot{y} = 0$ when $t = 0$,

$\dot{y} = A(7.746) - 2.8 = 0; \qquad A = 0.361$

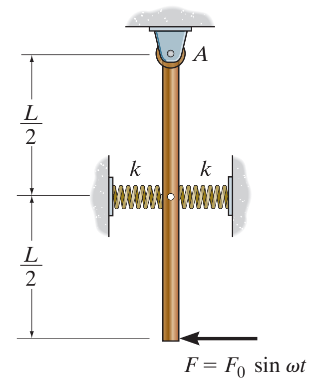
Expressing the results in mm, we have

$y = (361 \sin 7.75t + 100 \cos 7.75t - 350 \sin 8t)$ mm **Ans.**

Ans:
$y = (361 \sin 7.75t + 100 \cos 7.75t)$
$-350 \sin 8t)$ mm

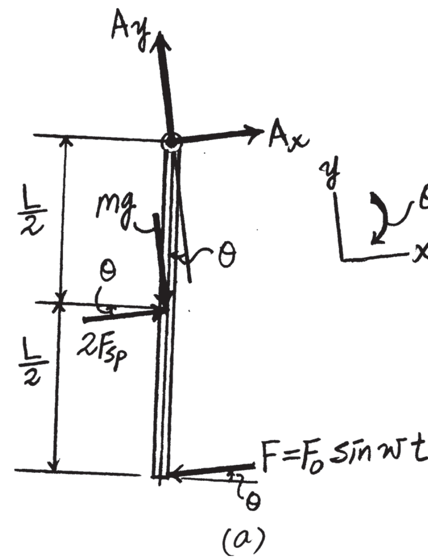
22–47.

The uniform rod has a mass of m. If it is acted upon by a periodic force of $F = F_0 \sin \omega t$, determine the amplitude of the steady-state vibration.



SOLUTION

Equation of Motion: When the rod rotates through a small angle θ, the springs compress and stretch $s = r_{AG}\theta = \frac{L}{2}\theta$. Thus, the force in each spring is $F_{sp} = ks = \frac{kL}{2}\theta$. The mass moment of inertia of the rod about point A is $I_A = \frac{1}{3}mL^2$. Referring to the free-body diagram of the rod shown in Fig. a,



$$+\Sigma M_A = I_A\alpha; \qquad F_O \sin \omega t \cos \theta(L) - mg \sin \theta\left(\frac{L}{2}\right) - 2\left(\frac{kL}{2}\theta\right)\cos\theta\left(\frac{L}{2}\right)$$

$$= \frac{1}{3}mL^2\dot{\theta}$$

Since θ is small, $\sin\theta \cong 0$ and $\cos\theta \cong 1$. Thus, this equation becomes

$$\frac{1}{3}mL\ddot{\theta} + \frac{1}{2}(mg + kL)\theta = F_O \sin \omega t$$

$$\ddot{\theta} + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)\theta = \frac{3F_O}{mL}\sin\omega t \qquad \textbf{(1)}$$

The particular solution of this differential equation is assumed to be in the form of

$$\theta_p = C \sin \omega t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2) twice,

$$\ddot{\theta}_p = -C\omega^2 \sin \omega t \qquad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$-C\omega^2 \sin\omega t + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)(C \sin \omega t) = \frac{3F_O}{mL}\sin\omega t$$

$$C\left[\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2\right]\sin\omega t = \frac{3F_O}{mL}\sin\omega t$$

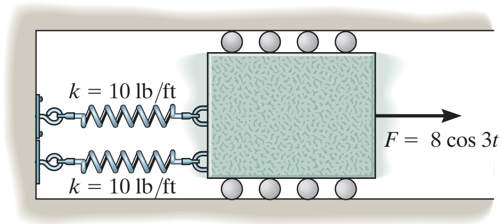
$$C = \frac{3F_O/mL}{\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2}$$

$$C = \frac{3F_O}{\frac{3}{2}(mg + Lk) - mL\omega^2} \qquad \textbf{Ans.}$$

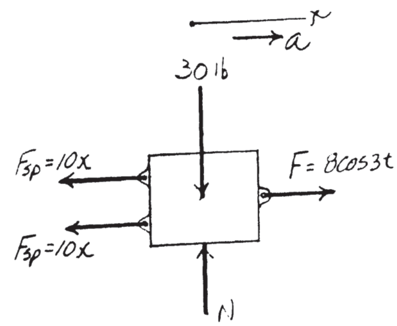
Ans:
$$C = \frac{3F_O}{\frac{3}{2}(mg + Lk) - mL\omega^2}$$

***22–48.**

The 30-lb block is attached to two springs having a stiffness of 10 lb/ft. A periodic force $F = (8 \cos 3t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



SOLUTION



Free-body Diagram: When the block is being displaced by amount x to the right, the *restoring force* that develops in both springs is $F_{sp} = kx = 10x$.

Equation of Motion:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -2(10x) + 8 \cos 3t = \frac{30}{32.2} a$$

$$a + 21.47x = 8.587 \cos 3t \qquad \textbf{[1]}$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substituting this value into Eq. [1], we have

$$\ddot{x} + 21.47x = 8.587 \cos 3t \qquad \textbf{[2]}$$

Since the friction will eventually dampen out the free vibration, we are only interested in the *particular solution* of the above differential equation which is in the form of

$$x_p = C \cos 3t$$

Taking second time derivative and substituting into Eq. [2], we have

$$-9C \cos 3t + 21.47C \cos 3t = 8.587 \cos 3t$$

$$C = 0.6888 \text{ ft}$$

Thus,

$$x_p = 0.6888 \cos 3t \qquad \textbf{[3]}$$

Taking the time derivative of Eq. [3], we have

$$v_p = \dot{x}_p = -2.0663 \sin 3t$$

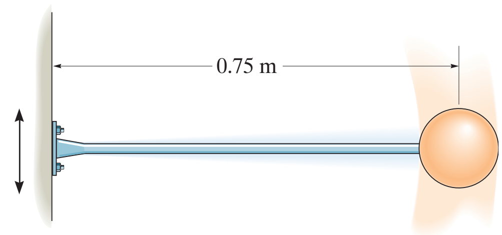
Thus,

$$\left(v_p\right)_{max} = 2.07 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$(v_p)_{max} = 2.07$ ft/s

22–49.

The light elastic rod supports a 4-kg sphere. When an 18-N vertical force is applied to the sphere, the rod deflects 14 mm. If the wall oscillates with harmonic frequency of 2 Hz and has an amplitude of 15 mm, determine the amplitude of vibration for the sphere.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{18}{0.014} = 1285.71 \text{ N/m}$$

$$\omega_0 = 2 \text{ Hz} = 2(2\pi) = 12.57 \text{ rad/s}$$

$$\delta_0 = 0.015 \text{ m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1285.71}{4}} = 17.93$$

Using Eq. 22–22, the amplitude is

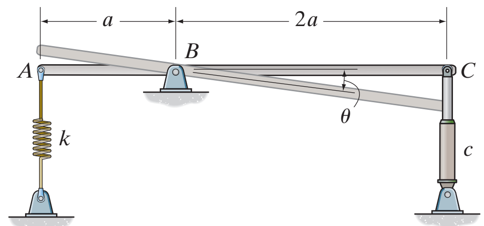
$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \right| = \left| \frac{0.015}{1 - \left(\frac{12.57}{17.93}\right)^2} \right|$$

$$(x_p)_{max} = 0.0295 \text{ m} = 29.5 \text{ mm}$$ **Ans.**

Ans:
$(x_p)_{\max} = 29.5$ mm

22–50.

Find the differential equation for small oscillations in terms of θ for the uniform rod of mass m. Also show that if $c < \sqrt{mk}/2$, then the system remains underdamped. The rod is in a horizontal position when it is in equilibrium.



SOLUTION

Equation of Motion: When the rod is in equilibrium, $\theta = 0°$, $F_c = c\dot{y}_c = 0$ and $\ddot{\theta} = 0$. writing the moment equation of motion about point B by referring to the free-body diagram of the rod, Fig. a,

$$+\Sigma M_B = 0; \qquad -F_A(a) - mg\left(\frac{a}{2}\right) = 0 \qquad F_A = \frac{mg}{2}$$

Thus, the initial stretch of the spring is $s_O = \dfrac{F_A}{k} = \dfrac{mg}{2k}$. When the rod rotates about point B through a small angle θ, the spring stretches further by $s_1 = a\theta$. Thus, the force in the spring is $F_A = k(s_0 + s_1) = k\left(\dfrac{mg}{2k} + a\theta\right)$. Also, the velocity of end C of the rod is $v_c = \dot{y}_c = 2a\dot{\theta}$. Thus, $F_c = c\dot{y}_c = c(2a\dot{\theta})$. The mass moment of inertia of the rod about B is $I_B = \dfrac{1}{12}m(3a)^2 + m\left(\dfrac{a}{2}\right)^2 = ma^2$. Again, referring to Fig. a and writing the moment equation of motion about B,

$$\Sigma M_B = I_B\alpha; \qquad k\left(\frac{mg}{2k} + a\theta\right)\cos\theta(a) + \left(2a\dot{\theta}\right)\cos\theta(2a) - mg\cos\theta\left(\frac{a}{2}\right)$$

$$= -ma^2\ddot{\theta}$$

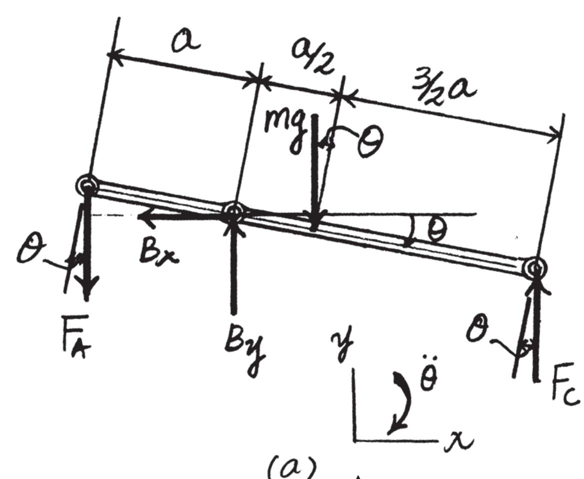
$$\ddot{\theta} + \frac{4c}{m}\cos\theta\dot{\theta} + \frac{k}{m}(\cos\theta)\theta = 0$$

Since θ is small, $\cos\theta \cong 1$. Thus, this equation becomes

$$\ddot{\theta} + \frac{4c}{m}\dot{\theta} + \frac{k}{m}\theta = 0$$ **Ans.**

Comparing this equation to that of the standard form,

$$\omega_n = \sqrt{\frac{k}{m}} \qquad c_{eq} = 4c$$



Thus,

$$c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk}$$

For the system to be underdamped,

$$c_{eq} < c_c$$

$$4c < 2\sqrt{mk}$$

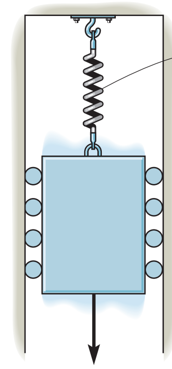
$$c < \frac{1}{2}\sqrt{mk}$$ **Ans.**

Ans:

$$\ddot{\theta} + \frac{4c}{m}\dot{\theta} + \frac{k}{m}\dot{\theta} = 0$$

22–51.

The 40-kg block is attached to a spring having a stiffness of 800 N/m. A force $F = (100 \cos 2t)$ N, where t is in seconds is applied to the block. Determine the maximum speed of the block for the steady-state vibration.



SOLUTION

For the steady-state vibration, the displacement is

$$y_p = \frac{F_0/k}{1 - (\omega_0/\omega_n)^2} \cos \omega t$$

Here $F_0 = 100$ N, $k = 800$ N/m, $\omega_0 = 2$ rad/s and

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{40}} = \sqrt{20} \text{ rad/s}.$$

Thus

$$y_p = \frac{100/800}{1 - (2/\sqrt{20})^2} \cos 2t$$

$$y_P = 0.15625 \cos 2t$$

Taking the time derivative of this equation

$$v_p = \dot{y}_p = -0.3125 \sin 2t \qquad \textbf{(2)}$$

v_p is maximum when $\sin 2t = 1$. Thus

$$(v_p)_{\max} = 0.3125 \text{ m/s} \qquad \textbf{Ans.}$$

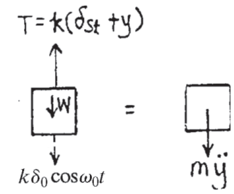
Ans:
$(v_p)_{\max} = 0.3125$ m/s

***22–52.**

Use a block-and-spring model like that shown in Fig. 22–14*a* but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

SOLUTION

$+\downarrow \Sigma F_y = ma_y; \quad k\delta_0 \cos \omega_0 t + W - k\delta_{st} - ky = m\ddot{y}$



Since $W = k\delta_{st}$,

$$\ddot{y} + \frac{k}{m}y = \frac{k\delta_0}{m}\cos \omega_0 t \qquad \textbf{(1)}$$

$y_C = A \sin \omega_n y + B \cos \omega_n y$ (General sol.)

$y_P = C \cos \omega_0 t$ (Particular sol.)

Substitute y_p into Eq. (1)

$$C(-\omega_0^2 + \frac{k}{m}) \cos \omega_0 t = \frac{k\delta_0}{m}\cos \omega_0 t$$

$$C = \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)}$$

Thus, $y = y_C + y_P$

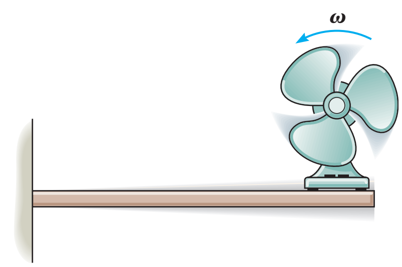
$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)}\cos \omega_0 t \qquad \textbf{Ans.}$$

Ans:

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)}\cos \omega_0 t$$

22–53.

The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the angular velocity of the fan blade at which resonance will occur. *Hint:* See the first part of Example 22.8.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

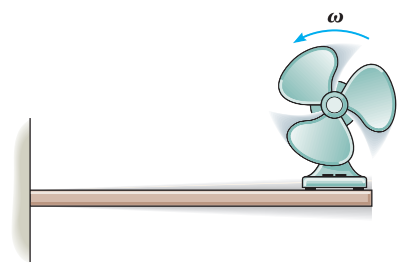
Resonance occurs when

$$\omega = \omega_n = 14.0 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 14.0 \text{ rad/s}$

22–54.

In Prob. 22–53, determine the amplitude of steady-state vibration of the fan if its angular velocity is 10 rad/s.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\,\omega^2 = 3.5(0.1)(10)^2 = 35 \text{ N}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

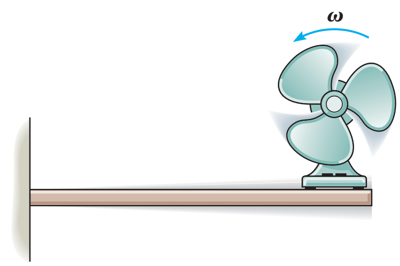
$$(x_p)_{\max} = \left| \frac{\frac{35}{4905}}{1 - \left(\frac{10}{14.01}\right)^2} \right| = 0.0146 \text{ m}$$

$$(x_p)_{\max} = 14.6 \text{ mm}$$ **Ans.**

Ans:
$(x_p)_{\max} = 14.6$ mm

22–55.

What will be the amplitude of steady-state vibration of the fan in Prob. 22–53 if the angular velocity of the fan blade is 18 rad/s? *Hint:* See the first part of Example 22.8.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(18)^2 = 113.4 \text{ N}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

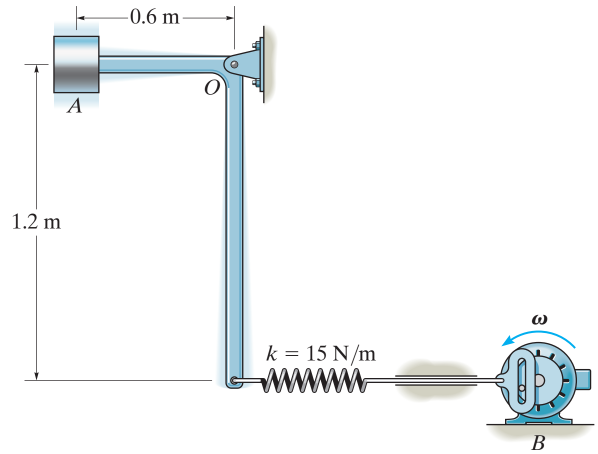
$$(x_p)_{\max} = \left| \frac{\frac{113.4}{4905}}{1 - \left(\frac{18}{14.01}\right)^2} \right| = 0.0355 \text{ m}$$

$$(x_p)_{\max} = 35.5 \text{ mm}$$ **Ans.**

Ans:
$(x_p)_{\max} = 35.5$ mm

***22–56.**

The small block at A has a mass of 4 kg and is mounted on the bent rod having negligible mass. If the rotor at B causes a harmonic movement $\delta_B = (0.1 \cos 15t)$ m, where t is in seconds, determine the steady-state amplitude of vibration of the block.



SOLUTION

$+\Sigma M_O = I_O\,\alpha; \qquad 4(9.81)(0.6) - F_s(1.2) = 4(0.6)^2\ddot{\theta}$

$$F_s = kx = 15(x + x_{st} - 0.1 \cos 15t)$$

$$x_{st} = \frac{4(9.81)(0.6)}{1.2(15)}$$

Thus,

$$-15(x - 0.1 \cos 15t)(1.2) = 4(0.6)^2\ddot{\theta}$$

$$x = 1.2\theta$$

$$\theta + 15\theta = 1.25 \cos 15t$$

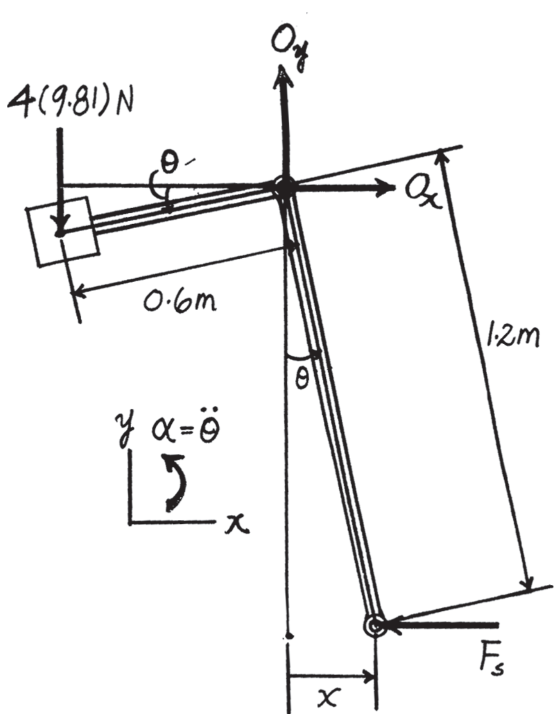
Set $x_p = C \cos 15t$

$$-C(15)^2 \cos 15t + 15(C \cos 15t) = 1.25 \cos 15t$$

$$C = \frac{1.25}{15 - (15)^2} = -0.00595 \text{ m}$$

$$\theta_{\max} = C = 0.00595 \text{ rad}$$

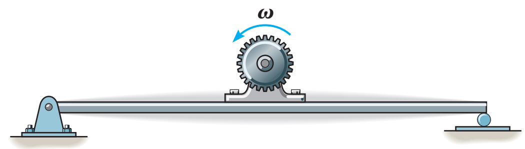
$$y_{\max} = (0.6 \text{ m})(0.00595 \text{ rad}) = 0.00357 \text{ rad}$$ **Ans.**



Ans:
$y_{\max} = 0.00357$ rad

22–57.

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. due to the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weights 150 lb. Neglect the mass of the beam.



SOLUTION

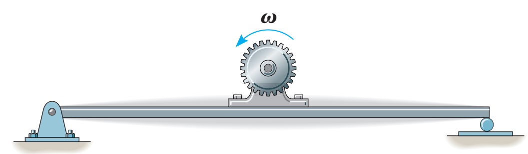
$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.66$$

Resonance occurs when $\omega = \omega_n = 19.7 \text{ rad/s}$ **Ans.**

Ans:
$\omega = 19.7 \text{ rad/s}$

22–58.

What will be the amplitude of steady-state vibration of the motor in Prob. 22–57 if the angular velocity of the flywheel is 20 rad/s?



SOLUTION

The constant value F_O of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)(20)^2 = 2.588 \text{ lb}$$

Hence $F = 2.588 \sin 20t$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

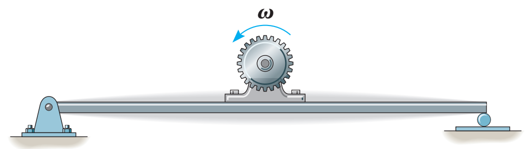
From Eq. 22–21, the amplitude of the steady state motion is

$$C = \left|\frac{F_0/k}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right| = \left|\frac{2.588/1800}{1 - \left(\frac{20}{19.657}\right)^2}\right| = 0.04085 \text{ ft} = 0.490 \text{ in.}$$ **Ans.**

Ans:
$C = 0.490$ in.

22–59.

Determine the angular velocity of the flywheel in Prob. 22–57 which will produce an amplitude of vibration of 0.25 in.



SOLUTION

The constant value F_O of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)\omega^2 = 0.006470\omega^2$$

$$F = 0.006470\omega^2 \sin \omega t$$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22.21, the amplitude of the steady-state motion is

$$C = \left|\frac{F_0/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right|$$

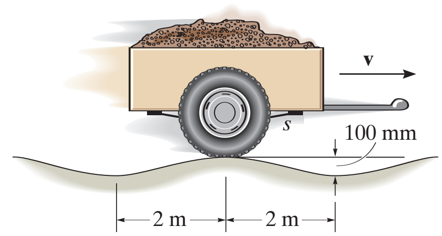
$$\frac{0.25}{12} = \left|\frac{0.006470\left(\frac{\omega^2}{1800}\right)}{1 - \left(\frac{\omega}{19.657}\right)^2}\right|$$

$$\omega = 19.0 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 19.0$ rad/s

***22–60.**

The 450-kg trailer is pulled with a constant speed over the surface of a bumpy road, which may be approximated by a cosine curve having an amplitude of 50 mm and wave length of 4 m. If the two springs s which support the trailer each have a stiffness of 800 N/m, determine the speed v which will cause the greatest vibration (resonance) of the trailer. Neglect the weight of the wheels.



SOLUTION

The amplitude is $\delta_0 = 50\text{ mm} = 0.05\text{ m}$

The wave length is $\lambda = 4\text{ m}$

$$k = 2(800) = 1600\text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1600}{450}} = 1.89\text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{1.89} = 3.33\text{ s}$$

For maximum vibration of the trailer, resonance must occur, i.e.,

$$\omega_0 = \omega_n$$

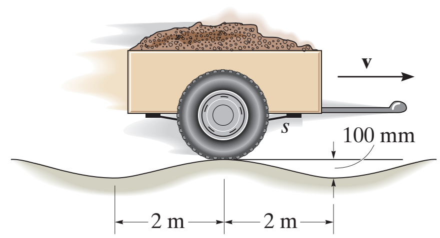
Thus, the trailer must travel $\lambda = 4\text{ m}$, in $\tau = 3.33\text{ s}$, so that

$$v_R = \frac{\lambda}{\tau} = \frac{4}{3.33} = 1.20\text{ m.s} \qquad \textbf{Ans.}$$

Ans:
$v_R = 1.20\text{ m.s}$

22–61.

Determine the amplitude of vibration of the trailer in Prob. 22–60 if the speed $v = 15$ km/h.



SOLUTION

$$v = 15 \text{ km/h} = \frac{15(1000)}{3600} \text{ m/s} = 4.17 \text{ m/s}$$

$$\delta_0 = 0.05 \text{ m}$$

As shown in Prob. 22–50, the velocity is inversely proportional to the period. Since $\frac{1}{\tau} = f$ the the velocity is proportional of f, ω_n and ω_0

Hence, the amplitude of motion is

$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \right| = \left| \frac{\delta_0}{1 - \left(\frac{v}{v_R}\right)^2} \right|$$

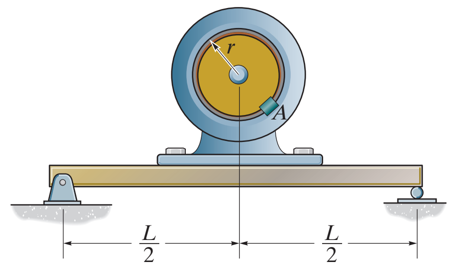
$$(x_p)_{max} = \left| \frac{0.05}{1 - \left(\frac{4.17}{1.20}\right)^2} \right| = 0.00453 \text{ m}$$

$$(x_p)_{max} = 4.53 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$(x_p)_{\max} = 4.53$ mm

22–62.

The motor of mass M is supported by a simply supported beam of negligible mass. If block A of mass m is clipped onto the rotor, which is turning at constant angular velocity of ω, determine the amplitude of the steady-state vibration. *Hint:* When the beam is subjected to a concentrated force of P at its mid-span, it deflects $\delta = PL^3/48EI$ at this point. Here E is Young's modulus of elasticity, a property of the material, and I is the moment of inertia of the beam's cross-sectional area.



SOLUTION

In this case, $P = k_{eq}\delta$. Then, $k_{eq} = \dfrac{P}{\delta} = \dfrac{P}{PL^3/48EI} = \dfrac{48EI}{L^3}$. Thus, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\frac{48EI}{L^3}}{M}} = \sqrt{\frac{48EI}{ML^3}}$$

Here, $F_O = ma_n = m(\omega^2 r)$. Thus,

$$Y = \frac{F_O/k_{eq}}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$$

$$Y = \frac{\dfrac{m(\omega^2 r)}{48EI/L^3}}{1 - \dfrac{\omega^2}{48EI/ML^3}}$$

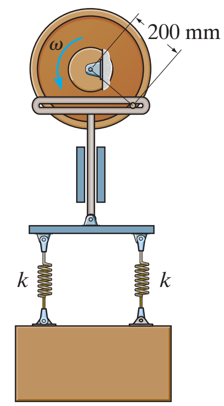
$$Y = \frac{mr\omega^2 L^3}{48EI - M\omega^2 L^3}$$ **Ans.**

Ans:

$$Y = \frac{mr\omega^2 L^3}{48EI - M\omega^2 L^3}$$

22–63.

The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\boldsymbol{\omega}$. If the amplitude of the steady-state vibration is observed to be 400 mm, and the springs each have a stiffness of $k = 2500$ N/m, determine the two possible values of $\boldsymbol{\omega}$ at which the wheel must rotate. The block has a mass of 50 kg.



SOLUTION

In this case, $k_{eq} = 2k = 2(2500) = 5000$ N/m Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{5000}{50}} = 10 \text{ rad/s}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{max} = \pm 0.4$ m, so that

$$(Y_P)_{max} = \frac{\delta_O}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\frac{\omega}{10}\right)^2}$$

$$\frac{\omega^2}{100} = 1 \pm 0.5$$

Thus,

$$\frac{\omega^2}{100} = 1.5 \qquad \omega = 12.2 \text{ rad/s} \qquad \textbf{Ans.}$$

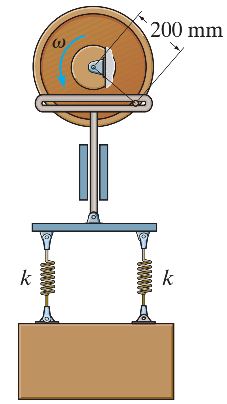
or

$$\frac{\omega^2}{100} = 0.5 \qquad \omega = 7.07 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 12.2$ rad/s
$\omega = 7.07$ rad/s

***22–64.**

The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\omega = 5$ rad/s. If the amplitude of the steady-state vibration is observed to be 400 mm, determine the two possible values of the stiffness k of the springs. The block has a mass of 50 kg.



SOLUTION

In this case, $k_{eq} = 2k$ Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2k}{50}} = \sqrt{0.04k}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{\max} = \pm 0.4$ m, so that

$$(Y_P)_{\max} = \frac{\delta_O}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\dfrac{5}{\sqrt{0.04k}}\right)^2}$$

$$\frac{625}{k} = 1 \pm 0.5$$

Thus,

$$\frac{625}{k} = 1.5 \qquad k = 417 \text{ N/m} \qquad \textbf{Ans.}$$

or

$$\frac{625}{k} = 0.5 \qquad k = 1250 \text{ N/m} \qquad \textbf{Ans.}$$

Ans:
$k = 417$ N/m
$k = 1250$ N/m

22–65.

A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \omega = 2$$

$$\phi' = \tan^{-1}\left(\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right) = \tan^{-1}\left(\frac{2(0.8)\left(\frac{2}{18.57}\right)}{1 - \left(\frac{2}{18.57}\right)^2}\right)$$

$$\phi' = 9.89^\circ$$ **Ans.**

Ans:
$\phi' = 9.89^\circ$

22–66.

Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22–65.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \quad \omega = 2$$

$$\text{MF} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_n}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}} = \frac{1}{\sqrt{\left[1 - \left(\frac{2}{18.57}\right)^2\right]^2 + \left[2(0.8)\left(\frac{2}{18.57}\right)\right]^2}}$$

MF = 0.997 **Ans.**

Ans:
MF = 0.997

22–67.

A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$ N, where v is in m/s.

SOLUTION

$$c = 50 \text{ N s/m} \quad k = 600 \text{ N/m} \quad m = 7 \text{ kg}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{7}} = 9.258 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(7)(9.258) = 129.6 \text{ N}\cdot\text{s/m}$$

Since $c < c_z$, the system is underdamped,

$$\omega_d = \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.258 \sqrt{1 - \left(\frac{50}{129.6}\right)^2} = 8.542 \text{ rad/s}$$

$$\frac{c}{2m} = \frac{50}{2(7)} = 3.751$$

From Eq. 22-32

$$y = D\left[e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t}\, \omega_d \cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right) e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d \cos(\omega_d t + \phi) - \frac{c}{2m} \sin(\omega_d t + \phi)\right]$$

Applying the initial condition at $t = 0$, $y = 0$ and $v = -0.6$ m/s.

$$0 = D[e^{-0} \sin(0 + \phi)] \qquad \text{since} \quad D \neq 0$$

$$\sin\phi = 0 \qquad \phi = 0°$$

$$-0.6 = De^{-0}\,[8.542 \cos 0° - 0]$$

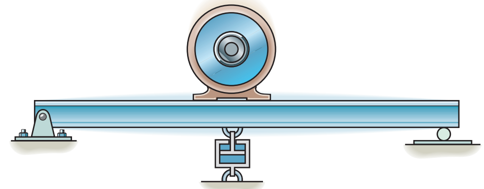
$$D = -0.0702 \text{ m}$$

$$y = [-0.0702\,e^{-3.57t} \sin(8.540)] \text{ m}$$ **Ans.**

Ans:
$y = \{-0.0702\,e^{-3.57t} \sin(8.540)\}$ m

***22–68.**

The 200-lb electric motor is fastened to the midpoint of the simply supported beam. It is found that the beam deflects 2 in. when the motor is not running. The motor turns an eccentric flywheel which is equivalent to an unbalanced weight of 1 lb located 5 in. from the axis of rotation. If the motor is turning at 100 rpm, determine the amplitude of steady-state vibration. The damping factor is $c/c_c = 0.20$. Neglect the mass of the beam.



SOLUTION

$$\delta = \frac{2}{12} = 0.167 \text{ ft}$$

$$\omega = 100\left(\frac{2\pi}{60}\right) = 10.47 \text{ rad/s}$$

$$k = \frac{200}{\frac{2}{12}} = 1200 \text{ lb/ft}$$

$$F_O = mr\omega^2 = \left(\frac{1}{32.2}\right)\left(\frac{5}{12}\right)(10.47)^2 = 1.419 \text{ lb}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{\frac{200}{32.2}}} = 13.90 \text{ rad/s}$$

$$C' = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)\right]^2}}$$

$$= \frac{\frac{1.419}{1200}}{\sqrt{\left[1 - \left(\frac{10.47}{13.90}\right)^2\right]^2 + \left[2(0.20)\left(\frac{10.47}{13.90}\right)\right]^2}}$$

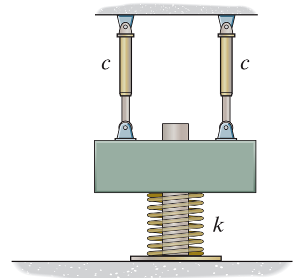
$$= 0.00224 \text{ ft}$$

$C' = 0.0269$ in. **Ans.**

Ans:
$C' = 0.0269$ in.

22–69.

Two identical dashpots are arranged parallel to each other, as shown. Show that if the damping coefficient $c < \sqrt{mk}$, then the block of mass m will vibrate as an underdamped system.



SOLUTION

When the two dash pots are arranged in parallel, the piston of the dashpots have the same velocity. Thus, the force produced is

$$F = c\dot{y} + c\dot{y} = 2c\dot{y}$$

The equivalent damping coefficient c_{eq} of a single dashpot is

$$c_{eq} = \frac{F}{\dot{y}} = \frac{2c\dot{y}}{\dot{y}} = 2c$$

For the vibration to occur (underdamped system), $c_{eq} < c_c$. However, $c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}}$. Thus,

$$c_{eq} < c_c$$

$$2c < 2m\sqrt{\frac{k}{m}}$$

$$c < \sqrt{mk}$$ **Ans.**

Ans:
$F = 2c\dot{y}$
$c_c = 2m\sqrt{\frac{k}{m}}$
$c < \sqrt{mk}$

22–70.

The damping factor, c/c_c, may be determined experimentally by measuring the successive amplitudes of vibrating motion of a system. If two of these maximum displacements can be approximated by x_1 and x_2, as shown in Fig. 22–16, show that $\ln\ x_1/x_2 = 2\pi(c/c_c)/\sqrt{1-(c/c_c)^2}$. The quantity $\ln\ x_1/x_2$ is called the *logarithmic decrement*.

SOLUTION

Using Eq. 22–32,

$$x = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

The maximum displacement is

$$x_{max} = De^{-\left(\frac{c}{2m}\right)t}$$

At $t = t_1$, and $t = t_2$

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1}$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t2}$$

Hence,

$$\frac{x_1}{x_2} = \frac{De^{-\left(\frac{c}{2m}\right)t1}}{De^{-\left(\frac{c}{2m}\right)t_2} = e^{-\left(\frac{c}{2m}\right)(t_1-t_2)}}$$

Since $\omega_d t_2 - \omega_d t_1 = 2\pi$

then $t_2 - t_1 = \dfrac{2\pi}{\omega_d}$

so that $\ln\left(\dfrac{x_1}{x_2}\right) = \dfrac{c\pi}{m\omega_d}$

Using Eq. 22–33, $c_c = 2m\omega_n$

$$\omega_d = \omega_n\sqrt{1-\left(\frac{c}{c_c}\right)^2} = \frac{c_c}{2m}\sqrt{1-\left(\frac{c}{c_r}\right)^2}$$

So that,

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$

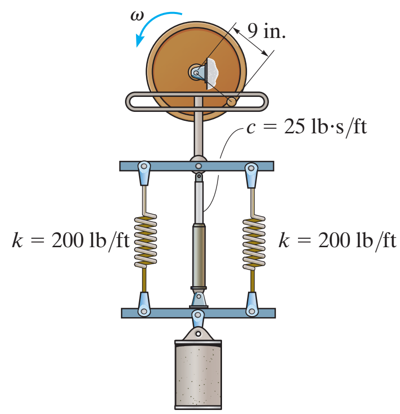
Q.E.D.

Ans:

$$\text{In}\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$

22–71.

If the amplitude of the 50-lb cylinder's steady-state vibration is 6 in., determine the wheel's angular velocity ω.



SOLUTION

In this case, $Y = \frac{6}{12} = 0.5$ ft, $\delta_O = \frac{9}{12} = 0.75$ ft, and $k_{eq} = 2k = 2(200) = 400$ lb/ft. Then

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{400}{(50/32.2)}} = 16.05 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2\left(\frac{50}{32.2}\right)(16.05) = 49.84 \text{ lb}\cdot\text{s/ft}$$

$$\frac{c}{c_c} = \frac{25}{49.84} = 0.5016$$

$$Y = \frac{\delta_O}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2(c/c_c)\omega}{\omega_n}\right)^2}}$$

$$0.5 = \frac{0.75}{\sqrt{\left[1-\left(\frac{\omega}{16.05}\right)^2\right]^2 + \left(\frac{2(0.5016)\omega}{16.05}\right)^2}}$$

$$15.07(10^{-6})\omega^4 - 3.858(10^{-3})\omega^2 - 1.25 = 0$$

Solving for the positive root of this equation,

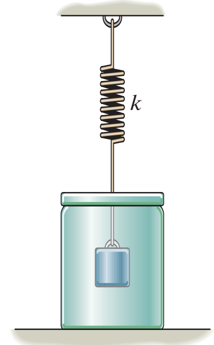
$$\omega^2 = 443.16$$

$$\omega = 21.1 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 21.1$ rad/s

***22–72.**

The block, having a weight of 12 lb, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.7|v|)$ lb, where v is in ft/s. If the block is pulled down 0.62 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 53$ lb/ft. Assume that positive displacement is downward.



SOLUTION

$$c = 0.7 \text{ lb} \cdot \text{s/ft} \qquad k = 53 \text{ lb/ft} \qquad m = \frac{12}{32.2} = 0.3727 \text{ slug}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{53}{0.3727}} = 11.925 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(0.3727)(11.925) = 8.889 \text{ lb} \cdot \text{s/ft}$$

Since $c < c_c$ the system is underdamped.

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 11.925\sqrt{1 - \left(\frac{0.7}{8.889}\right)^2} = 11.888 \text{ rad/s}$$

$$\frac{c}{2m} = \frac{0.7}{2(0.3727)} = 0.9392$$

From Eq. 22–32 $y = D\left[e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t}\omega_d \cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right)e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d \cos(\omega_d t + \phi) - \frac{c}{2m} \sin(\omega_d t + \phi)\right]$$

Appling the initial condition at $t = 0$, $y = 0.62$ ft and $v = 0$.

$$0.62 = D\left[e^{-0} \sin(0 + \phi)\right]$$

$$D \sin\phi = 0.62 \qquad \textbf{(1)}$$

$$0 = De^{-0}\left[11.888 \cos(0 + \phi) - 0.9392 \sin(0 + \phi)\right] \qquad \text{since } D \neq 0$$

$$11.888 \cos\phi - 0.9392 \sin\theta = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

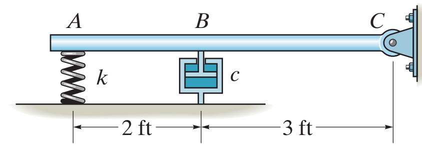
$$\phi = 85.5^\circ = 1.49 \text{ rad} \qquad D = 0.622 \text{ ft}$$

$$y = 0.622[e^{-0.939t} \sin(11.9\text{t} + 1.49)] \qquad \textbf{Ans.}$$

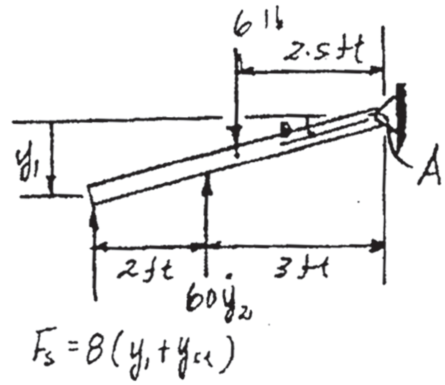
Ans:
$y = 0.622\,[e^{-0.939t} \sin(11.9\text{t} + 1.49)]$

22–73.

The bar has a weight of 6 lb. If the stiffness of the spring is $k = 8$ lb/ft and the dashpot has a damping coefficient $c = 60$ lb·s/ft, determine the differential equation which describes the motion in terms of the angle θ of the bar's rotation. Also, what should be the damping coefficient of the dashpot if the bar is to be critically damped?



SOLUTION



$$\circlearrowleft +\Sigma M_A = I_A\,\alpha; \qquad 6(2.5) - (60\dot{y}_2)(3) - 8(y_1 + y_{st})(5) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(5)^2\right]\ddot{\theta}$$

$$1.5528\ddot{\theta} + 180\dot{y}_2 + 40y_1 + 40y_{st} - 15 = 0 \qquad \textbf{[1]}$$

From equilibrium $40y_{st} - 15 = 0$. Also, for small θ, $y_1 = 5\theta$ and $y_2 = 3\theta$ hence $\dot{y}_2 = 3\dot{\theta}$.

From Eq. [1] $\quad 1.5528\ddot{\theta} + 180(3\dot{\theta}) + 40(5\theta) = 0$

$$1.55\ddot{\theta} + 540\dot{\theta} + 200\theta = 0 \qquad \textbf{Ans.}$$

By comparing the above differential equation to Eq. 22-27

$$m = 1.55 \qquad k = 200 \qquad \omega_n = \sqrt{\frac{200}{1.55}} = 11.35 \text{ rad/s} \qquad c = 9c_{d\cdot p}$$

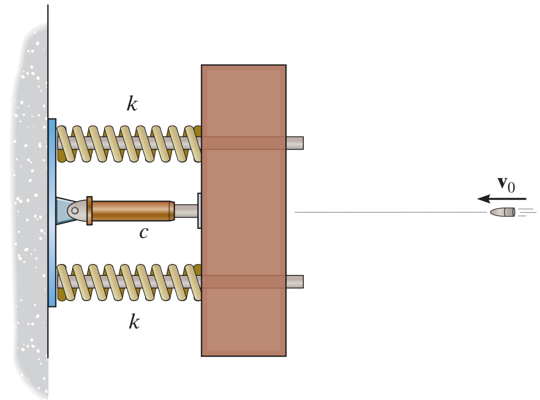
$$\left(\frac{9(c_{d\cdot p})_c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$(c_{d\cdot p})_c = \frac{2}{9}\sqrt{km} = \frac{2}{9}\sqrt{200(1.55)} = 3.92 \text{ lb}\cdot\text{s/ft} \qquad \textbf{Ans.}$$

Ans:
$1.55\ddot{\theta} + 540\dot{\theta} + 200\theta = 0$
$(c_{dp})_c = 3.92$ lb·s/ft

22–74.

A bullet of mass m has a velocity of $\mathbf{v}_0$ just before it strikes the target of mass M. If the bullet embeds in the target, and the vibration is to be critically damped, determine the dashpot's critical damping coefficient, and the springs' maximum compression. The target is free to move along the two horizontal guides that are "nested" in the springs.



SOLUTION

Since the springs are arranged in parallel, the equivalent stiffness of the single spring system is $k_{eq} = 2k$. Also, when the bullet becomes embedded in the target, $m_T = m + M$. Thus, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m_T}} = \sqrt{\frac{2k}{m + M}}$$

When the system is critically damped

$$c = c_c = 2m_T\omega_n = 2(m + M)\sqrt{\frac{2k}{m + M}} = \sqrt{8(m + M)k} \qquad \textbf{Ans.}$$

The equation that describes the critically dampened system is

$$x = (A + Bt)e^{-\omega_n t}$$

When $t = 0, x = 0$. Thus,

$$A = 0$$

Then,

$$x = Bte^{-\omega_n t} \qquad \textbf{(1)}$$

Taking the time derivative,

$$v = \dot{x} = Be^{-\omega_n t} - B\omega_n te^{-\omega_n t}$$

$$v = Be^{-\omega_n t}(1 - \omega_n t) \qquad \textbf{(2)}$$

Since linear momentum is conserved along the horizontal during the impact, then

$$(\overset{+}{\leftarrow}) \qquad mv_0 = (m + M)v$$

$$v = \left(\frac{m}{m + M}\right)v_0$$

Here, when $t = 0$, $v = \left(\frac{m}{m + M}\right)v_0$. Thus, Eq. (2) gives

$$B = \left(\frac{m}{m + M}\right)v_0$$

And Eqs. (1) and (2) become

$$x = \left[\left(\frac{m}{m + M}\right)v_0\right]te^{-\omega_n t} \qquad \textbf{(3)}$$

$$v = \left[\left(\frac{m}{m + M}\right)v_0\right]e^{-\omega_n t}(1 - \omega_n t) \qquad \textbf{(4)}$$

22–74. Continued

The maximum compression of the spring occurs when the block stops. Thus, Eq. (4) gives

$$0 = \left[\left(\frac{m}{m + M}\right)v_0\right](1 - \omega_n t)$$

Since $\left(\dfrac{m}{m + M}\right)v_0 \neq 0$, then

$$1 - \omega_n t = 0$$

$$t = \frac{1}{\omega_n} = \sqrt{\frac{m + M}{2k}}$$

Substituting this result into Eq. (3)

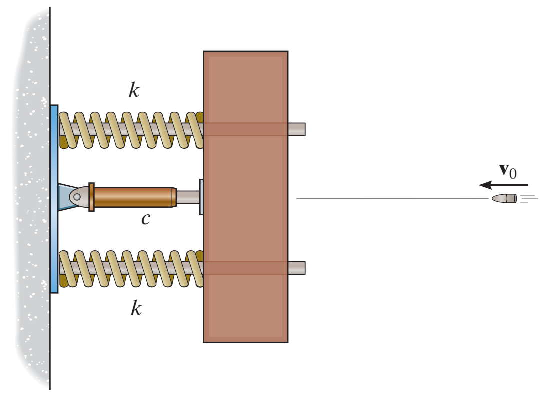
$$x_{\max} = \left[\left(\frac{m}{m + M}\right)v_0\right]\left(\sqrt{\frac{m + M}{2k}}\right)e^{-1}$$

$$= \left[\frac{m}{e}\sqrt{\frac{1}{2k(m + M)}}\right]v_0 \qquad \textbf{Ans.}$$

Ans:
$c_c = \sqrt{8(m + M)k}$
$x_{\max} = \left[\dfrac{m}{e}\sqrt{\dfrac{1}{2k(m + M)}}\right]v_0$

22–75.

A bullet of mass m has a velocity $\mathbf{v}_0$ just before it strikes the target of mass M. If the bullet embeds in the target, and the dashpot's damping coefficient is $0 < c \ll c_c$, determine the springs' maximum compression. The target is free to move along the two horizontal guides that are"nested" in the springs.



SOLUTION

Since the springs are arranged in parallel, the equivalent stiffness of the single spring system is $k_{eq} = 2k$. Also, when the bullet becomes embedded in the target, $m_T = m + M$. Thus, the natural circular frequency of the system

$$\omega_n = \sqrt{\frac{k_{eq}}{m_T}} = \sqrt{\frac{2k}{m + M}}$$

The equation that describes the underdamped system is

$$x = Ce^{-(c/2m_T)t}\sin(\omega_d t + \phi) \qquad \textbf{(1)}$$

When $t = 0$, $x = 0$. Thus, Eq. (1) gives

$$0 = C\sin\phi$$

Since $C \neq 0$, $\sin\phi = 0$. Then $\phi = 0$. Thus, Eq. (1) becomes

$$x = Ce^{-(c/2m_T)t}\sin\omega_d t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2),

$$v = \dot{x} = C\left[\omega_d e^{-(c/2m_T)t}\cos\omega_d t - \frac{c}{2m_T}e^{-(c/2m_T)t}\sin\omega_d t\right]$$

$$v = Ce^{-(c/2m_T)t}\left[\omega_d\cos\omega_d t - \frac{c}{2m_T}\sin\omega_d t\right] \qquad \textbf{(3)}$$

Since linear momentum is conserved along the horizontal during the impact, then

$$(\overset{+}{\leftarrow}) \qquad mv_0 = (m + M)v$$

$$v = \left(\frac{m}{m + M}\right)v_0$$

When $t = 0$, $v = \left(\frac{m}{m + M}\right)v_0$. Thus, Eq. (3) gives

$$\left(\frac{m}{m + M}\right)v_0 = C\omega_d \qquad C = \left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}$$

And Eqs. (2) becomes

$$x = \left[\left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}\right]e^{-(c/2m_T)t}\sin\omega_d t \qquad \textbf{(4)}$$

22–75. Continued

The maximum compression of the spring occurs when

$$\sin \omega_d t = 1$$

$$\omega_d t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2\omega_d}$$

Substituting this result into Eq. (4),

$$x_{\max} = \left[\left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}\right]e^{-[c/2(m+M)]\left(\frac{\pi}{2\omega_d}\right)}$$

However, $\omega_d = \sqrt{\dfrac{k_{eq}}{m_T} - \left(\dfrac{c}{2m_T}\right)^2} = \sqrt{\dfrac{2k}{m+M} - \dfrac{c^2}{4(m+M)^2}} = \dfrac{1}{2(m+M)}$ $\sqrt{8k(m+M) - c^2}$. Substituting this result into Eq. (5),

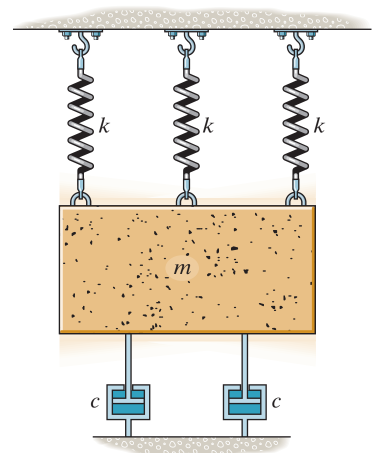
$$x_{\max} = \frac{2mv_0}{\sqrt{8k(m+M) - c^2}}e^{-\left[\frac{\pi c}{2\sqrt{8k(m+M) - c^2}}\right]} \qquad \textbf{Ans.}$$

Ans:

$$x_{\max} = \frac{2mv_0}{\sqrt{8k(m+M) - c^2}}\, e^{-\pi c/\left(2\sqrt{8k(m+M)-c^2}\right)}$$

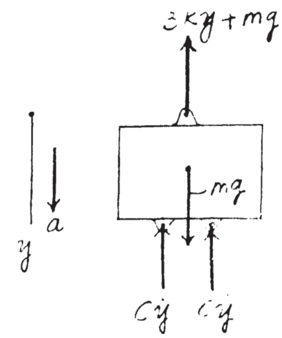
***22–76.**

Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N · s/m, $m = 25$ kg.



SOLUTION

Free-body Diagram: When the block is being displaced by an amount y vertically downward, the *restoring force* is developed by the three springs attached the block.



Equation of Motion:

$+\uparrow \Sigma F_x = 0; \qquad 3ky + mg + 2c\dot{y} - mg = -m\ddot{y}$

$$m\ddot{y} + 2c\dot{y} + 3ky = 0 \qquad \textbf{(1)}$$

Here, $m = 25$ kg, $c = 200$ N · s/m and $k = 100$ N/m. Substituting these values into Eq. (1) yields

$$25\ddot{y} + 400\dot{y} + 300y = 0$$

$$\ddot{y} + 16\dot{y} + 12y = 0 \qquad \textbf{Ans.}$$

Comparing the above differential equation with Eq. 22–27, we have $m = 1$kg, $c = 16$ N · s/m and $k = 12$ N/m. Thus, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{12}{1}} = 3.464$ rad/s

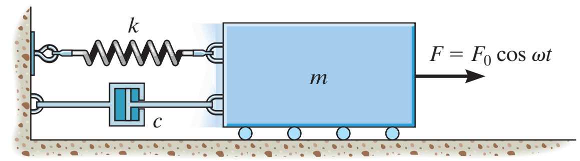
$$c_c = 2m\omega_n = 2(1)(3.464) = 6.928 \text{ N} \cdot \text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped.** **Ans.**

Ans:
$\ddot{y} + 16\dot{y} + 12y = 0$
Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped.**

22–77.

Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



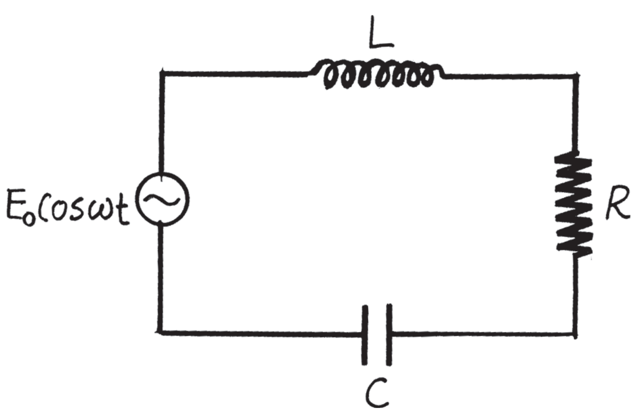
SOLUTION

For the block,

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

Using Table 22–1,

$$L\ddot{q} + R\dot{q} + \left(\frac{1}{C}\right)q = E_0 \cos \omega t$$ **Ans.**

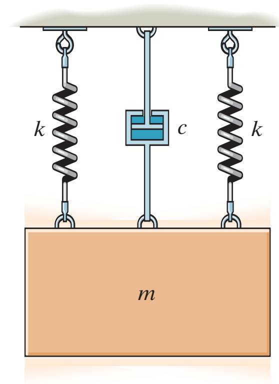


Ans:

$L\ddot{q} + R\dot{q} + \left(\frac{1}{C}\right)q = E_0 \cos \omega t$

22–78.

Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?



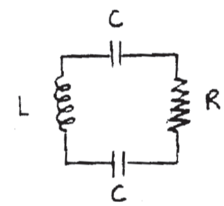
SOLUTION

For the block,

$$m\ddot{x} + c\dot{x} + 2k = 0$$

Using Table 22–1,

$$L\ddot{q} + R\dot{q} + \left(\frac{2}{C}\right)q = 0 \qquad \textbf{Ans.}$$

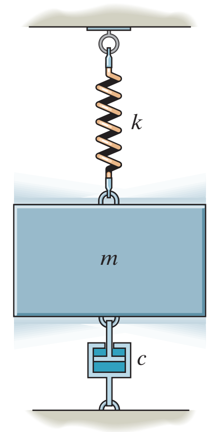


Ans:

$$L\ddot{q} + R\dot{q} + \left(\frac{2}{C}\right)q = 0$$

22–79.

Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



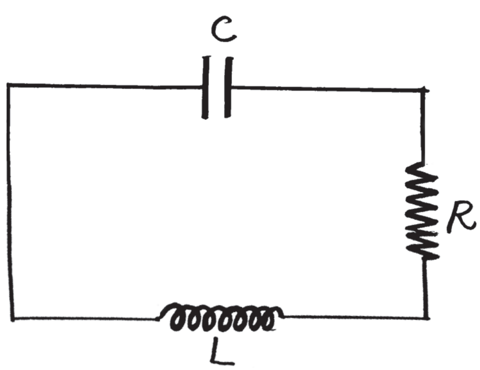
SOLUTION

For the block

$$m\ddot{y} + c\dot{y} + ky = 0$$

Using Table 22–1

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0 \qquad \textbf{Ans.}$$

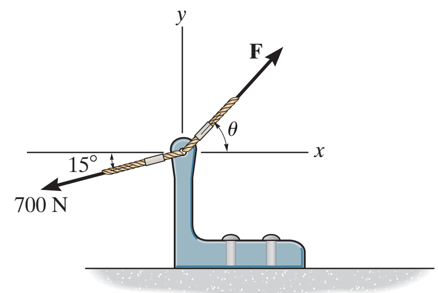


Ans:

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0$$

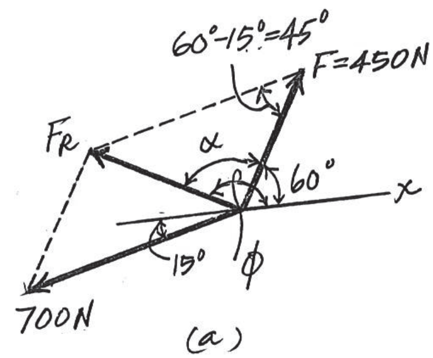
2–1.

If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45°}$$

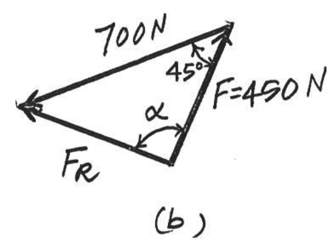
$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

This yields

$$\frac{\sin\alpha}{700} = \frac{\sin 45°}{497.01} \qquad \alpha = 95.19°$$

Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

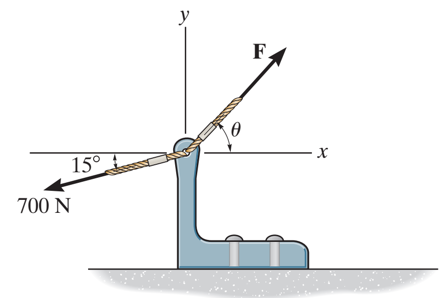
$$\phi = \alpha + 60° = 95.19° + 60° = 155° \qquad \textbf{Ans.}$$



Ans:
$F_R = 497$ N
$\phi = 155°$

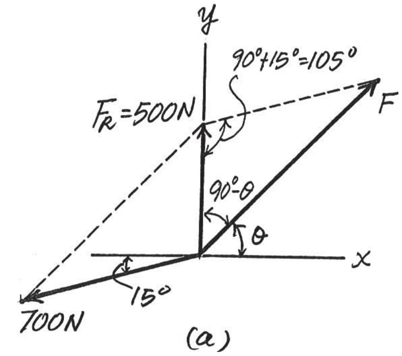
2–2.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



SOLUTION

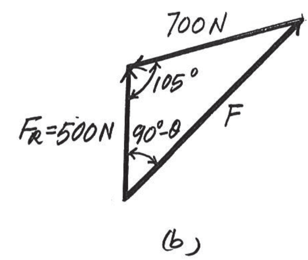
The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105^\circ}$$

$$= 959.78 \text{ N} = 960 \text{ N} \qquad \textbf{Ans.}$$



Applying the law of sines to Fig. b, and using this result, yields

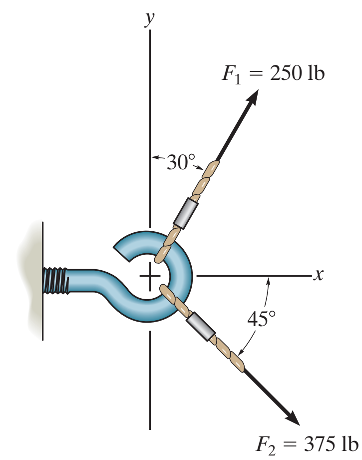
$$\frac{\sin(90^\circ + \theta)}{700} = \frac{\sin 105^\circ}{959.78}$$

$$\theta = 45.2^\circ \qquad \textbf{Ans.}$$

Ans:
$F = 960$ N
$\theta = 45.2^\circ$

2–3.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



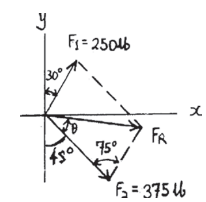
SOLUTION

$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb}$ **Ans.**

$\dfrac{393.2}{\sin 75^\circ} = \dfrac{250}{\sin \theta}$

$\theta = 37.89^\circ$

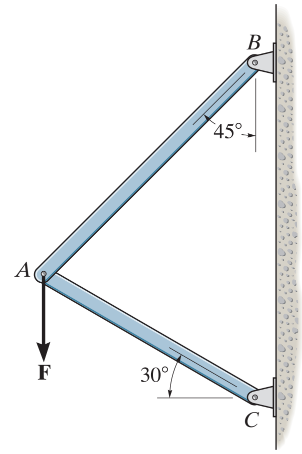
$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ$ **Ans.**



Ans:
$F_R = 393$ lb
$\phi = 353^\circ$

***2–4.**

The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

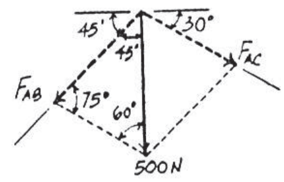
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{500}{\sin 75^\circ}$$

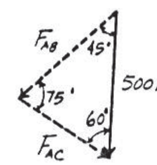
$$F_{AB} = 448 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{500}{\sin 75^\circ}$$

$$F_{AC} = 366 \text{ N} \qquad \textbf{Ans.}$$



(a)

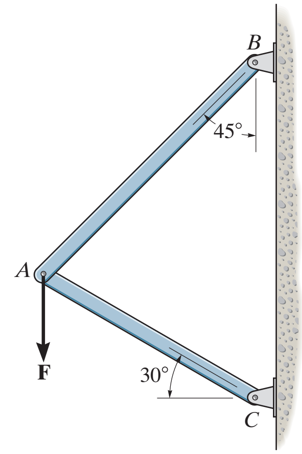


(b)

Ans:
$F_{AB} = 448$ N
$F_{AC} = 366$ N

2–5.

Solve Prob. 2-4 with $F = 350$ lb.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

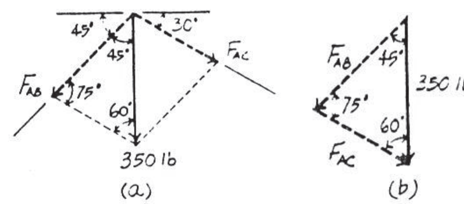
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

$$F_{AB} = 314 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

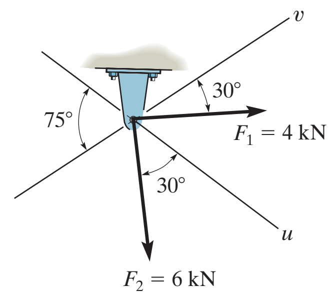
$$F_{AC} = 256 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$F_{AB} = 314$ lb
$F_{AC} = 256$ lb

2–6.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,

Trigonometry. Applying Law of cosines by referring to Fig. *b*,

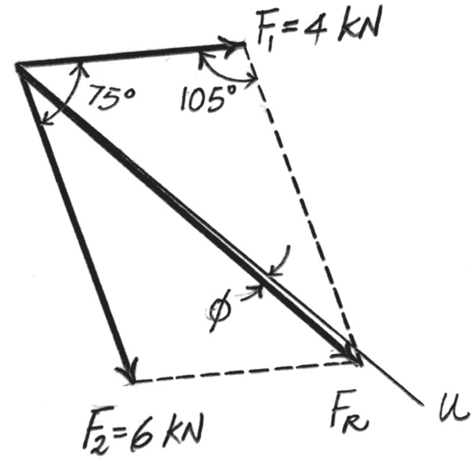
$$F_R = \sqrt{4^2 + 6^2 - 2(4)(6)\cos 105^\circ} = 8.026 \text{ kN} = 8.03 \text{ kN}$$ **Ans.**

Using this result to apply Law of sines, Fig. *b*,

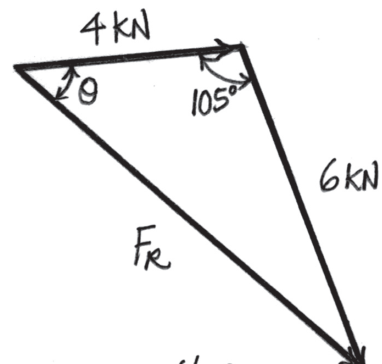
$$\frac{\sin\theta}{6} = \frac{\sin 105^\circ}{8.026}; \qquad \theta = 46.22^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured clockwise from the positive u axis is

$$\phi = 46.22^\circ - 45^\circ = 1.22^\circ$$ **Ans.**



(a)

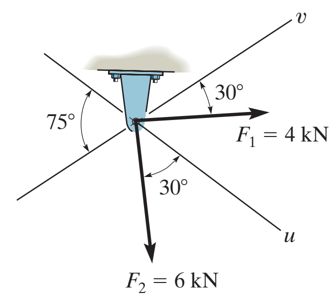


(b)

Ans:
$\phi = 1.22^\circ$

2–7.

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.



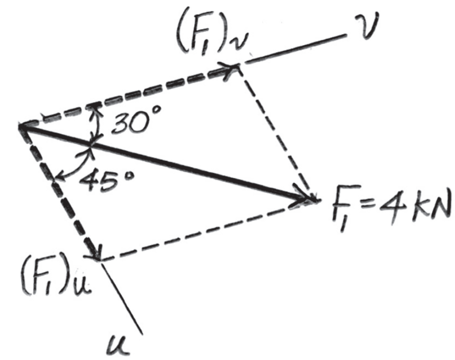
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. a,

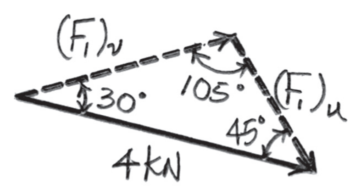
Trigonometry. Applying the sines law by referring to Fig. b.

$$\frac{(F_1)_v}{\sin 45°} = \frac{4}{\sin 105°}; \qquad (F_1)_v = 2.928 \text{ kN} = 2.93 \text{ kN} \qquad \textbf{Ans.}$$

$$\frac{(F_1)_u}{\sin 30°} = \frac{4}{\sin 105°}; \qquad (F_1)_u = 2.071 \text{ kN} = 2.07 \text{ kN} \qquad \textbf{Ans.}$$



(a)

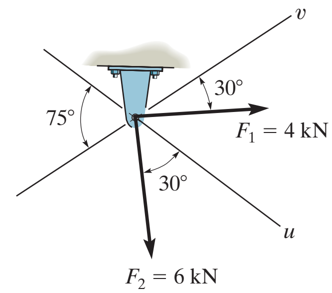


(b)

Ans:
$(F_1)_v = 2.93$ kN
$(F_1)_u = 2.07$ kN

***2–8.**

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.

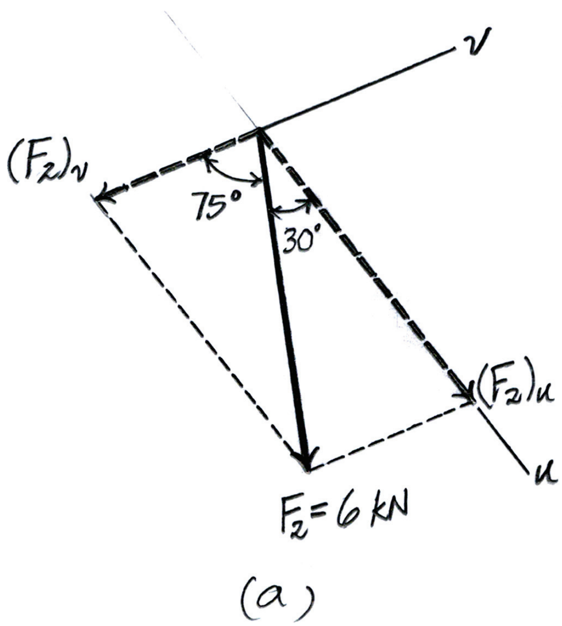


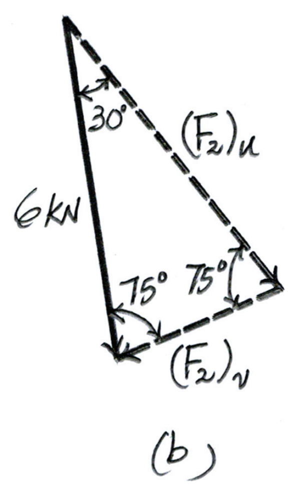
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the sines law of referring to Fig. *b*,

$$\frac{(F_2)_u}{\sin 75^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_u = 6.00 \text{ kN} \qquad \textbf{Ans.}$$

$$\frac{(F_2)_v}{\sin 30^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_v = 3.106 \text{ kN} = 3.11 \text{ kN} \qquad \textbf{Ans.}$$

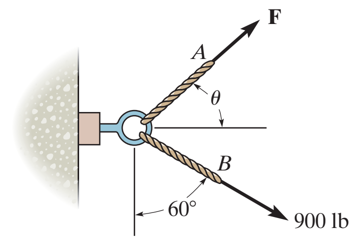




Ans:
$(F_2)_u = 6.00$ kN
$(F_2)_v = 3.11$ kN

2–9.

If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope A and the corresponding angle θ.



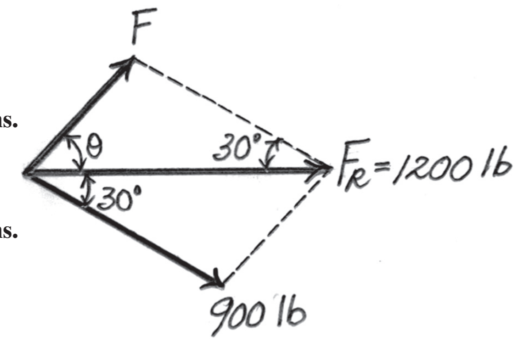
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the law of cosines by referring to Fig. *b*,

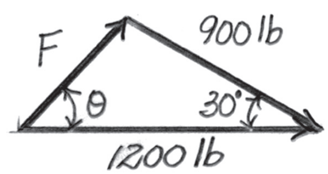
$$F = \sqrt{900^2 + 1200^2 - 2(900)(1200)\cos 30^\circ} = 615.94 \text{ lb} = 616 \text{ lb}$$ **Ans.**

Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin\theta}{900} = \frac{\sin 30^\circ}{615.94}; \qquad \theta = 46.94^\circ = 46.9^\circ$$ **Ans.**



(a)

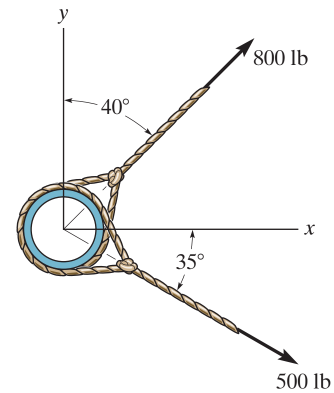


(b)

Ans:
$F = 616$ lb
$\theta = 46.9^\circ$

2–10.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the law of cosines by referring to Fig. *b*,

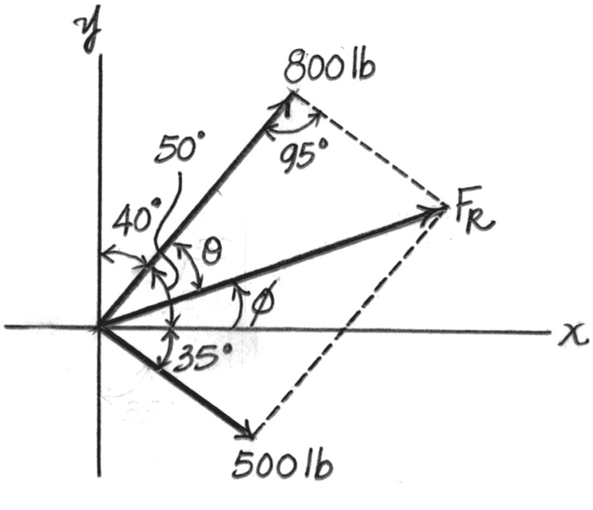
$$F_R = \sqrt{800^2 + 500^2 - 2(800)(500)\cos 95^\circ} = 979.66 \text{ lb} = 980 \text{ lb} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. *b*,

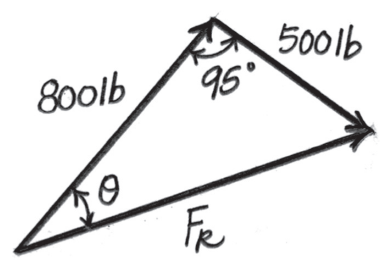
$$\frac{\sin\theta}{500} = \frac{\sin 95^\circ}{979.66}; \qquad \theta = 30.56^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\phi = 50^\circ - 30.56^\circ = 19.44^\circ = 19.4^\circ \qquad \textbf{Ans.}$$



(a)

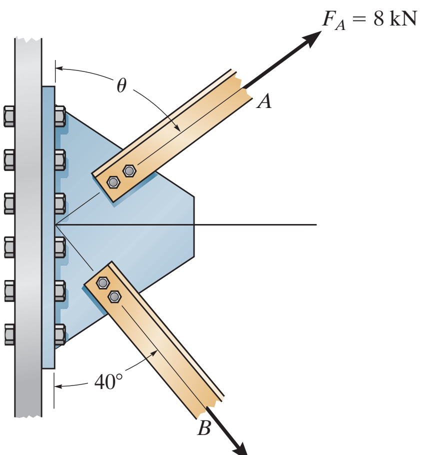


(b)

Ans:
$F_R = 980$ lb
$\phi = 19.4^\circ$

2–11.

The plate is subjected to the two forces at A and B as shown. If $\theta = 60^\circ$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. a.

Trigonometry: Using law of cosines (Fig. b), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100^\circ}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans.}$$

The angle θ can be determined using law of sines (Fig. b).

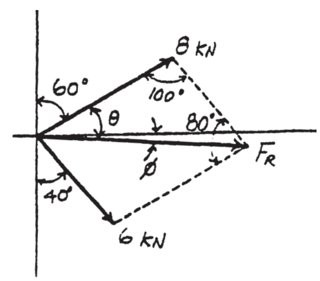
$$\frac{\sin \theta}{6} = \frac{\sin 100^\circ}{10.80}$$

$$\sin \theta = 0.5470$$

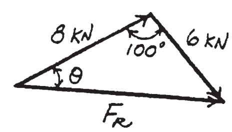
$$\theta = 33.16^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ \qquad \textbf{Ans.}$$



(a)

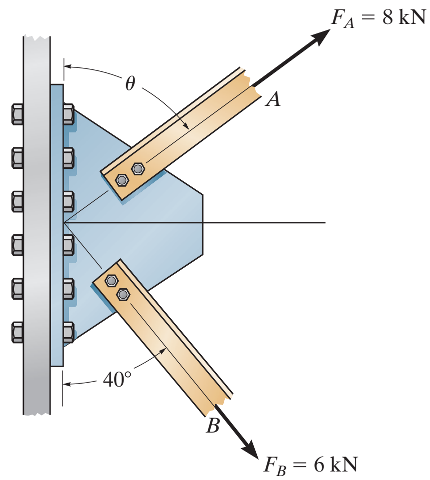


(b)

Ans:
$F_R = 10.8$ kN
$\phi = 3.16^\circ$

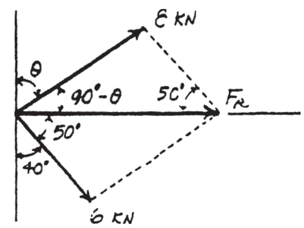
***2–12.**

Determine the angle of θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.



Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

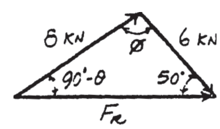
$$\sin(90^\circ - \theta) = 0.5745$$

$$\theta = 54.93^\circ = 54.9^\circ \qquad \textbf{Ans.}$$

From the triangle, $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93^\circ}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$

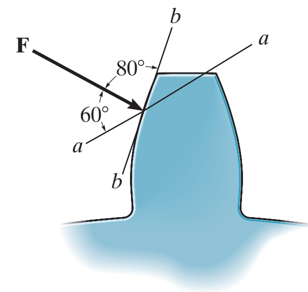


(b)

Ans:
$\theta = 54.9^\circ$
$F_R = 10.4$ kN

2–13.

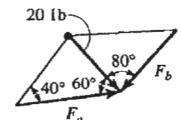
The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines *aa* and *bb*.



SOLUTION

$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \qquad F_a = 30.6 \text{ lb} \qquad \textbf{Ans.}$$

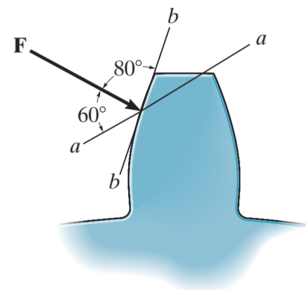
$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.9 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$F_a = 30.6$ lb
$F_b = 26.9$ lb

2–14.

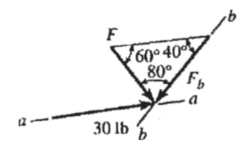
The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



SOLUTION

$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6 \text{ lb} \qquad \textbf{Ans.}$$

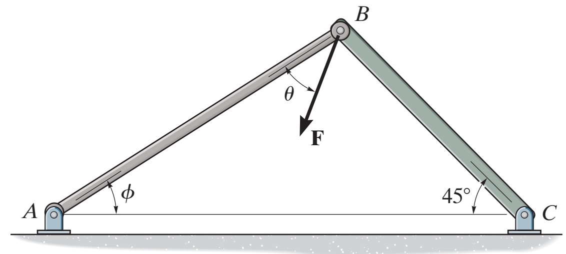
$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$F = 19.6$ lb
$F_b = 26.4$ lb

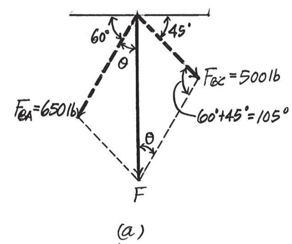
2–15.

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. a and b, respectively.



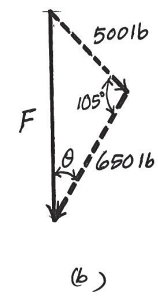
Applying the law of cosines to Fig. b,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

$$= 916.91 \text{ lb} = 917 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. b, yields

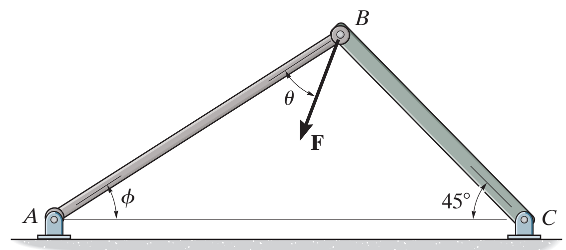
$$\frac{\sin\theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8° \qquad \textbf{Ans.}$$



Ans:
$F = 917$ lb
$\theta = 31.8°$

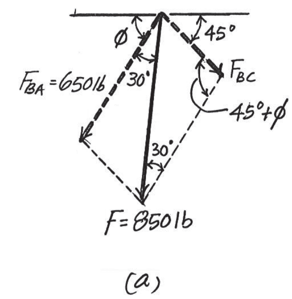
***2–16.**

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ ($0° \leq \phi \leq 45°$) and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. a and b, respectively.



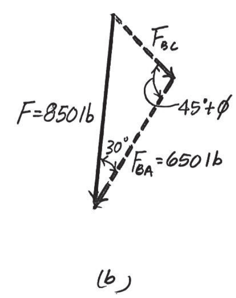
Applying the law of cosines to Fig. b,

$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30°}$$

$$= 433.64 \text{ lb} = 434 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the sine law to Fig. b, yields

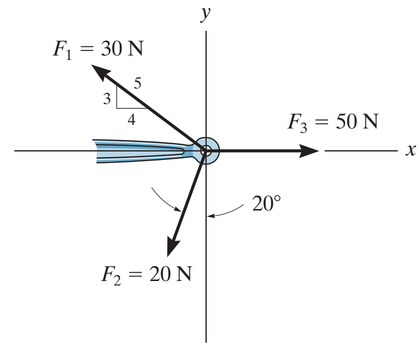
$$\frac{\sin(45° + \phi)}{850} = \frac{\sin 30°}{433.64} \qquad \phi = 33.5° \qquad \textbf{Ans.}$$



Ans:
$F_{BC} = 434$ lb
$\phi = 33.5°$

2–17.

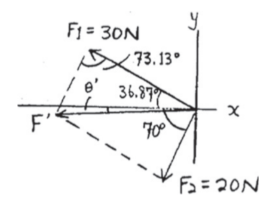
Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



SOLUTION

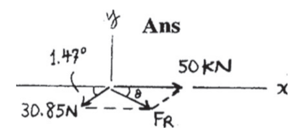
$$F' = \sqrt{(20)^2 + (30)^2 - 2(20)(30)\cos 73.13^\circ} = 30.85 \text{ N}$$

$$\frac{30.85}{\sin 73.13^\circ} = \frac{30}{\sin(70^\circ - \theta')}; \qquad \theta' = 1.47^\circ$$



$$F_R = \sqrt{(30.85)^2 + (50)^2 - 2(30.85)(50)\cos 1.47^\circ} = 19.18 = 19.2 \text{ N} \qquad \textbf{Ans.}$$

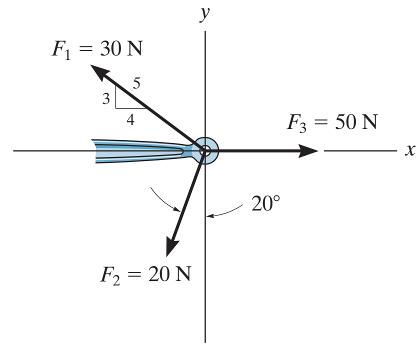
$$\frac{19.18}{\sin 1.47^\circ} = \frac{30.85}{\sin\theta}; \qquad \theta = 2.37^\circ \qquad \textbf{Ans.}$$



Ans:
$F_R = 19.2$ N
$\theta = 2.37^\circ$

2–18.

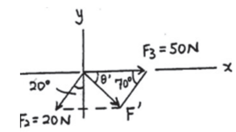
Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



SOLUTION

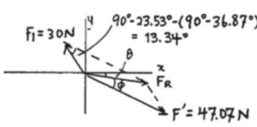
$$F' = \sqrt{(20)^2 + (50)^2 - 2(20)(50)\cos 70^\circ} = 47.07 \text{ N}$$

$$\frac{20}{\sin\theta'} = \frac{47.07}{\sin 70^\circ}; \qquad \theta' = 23.53^\circ$$



$$F_R = \sqrt{(47.07)^2 + (30)^2 - 2(47.07)(30)\cos 13.34^\circ} = 19.18 = 19.2 \text{ N}$$ **Ans.**

$$\frac{19.18}{\sin 13.34^\circ} = \frac{30}{\sin\phi}; \qquad \phi = 21.15^\circ$$

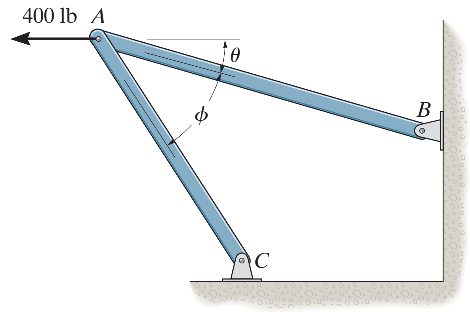


$$\theta = 23.53^\circ - 21.15^\circ = 2.37^\circ$$ **Ans.**

Ans:
$F_R = 19.2$ N
$\theta = 2.37^\circ$

2–19.

Determine the design angle θ ($0° \leq \theta \leq 90°$) for strut AB so that the 400-lb horizontal force has a component of 500 lb directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.



SOLUTION

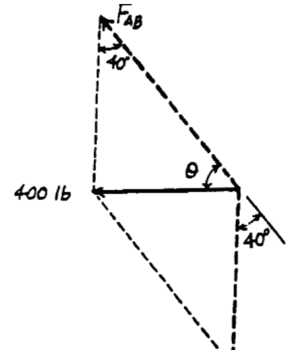
Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin\theta}{500} = \frac{\sin 40°}{400}$$

$$\sin\theta = 0.8035$$

$$\theta = 53.46° = 53.5° \qquad \textbf{Ans.}$$



(a)

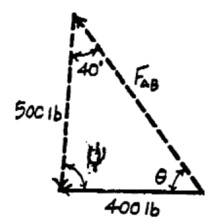
Thus,

$$\psi = 180° - 40° - 53.46° = 86.54°$$

Using law of sines (Fig. *b*)

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

$$F_{AB} = 621 \text{ lb} \qquad \textbf{Ans.}$$

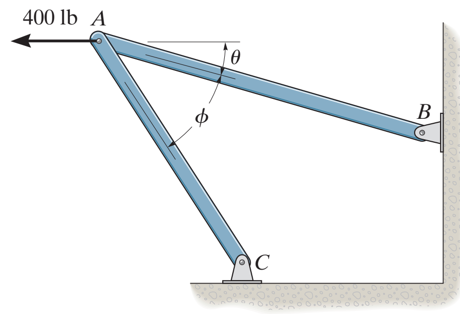


(b)

Ans:
$\theta = 53.5°$
$F_{AB} = 621$ lb

***2–20.**

Determine the design angle ϕ $(0° \le \phi \le 90°)$ between struts AB and AC so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from B towards A. Take $\theta = 30°$.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

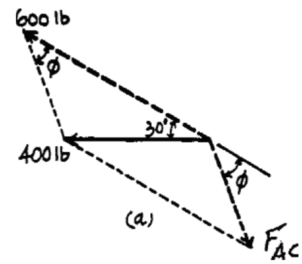
$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30°} = 322.97\text{ lb}$$

The angle ϕ can be determined using law of sines (Fig. *b*).

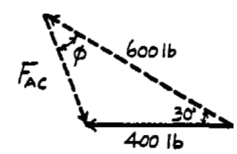
$$\frac{\sin\phi}{400} = \frac{\sin 30°}{322.97}$$

$$\sin\phi = 0.6193$$

$$\phi = 38.3° \qquad \textbf{Ans.}$$



(a)

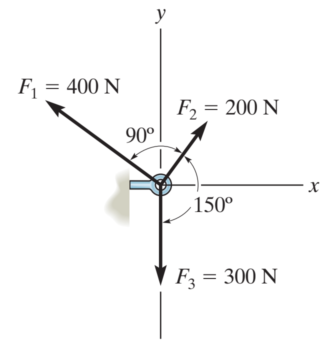


(b)

Ans:
$\phi = 38.3°$

2–21.

Determine the magnitude and direction of the resultant force, $\mathbf{F}_R$ measured counterclockwise from the positive x axis. Solve the problem by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



SOLUTION

Parallelogram Law. The parallelogram law of addition for $\mathbf{F}_1$ and $\mathbf{F}_2$ and then their resultant $\mathbf{F}'$ and $\mathbf{F}_3$ are shown in Figs. *a* and *b*, respectively.
Trigonometry. Referring to Fig. *c*,

$$F' = \sqrt{200^2 + 400^2} = 447.21 \text{ N} \qquad \theta' = \tan^{-1}\left(\frac{200}{400}\right) = 26.57^\circ$$

Thus $\phi' = 90^\circ - 30^\circ - 26.57^\circ = 33.43^\circ$

Using these results to apply the law of cosines by referring to Fig. *d*,

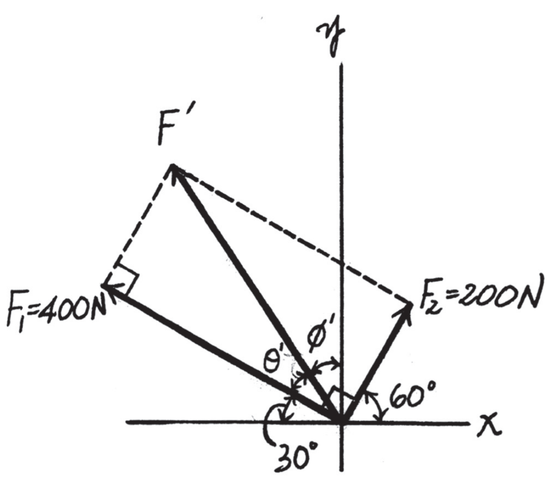
$$F_R = \sqrt{300^2 + 447.21^2 - 2(300)(447.21)\cos 33.43^\circ} = 257.05 \text{ N} = 257 \text{ kN} \quad \textbf{Ans.}$$

Then, apply the law of sines,

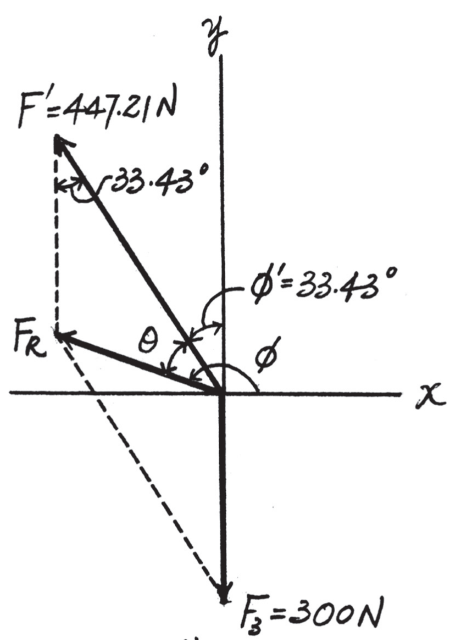
$$\frac{\sin\theta}{300} = \frac{\sin 33.43^\circ}{257.05}; \qquad \theta = 40.02^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

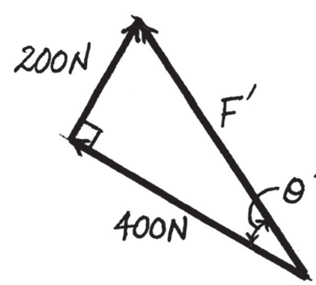
$$\phi = 90^\circ + 33.43^\circ + 40.02^\circ = 163.45^\circ = 163^\circ \quad \textbf{Ans.}$$



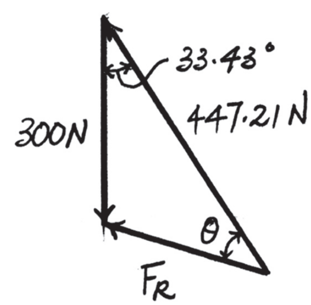
(a)



(b)



(C)

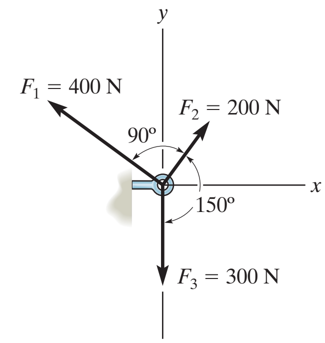


(d)

Ans:
$F_R = 257$ N
$\phi = 163^\circ$

2–22.

Determine the magnitude and direction of the resultant force, measured counterclockwise from the positive x axis. Solve l by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



SOLUTION

Parallelogram Law. The parallelogram law of addition for $\mathbf{F}_2$ and $\mathbf{F}_3$ and then their resultant $\mathbf{F}'$ and $\mathbf{F}_1$ are shown in Figs. *a* and *b*, respectively.
Trigonometry. Applying the law of cosines by referring to Fig. *c*,

$$F' = \sqrt{200^2 + 300^2 - 2(200)(300)\cos 30^\circ} = 161.48 \text{ N} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. *c*,

$$\frac{\sin\theta'}{200} = \frac{\sin 30^\circ}{161.48}; \qquad \theta' = 38.26^\circ$$

Using the results of $\mathbf{F}'$ and θ' to apply the law of cosines by referring to Fig. *d*,

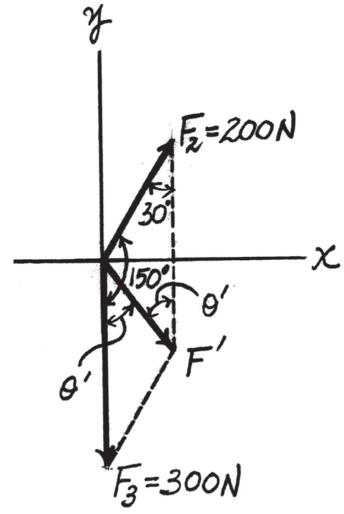
$$F_R = \sqrt{161.48^2 + 400^2 - 2(161.48)(400)\cos 21.74^\circ} = 257.05 \text{ N} = 257 \text{ N} \qquad \textbf{Ans.}$$

Then, apply the sines law,

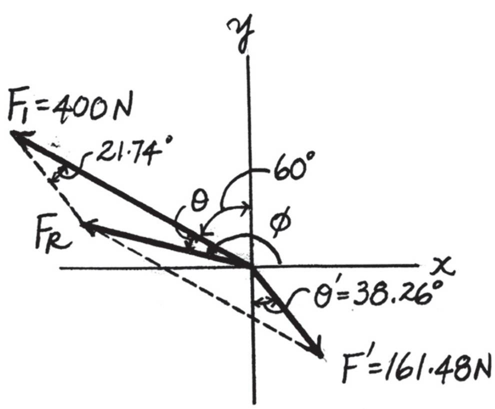
$$\frac{\sin\theta}{161.48} = \frac{\sin 21.74^\circ}{257.05}; \qquad \theta = 13.45^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

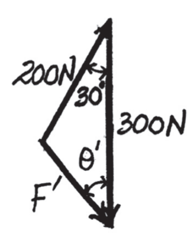
$$\phi = 90^\circ + 60^\circ + 13.45^\circ = 163.45^\circ = 163^\circ \qquad \textbf{Ans.}$$



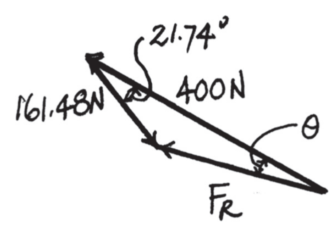
(a)



(b)



(c)

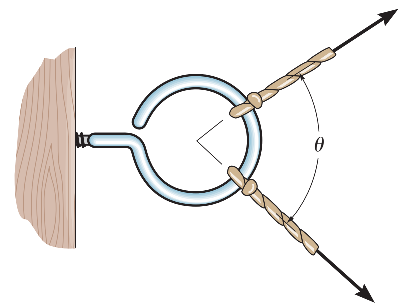


(d)

Ans:
$\phi = 163^\circ$
$F_R = 257$ N

2–23.

Two forces act on the screw eye. If $F_1 = 400\text{ N}$ and $F_2 = 600\text{ N}$, determine the angle $\theta(0° \leq \theta \leq 180°)$ between them, so that the resultant force has a magnitude of $F_R = 800\text{ N}$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

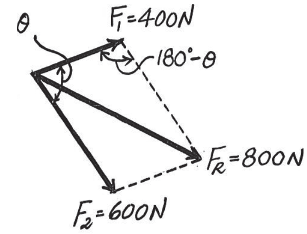
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - \theta°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - \theta)$$

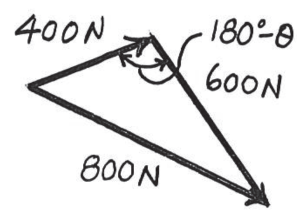
$$\cos(180° - \theta) = -0.25$$

$$180° - \theta = 104.48$$

$$\theta = 75.52° = 75.5° \qquad \textbf{Ans.}$$



(a)

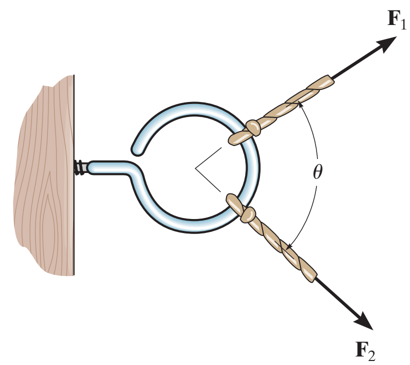


(b)

Ans:
$\theta = 75.5°$

***2–24.**

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



SOLUTION

$$\frac{F}{\sin\phi} = \frac{F}{\sin(\theta - \phi)}$$

$$\sin(\theta - \phi) = \sin\phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \qquad \textbf{Ans.}$$

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180^\circ - \theta)}$$

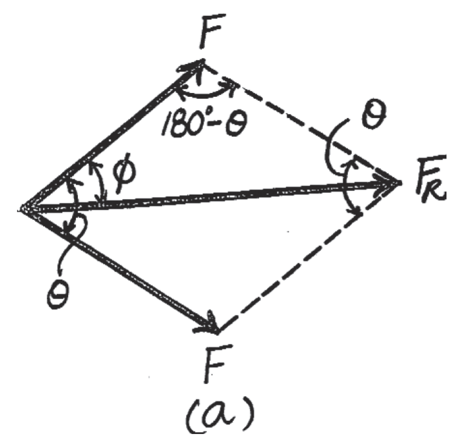
Since $\cos(180^\circ - \theta) = -\cos\theta$

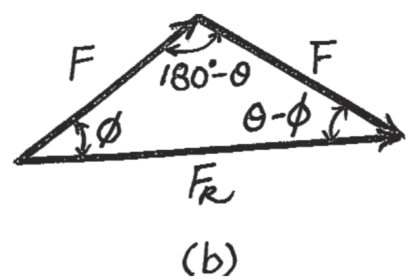
$$F_R = F\left(\sqrt{2}\right)\sqrt{1 + \cos\theta}$$

Since $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos\theta}{2}}$

Then

$$F_R = 2F\cos\left(\frac{\theta}{2}\right) \qquad \textbf{Ans.}$$





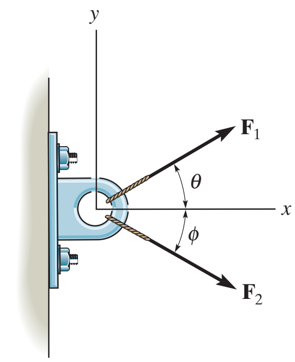
Ans:

$\phi = \frac{\theta}{2}$

$F_R = 2F\cos\left(\frac{\theta}{2}\right)$

2–25.

If $F_1 = 30$ lb and $F_2 = 40$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 60$ lb.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the law of cosine by referring to Fig. *b*,

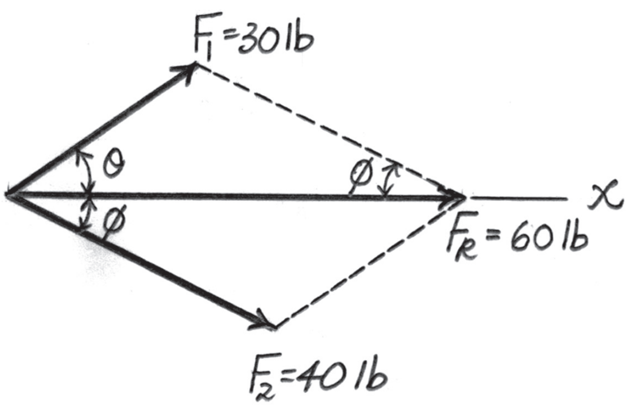
$$40^2 = 30^2 + 60^2 - 2(30)(60)\cos\theta$$

$$\theta = 36.34° = 36.3° \qquad \textbf{Ans.}$$

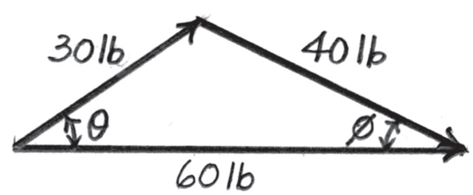
And

$$30^2 = 40^2 + 60^2 - 2(40)(60)\cos\phi$$

$$\phi = 26.38° = 26.4° \qquad \textbf{Ans.}$$



(a)

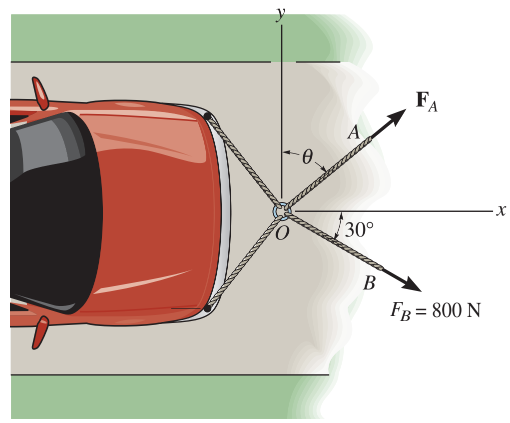


(b)

Ans:
$\theta = 36.3°$
$\phi = 26.4°$

2–26.

Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



SOLUTION

$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = F_A \sin\theta + 800\cos 30° = 1250$

$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = F_A \cos\theta - 800\sin 30° = 0$

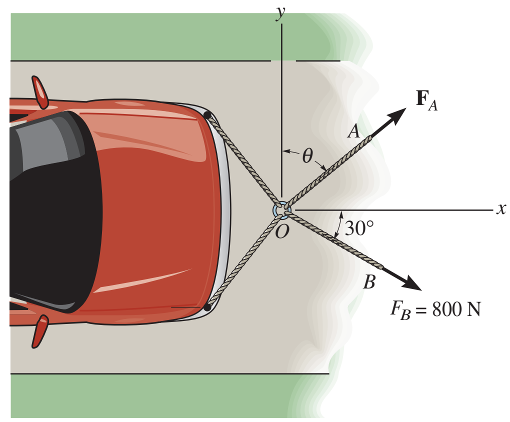
$\theta = 54.3°$ **Ans.**

$F_A = 686\text{ N}$ **Ans.**

Ans:
$\theta = 54.3°$
$F_A = 686\text{ N}$

2–27.

Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$.



SOLUTION

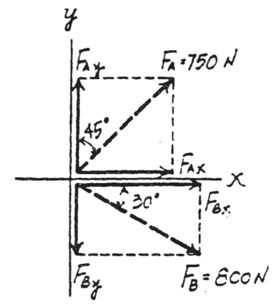
Scalar Notation: Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 750 \sin 45° + 800 \cos 30°$$

$$= 1223.15 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 750 \cos 45° - 800 \sin 30°$$

$$= 130.33 \text{ N} \uparrow$$



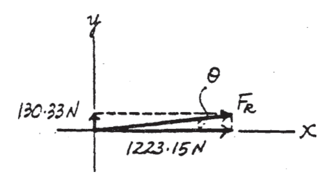
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \qquad \textbf{Ans.}$$

The directional angle θ measured counterclockwise from positive x axis is

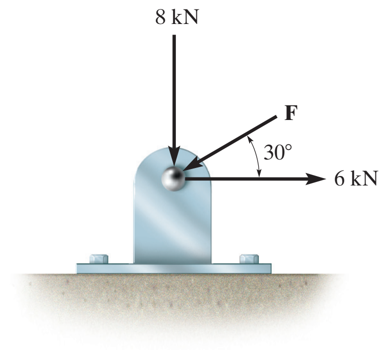
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08° \qquad \textbf{Ans.}$$



Ans:
$F_R = 1.23$ kN
$\theta = 6.08°$

***2–28.**

Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?**.**



SOLUTION

Parallelogram Law. The parallelogram laws of addition for 6 kN and 8 kN and then their resultant F' and F are shown in Figs. *a* and *b*, respectively. In order for F_R to be minimum, it must act perpendicular to **F**.

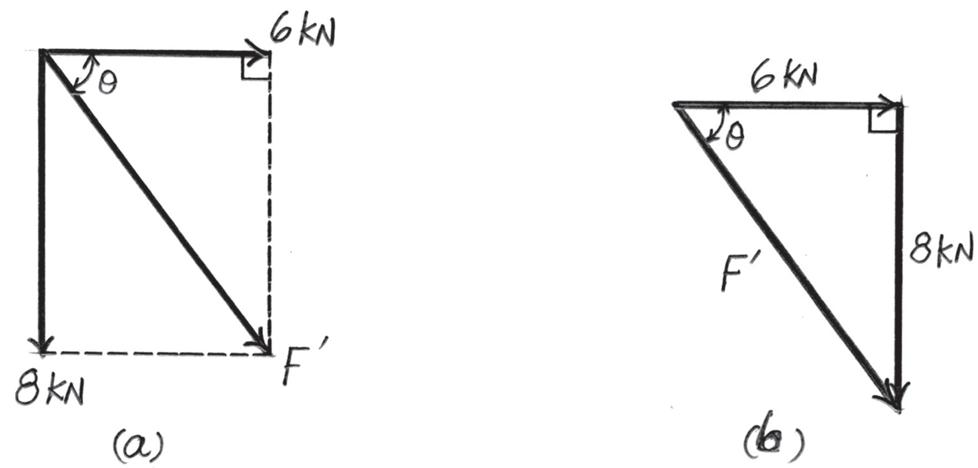
Trigonometry. Referring to Fig. *b*,

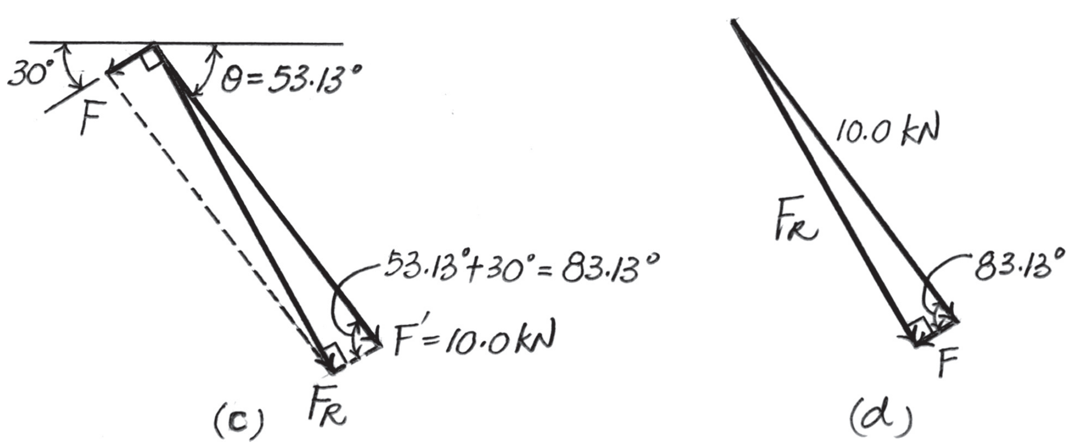
$$F' = \sqrt{6^2 + 8^2} = 10.0 \text{ kN} \qquad \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°.$$

Referring to Figs. *c* and *d*,

$$F_R = 10.0 \sin 83.13° = 9.928 \text{ kN} = 9.93 \text{ kN} \qquad \textbf{Ans.}$$

$$F = 10.0 \cos 83.13° = 1.196 \text{ kN} = 1.20 \text{ kN} \qquad \textbf{Ans.}$$

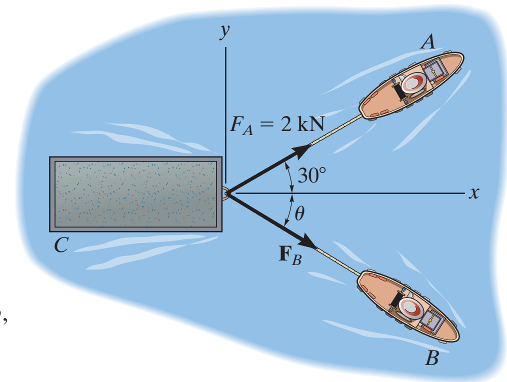




Ans:
$F_R = 9.93$ kN
$F = 1.20$ kN

2–29.

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

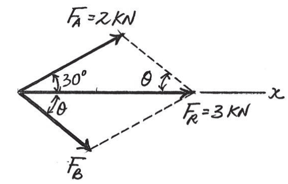
Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

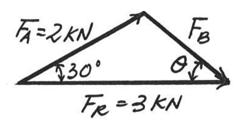
$$= 1.615\text{kN} = 1.61 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ \qquad \textbf{Ans.}$$



(a)

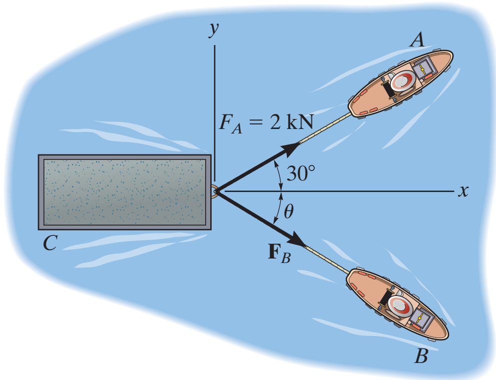


(b)

Ans:
$F_B = 1.61$ kN
$\theta = 38.3^\circ$

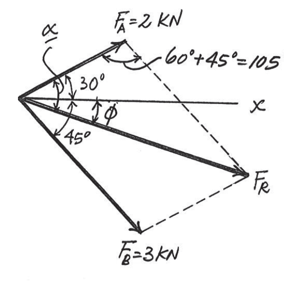
2–30.

If $F_B = 3$ kN and $\theta = 45^\circ$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.



SOLUTION

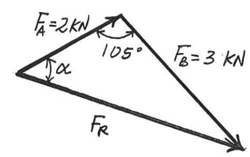
The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105^\circ}$$

$$= 4.013 \text{ kN} = 4.01 \text{ kN} \qquad \textbf{Ans.}$$



Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\alpha}{3} = \frac{\sin 105^\circ}{4.013} \qquad \alpha = 46.22^\circ$$

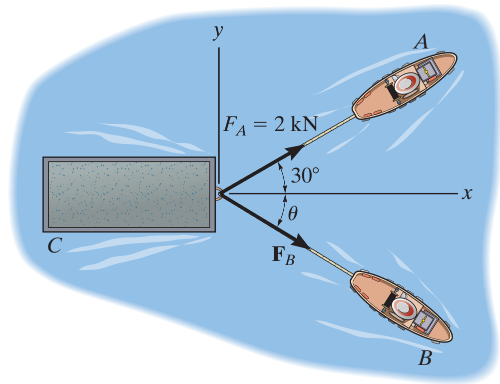
Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\phi = \alpha - 30^\circ = 46.22^\circ - 30^\circ = 16.2^\circ \qquad \textbf{Ans.}$$

Ans:
$F_R = 4.01$ kN
$\phi = 16.2^\circ$

2–31.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and F_B is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.



SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

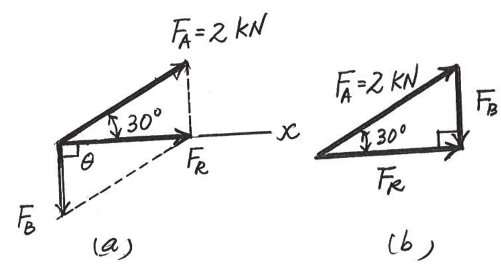
$$\theta = 90^\circ \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

By applying simple trigonometry to Fig. *b*,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN} \qquad \textbf{Ans.}$$

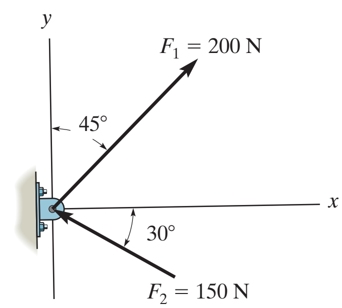
$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$\theta = 90^\circ$
$F_B = 1$ kN
$F_R = 1.73$ kN

***2–32.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 \sin 45^\circ - 150 \cos 30^\circ = 11.518 \text{ N} \rightarrow$$

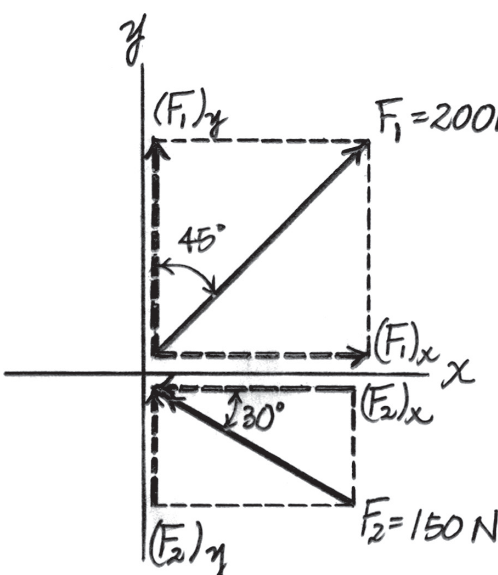
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 200 \cos 45^\circ + 150 \sin 30^\circ = 216.42 \text{ N} \uparrow$$

Referring to Fig. b, the magnitude of the resultant force F_R is

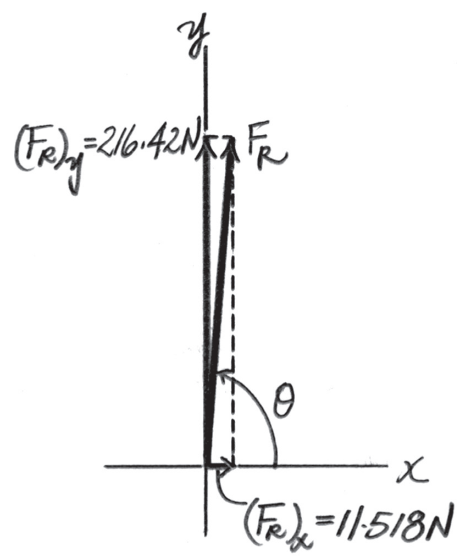
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{11.518^2 + 216.42^2} = 216.73 \text{ N} = 217 \text{ N} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{216.42}{11.518}\right) = 86.95^\circ = 87.0^\circ \qquad \textbf{Ans.}$$



(a)

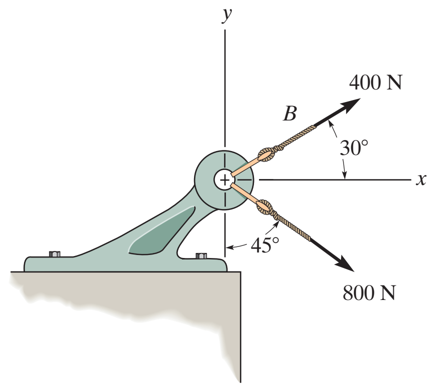


(b)

Ans:
$F_R = 217$ N
$\theta = 87.0^\circ$

2–33.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 400 \cos 30° + 800 \sin 45° = 912.10 \text{ N} \rightarrow$$

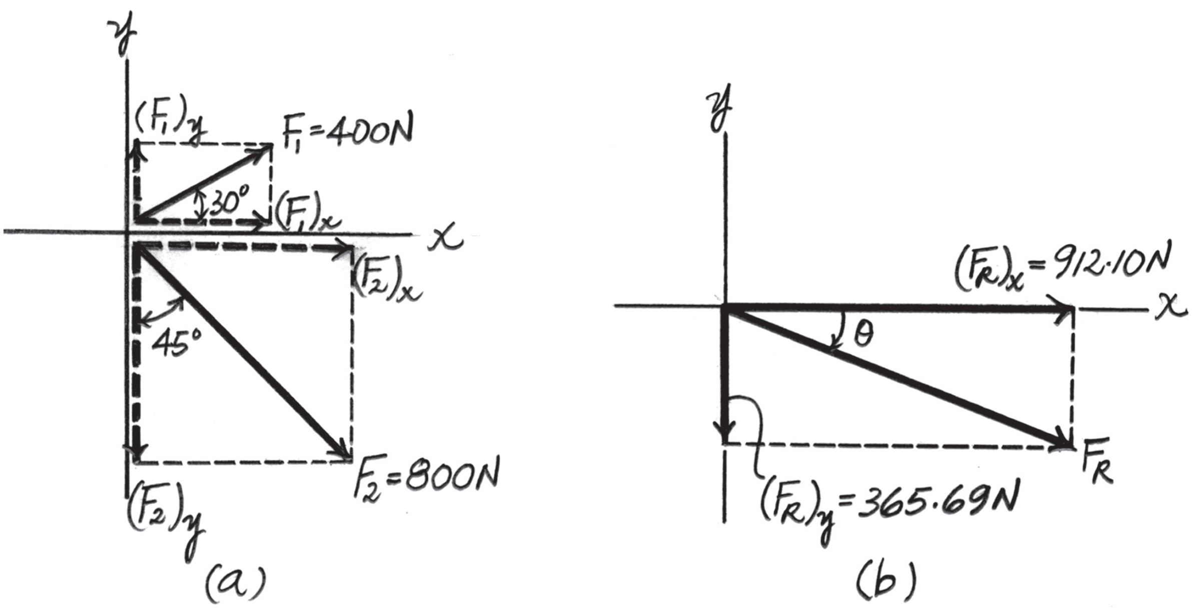
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 400 \sin 30° - 800 \cos 45° = -365.69 \text{ N} = 365.69 \text{ N}\downarrow$$

Referring to Fig. b, the magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{912.10^2 + 365.69^2} = 982.67 \text{ N} = 983 \text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

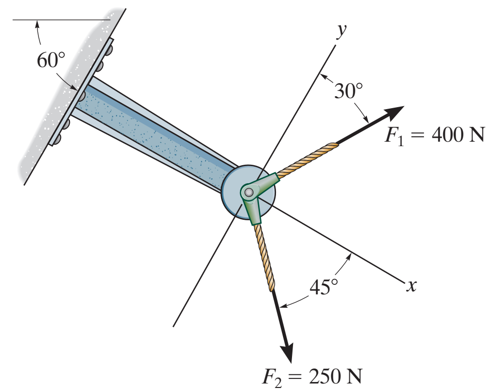
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{365.69}{912.10}\right) = 21.84° = 21.8° \qquad \textbf{Ans.}$$



Ans:
$F_R = 983$ N
$\theta = 21.8°$

2–34.

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



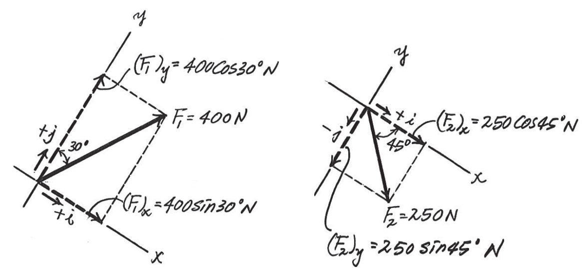
SOLUTION

$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\} \text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\} \text{ N}$$

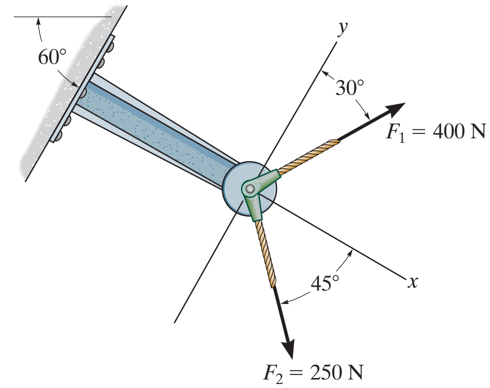
$$= \{177\mathbf{i} - 177\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$



Ans:
$\mathbf{F}_1 = \{200\mathbf{i} + 346\mathbf{j}\}$ N
$\mathbf{F}_2 = \{177\mathbf{i} - 177\mathbf{j}\}$ N

2–35.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

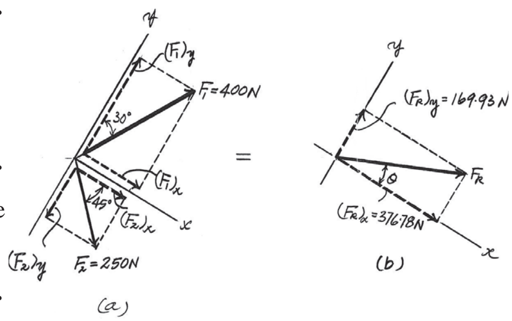
$$(F_1)_x = 400 \sin 30° = 200 \text{ N} \qquad (F_1)_y = 400 \cos 30° = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45° = 176.78 \text{ N} \qquad (F_2)_y = 250 \sin 45° = 176.78 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 + 176.78 = 376.78 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$



The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N} \qquad \textbf{Ans.}$$

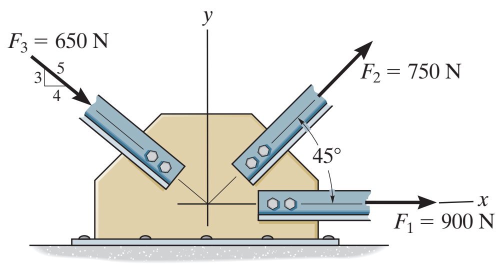
The direction angle θ of $\mathbf{F}_R$, Fig. b, measured counterclockwise from the positive axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{169.63}{376.78}\right) = 24.2° \qquad \textbf{Ans.}$$

Ans:
$F_R = 413$ N
$\theta = 24.2°$

***2–36.**

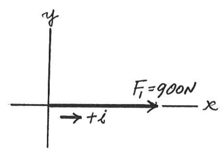
Resolve each force acting on the *gusset plate* into its x and y components, and express each force as a Cartesian vector.

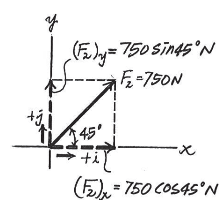


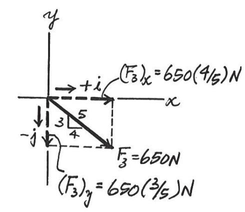
$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = \{750 \cos 45°(+\mathbf{i}) + 750 \sin 45°(+\mathbf{j})\}\text{ N}$$
$$= \{530\mathbf{i} + 530\mathbf{j}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\text{ N}$$
$$= \{520\,\mathbf{i} - 390\mathbf{j})\}\text{ N}$$ **Ans.**



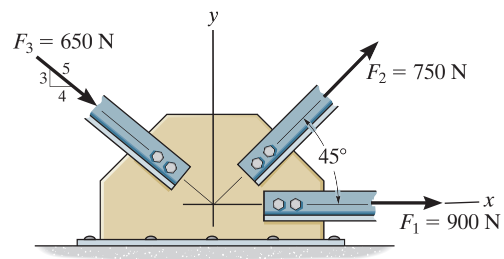




Ans:
$\mathbf{F}_1 = \{900\mathbf{i}\}$ N
$\mathbf{F}_2 = \{530\mathbf{i} + 530\mathbf{j}\}$ N
$\mathbf{F}_3 = \{520\mathbf{i} - 390\mathbf{j}\}$ N

2–37.

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900\text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750\cos 45^\circ = 530.33\text{ N} \qquad (F_2)_y = 750\sin 45^\circ = 530.33\text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520\text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33\text{ N} \rightarrow$$

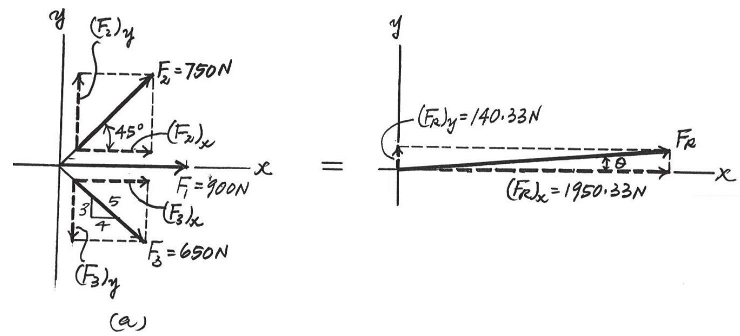
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33\text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955\text{ N} = 1.96\text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12^\circ \qquad \textbf{Ans.}$$

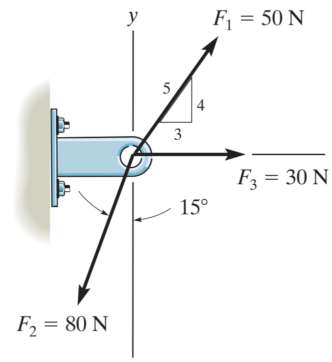


(a)

Ans:
$F_R = 1.96$ kN
$\theta = 4.12^\circ$

2–38.

Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



SOLUTION

Cartesian Notation. Referring to Fig. a,

$$\mathbf{F}_1 = (F_1)_x\,\mathbf{i} + (F_1)_y\,\mathbf{j} = 50\left(\frac{3}{5}\right)\mathbf{i} + 50\left(\frac{4}{5}\right)\mathbf{j} = \{30\,\mathbf{i} + 40\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -(F_2)_x\,\mathbf{i} - (F_2)_y\,\mathbf{j} = -80\sin 15°\,\mathbf{i} - 80\cos 15°\,\mathbf{j}$$

$$= \{-20.71\,\mathbf{i} - 77.27\,\mathbf{j}\}\text{ N}$$

$$= \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$F_3 = (F_3)_x\,\mathbf{i} = \{30\,\mathbf{i}\} \qquad \textbf{Ans.}$$

Thus, the resultant force is

$$\mathbf{F}_R = \Sigma\mathbf{F}\,; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (30\mathbf{i} + 40\,\mathbf{j}) + (-20.71\mathbf{i} - 77.27\mathbf{j}) + 30\mathbf{i}$$

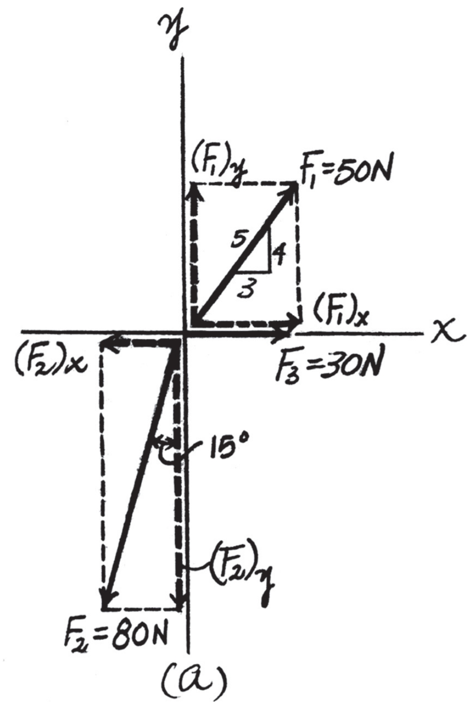
$$= \{39.29\,\mathbf{i} - 37.27\,\mathbf{j}\}\text{ N}$$

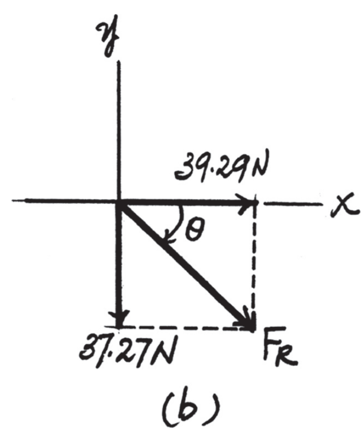
Referring to Fig. b, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{39.29^2 + 37.27^2} = 54.16\text{ N} = 54.2\text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left(\frac{37.27}{39.29}\right) = 43.49° - 43.5° \qquad \textbf{Ans.}$$

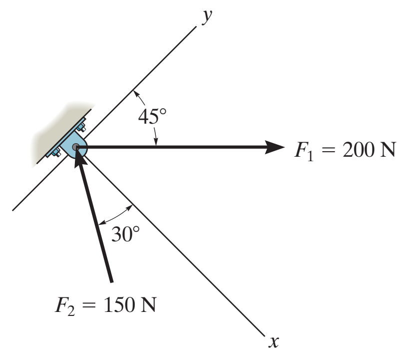




Ans:
$\mathbf{F}_1 = \{30\mathbf{i} + 40\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}$ N
$F_3 = \{30\,\mathbf{i}\}$
$F_R = 54.2$ N
$\theta = 43.5°$

2–39.

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

$F_{1x} = 200 \sin 45° = 141 \text{ N}$ **Ans.**

$F_{1y} = 200 \cos 45° = 141 \text{ N}$ **Ans.**

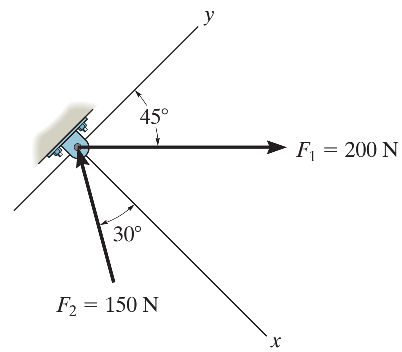
$F_{2x} = -150 \cos 30° = -130 \text{ N}$ **Ans.**

$F_{2y} = 150 \sin 30° = 75 \text{ N}$ **Ans.**

Ans:
$F_{1x} = 141$ N
$F_{1y} = 141$ N
$F_{2x} = -130$ N
$F_{2y} = 75$ N

***2–40.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$+\searrow F_{Rx} = \Sigma F_x;\qquad F_{Rx} = -150\cos 30° + 200\sin 45° = 11.518\text{ N}$

$\nearrow + F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 150\sin 30° + 200\cos 45° = 216.421\text{ N}$

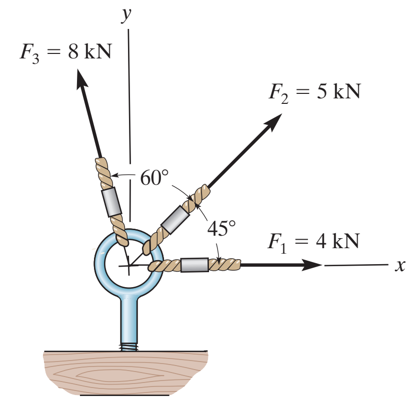
$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217\text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{216.421}{11.518}\right) = 87.0°$ **Ans.**

Ans:
$F_R = 217$ N
$\theta = 87.0°$

2–41.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 4 + 5\cos 45° - 8\sin 15° = 5.465 \text{ kN} \rightarrow$$

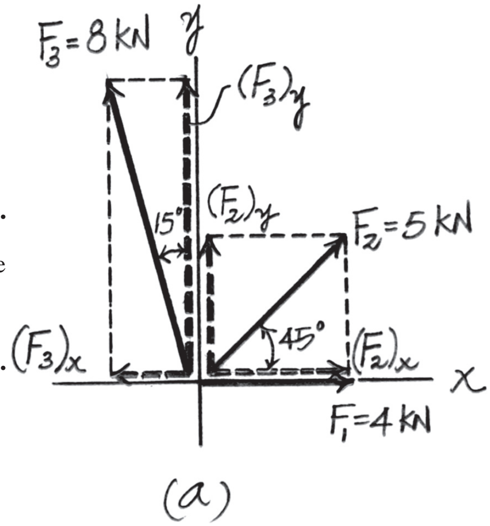
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 5\sin 45° + 8\cos 15° = 11.263 \text{ kN} \uparrow$$

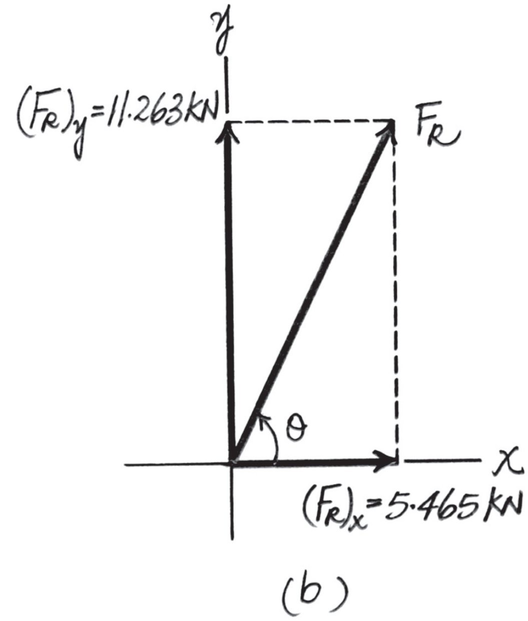
By referring to Fig. b, the magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5.465^2 + 11.263^2} = 12.52 \text{ kN} = 12.5 \text{ kN} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{11.263}{5.465}\right) = 64.12° = 64.1° \qquad \textbf{Ans.}$$

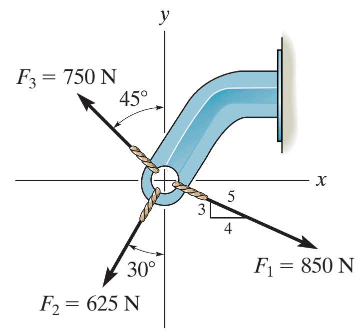




Ans:
$F_R = 12.5$ kN
$\theta = 64.1°$

2–42.

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = \frac{4}{5}(850)\,\mathbf{i} - \frac{3}{5}(850)\,\mathbf{j}$$

$$= \{680\,\mathbf{i} - 510\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -625\sin 30°\,\mathbf{i} - 625\cos 30°\,\mathbf{j}$$

$$= \{-312\,\mathbf{i} - 541\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

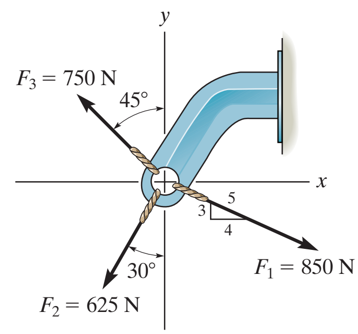
$$\mathbf{F}_3 = -750\sin 45°\,\mathbf{i} + 750\cos 45°\,\mathbf{j}$$

$$= \{-530\,\mathbf{i} + 530\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{680\mathbf{i} - 510\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-312\mathbf{i} - 541\mathbf{j}\}$ N
$\mathbf{F}_3 = \{-530\mathbf{i} + 530\mathbf{j}\}$ N

2–43.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

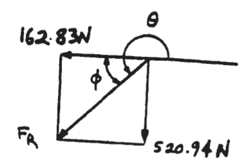
$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.83$ N

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.94$ N

$$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546 \text{ N} \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64°$$

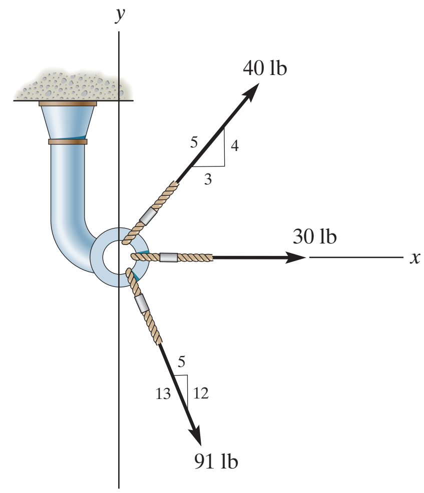
$$\theta = 180° + 72.64° = 253° \qquad \textbf{Ans.}$$



Ans:
$F_R = 546$ N
$\theta = 253°$

***2–44.**

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 40\left(\frac{3}{5}\right) + 91\left(\frac{5}{13}\right) + 30 = 89 \text{ lb} \rightarrow$$

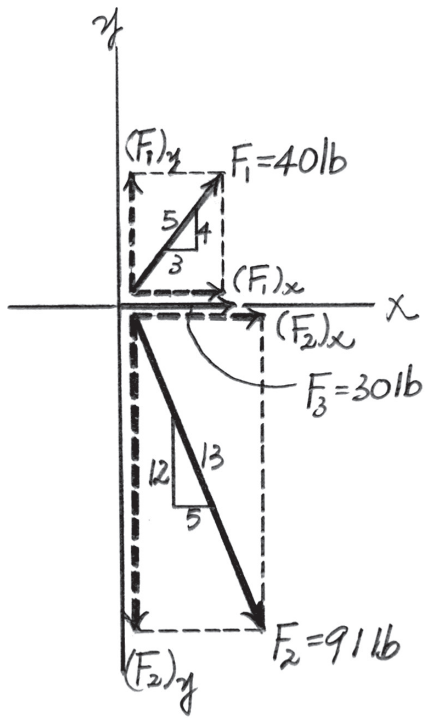
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 40\left(\frac{4}{5}\right) - 91\left(\frac{12}{13}\right) = -52 \text{ lb} = 52 \text{ lb}\downarrow$$

By referring to Fig. b, the magnitude of resultant force is

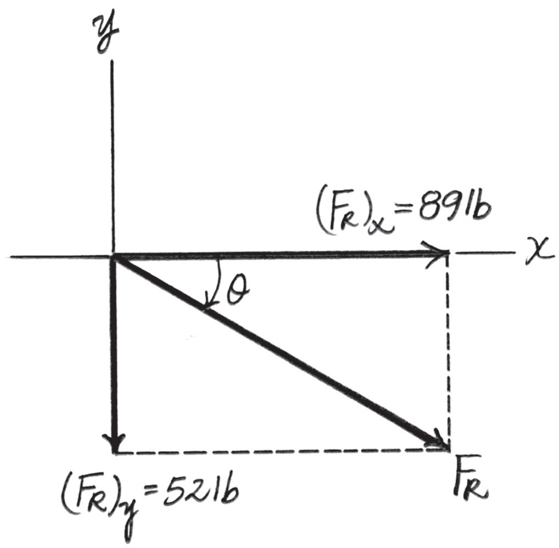
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{89^2 + 52^2} = 103.08 \text{ lb} = 103 \text{ lb} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{52}{89}\right) = 30.30° = 30.3° \qquad \textbf{Ans.}$$



(a)



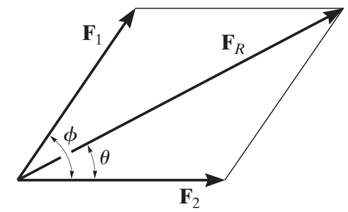
(b)

Ans:
$F_R = 103$ lb
$\theta = 30.3°$

2–45.

Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.

SOLUTION

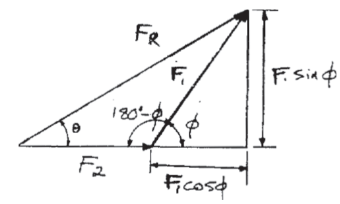


$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180^\circ - \phi)$$

Since $\cos(180^\circ - \phi) = -\cos\phi$,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi} \qquad \textbf{Ans.}$$

From the figure,



$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right) \qquad \textbf{Ans.}$$

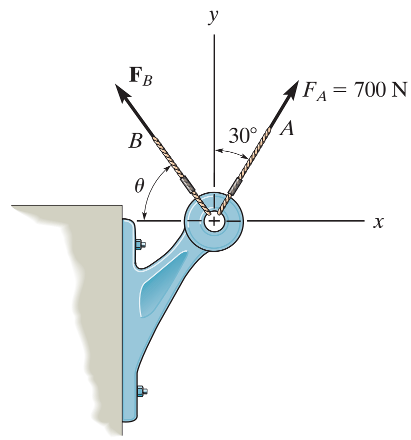
Ans:

$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$

$\theta = \tan^{-1}\left(\dfrac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$

2–46.

Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



SOLUTION

Scalar Notation: Suming the force components algebraically, we have

$\overset{+}{\rightarrow} F_{R_z} = \Sigma F_x;$ $\quad 0 = 700 \sin 30° - F_B \cos\theta$

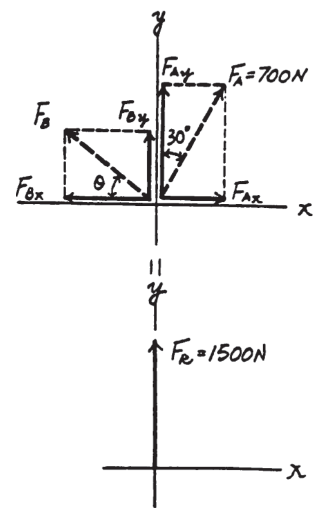
$$F_B \cos\theta = 350 \qquad \textbf{(1)}$$

$+\uparrow F_{R_y} = \Sigma F_y;$ $\quad 1500 = 700 \cos 30° + F_B \sin\theta$

$$F_B \sin\theta = 893.8 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields

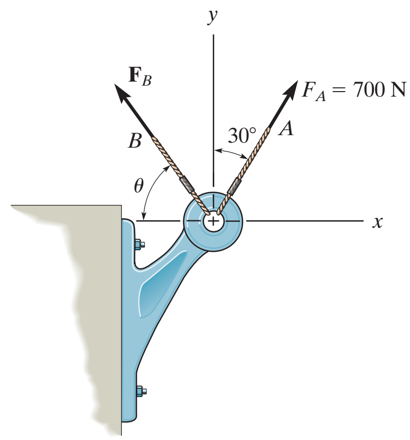
$$\theta = 68.6° \qquad F_B = 960 \text{ N}$$ **Ans.**



Ans:
$\theta = 68.6°$
$F_B = 960$ N

2–47.

Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.



SOLUTION

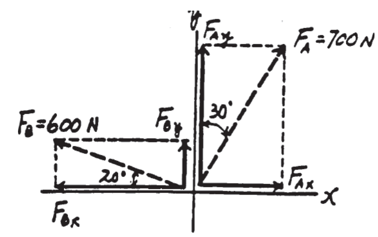
Scalar Notation: Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 700 \sin 30° - 600 \cos 20°$$

$$= -213.8 \text{ N} = 213.8 \text{ N} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 700 \cos 30° + 600 \sin 20°$$

$$= 811.4 \text{ N} \uparrow$$

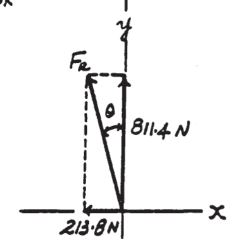


The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N} \qquad \textbf{Ans.}$$

The directional angle θ measured counterclockwise from positive y axis is

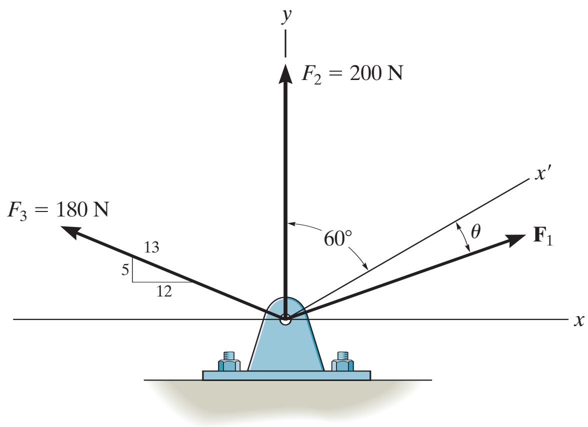
$$\theta = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \textbf{Ans.}$$



Ans:
$F_R = 839$ N
$\theta = 14.8°$

***2–48.**

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 800 N.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad 800 \sin 60° = F_1 \sin(60° + \theta) - \frac{12}{13}(180)$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad 800 \cos 60° = F_1 \cos(60° + \theta) + 200 + \frac{5}{13}(180)$

$60° + \theta = 81.34°$

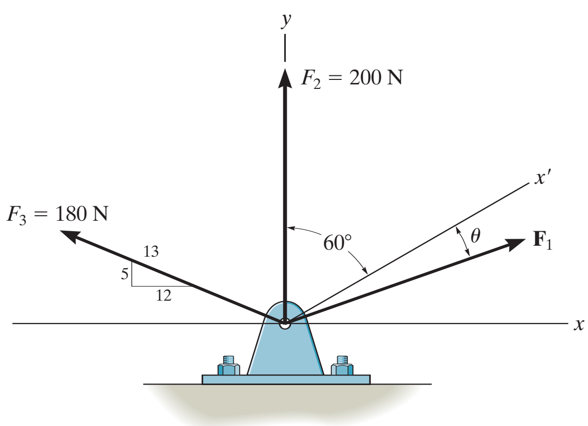
$\theta = 21.3°$ **Ans.**

$F_1 = 869\text{ N}$ **Ans.**

Ans:
$\theta = 21.3°$
$F_1 = 869\text{ N}$

2–49.

If $F_1 = 300$ N and $\theta = 10°$, determine the magnitude and direction, measured counterclockwise from the positive x' axis, of the resultant force acting on the bracket.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma Fx;\qquad F_{Rx} = 300 \sin 70° - \frac{12}{13}(180) = 115.8 \text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 300 \cos 70° + 200 + \frac{5}{13}(180) = 371.8 \text{ N}$

$$F_R = \sqrt{(115.8)^2 + (371.8)^2} = 389 \text{ N}$$ **Ans.**

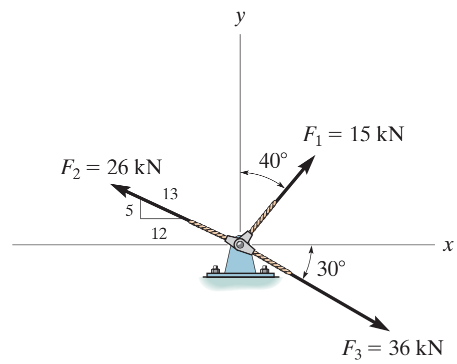
$$\phi = \tan^{-1}\left[\frac{371.8}{115.8}\right] = 72.71° \quad \measuredangle\theta$$

$$\phi' = 72.71° - 30° = 42.7°$$ **Ans.**

Ans:
$F_R = 389$ N
$\phi' = 42.7°$

2–50.

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$\mathbf{F}_1 = \{-200\,\mathbf{i}\}$ lb **Ans.**

$\mathbf{F}_2 = -250 \sin 30°\,\mathbf{i} + 250 \cos 30°\,\mathbf{j}$

$= \{-125\,\mathbf{i} + 217\,\mathbf{j}\}$ lb **Ans.**

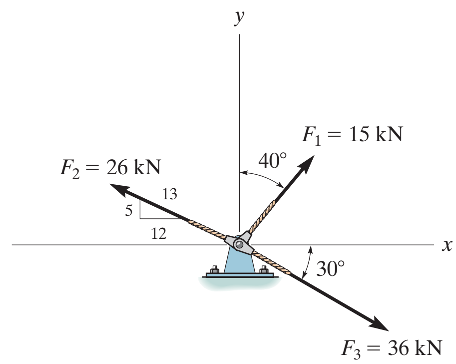
$\mathbf{F}_3 = 225 \cos 30°\,\mathbf{i} + 225 \sin 30°\,\mathbf{j}$

$= \{195\,\mathbf{i} + 112\,\mathbf{j}\}$ lb **Ans.**

Ans:
$\mathbf{F}_1 = \{-200\mathbf{i}\}$ lb
$\mathbf{F}_2 = \{-125\mathbf{i} + 217\mathbf{j}\}$ lb
$\mathbf{F}_3 = \{195\mathbf{i} + 112\mathbf{j}\}$ lb

2–51.

Determine the magnitude of the resultant force and its orientation measured counterclockwise from the positive x axis.



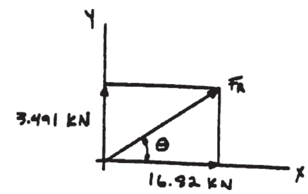
SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = 15 \sin 40° - \frac{12}{13}(26) + 36 \cos 30° = 16.82 \text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = 15 \cos 40° + \frac{5}{13}(26) - 36 \sin 30° = 3.491 \text{ kN}$

$$F_R = \sqrt{(16.82)^2 + (3.491)^2} = 17.2 \text{ kN}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{3.491}{16.82}\right) = 11.7°$$ **Ans.**



Also,

$\mathbf{F}_1 = \{15 \sin 40° \, \mathbf{i} + 15 \cos 40° \, \mathbf{j}\} \text{ kN} = \{9.64\mathbf{i} + 11.5\mathbf{j}\} \text{ kN}$

$\mathbf{F}_2 = \left\{-\frac{12}{13}(26)\mathbf{i} + \frac{5}{13}(26)\mathbf{j}\right\} \text{ kN} = \{-24\mathbf{i} + 10\mathbf{j}\} \text{ kN}$

$\mathbf{F}_3 = \{36 \cos 30°\mathbf{i} - 36 \sin 30°\mathbf{j}\} \text{ kN} = \{31.2\mathbf{i} - 18\mathbf{j}\} \text{ kN}$

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

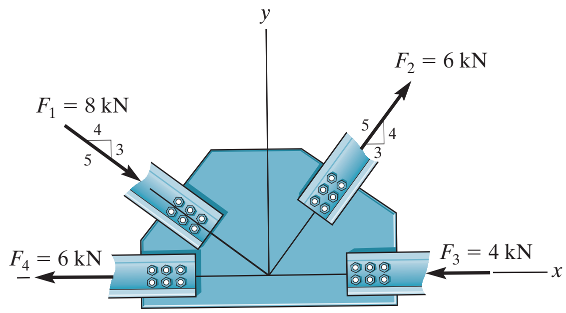
$\quad = \{9.64\mathbf{i} + 11.5\mathbf{j}\} + \{-24\mathbf{i} + 10\mathbf{j}\} + \{31.2\mathbf{i} - 18\mathbf{j}\}$

$\quad = \{16.8\mathbf{i} + 3.49\mathbf{j}\} \text{ kN}$

Ans:
$F_R = 17.2 \text{ kN}, \theta = 11.7°$

***2–52.**

Determine the x and y components of each force acting on the *gusset plate* of a bridge truss. Show that the resultant force is zero.



SOLUTION

Scalar Notation. Referring to Fig. a, the x and y components of each forces are

$$(F_1)_x = 8\left(\frac{4}{5}\right) = 6.40 \text{ kN} \rightarrow$$ **Ans.**

$$(F_1)_y = 8\left(\frac{3}{5}\right) = 4.80 \text{ kN} \downarrow$$ **Ans.**

$$(F_2)_x = 6\left(\frac{3}{5}\right) = 3.60 \text{ kN} \rightarrow$$ **Ans.**

$$(F_2)_y = 6\left(\frac{4}{5}\right) = 4.80 \text{ kN} \uparrow$$ **Ans.**

$$(F_3)_x = 4 \text{ kN} \leftarrow$$ **Ans.**

$$(F_3)_y = 0$$ **Ans.**

$$(F_4)_x = 6 \text{ kN} \leftarrow$$ **Ans.**

$$(F_4)_y = 0$$ **Ans.**

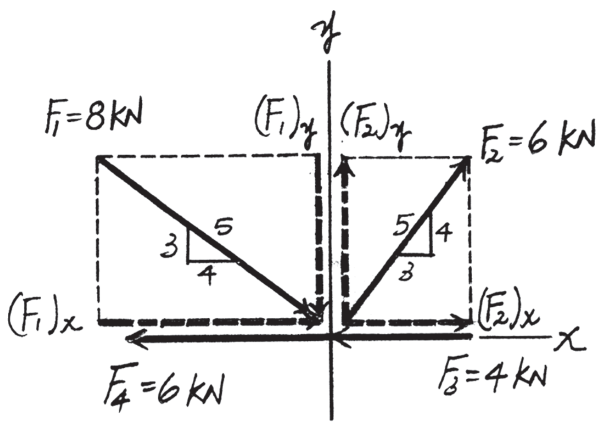
Summing these force components along x and y axes algebraically,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 6.40 + 3.60 - 4 - 6 = 0$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 4.80 - 4.80 = 0$$

Thus,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{O^2 + O^2} = O \qquad \textbf{(Q.E.D)}$$

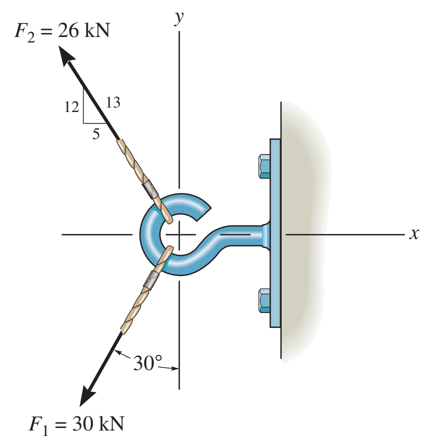


(a)

Ans:
$(F_1)_x = 6.40 \text{ kN} \rightarrow$
$(F_1)_y = 4.80 \text{ kN} \downarrow$
$(F_2)_x = 3.60 \text{ kN} \rightarrow$
$(F_2)_y = 4.80 \text{ kN} \uparrow$
$(F_3)_x = 4 \text{ kN} \leftarrow$
$(F_3)_y = 0$
$(F_4)_x = 6 \text{ kN} \leftarrow$
$(F_4)_y = 0$

2–53.

Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = -30 \sin 30° \,\mathbf{i} - 30 \cos 30° \,\mathbf{j}$$

$$= \{-15.0\,\mathbf{i} - 26.0\,\mathbf{j}\} \text{ kN} \qquad \textbf{Ans.}$$

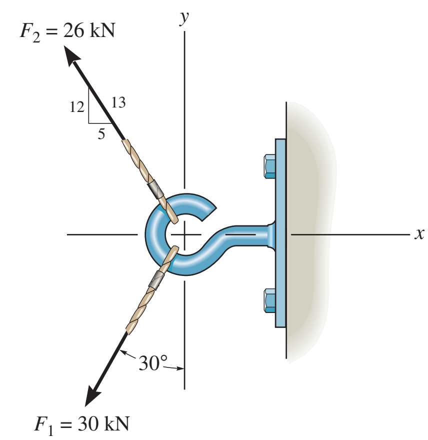
$$\mathbf{F}_2 = -\frac{5}{13}(26)\,\mathbf{i} + \frac{12}{13}(26)\,\mathbf{j}$$

$$= \{-10.0\,\mathbf{i} + 24.0\,\mathbf{j}\} \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{-15.0\mathbf{i} - 26.0\mathbf{j}\}$ kN
$\mathbf{F}_2 = \{-10.0\mathbf{i} + 24.0\mathbf{j}\}$ kN

2–54.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

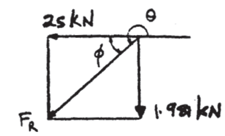
$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -30 \sin 30^\circ - \frac{5}{13}(26) = -25 \text{ kN}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -30 \cos 30^\circ + \frac{12}{13}(26) = -1.981 \text{ kN}$$

$$F_R = \sqrt{(-25)^2 + (-1.981)^2} = 25.1 \text{ kN} \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{1.981}{25}\right) = 4.53^\circ$$

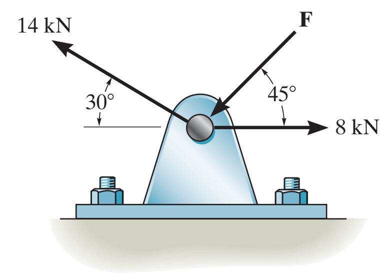
$$\theta = 180^\circ + 4.53^\circ = 185^\circ \qquad \textbf{Ans.}$$



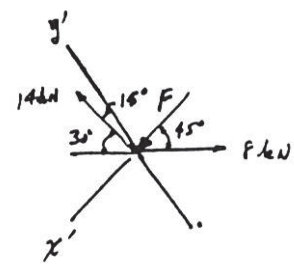
Ans:
$F_R = 25.1$ kN
$\theta = 185^\circ$

2–55.

Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



SOLUTION



$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rz} = 8 - F\cos 45° - 14\cos 30°$

$$= -4.1244 - F\cos 45°$$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -F\sin 45° + 14\sin 30°$

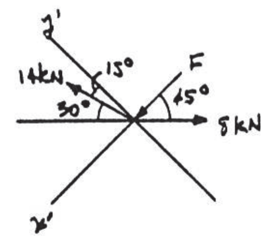
$$= 7 - F\sin 45°$$

$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad \textbf{(1)}$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$F = 2.03\text{ kN}$ **Ans.**

From Eq. (1); $\quad F_R = 7.87\text{ kN}$ **Ans.**



Also, from the figure require

$(F_R)_{x'} = 0 = \Sigma F_{x'};$ $\quad F + 14\sin 15° - 8\cos 45° = 0$

$F = 2.03\text{ kN}$ **Ans.**

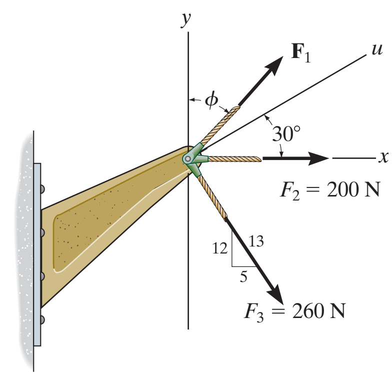
$(F_R)_{y'} = \Sigma F_{y'};$ $\quad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87\text{ kN}$ **Ans.**

Ans:
$F = 2.03\text{ kN}$
$F_R = 7.87\text{ kN}$

***2–56.**

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \sin\phi \qquad (F_1)_y = F_1 \cos\phi$

$(F_2)_x = 200 \text{ N} \qquad (F_2)_y = 0$

$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ N}$

$(F_R)_x = 450\cos 30^\circ = 389.71 \text{ N} \qquad (F_R)_y = 450\sin 30^\circ = 225 \text{ N}$

Resultant Force: Summing the force components algebraically along the x and y axes,

$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x;\quad 389.71 = F_1\sin\phi + 200 + 100$

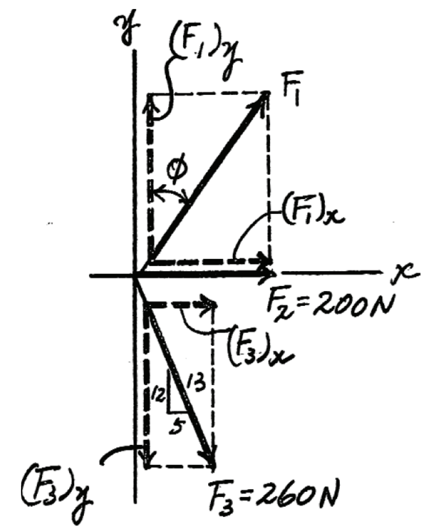
$$F_1\sin\phi = 89.71 \qquad \textbf{(1)}$$

$+\uparrow\Sigma(F_R)_y = \Sigma F_y;\quad 225 = F_1\cos\phi - 240$

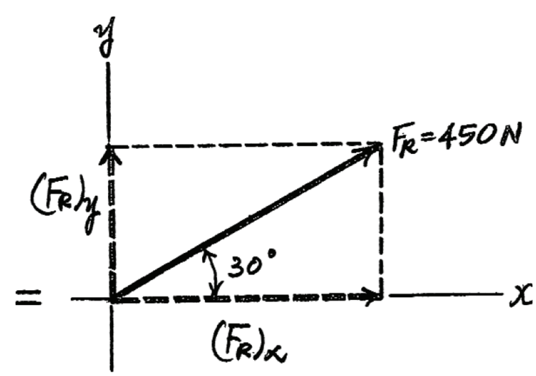
$$F_1\cos\phi = 465 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$\phi = 10.9^\circ \qquad F_1 = 474 \text{ N} \qquad \textbf{Ans.}$$



(a)

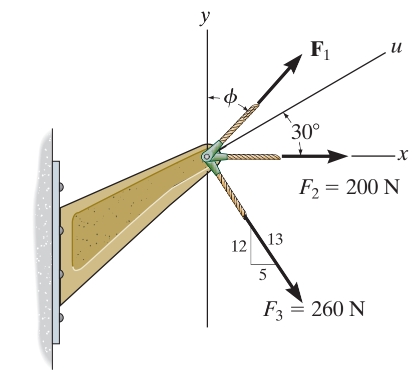


(b)

Ans:
$\phi = 10.9^\circ$
$F_1 = 474$ N

2–57.

If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \sin 30° = 0.5F_1 \qquad (F_1)_y = F_1 \cos 30° = 0.8660F_1$$

$$(F_2)_x = 200 \text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 0.5F_1 + 200 + 100 = 0.5F_1 + 300$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 0.8660F_1 - 240$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2}$$

$$= \sqrt{F_1^2 - 115.69F_1 + 147\,600} \qquad \textbf{(1)}$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147\,600 \qquad \textbf{(2)}$$

The first derivative of Eq. (2) is

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad \textbf{(3)}$$

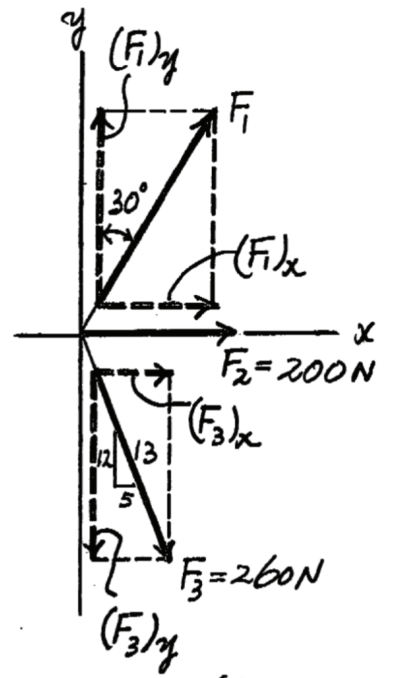
For $\mathbf{F}_R$ to be minimum, $\dfrac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

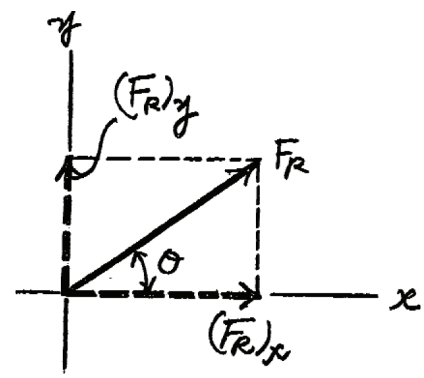
$$F_1 = 57.846 \text{ N} = 57.8 \text{ N} \qquad \textbf{Ans.}$$

from Eq. (1),

$$F_R = \sqrt{(57.846)^2 - 115.69(57.846) + 147\,600} = 380 \text{ N} \qquad \textbf{Ans.}$$



(a)

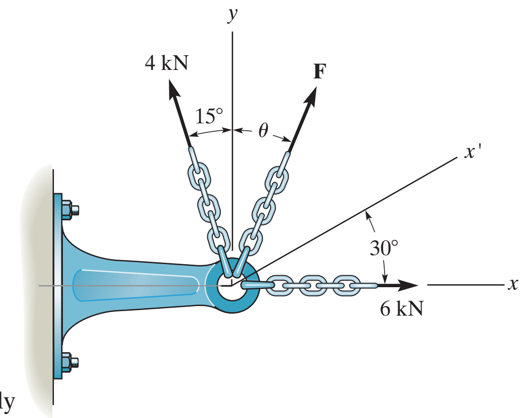


(b)

Ans:
$F_R = 380$ N
$F_1 = 57.8$ N

2–58.

Three forces act on the bracket. Determine the magnitude and direction θ of **F** so that the resultant force is directed along the positive x' axis and has a magnitude of 8 kN.



SOLUTION

Scalar Notation. Equating the force components along the x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad 8 \cos 30° = F \sin \theta + 6 - 4 \sin 15°$$

$$F \sin \theta = 1.9635 \qquad (1)$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad 8 \sin 30° = F \cos \theta + 4 \cos 15°$$

$$F \cos \theta = 0.1363 \qquad (2)$$

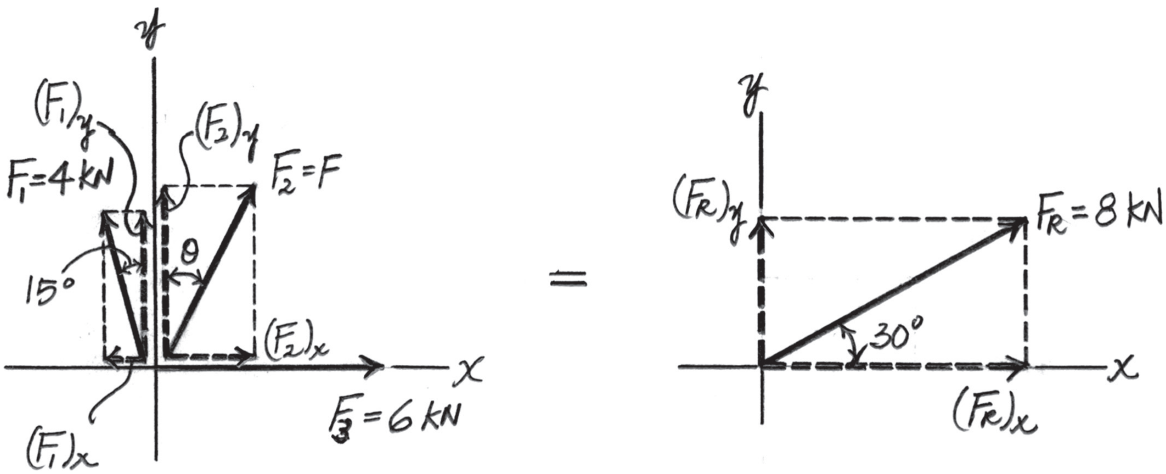
Divide Eq (1) by (2)

$$\tan \theta = 14.406 \qquad \theta = 86.03° = 86.0° \qquad \textbf{Ans.}$$

Substitute this result into Eq (1)

$$F \sin 86.03° = 1.9635$$

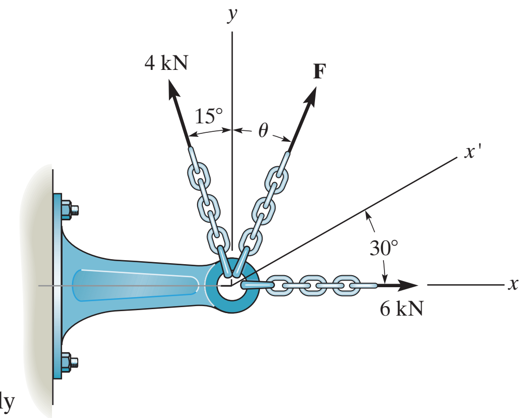
$$F = 1.968 \text{ kN} = 1.97 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$\theta = 86.0°$
$F = 1.97$ kN

2–59.

If $F = 5$ kN and $\theta = 30°$, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 5 \sin 30° + 6 - 4 \sin 15° = 7.465 \text{ kN} \rightarrow$$

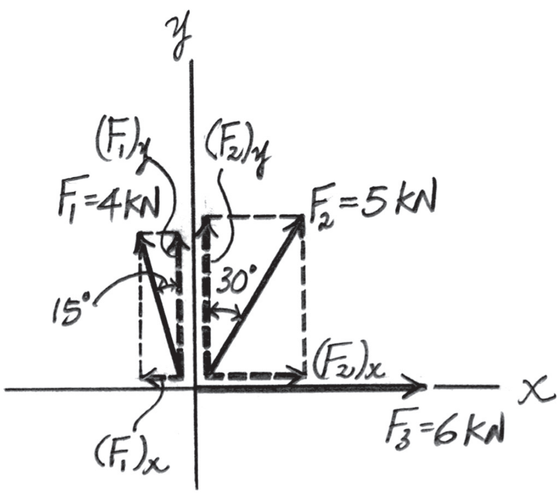
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 4 \cos 15° + 5 \cos 30° = 8.194 \text{ kN} \uparrow$$

By referring to Fig. b, the magnitude of the resultant force is

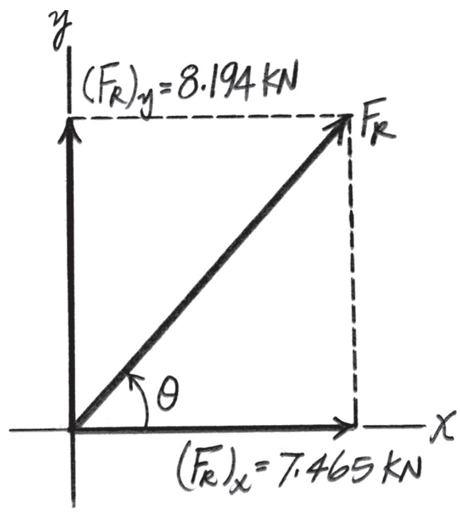
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{7.465^2 + 8.194^2} = 11.08 \text{ kN} = 11.1 \text{ kN} \quad \textbf{Ans.}$$

And its directional angle θ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{8.194}{7.465}\right) = 47.67° = 47.7° \quad \textbf{Ans.}$$



(a)

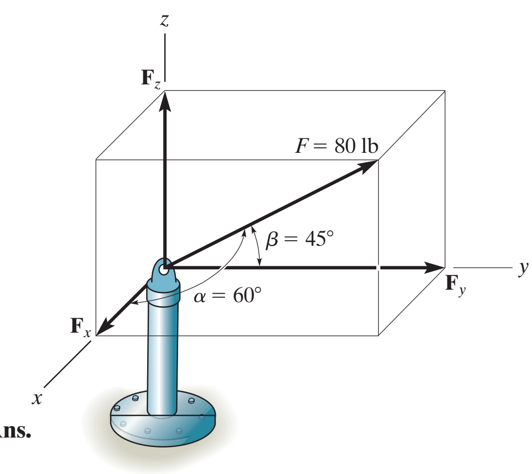


(b)

Ans:
$F_R = 11.1$ kN
$\theta = 47.7°$

***2–60.**

The force **F** has a magnitude of 80 lb and acts within the octant shown. Determine the magnitudes of the x, y, z components of **F**.



SOLUTION

$1 = \cos^2 60° + \cos^2 45° + \cos^2 \gamma$

Solving for the positive root, $\gamma = 60°$

$F_x = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

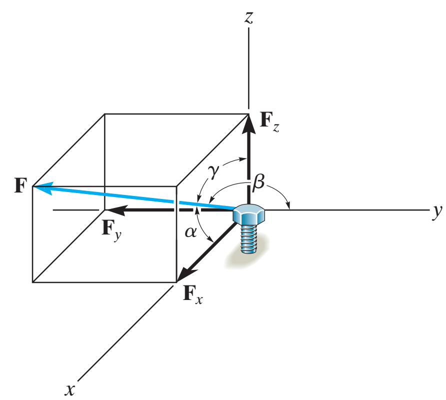
$F_y = 80 \cos 45° = 56.6 \text{ lb}$ **Ans.**

$F_z = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

Ans:
$F_x = 40.0$ lb
$F_y = 56.6$ lb
$F_z = 40.0$ lb

2–61.

The bolt is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60°$ and $\gamma = 45°$, determine the magnitudes of its components.



SOLUTION

$$\cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$\beta = 120°$$

$$F_x = |80 \cos 60°| = 40 \text{ N}$$ **Ans.**

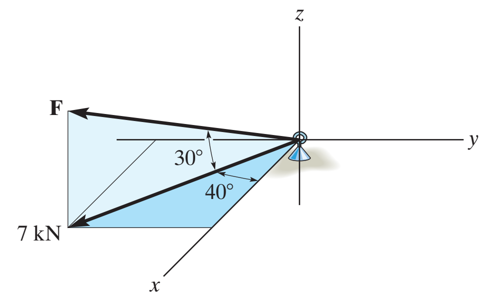
$$F_y = |80 \cos 120°| = 40 \text{ N}$$ **Ans.**

$$F_z = |80 \cos 45°| = 56.6 \text{ N}$$ **Ans.**

Ans:
$F_x = 40$ N
$F_y = 40$ N
$F_z = 56.6$ N

2–62.

Determine the magnitude and coordinate direction angles of the force **F** acting on the support. The component of **F** in the x–y plane is 7 kN.



SOLUTION

Coordinate Direction Angles. The unit vector of **F** is

$$\mathbf{u}_F = \cos 30° \cos 40° \mathbf{i} - \cos 30° \sin 40° \mathbf{j} + \sin 30° \, \mathbf{k}$$

$$= \{0.6634\mathbf{i} - 0.5567\mathbf{j} + 0.5\,\mathbf{k}\}$$

Thus,

$\cos \alpha = 0.6634; \quad \alpha = 48.44° = 48.4°$ **Ans.**

$\cos \beta = -0.5567; \quad \beta = 123.83° = 124°$ **Ans.**

$\cos \gamma = 0.5; \quad \gamma = 60°$ **Ans.**

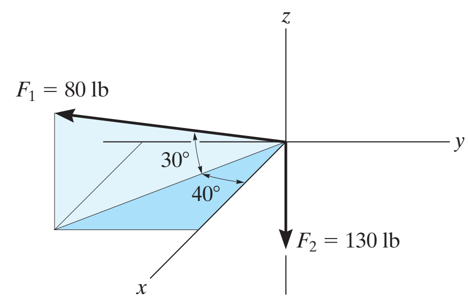
The magnitude of **F** can be determined from

$F \cos 30° = 7; \quad F = 8.083 \text{ kN} = 8.08 \text{ kN}$ **Ans.**

Ans:
$\alpha = 48.4°$
$\beta = 124°$
$\gamma = 60°$
$F = 8.08$ kN

2–63.

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40°\mathbf{i} - 80 \cos 30° \sin 40°\mathbf{j} + 80 \sin 30°\mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

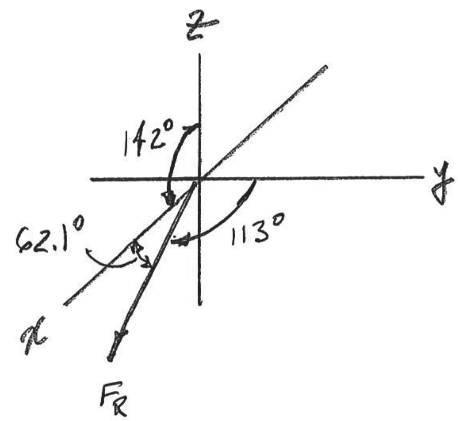
$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb

$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans.**

$\alpha = \cos^{-1}\left(\dfrac{53.1}{113.6}\right) = 62.1°$ **Ans.**

$\beta = \cos^{-1}\left(\dfrac{-44.5}{113.6}\right) = 113°$ **Ans.**

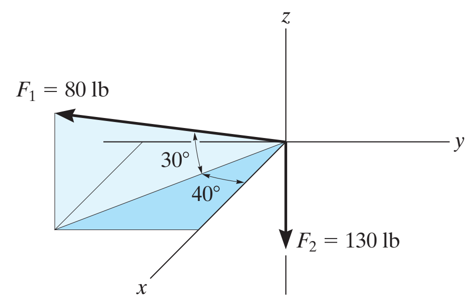
$\gamma = \cos^{-1}\left(\dfrac{-90.0}{113.6}\right) = 142°$ **Ans.**



Ans:
$F_R = 114$ lb
$\alpha = 62.1°$
$\beta = 113°$
$\gamma = 142°$

***2–64.**

Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\} \text{ lb}$$ **Ans.**

$$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$$ **Ans.**

$$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$$ **Ans.**

$$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$$ **Ans.**

$$\mathbf{F}_2 = \{-130\mathbf{k}\} \text{ lb}$$ **Ans.**

$$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$$ **Ans.**

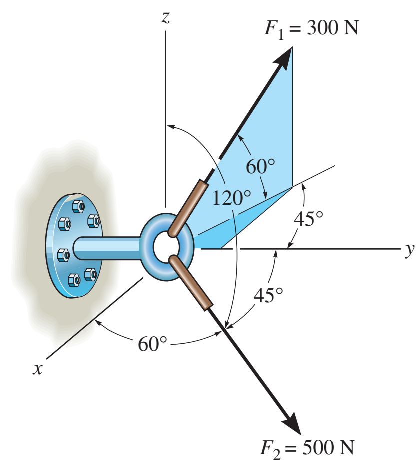
$$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$$ **Ans.**

$$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb
$\alpha_1 = 48.4°$
$\beta_1 = 124°$
$\gamma_1 = 60°$
$\mathbf{F}_2 = \{-130\text{k}\}$ lb
$\alpha_2 = 90°$
$\beta_2 = 90°$
$\gamma_2 = 180°$

2–65.

The screw eye is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$\mathbf{F}_1 = 300(-\cos 60°\ \sin 45°\,\mathbf{i} + \cos 60° \cos 45°\,\mathbf{j} + \sin 60°\,\mathbf{k})$

$= \{-106.07\,\mathbf{i} + 106.07\,\mathbf{j} + 259.81\,\mathbf{k}\}$ N

$= \{-106\,\mathbf{i} + 106\,\mathbf{j} + 260\,\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_2 = 500(\cos 60°\,\mathbf{i} + \cos 45°\,\mathbf{j} + \cos 120°\,\mathbf{k})$

$= \{250.0\,\mathbf{i} + 353.55\,\mathbf{j} - 250.0\,\mathbf{k}\}$ N

$= \{250\,\mathbf{i} + 354\,\mathbf{j} - 250\,\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= -106.07\,\mathbf{i} + 106.07\,\mathbf{j} + 259.81\,\mathbf{k} + 250.0\,\mathbf{i} + 353.55\,\mathbf{j} - 250.0\,\mathbf{k}$

$= 143.93\,\mathbf{i} + 459.62\,\mathbf{j} + 9.81\,\mathbf{k}$

$= \{144\,\mathbf{i} + 460\,\mathbf{j} + 9.81\,\mathbf{k}\}$ N **Ans.**

$F_R = \sqrt{143.93^2 + 459.62^2 + 9.81^2} = 481.73 \text{ N} = 482 \text{ N}$ **Ans.**

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}}{481.73} = 0.2988\mathbf{i} + 0.9541\mathbf{j} + 0.02036\mathbf{k}$$

$\cos\alpha = 0.2988$ $\quad \alpha = 72.6°$ **Ans.**

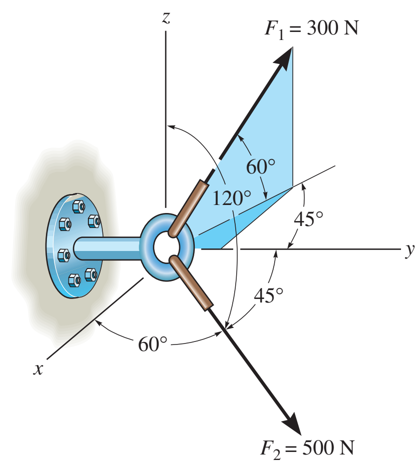
$\cos\beta = 0.9541$ $\quad \beta = 17.4°$ **Ans.**

$\cos\gamma = 0.02036$ $\quad \gamma = 88.8°$ **Ans.**

Ans:
$\mathbf{F}_1 = \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N
$\mathbf{F}_2 = \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$ N
$\mathbf{F}_R = \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$ N
$F_R = 482$ N
$\alpha = 72.6°$
$\beta = 17.4°$
$\gamma = 88.8°$

2–66.

Determine the coordinate direction angles of $\mathbf{F}_1$.



SOLUTION

$$\mathbf{F}_1 = 300(-\cos 60° \sin 45° \mathbf{i} + \cos 60° \cos 45° \mathbf{j} + \sin 60° \mathbf{k})$$

$$= \{-106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k}\} \text{ N}$$

$$= \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\} \text{ N}$$

$$\mathbf{u}_1 = \frac{\mathbf{F}_1}{300} = -0.3536\mathbf{i} + 0.3536\mathbf{j} + 0.8660\mathbf{k}$$

$\alpha_1 = \cos^{-1}(-0.3536) = 111°$ **Ans.**

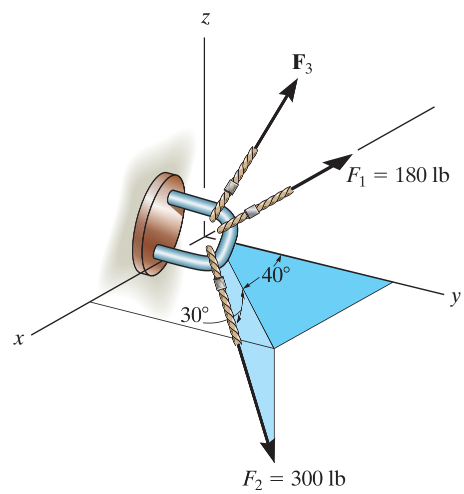
$\beta_1 = \cos^{-1}(0.3536) = 69.3°$ **Ans.**

$\gamma_1 = \cos^{-1}(0.8660) = 30.0°$ **Ans.**

Ans:
$\alpha_1 = 111°$
$\beta_1 = 69.3°$
$\gamma_1 = 30.0°$

2–67.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600 lb.



SOLUTION

$F_{Rx} = \Sigma F_x$; $\quad 0 = -180 + 300 \cos 30° \sin 40° + F_3 \cos \alpha$

$F_{Ry} = \Sigma F_y$; $\quad 600 = 300 \cos 30° \cos 40° + F_3 \cos \beta$

$F_{Rz} = \Sigma F_z$; $\quad 0 = -300 \sin 30° + F_3 \cos \gamma$

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

Solving:

$F_3 = 428$ lb **Ans.**

$\alpha = 88.3°$ **Ans.**

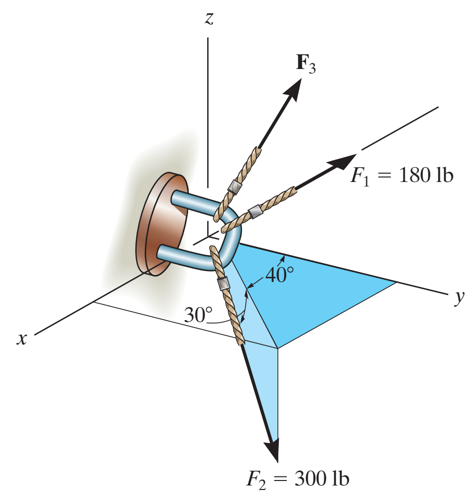
$\beta = 20.6°$ **Ans.**

$\gamma = 69.5°$ **Ans.**

Ans:
$F_3 = 428$ lb
$\alpha = 88.3°$
$\beta = 20.6°$
$\gamma = 69.5°$

***2–68.**

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



SOLUTION

$F_{Rx} = \Sigma F_x; \qquad 0 = -180 + 300 \cos 30^\circ \sin 40^\circ + F_3 \cos \alpha$

$F_{Ry} = \Sigma F_y; \qquad 0 = 300 \cos 30^\circ \cos 40^\circ + F_3 \cos \beta$

$F_{Rz} = \Sigma F_z; \qquad 0 = -300 \sin 30^\circ + F_3 \cos \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Solving:

$F_3 = 250 \text{ lb}$ **Ans.**

$\alpha = 87.0^\circ$ **Ans.**

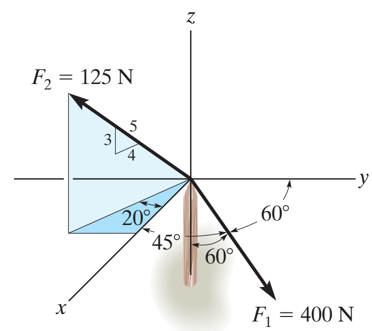
$\beta = 143^\circ$ **Ans.**

$\gamma = 53.1^\circ$ **Ans.**

Ans:
$F_3 = 250 \text{ lb}$
$\alpha = 87.0^\circ$
$\beta = 143^\circ$
$\gamma = 53.1^\circ$

2–69.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 400\,(\cos 45°\mathbf{i} + \cos 60°\mathbf{j} - \cos 60°\mathbf{k}) = \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\}\text{ N}$$

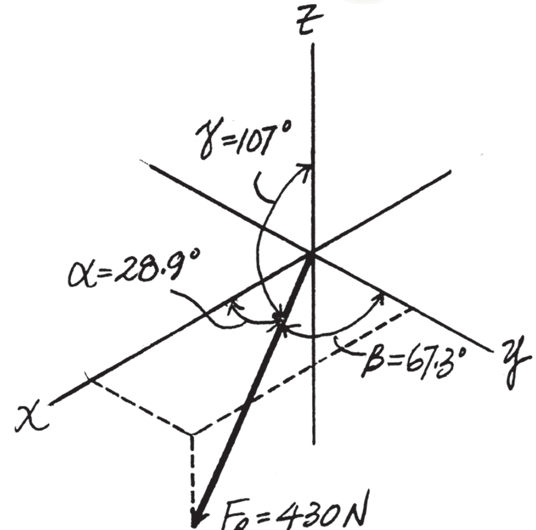
$$\mathbf{F}_2 = 125\left[\frac{4}{5}(\cos 20°)\mathbf{i} - \frac{4}{5}(\sin 20°)\mathbf{j} + \frac{3}{5}\mathbf{k}\right] = \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\} + \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$

$$= \{376.81\mathbf{i} + 165.80\mathbf{j} - 125.00\mathbf{k}\}\text{ N}$$



The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{376.81^2 + 165.80^2 + (-125.00)^2}$$

$$= 430.23\text{ N} = 430\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{376.81}{430.23}; \qquad \alpha = 28.86° = 28.9° \qquad \textbf{Ans.}$$

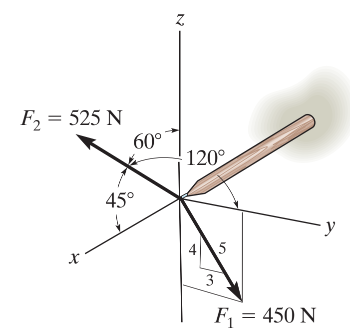
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{165.80}{430.23}; \qquad \beta = 67.33° = 67.3° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-125.00}{430.23}; \qquad \gamma = 106.89° = 107° \qquad \textbf{Ans.}$$

Ans:
$F_R = 430$ N
$\alpha = 28.9°$
$\beta = 67.3°$
$\gamma = 107°$

2–70.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.

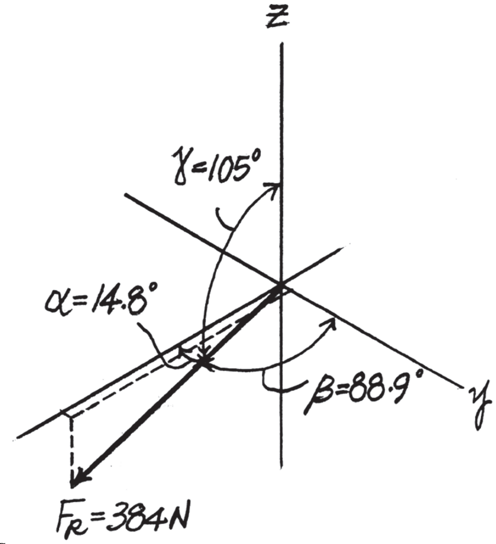


SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 450\left(\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}\right) = \{270\mathbf{j} - 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 525\,(\cos 45°\mathbf{i} + \cos 120°\mathbf{j} + \cos 60°\mathbf{k}) = \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}\text{ N}$$



Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= \{270\mathbf{j} - 360\mathbf{k}\} + \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}$$

$$= \{371.23\mathbf{i} + 7.50\mathbf{j} - 97.5\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{371.23^2 + 7.50^2 + (-97.5)^2}$$

$$= 383.89\text{ N} = 384\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{371.23}{383.89}; \qquad \alpha = 14.76° = 14.8° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{7.50}{383.89}; \qquad \beta = 88.88° = 88.9° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-97.5}{383.89}; \qquad \gamma = 104.71° = 105° \qquad \textbf{Ans.}$$

Ans:

$F_R = 384$ N

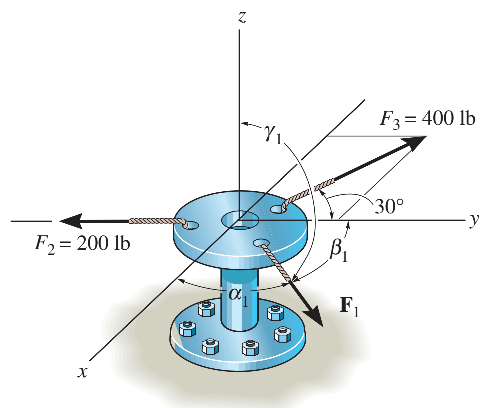
$\cos\alpha = \dfrac{371.23}{383.89}$; $\alpha = 14.8°$

$\cos\beta = \dfrac{7.50}{383.89}$; $\beta = 88.9°$

$\cos\gamma = \dfrac{-97.5}{383.89}$; $\gamma = 105°$

2–71.

Specify the magnitude and coordinate direction angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant of the three forces acting on the bracket is $\mathbf{F}_R = \{-350\mathbf{k}\}$ lb. Note that $\mathbf{F}_3$ lies in the x–y plane.



SOLUTION

$\mathbf{F}_1 = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$

$\mathbf{F}_2 = -200\,\mathbf{j}$

$\mathbf{F}_3 = -400 \sin 30°\,\mathbf{i} + 400 \cos 30°\,\mathbf{j}$

$\quad = -200\,\mathbf{i} + 346.4\,\mathbf{j}$

$\mathbf{F}_R = \Sigma\mathbf{F}$

$-350\,\mathbf{k} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k} - 200\,\mathbf{j} - 200\,\mathbf{i} + 346.4\,\mathbf{j}$

$0 = F_x - 200\,; \qquad F_x = 200 \text{ lb}$

$0 = F_y - 200 + 346.4\,; \qquad F_y = -146.4 \text{ lb}$

$F_z = -350 \text{ lb}$

$F_1 = \sqrt{(200)^2 + (-146.4)^2 + (-350)^2}$

$F_1 = 425.9 \text{ lb} = 429 \text{ lb}$ **Ans.**

$\alpha_1 = \cos^{-1}\left(\dfrac{200}{428.9}\right) = 62.2°$ **Ans.**

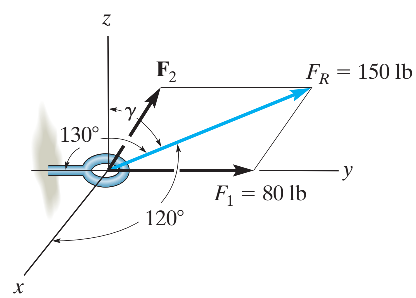
$\beta_1 = \cos^{-1}\left(\dfrac{-146.4}{428.9}\right) = 110°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\dfrac{-350}{428.9}\right) = 145°$ **Ans.**

Ans:
$F_1 = 429$ lb
$\alpha_1 = 62.2°$
$\beta_1 = 110°$
$\gamma_1 = 145°$

***2–72.**

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If the resultant force $\mathbf{F}_R$ has a magnitude of 150 lb and the coordinate direction angles shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_R$, γ can be determined from

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 120° + \cos^2 50° + \cos^2\gamma = 1$$

$$\cos\gamma = \pm 0.5804$$

Here $\gamma < 90°$, then

$$\gamma = 54.52°$$

Thus

$$\mathbf{F}_R = 150(\cos 120°\mathbf{i} + \cos 50°\mathbf{j} + \cos 54.52°\mathbf{k})$$

$$= \{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Also

$$\mathbf{F}_1 = \{80\mathbf{j}\}\text{ lb}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\} = \{80\mathbf{j}\} + \mathbf{F}_2$$

$$F_2 = \{-75.0\mathbf{i} + 16.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Thus, the magnitude of $\mathbf{F}_2$ is

$$F_2 = \sqrt{(F_2)_x + (F_2)_y + (F_2)_z} = \sqrt{(-75.0)^2 + 16.42^2 + 87.05^2}$$

$$= 116.07\text{ lb} = 116\text{ lb} \qquad \textbf{Ans.}$$

And its coordinate direction angles are

$$\cos\alpha_2 = \frac{(F_2)_x}{F_2} = \frac{-75.0}{116.07}; \qquad \alpha_2 = 130.25° = 130° \qquad \textbf{Ans.}$$

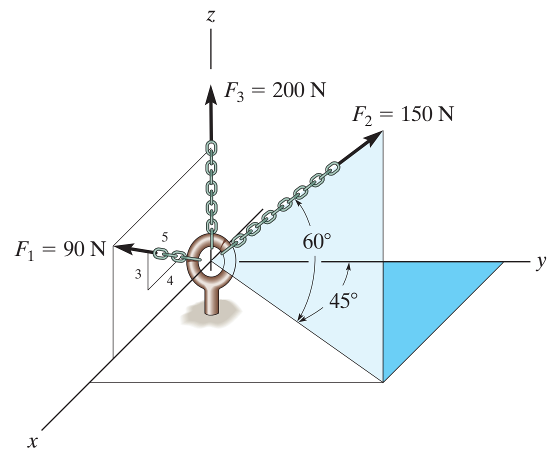
$$\cos\beta_2 = \frac{(F_2)_y}{F_2} = \frac{16.42}{116.07}; \qquad \beta_2 = 81.87° = 81.9° \qquad \textbf{Ans.}$$

$$\cos\gamma_2 = \frac{(F_2)_z}{F_2} = \frac{87.05}{116.07}; \qquad \gamma_2 = 41.41° = 41.4° \qquad \textbf{Ans.}$$

Ans:
$F_R = 116$ lb
$\cos\alpha_2 = 130°$
$\cos\beta_2 = 81.9°$
$\cos\gamma_2 = 41.4°$

2–73.

Express each force in Cartesian vector form.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\} \text{ N}$$ **Ans.**

$$\mathbf{F}_2 = 150\,(\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

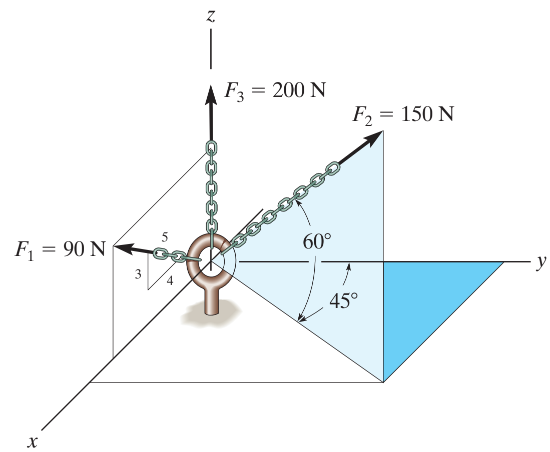
$$= \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\} \text{ N}$$ **Ans.**

$$\mathbf{F}_3 = \{200\,\mathbf{k}\}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{72.0\mathbf{i} + 54.0\mathbf{k}\}$ N
$\mathbf{F}_2 = \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\}$ N
$\mathbf{F}_3 = \{200\,\mathbf{k}\}$

2–74.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

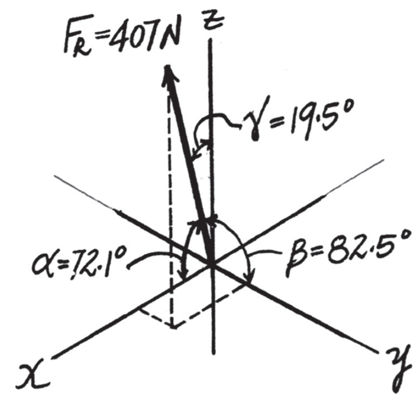
Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 150\,(\cos 60°\sin 45°\mathbf{i} + \cos 60°\cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{200\text{ k}\}\text{ N}$$



Resultant Force.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (72.0\mathbf{i} + 54.0\mathbf{k}) + (53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}) + (200\mathbf{k})$$

$$= \{125.03\mathbf{i} + 53.03\mathbf{j} + 383.90\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{125.03^2 + 53.03^2 + 383.90^2}$$

$$= 407.22\text{ N} = 407\text{ N} \qquad \textbf{Ans.}$$

And the coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{125.03}{407.22}; \qquad \alpha = 72.12° = 72.1° \qquad \textbf{Ans.}$$

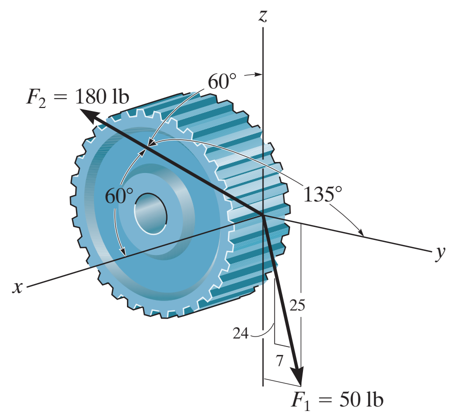
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{53.03}{407.22}; \qquad \beta = 82.52° = 82.5° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{383.90}{407.22}; \qquad \gamma = 19.48° = 19.5° \qquad \textbf{Ans.}$$

Ans:
$F_R = 407$ N
$\alpha = 72.1°$
$\beta = 82.5°$
$\gamma = 19.5°$

2–75.

The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\}\text{ lb}$$ **Ans.**

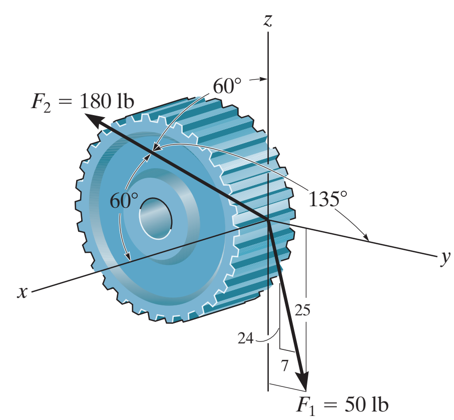
$$\mathbf{F}_2 = 180\cos 60°\mathbf{i} + 180\cos 135°\mathbf{j} + 180\cos 60°\mathbf{k}$$

$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{14.0\mathbf{j} - 48.0\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}$ lb

***2–76.**

The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



SOLUTION

$$F_{Rx} = 180\cos 60° = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180\cos 135° = -113$$

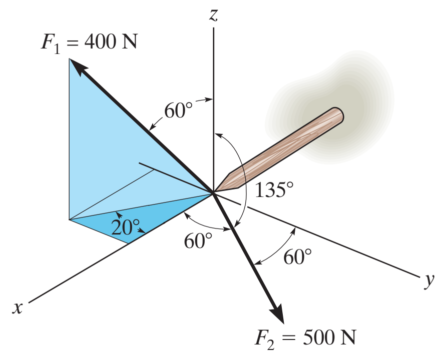
$$F_{Rz} = -\frac{24}{25}(50) + 180\cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$F_{Rx} = 90$
$F_{Ry} = -113$
$F_{Rz} = 42$
$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}$ lb

2–77.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

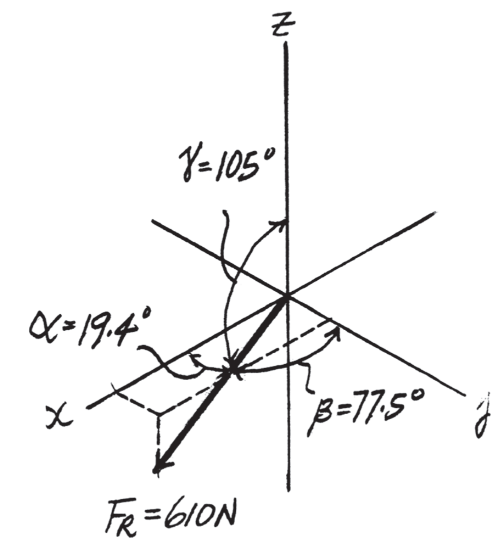
Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 400\,(\sin 60^\circ \cos 20^\circ\mathbf{i} - \sin 60^\circ \sin 20^\circ\mathbf{j} + \cos 60^\circ\mathbf{k})$$

$$= \{325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 500\,(\cos 60^\circ\mathbf{i} + \cos 60^\circ\mathbf{j} + \cos 135^\circ\mathbf{k})$$

$$= \{250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k}\}\text{ N}$$



Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}) + (250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k})$$

$$= \{575.52\mathbf{i} + 131.52\mathbf{j} - 153.55\,\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{575.52^2 + 131.52^2 + (-153.55)^2}$$

$$= 610.00\text{ N} = 610\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{575.52}{610.00} \qquad \alpha = 19.36^\circ = 19.4^\circ \qquad \textbf{Ans.}$$

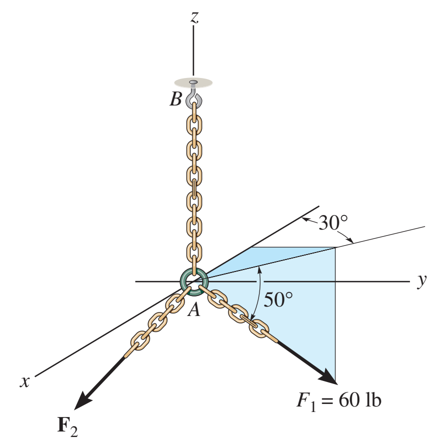
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{131.52}{610.00} \qquad \beta = 77.549^\circ = 77.5^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-153.55}{610.00} \qquad \gamma = 104.58^\circ = 105^\circ \qquad \textbf{Ans.}$$

Ans:
$F_R = 610$ N
$\alpha = 19.4^\circ$
$\beta = 77.5^\circ$
$\gamma = 105^\circ$

2–78.

The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_R = \{-100\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_1 = 60\{-\cos 50^\circ \cos 30^\circ\,\mathbf{i} + \cos 50^\circ \sin 30^\circ\,\mathbf{j} - \sin 50^\circ\,\mathbf{k}\}\text{ lb}$$

$$= \{-33.40\,\mathbf{i} + 19.28\,\mathbf{j} - 45.96\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = \{F_{2_x}\,\mathbf{i} + F_{2_y}\,\mathbf{j} + F_{2_z}\,\mathbf{k}\}\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$-100\mathbf{k} = \left\{\left(F_{2_x} - 33.40\right)\mathbf{i} + \left(F_{2_y} + 19.28\right)\mathbf{j} + \left(F_{2_z} - 45.96\right)\mathbf{k}\right\}$$

Equating **i, j** and **k** components, we have

$$F_{2_x} - 33.40 = 0 \qquad F_{2_x} = 33.40\text{ lb}$$

$$F_{2_y} + 19.28 = 0 \qquad F_{2_y} = -19.28\text{ lb}$$

$$F_{2_z} - 45.96 = -100 \qquad F_{2_z} = -54.04\text{ lb}$$

The magnitude of force $\mathbf{F}_2$ is

$$F_2 = \sqrt{F_{2_x}^2 + F_{2_y}^2 + F_{2_z}^2}$$

$$= \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2}$$

$$= 66.39\text{ lb} = 66.4\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles for $\mathbf{F}_2$ are

$$\cos\alpha = \frac{F_{2_x}}{F_2} = \frac{33.40}{66.39} \qquad \alpha = 59.8^\circ \qquad \textbf{Ans.}$$

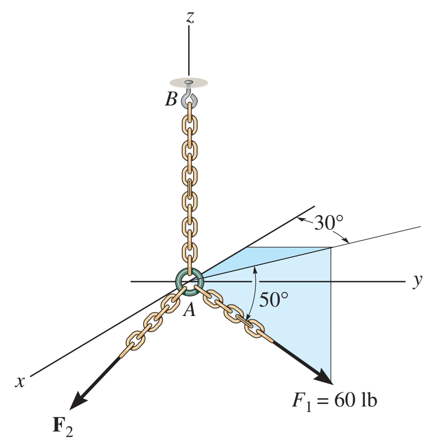
$$\cos\beta = \frac{F_{2_y}}{F_2} = \frac{-19.28}{66.39} \qquad \beta = 107^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{F_{2_z}}{F_2} = \frac{-54.04}{66.39} \qquad \gamma = 144^\circ \qquad \textbf{Ans.}$$

Ans:
$F_2 = 66.4$ lb
$\alpha = 59.8^\circ$
$\beta = 107^\circ$
$\gamma = 144^\circ$

2–79.

Determine the coordinate direction angles of the force $\mathbf{F}_1$ and indicate them on the figure.



SOLUTION

Unit Vector For Force $\mathbf{F}_1$:

$$\mathbf{u}_{F_1} = -\cos 50° \cos 30° \,\mathbf{i} + \cos 50° \sin 30° \,\mathbf{j} - \sin 50° \,\mathbf{k}$$

$$= -0.5567\,\mathbf{i} + 0.3214\,\mathbf{j} - 0.7660\,\mathbf{k}$$

Coordinate Direction Angles: From the unit vector obtained above, we have

$\cos\alpha = -0.5567 \qquad \alpha = 124°$ **Ans.**

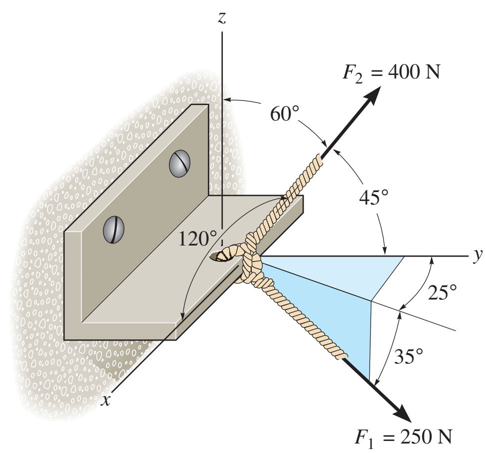
$\cos\beta = 0.3214 \qquad \beta = 71.3°$ **Ans.**

$\cos\gamma = -0.7660 \qquad \gamma = 140°$ **Ans.**

Ans:
$\alpha = 124°$
$\beta = 71.3°$
$\gamma = 140°$

***2–80.**

The bracket is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force $\mathbf{F}_R$. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_1 = 250\{\cos 35^\circ \sin 25^\circ \mathbf{i} + \cos 35^\circ \cos 25^\circ \mathbf{j} - \sin 35^\circ \mathbf{k}\} \text{ N}$$

$$= \{86.55\mathbf{i} + 185.60\mathbf{j} - 143.39\mathbf{k}\} \text{ N}$$

$$= \{86.5\mathbf{i} + 186\mathbf{j} - 143\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 400\{\cos 120^\circ \mathbf{i} + \cos 45^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}\} \text{ N}$$

$$= \{-200.0\mathbf{i} + 282.84\mathbf{j} + 200.0\mathbf{k}\} \text{ N}$$

$$= \{-200\mathbf{i} + 283\mathbf{j} + 200\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= \{(86.55 - 200.0)\mathbf{i} + (185.60 + 282.84)\mathbf{j} + (-143.39 + 200.0)\,\mathbf{k}\}$$

$$= \{-113.45\mathbf{i} + 468.44\mathbf{j} + 56.61\mathbf{k}\} \text{ N}$$

$$= \{-113\mathbf{i} + 468\mathbf{j} + 56.6\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$

$$= \sqrt{(-113.45)^2 + 468.44^2 + 56.61^2}$$

$$= 485.30 \text{ N} = 485 \text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{-113.45}{485.30} \qquad \alpha = 104^\circ \qquad \textbf{Ans.}$$

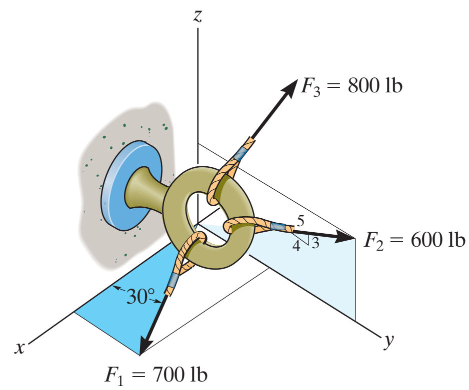
$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{468.44}{485.30} \qquad \beta = 15.1^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{56.61}{485.30} \qquad \gamma = 83.3^\circ \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{86.5\mathbf{i} + 186\mathbf{j} - 143\mathbf{k}\}$ N
$\mathbf{F}_2 = \{-200\mathbf{i} + 283\mathbf{j} + 200\mathbf{k}\}$ N
$\mathbf{F}_R = \{-113\mathbf{i} + 468\mathbf{j} + 56.6\mathbf{k}\}$ N
$F_R = 485$ N
$\alpha = 104^\circ$
$\beta = 15.1^\circ$
$\gamma = 83.3^\circ$

2–81.

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 60°$ and $\gamma_3 = 45°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



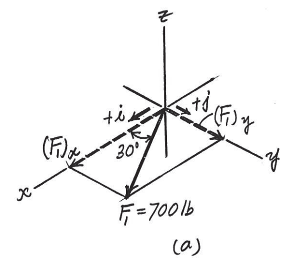
SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. a, b, and c, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

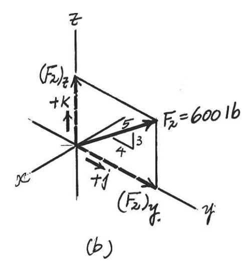
$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_3 = 800\cos 120°\mathbf{i} + 800\cos 60°\mathbf{j} + 800\cos 45°\mathbf{k} = [-400\mathbf{i} + 400\mathbf{j} + 565.69\mathbf{k}]\text{ lb}$$



Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (-400\mathbf{i} + 400\mathbf{j} + 565.69\mathbf{k})\\ &= [206.22\mathbf{i} + 1230\mathbf{j} + 925.69\mathbf{k}]\text{ lb}\end{aligned}$$



The magnitude of $\mathbf{F}_R$ is

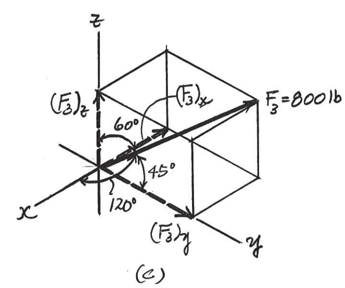
$$\begin{aligned}F_R &= \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}\\ &= \sqrt{(206.22)^2 + (1230)^2 + (925.69)^2} = 1553.16\text{ lb} = 1.55\text{ kip} \qquad \textbf{Ans.}\end{aligned}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{206.22}{1553.16}\right) = 82.4° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{1230}{1553.16}\right) = 37.6° \qquad \textbf{Ans.}$$

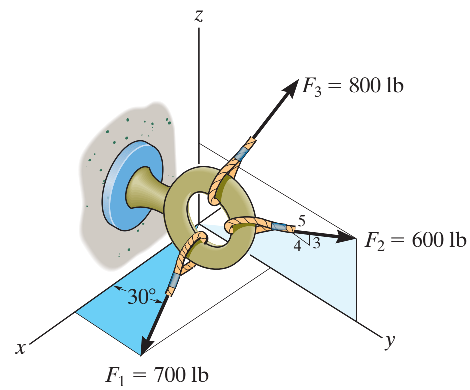
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{925.69}{1553.16}\right) = 53.4° \qquad \textbf{Ans.}$$



Ans:
$F_R = 1.55$ kip
$\alpha = 82.4°$
$\beta = 37.6°$
$\gamma = 53.4°$

2–82.

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 45°$ and $\gamma_3 = 60°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

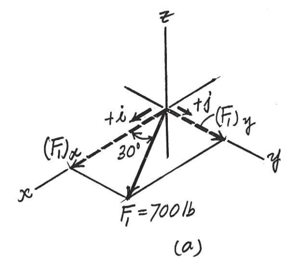
Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. a, b, and c, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

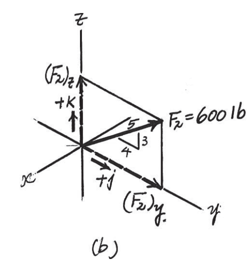
$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_3 = 800\cos 120°\mathbf{i} + 800\cos 45°\mathbf{j} + 800\cos 60°\mathbf{k} = \{-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}\}\text{ lb}$$

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= 606.22\mathbf{i} + 350\mathbf{j} + 480\mathbf{j} + 360\mathbf{k} - 400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k} \\ &= \{206.22\mathbf{i} + 1395.69\mathbf{j} + 760\mathbf{k}\}\text{ lb}\end{aligned}$$



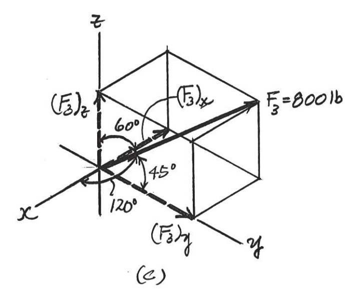
$$\begin{aligned}F_R &= \sqrt{(206.22)^2 + (1395.69)^2 + (760)^2} \\ &= 1602.52\text{ lb} = 1.60\text{ kip}\end{aligned}$$ **Ans.**



$$\alpha = \cos^{-1}\left(\frac{206.22}{1602.52}\right) = 82.6°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{1395.69}{1602.52}\right) = 29.4°$$ **Ans.**

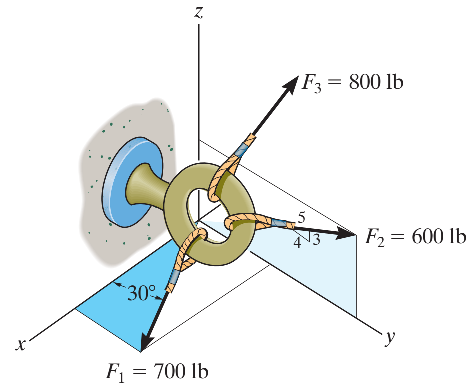
$$\gamma = \cos^{-1}\left(\frac{760}{1602.52}\right) = 61.7°$$ **Ans.**



Ans:
$F_R = 1.60$ kip
$\alpha = 82.6°$
$\beta = 29.4°$
$\gamma = 61.7°$

2–83.

If the direction of the resultant force acting on the eyebolt is defined by the unit vector $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, determine the coordinate direction angles of $\mathbf{F}_3$ and the magnitude of $\mathbf{F}_R$.



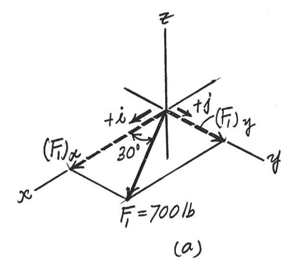
SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. *a*, *b*, and *c*, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_3 = 800\cos\alpha_3\mathbf{i} + 800\cos\beta_3\mathbf{j} + 800\cos\gamma_3\mathbf{k}$$



Since the direction of $\mathbf{F}_R$ is defined by $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, it can be written in Cartesian vector form as

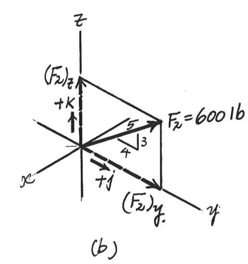
$$\mathbf{F}_R = F_R\mathbf{u}_{F_R} = F_R(\cos 30°\mathbf{j} + \sin 30°\mathbf{k}) = 0.8660F_R\mathbf{j} + 0.5F_R\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$0.8660F_R\mathbf{j} + 0.5F_R\mathbf{k} = (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (800\cos\alpha_3\mathbf{i} + 800\cos\beta_3\mathbf{j} + 800\cos\gamma_3\mathbf{k})$$

$$0.8660F_R\mathbf{j} + 0.5F_R\mathbf{k} = (606.22 + 800\cos\alpha_3)\mathbf{i} + (350 + 480 + 800\cos\beta_3)\mathbf{j} + (360 + 800\cos\gamma_3)\mathbf{k}$$

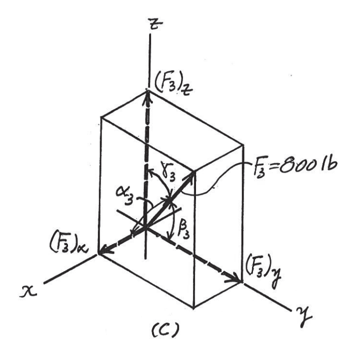


Equating the **i**, **j**, and **k** components, we have

$$0 = 606.22 + 800\cos\alpha_3$$
$$800\cos\alpha_3 = -606.22 \qquad \textbf{(1)}$$

$$0.8660F_R = 350 + 480 + 800\cos\beta_3$$
$$800\cos\beta_3 = 0.8660F_R - 830 \qquad \textbf{(2)}$$

$$0.5F_R = 360 + 800\cos\gamma_3$$
$$800\cos\gamma_3 = 0.5F_R - 360 \qquad \textbf{(3)}$$



Squaring and then adding Eqs. (1), (2), and (3), yields

$$800^2\,[\cos^2\alpha_3 + \cos^2\beta_3 + \cos^2\gamma_3] = F_R{}^2 - 1797.60F_R + 1{,}186{,}000 \qquad \textbf{(4)}$$

However, $\cos^2\alpha_3 + \cos^2\beta_3 + \cos^2\gamma_3 = 1$. Thus, from Eq. (4)

$$F_R{}^2 - 1797.60F_R + 546{,}000 = 0$$

Solving the above quadratic equation, we have two positive roots

$F_R = 387.09\text{ N} = 387\text{ N}$ **Ans.**

$F_R = 1410.51\text{ N} = 1.41\text{ kN}$ **Ans.**

From Eq. (1),

$\alpha_3 = 139°$ **Ans.**

Substituting $F_R = 387.09$ N into Eqs. (2), and (3), yields

$\beta_3 = 128° \qquad \gamma_3 = 102°$ **Ans.**

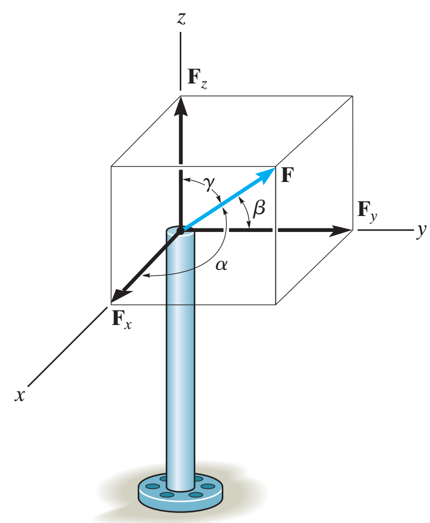
Substituting $F_R = 1410.51$ N into Eqs. (2), and (3), yields

$\beta_3 = 60.7° \qquad \gamma_3 = 64.4°$ **Ans.**

Ans:
$\alpha_3 = 139°$
$\beta_3 = 128°, \gamma_3 = 102°, F_{R1} = 387\text{ N}$
$\beta_3 = 60.7°, \gamma_3 = 64.4°, F_{R2} = 1.41\text{ kN}$

***2–84.**

The pole is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



SOLUTION

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 30° + \cos^2 75° = 1$$

$$\alpha = 64.67°$$

$F_x = 3\cos 64.67° = 1.28\text{ kN}$ **Ans.**

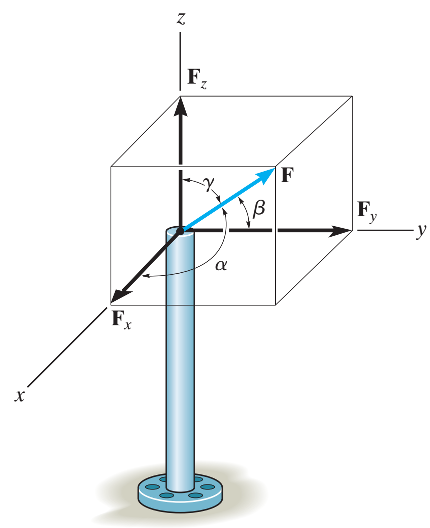
$F_y = 3\cos 30° = 2.60\text{ kN}$ **Ans.**

$F_z = 3\cos 75° = 0.776\text{ kN}$ **Ans.**

Ans:
$F_x = 1.28\text{ kN}$
$F_y = 2.60\text{ kN}$
$F_z = 0.776\text{ kN}$

2–85.

The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



SOLUTION

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

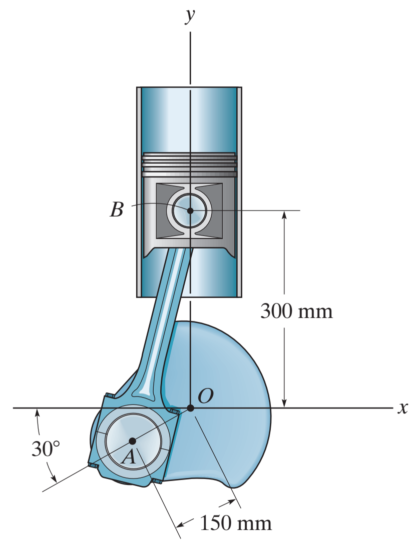
$$F = 2.02 \text{ kN}$$ **Ans.**

$$F_y = 2.02 \cos 75° = 0.523 \text{ kN}$$ **Ans.**

Ans:
$F = 2.02$ kN
$F_y = 0.523$ kN

2–86.

Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



SOLUTION

Position Vector. The coordinates of points A and B are $A(-150 \cos 30°, -150 \sin 30°)$ mm and $B(0, 300)$ mm respectively. Then

$$\mathbf{r}_{AB} = [0 - (-150 \cos 30°)]\mathbf{i} + [300 - (-150 \sin 30°)]\mathbf{j}$$

$$= \{129.90\mathbf{i} + 375\mathbf{j}\} \text{ mm}$$

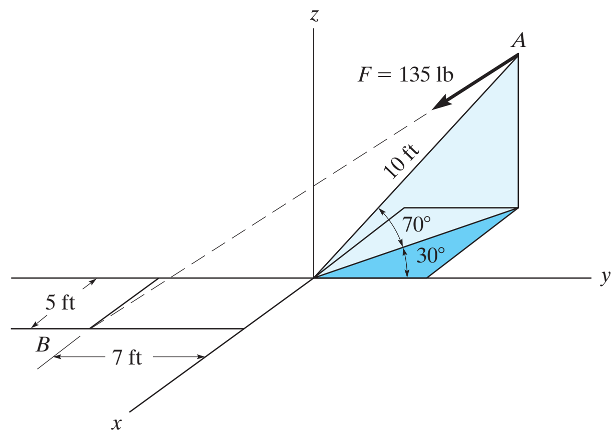
Thus, the magnitude of $\mathbf{r}_{AB}$ is

$$\mathbf{r}_{AB} = \sqrt{129.90^2 + 375^2} = 396.86 \text{ mm} = 397 \text{ mm}$$ **Ans.**

Ans:
$r_{AB} = 397$ mm

2–87.

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



SOLUTION

$$\mathbf{r}_{AB} = (5 + 10\cos 70^\circ \sin 30^\circ)\mathbf{i} + (-7 - 10\cos 70^\circ \cos 30^\circ)\mathbf{j} - 10\sin 70^\circ\mathbf{k}$$

$$\mathbf{r}_{AB} = \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(6.710)^2 + (-9.962)^2 + (-9.397)^2} = 15.25$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = (0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k})$$

$$\mathbf{F} = 135\mathbf{u}_{AB} = (59.40\mathbf{i} - 88.18\mathbf{j} - 83.18\mathbf{k})$$

$$= \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{59.40}{135}\right) = 63.9^\circ$$ **Ans.**

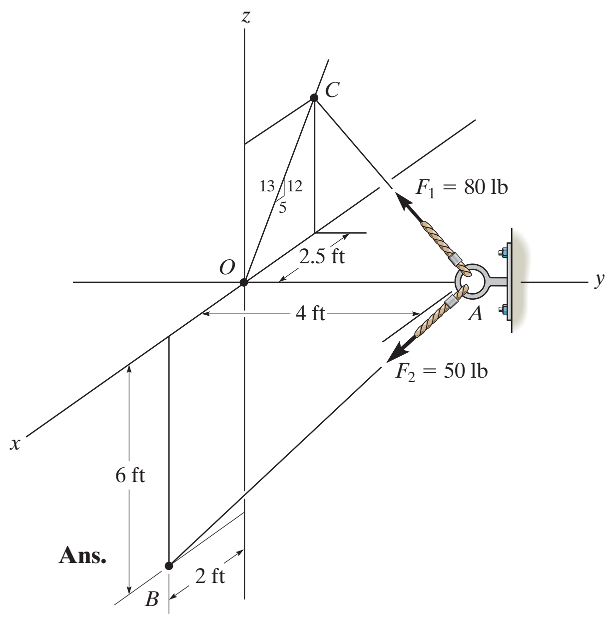
$$\beta = \cos^{-1}\left(\frac{-88.18}{135}\right) = 131^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-83.18}{135}\right) = 128^\circ$$ **Ans.**

Ans:
$\{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}$ lb
$\alpha = 63.9^\circ$
$\beta = 131^\circ$
$\gamma = 128^\circ$

***2–88.**

Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{r}_{AC} = \left\{-2.5\,\mathbf{i} - 4\,\mathbf{j} + \frac{12}{5}(2.5)\,\mathbf{k}\right\}\text{ ft}$$

$$\mathbf{F}_1 = 80\text{ lb}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = -26.20\,\mathbf{i} - 41.93\,\mathbf{j} + 62.89\,\mathbf{k}$$

$$= \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{r}_{AB} = \{2\,\mathbf{i} - 4\,\mathbf{j} - 6\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_2 = 50\text{ lb}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 13.36\,\mathbf{i} - 26.73\,\mathbf{j} - 40.09\,\mathbf{k}$$

$$= \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -12.84\,\mathbf{i} - 68.65\,\mathbf{j} + 22.80\,\mathbf{k}$$

$$= \{-12.8\,\mathbf{i} - 68.7\,\mathbf{j} + 22.8\,\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{(-12.84)^2\,(-68.65)^2 + (22.80)^2} = 73.47 = 73.5\text{ lb} \qquad \textbf{Ans.}$$

$$\alpha = \cos^{-1}\left(\frac{-12.84}{73.47}\right) = 100° \qquad \textbf{Ans.}$$

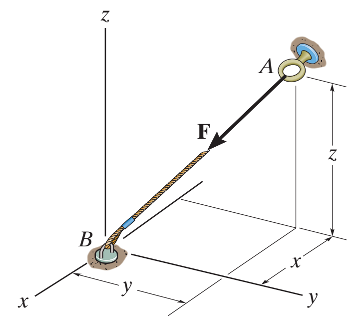
$$\beta = \cos^{-1}\left(\frac{-68.65}{73.47}\right) = 159° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{22.80}{73.47}\right) = 71.9° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}$ lb
$F_R = 73.5$ lb
$\alpha = 100°$
$\beta = 159°$
$\gamma = 71.9°$

2–89.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - (-x)]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$\frac{x}{9} = 0.5623 \qquad x = 5.06\text{ m} \qquad \textbf{Ans.}$$

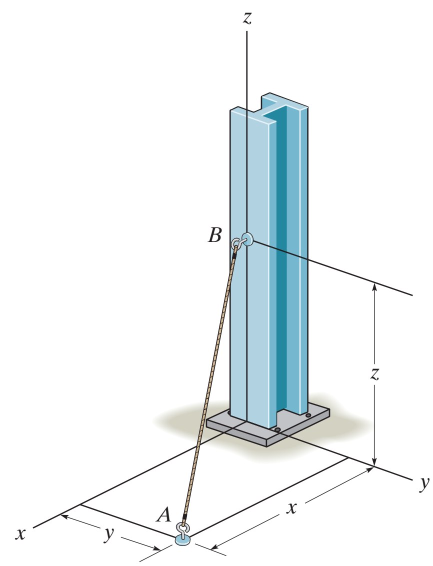
$$\frac{-y}{9} = -0.4016 \qquad y = 3.61\text{ m} \qquad \textbf{Ans.}$$

$$\frac{-z}{9} = 0.7229 \qquad z = 6.51\text{ m} \qquad \textbf{Ans.}$$

Ans:
$x = 5.06$ m
$y = 3.61$ m
$z = 6.51$ m

2–90.

The 8-m-long cable is anchored to the ground at A. If $x = 4$ m and $y = 2$ m, determine the coordinate z to the highest point of attachment along the column.



SOLUTION

$$\mathbf{r} = \{4\mathbf{i} + 2\mathbf{j} + z\mathbf{k}\}\text{ m}$$

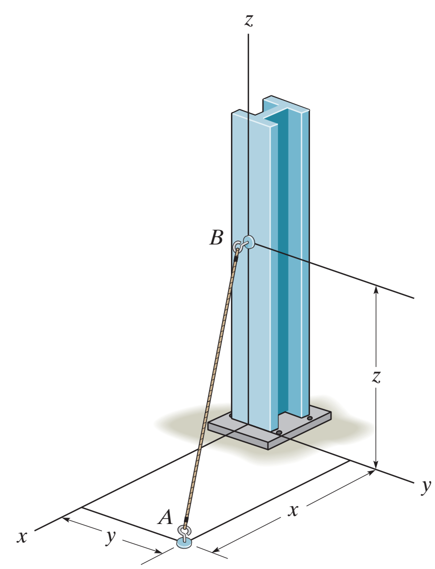
$$r = \sqrt{(4)^2 + (2)^2 + (z)^2} = 8$$

$$z = 6.63\text{ m}$$ **Ans.**

Ans:
6.63 m

2–91.

The 8-m-long cable is anchored to the ground at A. If $z = 5$ m, determine the location $+x, +y$ of point A. Choose a value such that $x = y$.



SOLUTION

$\mathbf{r} = \{x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}\}$ m

$r = \sqrt{(x)^2 + (y)^2 + (5)^2} = 8$

$x = y$, thus

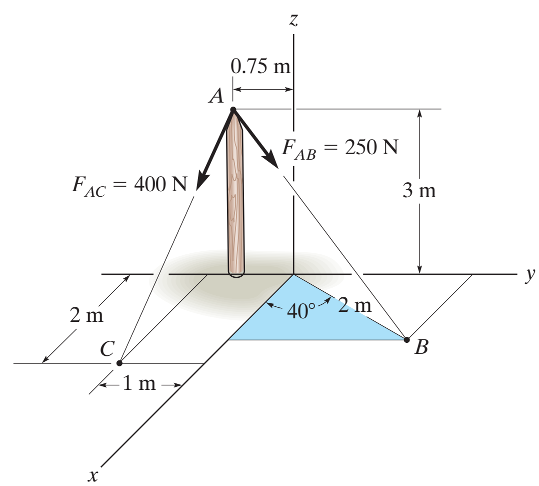
$2x^2 = 8^2 - 5^2$

$x = y = 4.42$ m **Ans.**

Ans:
4.42 m

***2–92.**

Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Unit Vectors. The coordinates for points A, B and C are $(0, -0.75, 3)$ m, $B(2\cos 40°, 2\sin 40°, 0)$ m and $C(2, -1, 0)$ m respectively.

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(2\cos 40° - 0)\mathbf{i} + [2\sin 40° - (-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2\cos 40° - 0)^2 + [2\sin 40° - (-0.75)]^2 + (0-3)^2}}$$

$$= 0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(2-0)\mathbf{i} + [-1-(-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2-0)^2 + [-1-(-0.75)]^2 + (0-3)^2}}$$

$$= 0.5534\mathbf{i} - 0.0692\mathbf{j} - 0.8301\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = \mathbf{F}_{AB}\mathbf{u}_{AB} = 250\,(0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k})$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\}\text{ N}$$

$$= \{97.3\mathbf{i} + 129\mathbf{j} - 191\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_{AC} = \mathbf{F}_{AC}\mathbf{u}_{AC} = 400\,(0.5534\mathbf{i} - 0.06917\mathbf{j} - 0.8301\mathbf{k})$$

$$= \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}\text{ N}$$

$$= \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\} + \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}$$

$$= \{318.67\mathbf{i} + 101.63\mathbf{j} - 522.58\,\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$\mathbf{F}_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{318.67^2 + 101.63^2 + (-522.58)^2}$$

$$= 620.46\text{ N} = 620\text{ N}$$

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{318.67}{620.46}; \qquad \alpha = 59.10° = 59.1° \qquad \textbf{Ans.}$$

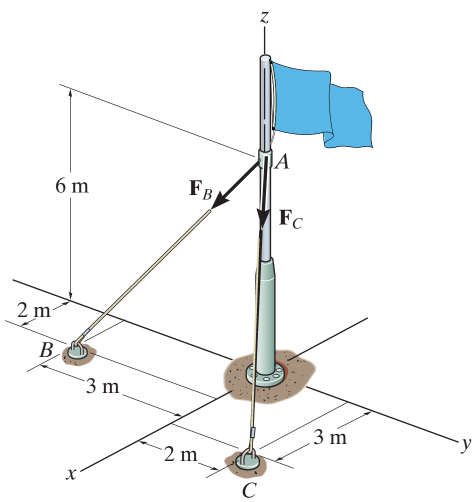
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{101.63}{620.46}; \qquad \beta = 80.57° = 80.6° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-522.58}{620.46}; \qquad \gamma = 147.38° = 147° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_{AB} = \{97.3\mathbf{i} - 129\mathbf{j} - 191\mathbf{k}\}$ N
$\mathbf{F}_{AC} = \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}$ N
$F_R = 620$ N
$\cos\alpha = 59.1°$
$\cos\beta = 80.6°$
$\cos\gamma = 147°$

2–93.

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 560\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 700\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

Resultant Force:

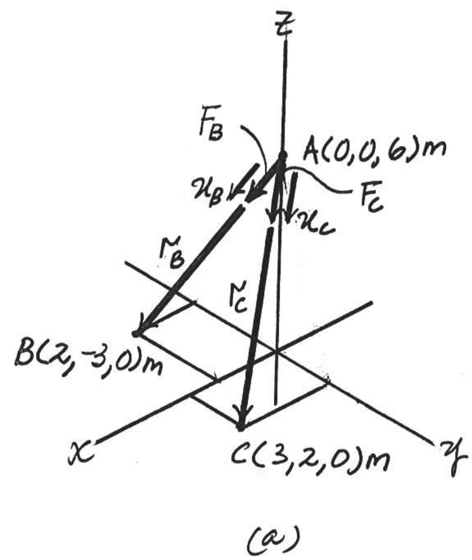
$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k})$$

$$= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(460)^2 + (-40)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}$$ **Ans.**



(a)

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9^\circ$$ **Ans.**

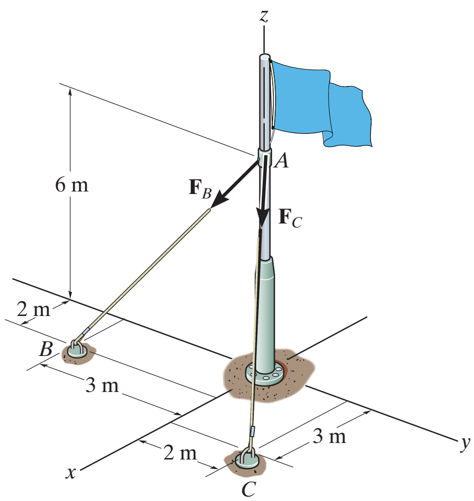
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157^\circ$$ **Ans.**

Ans:
$F_R = 1.17$ kN
$\alpha = 66.9^\circ$
$\beta = 92.0^\circ$
$\gamma = 157^\circ$

2–94.

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

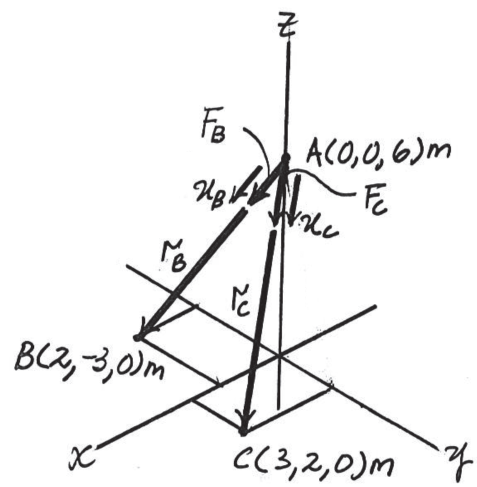
$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$



(a)

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$

$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0° \qquad \textbf{Ans.}$$

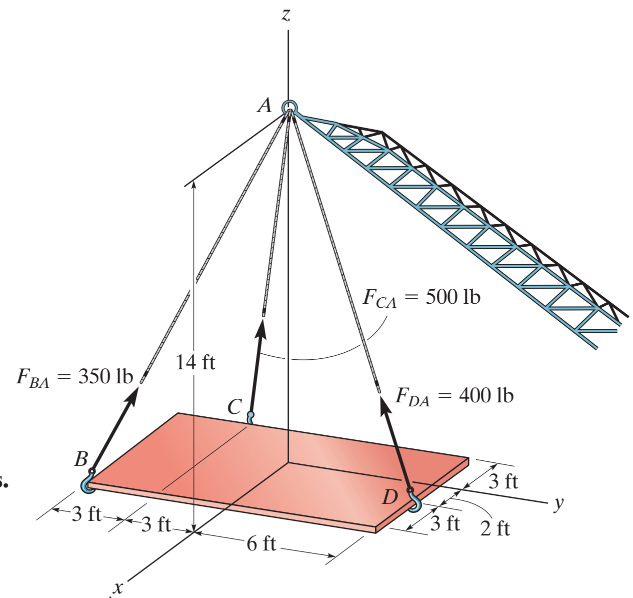
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157° \qquad \textbf{Ans.}$$

Ans:
$F_R = 1.17$ kN
$\alpha = 68.0°$
$\beta = 96.8°$
$\gamma = 157°$

2–95.

The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_{BA} = 350\left(\frac{\mathbf{r}_{BA}}{r_{BA}}\right) = 350\left(-\frac{5}{16.031}\mathbf{i} + \frac{6}{16.031}\mathbf{j} + \frac{14}{16.031}\mathbf{k}\right)$$

$$= \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_{CA} = 500\left(\frac{\mathbf{r}_{CA}}{r_{CA}}\right) = 500\left(\frac{3}{14.629}\mathbf{i} + \frac{3}{14.629}\mathbf{j} + \frac{14}{14.629}\mathbf{k}\right)$$

$$= \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}\text{ lb}$$ **Ans.**

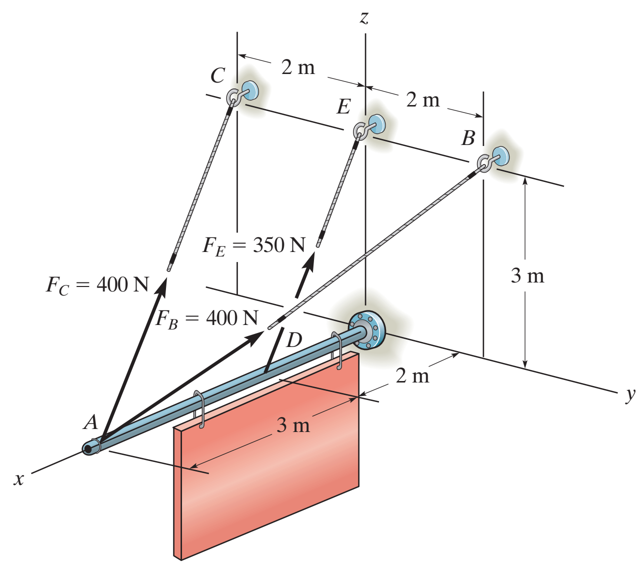
$$\mathbf{F}_{DA} = 400\left(\frac{\mathbf{r}_{DA}}{r_{DA}}\right) = 400\left(-\frac{2}{15.362}\mathbf{i} - \frac{6}{15.362}\mathbf{j} + \frac{14}{15.362}\mathbf{k}\right)$$

$$= \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_{BA} = \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}$ lb
$\mathbf{F}_{CA} = \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}$ lb
$\mathbf{F}_{DA} = \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}$ lb

***2–96.**

The three supporting cables exert the forces shown on the sign. Represent each force as a Cartesian vector.



SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{r}_E = (0 - 2)\mathbf{i} + (0 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_E = \sqrt{(-2)^2 + 0^2 + 3^2} = \sqrt{13}\text{ m}$$

$$\mathbf{F} = F_{\mathbf{u}} = F\left(\frac{\mathbf{r}}{r}\right)$$

$$\mathbf{F}_C = 400\left(\frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

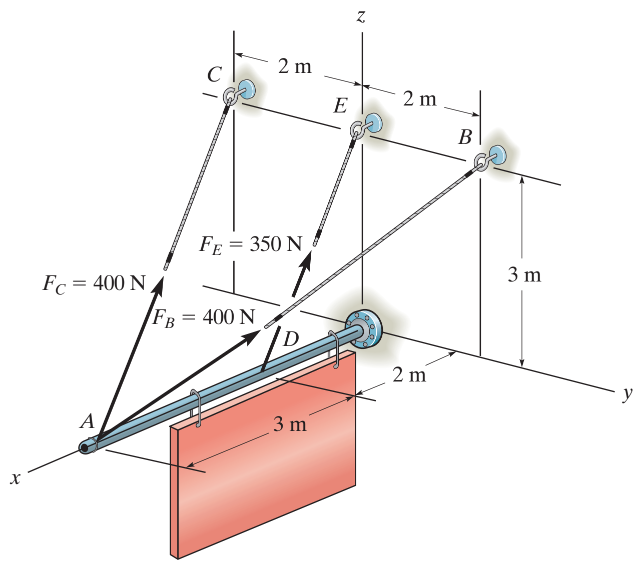
$$\mathbf{F}_B = 400\left(\frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_E = 350\left(\frac{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}}{\sqrt{13}}\right) = \{-194\mathbf{i} + 291\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_C = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_B = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_E = \{-194\mathbf{i} + 291\mathbf{k}\}$ N

2–97.

Determine the magnitude and coordinate direction angles of the resultant force of the two forces acting on the sign at point A.



SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + (3)^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_C = 400\left(\frac{\mathbf{r}_C}{r_C}\right) = 400\left(\frac{(-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_C = (-324.4428\mathbf{i} - 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_B = 400\left(\frac{\mathbf{r}_B}{r_B}\right) = 400\left(\frac{(-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_B = (-324.443\mathbf{i} + 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_C + \mathbf{F}_B = (-648.89\mathbf{i} + 389.33\mathbf{k})$$

$$F_R = \sqrt{(-648.89)^2 + (389.33)^2 + 0^2} = 756.7242$$

$F_R = 757$ N **Ans.**

$\alpha = \cos^{-1}\left(\frac{-648.89}{756.7242}\right) = 149.03 = 149°$ **Ans.**

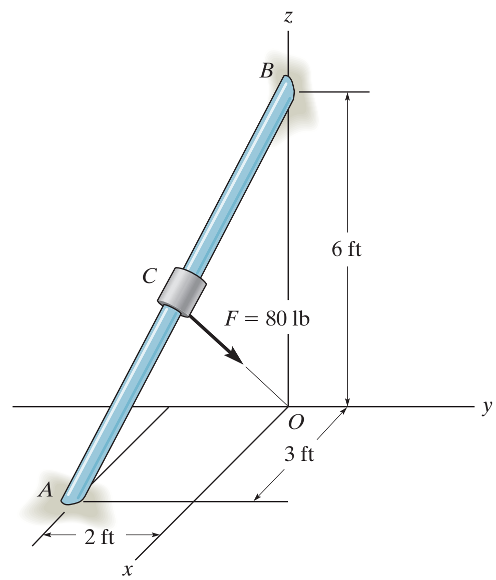
$\beta = \cos^{-1}\left(\frac{0}{756.7242}\right) = 90.0°$ **Ans.**

$\gamma = \cos^{-1}\left(\frac{389.33}{756.7242}\right) = 59.036 = 59.0°$ **Ans.**

Ans:
$F_R = 757$ N
$\alpha = 149°$
$\beta = 90.0°$
$\gamma = 59.0°$

2–98.

The force **F** has a magnitude of 80 lb and acts at the midpoint *C* of the thin rod. Express the force as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AB} = (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$$

$$\mathbf{r}_{CB} = \frac{1}{2}\mathbf{r}_{AB} = (-1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k})$$

$$\mathbf{r}_{CO} = \mathbf{r}_{BO} + \mathbf{r}_{CB}$$

$$= -6\mathbf{k} - 1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k}$$

$$= -1.5\mathbf{i} + 1\mathbf{j} - 3\mathbf{k}$$

$$r_{CO} = 3.5$$

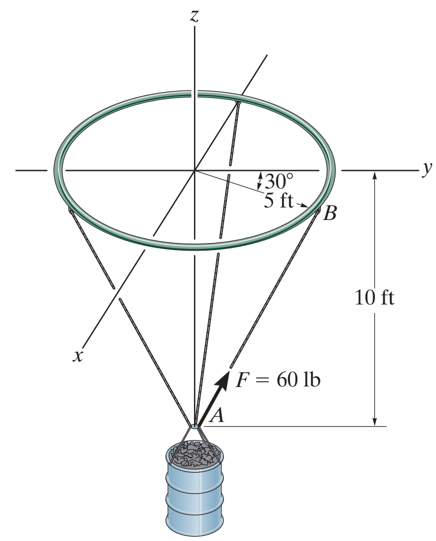
$$F = 80\left(\frac{\mathbf{r}_{CO}}{r_{CO}}\right) = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:

$F = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}$ lb

2–99.

The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



SOLUTION

Unit Vector: First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

$B\,(5 \sin 30°, 5 \cos 30°, 0)$ ft $= B\,(2.50, 4.330, 0)$ ft

Then

$$\mathbf{r}_{AB} = \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\}\text{ ft}$$

$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

Force Vector:

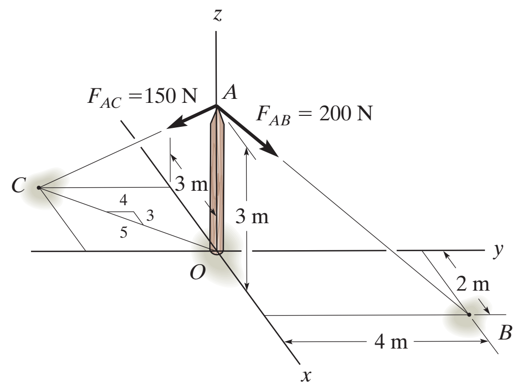
$$\mathbf{F} = F\mathbf{u}_{AB} = 60\,\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\}\text{ lb}$$

$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F} = \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}$ lb

***2–100.**

Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



SOLUTION

Unit Vector. The coordinates for points A, B and C are $A(0, 0, 3)$ m, $B(2, 4, 0)$ m and $C(-3, -4, 0)$ m respectively

$$\mathbf{r}_{AB} = (2-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = \{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 4^2 + (-3)^2}} = \frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}$$

$$\mathbf{r}_{AC} = (-3-0)\mathbf{i} + (-4-0)\mathbf{j} + (0-3)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (-3)^2}} = -\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 200\left(\frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}\right)$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 150\left(-\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}\right)$$

$$= \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}\text{ N}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} + \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}$$

$$= \{-2.896\mathbf{i} + 45.66\mathbf{j} - 188.59\,\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(-2.896)^2 + 45.66^2 + (-188.59)^2}$$

$$= 194.06\text{ N} = 194\text{ N} \qquad \textbf{Ans.}$$

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{-2.896}{194.06}; \qquad \alpha = 90.86^\circ = 90.9^\circ \qquad \textbf{Ans.}$$

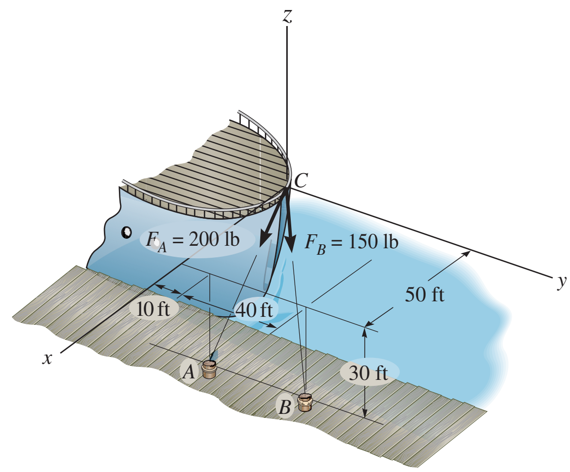
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{45.66}{194.06}; \qquad \beta = 76.39^\circ = 76.4^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-188.59}{194.06}; \qquad \gamma = 166.36^\circ = 166^\circ \qquad \textbf{Ans.}$$

Ans:
$F_R = 194$ N
$\cos\alpha = 90.9^\circ$
$\cos\beta = 76.4^\circ$
$\cos\gamma = 166^\circ$

2–101.

The two mooring cables exert forces on the stern of a ship as shown. Represent each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant.



SOLUTION

Unit Vector:

$$\mathbf{r}_{CA} = \{(50-0)\mathbf{i} + (10-0)\mathbf{j} + (-30-0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}\}\text{ ft}$$

$$r_{CA} = \sqrt{50^2 + 10^2 + (-30)^2} = 59.16\text{ ft}$$

$$\mathbf{u}_{CA} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}}{59.16} = 0.8452\mathbf{i} + 0.1690\mathbf{j} - 0.5071\mathbf{k}$$

$$\mathbf{r}_{CB} = \{(50-0)\mathbf{i} + (50-0)\mathbf{j} + (-30-0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i} + 50\mathbf{j} - 30\mathbf{k}\}\text{ ft}$$

$$r_{CB} = \sqrt{50^2 + 50^2 + (-30)^2} = 76.81\text{ ft}$$

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i} + 50\mathbf{j} - 30\mathbf{k}}{76.81} = 0.6509\mathbf{i} + 0.6509\mathbf{j} - 0.3906\mathbf{k}$$

Force Vector:

$$\mathbf{F}_A = F_A\mathbf{u}_{CA} = 200\{0.8452\mathbf{i} + 0.1690\mathbf{j} - 0.5071\mathbf{k}\}\text{ lb}$$

$$= \{169.03\mathbf{i} + 33.81\mathbf{j} - 101.42\mathbf{k}\}\text{ lb}$$

$$= \{169\mathbf{i} + 33.8\mathbf{j} - 101\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_B = F_B\mathbf{u}_{CB} = 150\{0.6509\mathbf{i} + 0.6509\mathbf{j} - 0.3906\mathbf{k}\}\text{ lb}$$

$$= \{97.64\mathbf{i} + 97.64\mathbf{j} - 58.59\mathbf{k}\}\text{ lb}$$

$$= \{97.6\mathbf{i} + 97.6\mathbf{j} - 58.6\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B$$

$$= \{(169.03 + 97.64)\mathbf{i} + (33.81 + 97.64)\mathbf{j} + (-101.42 - 58.59)\mathbf{k}\}\text{ lb}$$

$$= \{266.67\mathbf{i} + 131.45\mathbf{j} - 160.00\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{266.67^2 + 131.45^2 + (-160.00)^2}$$

$$= 337.63\text{ lb} = 338\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\cos\alpha = \frac{266.67}{337.63} \qquad \alpha = 37.8° \qquad \textbf{Ans.}$$

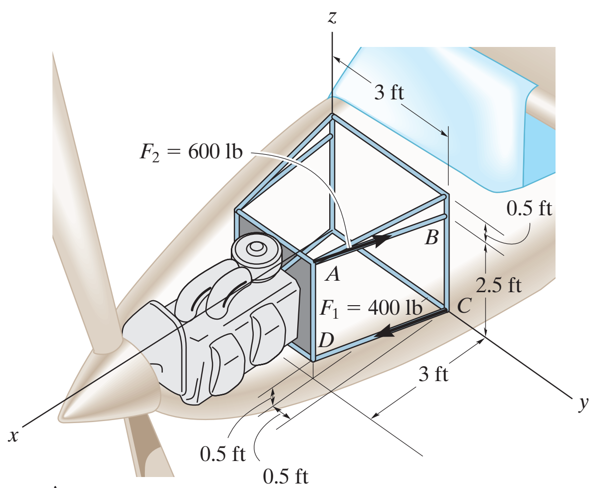
$$\cos\beta = \frac{131.45}{337.63} \qquad \beta = 67.1° \qquad \textbf{Ans.}$$

$$\cos\gamma = -\frac{160.00}{337.63} \qquad \gamma = 118° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_A = \{169\mathbf{i} + 33.8\mathbf{j} - 101\mathbf{k}\}$ lb
$\mathbf{F}_B = \{97.6\mathbf{i} + 97.6\mathbf{j} - 58.6\mathbf{k}\}$ lb
$F_R = 338$ lb
$\alpha = 37.8°$
$\beta = 67.1°$
$\gamma = 118°$

2–102.

The engine of the lightweight plane is supported by struts that are connected to the space truss that makes up the structure of the plane. The anticipated loading in two of the struts is shown. Express each of these forces as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = 400\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = 400\left(\frac{3}{3.0822}\,\mathbf{i} - \frac{0.5}{3.0822}\,\mathbf{j} + \frac{0.5}{3.0822}\,\mathbf{k}\right)$$

$$= \{389\,\mathbf{i} - 64.9\,\mathbf{j} + 64.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

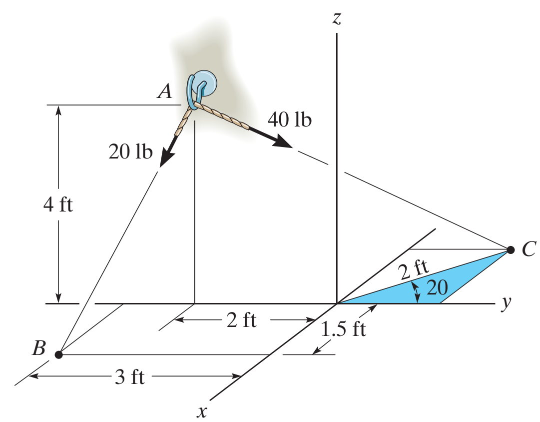
$$\mathbf{F}_2 = 600\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 600\left(-\frac{3}{3.0822}\,\mathbf{i} - \frac{0.5}{3.0822}\,\mathbf{j} + \frac{0.5}{3.0822}\,\mathbf{k}\right)$$

$$= \{-584\,\mathbf{i} + 97.3\,\mathbf{j} - 97.3\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{389\mathbf{i} - 64.9\mathbf{j} + 64.9\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{-584\mathbf{i} + 97.3\mathbf{j} - 97.3\mathbf{k}\}$ lb

2–103.

Determine the magnitude and coordinates on angles of the resultant force.



SOLUTION

$\mathbf{r}_{AC} = \{-2 \sin 20°\mathbf{i} + (2 + 2 \cos 20°)\mathbf{j} - 4\,\mathbf{k}\}$ ft

$\mathbf{u}_{AC} = \left(\dfrac{\mathbf{r}_{AC}}{r_{AC}}\right) = -0.1218\mathbf{i} + 0.6910\,\mathbf{j} - 0.7125\,\mathbf{k}$

$\mathbf{F}_{Ac} = 4\text{ lb}\mathbf{u}_{AC} = \{-4.874\mathbf{i} + 27.64\,\mathbf{j} - 28.50\,\mathbf{k}\}$ lb

$\mathbf{r}_{AB} = \{1.5\,\mathbf{i} - 1\,\mathbf{j} - 4\,\mathbf{k}\}$ ft

$\mathbf{u}_{AB} = \left(\dfrac{\mathbf{r}_{AB}}{r_{AB}}\right) = 0.3419\,\mathbf{i} + 0.2279\,\mathbf{j} - 0.9117\,\mathbf{k}$

$\mathbf{F}_{AB} = 20\text{ lb }\mathbf{u}_{AB} = \{6.838\mathbf{i} - 4.558\,\mathbf{j} - 18.23\,\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$

$\mathbf{F}_R = \{1.964\,\mathbf{i} + 23.08\,\mathbf{j} - 46.73\,\mathbf{k}\}$lb

$F_R = \sqrt{(1.964)^2 + (23.08)^2 + (-46.73)^2} = 52.16 = 52.2$ lb **Ans.**

$\alpha = \cos^{-1}\left(\dfrac{1.964}{52.16}\right) = 87.8°$ **Ans.**

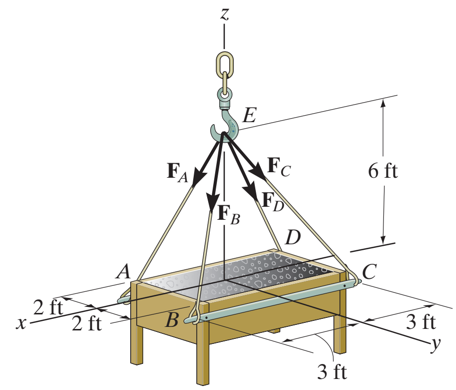
$\beta = \cos^{-1}\left(\dfrac{23.08}{52.16}\right) = 63.7°$ **Ans.**

$\gamma = \cos^{-1}\left(\dfrac{-46.73}{52.16}\right) = 154°$ **Ans.**

Ans:
$F_R = 52.2$ lb
$\alpha = 87.8°$
$\beta = 63.7°$
$\gamma = 154°$

***2–104.**

If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 70\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 70\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}]\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) + (30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k})$$
$$= \{-240\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

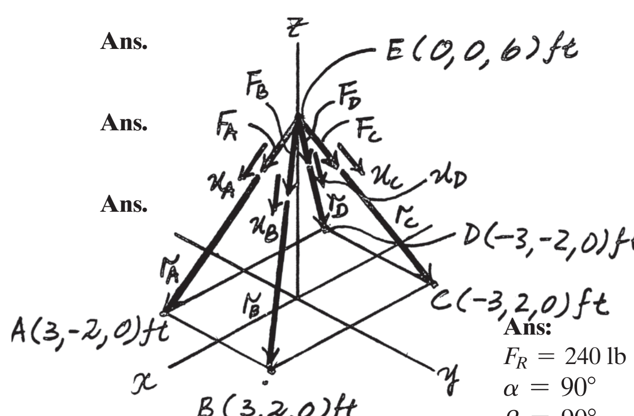
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{0 + 0 + (-240)^2} = 240\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90° \qquad \textbf{Ans.}$$

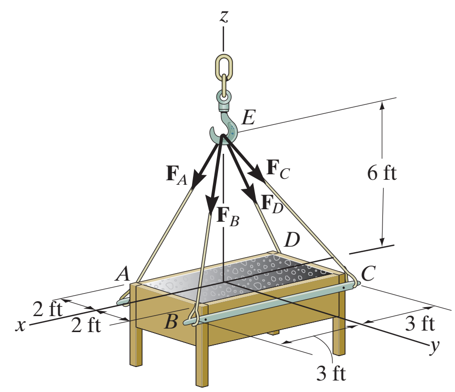
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-240}{240}\right) = 180° \qquad \textbf{Ans.}$$



Ans:
$F_R = 240$ lb
$\alpha = 90°$
$\beta = 90°$
$\gamma = 180°$

2–105.

If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_A, \mathbf{u}_B, \mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_A, \mathbf{F}_B, \mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Since the magnitudes of $\mathbf{F}_A, \mathbf{F}_B, \mathbf{F}_C$, and $\mathbf{F}_D$ are the same and denoted as F, the four vectors or forces can be written as

$$\mathbf{F}_A = F_A\mathbf{u}_A = F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

Resultant Force: The vector addition of $\mathbf{F}_A, \mathbf{F}_B, \mathbf{F}_C$, and $\mathbf{F}_D$ is equal to $\mathbf{F}_R$. Thus,

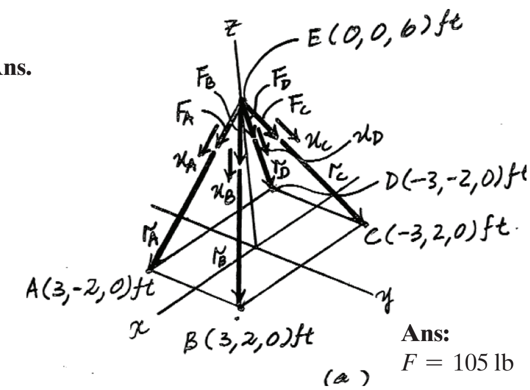
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$\{-360\mathbf{k}\} = \left[F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) + \left[F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right]\right.$$

$$-360\mathbf{k} = -\frac{24}{7}\mathbf{k}$$

Thus,

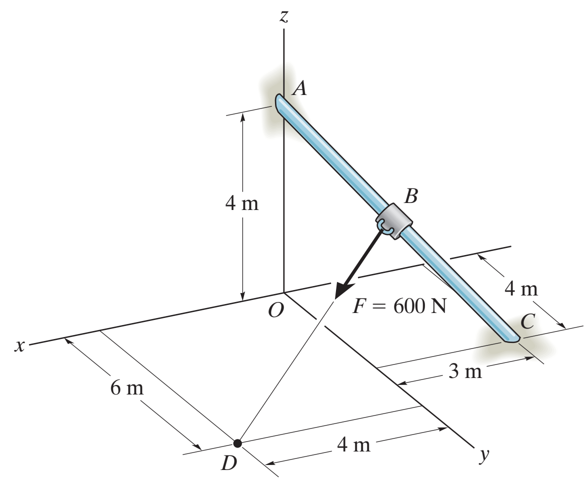
$$360 = \frac{24}{7}F \qquad F = 105 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$F = 105$ lb

2–106.

Express the force **F** in Cartesian vector form if it acts at the midpoint B of the rod.



SOLUTION

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{AC}}{2} = \frac{-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$$

$$\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$$

$$= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$$

$$= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

$$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887\text{ m}$$

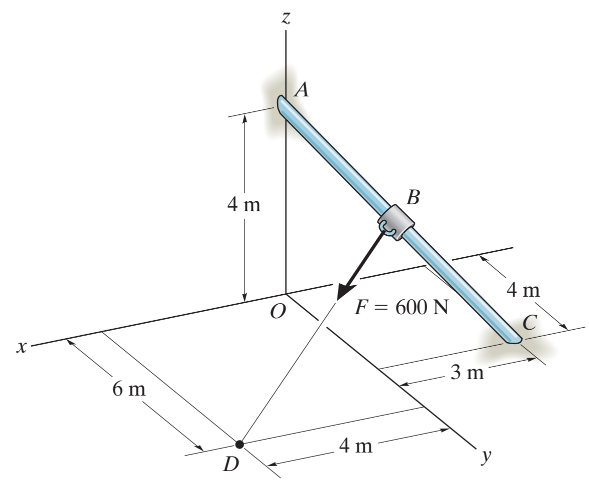
$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$$

$$\mathbf{F} = \{466\mathbf{i} + 339\mathbf{j} - 169\mathbf{k}\}\text{ N}$$ **Ans.**

Ans:
$\mathbf{F} = \{466\mathbf{i} + 339\mathbf{j} - 169\mathbf{k}\}$ N

2–107.

Express force **F** in Cartesian vector form if point *B* is located 3 m along the rod end *C*.



SOLUTION

$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

$r_{CA} = 6.403124$

$\mathbf{r}_{CB} = \dfrac{3}{6.403124}(\mathbf{r}_{CA}) = 1.4056\mathbf{i} - 1.8741\mathbf{j} + 1.8741\mathbf{k}$

$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$

$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$

$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$

$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$

$r_{BD} = \sqrt{(5.5914)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$

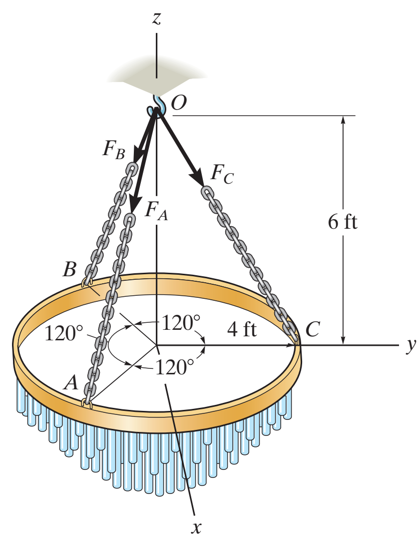
$\mathbf{F} = 600\left(\dfrac{\mathbf{r}_{BD}}{r_{BD}}\right) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$

$\mathbf{F} = \{476\mathbf{i} + 329\mathbf{j} - 159\mathbf{k}\}$ N **Ans.**

Ans:
$\mathbf{F} = \{476\mathbf{i} + 329\mathbf{j} - 159\mathbf{k}\}$ N

***2–108.**

The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{F}_A = 60\,\frac{(4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_B = 60\,\frac{(-4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\mathbf{k})}{\sqrt{(-4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{-28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_C = 60\,\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4)^2 + (-6)^2}}$$

$$= \{33.3\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-149.8\,\mathbf{k}\}\text{ lb}$$

$$F_R = 150\text{ lb} \qquad \textbf{Ans.}$$

$$\alpha = 90° \qquad \textbf{Ans.}$$

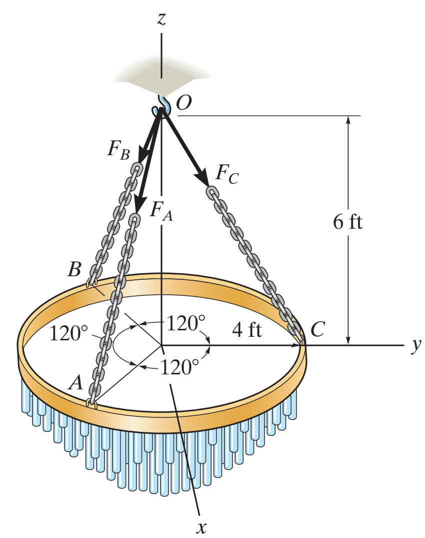
$$\beta = 90° \qquad \textbf{Ans.}$$

$$\gamma = 180° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_A = \{28.8\mathbf{i} - 16.6\mathbf{j} - 49.9\mathbf{k}\}$ lb
$\mathbf{F}_B = \{-28.8\mathbf{i} - 16.6\mathbf{j} - 49.9\mathbf{k}\}$ lb
$\mathbf{F}_C = \{33.3\mathbf{j} - 49.9\mathbf{k}\}$ lb
$F_R = 150$ lb
$\alpha = 90°$
$\beta = 90°$
$\gamma = 180°$

2–109.

The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



SOLUTION

$$\mathbf{F}_C = F\,\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547\,F\mathbf{j} - 0.8321\,F\mathbf{k}$$

$$\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$$

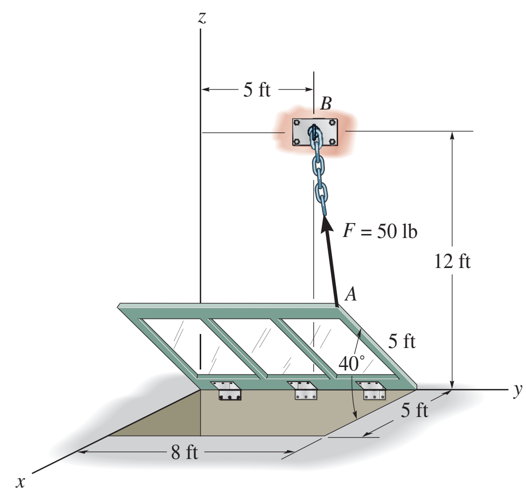
$$F_{Rz} = \Sigma F_z; \qquad 130 = 3(0.8321F)$$

$$F = 52.1 \text{ lb}$$ **Ans.**

Ans:
$F = 52.1$ lb

2–110.

The window is held open by chain AB. Determine the length of the chain, and express the 50-lb force acting at A along the chain as a Cartesian vector and determine its coordinate direction angles.



SOLUTION

Unit Vector: The coordinates of point A are

$$A(5\cos 40°, 8, 5\sin 40°)\text{ ft} = A(3.830, 8.00, 3.214)\text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(0 - 3.830)\mathbf{i} + (5 - 8.00)\mathbf{j} + (12 - 3.214)\mathbf{k}\}\text{ ft}$$

$$= \{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(-3.830)^2 + (-3.00)^2 + 8.786^2} = 10.043\text{ ft} = 10.0\text{ ft} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}}{10.043}$$

$$= -0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 50\{-0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}\}\text{ lb}$$

$$= \{-19.1\mathbf{i} - 14.9\mathbf{j} + 43.7\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

Coordinate Direction Angles: From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$$\cos\alpha = -0.3814 \qquad \alpha = 112° \qquad \textbf{Ans.}$$

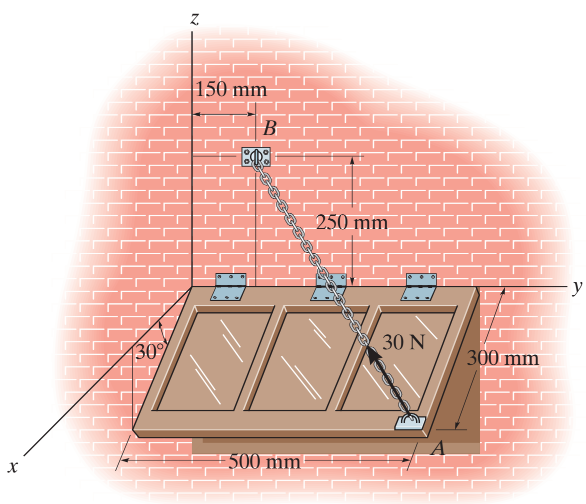
$$\cos\beta = -0.2987 \qquad \beta = 107° \qquad \textbf{Ans.}$$

$$\cos\gamma = 0.8748 \qquad \gamma = 29.0° \qquad \textbf{Ans.}$$

Ans:
$r_{AB} = 10.0$ ft
$\mathbf{F} = \{-19.1\mathbf{i} - 14.9\mathbf{j} + 43.7\mathbf{k}\}$ lb
$\alpha = 112°$
$\beta = 107°$
$\gamma = 29.0°$

2–111.

The window is held open by cable AB. Determine the length of the cable and express the 30-N force acting at A along the cable as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AB} = (0-300\cos 30°)\mathbf{i} + (150 - 500)\mathbf{j} + (250 + 300\sin 30°)\mathbf{k}$$

$$= -259.81\,\mathbf{i} - 350\,\mathbf{j} + 400\,\mathbf{k}$$

$$r_{AB} = \sqrt{(-259.81)^2 + (-350)^2 + (400)^2} = 591.61$$

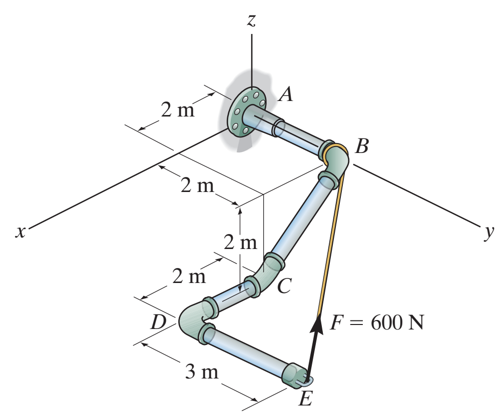
$$= 592 \text{ mm}$$ **Ans.**

$$\mathbf{F} = 30\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = \{-13.2\,\mathbf{i} - 17.7\,\mathbf{j} + 20.3\mathbf{k}\}\text{ N}$$ **Ans.**

Ans:
$r_{AB} = 592$ mm
$\mathbf{F} = \{-13.2\mathbf{i} - 17.7\mathbf{j} + 20.3\mathbf{k}\}$ N

***2–112.**

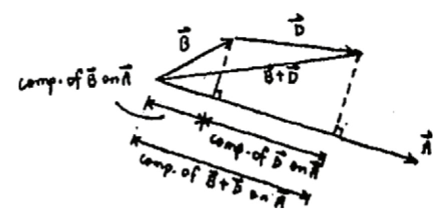
Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.



SOLUTION

Since the component of (**B** + **D**) is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \textbf{(QED)}$$

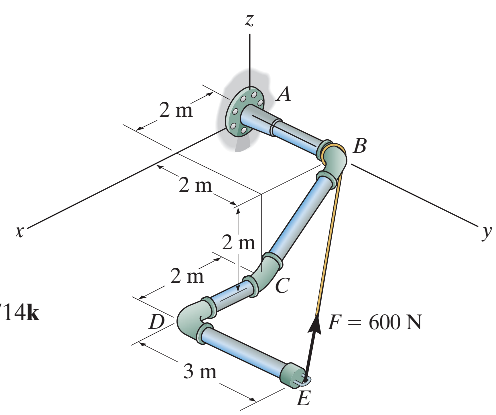


Also,

$$\begin{aligned}
\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) &= (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}] \\
&= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z) \\
&= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z) \\
&= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \textbf{(QED)}
\end{aligned}$$

2–113.

Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600\left(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}\right) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the component of **F** *p*arallel to segment DE of the pipe assembly is

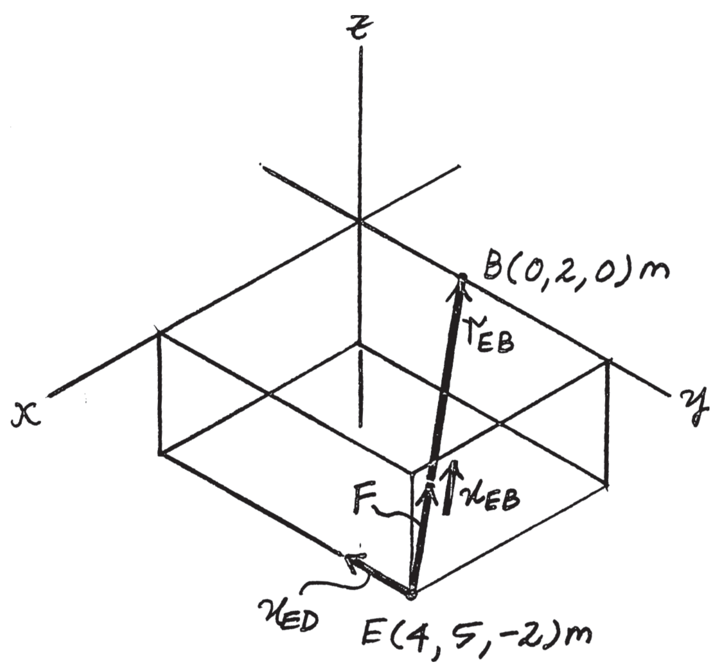
$$(F_{ED})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{ED} = \left(-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}\right)\cdot\left(-\mathbf{j}\right)$$

$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$

$$= 334.25 = 334\text{ N} \qquad \textbf{Ans.}$$

The component of **F** perpendicular to segment DE of the pipe assembly is

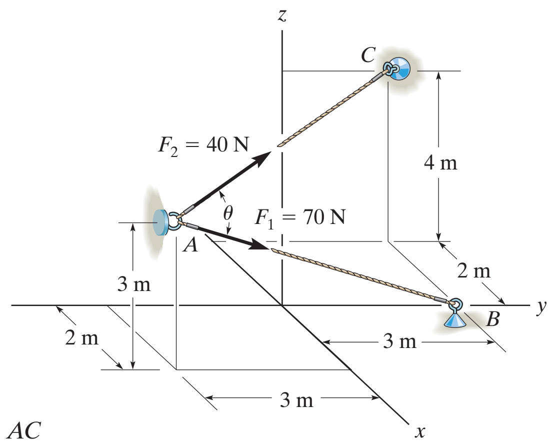
$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}{}^2} = \sqrt{600^2 - 334.25^2} = 498\text{ N} \qquad \textbf{Ans.}$$



Ans:
$(F_{ED})_{||} = 334$ N
$(F_{ED})_{\perp} = 498$ N

2–114.

Determine the angle θ between the two cables.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$ m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

The Angle θ Between AB and AC.

$$\mathbf{u}_{AB} \cdot \mathbf{u}_{AC} = \left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= \left(-\frac{2}{7}\right)\left(-\frac{4}{\sqrt{53}}\right) + \frac{6}{7}\left(\frac{6}{\sqrt{53}}\right) + \left(-\frac{3}{7}\right)\left(\frac{1}{\sqrt{53}}\right)$$

$$= \frac{41}{7\sqrt{53}}$$

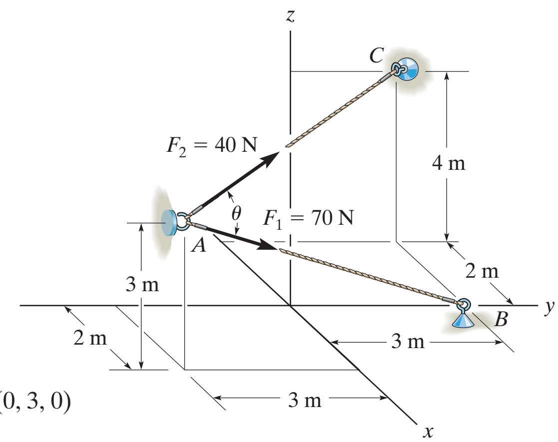
Then

$$\theta = \cos^{-1}\left(\mathbf{u}_{AB} \cdot \mathbf{u}_{AC}\right) = \cos^{-1}\left(\frac{41}{7\sqrt{53}}\right) = 36.43^\circ = 36.4^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 36.4^\circ$

2–115.

Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

Force Vector, For $\mathbf{F}_1$,

$$\mathbf{F}_1 = F_1\mathbf{u}_{AB} = 70\left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) = \{-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}\}\text{ N}$$

Projected Component of $\mathbf{F}_1$. Along AC, it is

$$(F_1)_{AC} = \mathbf{F}_1 \cdot \mathbf{u}_{AC} = (-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= (-20)\left(-\frac{4}{\sqrt{53}}\right) + 60\left(\frac{6}{\sqrt{53}}\right) + (-30)\left(\frac{1}{\sqrt{53}}\right)$$

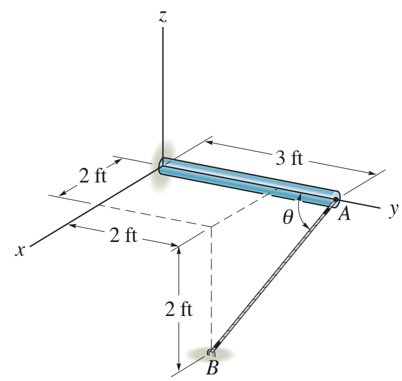
$$= 56.32\text{ N} = 56.3\text{ N}$$ **Ans.**

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{AC}$.

Ans:
$(F_1)_{AC} = 56.3$ N

***2–116.**

Determine the angle θ between the y axis of the pole and the wire AB.



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\} \text{ ft}$$

$$\mathbf{r}_{AB} = \{(2 - 0)\mathbf{i} + (2 - 3)\mathbf{j} + (-2 - 0)\mathbf{k}\} \text{ ft}$$

$$= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00 \text{ ft} \qquad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00 \text{ ft}$$

The Angles Between Two Vectors θ: The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k})$$

$$= 0(2) + (-3)(-1) + 0(-2)$$

$$= 3$$

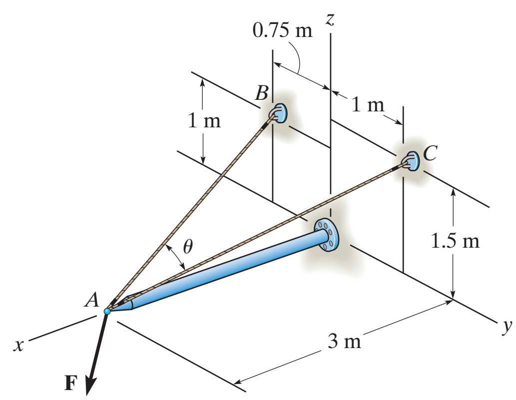
Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5° \qquad \textbf{Ans.}$$

Ans:
$\theta = 70.5°$

2–117.

Determine the magnitudes of the projected components of the force $\mathbf{F} = [60\mathbf{i} + 12\mathbf{j} - 40\mathbf{k}]$ N along the cables AB and AC.



SOLUTION

$$\mathbf{F} = \{60\,\mathbf{i} + 12\,\mathbf{j} - 40\,\mathbf{k}\}\text{ N}$$

$$\mathbf{u}_{AB} = \frac{-3\,\mathbf{i} - 0.75\,\mathbf{j} + 1\,\mathbf{k}}{\sqrt{(-3)^2 + (-0.75)^2 + (1)^2}}$$

$$= -0.9231\,\mathbf{i} - 0.2308\,\mathbf{j} + 0.3077\,\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{-3\,\mathbf{i} + 1\,\mathbf{j} + 1.5\,\mathbf{k}}{\sqrt{(-3)^2 + (1)^2 + (1.5)^2}}$$

$$= -0.8571\,\mathbf{i} + 0.2857\,\mathbf{j} + 0.4286\,\mathbf{k}$$

$$\text{Proj } F_{AB} = \mathbf{F}\cdot\mathbf{u}_{AB} = (60)(-0.9231) + (12)(-0.2308) + (-40)(0.3077)$$

$$= -70.46\text{ N}$$

$$|\text{Proj } F_{AB}| = 70.5\text{ N}$$ **Ans.**

$$\text{Proj } F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (60)(-0.8571) + (12)(0.2857) + (-40)(0.4286)$$

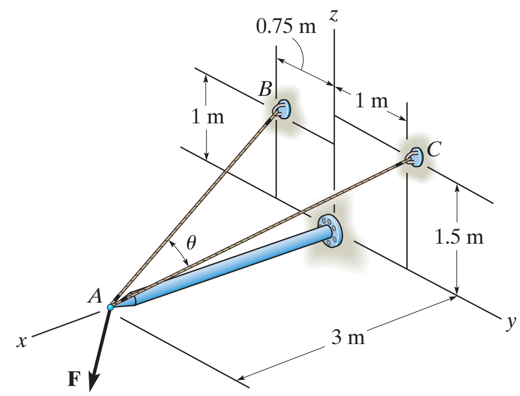
$$= -65.14\text{ N}$$

$$|\text{Proj } F_{AC}| = 65.1\text{ N}$$ **Ans.**

Ans:
$|\text{Proj } F_{AB}| = 70.5$ N
$|\text{Proj } F_{AC}| = 65.1$ N

2–118.

Determine the angle θ between cables AB and AC.



SOLUTION

$\mathbf{r}_{AB} = \{-3\,\mathbf{i} - 0.75\,\mathbf{j} + 1\,\mathbf{k}\}$ m

$r_{AB} = \sqrt{(-3)^2 + (-0.75)^2 + (1)^2} = 3.25$ m

$\mathbf{r}_{AC} = \{-3\,\mathbf{i} + 1\,\mathbf{j} + 1.5\,\mathbf{k}\}$ m

$r_{AC} = \sqrt{(-3)^2 + (1)^2 + (1.5)^2} = 3.50$ m

$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (-3)(-3) + (-0.75)(1) + (1)(1.5) = 9.75$

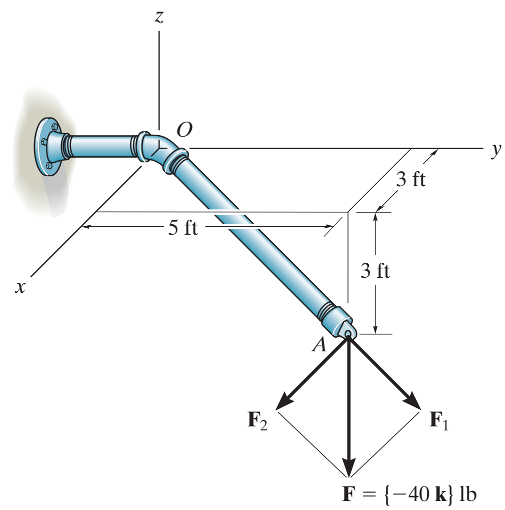
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{r_{AB}\, r_{AC}}\right) = \cos^{-1}\left(\frac{9.75}{(3.25)(3.50)}\right)$$

$\theta = 31.0°$ **Ans.**

Ans:
$\theta = 31.0°$

2–119.

A force of $\mathbf{F} = \{-40\mathbf{k}\}$ lb acts at the end of the pipe. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which are directed along the pipe's axis and perpendicular to it.



SOLUTION

$$\mathbf{u}_{OA} = \frac{3\,\mathbf{i} + 5\,\mathbf{j} - 3\mathbf{k}}{\sqrt{3^2 + 5^2 + (-3)^2}} = \frac{3\,\mathbf{i} + 5\,\mathbf{j} - 3\,\mathbf{k}}{\sqrt{43}}$$

$$F_1 = \mathbf{F}\cdot\mathbf{u}_{OA} = (-40\,\mathbf{k})\cdot\left(\frac{3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}}{\sqrt{43}}\right)$$

$$= 18.3 \text{ lb}$$ **Ans.**

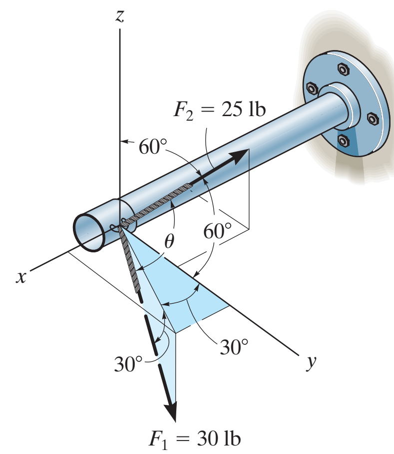
$$F_2 = \sqrt{F_2 - F_1^2}$$

$$F_2 = \sqrt{40^2 - 18.3^2} = 35.6 \text{ lb}$$ **Ans.**

Ans:
$F_1 = 18.3$ lb
$F_2 = 35.6$ lb

***2–120.**

Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R\mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k})\text{ lb}$$

$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\}\text{ lb}$$

Unit Vector: One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

***Projected Component of* $\mathbf{F}_1$ *Along the Line of Action of* $\mathbf{F}_2$:**

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

$$= -5.44\text{ lb}$$

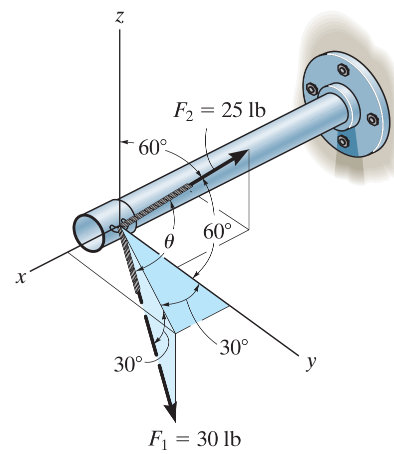
Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

Ans:
The magnitude is $(F_1)_{F_2} = 5.44$ lb

2–121.

Determine the angle θ between the two cables attached to the pipe.



SOLUTION

Unit Vectors:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

$$= -0.1812$$

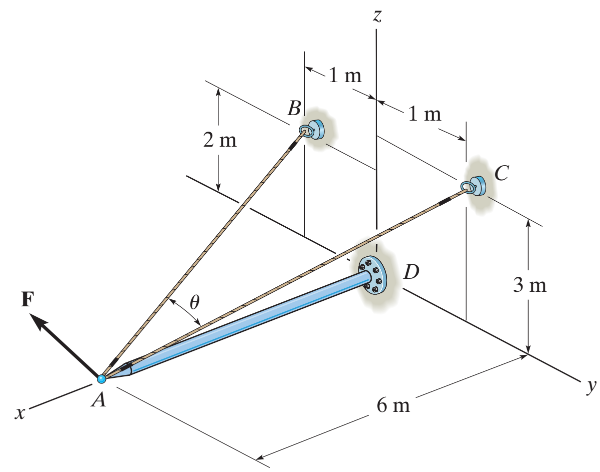
Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100° \qquad \textbf{Ans.}$$

Ans:
$\theta = 100°$

2–122.

Determine the angle θ between the cables AB and AC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(6, 0, 0)$ m, $B(0, -1, 2)$ m and $C(0, 1, 3)$ respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-6)\mathbf{i} + (-1-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-1-0)^2 + (2-0)^2}} = -\frac{6}{\sqrt{41}}\mathbf{i} - \frac{1}{\sqrt{41}}\mathbf{j} + \frac{2}{\sqrt{41}}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(0-6)\mathbf{i} + (1-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (1-0)^2 + (3-0)^2}} = -\frac{6}{\sqrt{46}}\mathbf{i} + \frac{1}{\sqrt{46}}\mathbf{j} + \frac{3}{\sqrt{46}}\mathbf{k}$$

The Angle θ Between AB and AC.

$$\mathbf{u}_{AB} \cdot \mathbf{u}_{AC} = \left(-\frac{6}{\sqrt{41}}\mathbf{i} - \frac{1}{\sqrt{41}}\mathbf{j} + \frac{2}{\sqrt{41}}\mathbf{k}\right) \cdot \left(-\frac{6}{\sqrt{46}}\mathbf{i} + \frac{1}{\sqrt{46}}\mathbf{j} + \frac{3}{\sqrt{46}}\mathbf{k}\right)$$

$$= \left(-\frac{6}{\sqrt{41}}\right)\left(-\frac{6}{\sqrt{46}}\right) + \left(-\frac{1}{\sqrt{41}}\right)\left(\frac{1}{\sqrt{46}}\right) + \frac{2}{\sqrt{41}}\left(\frac{3}{\sqrt{46}}\right)$$

$$= \frac{41}{\sqrt{1886}}$$

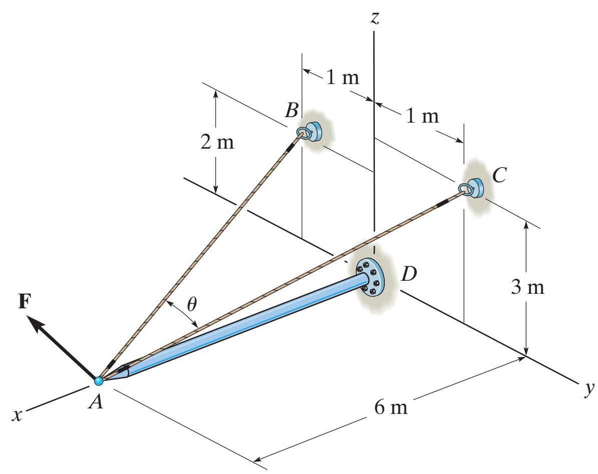
Then

$$\theta = \cos^{-1}\left(U_{AB} \cdot U_{AC}\right) = \cos^{-1}\left(\frac{41}{\sqrt{1886}}\right) = 19.24998^\circ = 19.2^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 19.2^\circ$

2–123.

Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable BA.



SOLUTION

Unit Vector. Here, the coordinates of points A and B are $A(6, 0, 0)$ m and $B(0, -1, 2)$ m respectively. Thus the unit vector along BA is

$$\mathbf{u}_{BA} = \frac{\mathbf{r}_{BA}}{r_{BA}} = \frac{(6-0)\mathbf{i} + [0-(-1)]\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(6-0)^2 + [0-(-1)]^2 + (0-2)^2}} = \frac{6}{\sqrt{41}}\mathbf{i} + \frac{1}{\sqrt{41}}\mathbf{j} - \frac{2}{\sqrt{41}}\mathbf{k}$$

Projected component of F. Along BA, it is

$$F_{BA} = \mathbf{F}\cdot\mathbf{u}_{BA} = (400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k})\cdot\left(\frac{6}{\sqrt{41}}\mathbf{i} + \frac{1}{\sqrt{41}}\mathbf{j} - \frac{2}{\sqrt{41}}\mathbf{k}\right)$$

$$= 400\left(\frac{6}{\sqrt{41}}\right) + (-200)\left(\frac{1}{\sqrt{41}}\right) + 500\left(-\frac{2}{\sqrt{41}}\right)$$

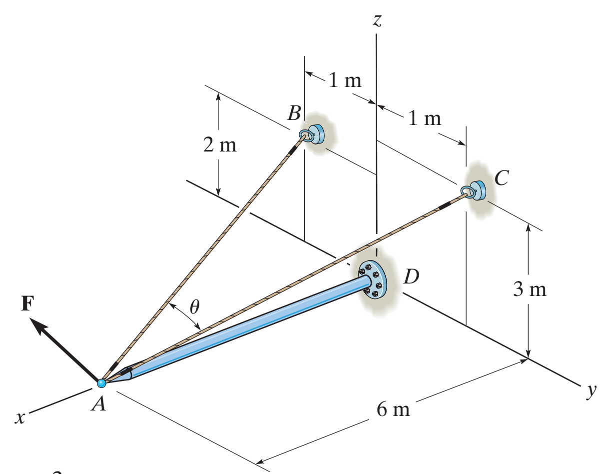
$$= 187.41 \text{ N} = 187 \text{ N}$$ **Ans.**

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{BA}$.

Ans:
$F_{BA} = 187$ N

***2–124.**

Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable CA.



SOLUTION

Unit Vector. Here, the coordinates of points A and C are $A(6, 0, 0)$ m and $C(0, 1, 3)$ m respectively. Thus, the unit vector along CA is

$$\mathbf{u}_{CA} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{(6-0)\mathbf{i} + (0-1)\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(6-0)^2 + (0-1)^2 + (0-3)^2}} = \frac{6}{\sqrt{46}}\mathbf{i} - \frac{1}{\sqrt{46}}\mathbf{j} - \frac{3}{\sqrt{46}}\mathbf{k}$$

Projected component of F. Along CA, it is

$$F_{CA} = \mathbf{F}\cdot\mathbf{u}_{CA} = (400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k})\cdot\left(\frac{6}{\sqrt{46}}\mathbf{i} - \frac{1}{\sqrt{46}}\mathbf{j} - \frac{3}{\sqrt{46}}\mathbf{k}\right)$$

$$= 400\left(\frac{6}{\sqrt{46}}\right) + (-200)\left(-\frac{1}{\sqrt{46}}\right) + 500\left(-\frac{3}{\sqrt{46}}\right)$$

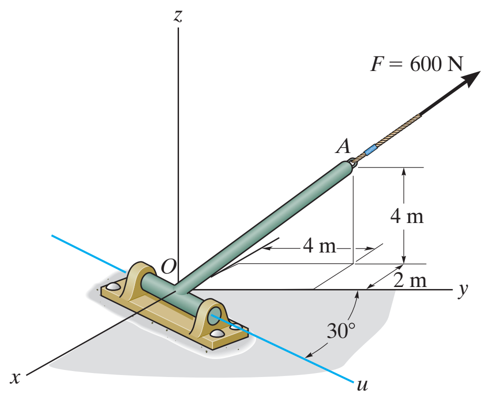
$$= 162.19 \text{ N} = 162 \text{ N}$$ **Ans.**

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{CA}$.

Ans:
$F_{CA} = 162$ N

2–125.

Determine the magnitude of the projection of force $F = 600$ N along the u axis.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_u$ must be determined first. From Fig. a,

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-2-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (4-0)^2 + (4-0)^2}} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_u = \sin 30°\mathbf{i} + \cos 30°\mathbf{j}$$

Thus, the force vectors $\mathbf{F}$ is given by

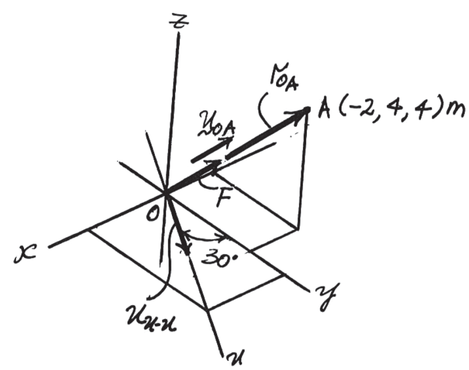
$$\mathbf{F} = F\,\mathbf{u}_{OA} = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ along the u axis is

$$\mathbf{F}_u = F \cdot \mathbf{u}_u = (-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}) \cdot (\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= (-200)(\sin 30°) + 400(\cos 30°) + 400(0)$$

$$= 246 \text{ N}$$ **Ans.**

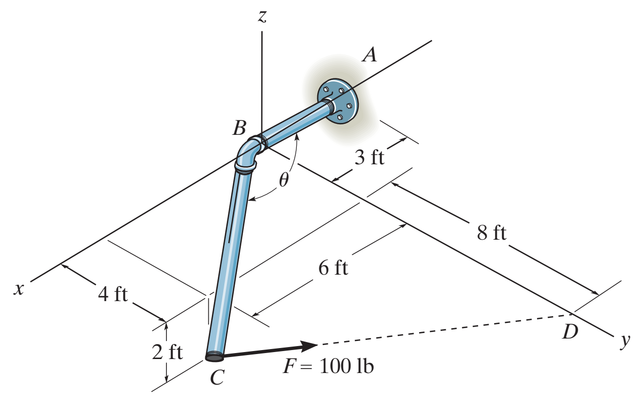


(a)

Ans:
$F_u = 246$ N

2–126.

Determine the magnitude of the projected component of the 100-lb force acting along the axis BC of the pipe.



SOLUTION

$$\vec{\gamma}_{BC} = \{6\hat{i} + 4\hat{j} - 2\hat{k}\}\ \text{ft}$$

$$\vec{F} = 100\,\frac{\{-6\hat{i} + 8\hat{j} + 2\hat{k}\}}{\sqrt{(-6)^2 + 8^2 + 2^2}}$$

$$= \{-58.83\hat{i} + 78.45\hat{j} + 19.61\hat{k}\}\ \text{lb}$$

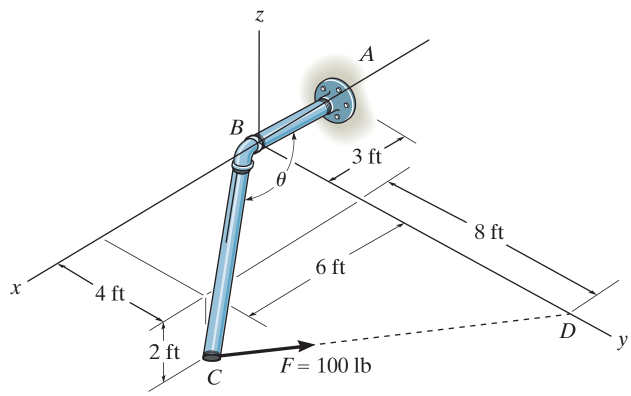
$$F_p = \vec{F} \cdot \vec{\mu}_{BC} = \vec{F} \cdot \frac{\vec{\gamma}_{BC}}{|\vec{\gamma}_{BC}|} = \frac{-78.45}{7.483} = -10.48$$

$$F_p = 10.5\ \text{lb}$$ **Ans.**

Ans:
$F_p = 10.5$ lb

2–127.

Determine the angle θ between pipe segments BA and BC.



SOLUTION

$$\vec{\gamma}_{BC} = \{6\hat{\imath} + 4\hat{\jmath} - 2\hat{k}\}\text{ ft}$$

$$\vec{\gamma}_{BA} = \{-3\hat{\imath}\}\text{ ft}$$

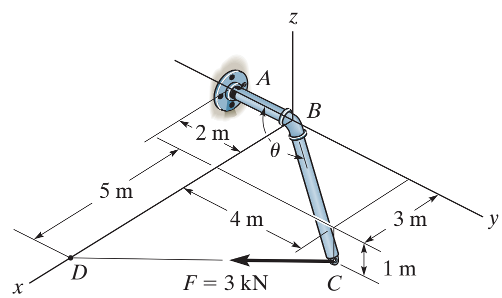
$$\theta = \cos^{-1}\left(\frac{\vec{\gamma}_{BC}\cdot\vec{\gamma}_{BA}}{|\vec{\gamma}_{BC}||\vec{\gamma}_{BA}|}\right) = \cos^{-1}\left(\frac{-18}{22.45}\right)$$

$\theta = 143°$ **Ans.**

Ans:
$\theta = 142°$

***2–128.**

Determine the angle θ between BA and BC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(0, -2, 0)$ m, $B(0, 0, 0)$ m and $C(3, 4, -1)$ m respectively. Thus, the unit vectors along BA and BC are

$$\mathbf{u}_{BA} = -\mathbf{j} \qquad \mathbf{u}_{BE} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{\mathbf{4}}{\sqrt{\mathbf{26}}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

The Angle θ Between BA and BC.

$$\mathbf{u}_{BA}\,\mathbf{u}_{BC} = (-\,\mathbf{j}) \cdot \left(\frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}\right)$$

$$= (-1)\left(\frac{4}{\sqrt{26}}\right) = -\frac{4}{\sqrt{26}}$$

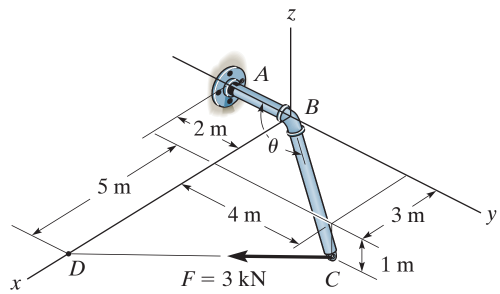
Then

$$\theta = \cos^{-1}(\mathbf{u}_{BA} \cdot \mathbf{u}_{BC}) = \cos^{-1}\left(-\frac{4}{\sqrt{26}}\right) = 141.67^\circ = 142^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 142^\circ$

2–129.

Determine the magnitude of the projected component of the 3 kN force acting along the axis BC of the pipe.



SOLUTION

Unit Vectors. Here, the coordinates of points B, C and D are $B\,(0, 0, 0)$ m, $C(3, 4, -1)$ m and $D(8, 0, 0)$. Thus the unit vectors along BC and CD are

$$\mathbf{u}_{BC} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

$$\mathbf{u}_{CD} = \frac{(8-3)\,\mathbf{i} + (0-4)\,\mathbf{j} + [0-(-1)]\,\mathbf{k}}{\sqrt{(8-3)^2 + (0-4)^2 + [0-(-1)]^2}} = \frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}$$

Force Vector. For $\mathbf{F}$,

$$\mathbf{F} = F\mathbf{u}_{CD} = 3\left(\frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}\right)$$

$$= \left(\frac{15}{\sqrt{42}}\,\mathbf{i} - \frac{12}{\sqrt{42}}\,\mathbf{j} + \frac{3}{\sqrt{42}}\,\mathbf{k}\right)\text{ kN}$$

Projected Component of F. Along BC, it is

$$\left|(F_{BC})\right| = \left|\mathbf{F}\cdot\mathbf{u}_{BC}\right| = \left|\left(\frac{15}{\sqrt{42}}\mathbf{i} - \frac{12}{\sqrt{42}}\mathbf{j} + \frac{3}{\sqrt{42}}\mathbf{k}\right)\cdot\left(\frac{3}{\sqrt{26}}\mathbf{i} + \frac{4}{\sqrt{26}}\mathbf{j} - \frac{1}{\sqrt{26}}\mathbf{k}\right)\right|$$

$$= \left|\left(\frac{15}{\sqrt{42}}\right)\left(\frac{3}{\sqrt{26}}\right) + \left(-\frac{12}{\sqrt{42}}\right)\left(\frac{4}{\sqrt{26}}\right) + \frac{3}{\sqrt{42}}\left(-\frac{1}{\sqrt{26}}\right)\right|$$

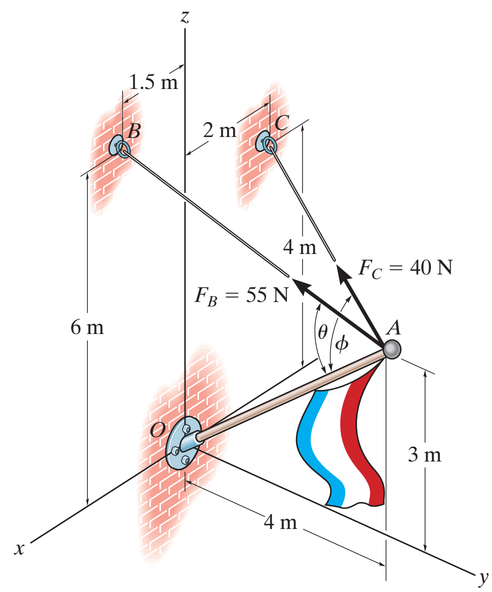
$$= \left|-\frac{6}{\sqrt{1092}}\right| = \left|-0.1816\text{ kN}\right| = 0.182\text{ kN}$$ **Ans.**

The negative signs indicate that this component points in the direction opposite to that of $\mathbf{u}_{BC}$.

Ans:
0.182 kN

2–130.

Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m ; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$ m

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4°$$ **Ans.**

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

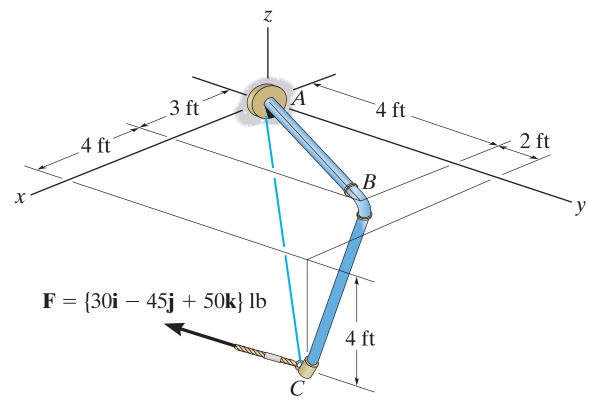
$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4°$$ **Ans.**

Ans:
$\theta = 74.4°$
$\phi = 55.4°$

2–131.

Determine the magnitudes of the components of **F** acting along and perpendicular to segment *BC* of the pipe assembly.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*

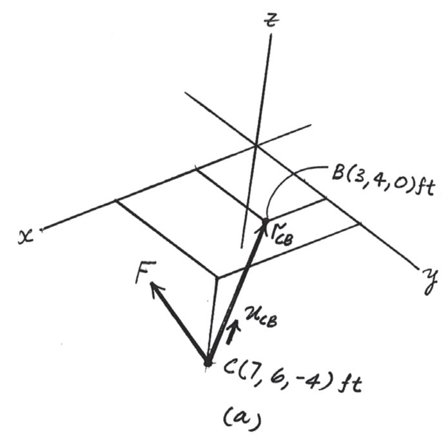
$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to segment *BC* of the pipe assembly is

$$(F_{BC})_{pa} = \mathbf{F}\cdot\mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

$$= 28.33 \text{ lb} = 28.3 \text{ lb} \qquad \textbf{Ans.}$$



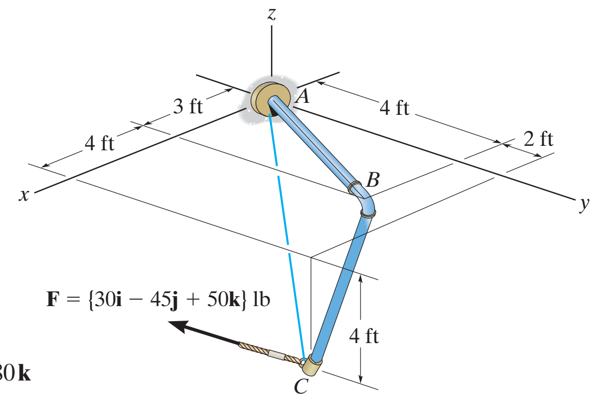
The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment *BC* of the pipe assembly can be determined from

$$(F_{BC})_{pr} = \sqrt{F^2 - (F_{BC})_{pa}^{\,2}} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$(F_{BC})_{||} = 28.3$ lb
$(F_{BC})_{\perp} = 68.0$ lb

***2–132.**

Determine the magnitude of the projected component of **F** along *AC*. Express this component as a Cartesian vector.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\,\mathbf{i} + 0.5970\,\mathbf{j} - 0.3980\,\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line *AC* is

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

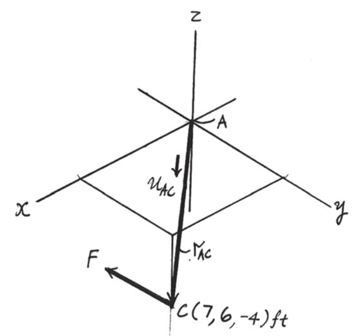
$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$

$$= 25.87 \text{ lb}$$ **Ans.**

Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

$$\mathrm{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$ **Ans.**

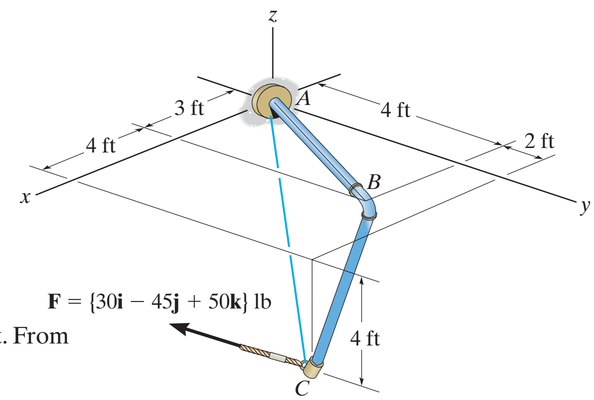


(a)

Ans:
$F_{AC} = 25.87$ lb
$F_{AC} = \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\}$ lb

2–133.

Determine the angle θ between the pipe segments BA and BC.

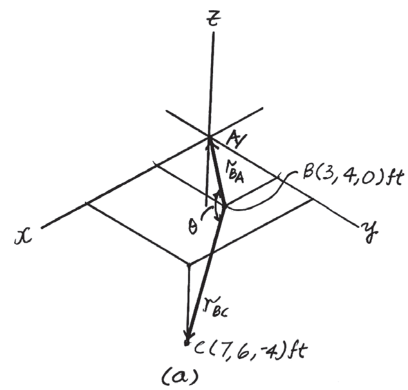


SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{BA} = (0-3)\mathbf{i} + (0-4)\mathbf{j} + (0-0)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{BC} = (7-3)\mathbf{i} + (6-4)\mathbf{j} + (-4-0)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\text{ ft}$$



The magnitude of $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ are

$$r_{BA} = \sqrt{(-3)^2 + (-4)^2} = 5\text{ ft}$$

$$r_{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6\text{ ft}$$

Vector Dot Product:

$$\begin{aligned}\mathbf{r}_{BA}\cdot\mathbf{r}_{BC} &= (-3\mathbf{i} - 4\mathbf{j})\cdot(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})\\ &= (-3)(4) + (-4)(2) + 0(-4)\\ &= -20\text{ ft}^2\end{aligned}$$

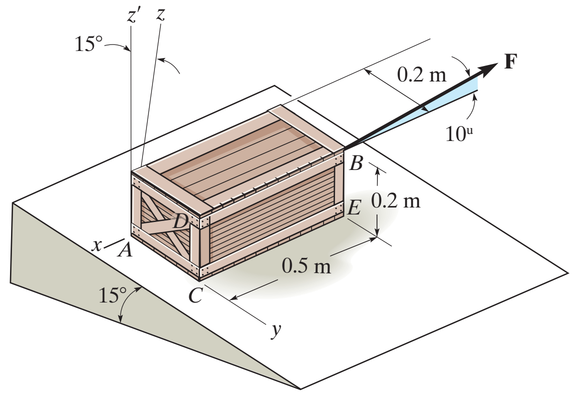
Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA}\cdot\mathbf{r}_{BC}}{r_{BA}\,r_{BC}}\right) = \cos^{-1}\left[\frac{-20}{5(6)}\right] = 132° \qquad \textbf{Ans.}$$

Ans:
$\theta = 132°$

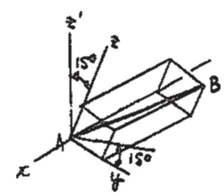
2–134.

If the force $F = 100$ N lies in the plane $DBEC$, which is parallel to the x–z plane, and makes an angle of 10° with the extended line DB as shown, determine the angle that **F** makes with the diagonal AB of the crate.



SOLUTION

Use the x, y, z axes.



$$\mathbf{u}_{AB} = \left(\frac{-0.5\mathbf{i} + 0.2\mathbf{j} + 0.2\mathbf{k}}{0.57446}\right)$$

$$= -0.8704\mathbf{i} + 0.3482\mathbf{j} + 0.3482\mathbf{k}$$

$$\mathbf{F} = -100\cos 10°\mathbf{i} + 100\sin 10°\mathbf{k}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{F}\cdot\mathbf{u}_{AB}}{F\,u_{AB}}\right)$$

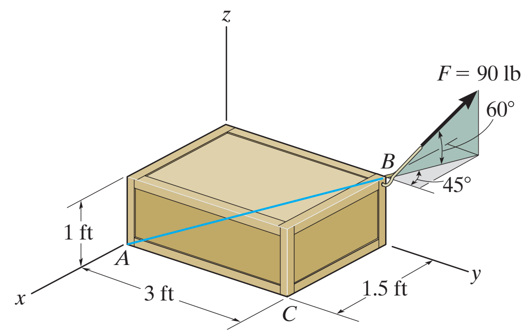
$$= \cos^{-1}\left(\frac{-100\,(\cos 10°)(-0.8704) + 0 + 100\sin 10°\,(0.3482)}{100(1)}\right)$$

$$= \cos^{-1}(0.9176) = 23.4°$$ **Ans.**

Ans:
$\theta = 23.4°$

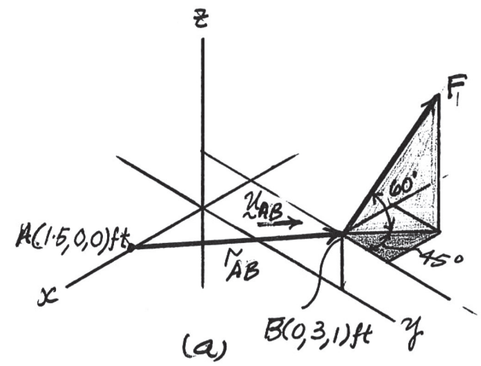
2–135.

Determine the magnitudes of the components of force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*



$$\mathbf{F} = 90(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0 - 1.5)\mathbf{i} + (3 - 0)\mathbf{j} + (1 - 0)\mathbf{k}}{\sqrt{(0 - 1.5)^2 + (3 - 0)^2 + (1 - 0)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to the diagonal AB is

$$[(F)_{AB}]_{\text{pa}} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

$$= 63.18 \text{ lb} = 63.2 \text{ lb}$$ **Ans.**

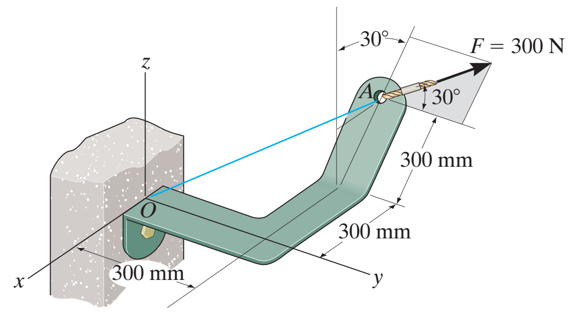
The magnitude of the component **F** perpendicular to the diagonal AB is

$$[(F)_{AB}]_{\text{pr}} = \sqrt{F^2 - [(F)_{AB}]_{\text{pa}}{}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb}$$ **Ans.**

Ans:
$[(F)_{AB}]_{||} = 63.2$ lb
$[(F)_{AB}]_{\perp} = 64.1$ lb

***2–136.**

Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300\sin 30^\circ \sin 30^\circ \mathbf{i} + 300\cos 30^\circ \mathbf{j} + 300\sin 30^\circ \cos 30^\circ \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F}\cdot\mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right)\cdot\mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75\text{ N}$$

$$F_y = \mathbf{F}\cdot\mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right)\cdot\mathbf{j}$$

$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260\text{ N}$$

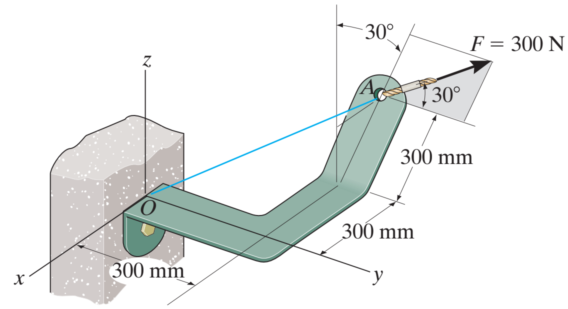
The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

$$F_x = 75\text{ N}, \qquad F_y = 260\text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_x = 75$ N
$F_y = 260$ N

2–137.

Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30° \sin 30°\mathbf{i} + 300 \cos 30°\mathbf{j} + 300 \sin 30° \cos 30°\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\}\ \text{N}$$

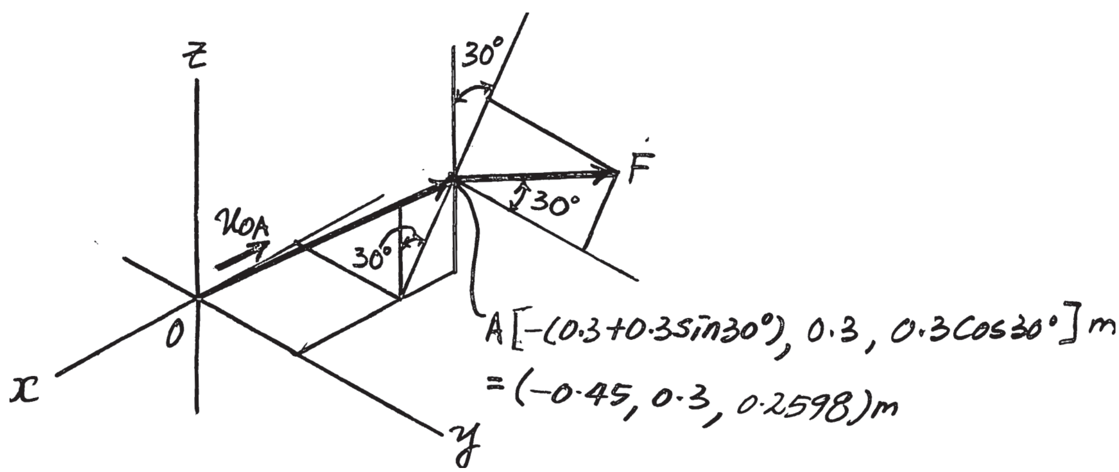
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line OA is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \left(-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}\right)$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

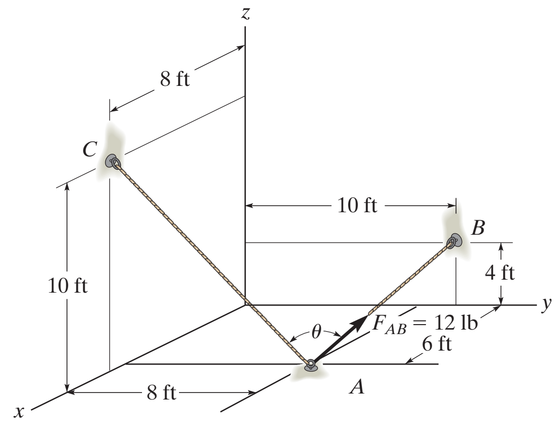
$$= 242\ \text{N}$$ **Ans.**



Ans:
$F_{OA} = 242$ N

2–138.

Determine the angle θ between the two cables.



SOLUTION

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}\, r_{AB}}\right)$$

$$= \cos^{-1}\left[\frac{(2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k}) \cdot (-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k})}{\sqrt{2^2 + (-8)^2 + 10^2}\,\sqrt{(-6)^2 + 2^2 + 4^2}}\right]$$

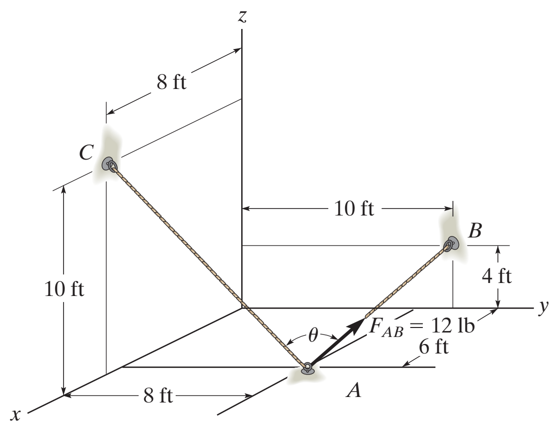
$$= \cos^{-1}\left(\frac{12}{96.99}\right)$$

$\theta = 82.9^\circ$ **Ans.**

Ans:
$\theta = 82.9^\circ$

2–139.

Determine the projected component of the force $F = 12$ lb acting in the direction of cable AC. Express the result as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AC} = \{2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_{AB} = 12\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 12\left(-\frac{6}{7.483}\,\mathbf{i} + \frac{2}{7.483}\,\mathbf{j} + \frac{4}{7.483}\,\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{-9.621\,\mathbf{i} + 3.207\,\mathbf{j} + 6.414\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{u}_{AC} = \frac{2}{12.961}\,\mathbf{i} - \frac{8}{12.961}\,\mathbf{j} + \frac{10}{12.961}\,\mathbf{k}$$

$$\text{Proj } F_{AB} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} = -9.621\left(\frac{2}{12.961}\right) + 3.207\left(-\frac{8}{12.961}\right) + 6.414\left(\frac{10}{12.961}\right)$$

$$= 1.4846$$

$$\text{Proj } \mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AC}$$

$$\text{Proj } \mathbf{F}_{AB} = (1.4846)\left[\frac{2}{12.962}\,\mathbf{i} - \frac{8}{12.962}\,\mathbf{j} + \frac{10}{12.962}\,\mathbf{k}\right]$$

$$\text{Proj } \mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
Proj $\mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}$ lb