

## CHAPTER 3

### Exercises

**E3.1**  $v(t) = q(t) / C = 10^{-6} \sin(10^5 t) / (2 \times 10^{-6}) = 0.5 \sin(10^5 t) \text{ V}$   
 $i(t) = C \frac{dv}{dt} = (2 \times 10^{-6})(0.5 \times 10^5) \cos(10^5 t) = 0.1 \cos(10^5 t) \text{ A}$

**E3.2** Because the capacitor voltage is zero at  $t = 0$ , the charge on the capacitor is zero at  $t = 0$ .

$$\begin{aligned} q(t) &= \int_0^t i(x) dx + 0 \\ &= \int_0^t 10^{-3} dx = 10^{-3} t \text{ for } 0 \leq t \leq 2 \text{ ms} \\ &= \int_0^{2 \times 10^{-3}} 10^{-3} dx + \int_{2 \times 10^{-3}}^t -10^{-3} dx = 4 \times 10^{-6} - 10^{-3} t \text{ for } 2 \text{ ms} \leq t \leq 4 \text{ ms} \end{aligned}$$

$$\begin{aligned} v(t) &= q(t) / C \\ &= 10^4 t \text{ for } 0 \leq t \leq 2 \text{ ms} \\ &= 40 - 10^4 t \text{ for } 2 \text{ ms} \leq t \leq 4 \text{ ms} \end{aligned}$$

$$\begin{aligned} p(t) &= i(t)v(t) \\ &= 10t \text{ for } 0 \leq t \leq 2 \text{ ms} \\ &= -40 \times 10^{-3} + 10t \text{ for } 2 \text{ ms} \leq t \leq 4 \text{ ms} \end{aligned}$$

$$\begin{aligned} w(t) &= Cv^2(t) / 2 \\ &= 5t^2 \text{ for } 0 \leq t \leq 2 \text{ ms} \\ &= 0.5 \times 10^{-7} (40 - 10^4 t)^2 \text{ for } 2 \text{ ms} \leq t \leq 4 \text{ ms} \end{aligned}$$

in which the units of charge, electrical potential, power, and energy are coulombs, volts, watts and joules, respectively. Plots of these quantities are shown in Figure 3.8 in the book.

**E3.3** Refer to Figure 3.10 in the book. Applying KVL, we have

$$V = V_1 + V_2 + V_3$$

Then using Equation 3.8 to substitute for the voltages we have

$$v(t) = \frac{1}{C_1} \int_0^t i(t) dt + v_1(0) + \frac{1}{C_2} \int_0^t i(t) dt + v_2(0) + \frac{1}{C_3} \int_0^t i(t) dt + v_3(0)$$

This can be written as

$$v(t) = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \int_0^t i(t) dt + v_1(0) + v_2(0) + v_3(0) \quad (1)$$

Now if we define

$$\frac{1}{C_{eq}} = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \quad \text{and} \quad v(0) = v_1(0) + v_2(0) + v_3(0)$$

we can write Equation (1) as

$$v(t) = \frac{1}{C_{eq}} \int_0^t i(t) dt + v(0)$$

Thus the three capacitances in series have an equivalent capacitance given by Equation 3.25 in the book.

**E3.4** (a) For series capacitances:

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2} = \frac{1}{1/2 + 1/1} = 2/3 \mu F$$

(b) For parallel capacitances:

$$C_{eq} = C_1 + C_2 = 1 + 2 = 3 \mu F$$

**E3.5** From Table 3.1 we find that the relative dielectric constant of polyester is 3.4. We solve Equation 3.26 for the area of each sheet:

$$A = \frac{Cd}{\epsilon} = \frac{Cd}{\epsilon_r \epsilon_0} = \frac{10^{-6} \times 15 \times 10^{-6}}{3.4 \times 8.85 \times 10^{-12}} = 0.4985 \text{ m}^2$$

Then the length of the strip is

$$L = A/W = 0.4985 / (2 \times 10^{-2}) = 24.93 \text{ m}$$

**E3.6** 
$$v(t) = L \frac{di(t)}{dt} = (10 \times 10^{-3}) \frac{d}{dt} [0.1 \cos(10^4 t)] = -10 \sin(10^4 t) \text{ V}$$

$$w(t) = \frac{1}{2} L i^2(t) = 5 \times 10^{-3} \times [0.1 \cos(10^4 t)]^2 = 50 \times 10^{-6} \cos^2(10^4 t) \text{ J}$$

**E3.7**

$$\begin{aligned}
i(t) &= \frac{1}{L} \int_0^t v(x) dx + i(0) = \frac{1}{150 \times 10^{-6}} \int_0^t v(x) dx \\
&= 6667 \int_0^t 7.5 \times 10^6 x dx = 25 \times 10^9 t^2 \text{ V for } 0 \leq t \leq 2 \mu\text{s} \\
&= 6667 \int_0^{2\text{E-}6} 7.5 \times 10^6 x dx = 0.1 \text{ V for } 2 \mu\text{s} \leq t \leq 4 \mu\text{s} \\
&= 6667 \left( \int_0^{2\text{E-}6} 7.5 \times 10^6 x dx + \int_{4\text{E-}6}^t (-15) dx \right) = 0.5 - 10^5 t \text{ V for } 4 \mu\text{s} \leq t \leq 5 \mu\text{s}
\end{aligned}$$

A plot of  $i(t)$  versus  $t$  is shown in Figure 3.19b in the book.

**E3.8** Refer to Figure 3.20a in the book. Using KVL we can write:

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

Using Equation 3.28 to substitute, this becomes

$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt} \quad (1)$$

Then if we define  $L_{eq} = L_1 + L_2 + L_3$ , Equation (1) becomes:

$$v(t) = L_{eq} \frac{di(t)}{dt}$$

which shows that the series combination of the three inductances has the same terminal equation as the equivalent inductance.

**E3.9** Refer to Figure 3.20b in the book. Using KCL we can write:

$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

Using Equation 3.32 to substitute, this becomes

$$i(t) = \frac{1}{L_1} \int_0^t v(t) dt + i_1(0) + \frac{1}{L_2} \int_0^t v(t) dt + i_2(0) + \frac{1}{L_3} \int_0^t v(t) dt + i_3(0)$$

This can be written as

$$v(t) = \left( \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \int_0^t v(t) dt + i_1(0) + i_2(0) + i_3(0) \quad (1)$$

Now if we define

$$\frac{1}{L_{eq}} = \left( \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \quad \text{and} \quad i(0) = i_1(0) + i_2(0) + i_3(0)$$

we can write Equation (1) as

$$i(t) = \frac{1}{L_{eq}} \int_0^t v(t) dt + i(0)$$

Thus, the three inductances in parallel have the equivalent inductance shown in Figure 3.20b in the book.

**E3.10** Refer to Figure 3.21 in the book.

(a) The 2-H and 3-H inductances are in series and are equivalent to a 5-H inductance, which in turn is in parallel with the other 5-H inductance. This combination has an equivalent inductance of  $1/(1/5 + 1/5) = 2.5$  H. Finally the 1-H inductance is in series with the combination of the other inductances so the equivalent inductance is  $1 + 2.5 = 3.5$  H.

(b) The 2-H and 3-H inductances are in series and have an equivalent inductance of 5 H. This equivalent inductance is in parallel with both the 5-H and 4-H inductances. The equivalent inductance of the parallel combination is  $1/(1/5 + 1/4 + 1/5) = 1.538$  H. This combination is in series with the 1-H and 6-H inductances so the overall equivalent inductance is  $1.538 + 1 + 6 = 8.538$  H.

**E3.11** The MATLAB commands including some explanatory comments are:

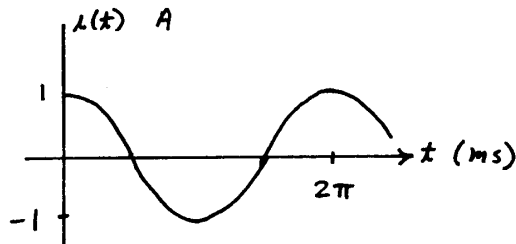
```
% We avoid using i alone as a symbol for current because
% we reserve i for the square root of -1 in MATLAB. Thus, we
% will use iC for the capacitor current.
syms t iC qC vC % Define t, iC, qC and vC as symbolic objects.
iC = 0.5*sin((1e4)*t);
ezplot(iC, [0 3*pi*1e-4])
qC=int(iC,t,0,t); % qC equals the integral of iC.
figure % Plot the charge in a new window.
ezplot(qC, [0 3*pi*1e-4])
vC = 1e7*qC;
figure % Plot the voltage in a new window.
ezplot(vC, [0 3*pi*1e-4])
```

The plots are very similar to those of Figure 3.5 in the book. An m-file (named Exercise\_3\_11) containing these commands can be found in the MATLAB folder on the OrCAD disk.

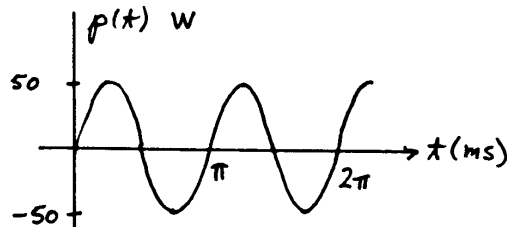
## Problems

- P3.1** Capacitors consist of two conductors separated by an insulating material. Frequently, the conductors are sheets of metal that are separated by a thin layer of the insulating material.
- P3.2** Because we have  $i = C dv / dt$  for a capacitance, the current is zero if the voltage is constant. Thus, we say that capacitances act as open circuits for constant (dc) voltages.
- P3.3** A dielectric material is an electrical insulator through which virtually no current flows, assuming normal operating voltages. Some examples of dielectrics mentioned in the text are air, Mylar, polyester, polypropylene, and mica. Some others are porcelain, glass, and certain types of oil.
- P3.4** Charge (usually in the form of electrons) flows in and accumulates on one plate. Meanwhile, an equal amount of charge flows out of the other plate. Thus, current seems to flow through a capacitor.

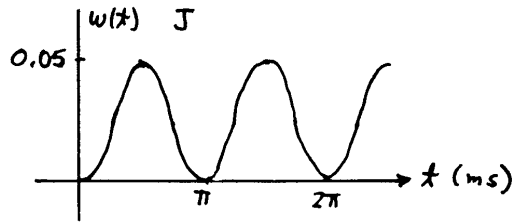
**P3.5\*** 
$$i(t) = C \frac{dv}{dt} = 10^{-5} \frac{d}{dt} (100 \sin 1000t) = \cos(1000t)$$



$$p(t) = v(t)i(t) = 100 \cos(1000t) \sin(1000t) = 50 \sin(2000t)$$



$$w(t) = \frac{1}{2} C [v(t)]^2 = 0.05 \sin^2(1000t)$$



**P3.6\***  $i = C \frac{dv}{dt}$

$$\frac{dv}{dt} = \frac{i}{C} = \frac{100 \times 10^{-6}}{2000 \times 10^{-6}} = 0.05 \text{ V/s} \quad \Delta t = \frac{\Delta v}{dv/dt} = \frac{100}{0.05} = 2000 \text{ s}$$

**P3.7\***  $v(t) = \frac{1}{C} \int_0^t i(t) dt + v(0) \quad v(t) = 2 \times 10^4 \int_0^t 3 \times 10^{-3} dt - 20$

$$v(t) = 60t - 20 \text{ V}$$

$$p(t) = i(t)v(t) = 3 \times 10^{-3}(60t - 20) \text{ W}$$

Evaluating at  $t = 0$ , we have  $p(0) = -60 \text{ mW}$ . Because the power has a negative value, the capacitor is delivering energy.

At  $t = 1 \text{ s}$ , we have  $p(1) = 120 \text{ mW}$ . Because the power is positive, we know that the capacitor is absorbing energy.

**P3.8\***  $W = \text{power} \times \text{time} = 5 \text{ hp} \times 746 \text{ W / hp} \times 3600 \text{ s}$

$$= 13.4 \times 10^6 \text{ J}$$

$$V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \times 13.4 \times 10^6}{0.01}} = 51.8 \text{ kV}$$

It turns out that a 0.01-F capacitor rated for this voltage would be much too large and massive for powering an automobile. Besides, to have reasonable performance, an automobile would need much more than 5 hp for an hour.

**P3.9** The net charge on each plate is  $Q = CV = (5 \times 10^{-6}) \times 100 = 500 \text{ } \mu\text{C}$ . One plate has a net positive charge and the other has a net negative charge so the net charge for both plates is zero.

**P3.10** 
$$v(t) = \frac{1}{C} \int_0^t i(t) dt + v(0) = \frac{1}{C} \int_0^t I_m \cos(\omega t) dt = \frac{I_m}{\omega C} [\sin(\omega t) - \sin(0)]$$
  

$$= \frac{I_m}{\omega C} \sin(\omega t)$$

Clearly for  $\omega \rightarrow \infty$ , the voltage becomes zero, so the capacitance becomes the equivalent of a short circuit.

**P3.11**

$$i(t) = C \frac{dv}{dt}$$

$$= 10^{-6} \frac{d}{dt} (100e^{-100t})$$

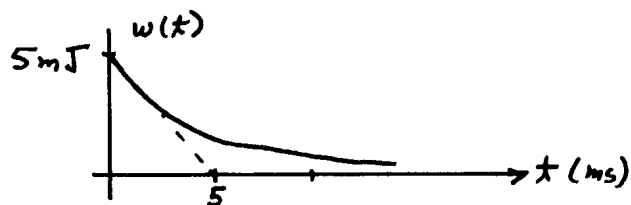
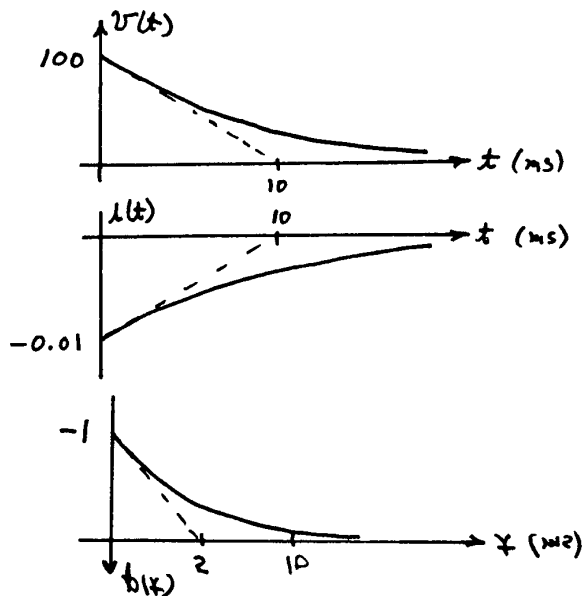
$$= -0.01e^{-100t} \text{ A}$$

$$p(t) = v(t)i(t)$$

$$= -e^{-200t} \text{ W}$$

$$w(t) = \frac{1}{2} C [v(t)]^2$$

$$= 5 \times 10^{-3} \times e^{-200t} \text{ J}$$

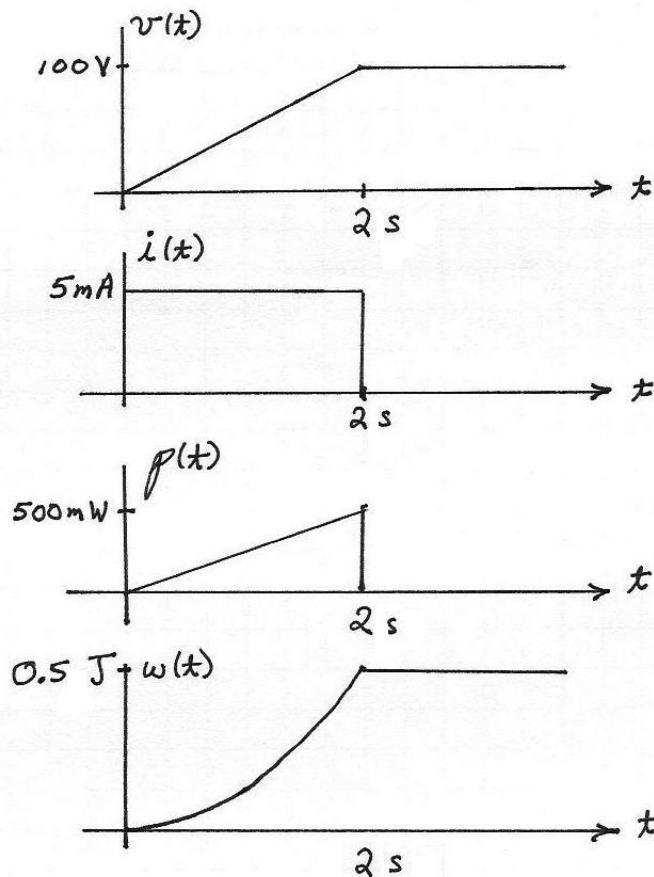


**P3.12**  $Q = Cv = 5 \times 10^{-6} \times 200 = 1 \text{ mC}$

$$W = \frac{1}{2} Cv^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (200)^2 = 0.1 \text{ J}$$

$$P = \frac{\Delta W}{\Delta t} = \frac{0.1}{10^{-6}} = 100 \text{ kW}$$

P3.13



P3.14

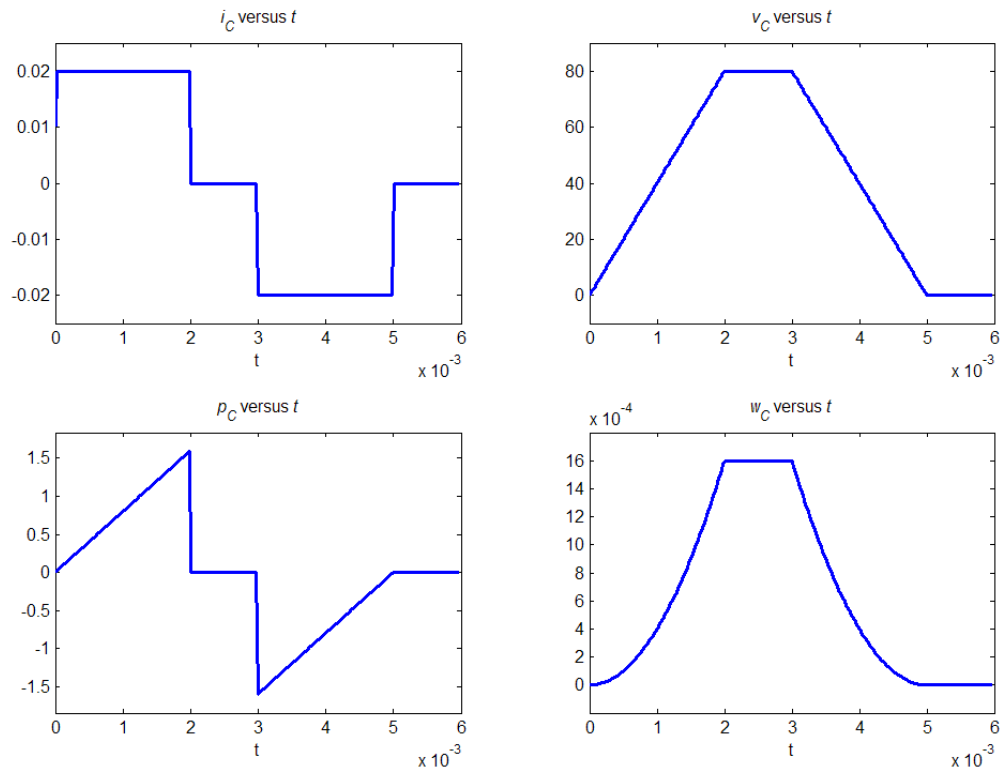
$$v(t) = \frac{1}{C} \int_0^t i(t) dt + v(0)$$

$$v(t) = 2 \times 10^6 \int_0^t i(t) dt \quad p(t) = v(t)i(t)$$

$$w(t) = \frac{1}{2} C v^2(t) = 0.25 \times 10^{-6} \times v^2(t)$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.





**P3.15** Because the switch is closed prior to  $t = 0$ , the initial voltage is zero, and we have

$$v(t) = \frac{1}{C} \int_0^t i(t) dt + v(0) = 2 \times 10^5 \int_0^t (3 \times 10^{-3}) dt + 0 = 600t$$

$$v(10 \times 10^{-3}) = 6 \text{ V}$$

$$p = vi = 1.8t \text{ W}$$

$$p(10 \times 10^{-3}) = 18 \text{ mW}$$

$$w(t) = \frac{1}{2} C v^2(t) = 0.9t^2 \text{ J}$$

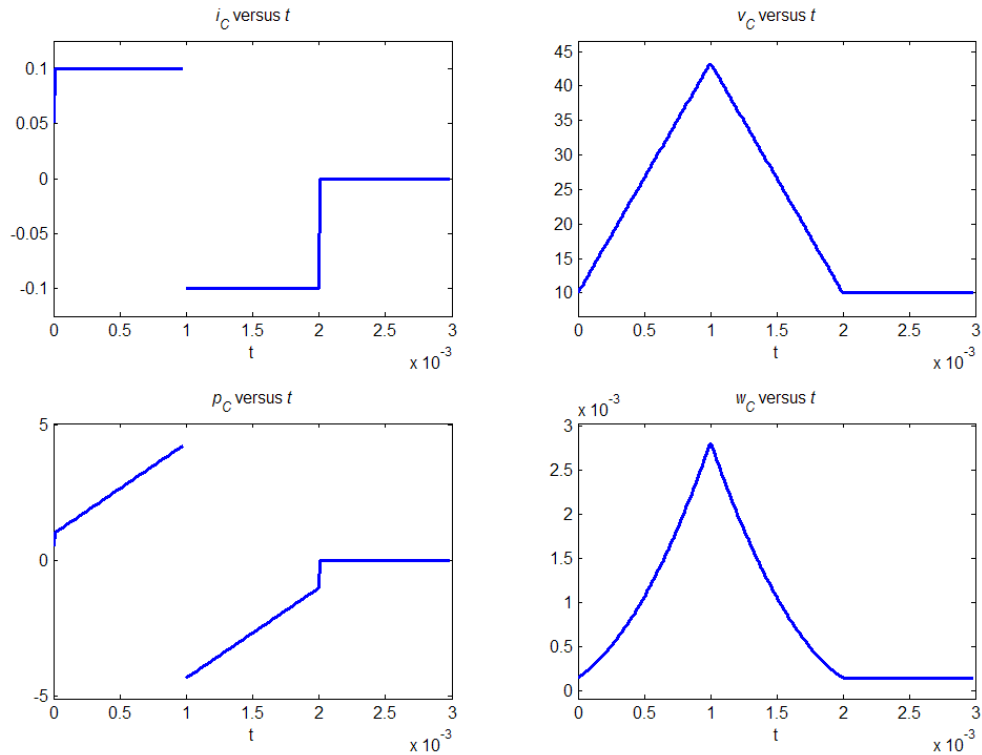
$$w(10 \times 10^{-3}) = 90 \text{ } \mu\text{J}$$

**P3.16** 
$$v(t) = \frac{1}{C} \int_0^t i(t) dt + v(0) = 0.333 \times 10^6 \int_0^t i(t) dt + 10$$

$$p(t) = v(t)i(t)$$

$$w(t) = \frac{1}{2} C v^2(t) = 1.5 \times 10^{-6} \times v^2(t)$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.



**P3.17** We can write

$$w = \frac{1}{2} C v^2 = 5 \times 10^{-6} v^2 = 200$$

Solving, we find  $v = 6325$  V. Then, because the stored energy is decreasing, the power is negative. Thus, we have

$$i = \frac{p}{v} = \frac{-500}{6325} = -79.06 \text{ mA}$$

The minus sign shows that the current actually flows opposite to the passive convention. Thus, current flows out of the positive terminal of the capacitor.

**P3.18** A capacitance initially charged to 10 V has

$$v(t) = \frac{1}{C} \int_0^t i(t) dt + 10$$

However, if the capacitance is infinite, this becomes  $v(t) = 10 \text{ V}$  which describes a 10-V voltage source. Thus, a very large capacitance initially charged to 10 V is an approximate 10-V voltage source.

**P3.19** 
$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt} [C(t)v(t)] = \frac{d}{dt} [200 \times 10^{-6} + 100 \times 10^{-6} \sin(200t)]$$
  

$$i(t) = 20 \cos(200t) \text{ mA}$$

**P3.20** We can write  $v(t_0) = V_f = \frac{1}{C} \int_{t_0}^{t_0 + \Delta t} i(t) dt$ . The integral represents the area of the current pulse, which has units of ampere seconds or coulombs and must equal  $V_f C$ . The pulse area represents the net charge transferred by the current pulse. Because a constant-amplitude pulse has the largest area for a given peak amplitude, we can say that the peak amplitude of the current pulse must be at least as large as  $V_f C / \Delta t$ .

We conclude that the area of the pulse remains constant and that the peak amplitude approaches infinity as  $\Delta t$  approaches zero. In the limit, this type of pulse is called an impulse.

**P3.21** By definition, the voltage across a short circuit must be zero. Since we have  $v = Ri$  for a resistor, zero resistance corresponds to a short circuit. For an initially uncharged capacitance, we have

$$v(t) = \frac{1}{C} \int_0^t i(t) dt$$

For the voltage to be zero for all values of current and time, the capacitance must be infinite. Thus, an infinite initially uncharged capacitance is equivalent to a short circuit.

For an open circuit, the current must be zero. This requires infinite resistance. However for a capacitance, we have

$$i(t) = C \frac{dv(t)}{dt}$$

Thus, a capacitance of zero is equivalent to an open circuit.

**P3.22** 
$$i(t) = C \frac{dv(t)}{dt}$$
  

$$= 20 \times 10^{-6} \frac{d}{dt} [3 \cos(10^5 t) + 2 \sin(10^5 t)]$$

$$= -6 \sin(10^5 t) + 4 \cos(10^5 t)$$

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= [3 \cos(10^5 t) + 2 \sin(10^5 t)][-6 \sin(10^5 t) + 4 \cos(10^5 t)] \end{aligned}$$

Evaluating at  $t = 0$ , we have  $p(0) = 12 \text{ W}$ . Because  $p(0)$  is positive, we know that the capacitor is absorbing energy at  $t = 0$ .

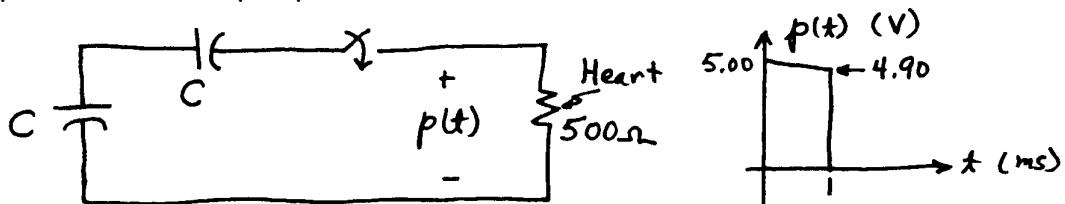
Evaluating at  $t_2 = \pi/2 \times 10^{-5}$ , we have  $p(t_2) = -12 \text{ W}$ . Because  $p(t_2)$  is negative, we know that the capacitor is supplying energy at  $t = t_2$ .

**P3.23** Capacitances in parallel are combined by adding their values. Thus, capacitances in parallel are combined as resistances in series are. Capacitances in series are combined by taking the reciprocal of the sum of the reciprocals of the individual capacitances. Thus, capacitances in series are combined as resistances in parallel are.

**P3.24\*** (a)  $C_{eq} = 1 + \frac{1}{1/2 + 1/2} = 2 \mu\text{F}$

(b) The two  $4\text{-}\mu\text{F}$  capacitances are in series and have an equivalent capacitance of  $\frac{1}{1/4 + 1/4} = 2 \mu\text{F}$ . This combination is in parallel with the  $2\text{-}\mu\text{F}$  capacitance, giving an equivalent of  $4 \mu\text{F}$ . Then the  $12 \mu\text{F}$  is in series, giving a capacitance of  $\frac{1}{1/12 + 1/4} = 3 \mu\text{F}$ . Finally, the  $5 \mu\text{F}$  is in parallel, giving an equivalent capacitance of  $C_{eq} = 3 + 5 = 8 \mu\text{F}$ .

**P3.25\*** As shown below, the two capacitors are placed in series with the heart to produce the output pulse.

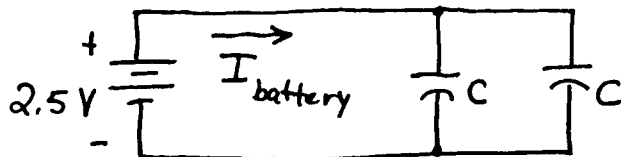


While the capacitors are connected, the average voltage supplied to the heart is 4.95 V. Thus, the average current is  $I_{pulse} = 4.95/500 = 9.9 \text{ mA}$ .

The charge removed from each capacitor during the pulse is

$$\Delta Q = 9.9 \text{ mA} \times 1 \text{ ms} = 9.9 \mu\text{C}. \text{ This results in a } 0.1 \text{ V change in voltage, so}$$

we have  $C_{eq} = \frac{C}{2} = \frac{\Delta Q}{\Delta V} = \frac{9.9 \times 10^{-6}}{0.1} = 99 \mu\text{F}$ . Thus, each capacitor has a capacitance of  $C = 198 \mu\text{F}$ . Then as shown below, the capacitors are placed in parallel with the 2.5-V battery to recharge them.



The battery must supply  $9.9 \mu\text{C}$  to each battery. Thus, the average current supplied by the battery is  $I_{battery} = \frac{2 \times 9.9 \mu\text{C}}{1\text{s}} = 19.8 \mu\text{A}$ . The ampere-hour rating of the battery is

$$19.8 \times 10^{-6} \times 5 \times 365 \times 24 = 0.867 \text{ Ampere hours}.$$

**P3.26** (a)  $C_{eq} = 3 + \frac{1}{1/2 + 1/1} + \frac{1}{1/2 + 1/(1+1)} = 4.667 \mu\text{F}$

(b)  $C_{eq} = \frac{1}{1/(2+1) + 1/(4+2)} = 2 \mu\text{F}$

**P3.27** We obtain the maximum capacitance of  $2 \mu\text{F}$  by connecting the  $1-\mu\text{F}$  capacitors in parallel. We obtain the minimum capacitance of  $1/2 \mu\text{F}$  by connecting the  $1-\mu\text{F}$  capacitors in series.

**P3.28**  $C_{eq} = \frac{1}{1/C_1 + 1/C_2} = 6 \mu\text{F}$

The charges stored on each capacitor and on the equivalent capacitance are equal because the current through each is the same.

$$Q = C_{eq} \times 50 \text{ V} = 300 \mu\text{C}$$

$$V_1 = \frac{Q}{C_1} = 20 \text{ V}$$

$$V_2 = \frac{Q}{C_2} = 30 \text{ V}$$

As a check, we verify that  $V_1 + V_2 = 50 \text{ V}$ .

**P3.29** The equivalent capacitance is  $C_{eq} = 100 \mu\text{F}$  and its initial voltage is  $150 \text{ V}$ .

The energies are  $w_1 = \frac{1}{2} CV_1^2 = \frac{1}{2} 200 \times 10^{-6} \times 50^2 = 0.25 \text{ J}$  and  $w_2 = 1.0 \text{ J}$ .

Thus, the total energy stored in the two capacitors is  $w_{\text{total}} = 1.25 \text{ J}$ .

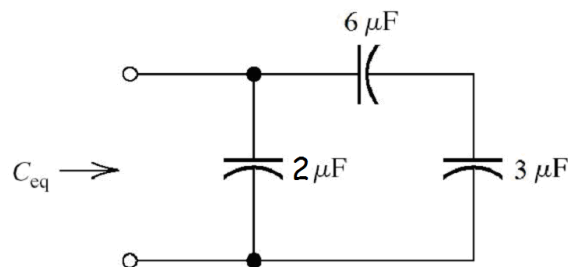
On the other hand, the energy stored in the equivalent capacitance is

$$w_{eq} = \frac{1}{2} C_{eq} V_{eq}^2 = \frac{1}{2} 100 \times 10^{-6} \times 150^2 = 1.125 \text{ J}.$$

The reason for the discrepancy is that all of the energy stored in the original capacitances cannot be accessed as long as they are connected in series. Net charge is trapped on the plates that are connected together.

**P3.30**

$$C_{eq} = 2 + \frac{1}{1/6 + 1/3} = 4 \mu\text{F}$$



**P3.31\***  $C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{15 \times 8.85 \times 10^{-12} \times 10 \times 10^{-2} \times 30 \times 10^{-2}}{0.01 \times 10^{-3}} = 0.398 \mu\text{F}$

**P3.32**  $C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{\epsilon_r \epsilon_0 WL}{d}$

(a) Thus if  $W$  and  $L$  are both doubled, the capacitance is increased by a factor of four resulting in  $C = 400 \text{ pF}$ .

(b) If  $d$  is halved, the capacitance is doubled resulting in  $C = 200 \text{ pF}$ .

(c) The relative dielectric constant of air is approximately unity. Thus, replacing air with oil increases  $\epsilon_r$  by a factor of 35 increasing the capacitance to  $3500 \text{ pF}$ .

**P3.33\*** The charge  $Q$  remains constant because the terminals of the capacitor are open-circuited.

$$Q = C_1 V_1 = 1000 \times 10^{-12} \times 1000 = 1 \mu\text{C}$$

$$W_1 = (1/2) C_1 (V_1)^2 = 500 \mu\text{J}$$

After the distance between the plates is doubled, the capacitance becomes  $C_2 = 500 \text{ pF}$ .

The voltage increases to  $V_2 = \frac{Q}{C_2} = \frac{10^{-6}}{500 \times 10^{-12}} = 2000 \text{ V}$  and the stored energy is  $W_2 = (1/2)C_2(V_2)^2 = 1000 \mu\text{J}$ . The force needed to pull the plates apart supplies the additional energy.

**P3.34** Using  $C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{\epsilon_r \epsilon_0 WL}{d}$  and  $V_{\max} = Kd$  to substitute into

$$W_{\max} = \frac{1}{2} C V_{\max}^2, \text{ we have } W_{\max} = \frac{1}{2} \frac{\epsilon_r \epsilon_0 WL}{d} K^2 d^2 = \frac{1}{2} \epsilon_r \epsilon_0 K^2 WLd.$$

However, the volume of the dielectric is  $\text{Vol} = WLd$ , so we have

$$W_{\max} = \frac{1}{2} \epsilon_r \epsilon_0 K^2 (\text{Vol})$$

Thus, we conclude that the maximum energy stored is independent of  $W$ ,  $L$ , and  $d$  if the volume is constant and if both  $W$  and  $L$  are much larger than  $d$ . To achieve large energy storage per unit volume, we should look for a dielectric having a large value for the product  $\epsilon_r K^2$ . The dielectric should have high relative dielectric constant and high breakdown strength.

**P3.35** The capacitance of the microphone is

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 10 \times 10^{-4}}{100[1 + 0.003 \cos(1000t)]10^{-6}} \\ \cong 88.5 \times 10^{-12} [1 - 0.003 \cos(1000t)]$$

The current flowing through the microphone is

$$i(t) = \frac{dq(t)}{dt} = \frac{d[CV]}{dt} \cong 53.1 \times 10^{-9} \sin(1000t) \text{ A}$$

**P3.36** Referring to Figure P3.36 in the book, we see that the transducer consists of two capacitors in parallel: one above the surface of the liquid and one below. Furthermore, the capacitance of each portion is proportional to its length and the relative dielectric constant of the material between the plates. Thus for the portion above the liquid, the capacitance in pF is

$$C_{\text{above}} = 200 \frac{100 - x}{100} \text{ pF}$$

in which  $x$  is the height of the liquid in cm. For the portion of the plates below the surface of the liquid:

$$C_{\text{below}} = 200(15) \frac{x}{100}$$

Then the total capacitance is:

$$C = C_{\text{above}} + C_{\text{below}}$$

$$C = 200 + 28x \text{ pF}$$

**P3.37** With the tank full, we have

$$w_{\text{full}} = \frac{1}{2} C_{\text{full}} V_{\text{full}}^2 = \frac{1}{2} \times 2500 \times 10^{-12} \times 1000^2 = 1.25 \text{ mJ}$$

$$Q = C_{\text{full}} V_{\text{full}} = 2500 \times 10^{-12} \times 1000 = 2.5 \text{ } \mu\text{C}$$

The charge cannot change when the tank is drained, so we have

$$V_{\text{empty}} = \frac{Q}{C_{\text{empty}}} = \frac{2.5 \times 10^{-6}}{100 \times 10^{-12}} = 25000 \text{ V}$$

$$w_{\text{empty}} = \frac{1}{2} C_{\text{empty}} V_{\text{empty}}^2 = \frac{1}{2} \times 100 \times 10^{-12} \times 25000^2 = 31.25 \text{ mJ}$$

The added energy is supplied from the gravitational potential energy of the insulating fluid. When there is liquid between the plates, the charge separation of the dielectric partly cancels the electrical forces of the charges on the plates. When the dielectric fluid drains, this cancellation effect is lost, which is why the voltage increases. The charge on the plates creates a small force of attraction on the fluid, and it is the force of gravity acting against this force of attraction as the fluid drains that accounts for the added energy.

**P3.38**

$$i_c(t) = C \frac{dv_c(t)}{dt} = 10^{-7} \frac{d}{dt} [10 \cos(100t)] = -10^{-4} \sin(100t)$$

$$v_r(t) = R i_c(t) = -10^{-3} \sin(100t)$$

$$v(t) = v_c(t) + v_r(t)$$

$$v(t) = 10 \cos(100t) - 10^{-3} \sin(100t)$$

Thus,  $v(t) = v_c(t)$  to within 1% accuracy, and the resistance can be neglected.

Repeating for  $v_c(t) = 0.1 \cos(10^7 t)$ , we find



$$i_c(t) = -0.1 \sin(10^7 t)$$

$$v_r(t) = -\sin(10^7 t)$$

$$v(t) = v_c(t) + v_r(t) = 0.1 \cos(10^7 t) - \sin(10^7 t)$$

Thus, in this case, the voltage across the parasitic resistance is larger in magnitude than the voltage across the capacitance.

**P3.39** The required volume is

$$\text{Vol} = Ad = \frac{2W_{\max}}{\epsilon_0 \epsilon_r K^2} = \frac{2 \times 132 \times 10^6}{8.85 \times 10^{-12} (32 \times 10^5)^2} = 2.9 \times 10^6 \text{ m}^3$$

Clearly an air dielectric capacitor is not a practical energy storage device for an electric car!

The thickness of the dielectric is

$$d_{\min} = \frac{V_{\max}}{K} = \frac{10^3}{32 \times 10^5} = 0.3125 \text{ mm}$$

**P3.40** Before the switch closes, the energies are

$$W_1 = (1/2)C_1(V_1)^2 = 50 \text{ mJ}$$

$$W_2 = (1/2)C_2(V_2)^2 = 50 \text{ mJ}$$

Thus, the total stored energy is 100 mJ. The charge on the top plate of  $C_1$  is  $Q_1 = C_1V_1 = +1000 \mu\text{C}$ . The charge on the top plate of  $C_2$  is  $Q_2 = C_2V_2 = -1000 \mu\text{C}$ . Thus, the total charge on the top plates is zero. When the switch closes, the charges cancel, the voltage becomes zero, and the stored energy becomes zero.

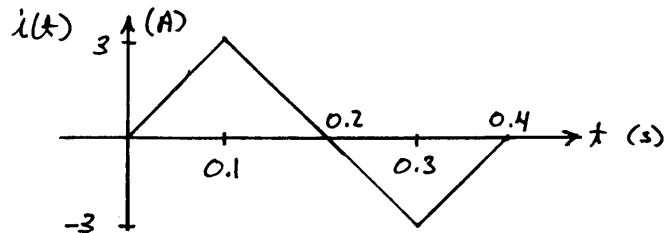
Where did the energy go? Usually, the resistance of the wires absorbs it. If the superconductors are used so that the resistance is zero, the energy can be accounted for by considering the inductance of the circuit. (It is not possible to have a real circuit that is precisely modeled by Figure P3.40; there is always resistance and inductance associated with the wires that connect the capacitances.)

**P3.41** A fluid-flow analogy for an inductor consists of an incompressible fluid flowing through a frictionless pipe of constant diameter. The pressure differential between the ends of the pipe is analogous to the voltage across the inductor and the flow rate of the liquid is proportional to the current. (If the pipe had friction, the electrical analog would have series resistance. If the ends of the pipe had different diameters, a pressure

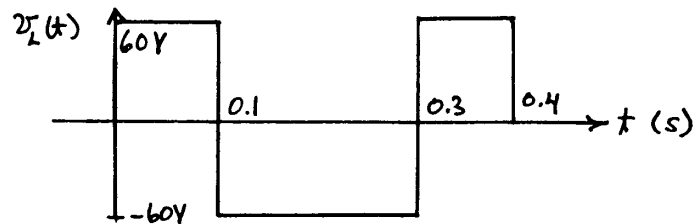
differential would exist for constant flow rate, whereas an inductance has zero voltage for constant flow rate.)

**P3.42** Inductors consist of coils of wire wound on coil forms such as toroids.

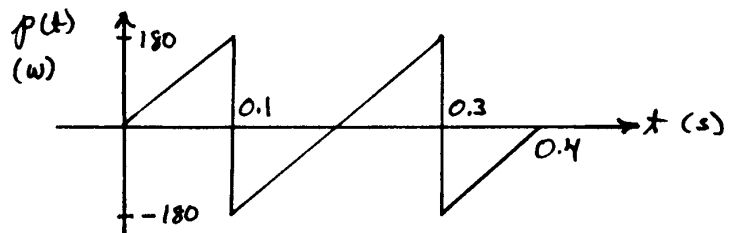
**P3.43\***  $L = 2 \text{ H}$



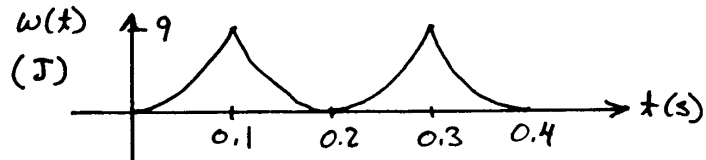
$$v_L(t) = L \frac{di_L(t)}{dt}$$



$$p(t) = v_L(t)i_L(t)$$



$$w(t) = \frac{1}{2} L [i_L(t)]^2$$



**P3.44\***  $v_L = L \frac{di}{dt} = 0.5 \frac{4}{0.2} = 10 \text{ V}$

**P3.45\***

$$i_L(t) = \frac{1}{L} \int_0^t v_L(t) dt + i_L(0) = 20 \times 10^3 \int_0^t 10 dt - 0.1 = 2 \times 10^5 t - 0.1 \text{ A}$$

Solving for the time that the current reaches  $+100 \text{ mA}$ , we have

$$i_L(t_x) = 0.1 = 2 \times 10^5 t_0 - 0.1$$

$$t_x = 1 \mu\text{s}$$

**P3.46** 
$$i_L(t_1) = \frac{1}{L} \int_0^{t_1} v_L(t) dt + i_L(0)$$

$$i_L(6) = \frac{1}{3} \int_0^6 10 dt + 0 = 20 \text{ A}$$

$$p(6) = v_L(6)i_L(6) = 200 \text{ W}$$

$$w(6) = \frac{1}{2} L i_L^2(6) = 600 \text{ J}$$

**P3.47** Because the energy stored in an inductor is  $w = \frac{1}{2} L i^2$ , the energy stored in the inductor decreases when the current magnitude decreases. Therefore, energy is flowing out of the inductor.

Because we have  $v_L(t) = L \frac{di_L(t)}{dt}$ , the voltage is zero when the current is constant. Thus the power, which is the product of current and voltage, is zero if the current is constant.

**P3.48** Because we have  $v_L(t) = L \frac{di_L(t)}{dt}$ , the voltage is zero when the current is constant. Thus, we say that inductors act as short circuits for steady dc currents.

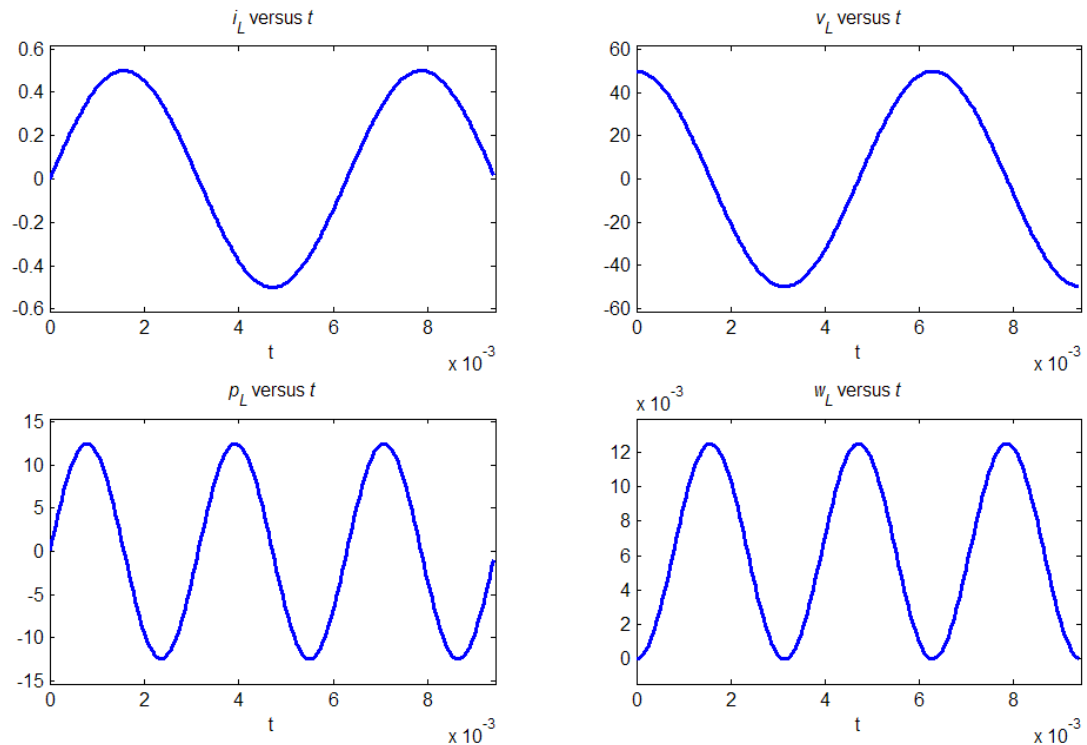
**P3.49**  $L = 0.1 \text{ H} \quad i_L(t) = 0.5 \sin(1000t) \text{ A}$

$$v_L(t) = L \frac{di_L(t)}{dt} = 50 \cos(1000t) \text{ V}$$

$$p(t) = v_L(t)i_L(t) = 25 \cos(1000t) \sin(1000t) = 12.5 \sin(2000t) \text{ W}$$

$$w(t) = \frac{1}{2} L [i_L(t)]^2 = 0.0125 \sin^2(1000t) \text{ J}$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.



**P3.50**  $L = 0.3 \text{ H}$

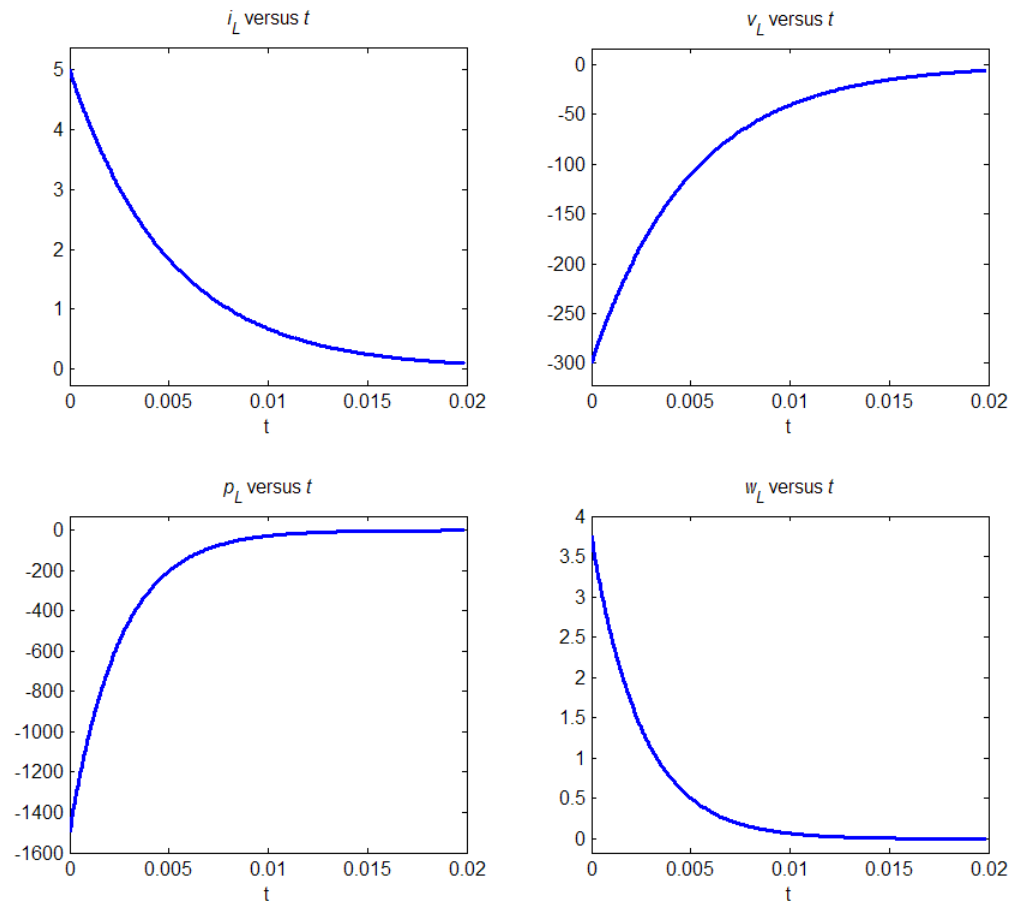
$$i_L(t) = 5e^{-200t}$$

$$v_L(t) = L \frac{di_L(t)}{dt} = -300e^{-200t} \text{ V}$$

$$p(t) = v_L(t)i_L(t) = (-300e^{-200t})(5e^{-200t}) = -1500e^{-400t} \text{ W}$$

$$w(t) = \frac{1}{2} L [i_L(t)]^2 = 3.75e^{-400t} \text{ J}$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s.



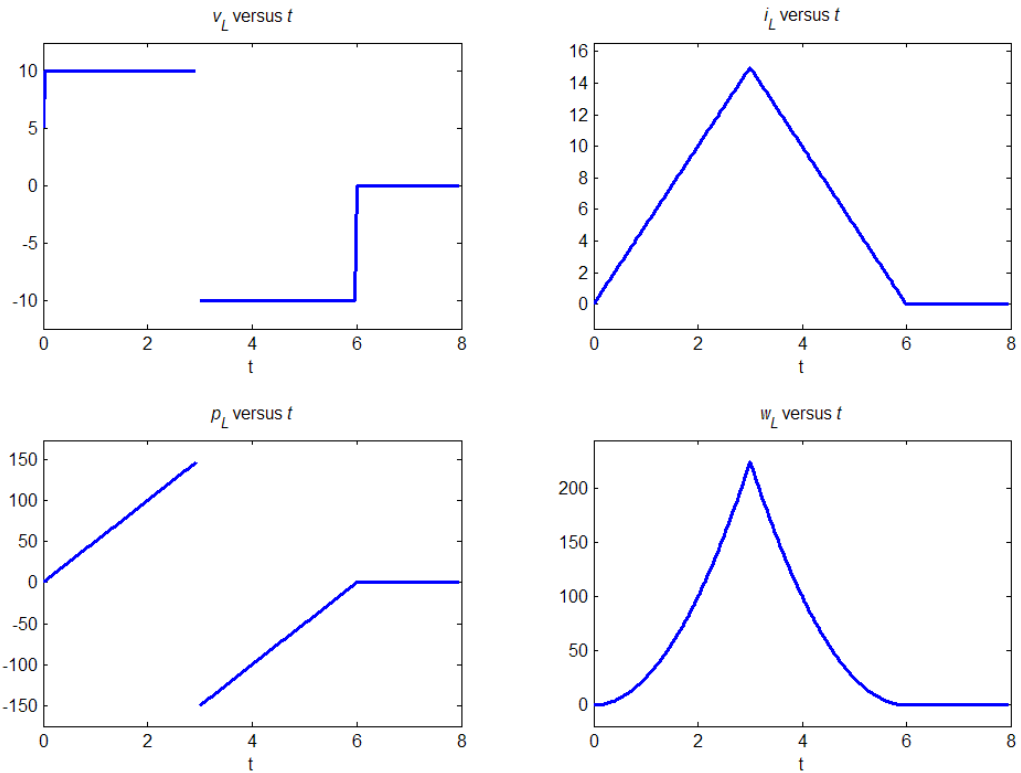
**P3.51**  $L = 2 \text{ H}$

$$i_L(t) = \frac{1}{L} \int_0^t v_L(t) dt + i_L(0) = \frac{1}{2} \int_0^t v_L(t) dt$$

$$p(t) = v_L(t) i_L(t)$$

$$w(t) = \frac{1}{2} L [i_L(t)]^2 = [i_L(t)]^2$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s



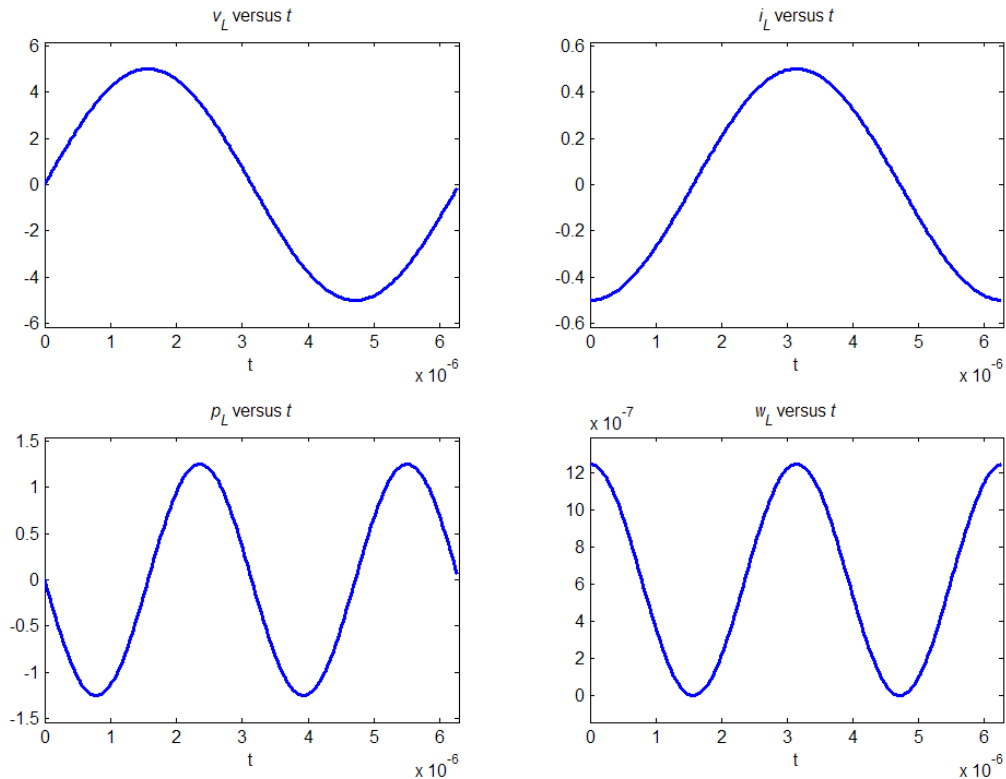
**P3.52**  $L = 10 \mu\text{H}$   $v_L(t) = 5 \sin(10^6 t)$

$$i_L(t) = \frac{1}{L} \int_0^t v_L(t) dt + i_L(0) = 10^5 \int_0^t 5 \sin(10^6 t) dt - 0.5 = -0.5 \cos(10^6 t) \text{ A}$$

$$p(t) = v_L(t)i_L(t) = -2.5 \sin(10^6 t) \cos(10^6 t) = -1.25 \sin(2 \times 10^6 t)$$

$$w(t) = \frac{1}{2} L [i_L(t)]^2 = 1.25 \cos^2(10^6 t) \mu\text{J}$$

The sketches should be similar to the following plots. The units for the quantities in these plots are A, V, W, J and s



**P3.53** For an inductor with an initial current of 10 A, we have

$$i(t) = \frac{1}{L} \int_{t_0}^t v(t) dt + 10. \text{ However for an infinite inductance, this becomes}$$

$i(t) = +10$  which is the specification for a 10-A current source. Thus, a very large inductance with an initial current of 10 A is an approximation to a 10-A current source.

**P3.54**  $w(4) = \frac{1}{2} L i_L^2(4) = 100 \text{ J} \Rightarrow |i_L(4)| = 10 \text{ A}$

Since a reference is not specified, we can choose  $i_L(4) = +10 \text{ A}$ . Also, because the stored energy is increasing, the power for the inductor carries a plus sign. Thus  $p(4) = 200 = v_L(4)i_L(4)$  and we have  $v_L(4) = 20 \text{ V}$ . Finally, because the current and voltage have the same algebraic signs, the current flows into the positive polarity.

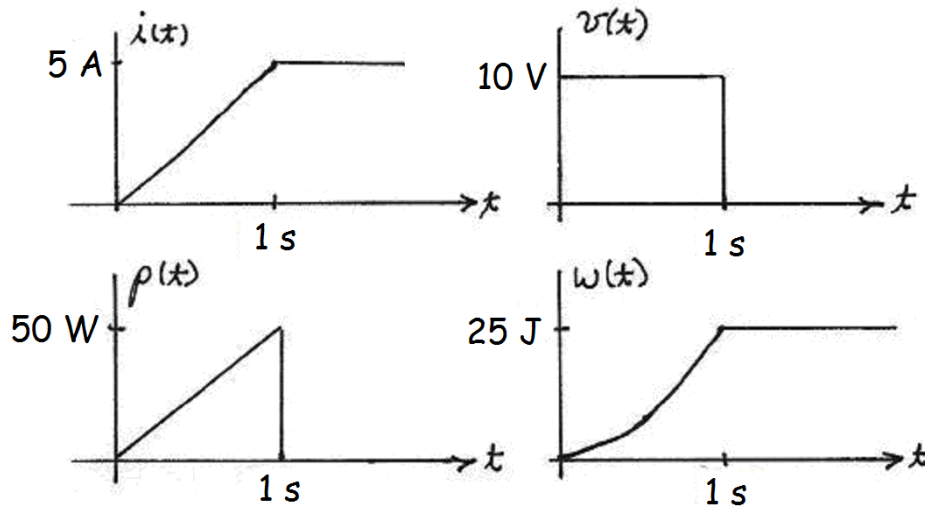
**P3.55** Because the current through an open circuit is zero by definition, and we have  $i(t) = \frac{1}{L} \int_{t_0}^t v(t) dt$  (assuming zero initial current), infinite inductance

corresponds to an open circuit.

Because the voltage across a short circuit is zero by definition, and

$v_L = L \frac{di_L(t)}{dt}$ , we see that  $L = 0$  corresponds to a short circuit.

**P3.56**



**P3.57** 
$$i(t) = \frac{1}{L} \int_0^t v(t) dt + i(0) = \frac{1}{L} \int_0^t V_m \cos(\omega t) dt = \frac{V_m}{\omega L} [\sin(\omega t) - \sin(0)]$$
$$= \frac{V_m}{\omega L} \sin(\omega t)$$

Clearly for  $\omega \rightarrow \infty$ , the current becomes zero, so the inductance becomes the equivalent of an open circuit.

**P3.58** We can write  $i_L(t_0 + \Delta t) = I_f = \frac{1}{L} \int_{t_0}^{t_0 + \Delta t} v_L(t) dt$ . The integral represents the area of the voltage pulse, which has units of volt seconds and must equal  $I_f L$ . Because a constant-amplitude pulse has the largest area for a given peak amplitude, we can say that the peak amplitude of the voltage pulse must be at least as large as  $I_f L / \Delta t$ .

We conclude that the area of the pulse remains constant and that the peak amplitude approaches infinity as  $\Delta t$  approaches zero. In the limit, this type of pulse is called an impulse.



**P3.59** Inductances are combined in the same way as resistances. Inductances in series are added. Inductances in parallel are combined by taking the reciprocal of the sum of the reciprocals of the several inductances.

**P3.60\*** (a)  $L_{eq} = 1 + \frac{1}{1/6 + 1/(1+2)} = 3 \text{ H}$

(b) 9 H in parallel with 18 H is equivalent to 6 H. Also, 20 H in parallel with 5 H is equivalent to 4 H. Finally, we have  $L_{eq} = \frac{1}{1/15 + 1/(6+4)} = 6 \text{ H}$

**P3.61** (a) The 2 H inductors and 0.5 H inductor have no effect because they are in parallel with a short circuit. Thus,  $L_{eq} = 1 \text{ H}$ .

(b) The two 2-H inductances in parallel are equivalent to 1 H. Also, the 1 H in parallel with the 3 H inductance is equivalent to 0.75 H. Thus,  

$$L_{eq} = 1 + \frac{1}{1/(1+1) + 1/(2+0.75)} = 2.158 \text{ H}.$$

**P3.62** If all four inductors are connected in series, we obtain the maximum inductance:

$$L_{\max} = 12 \text{ H}$$

By connecting all three inductors in parallel, we obtain the minimum inductance:

$$L_{\min} = \frac{1}{1/3 + 1/3 + 1/3 + 1/3} = 0.75 \text{ H}$$

**P3.63\*** 
$$i(t) = \frac{1}{L_{eq}} \int_0^t v(t) dt = \frac{L_1 + L_2}{L_1 L_2} \int_0^t v(t) dt$$

$$i_1(t) = \frac{1}{L_1} \int_0^t v(t) dt$$

Thus, we can write  $i_1(t) = \frac{L_2}{L_1 + L_2} i(t) = \frac{2}{3} i(t).$

Similarly, we have  $i_2(t) = \frac{L_1}{L_1 + L_2} i(t) = \frac{1}{3} i(t).$

This is similar to the current-division principle for resistances. Keep in mind that these formulas assume that the initial currents are zero.

- P3.64** Ordinarily, negative inductance is not practical. Thus, adding inductance in series always increases the equivalent inductance. However, placing inductance in parallel results in smaller inductance. Thus, we need to consider a parallel inductance such that

$$\frac{1}{1/L + 1/4} = 3$$

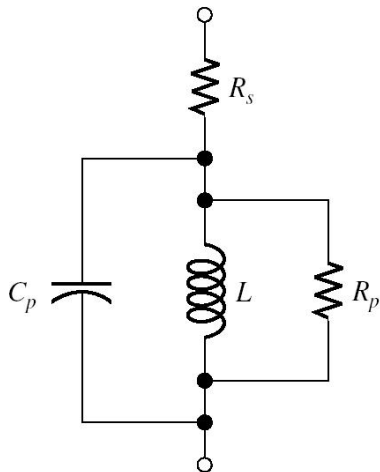
Solving, we find that  $L = 12 \text{ H}$ .

- P3.65** Ordinarily, negative inductance is not practical. Thus, adding inductance in series always increases the equivalent inductance. However, placing inductance in parallel results in smaller inductance. Thus, we need to consider a parallel inductance such that

$$\frac{1}{1/L + 1/10} = 6$$

Solving, we find that  $L = 15 \text{ H}$ .

**P3.66**



- P3.67** (a)  $v_R(t) = Ri(t) = 0.1 \cos(10^5 t)$   
 $v_L(t) = L \frac{di(t)}{dt} = -100 \sin(10^5 t)$   
 $v(t) = v_R(t) + v_L(t)$   
 $= 0.1 \cos(10^5 t) - 100 \sin(10^5 t)$

In this case, to obtain 1% accuracy, the resistance can be neglected.

(b) For  $i(t) = 0.1 \cos(10t)$ , we have

$$v_R(t) = 0.1 \cos(10t)$$

$$v_L(t) = -0.01 \sin(10t)$$

$$v(t) = 0.1 \cos(10t) - 0.01 \sin(10t)$$

Thus, in this case, the parasitic resistance cannot be neglected.

**P3.68** Because  $v_L = L \frac{di_L(t)}{dt} = 0$  for currents that are constant in time, we conclude that the inductance behaves as a short circuit for dc currents. The capacitance acts as an open circuit for constant voltages. Thus, the circuit simplifies to the single resistance  $R_s$ , which is given

$$\text{by } R_s = \frac{400 \text{ mV}}{100 \text{ mA}} = 4 \Omega.$$

**P3.69** 
$$v(t) = L \frac{di_L(t)}{dt} = 10 \times 10^{-3} \times 10^4 [\cos(10^4 t)] = 100 \cos(10^4 t) \text{ V}$$

$$i_C(t) = C \frac{dv(t)}{dt} = -\sin(10^4 t) \text{ A}$$

$$i(t) = i_C(t) + i_L(t) = 0$$

$$w_C(t) = \frac{1}{2} C v^2(t) = 5 \cos^2(10^4 t) = 2.5 + 2.5 \cos(2 \times 10^4 t) \text{ mJ}$$

$$w_L(t) = \frac{1}{2} L i_L^2(t) = 5 \sin^2(10^4 t) = 2.5 - 2.5 \cos(2 \times 10^4 t) \text{ mJ}$$

$$w(t) = w_C(t) + w_L(t) = 5 \text{ mJ}$$

The values in this circuit have been carefully selected so the source current is zero. Because the source current is zero and there are no resistances, there is no source or sink for energy in the circuit. Thus, we would expect the total energy to be constant, as the equations show. The total energy surges back and forth between the capacitance and the inductance. In a real circuit, the parasitic resistances would eventually absorb the energy. The circuit is analogous to a swinging pendulum or a ringing bell.

**P3.70** 
$$i(t) = C \frac{dv_C(t)}{dt} = 250 \times 10^{-6} \times 40 \times 1000 [-\sin(1000t)] = -10 \sin(1000t) \text{ A}$$

$$v_L(t) = L \frac{di(t)}{dt} = -40 \cos(1000t) \text{ V}$$

$$v(t) = v_C(t) + v_L(t) = 0$$

$$w_C(t) = \frac{1}{2} C v_C^2(t) = 200 \cos^2(1000t) = 100 + 100 \cos(2000t) \text{ mJ}$$

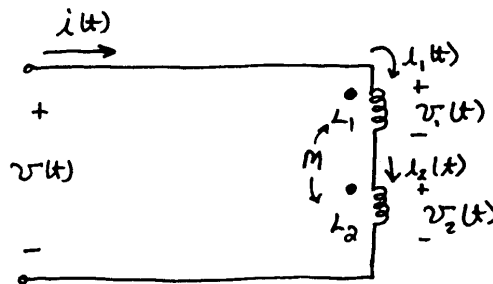
$$w_L(t) = \frac{1}{2} L i^2(t) = 200 \sin^2(1000t) = 100 - 100 \cos(2000t) \text{ mJ}$$

$$w(t) = w_C(t) + w_L(t) = 200 \text{ mJ}$$

The values in this circuit have been carefully selected so the source voltage is zero. Because the source voltage is zero and there are no resistances, there is no source or sink for energy in the circuit. Thus, we would expect the total energy to be constant, as the equations show. The total energy surges back and forth between the capacitance and the inductance. In a real circuit, the parasitic resistances would eventually absorb the energy. The circuit is analogous to a swinging pendulum or a ringing bell.

**P3.71** When a time-varying current flows in a coil, a time-varying magnetic field is produced. If some of this field links a second coil, voltage is induced in it. Thus, time-varying current in one coil results in a contribution to the voltage across a second coil.

**P3.72\*** (a)



As in Figure 3.23a, we can write

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

However, for the circuit at hand, we have  $i(t) = i_1(t) = i_2(t)$ .

Thus,

$$v_1(t) = (L_1 + M) \frac{di(t)}{dt}$$

$$v_2(t) = (L_2 + M) \frac{di(t)}{dt}$$

Also, we have  $v(t) = v_1(t) + v_2(t)$ .

Substituting, we obtain  $v(t) = (L_1 + 2M + L_2) \frac{di(t)}{dt}$ .

Thus, we can write  $v(t) = L_{eq} \frac{di(t)}{dt}$ , in which

$$L_{eq} = L_1 + 2M + L_2.$$

(b) Similarly, for the dot at the bottom end of  $L_2$ , we have

$$L_{eq} = L_1 - 2M + L_2$$

**P3.73** (a) Refer to Figures 3.23 and P3.73. For the dots as shown in Figure P3.73, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = 40 \cos(20t) - 15 \sin(30t) \text{ V}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = 20 \cos(20t) - 45 \sin(30t) \text{ V}$$

(b) With the dot moved to the bottom of  $L_2$ , we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt} = 40 \cos(20t) + 15 \sin(30t) \text{ V}$$

$$v_2(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = -20 \cos(20t) - 45 \sin(30t)$$

**P3.74** In general, we have

$$v_1(t) = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$v_2(t) = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Substituting the given information, we have

$$v_1(t) = -4000 \sin(1000t)$$

and

$$2000 \sin(1000t) = \mp 2000 M \sin(1000t)$$

We deduce that  $M = 1 \text{ H}$ . Furthermore, because the lower of the two algebraic signs applies, we know that the currents are referenced into unlike (i.e., one dotted and one undotted) terminals. Similarly the positive reference for the voltages is at unlike terminals.

**P3.75** With a short circuit across the terminals of the second coil, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = 0 = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

Solving the second equation for  $di_2(t)/dt$  and substituting into the first equation, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - \frac{M^2}{L_2} \frac{di_1(t)}{dt} = \frac{L_1 L_2 - M^2}{L_2} \frac{di_1(t)}{dt}$$

from which we can see that

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_2}$$

**P3.76** Because of the parallel connection, we have  $v_1(t) = v_2(t) = v(t)$  and the equations for the mutually coupled inductors become

$$v(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$

$$v(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

Using determinants to solve for the derivatives, we have

$$\frac{di_1(t)}{dt} = \frac{\begin{vmatrix} v(t) & -M \\ v(t) & L_2 \end{vmatrix}}{\begin{vmatrix} L_1 & -M \\ -M & L_2 \end{vmatrix}} = \frac{L_2 + M}{L_1 L_2 - M^2} v(t) \quad \frac{di_2(t)}{dt} = \frac{\begin{vmatrix} L_1 & v(t) \\ -M & v(t) \end{vmatrix}}{\begin{vmatrix} L_1 & -M \\ -M & L_2 \end{vmatrix}} = \frac{L_1 + M}{L_1 L_2 - M^2} v(t)$$

Then, we have

$$i(t) = i_1(t) + i_2(t)$$

$$\frac{di(t)}{dt} = \frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{L_1 + L_2 + 2M}{L_1 L_2 - M^2} v(t)$$

from which we conclude that

$$L_{eq} = \frac{L_1 L_2 + M^2}{L_1 + L_2 - 2M}$$

**P3.77**  $v_L = L \frac{di_L}{dt} = 0.2 \frac{d}{dt} [\exp(-2t) \sin(4\pi t)]$

$$v_L = 0.8\pi \exp(-2t) \cos(4\pi t) - 0.4 \exp(-2t) \sin(4\pi t)$$

A sequence of MATLAB commands that produces the desired plots is

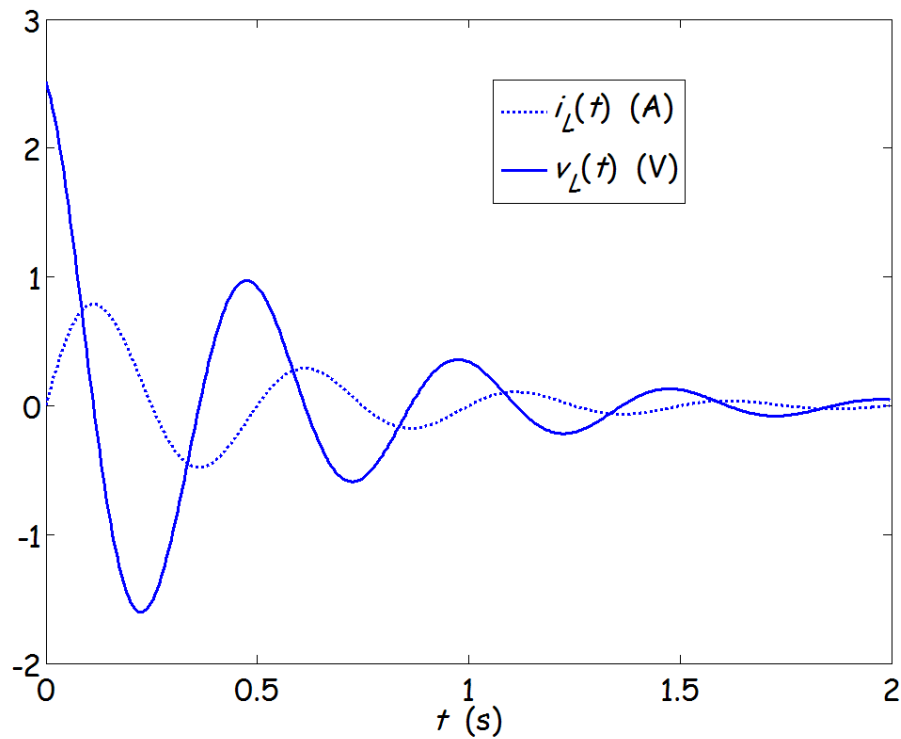
syms iL vL t

```

iL = exp(-2*t)*sin(4*pi*t);
vL = 0.2*diff(iL,t);
ezplot(iL, [0 2])
hold on
ezplot(vL, [0 2])

```

The result is:



**P3.78** Using either tables or integration by parts, we have

$$i_L(t) = \frac{1}{L} \int_0^t v_L(t) dt + i_L(0) = \frac{1}{L} \int_0^t t \exp(-t) dt = \frac{1}{L} [-\exp(-t)(t+1)]_0^t$$

$$i_L(t) = 1 - t \exp(-t) - \exp(-t)$$

A sequence of MATLAB commands to verify our result for  $i_L(t)$  and obtain the desired plots is

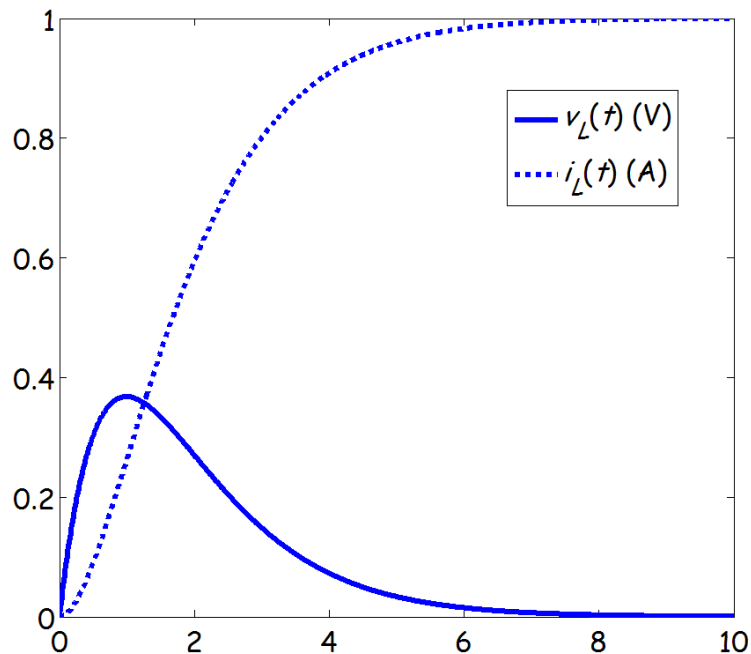
```

syms iL vL t
vL = t*exp(-t);
iL = int(vL,t,0,t)
ezplot(vL, [0 10])

```

hold on  
ezplot(iL, [0 10])

The result is:



### Practice Test

**T3.1**

$$v_{ab}(t) = \frac{1}{C} \int_0^t i_{ab}(t) dt + v_c(0) = 10^{-5} \int_0^t 0.3 \exp(-2000t) dt$$

$$v_{ab}(t) = -15 \exp(-2000t) \Big|_0^t$$

$$v_{ab}(t) = 15 - 15 \exp(-2000t) \text{ V}$$

$$w_c(\infty) = \frac{1}{2} C v_c^2(\infty) = \frac{1}{2} 10^{-5} (15)^2 = 1.125 \text{ mJ}$$

**T3.2**

The  $6\text{-}\mu\text{F}$  and  $3\text{-}\mu\text{F}$  capacitances are in series and have an equivalent capacitance of

$$C_{eq1} = \frac{1}{1/6 + 1/3} = 2 \text{ }\mu\text{F}$$



$C_{eq1}$  is in parallel with the 4- $\mu\text{F}$  capacitance, and the combination has an equivalent capacitance of

$$C_{eq2} = C_{eq1} + 4 = 6 \mu\text{F}$$

$C_{eq2}$  is in series with the 12- $\mu\text{F}$  and the combination, has an equivalent capacitance of

$$C_{eq3} = \frac{1}{1/12 + 1/6} = 4 \mu\text{F}$$

Finally,  $C_{eq3}$  is in parallel with the 1- $\mu\text{F}$  capacitance, and the equivalent capacitance is

$$C_{eq} = C_{eq3} + 1 = 5 \mu\text{F}$$

**T3.3** 
$$C = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{80 \times 8.85 \times 10^{-12} \times 2 \times 10^{-2} \times 3 \times 10^{-2}}{0.1 \times 10^{-3}} = 4248 \text{ pF}$$

**T3.4** 
$$v_{ab}(t) = L \frac{di_{ab}}{dt} = 2 \times 10^{-3} \times 0.3 \times 2000 \cos(2000t) = 1.2 \cos(2000t) \text{ V}$$

The maximum value of  $\sin(2000t)$  is unity. Thus the peak current is 0.3 A, and the peak energy stored is

$$w_{peak} = \frac{1}{2} L i_{peak}^2 = \frac{1}{2} \times 2 \times 10^{-3} (0.3)^2 = 90 \mu\text{J}$$

**T3.5** The 2-H and 4-H inductances are in parallel and the combination has an equivalent inductance of

$$L_{eq1} = \frac{1}{1/2 + 1/4} = 1.333 \text{ H}$$

Also, the 3-H and 5-H inductances are in parallel, and the combination has an equivalent inductance of

$$L_{eq2} = \frac{1}{1/3 + 1/5} = 1.875 \text{ H}$$

Finally,  $L_{eq1}$  and  $L_{eq2}$  are in series. The equivalent inductance between terminals  $a$  and  $b$  is

$$L_{eq} = L_{eq1} + L_{eq2} = 3.208 \text{ H}$$

**T3.6** For these mutually coupled inductances, we have

$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$

$$v_2(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

in which the currents are referenced into the positive polarities. Thus the currents are

$$i_1(t) = 2 \cos(500t) \quad \text{and} \quad i_2(t) = -2 \exp(-400t)$$

Substituting the inductance values and the current expressions we have

$$v_1(t) = -40 \times 10^{-3} \times 1000 \sin(500t) - 20 \times 10^{-3} \times 800 \exp(-400t)$$

$$v_1(t) = -40 \sin(500t) - 16 \exp(-400t) \text{ V}$$

$$v_2(t) = 20 \times 10^{-3} \times 1000 \sin(500t) - 30 \times 10^{-3} \times 800 \exp(-400t)$$

$$v_2(t) = 20 \sin(500t) - 24 \exp(-400t) \text{ V}$$

**T3.7** One set of commands is

```
syms vab iab t
iab = 3*(10^5)*(t^2)*exp(-2000*t);
vab = (1/20e-6)*int(iab,t,0,t)
subplot(2,1,1)
ezplot(iab, [0 5e-3]), title('\iti_a_b\rm (A) versus \itt\rm (s)')
subplot(2,1,2)
ezplot(vab, [0 5e-3]), title('\itv_a_b\rm (V) versus \itt\rm (s)')
```

The results are

$$v_{ab} = \frac{15}{4} - \frac{15}{4} \exp(-2000t) - 7500 \exp(-2000t) - 7.5 \times 10^6 t^2 \exp(-2000t)$$

