

Chapter 2

Discrete-Time Signals and Systems

P2.1 Generate the following sequences using the basic MATLAB signal functions and the basic MATLAB signal operations discussed in this chapter. Plot signal samples using the stem function.

1. $x_1(n) = 3\delta(n+2) + 2\delta(n) - \delta(n-3) + 5\delta(n-7), -5 \leq n \leq 15$

```
% P0201a: x1(n) = 3*delta(n + 2) + 2*delta(n) - delta(n - 3) +
%           5*delta(n - 7), -5 <= n <= 15.
clc; close all;
x1 = 3*impseq(-2,-5,15) + 2*impseq(0,-5,15) - impseq(3,-5,15) + 5*impseq(7,-5,15);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201a'); n1 = [-5:15];
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-1,max(x1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n1,'FontSize',8);
print -deps2 ../EPSFILES/P0201a;
```

The plots of $x_1(n)$ is shown in Figure 2.1.

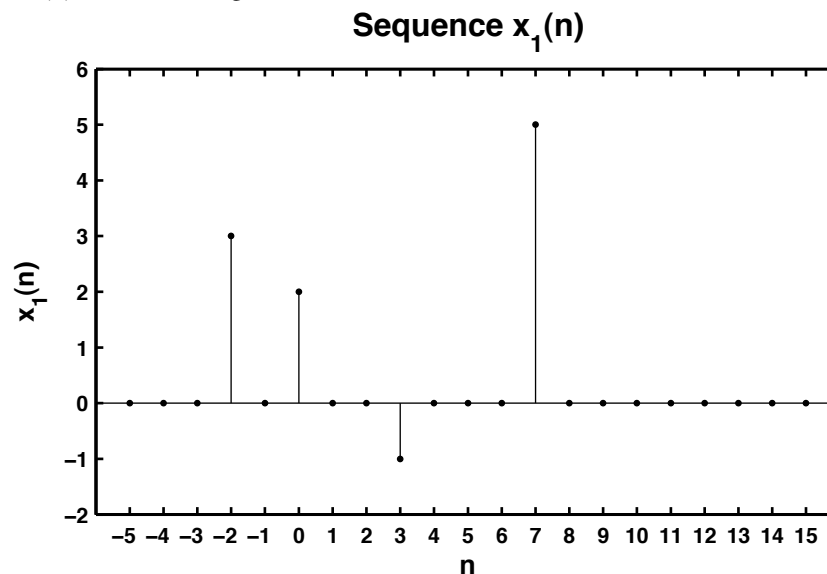


Figure 2.1: Problem P2.1.1 sequence plot

$$2. x_2(n) = \sum_{k=-5}^5 e^{-|k|} \delta(n-2k), -10 \leq n \leq 10.$$

```
% P0201b: x2(n) = sum_{k = -5}^{5} e^{-|k|} * delta(n - 2k), -10 <= n <= 10
clc; close all;
```

```
n2 = [-10:10]; x2 = zeros(1,length(n2));
for k = -5:5
    x2 = x2 + exp(-abs(k))*impseq(2*k, -10,10);
end
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201b');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n2);
print -deps2 ../EPSFILES/P0201b;
```

The plots of $x_2(n)$ is shown in Figure 2.2.

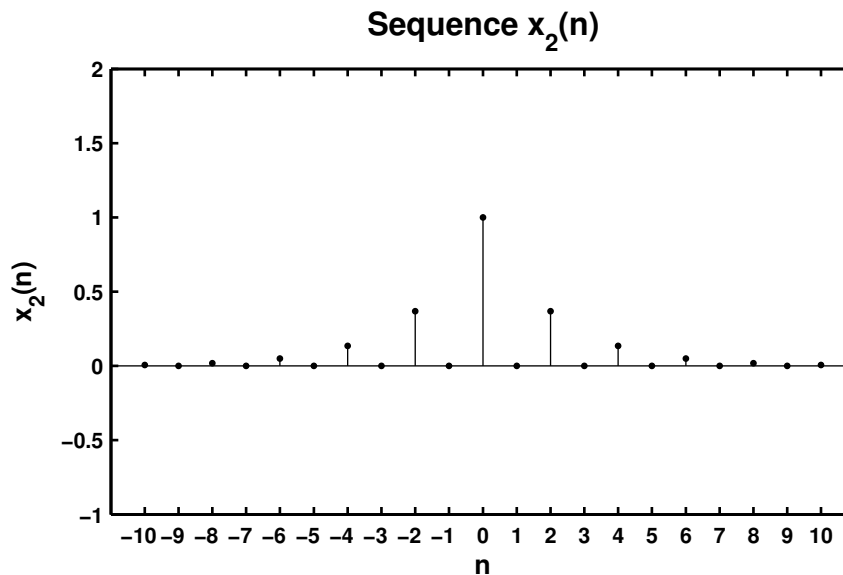


Figure 2.2: Problem P2.1.2 sequence plot

3. $x_3(n) = 10u(n) - 5u(n-5) - 10u(n-10) + 5u(n-15).$

```
% P0201c:  $x_3(n) = 10u(n) - 5u(n-5) + 10u(n-10) + 5u(n-15).$ 
clc; close all;

x3 = 10*stepseq(0,0,20) - 5*stepseq(5,0,20) - 10*stepseq(10,0,20) ...
    + 5*stepseq(15,0,20);
n3 = [0:20];
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-1,max(x3)+2]);
ytick = [-6:2:12];
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n3);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0201c;
```

The plots of $x_3(n)$ is shown in Figure 2.3.

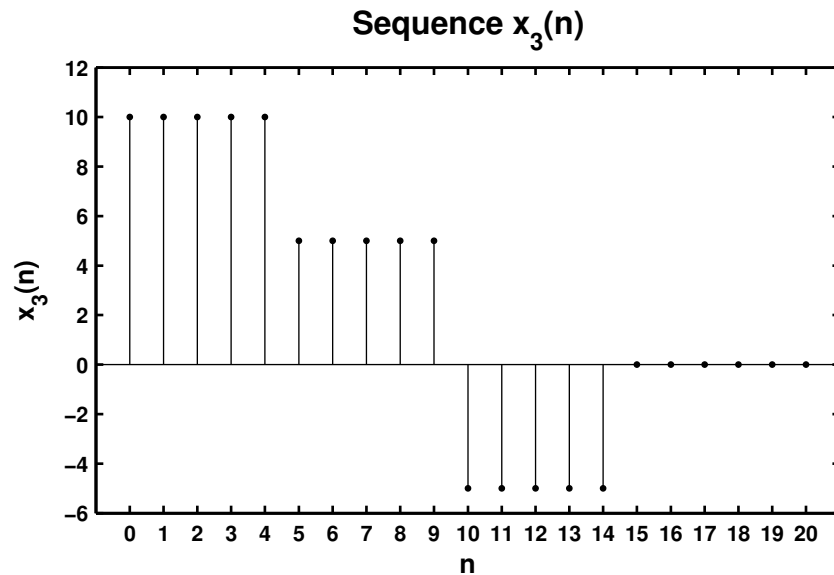


Figure 2.3: Problem P2.1.3 sequence plot

4. $x_4(n) = e^{0.1n}[u(n+20) - u(n-10)]$.

```
% P0201d: x4(n) = e ^ {0.1n} [u(n + 20) - u(n - 10)].
clc; close all;

n4 = [-25:15];
x4 = exp(0.1*n4).*(stepseq(-20,-25,15) - stepseq(10,-25,15));

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-2,max(n4)+2,min(x4)-1,max(x4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n)','FontSize',TFS); ntick = [n4(1):5:n4(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201d; print -deps2 ../Latex/P0201d;
```

The plots of $x_4(n)$ is shown in Figure 2.4.

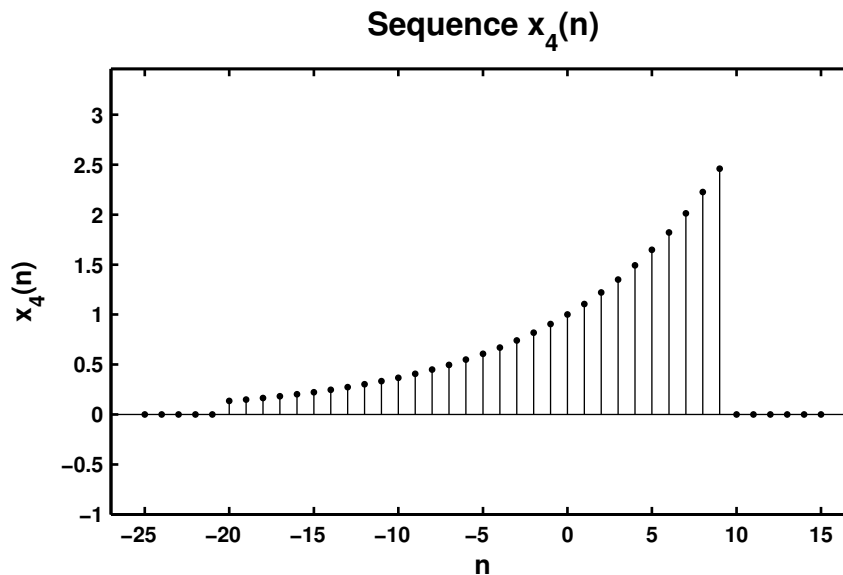


Figure 2.4: Problem P2.1.4 sequence plot

5. $x_5(n) = 5[\cos(0.49\pi n) + \cos(0.51\pi n)]$, $-200 \leq n \leq 200$. Comment on the waveform shape.

```
% P0201e: x5(n) = 5[cos(0.49*pi*n) + cos(0.51*pi*n)], -200 <= n <= 200.
clc; close all;

n5 = [-200:200]; x5 = 5*(cos(0.49*pi*n5) + cos(0.51*pi*n5));

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201e');
Hs = stem(n5,x5,'filled'); set(Hs,'markersize',2);
axis([min(n5)-10,max(n5)+10,min(x5)-2,max(x5)+2]);
xlabel('n','FontSize',LFS); ylabel('x_5(n)','FontSize',LFS);
title('Sequence x_5(n)','FontSize',TFS);
ntick = [n5(1): 40:n5(end)]; ytick = [-12 -10:5:10 12];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0201e; print -deps2 ../Latex/P0201e;
```

The plots of $x_5(n)$ is shown in Figure 2.5.

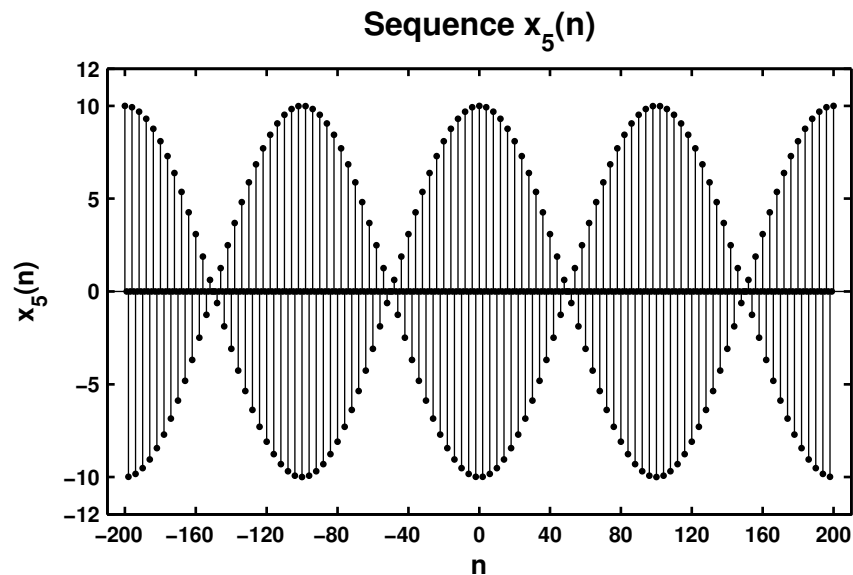


Figure 2.5: Problem P2.1.5 sequence plot

6. $x_6(n) = 2 \sin(0.01\pi n) \cos(0.5\pi n)$, $-200 \leq n \leq 200$.

```
%P0201f: x6(n) = 2 sin(0.01*pi*n) cos(0.5*pi*n), -200 <= n <= 200.
clc; close all;

n6 = [-200:200]; x6 = 2*sin(0.01*pi*n6).*cos(0.5*pi*n6);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201f');
Hs = stem(n6,x6,'filled'); set(Hs,'markersize',2);
axis([min(n6)-10,max(n6)+10,min(x6)-1,max(x6)+1]);
xlabel('n','FontSize',LFS); ylabel('x_6(n)','FontSize',LFS);
title('Sequence x_6(n)','FontSize',TFS);
ntick = [n6(1): 40:n6(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201f; print -deps2 ../Latex/P0201f;
```

The plots of $x_6(n)$ is shown in Figure 2.6.

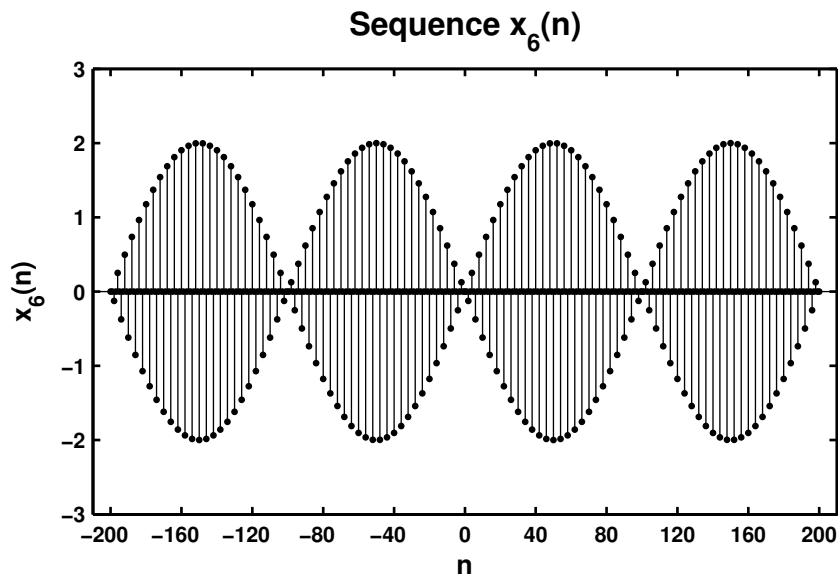


Figure 2.6: Problem P2.1.6 sequence plot

7. $x_7(n) = e^{-0.05n} \sin(0.1\pi n + \pi/3)$, $0 \leq n \leq 100$.

```
% P0201g: x7(n) = e ^ {-0.05*n}*sin(0.1*pi*n + pi/3), 0 <= n <=100.
clc; close all;

n7 = [0:100]; x7 = exp(-0.05*n7).*sin(0.1*pi*n7 + pi/3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201g');
Hs = stem(n7,x7,'filled'); set(Hs,'markersize',2);
axis([min(n7)-5,max(n7)+5,min(x7)-1,max(x7)+1]);
xlabel('n','FontSize',LFS); ylabel('x_7(n)','FontSize',LFS);
title('Sequence x_7(n)','FontSize',TFS);
ntick = [n7(1): 10:n7(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201g;
```

The plots of $x_7(n)$ is shown in Figure 2.7.

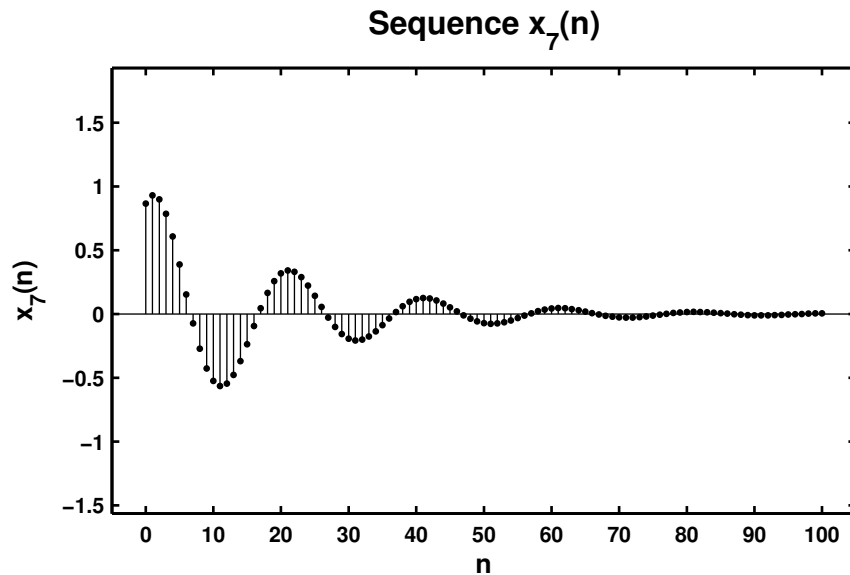


Figure 2.7: Problem P2.1.7 sequence plot

8. $x_8(n) = e^{0.01n} \sin(0.1\pi n)$, $0 \leq n \leq 100$.

```
% P0201h: x8(n) = e ^ {0.01*n}*sin(0.1*pi*n), 0 <= n <=100.
clc; close all;

n8 = [0:100]; x8 = exp(0.01*n8).*sin(0.1*pi*n8);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201h');
Hs = stem(n8,x8,'filled'); set(Hs,'markersize',2);
axis([min(n8)-5,max(n8)+5,min(x8)-1,max(x8)+1]);
xlabel('n','FontSize',LFS); ylabel('x_8(n)','FontSize',LFS);
title('Sequence x_8(n)','FontSize',TFS);
ntick = [n8(1): 10:n8(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201h
```

The plots of $x_8(n)$ is shown in Figure 2.8.

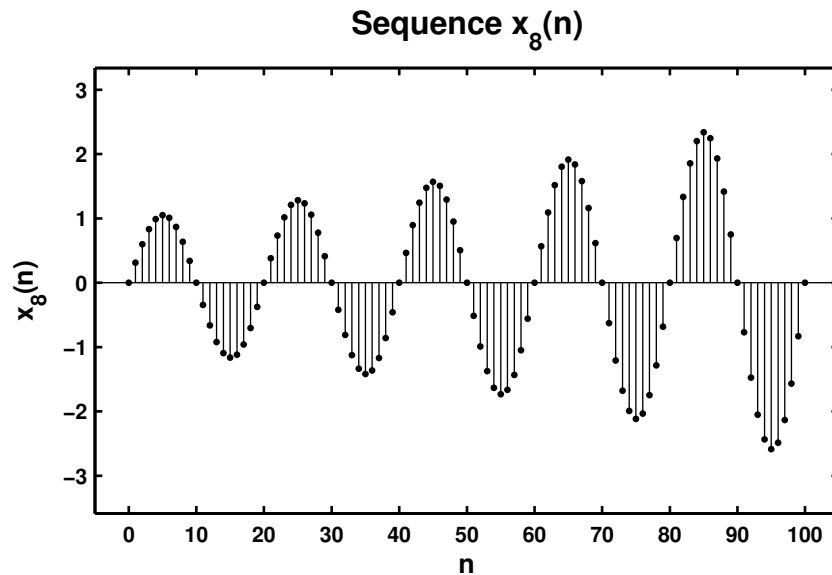


Figure 2.8: Problem P2.1.8 sequence plot

P2.2 Generate the following random sequences and obtain their histogram using the `hist` function with 100 bins. Use the `bar` function to plot each histogram.

1. $x_1(n)$ is a random sequence whose samples are independent and uniformly distributed over $[0, 2]$ interval. Generate 100,000 samples.

```
% P0202a: x1(n) = uniform[0,2]
clc; close all;

n1 = [0:100000-1]; x1 = 2*rand(1,100000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202a');
[h1,x1out] = hist(x1,100); bar(x1out, h1);
axis([-0.1 2.1 0 1200]);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_1(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202a;
```

The plots of $x_1(n)$ is shown in Figure 2.9.

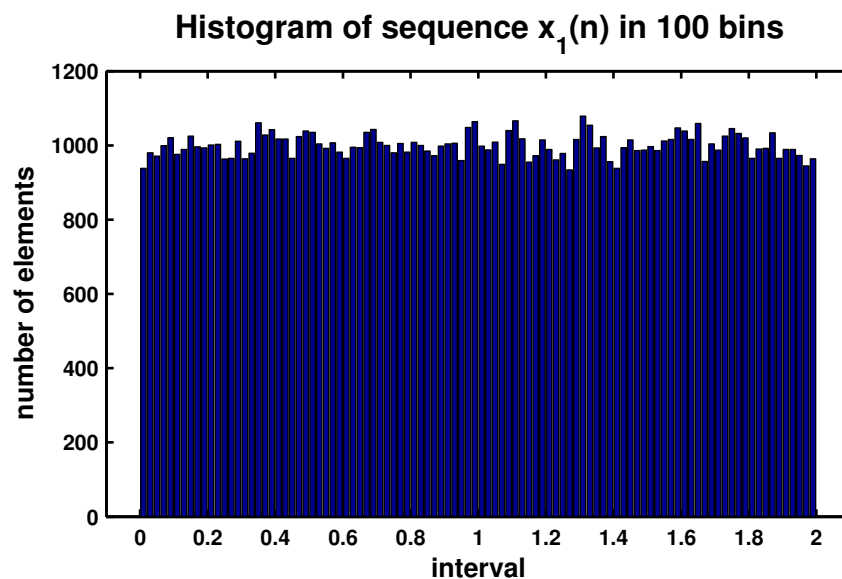


Figure 2.9: Problem P2.2.1 sequence plot

2. $x_2(n)$ is a Gaussian random sequence whose samples are independent with mean 10 and variance 10. Generate 10,000 samples.

```
% P0202b:  $x_2(n)$  = gaussian{10,10}
clc; close all;

n2 = [1:10000]; x2 = 10 + sqrt(10)*randn(1,10000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202b');
[h2,x2out] = hist(x2,100); bar(x2out,h2);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence  $x_2(n)$  in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202b;
```

The plots of $x_2(n)$ is shown in Figure 2.10.

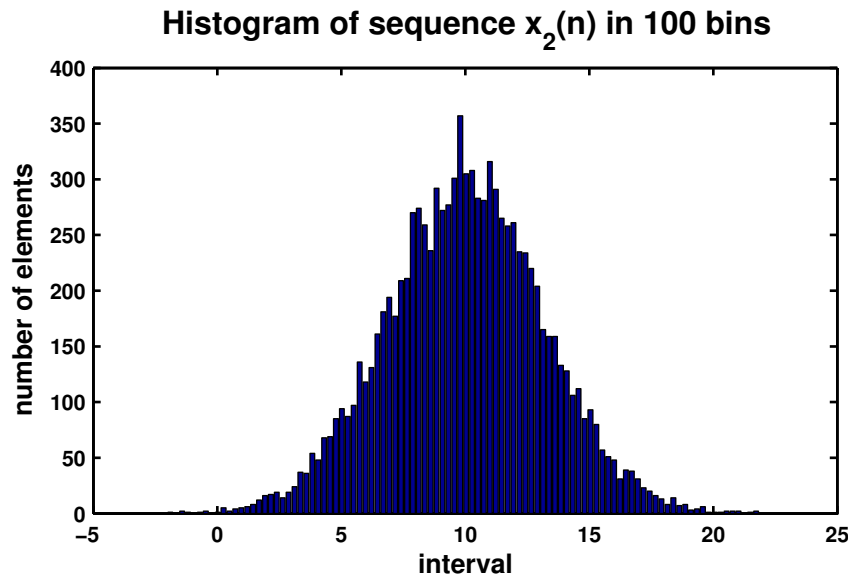


Figure 2.10: Problem P2.2.2 sequence plot

3. $x_3(n) = x_1(n) + x_1(n-1)$ where $x_1(n)$ is the random sequence given in part 1 above. Comment on the shape of this histogram and explain the shape.

```
% P0202c:  $x_3(n) = x_1(n) + x_1(n-1)$  where  $x_1(n) = \text{uniform}[0,2]$ 
clc; close all;

n1 = [0:100000-1]; x1 = 2*rand(1,100000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202c');
[x11,n11] = sigshift(x1,n1,1);
[x3,n3] = sigadd(x1,n1,x11,n11);
[h3,x3out] = hist(x3,100);
bar(x3out,h3); axis([-0.5 4.5 0 2500]);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence  $x_3(n)$  in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202c;
```

The plots of $x_3(n)$ is shown in Figure 2.11.

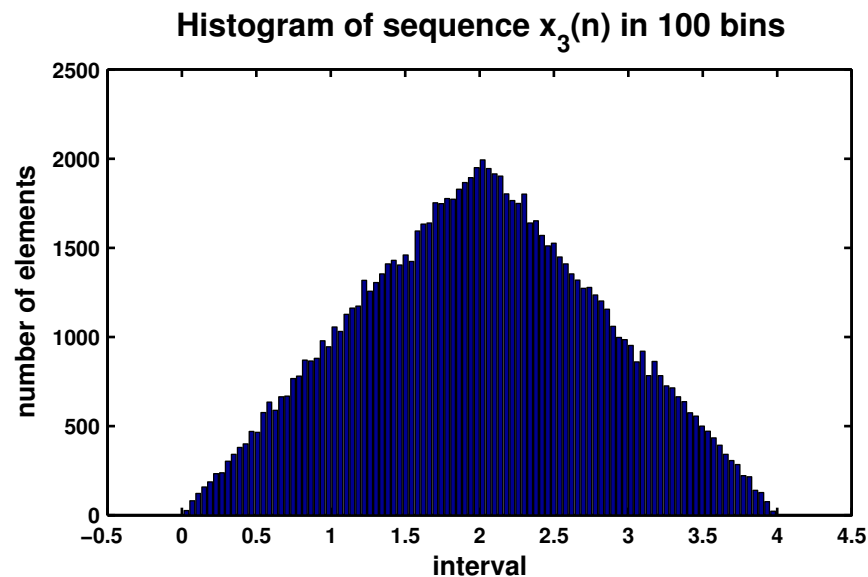


Figure 2.11: Problem P2.2.3 sequence plot

4. $x_4(n) = \sum_{k=1}^4 y_k(n)$ where each random sequence $y_k(n)$ is independent of others with samples uniformly distributed over $[-0.5, 0.5]$. Comment on the shape of this histogram.

```
%P0202d: x4(n) = sum_{k=1} ^ {4} y_k(n), where each independent of others
%          with samples uniformly distributed over [-0.5,0.5];
clc; close all;
```

```
y1 = rand(1,100000) - 0.5; y2 = rand(1,100000) - 0.5;
y3 = rand(1,100000) - 0.5; y4 = rand(1,100000) - 0.5;
x4 = y1 + y2 + y3 + y4;
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202d');
[h4,x4out] = hist(x4,100); bar(x4out,h4);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_4(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202d;
```

The plots of $x_4(n)$ is shown in Figure 2.12.

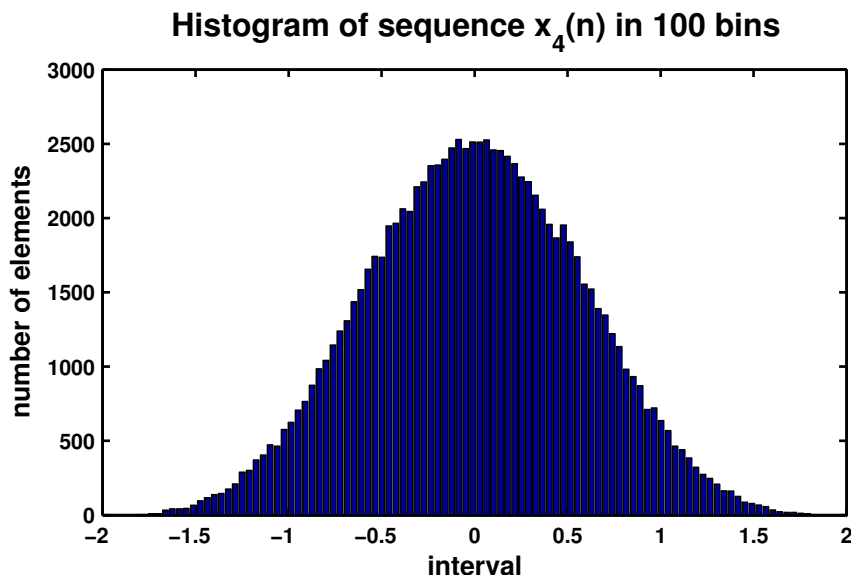


Figure 2.12: Problem P2.2.4 sequence plot

P2.3 Generate the following periodic sequences and plot their samples (using the `stem` function) over the indicated number of periods.

1. $\tilde{x}_1(n) = \{\dots, -2, -1, 0, 1, 2, \dots\}_{\text{periodic}}$. Plot 5 periods.

```
% P0203a: x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;
```

```
n1 = [-12:12]; x1 = [-2,-1,0,1,2];
x1 = x1*ones(1,5); x1 = (x1(:))';
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203a');
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-1,max(x1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS);
ntick = [n1(1):2:n1(end)]; ytick = [min(x1) - 1:max(x1) + 1];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0203a
```

The plots of $\tilde{x}_1(n)$ is shown in Figure 2.13.

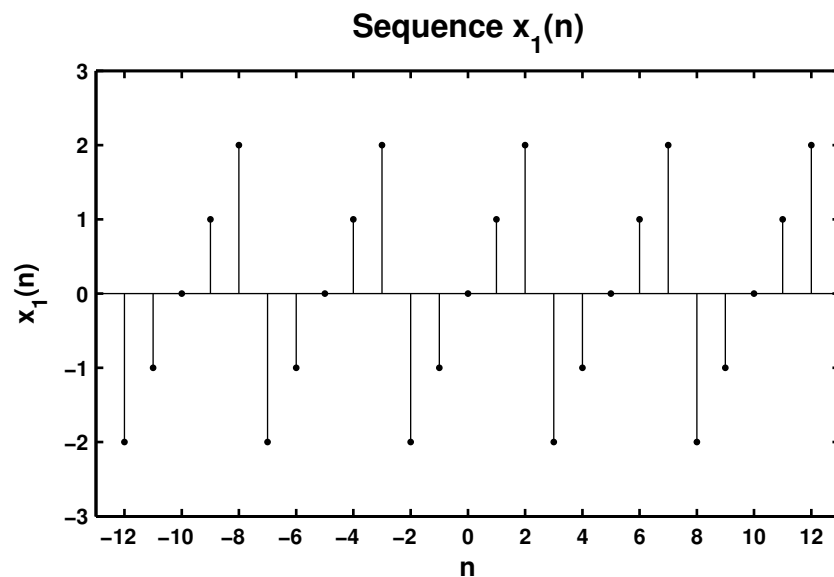


Figure 2.13: Problem P2.3.1 sequence plot

2. $\tilde{x}_2(n) = e^{0.1n}[u(n) - u(n-20)]_{\text{periodic}}$. Plot 3 periods.

```
% P0203b: x2 = e ^ {0.1n} [u(n) - u(n-20)] periodic. 3 periods
clc; close all;
```

```
n2 = [0:21]; x2 = exp(0.1*n2).*(stepseq(0,0,21)-stepseq(20,0,21));
x2 = x2*ones(1,3); x2 = (x2(:))'; n2 = [-22:43];
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203b');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-2,max(n2)+4,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS);
ntick = [n2(1):4:n2(end)-5 n2(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../Chap2_EPSFILES/P0203b;
```

The plots of $\tilde{x}_2(n)$ is shown in Figure 2.14.

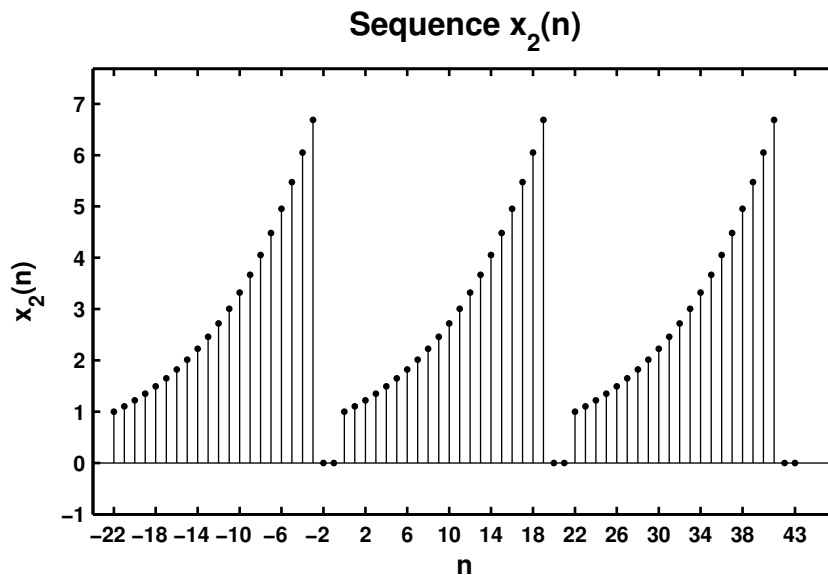


Figure 2.14: Problem P2.3.2 sequence plot

3. $\tilde{x}_3(n) = \sin(0.1\pi n)[u(n) - u(n - 10)]$. Plot 4 periods.

```
% P0203c: x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;
```

```
n3 = [0:11]; x3 = sin(0.1*pi*n3).*(stepseq(0,0,11)-stepseq(10,0,11));
x3 = x3*ones(1,4); x3 = (x3(:))'; n3 = [-12:35];
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-0.5,max(x3)+0.5]);
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
ntick = [n3(1):4:n3(end)-3 n3(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0203c;
```

The plots of $\tilde{x}_3(n)$ is shown in Figure 2.15.

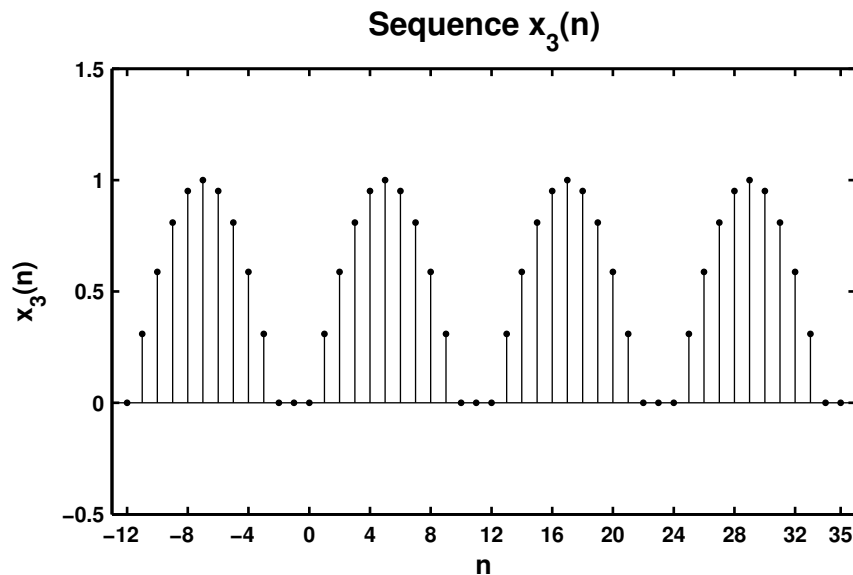


Figure 2.15: Problem P2.3.3 sequence plot

4. $\tilde{x}_4(n) = \{\dots, 1, 2, 3, \dots\}_{\text{periodic}} + \{\dots, 1, 2, 3, 4, \dots\}_{\text{periodic}}, 0 \leq n \leq 24$. What is the period of $\tilde{x}_4(n)$?

```
% P0203d x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2,...} periodic. 5 periods
clc; close all;
```

```
n4 = [0:24]; x4a = [1 2 3]; x4a = x4a'*ones(1,9); x4a = (x4a(:))';
x4b = [1 2 3 4]; x4b = x4b'*ones(1,7); x4b = (x4b(:))';
x4 = x4a(1:25) + x4b(1:25);
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-1,max(n4)+1,min(x4)-1,max(x4)+1]);
xlabel('n', 'FontSize', LFS); ylabel('x_4(n)', 'FontSize', LFS);
title('Sequence x_4(n):Period = 12','FontSize',TFS);
ntick = [n4(1) :2:n4(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0203d;
```

The plots of $\tilde{x}_4(n)$ is shown in Figure 2.16. From the figure, the fundamental period of $\tilde{x}_4(n)$ is 12.

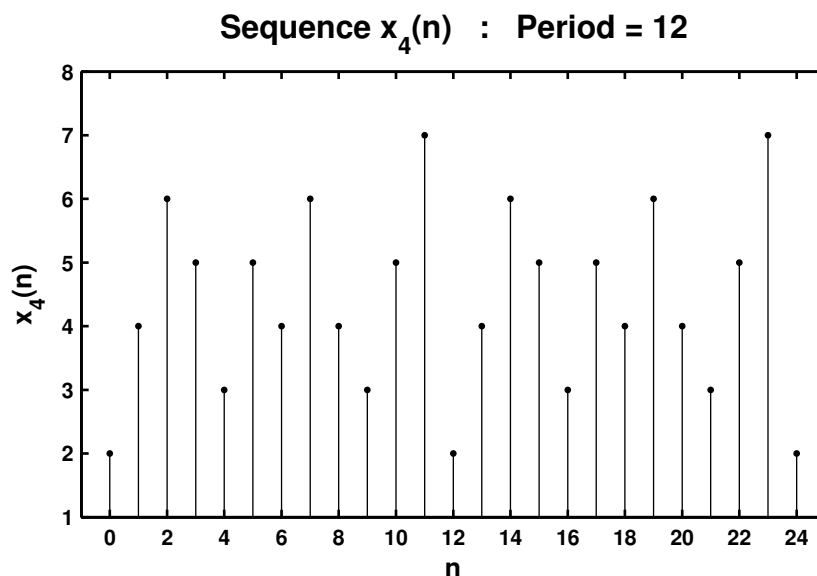


Figure 2.16: Problem P2.3.4 sequence plot

P2.4 Let $x(n) = \{2, 4, -3, 1, -5, 4, 7\}$. Generate and plot the samples (use the stem function) of the following sequences.

1. $x_1(n) = 2x(n-3) + 3x(n+4) - x(n)$

```
% P0204a: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
% x1(n) = 2x(n - 3) + 3x(n + 4) - x(n)
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7];
[x11,n11] = sigshift(x,n,3);           % shift by 3
[x12,n12] = sigshift(x,n,-4);          % shift by -4
[x13,n13] = sigadd(2*x11,n11,3*x12,n12); % add two sequences
[x1,n1] = sigadd(x13,n13,-x,n);        % add two sequences

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204a');
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-3,max(x1)+1]);
xlabel('n','FontSize',LFS);
ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS); ntick = n1;
ytick = [min(x1)-3:5:max(x1)+1];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204a;
```

The plots of $x_1(n)$ is shown in Figure 2.17.

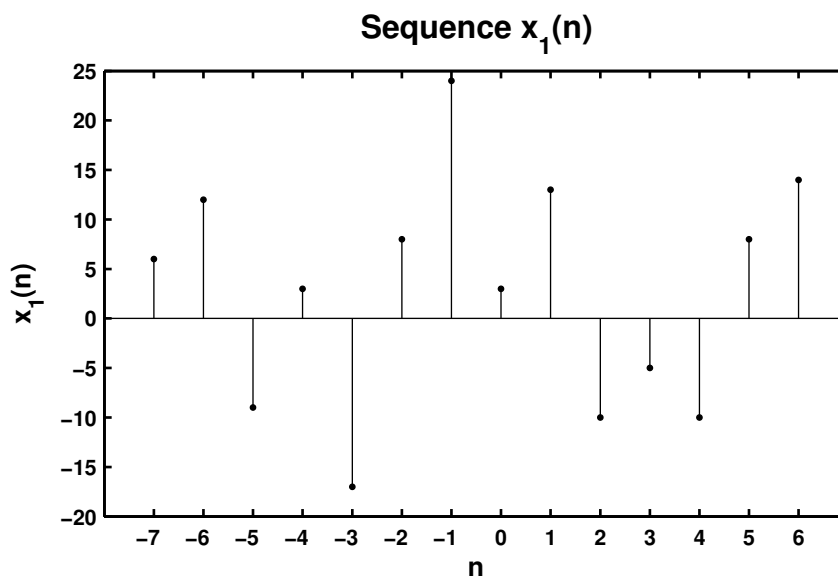


Figure 2.17: Problem P2.4.1 sequence plot

2. $x_2(n) = 4x(4+n) + 5x(n+5) + 2x(n)$

```
% P0204b: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
% x2(n) = 4x(4 + n) + 5x(n + 5) + 2x(n)
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204b');
n = [-3:3]; x = [2,4,-3,1,-5,4,7];

[x21,n21] = sigshift(x,n,-4);           % shift by -4
[x22,n22] = sigshift(x,n,-5);           % shift by -5
[x23,n23] = sigadd(4*x21,n21,5*x22,n22); % add two sequences
[x2,n2] = sigadd(x23,n23,2*x,n);        % add two sequences

Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-4,max(x2)+6]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS); ntick = n2;
ytick = [-25 -20:10:60 65];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204b;
```

The plots of $x_2(n)$ is shown in Figure 2.18.

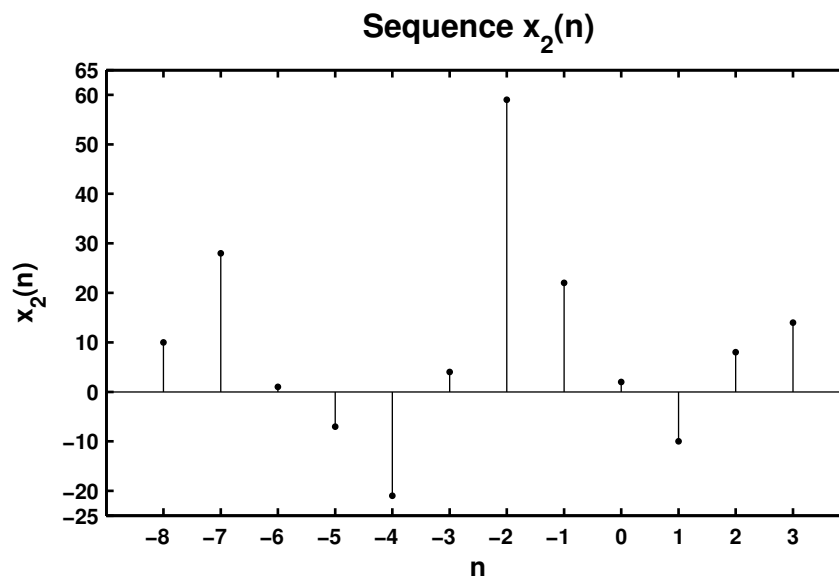


Figure 2.18: Problem P2.4.2 sequence plot

3. $x_3(n) = x(n+3)x(n-2) + x(1-n)x(n+1)$

```
% P0204c: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
%          x3(n) = x(n+3)x(n-2) + x(1-n)x(n+1)
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7]; % given sequence x(n)
[x31,n31] = sigshift(x,n,-3);      % shift sequence by -3
[x32,n32] = sigshift(x,n,2);       % shift sequence by 2
[x33,n33] = sigmult(x31,n31,x32,n32); % multiply 2 sequences
[x34,n34] = sigfold(x,n);          % fold x(n)
[x34,n34] = sigshift(x34,n34,1);   % shift x(-n) by 1
[x35,n35] = sigshift(x,n,-1);      % shift x(n) by -1
[x36,n36] = sigmult(x34,n34,x35,n35); % multiply 2 sequences
[x3,n3] = sigadd(x33,n33,x36,n36); % add 2 sequences

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-10,max(x3)+10]);
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
ntick = n3; ytick = [-30:10:60];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204c;
```

The plots of $x_3(n)$ is shown in Figure 2.19.

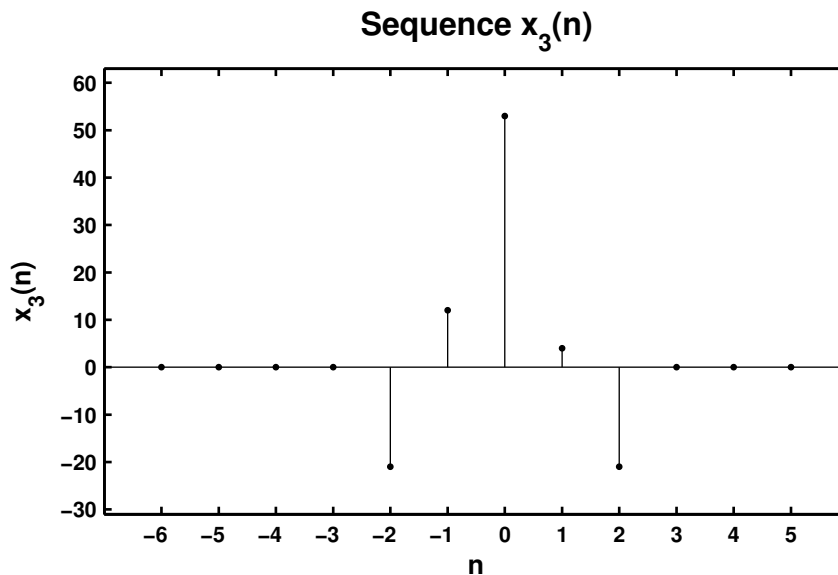


Figure 2.19: Problem P2.4.3 sequence plot

4. $x_4(n) = 2e^{0.5n}x(n) + \cos(0.1\pi n)x(n+2)$, $-10 \leq n \leq 10$

```
% P0204d: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
%          x4(n) = 2*e^{0.5n}*x(n)+cos(0.1*pi*n)*x(n+2), -10 <=n< =10
clc; close all;
```

```
n = [-3:3]; x = [2,4,-3,1,-5,4,7];          % given sequence x(n)
n4 = [-10:10]; x41 = 2*exp(0.5*n4); x412 = cos(0.1*pi*n4);
[x42,n42] = sigmult(x41,n4,x,n);
[x43,n43] = sigshift(x,n,-2);
[x44,n44] = sigmult(x412,n42,x43,n43);
[x4,n4] = sigadd(x42,n42,x44,n44);
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-1,max(n4)+1,min(x4)-11,max(x4)+10]);
xlabel('n','FontSize',LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n)','FontSize',TFS);
ntick = n4; ytick = [-20:10:70];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204d;
```

The plot of $x_4(n)$ is shown in Figure 2.20.

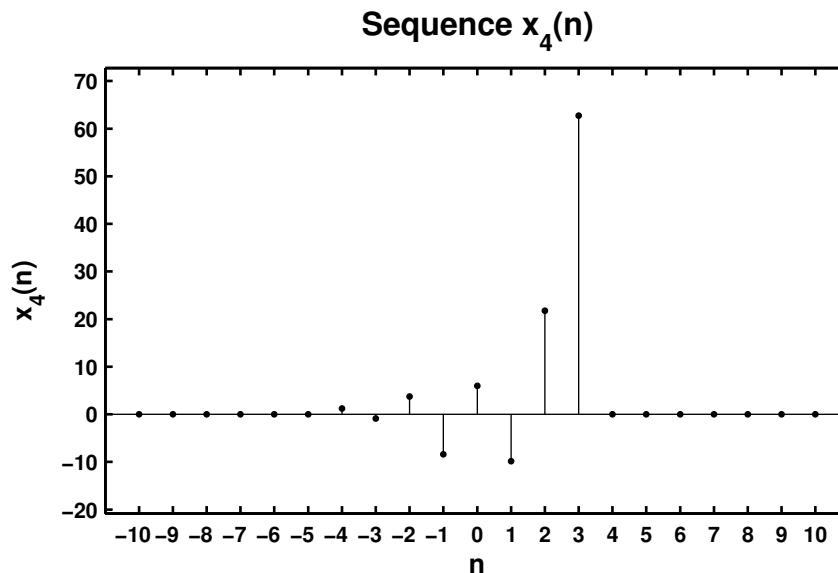


Figure 2.20: Problem P2.4.4 sequence plot

P2.5 The complex exponential sequence $e^{j\omega_0 n}$ or the sinusoidal sequence $\cos(\omega_0 n)$ are periodic if the *normalized* frequency $f_0 \triangleq \frac{\omega_0}{2\pi}$ is a rational number; that is, $f_0 = \frac{K}{N}$, where K and N are integers.

1. Analytical proof: The exponential sequence is periodic if

$$e^{j2\pi f_0(n+N)} = e^{j2\pi f_0 n} \text{ or } e^{j2\pi f_0 N} = 1 \Rightarrow f_0 N = K \text{ (an integer)}$$

which proves the result.

2. $x_1 = \exp(0.1\pi n)$, $-100 \leq n \leq 100$.

```
% P0205b: x1(n) = e^{0.1*j*pi*n} -100 <=n <=100
clc; close all;
n1 = [-100:100]; x1 = exp(0.1*j*pi*n1);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0205b');
subplot(2,1,1); Hs1 = stem(n1,real(x1),'filled'); set(Hs1,'markersize',2);
axis([min(n1)-5,max(n1)+5,min(real(x1))-1,max(real(x1))+1]);
xlabel('n','FontSize',LFS); ylabel('Real(x_1(n))','FontSize',LFS);
title(['Real part of sequence x_1(n) = ' ...
      'exp(0.1 \times j \times pi \times n) ' char(10) ...
      ' Period = 20, K = 1, N = 20'],'FontSize',TFS);
ntick = [n1(1):20:n1(end)]; set(gca,'XTickMode','manual','XTick',ntick);
subplot(2,1,2); Hs2 = stem(n1,imag(x1),'filled'); set(Hs2,'markersize',2);
axis([min(n1)-5,max(n1)+5,min(real(x1))-1,max(real(x1))+1]);
xlabel('n','FontSize',LFS); ylabel('Imag(x_1(n))','FontSize',LFS);
title(['Imaginary part of sequence x_1(n) = ' ...
      'exp(0.1 \times j \times pi \times n) ' char(10) ...
      ' Period = 20, K = 1, N = 20'],'FontSize',TFS);
ntick = [n1(1):20:n1(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0205b; print -deps2 ../Latex/P0205b;
```

The plots of $x_1(n)$ is shown in Figure 2.21. Since $f_0 = 0.1/2 = 1/20$ the sequence is periodic. From the plot in Figure 2.21 we see that in one period of 20 samples $x_1(n)$ exhibits cycle. This is true whenever K and N are relatively prime.

3. $x_2 = \cos(0.1n)$, $-20 \leq n \leq 20$.

```
% P0205c: x2(n) = cos(0.1n), -20 <= n <= 20
clc; close all;
n2 = [-20:20]; x2 = cos(0.1*n2);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0205c');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title(['Sequence x_2(n) = cos(0.1 \times n) ' char(10) ...
      'Not periodic since f_0 = 0.1 / (2 \times pi) ' ...
      ' is not a rational number'],'FontSize',TFS);
ntick = [n2(1):4:n2(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0205c;
```

The plots of $x_1(n)$ is shown in Figure 2.22. In this case f_0 is not a rational number and hence the sequence $x_2(n)$ is not periodic. This can be clearly seen from the plot of $x_2(n)$ in Figure 2.22.

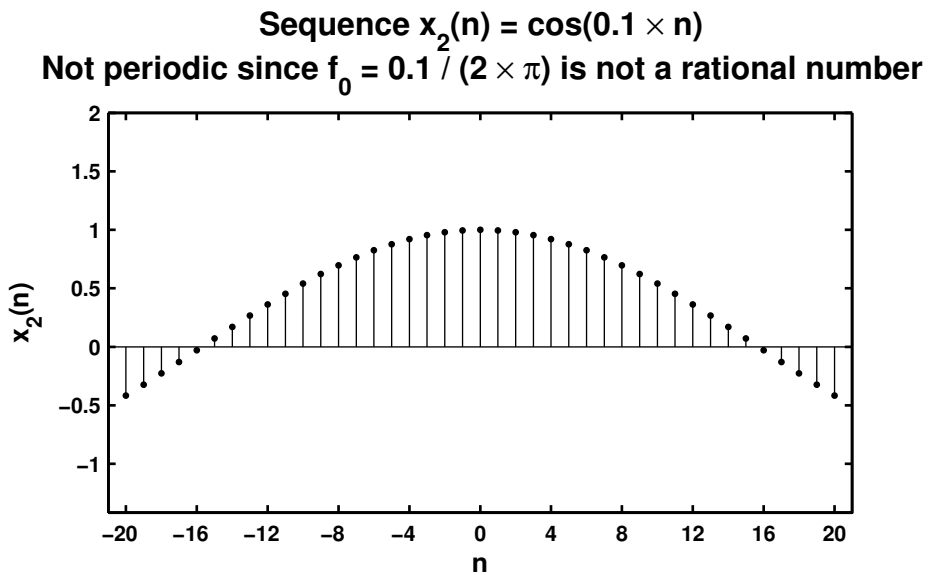


Figure 2.21: Problem P2.5.2 sequence plots

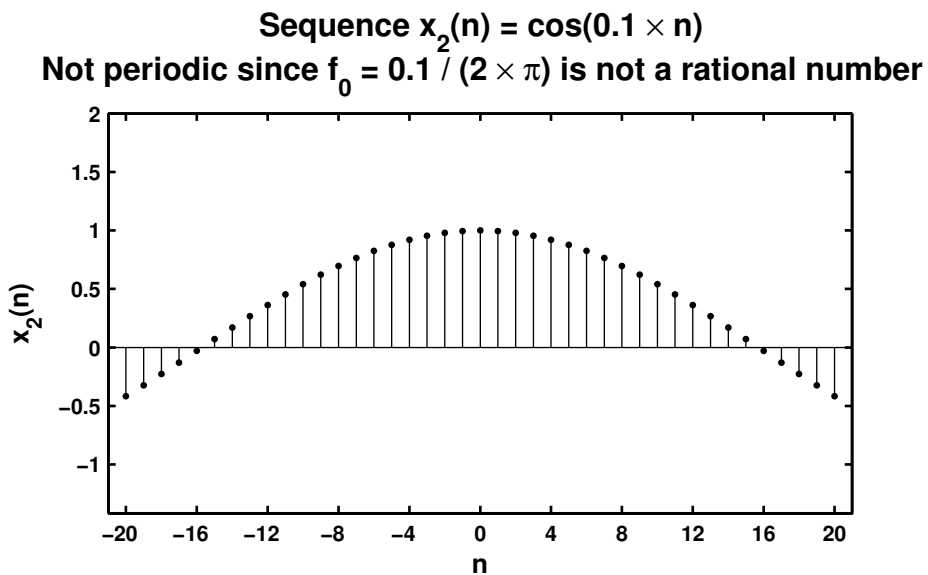


Figure 2.22: Problem P2.5.3 sequence plots

P2.6 Using the `evenodd` function decompose the following sequences into their even and odd components. Plot these components using the `stem` function.

1. $x_1(n) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

```
% P0206a: % Even odd decomposition of x1(n) = [0 1 2 3 4 5 6 7 8 9];
%
%                                     n = 0:9;
clc; close all;

x1 = [0 1 2 3 4 5 6 7 8 9]; n1 = [0:9]; [xe1,xo1,m1] = evenodd(x1,n1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206a');
subplot(2,1,1); Hs = stem(m1,xe1,'filled'); set(Hs,'markersize',2);
axis([min(m1)-1,max(m1)+1,min(xe1)-1,max(xe1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of x_1(n)','FontSize',TFS);
ntick = [m1(1):m1(end)]; ytick = [-1:5];
set(gca,'XTick',ntick);set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m1,xo1,'filled'); set(Hs,'markersize',2);
axis([min(m1)-1,max(m1)+1,min(xo1)-2,max(xo1)+2]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of x_1(n)','FontSize',TFS);
ntick = [m1(1):m1(end)]; ytick = [-6:2:6];
set(gca,'XTick',ntick);set(gca,'YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0206a; print -deps2 ../Latex/P0206a;
```

The plots of $x_1(n)$ is shown in Figure 2.23.

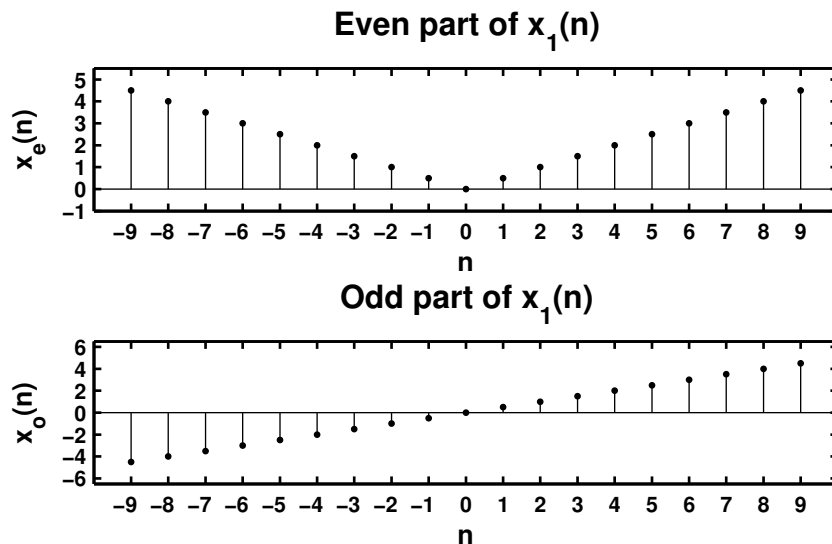


Figure 2.23: Problem P2.6.1 sequence plot

2. $x_2(n) = e^{0.1n}[u(n+5) - u(n-10)]$.

```
% P0206b: Even odd decomposition of  $x_2(n) = e^{0.1n} [u(n+5) - u(n-10)]$ ;
clc; close all;

n2 = [-8:12]; x2 = exp(0.1*n2).*(stepseq(-5,-8,12) - stepseq(10,-8,12));
[xe2,xo2,m2] = evenodd(x2,n2);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206b');
subplot(2,1,1); Hs = stem(m2,xo2,'filled'); set(Hs,'markersize',2);
axis([min(m2)-1,max(m2)+1,min(xo2)-1,max(xo2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Even part of  $x_2(n) = \exp(0.1n) [u(n+5) - u(n-10)]$ ',...
      'FontSize',TFS);
ntick = [m2(1):2:m2(end)]; set(gca,'XTick',ntick);
subplot(2,1,2); Hs = stem(m2,xo2,'filled'); set(Hs,'markersize',2);
axis([min(m2)-1,max(m2)+1,min(xo2)-1,max(xo2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of  $x_2(n) = \exp(0.1n) [u(n+5) - u(n-10)]$ ',...
      'FontSize',TFS);
ntick = [m2(1):2:m2(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206b; print -deps2 ../Latex/P0206b;
```

The plots of $x_2(n)$ is shown in Figure 2.24.

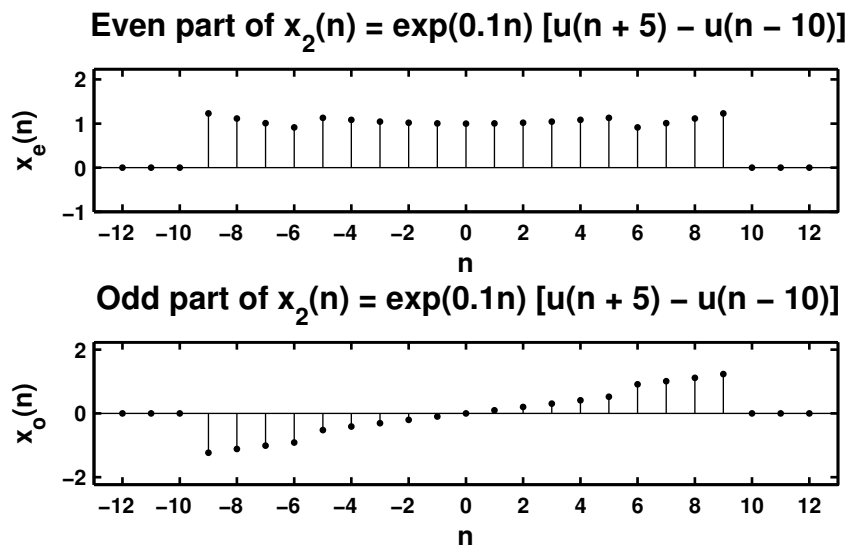


Figure 2.24: Problem P2.6.2 sequence plot

3. $x_3(n) = \cos(0.2\pi n + \pi/4)$, $-20 \leq n \leq 20$.

```
% P0206c: Even odd decomposition of  $x_2(n) = \cos(0.2\pi n + \pi/4)$ ;
%
%                                      $-20 \leq n \leq 20$ ;
clc; close all;

n3 = [-20:20]; x3 = cos(0.2*pi*n3 + pi/4);
[xe3,xo3,m3] = evenodd(x3,n3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206c');
subplot(2,1,1); Hs = stem(m3,x3,'filled'); set(Hs,'markersize',2);
axis([min(m3)-2,max(m3)+2,min(xe3)-1,max(xe3)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of  $x_3(n) = \cos(0.2 \times \pi \times n + \pi/4)$ ',...
      'FontSize',TFS);
ntick = [m3(1):4:m3(end)]; set(gca,'XTick',ntick);
subplot(2,1,2); Hs = stem(m3,xo3,'filled'); set(Hs,'markersize',2);
axis([min(m3)-2,max(m3)+2,min(xo3)-1,max(xo3)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of  $x_3(n) = \cos(0.2 \times \pi \times n + \pi/4)$ ',...
      'FontSize',TFS);
ntick = [m3(1):4 :m3(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206c; print -deps2 ../Latex/P0206c;
```

The plots of $x_3(n)$ is shown in Figure 2.25.

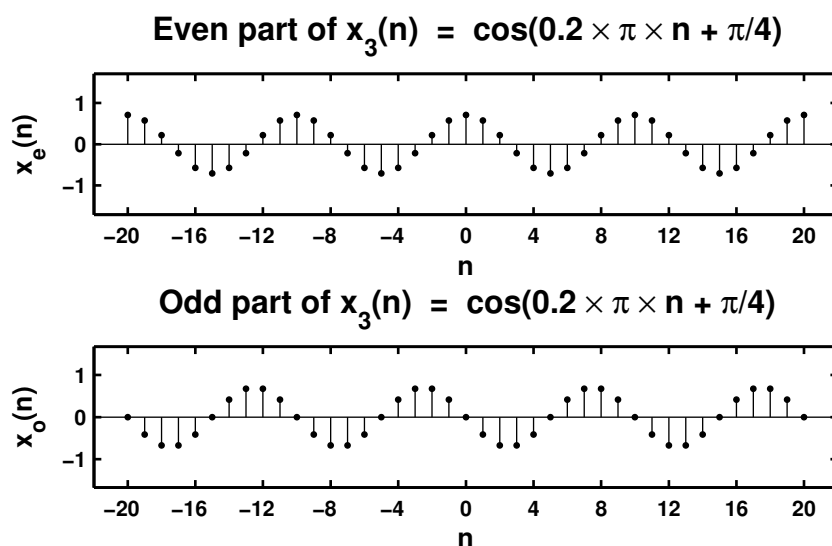


Figure 2.25: Problem P2.6.3 sequence plot

4. $x_4(n) = e^{-0.05n} \sin(0.1\pi n + \pi/3)$, $0 \leq n \leq 100$.

```
% P0206d: x4(n) = e ^ {-0.05*n}*sin(0.1*pi*n + pi/3), 0 <= n <= 100
clc; close all;

n4 = [0:100]; x4 = exp(-0.05*n4).*sin(0.1*pi*n4 + pi/3);
[xe4,xo4,m4] = evenodd(x4,n4);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206d');
subplot(2,1,1); Hs = stem(m4,x4,'filled'); set(Hs,'markersize',2);
axis([min(m4)-10,max(m4)+10,min(xe4)-1,max(xe4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title(['Even part of x_4(n) = ' ...
      'exp(-0.05 \times n) \times sin(0.1 \times \pi \times n + ' ...
      '\pi/3)'], 'FontSize',TFS);
ntick = [m4(1):20:m4(end)]; set(gca,'XTick',ntick);

subplot(2,1,2); Hs = stem(m4,xo4,'filled'); set(Hs,'markersize',2);
axis([min(m4)-10,max(m4)+10,min(xo4)-1,max(xo4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title(['Odd part of x_4(n) = ' ...
      'exp(-0.05 \times n) \times sin(0.1 \times \pi \times n + ' ...
      '\pi/3)'], 'FontSize',TFS);
ntick = [m4(1):20 :m4(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206d; print -deps2 ../Latex/P0206d;
```

The plots of $x_1(n)$ are shown in Figure 2.26.

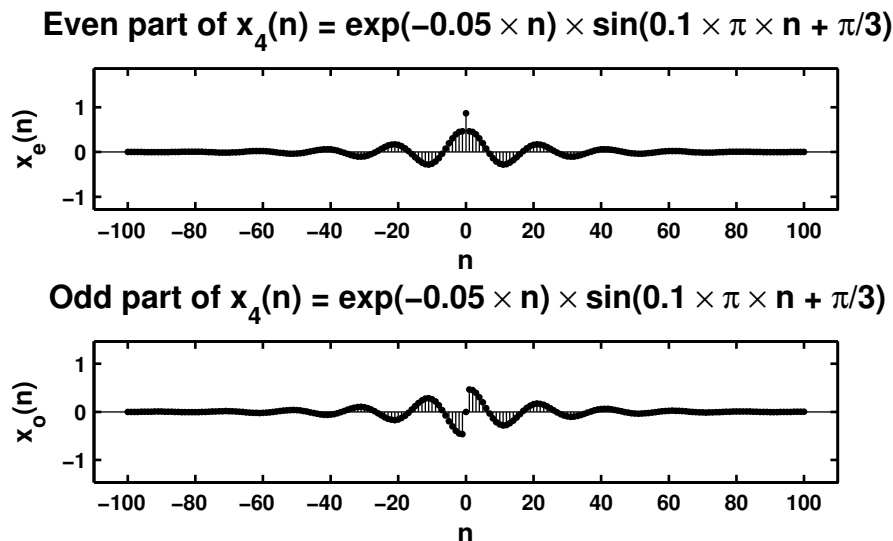


Figure 2.26: Problem P2.6.1 sequence plot

P2.7 A complex-valued sequence $x_e(n)$ is called *conjugate-symmetric* if $x_e(n) = x_e^*(-n)$ and a complex-valued sequence $x_o(n)$ is called *conjugate-antisymmetric* if $x_o(n) = -x_o^*(-n)$. Then any arbitrary complex-valued sequence $x(n)$ can be decomposed into $x(n) = x_e(n) + x_o(n)$ where $x_e(n)$ and $x_o(n)$ are given by

$$x_e(n) = \frac{1}{2} [x(n) + x^*(-n)] \quad \text{and} \quad x_o(n) = \frac{1}{2} [x(n) - x^*(-n)] \quad (2.1)$$

respectively.

1. Modify the `evenodd` function discussed in the text so that it accepts an arbitrary sequence and decomposes it into its conjugate-symmetric and conjugate-antisymmetric components by implementing (2.1).

```
function [xe , xo , m] = evenodd_c(x , n)
% Complex-valued signal decomposition into even and odd parts (version-2)
% -----
%[xe , xo , m] = evenodd_c(x , n);
%
%[xc , nc] = sigfold(conj(x) , n);
%[xe , m] = sigadd(0.5 * x , n , 0.5 * xc , nc);
%[xo , m] = sigadd(0.5 * x , n , -0.5 * xc , nc);

2. x(n) = 10exp([-0.1 + j0.2π]n), 0 ≤ n ≤ 10

% P0207b: Decomposition of x(n) = 10*e ^ {(-0.1 + j*0.2*pi)*n},
%                                     0 <= n <= 10
% into its conjugate symmetric and conjugate antisymmetric parts.
clc; close all;

n = [0:10]; x = 10*exp((-0.1+j*0.2*pi)*n); [xe,xo,neo] = evenodd(x,n);
Re_xe = real(xe); Im_xe = imag(xe); Re_xo = real(xo); Im_xo = imag(xo);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0207b');
subplot(2,2,1); Hs = stem(neo,Re_xe); set(Hs,'markersize',2);
ylabel('Re[x_e(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,12]);
ytick = [-5:5:15]; set(gca,'YTick',ytick);
title(['Real part of' char(10) 'even sequence x_e(n)'],'FontSize',TFS);

subplot(2,2,3); Hs = stem(neo,Im_xe); set(Hs,'markersize',2);
ylabel('Im[x_e(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,5]);
ytick = [-5:1:5]; set(gca,'YTick',ytick);
title(['Imaginary part of' char(10) 'even sequence x_e(n)'],'FontSize',TFS);

subplot(2,2,2); Hs = stem(neo,Re_xo); set(Hs,'markersize',2);
ylabel('Re[x_o(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,+5]);
ytick = [-5:1:5]; set(gca,'YTick',ytick);
title(['Real part of' char(10) 'odd sequence x_o(n)'],'FontSize',TFS);

subplot(2,2,4); Hs = stem(neo,Im_xo); set(Hs,'markersize',2);
ylabel('Im[x_o(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-5,5]);
ytick = [-5:1:5]; set(gca,'YTick',ytick);
title(['Imaginary part of' char(10) 'odd sequence x_o(n)'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0207b;%print -deps2 ../Latex/P0207b;
```

The plots of $x(n)$ are shown in Figure 2.27.

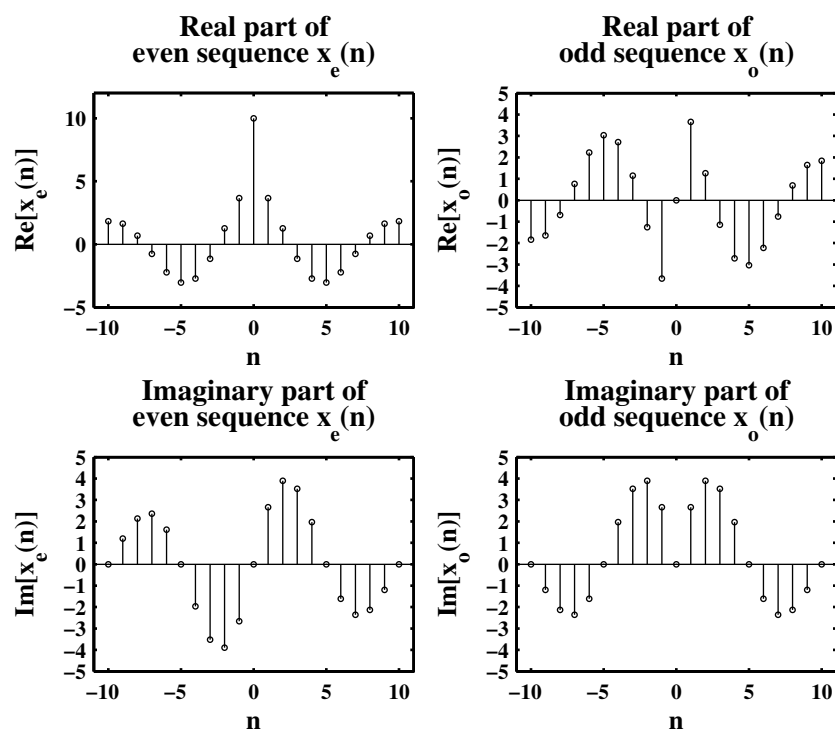


Figure 2.27: Problem P2.7.2 sequence plot

P2.8 The operation of *signal dilation* (or *decimation* or *down-sampling*) is defined by $y(n) = x(nM)$ in which the sequence $x(n)$ is down-sampled by an integer factor M .

1. MATLAB function:

```
function [y,m] = dnsample(x,n,M)
% [y,m] = dnsample(x,n,M)
% Downsample sequence x(n) by a factor M to obtain y(m)
mb = ceil(n(1)/M)*M; me = floor(n(end)/M)*M;
nb = find(n==mb); ne = find(n==me);
y = x(nb:M:ne); m = fix((mb:M:me)/M);
```

2. $x_1(n) = \sin(0.125\pi n)$, $-50 \leq n \leq 50$. Decimation by a factor of 4.

```
% P0208b: x1(n) = sin(0.125*pi*n), -50 <= n <= 50
%           Decimate x(n) by a factor of 4 to obtain y(n)
clc; close all;
n1 = [-50:50]; x1 = sin(0.125*pi*n1); [y1,m1] = dnsample(x1,n1,4);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0208b');
subplot(2,1,1); Hs = stem(n1,x1); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title('Original sequence x_1(n)','FontSize',TFS);
axis([min(n1)-5,max(n1)+5,min(x1)-0.5,max(x1)+0.5]);
ytick = [-1.5:0.5:1.5]; ntick = [n1(1):10:n1(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m1,y1); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n) = x(4n)','FontSize',LFS);
title('y_1(n) = Original sequence x_1(n) decimated by a factor of 4',...
      'FontSize',TFS);
axis([min(m1)-2,max(m1)+2,min(y1)-0.5,max(y1)+0.5]);
ytick = [-1.5:0.5:1.5]; ntick = [m1(1):2:m1(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0208b;
```

The plots of $x_1(n)$ and $y_1(n)$ are shown in Figure 2.28. Observe that the original signal $x_1(n)$ can be recovered.

3. $x(n) = \sin(0.5\pi n)$, $-50 \leq n \leq 50$. Decimation by a factor of 4.

```
% P0208c: x2(n) = sin(0.5*pi*n), -50 <= n <= 50
%           Decimate x2(n) by a factor of 4 to obtain y2(n)
clc; close all;
n2 = [-50:50]; x2 = sin(0.5*pi*n2); [y2,m2] = dnsample(x2,n2,4);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0208c');
subplot(2,1,1); Hs = stem(n2,x2); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
axis([min(n2)-5,max(n2)+5,min(x2)-0.5,max(x2)+0.5]);
title('Original sequence x_2(n)','FontSize',TFS);
ytick = [-1.5:0.5:1.5]; ntick = [n2(1):10:n2(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m2,y2); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n) = x(4n)','FontSize',LFS);
axis([min(m2)-1,max(m2)+1,min(y2)-1,max(y2)+1]);
title('y_2(n) = Original sequence x_2(n) decimated by a factor of 4',...
      'FontSize',TFS);
ntick = [m2(1):2:m2(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0208c; print -deps2 ../Latex/P0208c;
```

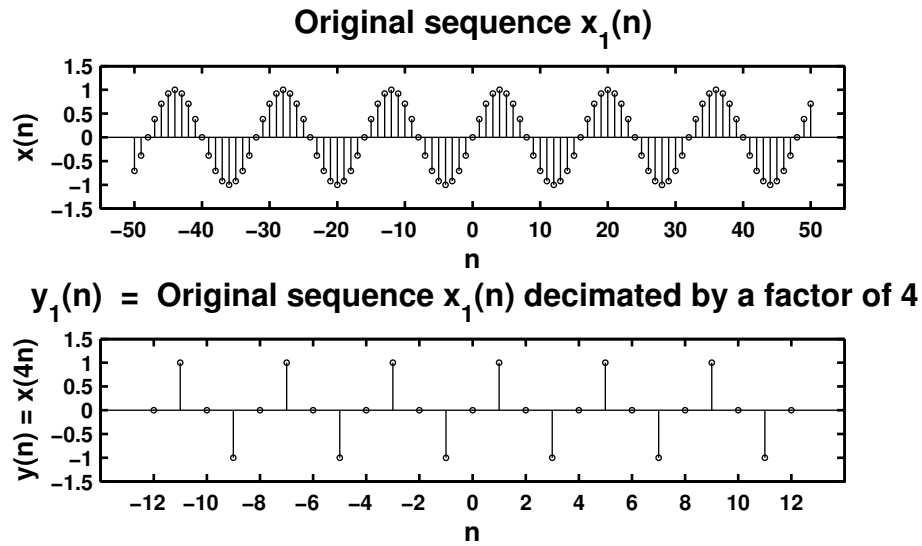


Figure 2.28: Problem P2.8.2 sequence plot

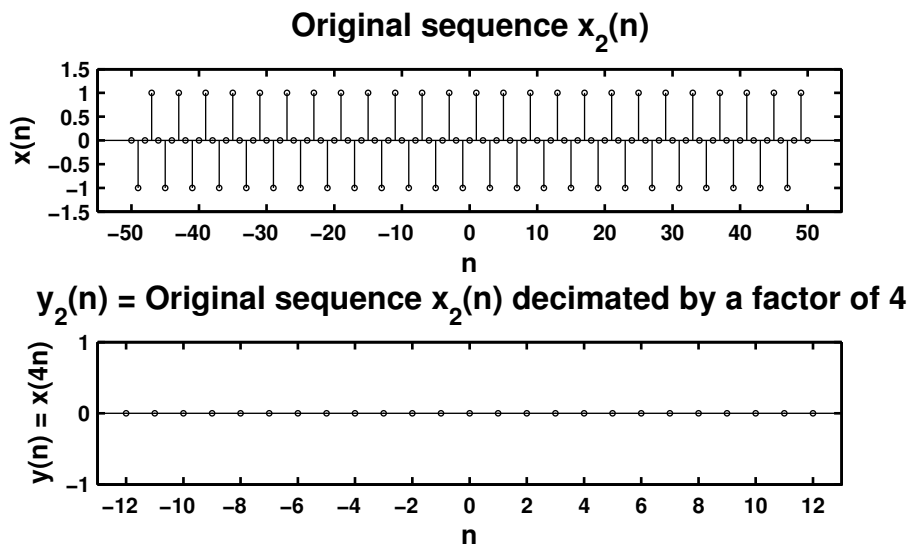


Figure 2.29: Problem P2.8.3 sequence plot

The plots of $x_2(n)$ and $y_2(n)$ are shown in Figure 2.29. Observe that the downsampled signal is a signal with zero frequency. Thus the original signal $x_2(n)$ is lost.

P2.9 The autocorrelation sequence $r_{xx}(\ell)$ and the crosscorrelation sequence $r_{xy}(\ell)$ for the sequences:

$$x(n) = (0.9)^n, \quad 0 \leq n \leq 20; \quad y(n) = (0.8)^{-n}, \quad -20 \leq n \leq 0$$

```
% P0209a: autocorrelation of sequence x(n) = 0.9 ^ n, 0 <= n <= 20
% using the conv_m function
clc; close all;

nx = [0:20]; x = 0.9 .^ nx; [xf,nxf] = sigfold(x,nx);
[rxx,nrxx] = conv_m(x,nx,xf,nxf);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0209a');
Hs = stem(nrxx,rxx); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('r_x_x(n)','FontSize',LFS);
title('Autocorrelation of x(n)','FontSize',TFS);
axis([min(nrxx)-1,max(nrxx)+1,min(rxx),max(rxx)+1]);
ntick = [nrxx(1):4:nrxx(end)]; set(gca,'XTick',ntick);
%print -deps2 ../CHAP2_EPSFILES/P0209a;
```

The plot of the autocorrelation is shown in Figure 2.30.

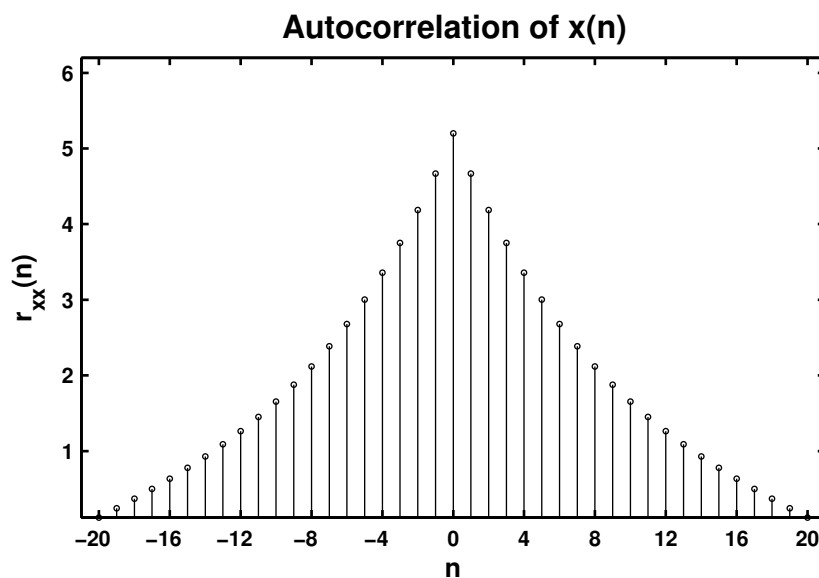


Figure 2.30: Problem P2.9 autocorrelation plot

```
% P0209b: crosscorrelation of sequence x(n) = 0.9 ^ n, 0 <= n <= 20
% with sequence y = 0.8.^n, -20 <= n <= 0 using the conv_m function
clc; close all;

nx = [0:20]; x = 0.9 .^ nx; ny = [-20:0]; y = 0.8 .^ (-ny);
[yf,nyf] = sigfold(y,ny); [rxy,nrxy] = conv_m(x,nx,yf,nyf);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0209b');
Hs = stem(nrxy,rxy); set(Hs,'markersize',3);
xlabel('n','FontSize',LFS); ylabel('r_x_y(n)','FontSize',LFS);
title('Crosscorrelation of x(n) and y(n)','FontSize',TFS);
```

```
axis([min(nrxy)-1,max(nrxy)+1,floor(min(rxy)),ceil(max(rxy))]);
ytick = [0:1:4]; ntick = [nrxy(1):2:nrxy(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
print -deps2 ../epsfiles/P0209b;
```

The plot of the crosscorrelation is shown in Figure 2.31.

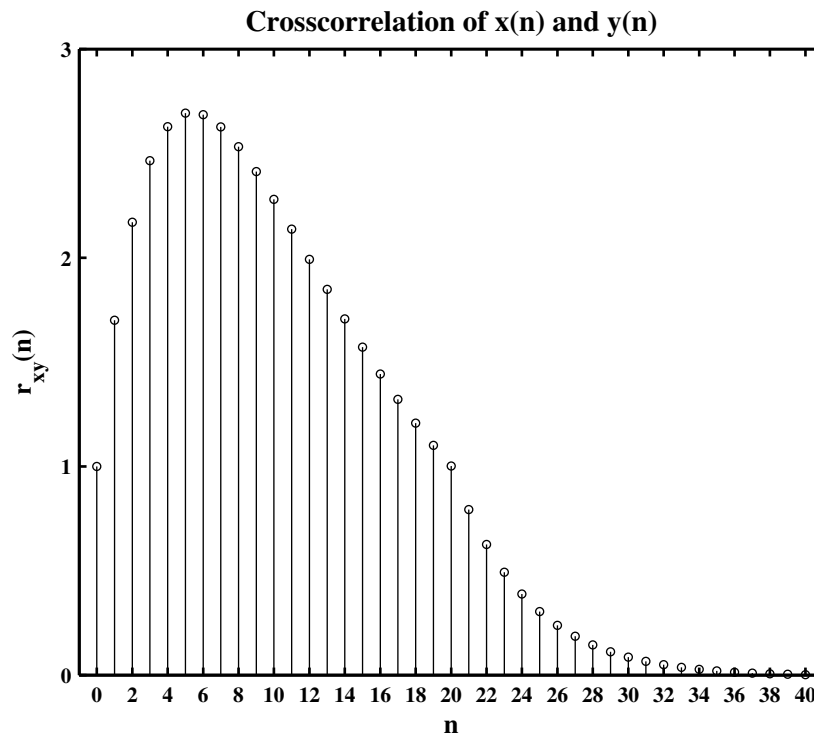


Figure 2.31: Problem P2.9 crosscorrelation plot

P2.10 In a certain concert hall, echoes of the original audio signal $x(n)$ are generated due to the reflections at the walls and ceiling. The audio signal experienced by the listener $y(n)$ is a combination of $x(n)$ and its echoes. Let $y(n) = x(n) + \alpha x(n - k)$ where k is the amount of delay in samples and α is its relative strength. We want to estimate the delay using the correlation analysis.

1. Determine analytically the autocorrelation $r_{yy}(\ell)$ in terms of the autocorrelation $r_{xx}(\ell)$.

Consider the autocorrelation $r_{yy}(\ell)$ of $y(n)$:

$$\begin{aligned}
 r_{yy}(\ell) &= \sum_{\ell} y(n)y(n-\ell) = \sum_{\ell} \{x(n) + \alpha x(n-k)\} \{x(n-\ell) + \alpha x(n-k-\ell)\} \\
 &= \sum_{\ell} x(n)x(n-\ell) + \alpha \sum_{\ell} x(n)x(n-k-\ell) + \alpha \sum_{\ell} x(n-k)x(n-\ell) \\
 &\quad + \alpha^2 \sum_{\ell} x(n-k)x(n-k-\ell) \\
 &= r_{xx}(\ell) + \alpha r_{xx}(\ell+k) + \alpha r_{xx}(\ell-k) + \alpha^2 r_{xx}(\ell) \\
 &= (1 + \alpha^2)r_{xx}(\ell) + \alpha[r_{xx}(\ell+k) + r_{xx}(\ell-k)]
 \end{aligned}$$

2. Let $x(n) = \cos(0.2\pi n) + 0.5 \cos(0.6\pi n)$, $\alpha = 0.1$, and $k = 50$. Generate 200 samples of $y(n)$ and determine its autocorrelation. Can you obtain α and k by observing $r_{yy}(\ell)$?

MATLAB script:

```

% P0210c: autocorrelation of sequence y(n) = x(n) + alpha*x(n - k)
%                                     alpha = 0.1, k = 50
% x(n) = cos(0.2*pi*n) + 0.5*cos(0.6*pi*n)
clc; close all;
Hf_1 = figure;
set(Hf_1, 'NumberTitle', 'off', 'Name', 'P0210c');
alpha = .1;
n = [-100:100];
%x = 3*rand(1,201)-1.5;
x = cos(0.2*pi*n) + 0.5*cos(0.6*pi*n);
[xf,nxf] = sigfold(x,n);
[rxx,nrxx] = conv_m(x,n,xf,nxf);

y = filter([1 zeros(1,49) alpha], 1, x);
[yf,nyf] = sigfold(y,n);
[ryx,nryx] = conv_m(y,n,xf,nxf);
[ryy,nryy] = conv_m(y,n,yf,nyf);

subplot(2,1,1);
Hs = stem(nrxx,rxx,'filled');
set(Hs,'markersize',2);
xlabel('n','FontSize',LFS);
ylabel('r_y_y(n)','FontSize',LFS);
title('autocorrelation of sequence x(n)','FontSize',TFS);
axis([-210, 210,-200,200]);
ntick = [-200:50:200];
ytick = [-200:50:200];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);

subplot(2,1,2);
Hs = stem(nryx,ryx,'filled');
```

```

set(Hs,'markersize',2);
xlabel('n','FontSize',LFS);
ylabel('r_y_y(n)','FontSize',LFS);
title('autocorrelation of sequence y(n) = x(n) + 0.1 \times x(n - 50)',...
      'FontSize',TFS);
axis([-210,210,-200,200]);
ntick = [-200:50:200];
ytick = [-200:50:200];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);

print -deps2 ../CHAP2_EPSFILES/P0210b;

```

Stem plots of the autocorrelations are shown in Figure 2.32. We observe that it is not possible to visually separate components of $r_{xx}(\ell)$ in $r_{yy}(\ell)$ and hence it is not possible to estimate α or k .

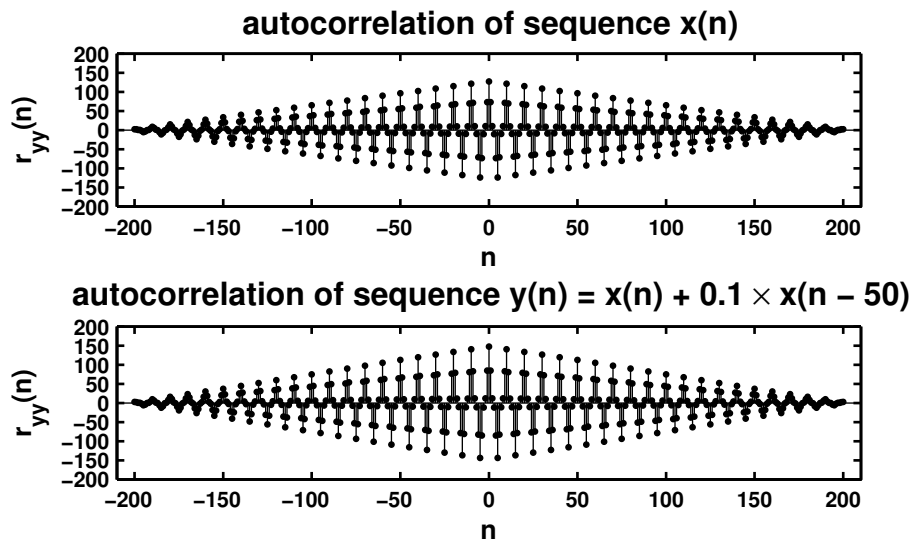


Figure 2.32: Problem P2.10 autocorrelation plot

P2.11 Linearity of discrete-time systems.**System-1:** $T_1[x(n)] = x(n)u(n)$

1. Analytic determination of linearity:

$$\begin{aligned} T_1[a_1x_1(n) + a_2x_2(n)] &= \{a_1x_1(n) + a_2x_2(n)\}u(n) = a_1x_1(n)u(n) + a_2x_2(n)u(n) \\ &= a_1T_1[x_1(n)] + a_2T_1[x_2(n)] \end{aligned}$$

Hence the system $T_1[x(n)]$ is **linear**.

2. MATLAB script:

```
% P0211a: To prove that the system T1[x(n)] = x(n)u(n) is linear
clear; clc; close all;
```

```
n = 0:100; x1 = rand(1,length(n));
x2 = sqrt(10)*randn(1,length(n)); u = stepseq(0,0,100);
y1 = x1.*u; y2 = x2.*u; y = (x1 + x2).*u;
diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-1 is Linear *** ');
else
    disp(' *** System-1 is NonLinear *** ');
end
MATLAB verification:
>> *** System-1 is Linear ***
```

System-2: $T_2[x(n)] = x(n) + nx(n+1)$

1. Analytic determination of linearity:

$$\begin{aligned} T_2[a_1x_1(n) + a_2x_2(n)] &= \{a_1x_1(n) + a_2x_2(n)\} + n\{a_1x_1(n+1) + a_2x_2(n+1)\} \\ &= a_1\{x_1(n) + nx_1(n+1)\} + a_2\{x_2(n) + nx_2(n+1)\} \\ &= a_1T_2[x_1(n)] + a_2T_2[x_2(n)] \end{aligned}$$

Hence the system is $T_2[x(n)]$ **linear**.

2. MATLAB script:

```
% P0211b: To prove that the system T2[x(n)] = x(n) + n*x(n+1) is linear
clear; clc; close all;
```

```
n = 0:100; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
z = n; [x11,nx11] = sigshift(x1,n,-1);
[x111,nx111] = sigmult(z,n,x11,nx11); [y1,ny1] = sigadd(x1,n,x111,nx111);

[x21,nx21] = sigshift(x2,n,-1); [x211,nx211] = sigmult(z,n,x21,nx21);
[y2,ny2] = sigadd(x2,n,x211,nx211);
xs = x1 + x2; [xs1,nxs1] = sigshift(xs,n,-1);
[xs11,nxs11] = sigmult(z,n,xs1,nxs1); [y,ny] = sigadd(xs,n,xs11,nxs11);
diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-2 is Linear *** ');
else
    disp(' *** System-2 is NonLinear *** ');
end
MATLAB verification:
>> *** System-2 is Linear ***
```

System-3: $T_3[x(n)] = x(n) + \frac{1}{2}x(n-2) - \frac{1}{3}x(n-3)x(2n)$

1. Analytic determination of linearity:

$$\begin{aligned} T_3[a_1x_1(n) + a_2x_2(n)] &= a_1x_1(n) + a_2x_2(n) + \frac{1}{2}\{a_1x_1(n-2) + a_2x_2(n-2)\} \\ &\quad - \frac{1}{3}\{a_1x_1(n-3) + a_2x_2(n-3)\}\{a_1x_1(2n) + a_2x_2(2n)\} \\ &= a_1\{x_1(n) + \frac{1}{2}x_1(n-2) - \frac{1}{3}a_1x_1(n-3)x_1(2n)\} \\ &\quad + a_2\{x_2(n) + \frac{1}{2}x_2(n-2) - \frac{1}{3}a_2x_2(n-3)x_2(2n)\} \\ &\quad + \frac{1}{3}\{a_1x_1(n-3)a_2x_2(2n) + a_2x_2(n-3)a_1x_1(2n)\} \end{aligned}$$

which clearly is not equal to $a_1T_3[x_1(n)] + a_2T_3[x_2(n)]$. The product term in the input-output equation makes the system $T_3[x(n)]$ **nonlinear**.

2. MATLAB script:

```
% P0211c: To prove that the system T3[x(n)] = x(n) + 1/2*x(n - 2)
%          - 1/3*x(n - 3)*x(2n)
%          is linear
clear; clc; close all;

n = [0:100]; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
[x11,nx11] = sigshift(x1,n,2); x11 = 1/2*x11; [x12,nx12] = sigshift(x1,n,3);
x12 = -1/3*x12; [x13,nx13] = dnsample(x1,n,2);
[x14,nx14] = sigmult(x12,nx12,x13,nx13);
[x15,nx15] = sigadd(x11,nx11,x14,nx14);
[y1,ny1] = sigadd(x1,n,x15,nx15); [x21,nx21] = sigshift(x2,n,2);
x21 = 1/2*x21; [x22,nx22] = sigshift(x2,n,3);
x22 = -1/3*x22; [x23,nx23] = dnsample(x2,n,2);
[x24,nx24] = sigmult(x22,nx22,x23,nx23);
[x25,nx25] = sigadd(x21,nx21,x24,nx24); [y2,ny2] = sigadd(x2,n,x25,nx25);
xs = x1 + x2; [xs1,nxs1] = sigshift(xs,n,2);
xs1 = 1/2*xs1; [xs2,nxs2] = sigshift(xs,n,3); xs2 = -1/3*xs2;
[xs3,nxs3] = dnsample(xs,n,2); [xs4,nxs4] = sigmult(xs2,nxs2,xs3,nxs3);
[xs5,nxs5] = sigadd(xs1,nxs1,xs4,nxs4);
[y,ny] = sigadd(xs,n,xs5,nxs5); diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-3 is Linear *** ');
else
    disp(' *** System-3 is NonLinear *** ');
end
MATLAB verification:
>> *** System-3 is NonLinear ***
```

System-4: $T_4[x(n)] = \sum_{k=-\infty}^{n+5} 2x(k)$

1. Analytic determination of linearity:

$$\begin{aligned} T_4[a_1x_1(n) + a_2x_2(n)] &= \sum_{k=-\infty}^{n+5} 2\{a_1x_1(k) + a_2x_2(k)\} = a_1 \sum_{k=-\infty}^{n+5} 2x_1(k) + a_2 \sum_{k=-\infty}^{n+5} 2x_2(k) \\ &= a_1T_4[x_1(n)] + a_2T_4[x_2(n)] \end{aligned}$$

Hence the system $T_4[x(n)]$ is **linear**.

2. MATLAB script:

```
% P0211d: To prove that the system T4[x(n)] = sum_{k=-infinity}^{n+5} 2*x(k)
%          is linear
clear; clc; close all;
```

```

n = [0:100]; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
y1 = cumsum(x1); ny1 = n - 5; y2 = cumsum(x2); ny2 = n - 5; xs = x1 + x2;
y = cumsum(xs); ny = n - 5; diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-4 is Linear *** ');
else
    disp(' *** System-4 is NonLinear *** ');
end
MATLAB verification:
>> *** System-4 is Linear ***

```

System-5: $T_5[x(n)] = x(2n)$

1. Analytic determination of linearity:

$$T_5[a_1x_1(n) + a_2x_2(n)] = a_1x_1(2n) + a_2x_2(2n) = a_1T_5[x_1(n)] + a_2T_5[x_2(n)]$$

Hence the system $T_5[x(n)]$ is **linear**.

2. MATLAB script:

```

% P0211e: To prove that the system T5[x(n)] = x(2n) is linear
clear; clc; close all;

n = 0:100; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
[y1,ny1] = dnsample(x1,n,2); [y2,ny2] = dnsample(x2,n,2); xs = x1 + x2;
[y,ny] = dnsample(xs,n,2); diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-5 is Linear *** ');
else
    disp(' *** System-5 is NonLinear *** ');
end

```

MATLAB verification:
>> *** System-5 is Linear ***

System-6: $T_6[x(n)] = \text{round}[x(n)]$

1. Analytic determination of linearity:

$$T_6[a_1x_1(n) + a_2x_2(n)] = \text{round}[a_1x_1(n) + a_2x_2(n)] \neq a_1 \text{round}[x_1(n)] + a_2 \text{round}[x_2(n)]$$

Hence the system $T_6[x(n)]$ is **nonlinear**.

2. MATLAB script:

```

% P0211f: To prove that the system T6[x(n)] = round(x(n)) is linear
clear; clc; close all;

n = 0:100; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
y1 = round(x1); y2 = round(x2); xs = x1 + x2;
y = round(xs); diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-6 is Linear *** ');
else
    disp(' *** System-6 is NonLinear *** ');
end
MATLAB verification:
>> *** System-6 is NonLinear ***

```

P2.12 Time-invariance of discrete-time systems.**System-1:** $T_1[x(n)] \triangleq y(n) = x(n)u(n)$

1. Analytic determination of time-invariance:

$$T_1[x(n-k)] = x(n-k)u(n) \neq x(n-k)u(n-k) = y(n-k)$$

Hence the system $T_1[x(n)]$ is **time-varying**.

2. MATLAB script:

```
% P0212a: To determine whether T1[x(n)] = x(n)u(n) is time invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n)); u = stepseq(0,0,100);
y = x.*u; [y1,ny1] = sigshift(y,n,1); [x1,nx1] = sigshift(x,n,1);
[y2,ny2] = sigmult(x1,nx1,u,n); [diff,ndiff] = sigadd(y1,ny1,-y2,ny2);
diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-1 is Time-Invariant *** ');
else
    disp(' *** System-1 is Time-Varying *** ');
end
MATLAB verification:
>> *** System-1 is Time-Varying ***
```

System-2: $T_2[x(n)] \triangleq y(n) = x(n) + nx(n+1)$

1. Analytic determination of time-invariance:

$$T_2[x(n-k)] = x(n-k) + nx(n-k+1) \neq x(n-k) + (n-k)x(n-k+1) = y(n-k)$$

Hence the system is $T_2[x(n)]$ **time-varying**.

2. MATLAB script:

```
% P0212b: To determine whether the system T2[x(n)] = x(n) + n*x(n+1) is
% time-invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n));
z = n; [x1,nx1] = sigshift(x,n,-1);
[x11,nx11] = sigmult(z,n,x1,nx1); [y,ny] = sigadd(x,n,x11,nx11);
[y1,ny1] = sigshift(y,ny,1); [xs,nxs] = sigshift(x,n,1);
[xs1,nxs1] = sigshift(xs,nxs,-1); [xs11,nxs11] = sigmult(z,n,xs1,nxs1);
[y2,ny2] = sigadd(xs,nxs,xs11,nxs11); [diff,ndiff] = sigadd(y1,ny1,-y2,ny2);
diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-2 is Time-Invariant *** ');
else
    disp(' *** System-2 is Time-Varying *** ');
end
MATLAB verification:
>> *** System-1 is Time-Varying ***
```

System-3: $T_3[x(n)] \triangleq y(n) = x(n) + \frac{1}{2}x(n-2) - \frac{1}{3}x(n-3)x(2n)$

1. Analytic determination of time-invariance:

$$\begin{aligned} T_3[x(n-k)] &= x(n-k) + \frac{1}{2}x(n-k-2) - \frac{1}{3}x(n-k-3)x(2n-k) \\ &\neq x(n-k) + \frac{1}{2}x(n-k-2) - \frac{1}{3}x(n-k-3)x(2n-2k) = y(n-k) \end{aligned}$$

Hence the system is $T_3[x(n)]$ **time-varying**.

2. MATLAB script:

```
% P0212c: To find whether the system T3[x(n)] = x(n) + 1/2*x(n - 2)
%               - 1/3*x(n - 3)*x(2n)
%               is time invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n)); [x1,nx1] = sigshift(x,n,2);
x1 = 1/2*x1; [x2,nx2] = sigshift(x,n,3); x2 = -1/3*x2;
[x3,nx3] = dnsample(x,n,2); [x4,nx4] = sigmult(x2,nx2,x3,nx3);
[x5,nx5] = sigadd(x1,nx1,x4,nx4); [y,ny] = sigadd(x,n,x5,nx5);
[y1,ny1] = sigshift(y,ny,1); [xs,nxs] = sigshift(x,n,1);
[xs1,nxs1] = sigshift(xs,nxs,2); xs1 = 1/2*xs1;
[xs2,nxs2] = sigshift(xs,nxs,3); xs2 = -1/3*xs2;
[xs3,nxs3] = dnsample(xs,nxs,2); [xs4,nxs4] = sigmult(xs2,nxs2,xs3,nxs3);
[xs5,nxs5] = sigadd(xs1,nxs1,xs4,nxs4); [y2,ny2] = sigadd(xs,nxs,xs5,nxs5);
[diff,ndiff] = sigadd(y1,ny1,-y2,ny2); diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-3 is Time-Invariant *** ');
else
    disp(' *** System-3 is Time-Varying *** ');
end
MATLAB verification:
>> *** System-3 is Time-Varying ***
```

System-4: $T_4[x(n)] \triangleq y(n) = \sum_{k=-\infty}^{n+5} 2x(k)$

1. Analytic determination of time-invariance:

$$T_4[x(n-\ell)] = \sum_{k=-\infty}^{n+5} 2x(k-\ell) = \sum_{k=-\infty}^{n-\ell+5} 2x(k) = y(n-\ell)$$

Hence the system $T_4[x(n)]$ is **time-invariant**.

2. MATLAB script:

```
% P0212d: To find whether the system T4[x(n)] = sum_{k=-infinity}^{n+5} 2*x(k)
%               is time-invariant
clear; clc; close all;
n = 0:100; x = sqrt(10)*randn(1,length(n)); y = cumsum(x); ny = n - 5;
[y1,ny1] = sigshift(y,ny,-1); [xs,nxs] = sigshift(x,n,-1); y2 = cumsum(xs);
ny2 = nxs - 5; [diff,ndiff] = sigadd(y1,ny1,-y2,ny2); diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-4 is Time-Invariant *** ');
else
    disp(' *** System-4 is Time-Varying *** ');
end
MATLAB verification:
>> *** System-4 is Time-Invariant ***
```

System-5: $T_5[x(n)] \triangleq y(n) = x(2n)$

1. Analytic determination of time-invariance:

$$T_5[x(n-k)] = x(2n-k) \neq x[2(n-k)] = y(n-k)$$

Hence the system $T_5[x(n)]$ is **time-varying**.

2. MATLAB script:

```
% P0212e: To determine whether the system  $T_5[x(n)] = x(2n)$  is time-invariant
clear; clc; close all;
n = 0:100; x = sqrt(10)*randn(1,length(n)); [y,ny] = dnsample(x,n,2);
[y1,ny1] = sigshift(y,ny,1); [xs,nxs] = sigshift(x,n,1);
[y2,ny2] = dnsample(xs,nxs,2); [diff,ndiff] = sigadd(y1,ny1,-y2,ny2);
diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-5 is Time-Invariant *** ');
else
    disp(' *** System-5 is Time-Varying *** ');
end
MATLAB verification:
>> *** System-5 is Time-Varying ***
```

System-6: $T_6[x(n)] \triangleq y(n) = \text{round}[x(n)]$

1. Analytic determination of time-invariance:

$$T_6[x(n-k)] = \text{round}[x(n-k)] = y(n-k)$$

Hence the system $T_6[x(n)]$ is **time-invariant**.

2. MATLAB script:

```
% P0212f: To determine if the system  $T_6[x(n)] = \text{round}(x(n))$  is time-invariant
clear; clc; close all;
n = 0:100; x = sqrt(10)*randn(1,length(n)); y = round(x); ny = n;
[y1,ny1] = sigshift(y,ny,1); [xs,nxs] = sigshift(x,n,1); y2 = round(xs);
ny2 = nx; [diff,ndiff] = sigadd(y1,ny1,-y2,ny2); diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-6 is Time-Invariant *** ');
else
    disp(' *** System-6 is Time-Varying *** ');
end
MATLAB verification:
>> *** System-6 is Time-Invariant ***
```

P2.13 Stability and Causality of Discrete-time Systems

System-1: $T_1[x(n)] \triangleq y(n) = x(n)u(n)$: This system is **stable** since $|y(n)| = |x(n)|$. It is also **causal** since the output depends only on the present value of the input.

System-2: $T_2[x(n)] \triangleq y(n) = x(n) + n x(n + 1)$: This system is **unstable** since

$$|y(n)| \leq |x(n)| + |n| |x(n + 1)| \nearrow \infty \text{ as } n \nearrow \infty \text{ for } |x(n)| < \infty$$

It is also **noncausal** since the output $y(n)$ depends on the future input value $x(n + 1)$ for $n > 0$.

System-3: $T_3[x(n)] \triangleq y(n) = x(n) + \frac{1}{2}x(n - 2) - \frac{1}{3}x(n - 3)x(2n)$: This system is **stable** since

$$|y(n)| \leq |x(n)| + \frac{1}{2}|x(n - 2)| + \frac{1}{3}|x(n - 3)| |x(2n)| < \infty \text{ for } |x(n)| < \infty$$

It is however is **noncausal** since $y(1)$ needs $x(2)$ which is a future input value.

System-4: $T_4[x(n)] \triangleq y(n) = \sum_{k=-\infty}^{n+5} 2x(k)$: This system is **unstable** since

$$|y(n)| \leq 2 \sum_{k=-\infty}^{n+5} |x(k)| \nearrow \infty \text{ as } n \nearrow \infty \text{ for } |x(n)| < \infty$$

It is also **noncausal** since the output $y(n)$ depends on the future input value $x(n + 5)$ for $n > 0$.

System-5: $T_5[x(n)] \triangleq y(n) = x(2n)$: This system is **stable** since $|y(n)| = |x(2n)| < \infty$ for $|x(n)| < \infty$. It is however **noncausal** since $y(1)$ needs $x(2)$ which is a future input value.

System-6: $T_6[x(n)] \triangleq y(n) = \text{round}[x(n)]$: This system is **stable** and **causal**.

P2.14 Properties of linear convolution.

$$\begin{aligned}
x_1(n) * x_2(n) &= x_2(n) * x_1(n) && : \text{Commutation} \\
[x_1(n) * x_2(n)] * x_3(n) &= x_1(n) * [x_2(n) * x_3(n)] && : \text{Association} \\
x_1(n) * [x_2(n) + x_3(n)] &= x_1(n) * x_2(n) + x_1(n) * x_3(n) && : \text{Distribution} \\
x(n) * \delta(n - n_0) &= x(n - n_0) && : \text{Identity}
\end{aligned}$$

1. Commutation:

$$\begin{aligned}
x_1(n) * x_2(n) &= \sum_{k=-\infty}^{\infty} x_1(k) \underbrace{x_2(n-k)}_{=m} = \sum_{m=-\infty}^{\infty} x_1(n-m) x_2(m) \\
&= \sum_{m=-\infty}^{\infty} x_2(m) x_1(n-m) = x_2(n) * x_1(n)
\end{aligned}$$

Association:

$$\begin{aligned}
[x_1(n) * x_2(n)] * x_3(n) &= \left[\sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k) \right] * x_3(n) \\
&= \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1(k) x_2(m-k) x_3(n-m) \\
&= \sum_{k=-\infty}^{\infty} x_1(k) \left[\sum_{m=-\infty}^{\infty} \underbrace{x_2(m-k)}_{=\ell} x_3(n-m) \right] \\
&= \sum_{k=-\infty}^{\infty} x_1(k) \left[\sum_{m=-\infty}^{\infty} x_2(\ell) x_3(n-k-\ell) \right] \\
&= \sum_{k=-\infty}^{\infty} x_1(k) [x_2(n-k) * x_3(n-k)] = x_1(n) * [x_2(n) * x_3(n)]
\end{aligned}$$

Distribution:

$$\begin{aligned}
x_1(n) * [x_2(n) + x_3(n)] &= \sum_{k=-\infty}^{\infty} x_1(k) [x_2(n-k) + x_3(n-k)] \\
&= \sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k) + \sum_{k=-\infty}^{\infty} x_1(k) x_3(n-k) \\
&= x_1(n) * x_2(n) + x_1(n) * x_3(n)
\end{aligned}$$

Identity:

$$x(n) * \delta(n - n_0) = \sum_{k=-\infty}^{\infty} x(k) \delta(n - n_0 - k) = x(n - n_0)$$

since $\delta(n - n_0 - k) = 1$ for $k = n - n_0$ and zero elsewhere.

2. Verification using MATLAB:

Commutation MATLAB script:

```
% P0214a: To prove the Commutation property of convolution
%           i.e. conv(x1(n),x2(n)) = conv(x2(n), x1(n))
clear; clc; close all;

n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4);
n11 = n1; [x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14;
x21 = 0.9 .^ -n2; [x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25);
x24 = x22 - x23; x2 = x21.*x24;
x3 = round((rand(1,21)*2 - 1)*5);

% Commutative property
[y1,ny1] = conv_m(x1,n1,x2,n2); [y2,ny2] = conv_m(x2,n2,x1,n1);
ydiff = max(abs(y1 - y2)), ndiff = max(abs(ny1 - ny2)),
MATLAB verification:
ydiff =
    0
ndiff =
    0
```

Association MATLAB script:

```
% P0214b: To prove the Association property of convolution
%           i.e. conv(conv(x1(n),x2(n)),x3(n)) = conv(x1(n),conv(x2(n),x3(n)))
clear; clc; close all;

n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4); n11 = n1;
[x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14;
x21 = 0.9 .^ -n2; [x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25);
x24 = x22 - x23; x2 = x21.*x24; x3 = round((rand(1,21)*2 - 1)*5);

% Association property
[y1,ny1] = conv_m(x1,n1,x2,n2); [y1,ny1] = conv_m(y1,ny1,x3,n3);
[y2,ny2] = conv_m(x2,n2,x3,n3); [y2,ny2] = conv_m(x1,n1,y2,ny2);
ydiff = max(abs(y1 - y2)), ndiff = max(abs(ny1 - ny2)),
MATLAB verification:
ydiff =
    0
ndiff =
    0
```

Distribution MATLAB script:

```
% P0214c: To prove the Distribution property of convolution
%           i.e. conv(x1(n),(x2(n)+x3(n)))=conv(x1(n),x2(n))+conv(x1(n),x3(n))
clear; clc; close all;

n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4); n11 = n1;
[x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14; x21 = 0.9 .^ -n2;
[x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25); x24 = x22 - x23;
x2 = x21.*x24; x3 = round((rand(1,21)*2 - 1)*5);

% Distributive property
```

```
[y1,ny1] = sigadd(x2,n2,x3,n3); [y1,ny1] = conv_m(x1,n1,y1,ny1);
[y2,ny2] = conv_m(x1,n1,x2,n2); [y3,ny3] = conv_m(x1,n1,x3,n3);
[y4,ny4] = sigadd(y2,ny2,y3,ny3); ydiff = max(abs(y1 - y4)),
ndiff = max(abs(ny1 - ny4)),
```

MATLAB verification:

```
ydiff =
    0
ndiff =
    0
```

Identity MATLAB script:

```
% P0214d: To prove the Identity property of convolution
%           i.e. conv(x(n),delta(n - n0)) = x(n - n0)
clear; clc; close all;

n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4); n11 = n1;
[x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14; x21 = 0.9 .^ -n2;
[x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25); x24 = x22 - x23;
x2 = x21.*x24; x3 = round((rand(1,21)*2 - 1)*5);
```

```
% Identity property
n0 = fix(100*rand(1,1)-0.5); [dl,ndl] = impseq(n0,n0,n0);
[y11,ny11] = conv_m(x1,n1,dl,ndl); [y12,ny12] = sigshift(x1,n1,n0);
y1diff = max(abs(y11 - y12)), ny1diff = max(abs(ny11 - ny12)),
```

```
[y21,ny21] = conv_m(x2,n2,dl,ndl); [y22,ny22] = sigshift(x2,n2,n0);
y2diff = max(abs(y21 - y22)), ny2diff = max(abs(ny21 - ny22)),
```

```
[y31,ny31] = conv_m(x3,n3,dl,ndl); [y32,ny32] = sigshift(x3,n3,n0);
y3diff = max(abs(y31 - y32)), ny3diff = max(abs(ny31 - ny32)),
```

MATLAB verification:

```
ydiff =
    0
ndiff =
    0
```

P2.15 Convolutions using conv_m function.

1. $x(n) = \{2, -4, 5, 3, -1, -2, 6\}$, $h(n) = \{1, -1, 1, -1, 1\}$: MATLAB script:

```

n1 = -3:3; x = [2 -4 5 3 -1 -2 6]; n2 = -1:3; h = [1 -1 1 -1 1];
[y,n] = conv_m(x,n1,h,n2); y, n
y =
     2     -6     11     -8     7     -7     9     -4     7     -8     6
n =
    -4     -3     -2     -1     0     1     2     3     4     5     6

```

2. $x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$: MATLAB script:

```

n1 = -3:3; x = [1 1 0 1 1]; n2 = -3:0; h = [1 -2 -3 4];
[y,n] = conv_m(x,n1,h,n2); y, n,
y =
     1     -1     -5     2     3     -5     1     4
n =
    -6     -5     -4     -3     -2     -1     0     1

```

3. $x(n) = (1/4)^{-n}[u(n+1) - u(n-4)]$, $h(n) = u(n) - u(n-5)$: MATLAB script:

```

n1 = -2:5; [x11,nx11] = stepseq(-1,-2,5); [x12,nx12] = stepseq(4,-2,5);
[x13,n13] = sigadd(x11,nx11,-x12,nx12); x14 = 0.25.^-n1; n14 = n1;
x = x14 .* x13;
n2 = 0:6; [h11,nh11] = stepseq(0,0,6); [h12,nh12] = stepseq(5,0,6); h=h11-h12;
[y,n] = conv_m(x,n1,h,n2); y, n,
y = 0     0.2500     1.2500     5.2500    21.2500    85.2500    85.0000    84.0000
    80.0000    64.0000         0         0         0         0
n = -2     -1     0     1     2     3     4     5     6     7     8     9    10
    11

```

4. $x(n) = n/4[u(n) - u(n-6)]$, $h(n) = 2[u(n+2) - u(n-3)]$: MATLAB script:

```

n1 = 0:7; [x11,nx11] = stepseq(0,0,7); [x12,nx12] = stepseq(6,0,7);
[x13,n13] = sigadd(x11,nx11,-x12,nx12); x14 = n1/4; n14 = n1; x = x14 .* x13;
n2 = -3:4; [h11,nh11] = stepseq(-2,-3,4); [h12,nh12] = stepseq(3,-3,4);
h = 2 * (h11 - h12); [y,n] = conv_m(x,n1,h,n2); y, n,
y = 0         0     0.5000     1.5000     3.0000     5.0000     7.5000     7.0000
    6.0000    4.5000     2.5000         0         0         0         0
n = -3     -2     -1     0     1     2     3     4     5     6     7     8     9
    10    11

```

P2.16 Let $x(n) = (0.8)^n u(n)$, $h(n) = (-0.9)^n u(n)$, and $y(n) = h(n) * x(n)$.

1. Convolution $y(n) = h(n) * x(n)$:

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{k=0}^{\infty} (-0.9)^k (0.8)^{n-k} u(n-k) \\ &= \left[\sum_{k=0}^n (-0.9)^k (0.8)^n (0.8)^{-k} \right] u(n) = (0.8)^n \left[\sum_{k=0}^n \left(-\frac{9}{8} \right)^k \right] u(n) \\ &= \frac{0.8^{n+1} - (-0.9)^{n+1}}{1.7} \end{aligned}$$

MATLAB script:

```
clc; close all; run defaultsettings;
n = [0:50]; x = 0.8.^n; h = (-0.9).^n;
Hf_1 = figure; set(Hf_1, 'NumberTitle', 'off', 'Name', 'P0216');
```

```
% (a) Plot of the analytical convolution
y1 = ((0.8).^(n+1) - (-0.9).^(n+1))/(0.8+0.9);
subplot(1,3,1); Hs1 = stem(n,y1,'filled'); set(Hs1,'markersize',2);
title('Analytical'); xlabel('n'); ylabel('y(n)');
```

2. Computation using convolution of truncated sequences: MATLAB script

```
% (b) Plot using the conv function and truncated sequences
x2 = x(1:26); h2 = h(1:26); y2 = conv(h2,x2);
subplot(1,3,2); Hs2 = stem(n,y2,'filled'); set(Hs2,'markersize',2);
title('Using conv function'); xlabel('n'); %ylabel('y(n)');
```

3. To use the MATLAB's filter function we have to represent the $h(n)$ sequence by coefficients an equivalent difference equation. MATLAB script:

```
% (c) Plot of the convolution using the filter function
y3 = filter([1],[1,0.9],x);
subplot(1,3,3); Hs3 = stem(n,y3,'filled'); set(Hs3,'markersize',2);
title('Using filter function'); xlabel('n'); %ylabel('y(n)');
```

The plots of this solution are shown in Figure 2.33. The analytical solution to the convolution in 1 is the exact answer. In the filter function approach of 2, the infinite-duration sequence $x(n)$ is exactly represented by coefficients of an equivalent filter. Therefore, the filter solution should be exact except that it is evaluated up to the length of the input sequence. The truncated-sequence computation in 3 is correct up to the first 26 samples and then it degrades rapidly.

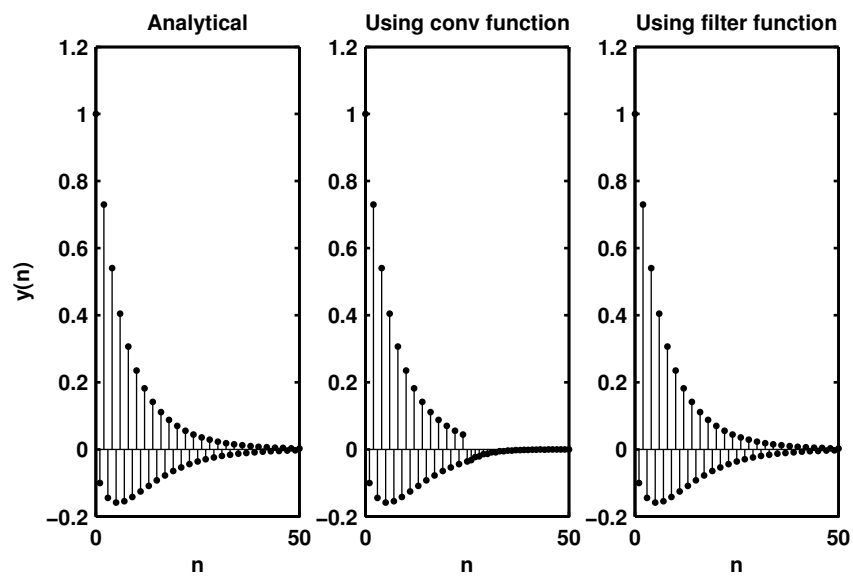


Figure 2.33: Problem P2.16 convolution plots

P2.17 Linear convolution as a matrix-vector multiplication. Consider the sequences

$$x(n) = \{1, 2, 3, 4, 5\} \text{ and } h(n) = \{6, 7, 8, 9\}$$

1. The linear convolution of the above two sequences is

$$y(n) = \{6, 19, 40, 70, 100, 94, 76, 45\}$$

2. The vector representation of the above operation is:

$$\underbrace{\begin{bmatrix} 6 \\ 19 \\ 40 \\ 70 \\ 100 \\ 94 \\ 76 \\ 45 \end{bmatrix}}_y = \underbrace{\begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 7 & 6 & 0 & 0 & 0 \\ 8 & 7 & 6 & 0 & 0 \\ 9 & 8 & 7 & 6 & 0 \\ 0 & 9 & 8 & 7 & 6 \\ 0 & 0 & 9 & 8 & 7 \\ 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}}_x$$

- (a) Note that the matrix $\mathbf{H}$ has an interesting structure. Each diagonal of $\mathbf{H}$ contains the same number. Such a matrix is called a Toeplitz matrix. It is characterized by the following property

$$[\mathbf{H}]_{i,j} = [\mathbf{H}]_{i-j}$$

which is similar to the definition of time-invariance.

- (b) Note carefully that the first column of $\mathbf{H}$ contains the impulse response vector $h(n)$ followed by number of zeros equal to the number of $x(n)$ values minus one. The first row contains the first element of $h(n)$ followed by the same number of zeros as in the first column. Using this information and the above property we can generate the whole Toeplitz matrix.

P2.18 MATLAB function conv_tp:

(a) The MATLAB function conv_tp:

```

function [y,H]=conv_tp(h,x)
% Linear Convolution using Toeplitz Matrix
% -----
% [y,H] = conv_tp(h,x)
% y = output sequence in column vector form
% H = Toeplitz matrix corresponding to sequence h so that y = Hx
% h = Impulse response sequence in column vector form
% x = input sequence in column vector form
%
Nx = length(x); Nh = length(h);
hc = [h; zeros(Nx-1, 1)]; hr = [h(1),zeros(1,Nx-1)];
H = toeplitz(hc,hr); y = H*x;

```

(b) MATLAB verification:

```

x = [1,2,3,4,5]'; h = [6,7,8,9]';
[y,H] = conv_tp(h,x); y = y', H
y =
    6    19    40    70   100    94    76    45
H =
    6     0     0     0     0
    7     6     0     0     0
    8     7     6     0     0
    9     8     7     6     0
    0     9     8     7     6
    0     0     9     8     7
    0     0     0     9     8
    0     0     0     0     9

```

P2.19 A linear and time-invariant system is described by the difference equation

$$y(n) - 0.5y(n-1) + 0.25y(n-2) = x(n) + 2x(n-1) + x(n-3)$$

(a) Impulse response using the Using the filter function.

```
% P0219a: System response using the filter function
clc; close all;

b = [1 2 0 1]; a = [1 -0.5 0.25]; [delta,n] = impseq(0,0,100);
h = filter(b,a,delta);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0219a');
Hs = stem(n,h,'filled'); set(Hs,'markersize',2);
axis([min(n)-5,max(n)+5,min(h)-0.5,max(h)+0.5]);
xlabel('n','FontSize',LFS); ylabel('h(n)','FontSize',LFS);
title('Impulse response','FontSize',TFS);
print -deps2 ../EPSFILES/P0219a.eps;
```

The plots of the impulse response $h(n)$ is shown in Figure 2.34.

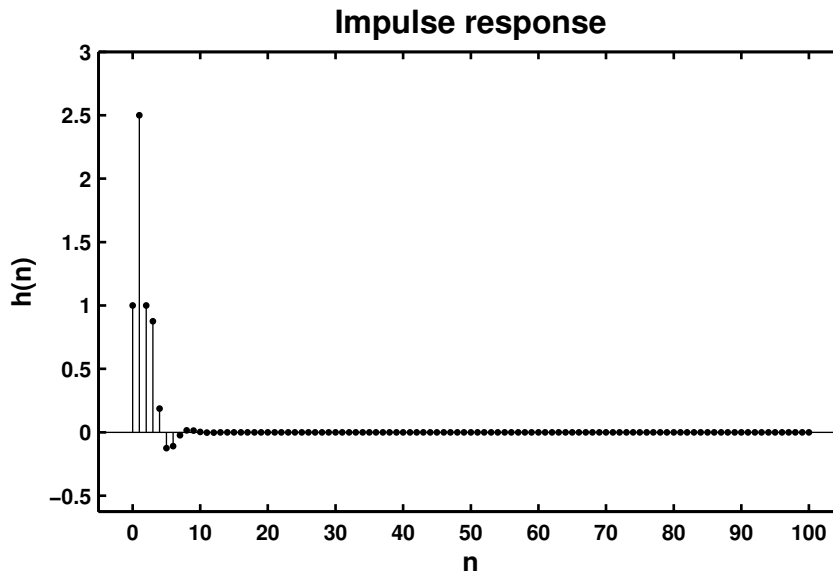


Figure 2.34: Problem P2.19.1 impulse response plot

(b) Clearly from Figure 2.34 the system is stable.

(c) Response $y(n)$ when the input is $x(n) = [5 + 3 \cos(0.2\pi n) + 4 \sin(0.6\pi n)] u(n)$:

```
% P0219c: Output response of a system using the filter function.
clc; close all;

b = [1 2 0 1]; a = [1 -0.5 0.25]; n = 0:200;
x = 5*ones(size(n))+3*cos(0.2*pi*n)+4*sin(0.6*pi*n); y = filter(b,a,x);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0219c');
Hs = stem(n,y,'filled'); set(Hs,'markersize',2); axis([-10,210,0,50]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output response','FontSize',TFS);
print -deps2 ../EPSFILES/P0219c.eps;
```

The plots of the response $y(n)$ is shown in Figure 2.35.

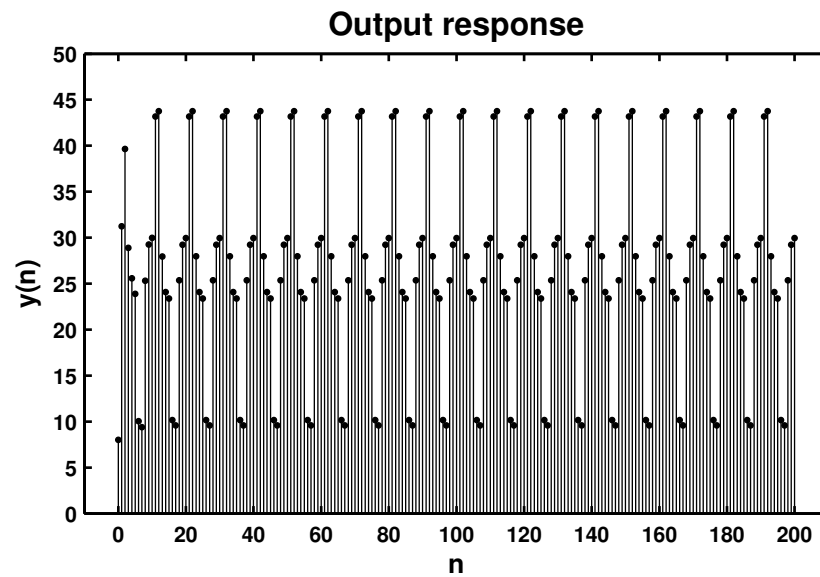


Figure 2.35: Problem P2.19.3 response plot

P2.20 A “simple” digital differentiator: $y(n] = x(n] - x(n - 1]$

(a) Response to a rectangular pulse $x(n] = 5[u(n] - u(n - 20)]$:

```
% P0220a: Simple Differentiator response to a rectangular pulse
clc; close all;

a = 1; b = [1 -1]; n1 = 0:22;
[x11,nx11] = stepseq(0,0,22); [x12,nx12] = stepseq(20,0,22);
x1 = 5*(x11 - x12); y1 = filter(b,a,x1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0220a');
Hs = stem(n1,y1,'filled'); set(Hs,'markersize',2); axis([-1,23,-6,6]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); ytick = [-6:6];
title('Output response for rectangular pulse ','FontSize',TFS);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0220a.eps;
```

The plots of the response $y(n]$ is shown in Figure 2.36.

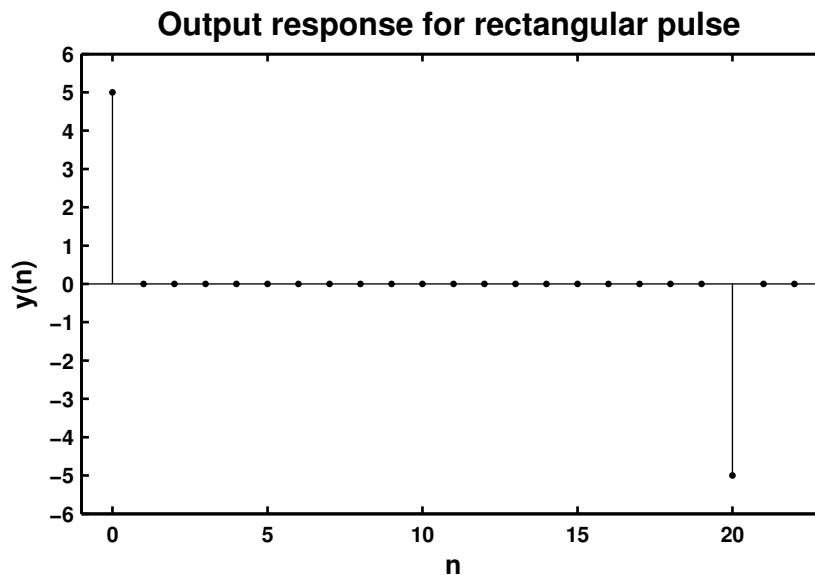


Figure 2.36: Problem P2.20.1 response plot

(b) Response to a triangular pulse $x(n) = n[u(n) - u(n - 10)] + (20 - n)[u(n - 10) - u(n - 20)]$:

```
% P0220b: Simple Differentiator response to a triangular pulse
clc; close all;

a = 1; b = [1 -1]; n2 = 0:21; [x11,nx11] = stepseq(0,0,21);
[x12,nx12] = stepseq(10,0,21); [x13,nx13] = stepseq(20,0,21);
x2 = n2.*(x11 - x12) + (20 - n2).*(x12 - x13); y2 = filter(b,a,x2);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0220b');
Hs = stem(n2,y2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(y2)-0.5,max(y2) + 0.5]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output response for triangular pulse','FontSize',TFS);
print -deps2 ../EPSFILES/P0220b.eps;
```

The plots of the response $y(n)$ is shown in Figure 2.37.

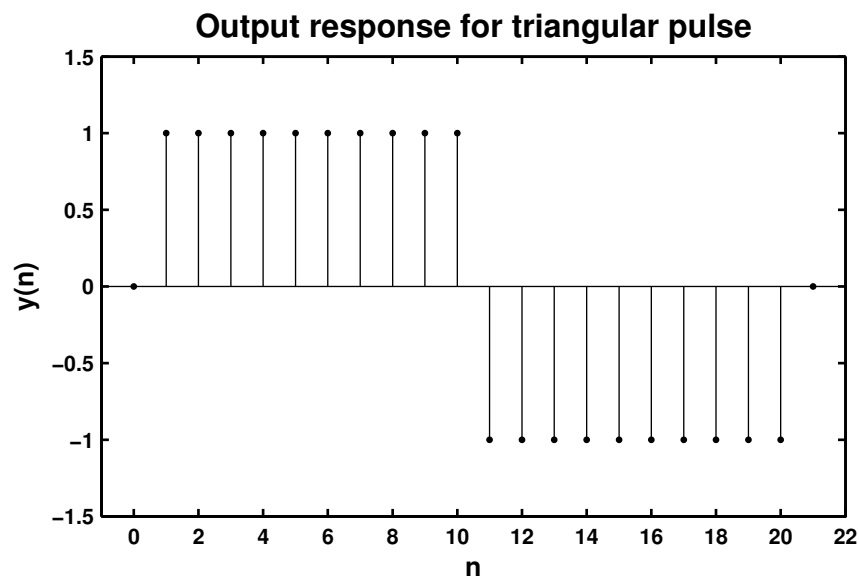


Figure 2.37: Problem P2.20.2 response plot

- (c) Response to a sinusoidal pulse $x(n) = \sin\left(\frac{\pi n}{25}\right)[u(n) - u(n - 100)]$:

```
% P0220cSimple Differentiator response to a sinusoidal pulse
clc; close all;

a = 1; b = [1 -1]; n3 = 0:101; [x11,nx11] = stepseq(0,0,101);
[x12,nx12] = stepseq(100,0,101); x13 = x11-x12; x3 = sin(pi*n3/25).*x13;
y3 = filter(b,a,x3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0220c');
Hs = stem(n3,y3,'filled'); set(Hs,'markersize',2);
axis([-5,105,-0.15,0.15]); ytick = [-0.15:0.05:0.15];
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output response for sinusoidal pulse','FontSize',TFS);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0220c.eps;
```

The plots of the response $y(n)$ is shown in Figure 2.38.

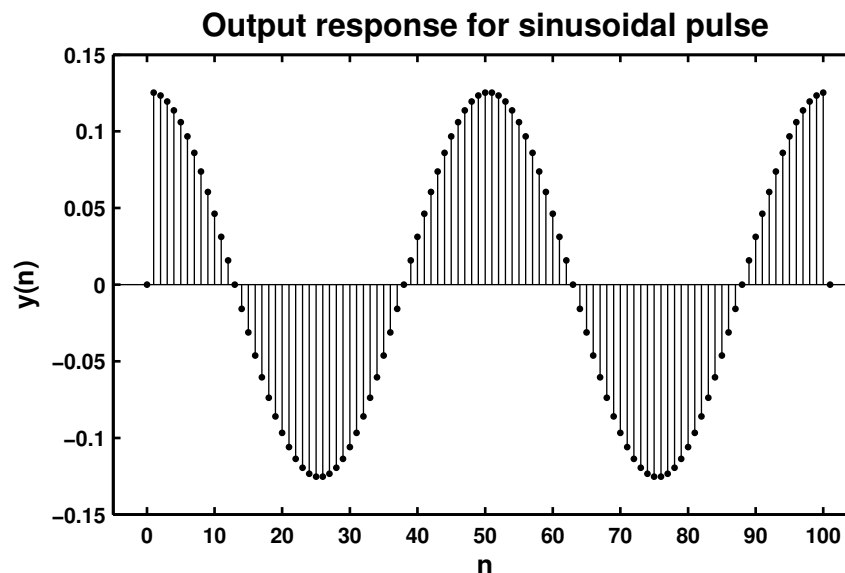


Figure 2.38: Problem P2.20.3 response plot