Chapter 2 Solutions

2.1.

a) 1000010
b) 110001
c) 1000000001
d) 1101100000
e) 11101101001
f) 11111011111

2.2.

a) 30_{10}, 36_8, $1E_{16}$
b) 26, 32, 1A
c) 291, 443, 123
d) 91, 133, 5B
e) 878_{10}, 1556_8, $36E_{16}$
f) 1514, 2752, 5EA

2.3.

a) 01100110
b) 11100011
c) 0010111111101000
d) 011111000010
e) 0101101000101101
f) 1110000010001011

2.4.

a) 000011101010
b) 111100010110
c) 000010011100
d) 101111000100
e) 111000101000

2.5.

	Decimal	Octal	Hexadecimal
a)	–53	713	CB
b)	30	36	1E
c)	–19	55	ED
d)	–167	7531	F59
e)	428	654	1AC

2.6.

a) 11100101; 229
b) 10110001; 177
c) ~~1~~11010110; 214
d) ~~1~~01011101; 93

2.7.

a) 11100101; –27
b) 10110001; –79

c) ~~1~~11010110; –42
d) ~~1~~01011101; 93

2.8.

a) 01101111; 111
b) 11001001; 201
c) 11110000; 240
d) 10110001; 177

2.9.

a) 01101111; 111
b) 11001001; –55
c) 11110000; –16
d) 10110001; –79

2.10.

Binary calculations	**Unsigned decimal calculations**	**Signed decimal calculations**
1001 + 0011 = 1100 No overflow	9 + 3 = 12 No overflow error	–7 + 3 = –4 No overflow error
0110 + 1011 = ~~1~~0001 Overflow	6 + 11 = 1 Overflow error	6 + (–5) = 1 No overflow error
0101 + 0110 = 1011 No overflow	5 + 6 = 11 No overflow error	5 + 6 = –5 Overflow error
0101 – 0110 = 1111 No overflow	5 – 6 = 15 Overflow error	5 – 6 = –1 No overflow error
1011 – 0101 = 0110 No overflow	11 – 5 = 6 No overflow error	–5 – 5 = 6 Overflow error

2.11.

x	y	z	$x'y'z'$	$x'yz$	$xy'z'$	xyz	F
0	0	0	1	0	0	0	1
0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	0
0	1	1	0	1	0	0	1
1	0	0	0	0	1	0	1
1	0	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	1	1	0	0	0	1	1

(a)

x	y	z	$xy'z$	$x'yz'$	xyz	xyz'	F
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	1	0	0	1
0	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	0	0	1
1	1	0	0	0	0	1	1
1	1	1	0	0	1	0	1

(b)

w	x	y	z	$w'xy'z$	$w'xyz$	$wxy'z$	$wxyz$	F
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	1	0	0	0	1
0	1	1	0	0	0	0	0	0
0	1	1	1	0	1	0	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	1	0	1
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	1	1

(c)

w	x	y	z	$wxy'z$	$w'yz'$	wxz	xyz'	F
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	1	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	1	0	1	1
0	1	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	0	1	1	0	1	0	1
1	1	1	0	0	0	0	1	1
1	1	1	1	0	0	1	0	1

(d)

x	y	z	xy'	$x'y'z$	xyz'	F
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	0	0	1
1	0	1	1	0	0	1
1	1	0	0	0	1	1
1	1	1	0	0	0	0

(e)

w	x	y	z	$w'z'$	$w'xy$	$wx'z$	$wxyz$	F
0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	0	1	0	0	0	1
0	1	0	1	0	0	0	0	0
0	1	1	0	1	1	0	0	1
0	1	1	1	0	1	0	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0	1
1	0	1	0	0	0	0	0	0
1	0	1	1	0	0	1	0	1
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	1	1

(f)

x	y	z	x'	y'	$x+y'$	yz	$(yz)'$	$[(x+y')\,(yz)']$	xy'	$x'y$	$(xy'+x'y)$	F
0	0	0	1	1	1	0	1	1	0	0	0	0
0	0	1	1	1	1	0	1	1	0	0	0	0
0	1	0	1	0	0	0	1	0	0	1	1	0
0	1	1	1	0	0	1	0	0	0	1	1	0
1	0	0	0	1	1	0	1	1	1	0	1	1
1	0	1	0	1	1	0	1	1	1	0	1	1
1	1	0	0	0	1	0	1	1	0	0	0	0
1	1	1	0	0	1	1	0	0	0	0	0	0

(g)

N_3	N_2	N_1	N_0	$N_3'N_2'N_1N_0'$	$N_3'N_2'N_1N_0$	$N_3N_2'N_1N_0'$	$N_3N_2'N_1N_0$	$N_3N_2N_1'N_0'$	$N_3N_2N_1N_0$	F
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	1
0	0	1	1	0	1	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0	0	0	1
1	0	1	1	0	0	0	1	0	0	1
1	1	0	0	0	0	0	0	1	0	1
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	1	1

(h)

2.12.

(a) $F = a'bc' + a'bc + abc'$

(b) $F = w'x'yz' + w'xy'z' + w'xy'z + w'xyz + wx'y'z + wx'yz' + wxy'z' + wxy'z + wxyz$

(c) $F_1 = w'x'y'z' + w'x'yz + w'xy'z + w'xyz' + wx'y'z + wx'yz' + wxy'z' + wxyz$

$F_2 = w'x'y'z' + w'x'y'z + w'x'yz' + w'x'yz + w'xy'z + wx'y'z' + wx'y'z + wxy'z' + wxy'z + wxyz' + wxyz$

(d) $F = N_3'N_2'N_1N_0' + N_3'N_2'N_1N_0 + N_3'N_2N_1N_0' + N_3N_2'N_1N_0' + N_3N_2'N_1N_0 + N_3N_2N_1'N_0' + N_3N_2N_1N_0$

2.14.

(a)

w	x	y	z	$w'z'$	$w'xy$	$wx'z$	$wxyz$	*Left Side*
0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	0	1	0	0	0	1
0	1	0	1	0	0	0	0	0
0	1	1	0	1	1	0	0	1
0	1	1	1	0	1	0	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0	1
1	0	1	0	0	0	0	0	0
1	0	1	1	0	0	1	0	1
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	1	1

$w'z'$	xyz	$wx'y'z$	wyz	*Right Side*
1	0	0	0	1
0	0	0	0	0
1	0	0	0	1
0	0	0	0	0
1	0	0	0	1
0	0	0	0	0
1	0	0	0	1
0	1	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	0	0	0
0	0	0	1	1
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	1	0	1	1

(b)

y	z	z	y'	yz'	*Left Side*
0	0	0	1	0	1
0	1	1	1	0	1
1	0	0	0	1	1
1	1	1	0	0	1

Right Side
1
1
1
1

(c)

x	y	z	$xy'z'$	x'	xyz'	*Left Side*
0	0	0	0	1	0	1
0	0	1	0	1	0	1
0	1	0	0	1	0	1
0	1	1	0	1	0	1
1	0	0	1	0	0	1
1	0	1	0	0	0	0
1	1	0	0	0	1	1
1	1	1	0	0	0	0

x'	z'	*Right Side*
1	1	1
1	0	1
1	1	1
1	0	1
0	1	1
0	0	0
0	1	1
0	0	0

(d)

x	y	z	xy	$x'z$	yz	*Left Side*
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	0	1	1	1
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	1
1	1	1	1	0	1	1

xy	$x'z$	*Right Side*
0	0	0
0	1	1
0	0	0
0	1	1
0	0	0
0	0	0
1	0	1
1	0	1

(e)

w	x	y	z	$w'x'yz'$	$w'x'yz$	$wx'yz'$	$wx'yz$	$wxyz$	*Left Side*
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	1
0	0	1	1	0	1	0	0	0	1
0	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
1	0	1	0	0	0	1	0	0	1
1	0	1	1	0	0	0	1	0	1
1	1	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	1	1

x'	wz	$(x' + wz)$	*Right Side*
1	0	1	0
1	0	1	0
1	0	1	1
1	0	1	1
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
1	0	1	0
1	1	1	0
1	0	1	1
1	1	1	1
0	0	0	0
0	1	1	0
0	0	0	0
0	1	1	1

(f)

w	x	y	z	$w'xy'z$	$w'xyz$	$wxy'z$	$wxyz$	Left Side	Right Side
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	0	1	1	0	0	0	1	1
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	1	0	0	1	1
1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0
1	1	0	1	0	0	1	0	1	1
1	1	1	0	0	0	0	0	0	0
1	1	1	1	0	0	0	1	1	1

(g)

x_i	y_i	c_i	x_iy_i	$x_i + y_i$	$c_i(x_i + y_i)$	Left Side	$x_iy_ic_i$	$x_iy_ic_i'$	$x_iy_i'c_i$	$x_i'y_ic_i$	Right Side
0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0	0	0	0
0	1	1	0	1	1	1	0	0	0	1	1
1	0	0	0	1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	0	0	1	0	1
1	1	0	1	1	0	1	0	1	0	0	1
1	1	1	1	1	1	1	1	0	0	0	1

(h)

x_i	y_i	c_i	x_iy_i	$x_i + y_i$	$c_i(x_i + y_i)$	Left Side	x_iy_i	$x_i \oplus y_i$	$c_i(x_i \oplus y_i)$	Right Side
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	1	0	0
0	1	1	0	1	1	1	0	1	1	1
1	0	0	0	1	0	0	0	1	0	0
1	0	1	0	1	1	1	0	1	1	1
1	1	0	1	1	0	1	1	0	0	1
1	1	1	1	1	1	1	1	0	0	1

2.19.

(a) $w'z' + w'xy + wx'z + wxyz$

$= w'x'y'z' + w'x'yz' + w'xy'z' + w'xyz' + w'xyz' + w'xyz + wx'y'z + wx'yz + wxyz$

$= w'x'y'z' + w'x'yz' + w'xy'z' + w'xyz' + w'xyz + wx'y'z + wx'yz + wxyz$

$= w'z' + w'xyz + wx'y'z + wx'yz + wxyz$

$= w'z' + (w'+w)xyz + wx'y'z + w(x'+x)yz$

$= w'z' + xyz + wx'y'z + wyz$

(b) $z + y' + yz'$

$= z(y'+y) + (z'+z)y' + yz'$

$$\begin{aligned}
&= zy' + zy + z'y' + \cancel{zy'} + yz' \\
&= z(y'+y) + z'(y'+y) \\
&= z + z' \\
&= 1
\end{aligned}$$

(c) $x\,y'\,z' + x' + x\,y\,z'$

$$\begin{aligned}
&= x\,z'\,(y' + y) + x' \\
&= x\,z' + x' \\
&= x\,z' + 1\,x' \\
&= (x + 1)(x + x')(z' + 1)(z' + x') \\
&= 1 \bullet 1 \bullet 1\,(z' + x') \\
&= x' + z'
\end{aligned}$$

(d) $xy + x'z + yz$

$$\begin{aligned}
&= xy(z'+z) + x'(y'+y)z + (x'+x)yz \\
&= xyz' + xyz + x'y'z + x'yz + \cancel{x'yz} + \cancel{xyz} \\
&= xy(z'+z) + x'(y'+y)z \\
&= xy(1) + x'(1)z \\
&= xy + x'z
\end{aligned}$$

(e) $w'x'yz' + w'x'yz + wx'yz' + wx'yz + wxyz$

$$\begin{aligned}
&= [w'x'yz' + w'x'yz + wx'yz' + wx'yz] + [wx'yz + wxyz] \\
&= x'y(w'z' + w'z + wz' + wz) + w(x'+x)yz \\
&= x'y + wyz \\
&= y(x' + wz)
\end{aligned}$$

(f) $w'xy'z + w'xyz + wxy'z + wxyz$

$$\begin{aligned}
&= xy'z(w' + w) + xyz(w' + w) \\
&= xy'z + xyz \\
&= xz(y + y') \\
&= xz
\end{aligned}$$

(g) $x_iy_i + c_i(x_i + y_i)$

$$\begin{aligned}
&= x_iy_i + x_ic_i + y_ic_i \\
&= x_iy_i(c_i + c_i') + x_i(y_i + y_i')c_i + (x_i + x_i')y_ic_i \\
&= x_iy_ic_i + x_iy_ic_i' + \cancel{x_iy_ic_i} + x_iy_i'c_i + \cancel{x_iy_ic_i} + x_i'y_ic_i \\
&= x_iy_ic_i + x_iy_ic_i' + x_iy_i'c_i + x_i'y_ic_i
\end{aligned}$$

(h) $x_iy_i + c_i(x_i + y_i)$

$$\begin{aligned}
&= x_iy_i + x_ic_i + y_ic_i \\
&= x_iy_i(c_i + c_i') + x_i(y_i + y_i')c_i + (x_i + x_i')y_ic_i \\
&= x_iy_ic_i + x_iy_ic_i' + \cancel{x_iy_ic_i} + x_iy_i'c_i + \cancel{x_iy_ic_i} + x_i'y_ic_i \\
&= x_iy_ic_i + x_iy_ic_i' + x_iy_i'c_i + x_i'y_ic_i \\
&= x_iy_i(c_i + c_i') + c_i(x_iy_i' + x_i'y_i) \\
&= x_iy_i + c_i(x_i \oplus y_i)
\end{aligned}$$

2.20.

(a) $x'y'z' + x'yz + xy'z' + xyz$

$$\begin{aligned}
&= (x + x')y'z' + (x + x')yz \\
&= y'z' + yz \\
&= y \odot z
\end{aligned}$$

(b) $xy'z + x'yz' + xyz + xyz'$

$$\begin{aligned}
&= (x + x')yz' + x(y + y')z \\
&= yz' + xz
\end{aligned}$$

(c) $w'xy'z + w'xyz + wxy'z + wxyz$
$= xz(w'y' + w'y + wy' + wy)$
$= xz$

(d) $wxy'z + w'yz' + wxz + xyz'$
$= wxz + yz'(x+w')$

(e) $xy' + x'y'z + xyz'$
$= xy'z + xy'z' + x'y'z + xyz'$
$= xy' + y'z + xz'$
$= y'(x + z) + xz'$

(f) $w'z' + w'xy + wx'z + wxyz$
$= w'z' + w'xyz + w'xyz' + wx'z + wxyz$
$= w'z' + xyz(w' + w) + w'xyz' + wx'z$
$= w'z' + xyz + wx'z$
$= w'z' + z(xy + wx')$

(g) $[(x+y')\,(yz)'\,]\,(xy' + x'y)$
$= [(x+y')\,(y'+z')]\,(xy' + x'y)$
$= [xy' + xz' + y' + y'z']\,(xy' + x'y)$
$= [xz' + y']\,(xy' + x'y)$
$= xy'z' + xx'yz' + y'xy' + y'x'y$
$= xy'z' + xy'$
$= xy'$

(h) $N_3'N_2'N_1N_0' + N_3'N_2'N_1N_0 + N_3N_2'N_1N_0' + N_3N_2'N_1N_0 + N_3N_2N_1'N_0' + N_3N_2N_1N_0$
$=$

2.21.

$F = (x' + y' + x'y' + xy)\,(x' + yz)$
$= (x' \bullet 1 + y' \bullet 1 + x'y' + xy)\,(x' + yz)$ by Theorem 6a
$= (x'\,(y + y') + y'\,(x + x') + x'y' + xy)\,(x' + yz)$ by Theorem 9b
$= (x'y + x'y' + y'x + y'x' + x'y' + xy)\,(x' + yz)$ by Theorem 12a
$= (x'y + x'y' + y'x + \cancel{y'x'} + \cancel{x'y'} + xy)\,(x' + yz)$ by Theorem 7b
$= (x'\,(y + y') + x\,(y + y'))\,(x' + yz)$ by Theorem 12a
$= (x' \bullet 1 + x \bullet 1)\,(x' + yz)$ by Theorem 9b
$= (x' + x)\,(x' + yz)$ by Theorem 6a
$= 1\,(x' + yz)$ by Theorem 9b
$= (x' + yz)$ by Theorem 6a

2.22.

For three variables (x, y, z), there is a total of eight (2^3) minterms. The function has five minterms, therefore, the inverted function will have three (8-5=3) minterms. Hence, implementing the inverted function and then adding a NOT gate at the final output will result in a smaller circuit. The circuit requires 3 AND gates, 1 OR gate, and 1 NOT gate.

2.23.

w	x	y	z	4 AND	4 NAND	4 NOR	4 XOR	4 XNOR
0	0	0	0	0	1	1	0	1
0	0	0	1	0	1	0	1	0

v	w	x	y	z	5 XOR	5 XNOR
0	0	0	0	0	0	0
0	0	0	0	1	1	1

0	0	1	0	0	1	0	1	0
0	0	1	1	0	1	0	0	1
0	1	0	0	0	1	0	1	0
0	1	0	1	0	1	0	0	1
0	1	1	0	0	1	0	0	1
0	1	1	1	0	1	0	1	0
1	0	0	0	0	1	0	1	0
1	0	0	1	0	1	0	0	1
1	0	1	0	0	1	0	0	1
1	0	1	1	0	1	0	1	0
1	1	0	0	0	1	0	0	1
1	1	0	1	0	1	0	1	0
1	1	1	0	0	1	0	1	0
1	1	1	1	1	0	0	0	1
				(a)	(b)	(c)	(d)	(e)

0	0	0	1	0	1	1
0	0	0	1	1	0	0
0	0	1	0	0	1	1
0	0	1	0	1	0	0
0	0	1	1	0	0	0
0	0	1	1	1	1	1
0	1	0	0	0	1	1
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	0	1	1	1	1
0	1	1	0	0	0	0
0	1	1	0	1	1	1
0	1	1	1	0	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	1
1	0	0	0	1	0	0
1	0	0	1	0	0	0
1	0	0	1	1	1	1
1	0	1	0	0	0	0
1	0	1	0	1	1	1
1	0	1	1	0	1	1
1	0	1	1	1	0	0
1	1	0	0	0	0	0
1	1	0	0	1	1	1
1	1	0	1	0	1	1
1	1	0	1	1	0	0
1	1	1	0	0	1	1
1	1	1	0	1	0	0
1	1	1	1	0	0	0
1	1	1	1	1	1	1
					(f)	(g)

2.24.

w	x	y	z	$(x \odot y)'$	$(xyz)'$	$(x \odot y)' + (xyz)'$	$(w' + x + z)$	F
0	0	0	0	0	1	1	1	1
0	0	0	1	0	1	1	1	1
0	0	1	0	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1	1
0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1
0	1	1	1	0	0	0	1	0
1	0	0	0	0	1	1	0	0
1	0	0	1	0	1	1	1	1
1	0	1	0	1	1	1	0	0
1	0	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	1	1	1	1
1	1	1	1	0	0	0	1	0

(a)

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b)

w	x	y	z	$w'xy'z$	$(x \oplus y)$	$w'z\,(y \oplus x)$	$[w'xy'z + w'z\,(y \oplus x)]$	F
0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	1
0	0	1	0	0	1	0	0	1
0	0	1	1	0	1	1	1	0
0	1	0	0	0	1	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	0	0	0	0	0	1
0	1	1	1	0	0	0	0	1
1	0	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0	1
1	0	1	0	0	1	0	0	1
1	0	1	1	0	1	0	0	1
1	1	0	0	0	1	0	0	1
1	1	0	1	0	1	0	0	1
1	1	1	0	0	0	0	0	1
1	1	1	1	0	0	0	0	1

(c)

2.25.

a)

w	x	y	z	$(x \odot y)'$	$(xyz)'$	$(x \odot y)' + (xyz)'$	$(w' + x + z)$	F
0	0	0	0	0	1	1	1	1
0	0	0	1	0	1	1	1	1
0	0	1	0	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1	1
0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	1	1	1
0	1	1	1	0	0	0	1	0
1	0	0	0	0	1	1	0	0
1	0	0	1	0	1	1	1	1
1	0	1	0	1	1	1	0	0
1	0	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	1	1	1	1
1	1	1	1	0	0	0	1	0

(a)

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b)

w	x	y	z	$w'xy'z$	$(x \oplus y)$	$w'z\,(y \oplus x)$	$[w'xy'z + w'z\,(y \oplus x)]$	F
0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	1
0	0	1	0	0	1	0	0	1
0	0	1	1	0	1	1	1	0
0	1	0	0	0	1	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	0	0	0	0	0	1
0	1	1	1	0	0	0	0	1
1	0	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0	1
1	0	1	0	0	1	0	0	1
1	0	1	1	0	1	0	0	1
1	1	0	0	0	1	0	0	1
1	1	0	1	0	1	0	0	1
1	1	1	0	0	0	0	0	1
1	1	1	1	0	0	0	0	1

(c)

b)

$$\begin{aligned}
F &= [(x \odot y)' + (xyz)'] \, (w' + x + z) \\
&= [xy' + x'y + x' + y' + z')] \, (w' + x + z) \\
&= (x' + y' + z') \, (w' + x + z) \\
&= (ww' + x' + y' + z') \, (w' + x + yy' + z) \\
&= (w + x' + y' + z') \, (w' + x' + y' + z') \, (w' + x + y + z) \, (w' + x + y' + z) \\
&= \Pi(M_7 + M_8 + M_{10} + M_{15})
\end{aligned}$$

(a)

$$\begin{aligned}
F &= x \oplus y \oplus z \\
&= (xy' + x'y)z' + (xy' + x'y)'z \\
&= xy'z' + x'yz' + xy'z + x'y'z \\
&= (x+y+z)(x+y'+z')(x'+y'+z)(x'+y'+z') \\
&= \Pi(M_0 + M_3 + M_6 + M_7)
\end{aligned}$$

(b)

$$\begin{aligned}
F &= [w'xy'z + w'z\,(y \oplus x)]' \\
&= [w'xy'z]' \, [w'z\,(y \oplus x)]' \\
&= [w+x'+y+z'] \, [w+z'+ (y \oplus x)'] \\
&= [w+x'+y+z'] \, [w+z'+ xy + x'y'] \\
&= [w+x'+y+z'] \, [w+x+y'+z'] \, \cancel{[w+x'+y+z']} \\
&= \Pi(\, M_3 + M_5)
\end{aligned}$$

(c)

2.26.

$$\begin{aligned}
F &= [(x \odot y)' + (xyz)'] \, (w' + x + z) \\
&= [xy' + x'y + x' + y' + z')] \, (w' + x + z) \\
&= (x' + y' + z') \, (w' + x + z) \\
&= x'w' + \cancel{x'x} + x'z + y'w' + y'x + y'z + z'w' + z'x + \cancel{z'z} \\
&= w'x' + x'z + w'y' + xy' + y'z + w'z' + xz' \\
&= (x \oplus z) + xy' + w'x' \\
&\text{or } (x \oplus z) + xy' + w'z' \\
&\text{or } (x \oplus z) + y'z + w'x' \\
&\text{or } (x \oplus z) + y'z + w'z'
\end{aligned}$$

(a)

$$F = x \oplus y \oplus z$$

(b)

$$
\begin{aligned}
F &= [w'xy'z + w'z\,(y \oplus x)]' \\
&= [w'xy'z]'\,[w'z\,(y \oplus x)]' \\
&= [w+x'+y+z']\,[w+z'+ (y \oplus x)'] \\
&= [w+x'+y+z']\,[w+z'+ xy + x'y'] \\
&= w + \cancel{wz'} + \cancel{wxy} + \cancel{wx'y'} + \cancel{wx'} + \cancel{x'z'} + x'y' + \cancel{wy} + \cancel{yz'} + xy + \cancel{wz'} + z' + \cancel{xyz'} + \cancel{x'y'z'} \\
&= w + z' + x'y' + xy \\
&= w + z' + (x \odot y)
\end{aligned}
$$

(c)

2.27.

x	y	*Left Side* $x \oplus y$	$x \odot y$	**Right Side** $(x \odot y)'$
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	0	1	0

(a)

x	y	y'	*Left Side* $x \oplus y'$	*Right Side* $x \odot y$
0	0	1	1	1
0	1	0	0	0
1	0	1	0	0
1	1	0	1	1

(b)

w	x	y	z	$w \oplus x$	$y \oplus z$	**Left Side** $(w \oplus x) \odot (y \oplus z)$	$w \odot x$	$y \odot z$	**Right Side** $(w \odot x) \odot (y \odot z)$	**Right Side** $(((w \odot x) \odot y) \odot z)$
0	0	0	0	0	0	1	1	1	1	1
0	0	0	1	0	1	0	1	0	0	0
0	0	1	0	0	1	0	1	0	0	0
0	0	1	1	0	0	1	1	1	1	1
0	1	0	0	1	0	0	0	1	0	0
0	1	0	1	1	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0	1	1
0	1	1	1	1	0	0	0	1	0	0
1	0	0	0	1	0	0	0	1	0	0
1	0	0	1	1	1	1	0	0	1	1
1	0	1	0	1	1	1	0	0	1	1
1	0	1	1	1	0	0	0	1	0	0
1	1	0	0	0	0	1	1	1	1	1
1	1	0	1	0	1	0	1	0	0	0
1	1	1	0	0	1	0	1	0	0	0
1	1	1	1	0	0	1	1	1	1	1

(c)

x	y	z	$(xy)'$	$((xy)'x)'$	$((xy)'y)'$	$[((xy)'x)'((xy)'y)']$	*Left Side* $[((xy)'x)'((xy)'y)']'$	*Right Side* $x \oplus y$
0	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	1	0	1	1	0	0	1	1
0	1	1	1	1	0	0	1	1
1	0	0	1	0	1	0	1	1
1	0	1	1	0	1	0	1	1
1	1	0	0	1	1	1	0	0
1	1	1	0	1	1	1	0	0

(d)

2.28.

$$
\begin{aligned}
(x \oplus y) &= xy' + x'y \\
&= xx' + xy' + x'y + yy' \\
&= (x + y)\,(x' + y') \\
&= (x'y')'\,(xy)' \\
&= [(x'y') + (xy)]' \\
&= (x \odot y)'
\end{aligned}
$$

(a)

$$
\begin{aligned}
x \oplus y' &= xy + x'y' \\
&= x \odot y
\end{aligned}
$$

(b)

$$
\begin{aligned}
&[((xy)'x)'\,((xy)'y)'\,]' \\
&\quad = ((xy)'x) + ((xy)'y) \\
&\quad = (x' + y'\,)x + (x' + y'\,)y \\
&\quad = \cancel{xx'} + xy' + x'y + \cancel{y'y} \\
&\quad = xy' + x'y \\
&\quad = x \oplus y
\end{aligned}
$$

(d)

2.29.

$$
\begin{aligned}
&x \oplus y \oplus z \\
&\quad = (x \oplus y) \oplus z \\
&\quad = (x'y + xy'\,) \oplus z \\
&\quad = (x'y + xy'\,)z' + (x'y + xy'\,)'z \\
&\quad = x'yz' + xy'z' + (x'y)'\,(xy'\,)'z \\
&\quad = x'yz' + xy'z' + (x{+}y'\,)\,(x'{+}y)\,z \\
&\quad = x'yz' + xy'z' + \cancel{xx'z} + xyz + x'y'z + \cancel{y'yz} \\
&\quad = x'y'z + x'yz' + xy'z' + xyz
\end{aligned}
$$

2.30.

$$
\begin{aligned}
x \oplus y \oplus z &= (x \oplus y) \oplus z \\
&= (x'y + xy') \oplus z \\
&= (x'y + xy')'\,z + (x'y + xy')\,z' \\
&= (x'y)' \cdot (xy')'\,z + x'yz' + xy'z' \\
&= (x + y') \cdot (x' + y)\,z + x'yz' + xy'z' \\
&= \cancel{xx'z} + xyz + x'y'z + \cancel{y'yz} + x'yz' + xy'z' \\
&= (xy + x'y')\,z + (x'y + xy')\,z' \\
&= (xy + x'y')\,z + (xy + x'y')'\,z' \\
&= (x{\odot}y)\,z + (x{\odot}y)'\,z' \\
&= x{\odot}y{\odot}z
\end{aligned}
$$

2.31.

(a) $F(x,y,z) = \Sigma(m_0, m_3, m_4, m_7)$
(b) $F(x,y,z) = \Sigma(m_2, m_5, m_6, m_7)$
(c) $F(w,x,y,z) = \Sigma(m_5, m_7, m_{13}, m_{15})$
(d) $F(w,x,y,z) = \Sigma(m_2, m_6, m_{13}, m_{14}, m_{15})$
(e) $F(x,y,z) = \Sigma(m_1, m_4, m_5, m_6)$
(f) $F(w,x,y,z) = \Sigma(m_0, m_2, m_4, m_6, m_7, m_9, m_{11}, m_{15})$
(g) $F(x,y,z) = \Sigma(m_4, m_5)$
(h) $F(N_3,N_2,N_1,N_0) = \Sigma(m_2, m_3, m_{10}, m_{11}, m_{12}, m_{15})$

(a)

(a) $F(x,y,z) = \Pi(M_1, M_2, M_5, M_6)$
(b) $F(x,y,z) = \Pi(M_0, M_1, M_3, M_4)$
(c) $F(w,x,y,z) = \Pi(M_0, M_1, M_2, M_3, M_4, M_6, M_8, M_9, M_{10}, M_{11}, M_{12}, M_{14})$
(d) $F(w,x,y,z) = \Pi(M_0, M_1, M_3, M_4, M_5, M_7, M_8, M_9, M_{10}, M_{11}, M_{12})$
(e) $F(x,y,z) = \Pi(M_0, M_2, M_3, M_7)$
(f) $F(w,x,y,z) = \Pi(M_1, M_3, M_5, M_8, M_{10}, M_{12}, M_{13}, M_{14})$
(g) $F(x,y,z) = \Pi(M_0, M_1, M_2, M_3, M_6, M_7)$
(h) $F(N_3,N_2,N_1,N_0) = \Pi(M_0, M_1, M_4, M_5, M_6, M_7, M_8, M_9, M_{13}, M_{14})$

(b)

2.32.

(a) $F(x,y,z) = x'y'z + x'yz + xyz$

(b) $F(w,x,y,z) = w'x'y'z + w'x'yz + w'xyz$

(c) $F(x,y,z) = (x+y+z')\ (x+y'+z')\ (x'+y'+z')$

(d) $F(w,x,y,z) = (w+x+y+z')\ (w+x+y'+z')\ (w+x'+y'+z')$

(e) $F'(x,y,z) = x'y'z' + x'yz' + xy'z' + xy'z + xyz'$

(f) $F(x,y,z) = (x+y+z)\ (x+y'+z)\ (x'+y+z)\ (x'+y+z')\ (x'+y'+z)$

2.33.

F' is expressed as a sum of its 0-minterms. Therefore, F is the sum of its 1-minterms = $\Sigma(0, 2, 4, 5, 6)$. Using three variables, the truth table is as follows:

x	y	z	Minterms	F
0	0	0	$m_0{=}x'\,y'\,z'$	1
0	0	1	$m_1{=}x'\,y'\,z$	0
0	1	0	$m_2{=}x'\,y\,z'$	1
0	1	1	$m_3{=}x'\,y\,\mathrm{z}$	0
1	0	0	$m_4{=}x\,y'\,z'$	1
1	0	1	$m_5{=}x\,y'\,z$	1
1	1	0	$m_6{=}x\,y\,z'$	1
1	1	1	$m_7{=}x\,y\,z$	0

2.34.

$F = \Sigma(3, 4, 5) = m_3 + m_4 + m_5$
$= x'yz + xy'z' + xy'z$
$=$ ~~$(x' + x + x)(x' + x + y')(x' + x + z)$~~
~~$(x' + y' + x)$~~$(x' + y' + y')(x' + y' + z)$
~~$(x' + z' + x)$~~$(x' + z' + y')$~~$(x' + z' + z)$~~
$(y + x + x)$~~$(y + x + y')$~~$(y + x + z)$
~~$(y + y' + x)(y + y' + y')(y + y' + z)$~~
$(y + z' + x)$~~$(y + z' + y')(y + z' + z)$~~
$(z + x + x)(z + x + y')(z + x + z)$
$(z + y' + x)(z + y' + y')(z + y' + z)$
~~$(z + z' + x)(z + z' + y')(z + z' + z)$~~
$= (x' + y' + z)\ (x' + y' + z')\ (x + y + z)\ (x + y + z')\ (x + y' + z)$

2.35.

a)

Product-of-sums (AND-of-OR) format is obtained by using the *duality principle* or De Morgan's Theorem:
$F' = (x'+y+z) \bullet (x'+y+z') \bullet (x'+y'+z) \bullet (x'+y'+z')$

b)

Sum-of-products (OR-of-AND) format is obtained by first constructing the truth table for F and then inverting the 0's and 1's to get F'. Then we simply use the AND terms where $F' = 1$.

x	y	z	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$F' = x'y'z' + x'y'z + x'yz' + x'yz$

2.36.

a)

$$\begin{aligned}
F &= w \odot x \odot y \odot z \\
&= (wx + w'x') \odot y \odot z \\
&= [(wx + w'x')y + (wx + w'x')'y']\, z + [(wx + w'x')y + (wx + w'x')'y']'z' \\
&= wxyz + w'x'yz + (wx)'(w'x')'y'z + [(wx + w'x')y + (wx + w'x')'y']'z' \\
&= m_{15} + m_3 + (w'+x')(w+x)y'z + [(wx + w'x')y + (wx + w'x')'y']'z' \\
&= m_{15} + m_3 + w'xy'z + wx'y'z + [(wx + w'x')y + (wx + w'x')'y']'z' \\
&= m_{15} + m_3 + m_5 + m_9 + [(wx + w'x')\, y]'\,[(wx + w'x')'y']'z' \\
&= m_{15} + m_3 + m_5 + m_9 + [(wx + w'x')' + y']\; [(wx + w'x') + y]\, z' \\
&= m_{15} + m_3 + m_5 + m_9 + [(wx)'(w'x')' + y']\; [wxz' + w'x'z' + yz'] \\
&= m_{15} + m_3 + m_5 + m_9 + [(w'+x')(w+x) + y']\; [wxz' + w'x'z' + yz'] \\
&= m_{15} + m_3 + m_5 + m_9 + [w'x + wx' + y']\; [wxz' + w'x'z' + yz'] \\
&= m_{15} + m_3 + m_5 + m_9 + w'xyz' + wx'yz' + wxy'z' + w'x'y'z' \\
&= m_{15} + m_3 + m_5 + m_9 + m_6 + m_{10} + m_{12} + m_0
\end{aligned}$$

2.37.

a)

```
module P2_24a (
   input w,x,y,z,
   output F
);
   assign F = (~(x^y) | ~(x&y&z)) & (~w|x|z);
endmodule
```

b)

```
module P2_24b (
   input x,y,z,
   output F
);
   assign F = x^y^z;
endmodule
```

c)

```
module P2_24c (
   input w,x,y,z,
   output F
);
   assign F = ~((~w&x&~y&z) | (~w&z&(y^x)));
endmodule
```

2.38.

a)

```
LIBRARY IEEE;
USE IEEE.STD_LOGIC_1164.all;

ENTITY P2_24a IS PORT (
   w,x,y,z: IN STD_LOGIC;
   F: OUT STD_LOGIC);
END P2_24a;

ARCHITECTURE Dataflow OF P2_24a IS
BEGIN
   F <= (NOT (x XOR y) OR NOT (x AND y AND z)) AND (NOT w OR x OR z);
END Dataflow;
```

b)

```
LIBRARY IEEE;
USE IEEE.STD_LOGIC_1164.all;

ENTITY P2_24b IS PORT (
   x,y,z:   IN STD_LOGIC;
   F: OUT STD_LOGIC);
END P2_24b;

ARCHITECTURE Dataflow OF P2_24b IS
BEGIN
   F <= x XOR y XOR z;
END Dataflow;
```

c)

```
LIBRARY IEEE;
USE IEEE.STD_LOGIC_1164.all;

ENTITY P2_24c IS PORT (
   w,x,y,z: IN STD_LOGIC;
   F: OUT STD_LOGIC);
END P2_24c;

ARCHITECTURE Dataflow OF P2_24c IS
BEGIN
   F <= NOT((NOT w AND x AND NOT y AND z) OR (NOT w AND z AND (y XOR x)));
END Dataflow;
```

2.39.

```
// this is a Verilog behavioral model of the car security system

module Siren (
   input M, D, V,
   output S
);

   wire term1, term2, term3;

   always @ (M or D or V) begin
      term1 = (M & ~D & V);
      term2 = (M & D & ~V);
      term3 = (M & D & V);
      S = term1 | term2 | term3;
   end

endmodule
```

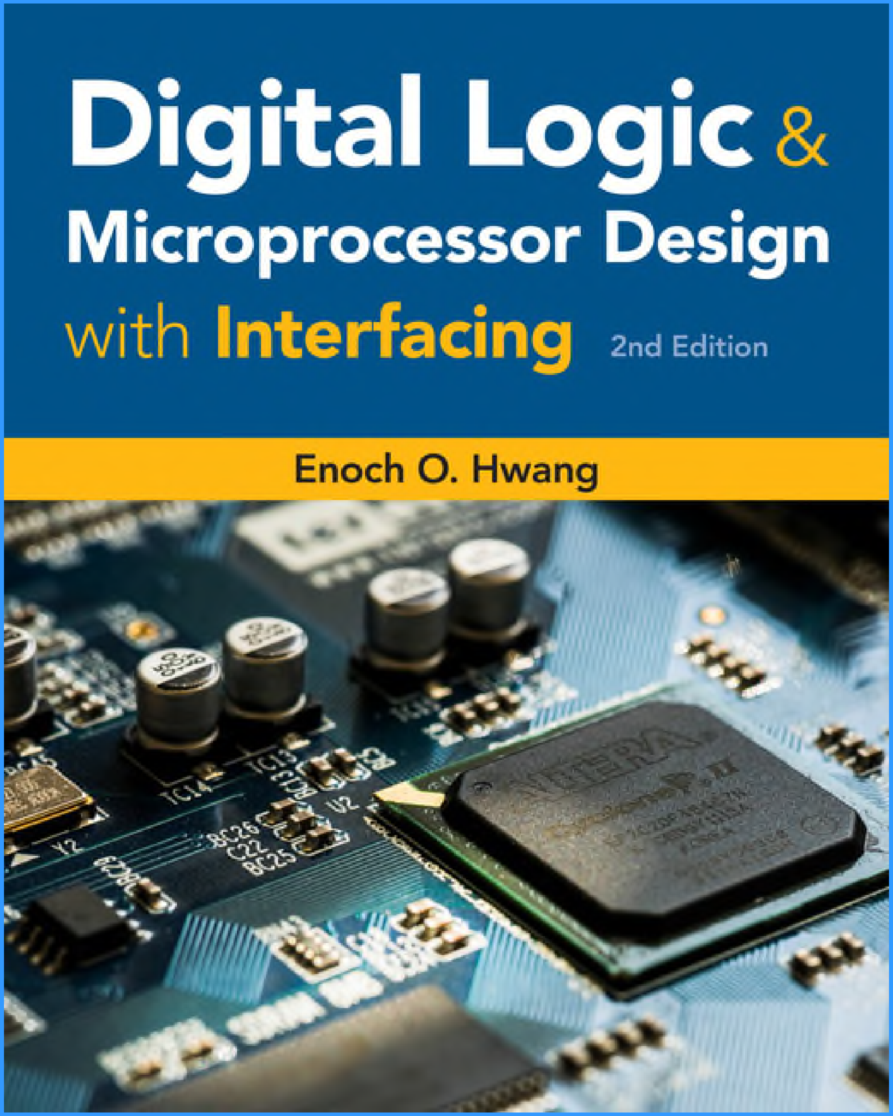
2.40.

```
LIBRARY IEEE;
USE IEEE.STD_LOGIC_1164.ALL;

ENTITY Siren IS PORT (
   M, D, V: IN STD_LOGIC;
   S: OUT STD_LOGIC);
END Siren;

ARCHITECTURE Behavioral OF Siren IS
BEGIN
   PROCESS(M, D, V)
   BEGIN
      S <= (M AND NOT D AND V) OR (M AND D AND NOT V) OR (M AND D AND V);
   END PROCESS;

END Behavioral;
```



Chapter 2
Fundamentals of Digital Circuits

Binary Number

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

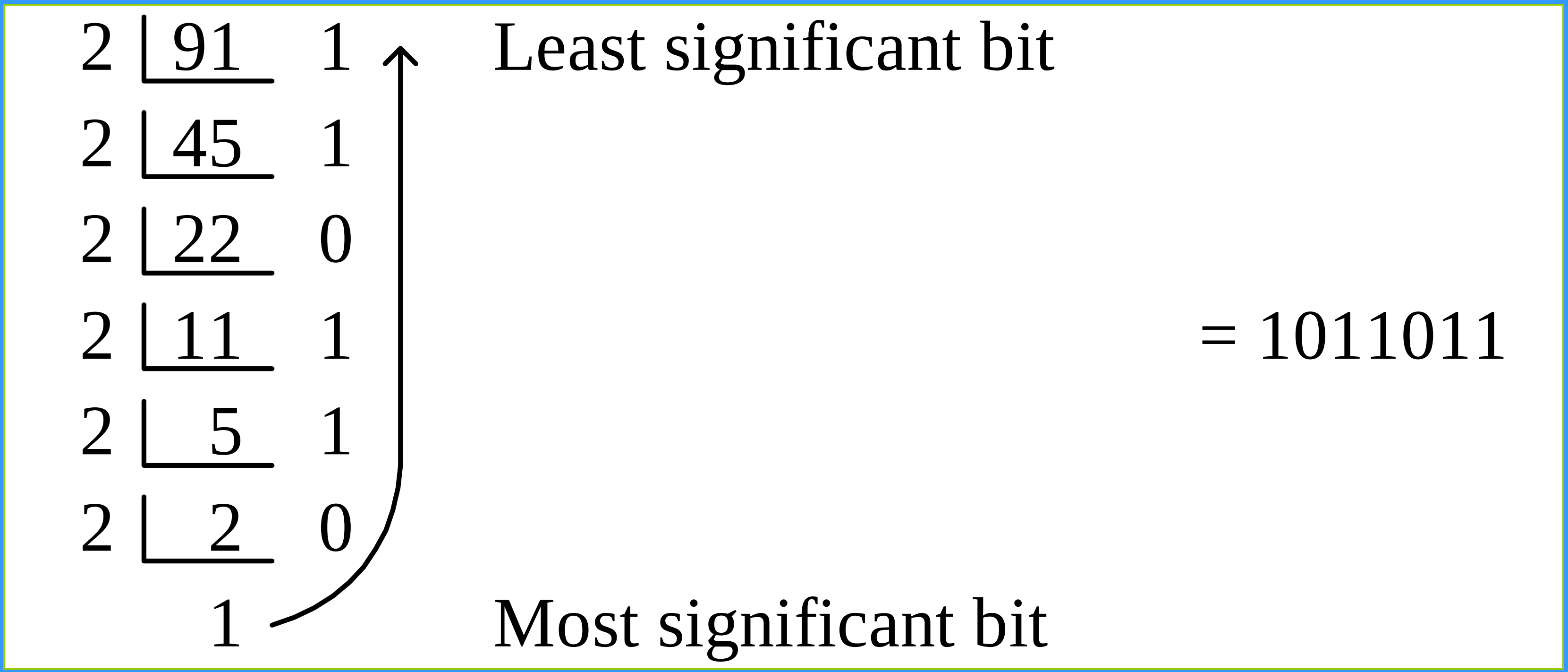
Value of a Binary Number

- For decimal number:
- $658_{10} = (6 \times 10^2) + (5 \times 10^1) + (8 \times 10^0)$
  $= 600 + 50 + 8 = 658_{10}$

- For binary number:
- $1011011_2 = (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

  $= 64 + 0 + 16 + 8 + 0 + 2 + 1 = 91_{10}$

Convert Decimal to Digital



2	91	1	Least significant bit
2	45	1	
2	22	0	
2	11	1	= 1011011
2	5	1	
2	2	0	
	1		Most significant bit

Octal and Hex

- Octal

001 110 011

1 6 3

5 7 2 4

101 111 010 100

$5724_8 = (5 \times 8^3) + (7 \times 8^2) + (2 \times 8^1) + (4 \times 8^0)$

$= 2560 + 448 + 16 + 4$

$= 3028_{10}$

- Hex

0110 1101 1011

6 D B

5 C F

0101 1100 1111

$5CA_{16} = (5 \times 16^2) + (C \times 16^1) + (F \times 16^0)$

$= (5 \times 16^2) + (12 \times 16^1) + (15 \times 16^0)$

$= 1280 + 192 + 15$

$= 1487_{10}$

Binary Number Arithmetic

$$\begin{array}{ccccc}
 & 1 & 0 & 0 & 1 \\
+ & 0 & 0 & {}_{1\nwarrow}1 & {}_{1\nwarrow}1 \\
\hline
 & 1 & 1 & 0 & 0
\end{array}$$

$$\begin{array}{cccccccc}
 & 1 & 1 & 0 & 1 & = & & 13 \\
+ & 1 & {}_{1\nwarrow}0 & {}_{1\nwarrow}1 & {}_{1\nwarrow}1 & = & + & 11 \\
\hline
1 & 1 & 0 & 0 & 0 & = & & 24
\end{array}$$

$$\begin{array}{cccccccc}
 & 1^{\curvearrowright} & \overset{2}{0}^{\curvearrowright} & \overset{2}{0} & 1 & = & & 9 \\
- & 0 & 0 & 1 & 1 & = & - & 3 \\
\hline
 & 0 & 1 & 1 & 0 & = & & 6
\end{array}$$

$$\begin{array}{cccccccc}
 & 1 & 1^{\curvearrowright} & \overset{2}{0} & 1 & = & & 13 \\
- & 1 & 0 & 1 & 1 & = & - & 11 \\
\hline
 & 0 & 0 & 1 & 0 & = & & 2
\end{array}$$

Negative Number

- Signed or unsigned number representation
- Use two’s complement representation for signed numbers
- For signed numbers, the MSB tells whether the number is positive or negative
  - 0 = positive
  - 1 = negative
- If signed number is positive then you can find the value just like for unsigned numbers

Negative Number

- If signed number is negative then you need to do two steps to find its value:
  - (1) Flip all the 1 bits to 0's and all the 0 bits to 1's.
  - (2) Add a 1 to the result obtained from step (1).
- The negated value obtained from step (2) is the value of the original signed number

1001	(original number – MSB is a 1)
(1) 0110	(flip all the bits)
(2) 0111	(add a 1 to the previous number)

0111 = 7, therefore 1001 = –7

Negative Number

4-bit Binary	Two's Complement
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	– 8
1001	– 7
1010	– 6
1011	– 5
1100	– 4
1101	– 3
1110	– 2
1111	– 1

For 4-bit unsigned number:
range is 0 to $2^4 - 1$
= 0 to 15

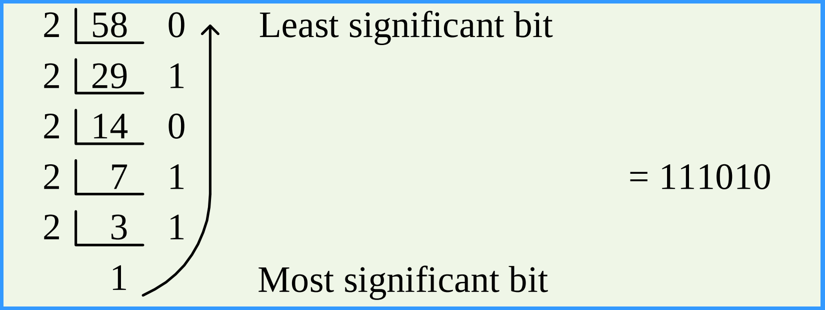
For 4-bit signed number:
range is -2^3 to $2^3 - 1$
= –8 to 7

For n-bit unsigned number:
range is 0 to $2^n - 1$

For *n*-bit signed number:
range is -2^{n-1} to $2^{n-1} - 1$

Example

- Find the two’s complement number for –58
  - Start with +58



  - Binary for +58 is 0111010. Need to add a leading 0, otherwise it is a negative number!

    0111010 = 58

    1000101 Flip all the bits

    1000110 add a 1 to the previous number

    Therefore, 1000110 = –58

Sign Extension

- For unsigned numbers, extend with 0
- For signed numbers, extend with the MSB

	Original Number	Sign Extended	Original Number	Sign Extended
	10010	11110010	0101	00000101
Flip bits	01101	00001101		
Add 1	01110	00001110		
Value	– 14	– 14	5	5

Signed Number Arithmetic

$$\begin{array}{ccccccc}
 & 0 & 0 & 1 & 1 & = & 3 \\
+ & 1 & 0_{1\nwarrow} & 0_{1\nwarrow} & 1 & = & +(-7) \\
\hline
 & 1 & 1 & 0 & 0 & = & -4
\end{array}$$

$$\begin{array}{ccccccc}
3 & = & & 0 & 0 & 1 & 1 \\
+(-3) & = & + & 1_{1\nwarrow} & 1_{1\nwarrow} & 0_{1\nwarrow} & 1 \\
\hline
0 & = & \cancel{1} & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{cccccccc}
 & 6 & = & & 0 & 1 & 1 & 0 \\
+ & 3 & = & + & 0_{1\nwarrow} & 0_{1\nwarrow} & 1 & 1 \\
\hline
 & 9 & \neq & & 1 & 0 & 0 & 1
\end{array}$$

$$\begin{array}{ccccccccc}
 & 6 & = & & 0 & 0 & 1 & 1 & 0 \\
+ & 3 & = & + & 0 & 0_{1\nwarrow} & 0_{1\nwarrow} & 1 & 1 \\
\hline
 & 9 & = & & 0 & 1 & 0 & 0 & 1
\end{array}$$

Binary Arithmetic

1. Perform the following 4-bit unsigned number addition. Is there an overflow error?

$$\begin{array}{r l} 0\ 1\ 0\ 1 & = \\ \underline{+\ 1\ 0\ 1\ 1} & = \\ & = \end{array}$$

Binary Arithmetic

1. Perform the following 4-bit unsigned number addition. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{+\ 1\ 0\ 1\ 1} & = & \underline{+11} \\ 1\ 0\ 0\ 0\ 0 & = & \end{array}$$

Binary Arithmetic

1. Perform the following 4-bit unsigned number addition. Is there an overflow error?

$$\begin{array}{r c r} 0\;1\;0\;1 & = & 5 \\ \underline{+\;\;1\;0\;1\;1} & = & \underline{+11} \\ 1\;0\;0\;0\;0 & = & 16 \end{array}$$

Yes, there is an overflow error

Binary Arithmetic

2. Perform the following 4-bit unsigned number addition. Is there an overflow error?

$$\begin{array}{rl} 0\ \ 1\ \ 0\ \ 1 & = \\ \underline{+\ \ 0\ \ 1\ \ 1\ \ 0} & = \\ & = \end{array}$$

Binary Arithmetic

2. Perform the following 4-bit unsigned number addition. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{+\ 0\ 1\ 1\ 0} & = & \underline{+\ 6} \\ 1\ 0\ 1\ 1 & = & 11 \end{array}$$

No, there is no overflow error

Binary Arithmetic

3. Perform the following 4-bit signed number addition. Is there an overflow error?

$$\begin{array}{r} 0\ 1\ 0\ 1 \\ +\ 1\ 0\ 1\ 1 \\ \hline \end{array}$$

=

=

=

Binary Arithmetic

3. Perform the following 4-bit signed number addition. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{+\ 1\ 0\ 1\ 1} & = & \underline{+(-5)} \\ 1\ 0\ 0\ 0\ 0 & = & 0 \end{array}$$

No, there is no overflow error

Binary Arithmetic

4. Perform the following 4-bit signed number addition. Is there an overflow error?

$$\begin{array}{rl} 0\ \ 1\ \ 0\ \ 1 & \quad = \\ \underline{+\ \ 0\ \ 1\ \ 1\ \ 0} & \quad = \\ & \quad = \end{array}$$

Binary Arithmetic

4. Perform the following 4-bit signed number addition. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{+\ 0\ 1\ 1\ 0} & = & \underline{+\ 6} \\ 1\ 0\ 1\ 1 & = & -5 \end{array}$$

Yes, there is an overflow error

Binary Arithmetic

5. Perform the following 4-bit unsigned number subtraction. Is there an overflow error?

$$\begin{array}{cccccccc} & 0 & 1 & 0 & 1 & & = \\ - & 0 & 1 & 1 & 0 & & = \\ \cline{1-5} & & & & & & = \end{array}$$

Binary Arithmetic

5. Perform the following 4-bit unsigned number subtraction. Is there an overflow error?

0 1 0 1	=	5
– 0 1 1 0	=	– 6
1 1 1 1	=	

Binary Arithmetic

5. Perform the following 4-bit unsigned number subtraction. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{-\ \ 0\ 1\ 1\ 0} & = & \underline{-\ \ 6} \\ 1\ 1\ 1\ 1 & = & 15 \end{array}$$

Yes, there is an overflow error

Binary Arithmetic

6. Perform the following 4-bit signed number subtraction. Is there an overflow error?

$$\begin{array}{rcl} & 0\ 1\ 0\ 1 & = \\ - & \underline{0\ 1\ 1\ 0} & = \\ & & = \end{array}$$

Binary Arithmetic

6. Perform the following 4-bit signed number subtraction. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{-\ \ 0\ 1\ 1\ 0} & = & \underline{-\ \ 6} \\ 1\ 1\ 1\ 1 & = & \end{array}$$

Binary Arithmetic

6. Perform the following 4-bit signed number subtraction. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{-\ \ 0\ 1\ 1\ 0} & = & \underline{-\ \ 6} \\ 1\ 1\ 1\ 1 & = & -1 \end{array}$$

No, there is no overflow error

Binary Arithmetic

7. Perform the following 4-bit signed number subtraction. Is there an overflow error?

$$\begin{array}{rl} 0\ 1\ 0\ 1 & \quad = \\ \underline{-\ \ 1\ 0\ 0\ 0} & \quad = \\ & \quad = \end{array}$$

Binary Arithmetic

7. Perform the following 4-bit signed number subtraction. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{-\ \ 1\ 0\ 0\ 0} & = & \underline{-(-8)} \\ 1\ 1\ 0\ 1 & = & \end{array}$$

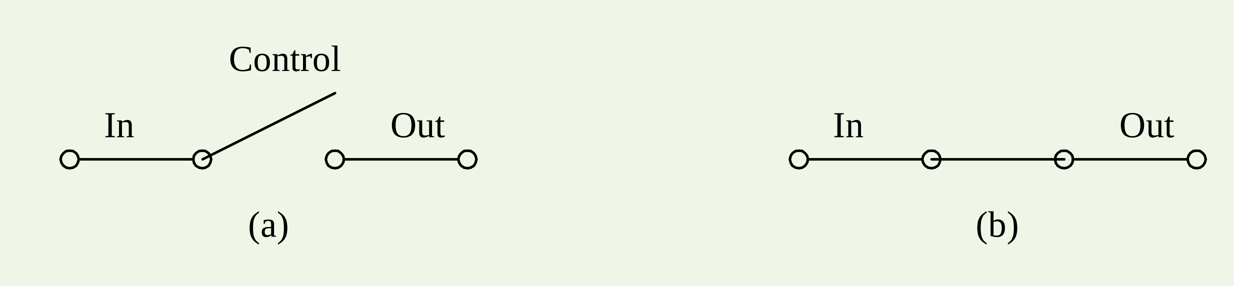
Binary Arithmetic

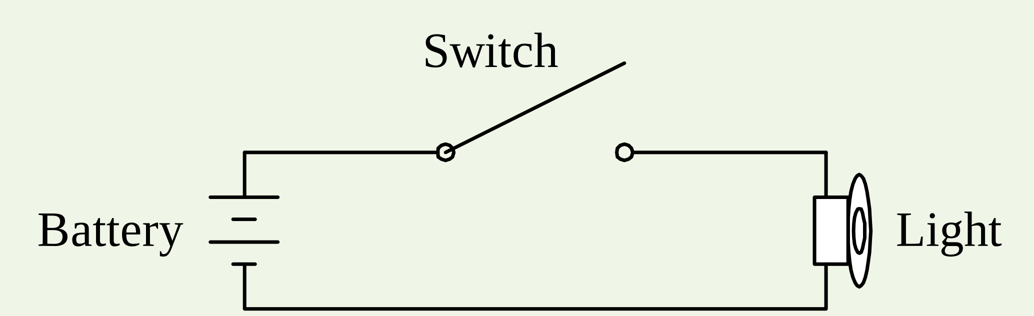
7. Perform the following 4-bit signed number subtraction. Is there an overflow error?

$$\begin{array}{rcr} 0\ 1\ 0\ 1 & = & 5 \\ \underline{-\ \ 1\ 0\ 0\ 0} & = & \underline{\ -(-8)} \\ 1\ 1\ 0\ 1 & = & -3 \end{array}$$

Yes, there is an overflow error

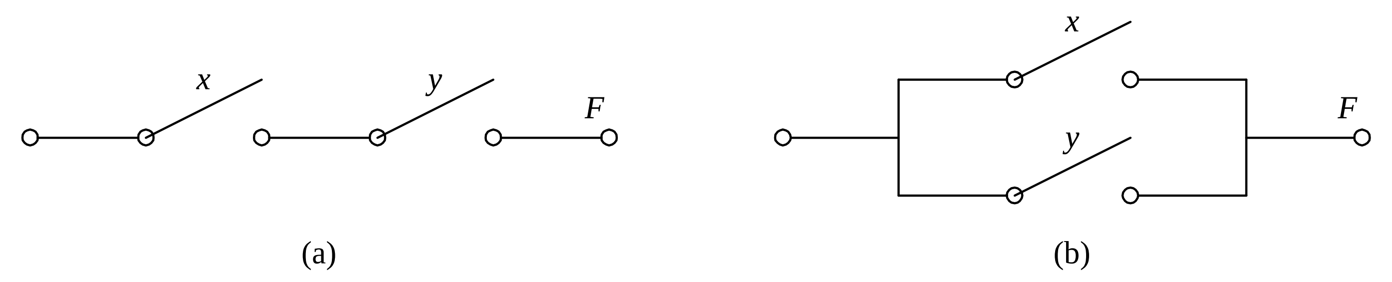
Binary Switch





L=x

Basic Logic

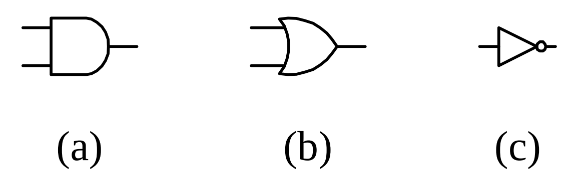


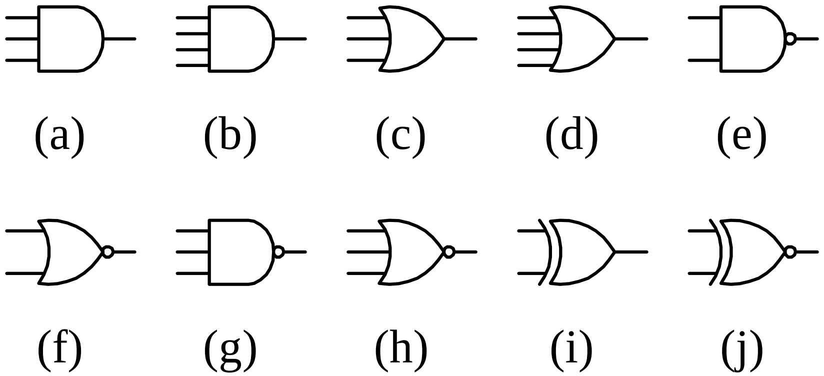
(a) (b)

- AND F = x and y $F = x \bullet y$ $F = xy$
- OR   F = x or y $F = x + y$
- NOT $F = x'$ $F = \overline{x}$
- Precedence from high to low: NOT/AND/OR
- $F = xy + z'$ $F = x(y + z)'$

Logic Gate

- **Logic gates** are the actual physical devices that implement the logical operators
- Using Logic Symbol to denote logic gates





- NAND
- NOR
- XOR
- XNOR

Logic Gate

- NAND $F = (xy)'$
- NOR $F = (x + y)'$
- XOR $F = x \oplus y = x'y + xy'$
- XNOR $F = x \odot y = x'y' + xy$

For even number of inputs xor = xnor′

$$(x \oplus y) = (x \odot y)'$$

For odd number of inputs xor = xnor

$$(x \oplus y \oplus z) = (x \odot y \odot z)$$

Truth Table

x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

x	F
0	1
1	0

		2-NAND	2-NOR	2-XOR	2-XNOR
x	y	$(x \bullet y)'$	$(x+y)'$	$x \oplus y$	$x \odot y$
0	0	1	1	0	1
0	1	1	0	1	0
1	0	1	0	1	0
1	1	0	0	0	1

Truth Table

			3-AND	3-OR	3-NAND	3-NOR	3-XOR	3-XNOR
x	y	z	$x \bullet y \bullet z$	$x + y + z$	$(x \bullet y \bullet z)'$	$(x + y + z)'$	$x \oplus y \oplus z$	$x \odot y \odot z$
0	0	0	0	0	1	1	0	0
0	0	1	0	1	1	0	1	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	0	1	1	0	0	0
1	1	0	0	1	1	0	0	0
1	1	1	1	1	0	0	1	1

NAND Gate

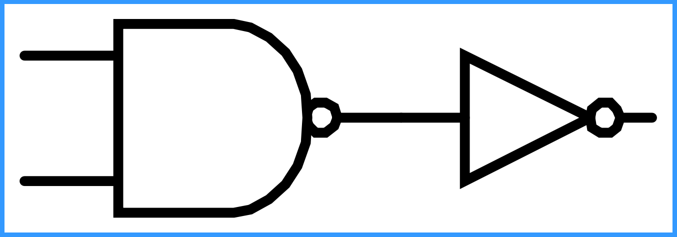
- How can you use a NAND gate to work like an AND gate?

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

NAND Gate

- How can you use a NAND gate to work like an AND gate?

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0



NAND Gate

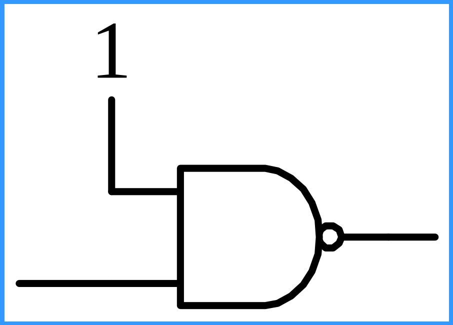
- How can you use a NAND gate to work like a NOT gate?

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

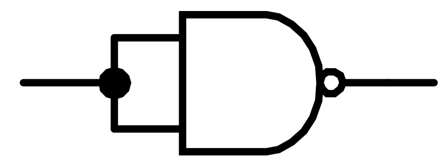
NAND Gate

- How can you use a NAND gate to work like a NOT gate?

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0



or



NAND Gate

- How can you use a NAND gate to work like an OR gate?

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

NAND Gate

- How can you use a NAND gate to work like an OR gate?

Use DeMorgan’s Theorem

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

NAND Gate

- How can you use a NAND gate to work like an OR gate?

Use DeMorgan's Theorem

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

$x+y = (x+y)''$

$=$

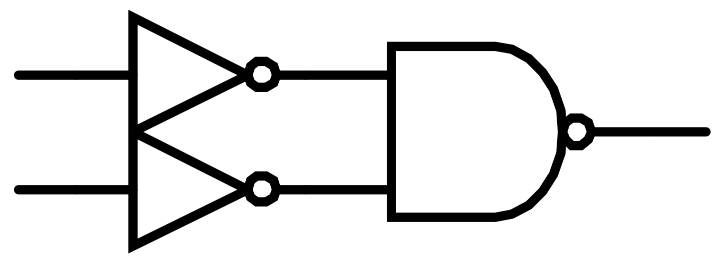
NAND Gate

- How can you use a NAND gate to work like an OR gate?

Use DeMorgan's Theorem

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

$$x + y = (x + y)''$$
$$= (x'y')'$$



XOR Gate

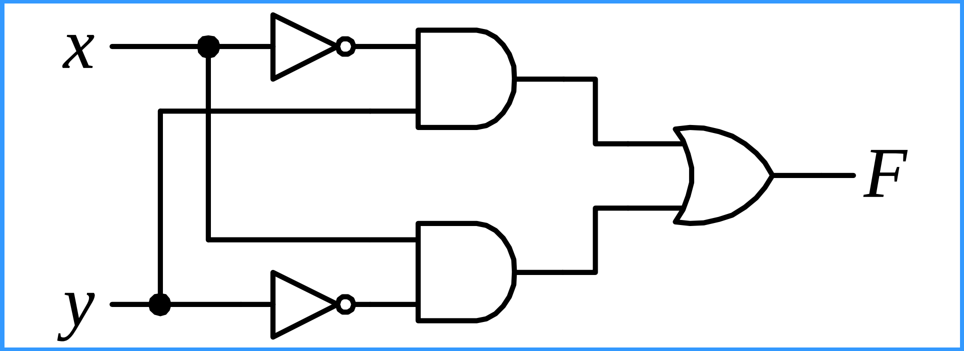
- Use AND, OR, and NOT gates to implement the XOR gate

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

XOR Gate

- Use AND, OR, and NOT gates to implement the XOR gate

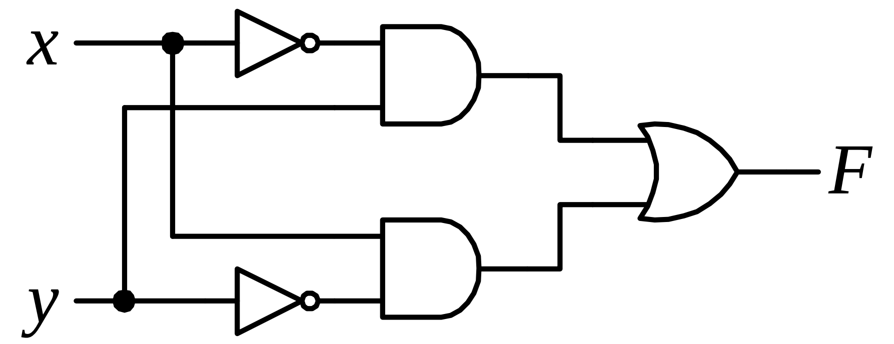
x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

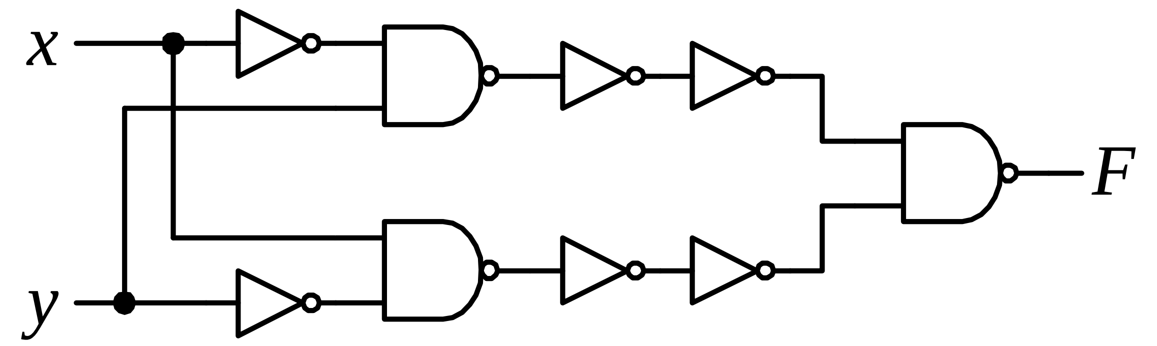


XOR Gate

- Use only NAND gates to implement the XOR gate

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

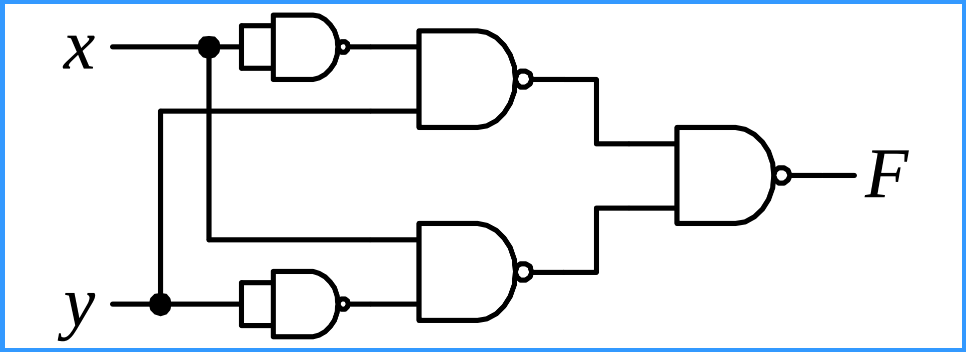




XOR Gate

- Use only NAND gates to implement the XOR gate

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0



Boolean Algebra

- Circuits built with binary switches can be described using Boolean algebra.
- Let B = {0,1} be the Boolean algebra. We have axioms, single variable theorems, and two or three variable theorems.
- Can be used to reduce circuit size.

Boolean Algebra

1a.	$0 \bullet 0 = 0$	1b.	$1 + 1 = 1$	
2a.	$1 \bullet 1 = 1$	2b.	$0 + 0 = 0$	
3a.	$0 \bullet 1 = 1 \bullet 0 = 0$	3b.	$1 + 0 = 0 + 1 = 1$	
4a.	$0' = 1$	4b.	$1' = 0$	

(a)

5a.	$x \bullet 0 = 0$	5b.	$x + 1 = 1$	Null Element
6a.	$x \bullet 1 = 1 \bullet x = x$	6b.	$x + 0 = 0 + x = x$	Identity
7a.	$x \bullet x = x$	7b.	$x + x = x$	Idempotent
8a.	$(x')' = x$			Double Complement
9a.	$x \bullet x' = 0$	9b.	$x + x' = 1$	Inverse

(b)

10a.	$x \bullet y = y \bullet x$	10b.	$x + y = y + x$	Commutative
11a.	$(x \bullet y) \bullet z = x \bullet (y \bullet z)$	11b.	$(x + y) + z = x + (y + z)$	Associative
12a.	$(x \bullet y) + (x \bullet z) = x \bullet (y + z)$	12b.	$(x + y) \bullet (x + z) = x + (y \bullet z)$	Distributive
13a.	$(x \bullet y)' = x' + y'$	13b.	$(x + y)' = x' \bullet y'$	DeMorgan’s

(c)

Reduce Logic Expression using Boolean Algebra

- Use Boolean algebra to reduce the expression $x + (x \bullet y)$ as much as possible

$$
\begin{aligned}
x + (x \bullet y) &= (x \bullet 1) + (x \bullet y) \\
&= x \bullet (1 + y) \\
&= x \bullet (1) \\
&= x
\end{aligned}
$$

Boolean Algebra

- Use Boolean algebra to reduce the equation as much as possible

$$F = (x'yz) + (xy'z) + (xyz') + (xyz)$$

Boolean Algebra

- Use Boolean algebra to reduce the equation as much as possible

$$F = (x'yz) + (xy'z) + (xyz') + (xyz)$$

$$= (x'yz) + (xy'z) + (xyz') + (xyz) + (xyz) + (xyz)$$

Boolean Algebra

- Use Boolean algebra to reduce the equation as much as possible

$$
\begin{aligned}
F &= (x'yz) + (xy'z) + (xyz') + (xyz) \\
&= (x'yz) + (xy'z) + (xyz') + (xyz) + (xyz) + (xyz) \\
&= (x'yz) + (xyz) + (xy'z) + (xyz) + (xyz') + (xyz) \\
&= [(x'yz) + (xyz)] + [(xy'z) + (xyz)] + [(xyz') + (xyz)]
\end{aligned}
$$

Boolean Algebra

- Use Boolean algebra to reduce the equation as much as possible

$$
\begin{aligned}
F &= (x'yz) + (xy'z) + (xyz') + (xyz) \\
&= (x'yz) + (xy'z) + (xyz') + (xyz) + (xyz) + (xyz) \\
&= (x'yz) + (xyz) + (xy'z) + (xyz) + (xyz') + (xyz) \\
&= [(x'yz) + (xyz)] + [(xy'z) + (xyz)] + [(xyz') + (xyz)] \\
&= yz(x' + x) + xz(y' + y) + xy(z' + z) \\
&= yz(1) + xz(1) + xy(1) \\
&= yz + xz + xy \\
&= z(y + x) + xy
\end{aligned}
$$

Duality Principle

- Dual: changing AND with OR and vice versa, Changing 0 with 1 and vice versa

$$(x \bullet y' \bullet z) + (x \bullet y \bullet z') + (y \bullet z) + 0$$

$$(x + y' + z) \bullet (x + y + z') \bullet (y + z) \bullet 1$$

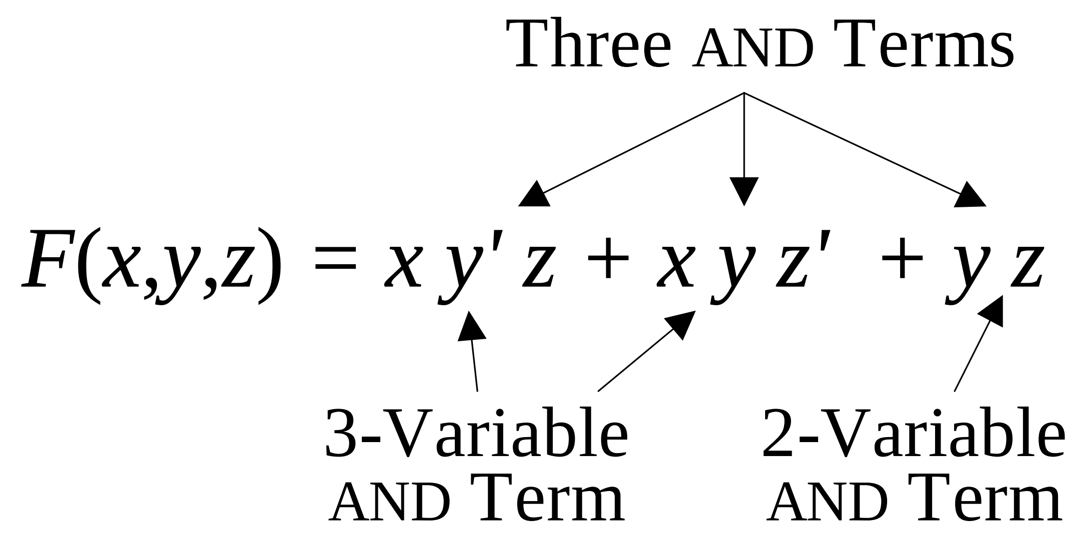
- Duality Principle: if a Boolean expression is true, then its dual is also true

$x + 1 = 1$ is true. $x \bullet 0 = 0$ is true

- The inverse of a Boolean expression can be obtained by taking the dual of that expression and then complementing each variable.

Boolean Function and Inverse

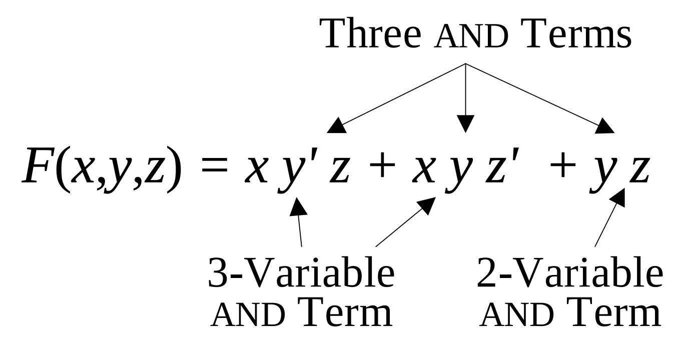
- Boolean function: logic expression to describe digital circuit.



Sum-of-product

Boolean Function and Inverse

- We are mainly interested in when a function evaluates to a 1



Three AND Terms

$$F(x,y,z) = x\,y'\,z + x\,y\,z' + y\,z$$

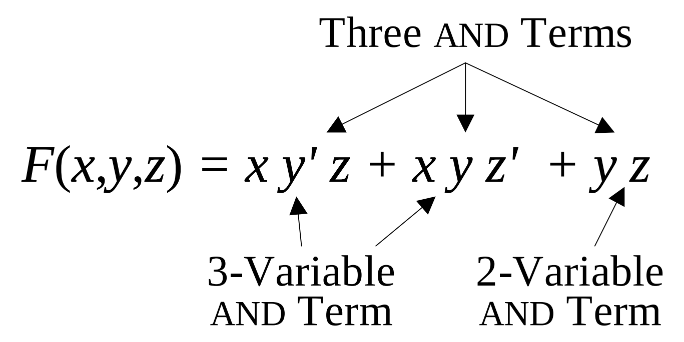
3-Variable AND Term

2-Variable AND Term

  - F = 1 when any one of the three AND terms evaluate to a 1
  - The first AND term, $xy'z$, equals 1 if

$$x = 1,\ y = 0, \text{ and } z = 1$$

Boolean Function and Inverse



Three AND Terms

$$F(x,y,z) = x\,y'\,z + x\,y\,z' + y\,z$$

3-Variable AND Term

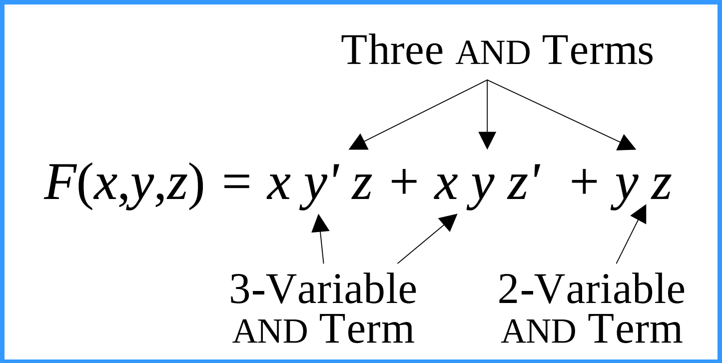
2-Variable AND Term

- The last AND term, *yz*, equals 1 if

  y = 1 and *z* = 1

  the missing variable, *x*, means it doesn't matter what its value is, so it can be either 0 or 1

Boolean Function and Inverse



Three AND Terms

$$F(x,y,z) = x\,y'\,z + x\,y\,z' + y\,z$$

3-Variable AND Term

2-Variable AND Term

- Putting everything together, F = 1 when

$x = 1$, $y = 0$, and $z = 1$

or $x = 1$, $y = 1$, and $z = 0$

or $x = 0$, $y = 1$, and $z = 1$

or $x = 1$, $y = 1$, and $z = 1$

Boolean Function and Inverse

- It is more convenient to summarize this verbal description of a function with a truth table

x	y	z	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Boolean Function and Inverse

- The inverse of a function, *F'*, can be obtained easily from the truth table

x	y	z	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

- Look at the rows where $F' = 1$
- $F' = (x'y'z') + (x'y'z) + (x'yz') + (xy'z')$

Boolean Function and Inverse

- To get F' using Boolean Algebra requires using DeMorgan’s Theorem twice

$$F = xy'z + xyz' + yz$$

$$\begin{aligned} F' &= (xy'z + xyz' + yz)' \\ &= (xy'z)' \bullet (xyz')' \bullet (yz)' \\ &= (x'+y+z') \bullet (x'+y'+z) \bullet (y'+z') \end{aligned}$$

Boolean Function and Inverse

- We have two different equations for F'

$$F' = (x'y'z') + (x'y'z) + (x'yz') + (xy'z')$$

sum-of-products

$$F' = (x'+y+z') \bullet (x'+y'+z) \bullet (y'+z')$$

product-of-sums

Minterms and Maxterms

- Minterm
  - Is a product term that contains all the variables in a function
  - The variable is negated (primed) if the value is a 0
- Maxterm
  - Is a sum term that contains all the variables in a function
  - The variable is negated (primed) if the value is a 1

Minterms and Maxterms

- m_i for minterms
- M_i for maxterms where $0 \leq i < 2^n$ for n variables

x	y	z	Minterm	Notation		Maxterm	Notation
0	0	0	x' y' z'	m_0		x + y + z	M_0
0	0	1	x' y' z	m_1		x + y + z'	M_1
0	1	0	x' y z'	m_2		x + y' + z	M_2
0	1	1	x' y z	m_3		x + y' + z'	M_3
1	0	0	x y' z'	m_4		x' + y + z	M_4
1	0	1	x y' z	m_5		x' + y + z'	M_5
1	1	0	x y z'	m_6		x' + y' + z	M_6
1	1	1	x y z	m_7		x' + y' + z'	M_7

Minterm/Maxterm Example

- $F = xy'z + xyz' + yz$
  $= x'yz + xy'z + xyz' + xyz$

x	y	z	F	F'	Minterm	Notation
0	0	0	0	1	x' y' z'	m_0
0	0	1	0	1	x' y' z	m_1
0	1	0	0	1	x' y z'	m_2
0	1	1	1	0	x' y z	m_3
1	0	0	0	1	x y' z'	m_4
1	0	1	1	0	x y' z	m_5
1	1	0	1	0	x y z'	m_6
1	1	1	1	0	x y z	m_7

$$F(x, y, z) = m_3 + m_5 + m_6 + m_7$$

$$F(x, y, z) = \Sigma(3, 5, 6, 7)$$

$$F'(x, y, z) = \Sigma(0, 1, 2, 4)$$

- $F = xy'z + xyz' + yz$
  $= (x + y + z) \bullet (x + y + z') \bullet (x + y' + z) \bullet (x' + y + z)$

x	y	z	F	F'	Maxterm	Notation
0	0	0	0	1	$x + y + z$	M_0
0	0	1	0	1	$x + y + z'$	M_1
0	1	0	0	1	$x + y' + z$	M_2
0	1	1	1	0	$x + y' + z'$	M_3
1	0	0	0	1	$x' + y + z$	M_4
1	0	1	1	0	$x' + y + z'$	M_5
1	1	0	1	0	$x' + y' + z$	M_6
1	1	1	1	0	$x' + y' + z'$	M_7

$$F(x, y, z) = M_0 \bullet M_1 \bullet M_2 \bullet M_4$$

$$F(x, y, z) = \Pi(0, 1, 2, 4)$$

$$F'(x, y, z) = \Pi(3, 5, 6, 7)$$

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- Write out the full function

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- Write out the full function

$$F(x, y, z) = x'y'z + x'yz' + x'yz + x'yz' + xyz' + xyz$$

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F using Π?

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F using Π?
- $F(x, y, z) = \Pi(0, 4)$

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F using Π?
- $F(x, y, z) = \Pi(0, 4)$
- Write out the full function for $\Pi(0, 4)$

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F using Π?
- $F(x, y, z) = \Pi(0, 4)$
- Write out the full function for $\Pi(0, 4)$
- $F(x, y, z) = (x + y + z) \bullet (x' + y + z)$

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F' using Σ?
- $F' = \Sigma(0, 4)$

Minterm/Maxterm Example

- Given $F(x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F' using Π?
- $F' = \Pi(1, 2, 3, 5, 6, 7)$

Minterm/Maxterm Example

- Given $F(w, x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- Write out the full function

Minterm/Maxterm Example

- Given $F(w, x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F using Π?

Minterm/Maxterm Example

- Given $F(w, x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F using Π?
- Write out the full function for Π of 0-Maxterms

Minterm/Maxterm Example

- Given $F(w, x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F' using Σ?

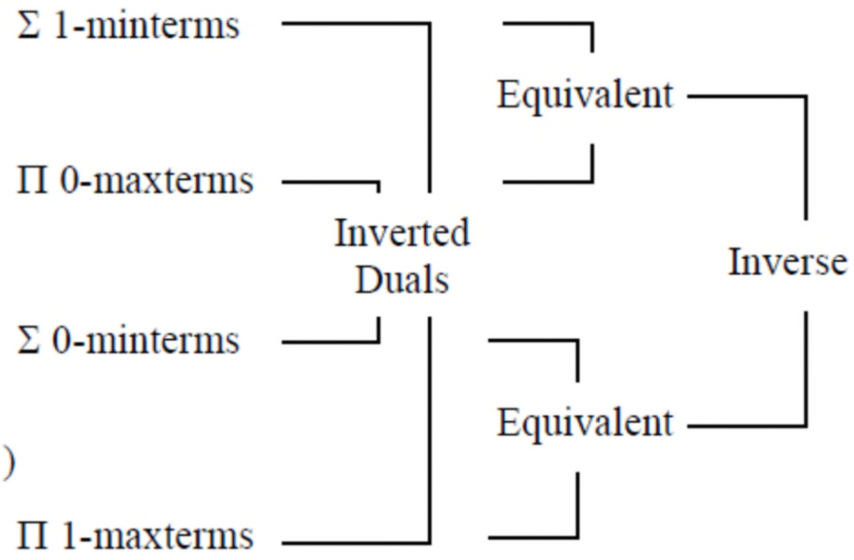
Minterm/Maxterm Example

- Given $F(w, x, y, z) = \Sigma(1, 2, 3, 5, 6, 7)$
- What is F' using Π?

Minterms/Maxterms Relationship

$$
\begin{aligned}
F(x, y, z) &= x'yz + xy'z + xyz' + xyz \\
&= m_3 + m_5 + m_6 + m_7 \\
&= \Sigma(3, 5, 6, 7) \\
&= (x+y+z) \bullet (x+y+z') \bullet (x+y'+z) \bullet (x'+y+z) \\
&= M_0 \bullet M_1 \bullet M_2 \bullet M_4 \\
&= \Pi(0, 1, 2, 4)
\end{aligned}
$$

$$
\begin{aligned}
F'(x, y, z) &= x'y'z' + x'y'z + x'yz' + xy'z' \\
&= m_0 + m_1 + m_2 + m_4 \\
&= \Sigma(0, 1, 2, 4) \\
&= (x+y'+z') \bullet (x'+y+z') \bullet (x'+y'+z) \bullet (x'+y'+z') \\
&= M_3 \bullet M_5 \bullet M_6 \bullet M_7 \\
&= \Pi(3, 5, 6, 7)
\end{aligned}
$$


Σ 1-minterms

Π 0-maxterms

Σ 0-minterms

Π 1-maxterms

Inverted Duals

Equivalent

Equivalent

Inverse

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Write it in the Σ of minterms format and Π of maxterms format
- Use Truth Table

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Use Truth Table

x	y	z	F	Minterm	Notation
0	0	0		x' y' z'	m_0
0	0	1		x' y' z	m_1
0	1	0		x' y z'	m_2
0	1	1		x' y z	m_3
1	0	0		x y' z'	m_4
1	0	1		x y' z	m_5
1	1	0		x y z'	m_6
1	1	1		x y z	m_7

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Use Truth Table

x	y	z	F	Minterm	Notation
0	0	0	0	x' y' z'	m_0
0	0	1	1	x' y' z	m_1
0	1	0	1	x' y z'	m_2
0	1	1	1	x' y z	m_3
1	0	0	0	x y' z'	m_4
1	0	1	0	x y' z	m_5
1	1	0	1	x y z'	m_6
1	1	1	1	x y z	m_7

$$F = \Sigma(1, 2, 3, 6, 7) \text{ and } F = \Pi(0, 4, 5)$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Write it in the Σ of minterms format
- Use Boolean algebra

$$F = y + x'z$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Write it in the Σ of minterms format
- Use Boolean algebra

$$
\begin{aligned}
F &= y + x'z \\
&= y(x+x')(z+z') + x'z(y+y') \\
&= xyz + xyz' + x'yz + x'yz' + \cancel{x'yz} + x'y'z \\
&= m_7 + m_6 + m_3 + m_2 + m_1 \\
&= \Sigma(1, 2, 3, 6, 7)
\end{aligned}
$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Write it in the Π of maxterms format
- Use Boolean algebra

$$F = y + x'z$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Write it in the Π of maxterms format
- Use Boolean algebra

$$
\begin{aligned}
F &= y + x'z \\
&= (y+x')(y+z) \\
&= (y+x'+zz')(y+z+xx') \\
&= (x'+y+z)\,(x'+y+z')\,(x+y+z)\,\cancel{(x'+y+z)} \\
&= M_4 \bullet M_5 \bullet M_0 = \Pi(0, 4, 5)
\end{aligned}
$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Write *F'* in the Σ of minterms format
- Use Boolean algebra

$$F' = (y + x'z)'$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$

$$
\begin{aligned}
F' &= (y + x'z)' \\
&= y' \bullet (x'z)' \\
&= y' \bullet (x+z') \\
&= y'x + y'z' \\
&= y'x(z+z') + y'z'(x+x') \\
&= xy'z + xy'z' + \cancel{xy'z'} + x'y'z' \\
&= m_5 + m_4 + m_0 \\
&= \Sigma\,(0, 4, 5)
\end{aligned}
$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$
- Write F' in the Π of maxterms format
- Use Boolean algebra

$$F' = (y + x'z)'$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$

$$
\begin{aligned}
F' &= (y + x'z)' \\
&= y' \bullet (x'z)' \\
&= y' \bullet (x+z') \\
&= (y' + xx' + zz') \bullet (x+z' + yy') \\
&= (x+y'+z)\,(x+y'+z')\,(x'+y'+z)\,(x'+y'+z') \\
&\qquad (x+y+z')\,\cancel{(x+y'+z')} \\
&= M_2 \bullet M_3 \bullet M_6 \bullet M_7 \bullet M_1 \\
&= \Pi(1, 2, 3, 6, 7)
\end{aligned}
$$

Converting to Minterms/Maxterms

- Given $F(x, y, z) = y + x'z$

$$F = \Sigma(1, 2, 3, 6, 7) = \Pi(0, 4, 5)$$
$$F' = \Sigma(0, 4, 5) = \Pi(1, 2, 3, 6, 7)$$

Canonical, Standard, and Non-standard Forms

- Canonical: Boolean function expressed in sum-of-minterms or product-of-maxterms

$$F = x'yz + xy'z + xyz' + xyz$$

$$F' = (x+y'+z') \bullet (x'+y+z') \bullet (x'+y'+z) \bullet (x'+y'+z')$$

$$F_1(x, y, z) = \Sigma(0, 1, 2, 3, 4, 5) \qquad F_2(x, y, z) = \Pi(6, 7)$$

$$F_1(x, y, z) = \Sigma(3, 5, 6) \qquad F_2(x, y, z) = \Pi(3, 5, 6)$$

- Standard: sum-of-products or products-of-sum has at least one minterm/maxterm

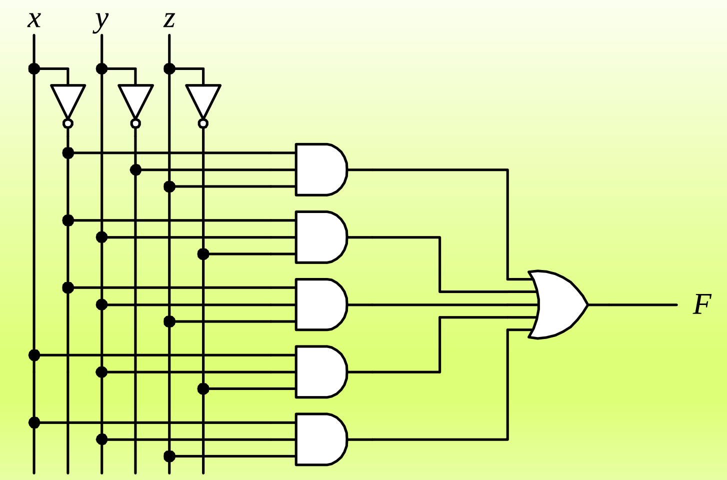
$$F = xy'z + xyz' + yz$$

- Non-standard: not in sum-of-product or product-of-sum format

$$F = x(y'z + yz') + yz$$

Digital Circuit

- Digital circuit is a connection of two or more logic gates
- Digital network can be described using schematic diagrams, Boolean expressions, or truth tables



x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$F(x, y, z) = x'y'z + x'yz' + x'yz + xyz' + xyz$

Design a Car Security System

- Input: D = Door switch

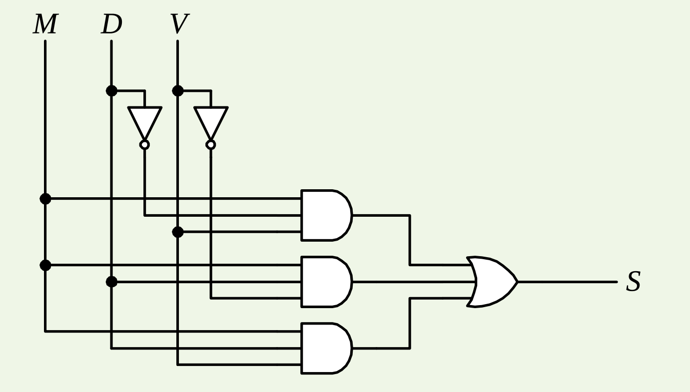
  V = Vibration sensor

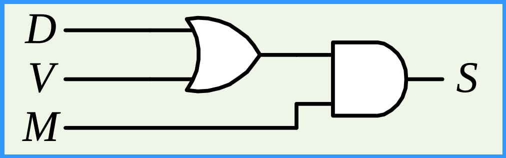
  M = Motion sensor

  Output: S = Siren

S= (M D ‘ V) + (M D V') + (M D V)
= M (D' V + D V' + D V)
= M (D' V + D V' + D V + D V)
= M (D(V' + V) + V(D' + D))
= M (D(1) + V(1))
= M (D + V)

M	D	V	S
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1





Verilog Code for Car System

```
// this is a Verilog dataflow model of the car security system
module Siren (
            input M,
            input D,
            input V,
            output S
);

            wire term1, term2, term3;

            assign term1 = (M & !D & V);
            assign term2 = (M & D & !V);
            assign term3 = (M & D & V);
            assign S = term1 | term2 | term3;

endmodule
```

Verilog Code for Car System

```
// this is a Verilog dataflow model of the car security system
module Siren (
            input M,
            input D,
            input V,
            output S
);

            assign S = (M & !D & V) | (M & D & !V) | (M & D & V);

endmodule
```

Verilog Code for Car System

```
// this is a Verilog dataflow model of the car security system
module Siren (
        input M,
        input D,
        input V,
        output S
);

        assign S = M & (D | V);

endmodule
```

Verilog Code for Car System

```
// this is a Verilog structural model of the car security system
module Siren (
            input M,
            input D,
            input V,
            output S
);

            wire w1;

            or (w1, D, V);
            and (S, M, w1);

endmodule
```

VHDL Code for Car System

```
// this is a VHDL dataflow model of the car security system
LIBRARY IEEE;
USE IEEE.STD_LOGIC_1164.ALL;
ENTITY Siren IS PORT (
        M:      IN STD_LOGIC;
        D:      IN STD_LOGIC;
        V:      IN STD_LOGIC;
        S:      OUT STD_LOGIC);
END Siren;
ARCHITECTURE Dataflow OF Siren IS
        SIGNAL term_1, term_2, term_3: STD_LOGIC;
BEGIN
        term_1 <= M AND (NOT D) AND V;
        term_2 <= M AND D AND (NOT V);
        term_3 <= M AND D AND V;
        S <= term_1 OR term_2 OR term_3;
END Dataflow;
```

VHDL Code for Car System

```
// this is a VHDL dataflow model of the car security system
LIBRARY IEEE;
USE IEEE.STD_LOGIC_1164.ALL;
ENTITY Siren IS PORT (
        M:      IN STD_LOGIC;
        D:      IN STD_LOGIC;
        V:      IN STD_LOGIC;
        S:      OUT STD_LOGIC);
END Siren;
ARCHITECTURE Dataflow OF Siren IS
BEGIN
        S <= M AND (D OR V);
END Dataflow;
```