

## Chapter 3: Interest Rate and Economic Equivalence

### Types of Interest

3.1

$$\$20,000 = \$10,000(1 + 0.075N)$$

- Simple interest:  $(1 + 0.075N) = 2$

$$N = \frac{1}{0.075} = 13.33 \approx 14 \text{ years}$$

- Compound interest:

$$\$20,000 = \$10,000(1 + 0.07)^N$$

$$(1 + 0.07)^N = 2$$

$$N = 10.24 \approx 11 \text{ years}$$

3.2

- Simple interest:

$$I = iPN = (0.06)(\$5,000)(5) = \$1,500$$

- Compound interest:

$$I = P[(1 + i)^N - 1] = \$5,000(1.3382 - 1) = \$1,691$$

3.3

- Option 1: Compound interest with 8%:

$$F = \$3,000(1 + 0.08)^5 = \$3,000(1.4693) = \$4,408$$

- Option 2: Simple interest with 9%:

$$\$3,000(1 + 0.09 \times 5) = \$3,000(1.45) = \$4,350$$

$\therefore$  Option 1 is better.

3.4

End of Year	Principal Repayment	Interest payment	Remaining Balance
0			\$10,000
1	\$1,638	\$1,000	\$8,362
2	\$1,802	\$836	\$6,560
3	\$1,982	\$656	\$4,578
4	\$2,180	\$458	\$2,398
5	\$2,398	\$240	\$0

### Equivalence Concept

3.5

$$P = \$18,000(P / F, 5\%, 5) = \$18,000(0.7835) = \$14,103$$

3.6

$$F = \$25,000(F / P, 8\%, 3) = \$25,000(1.2597) = \$31,493$$

3.7

$$F = \$100(F / P, 10\%, 10) + \$200(F / P, 10\%, 8) = \$688$$

3.8

$$\$1,000(F / P, i, 2) = \$1,200$$

$$\$1,000(1 + i)^2 = \$1,200$$

$$i = \sqrt{1.2} - 1$$

$$i = 9.54\%$$

### Single Payments (Use of $F/P$ or $P/F$ Factors)

3.9

$$F = \$180,000(F / P, 6\%, 10) = \$322,353$$

3.10

$$(a) F = \$7,000(F / P, 6\%, 5) = \$9,368$$

$$(b) F = \$3,250(F / P, 5\%, 15) = \$6,757$$

$$(c) F = \$18,000(F / P, 8\%, 33) = \$228,169$$

$$(d) F = \$20,000(F / P, 9\%, 8) = \$39,851$$

3.11

$$P = \$300,000(P / F, 8\%, 10) = \$138,958$$

3.12

$$(a) \quad P = \$15,500(P / F, 14\%, 8) = \$5,434$$

$$(b) \quad P = \$18,000(P / F, 4\%, 12) = \$11,243$$

$$(c) \quad P = \$20,000(P / F, 8\%, 9) = \$10,005$$

$$(d) \quad P = \$55,000(P / F, 11\%, 4) = \$36,230$$

3.13

$$(a) \quad P = \$12,000(P / F, 13\%, 4) = \$7,360$$

$$(b) \quad F = \$30,000(F / P, 13\%, 5) = \$55,273$$

3.14

$$F = 3P = P(1 + 0.06)^N$$

$$\log 3 = N \log(1.06)$$

$$N = 18.85 \approx 19 \text{ years}$$

3.15

$$F = 2P = P(1 + 0.08)^N$$

- $\log 2 = N \log(1.08)$

$$N = 9 \text{ years}$$

- Rule of 72:  $72 / 8 = 9 \text{ years}$

3.16

(a) Single-payment compound amount  $(F / P, i, N)$  factors for

$N$	9%	10%
35	20.4140	28.1024
40	31.4094	45.2593

To find  $(F / P, 9.5\%, 38)$ , first, interpolate for  $n = 38$ :

$N$	9%	10%
38	27.0112	38.3965

Then, interpolate for  $i = 9.5\%$  :

$$(F/P, 9.5\%, 38) = 32.7039$$

As compared to formula determination

$$(F/P, 9.5\%, 38) = 31.4584$$

(b) Single-payment compound amount  $(P/F, 8\%, N)$  factors for

$N$	45	50
	0.0313	0.0213

Then, interpolate for  $N = 47$

$$(P/F, 8\%, 47) = 0.0273$$

As compared to the value from the interest formula:

$$(P/F, 8\%, 47) = 0.0269$$

3.17

$$(a) \quad \$18(1+i)^{44} = \$92,400$$

$$i = 21.43\%$$

$$(b) \quad F = \$97.8(F/P, 21.43\%, 22) = \$7,007 \text{ billion}$$

## Uneven Payment Series

3.18

$$\$1,000 + \frac{\$1,000}{1.1} + \frac{\$1,500}{1.1^3} = \frac{\$1,210}{1.1^2} + \frac{X}{1.1^4}$$

$$X = \$2,981$$

3.19

$$P = \frac{\$25,000}{1.07^2} + \frac{\$33,000}{1.07^3} + \frac{\$46,000}{1.07^4} + \frac{\$38,000}{1.07^5} = \$110,961$$

3.20

$$F = \$2,000(F/P, 6\%, 10) + \$2,500(F/P, 6\%, 8) + \$3,000(F/P, 6\%, 6) = \$11,822$$

3.21

$$\begin{aligned}
 P &= \$3,000,000 + \$2,400,000(P/F, 8\%, 1) + \cdots \\
 &\quad + \$3,000,000(P/F, 8\%, 10) \\
 &= \$20,734,618
 \end{aligned}$$

Or,

$$\begin{aligned}
 P &= \$3,000,000 + \$2,400,000(P/A, 8\%, 5) \\
 &\quad + \$3,000,000(P/A, 8\%, 5)(P/F, 8\%, 5) \\
 &= \$20,734,618
 \end{aligned}$$

3.22

$$P = \$8,000(P/F, 6\%, 2) + \$6,000(P/F, 6\%, 5) + \$4,000(P/F, 6\%, 7) = \$14,264$$

## Equal Payment Series

3.23

(a) With deposits made at the end of each year

$$F = \$2,000(F/A, 8\%, 15) = \$54,304$$

(b) With deposits made at the beginning of each year

$$F = \$2,000(F/A, 8\%, 15)(1.08) = \$58,649$$

3.24

$$F = \$10,000(F/A, 6\%, 20) = \$367,856$$

3.25

$$(a) \quad F = \$6,000(F/A, 8\%, 5) = \$35,200$$

$$(b) \quad F = \$4,000(F/A, 6.25\%, 12) = \$68,473$$

$$(c) \quad F = \$9,000(F/A, 9.45\%, 20) = \$484,359$$

$$(d) \quad F = \$3,000(F/A, 11.75\%, 12) = \$71,308$$

3.26

$$(a) \quad A = \$32,000(A/F, 8\%, 15) = \$1,179$$

$$(b) \quad A = \$55,000(A/F, 6\%, 10) = \$4,173$$

$$(c) \quad A = \$35,000(A / F, 7\%, 20) = \$853.8$$

$$(d) \quad A = \$8,000(A / F, 11\%, 4) = \$1,699$$

3.27

$$\$50,000(A / F, 6\%, 10) = \$3,793.40$$

3.28

$$\$35,000 = \$2,000(F / A, 6\%, N)$$

$$(F / A, 6\%, N) = 17.5$$

$$N = 12.32 \approx 13 \text{ years}$$

3.29

$$\$15,000 = A(F / A, 11\%, 5)$$

$$A = \$2,408.57$$

3.30

$$\$5,000 = \$500(F / P, 7\%, 5) + A(F / A, 7\%, 5)$$

$$A = \$747.51$$

3.31

$$(a) \quad A = \$12,000(A / P, 4\%, 6) = \$2,289.14$$

$$(b) \quad A = \$3,500(A / P, 6.7\%, 7) = \$642.66$$

$$(c) \quad A = \$6,500(A / P, 3.5\%, 5) = \$1,439.63$$

$$(d) \quad A = \$32,000(A / P, 8.5\%, 15) = \$3,853.47$$

3.32

(a) The capital recovery factor  $(A / P, i, N)$  for

$N$	6%	7%
35	0.0690	0.0772
40	0.0665	0.0750

To find  $(A / P, 6.25\%, 38)$ , first, interpolate for  $N = 38$ :

$N$	6%	7%
38	0.0675	0.0759

Then, interpolate for  $i = 6.25\%$  ;

$$(A / P, 6.25\%, 38) = 0.0696 :$$

As compared to the value from the interest formula:

$$(A / P, 6.25\%, 38) = 0.0694$$

(b) The equal payment series present-worth factor  $(P / A, i, 85)$  for

$i$	9%	10%
	11.1038	9.9970

Then, interpolate for  $i = 9.25\%$  :

$$(P / A, 9.25\%, 85) = 10.8271$$

As compared to the value from the interest formula:

$$(P / A, 9.25\%, 85) = 10.8049$$

3.33

- Equal annual payment:

$$A = \$50,000(A / P, 12\%, 3) = \$20,817.45$$

- Interest payment for the second year:

End of Year	Principal Repayment	Interest payment	Remaining Balance
0			\$50,000
1	\$14,817.45	\$6,000	\$35,182.55
2	\$16,595.54	\$4,221.91	\$18,587.01
3	\$18,587.01	\$2,230.44	0

3.34

$$A = \$10,000(A / P, 9\%, 10) = \$1,558.2$$

3.35

(a)  $P = \$1,000(P / A, 6.8\%, 8) = \$6,017.86$

(b)  $P = \$3,500(P / A, 9.5\%, 12) = \$24,443.44$

(c)  $P = \$1,900(P / A, 8.25\%, 9) = \$11,746.68$

$$(d) \quad P = \$9,300(P / A, 7.75\%, 5) = \$37,378.16$$

3.36

$$P = \$35,000(P / A, 12\%, 10) = \$197,758$$

Since  $\$200,000 > \$197,758$ , You should not purchase the equipment.

3.37

(a)

$$\begin{aligned} P &= \$3,875,000 + \$3,125,000(P / F, 6\%, 1) + \dots + \$8,875,000(P / F, 6\%, 7) \\ &= \$39,547,241.99 \end{aligned}$$

(b)

$$P = \$1,375,000 + \$1,375,000(P / A, 6\%, 7) = \$9,050,774.48$$

Since  $\$9,050,774.48 > \$8,000,000$ , the prorated payment option is better choice.

## Linear Gradient Series

3.38

$$\begin{aligned} F &= \$10,000(F / A, 8\%, 5) + \$3,000(F / G, 8\%, 5) \\ &= \$10,000(F / A, 8\%, 5) + \$3,000(A / G, 8\%, 5)(F / A, 8\%, 5) \\ &= \$91,163.55 \end{aligned}$$

3.39

$$\begin{aligned} F &= \$7,500(F / A, 8\%, 5) - \$1,500(F / G, 8\%, 5) \\ &= \$7,500(F / A, 8\%, 5) - \$1,500(P / G, 8\%, 5)(F / P, 8\%, 5) \\ &= \$27,750.74 \end{aligned}$$

3.40

$$\begin{aligned} P &= \$100 + [\$100(F / A, 9\%, 7) + \$50(F / A, 9\%, 6) + \$50(F / A, 9\%, 4) \\ &\quad + \$50(F / A, 9\%, 2)](P / F, 9\%, 7) \\ &= \$991.32 \end{aligned}$$

3.41

$$\begin{aligned} A &= \$15,000 - \$1,000(A / G, 8\%, 12) \\ &= \$10,404.25 \end{aligned}$$



3.42

$$P = \$1,000(P / A, 6\%, 5) + \$250(P / G, 6\%, 5) \\ = \$6,196$$

3.43

Using the geometric gradient series present worth factor, we can establish the equivalence between the loan amount \$120,000 and the balloon payment series as

$$\$120,000 = A_1 (P / A_1, 10\%, 9\%, 5) = 4.6721A_1$$

$$A_1 = \$25,684.38$$

Payment series

$N$	Payment
1	\$25,684.38
2	\$28,252.82
3	\$31,078.10
4	\$34,185.91
5	\$37,604.51

3.44

$$F = \$6,000(P / A_1, 5\%, 7\%, 30)(F / P, 7\%, 30) \\ = \$987,093.8$$

3.45

$$(a) \quad P = \$6,000,000(P / A_1, -10\%, 12\%, 7) = \$21,372,076$$

- (b) Note that the oil price increases at the annual rate of 5% while the oil production decreases at the annual rate of 10%. Therefore, the annual revenue can be expressed as follows:

$$A_n = \$60(1 + 0.05)^{n-1}100,000(1 - 0.1)^{n-1} \\ = \$6,000,000(0.945)^{n-1} \\ = \$6,000,000(1 - 0.055)^{n-1}$$

This revenue series is equivalent to a decreasing geometric gradient series with  $g = -5.5\%$ . So,

$$P = \$6,000,000(P / A_1, -5.5\%, 12\%, 7) = \$23,847,897$$

- (c) Computing the present worth of the remaining series ( $A_4, A_5, A_6, A_7$ ) at the end of period 3 gives

$$A_4 = 6,000,000(1 - 0.055)^3 = 5,063,451.75$$

$$P = \$5,063,451.75(P / A_4, -5.5\%, 12\%, 4) = \$14,269,627.82$$

3.46

$$\begin{aligned} P &= \sum_{n=1}^{20} A_n(1+i)^{-n} \\ &= \sum_{n=1}^{20} (2,000,000)n(1.06)^{n-1}(1.06)^{-n} \\ &= (2,000,000/1.06) \sum_{n=1}^{20} n \left( \frac{1.06}{1.06} \right)^n \\ &= \$396,226,415 \end{aligned}$$

3.47

- (a) The withdrawal series would be

Period	Withdrawal
11	\$12,000
12	\$12,000(1.08)
13	\$12,000(1.08)(1.08)
14	\$12,000(1.08)(1.08)(1.08)
15	\$12,000(1.08)(1.08)(1.08)(1.08)

$$P_{10} = \$12,000(P / A_1, 8\%, 12\%, 5) = \$49,879.14$$

Assuming that each deposit is made at the end of each year, then:

$$\$49,879.14 = A(F / A, 12\%, 10)$$

$$A = \$2,842.32$$

- (b)  $P_{10} = \$12,000(P / A_1, 8\%, 9\%, 5) = \$54,045.08$

$$\$54,045.08 = A(F / A, 9\%, 10)$$

$$A = \$3,557.25$$

## Various Interest Factor Relationships

3.48

$$(a) \quad (P/F, 8\%, 67) = (P/F, 8\%, 50)(P/F, 8\%, 17) = 0.0058$$

$$(P/F, 8\%, 67) = (1 + 0.08)^{-67} = 0.0058$$

$$(b) \quad (A/P, i, N) = \frac{i}{1 - (P/F, i, N)}$$

$$(P/F, 8\%, 42) = (P/F, 8\%, 40)(P/F, 8\%, 2) = 0.0394$$

$$(A/P, 8\%, 42) = \frac{0.08}{1 - 0.0394} = 0.0833$$

$$(A/P, 8\%, 42) = \frac{0.08(1.08)^{42}}{(1.08)^{42} - 1} = 0.0833$$

$$(c) \quad (P/A, i, N) = \frac{1 - (P/F, i, N)}{i} = \frac{1 - (P/F, 8\%, 100)(P/F, 8\%, 35)}{0.08} = 12.4996$$

$$(P/A, 8\%, 135) = \frac{(1.08)^{135} - 1}{0.08(1.08)^{135}} = 12.4996$$

3.49

(a)

$$(F/P, i, N) = i(F/A, i, N) + 1$$

$$(1+i)^N = i \frac{(1+i)^N - 1}{i} + 1$$

$$= (1+i)^N - 1 + 1$$

$$= (1+i)^N$$

(b)

$$(P/F, i, N) = 1 - (P/A, i, N)i$$

$$(1+i)^{-N} = 1 - i \frac{(1+i)^N - 1}{i(1+i)^N}$$

$$= \frac{(1+i)^N}{(1+i)^N} - \frac{(1+i)^N - 1}{(1+i)^N}$$

$$= (1+i)^{-N}$$

(c)

$$(A/F, i, N) = (A/P, i, N) - i$$

$$\begin{aligned}\frac{i}{(1+i)^N - 1} &= \frac{i(1+i)^N}{(1+i)^N - 1} - i = \frac{i(1+i)^N}{(1+i)^N - 1} - \frac{i[(1+i)^N - 1]}{(1+i)^N - 1} \\ &= \frac{i}{(1+i)^N - 1}\end{aligned}$$

(d)

$$\begin{aligned}(A/P, i, N) &= \frac{i}{[1 - (P/F, i, N)]} \\ \frac{i(1+i)^N}{(1+i)^N - 1} &= \frac{i}{\frac{(1+i)^N}{(1+i)^N} - \frac{1}{(1+i)^N}} \\ &= \frac{i(1+i)^N}{(1+i)^N - 1}\end{aligned}$$

(e) , (f) , (g) Divide the numerator and denominator by  $(1+i)^N$  and take the limit  $N \rightarrow \infty$ .

## Equivalence Calculations

3.50

$$\begin{aligned}P &= [\$100(F/A, 12\%, 9) + \$50(F/A, 12\%, 7) + \$50(F/A, 12\%, 5)](P/F, 12\%, 10) \\ &= \$740.49\end{aligned}$$

3.51

$$\begin{aligned}P(1.08) + \$200 &= \$200(P/F, 8\%, 1) + \$120(P/F, 8\%, 2) + \$120(P/F, 8\%, 3) \\ &\quad + \$300(P/F, 8\%, 4) \\ P &= \$373.92\end{aligned}$$

3.52

$$\begin{aligned}A(P/A, 13\%, 5) &= \$100(P/A, 13\%, 5) + \$20(P/A, 13\%, 3)(P/F, 13\%, 2) = \\ \$351.72 + \$36.98 &= (3.5172)A \\ A &= \$110.51\end{aligned}$$

3.53

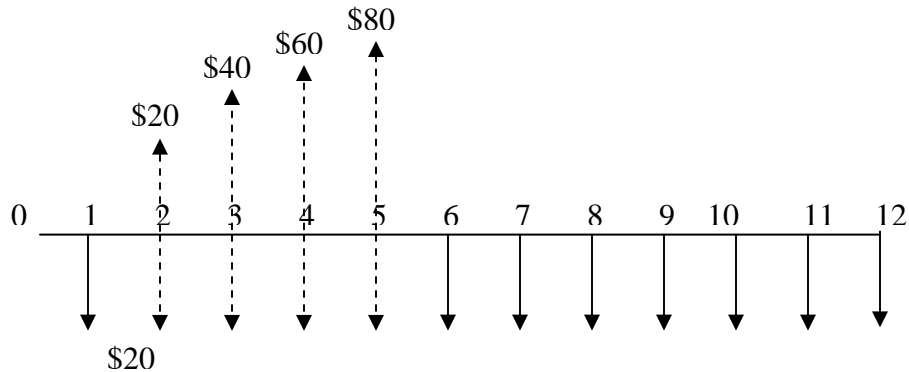
$$P_1 = \$200 + \$100(P/A, 6\%, 5) + \$50(P/F, 6\%, 1) + \$50(P/F, 6\%, 4) + \$100(P/F, 6\%, 5) \\ = \$782.75$$

$$P_2 = X(P/A, 6\%, 5) = \$782.75$$

$$X = \$185.82$$

3.54

$$P = \$20(P/G, 10\%, 5) - \$20(P/A, 10\%, 12) \\ = \$0.96$$



3.55

Establish economic equivalent at  $N = 8$ :

$$C(F/A, 8\%, 8) - C(F/A, 8\%, 2)(F/P, 8\%, 3) = \$6,000(P/A, 8\%, 2)$$

$$10.6366C - (2.08)(1.2597)C = \$6,000(1.7833)$$

$$8.0164C = \$10,699.80$$

$$C = \$1,334.73$$

3.56

The original cash flow series is

$N$	$A_N$	$N$	$A_N$
0	0	6	\$900
1	\$800	7	\$920
2	\$820	8	\$300
3	\$840	9	\$300
4	\$860	10	\$300 - \$500
5	\$880		

3.57

$$\$300(F / A, 10\%, 8) + \$200(F / A, 10\%, 3) = 2C(F / P, 10\%, 8) + C(F / A, 10\%, 7)$$

$$\$4,092.77 = 2C(2.1436) + C(9.4872)$$

$$C = \$297.13$$

3.58

Establishing equivalence at  $N = 5$ 

$$\$200(F / A, 8\%, 5) - \$50(F / P, 8\%, 1)$$

$$= X(F / A, 8\%, 5) - (\$200 + X)[(F / P, 8\%, 2) + (F / P, 8\%, 1)]$$

$$\$1,119.32 = X(5.8666) - (\$200 + X)(2.2464)$$

$$X = \$433.29$$

3.59

Computing equivalence at  $N = 5$ 

$$X = \$3,000(F / A, 9\%, 5) + \$3,000(P / A, 9\%, 5) = \$29,623.08$$

3.60 (b), (d), and (f)

3.61 (b), (d), and (e)

3.62

$$A_1 = (\$50 + \$50(A / G, 10\%, 5) - [\$50 + \$50(P / F, 10\%, 1)](A / P, 10\%, 5) = \$115.32$$

$$A_2 = A + A(A / P, 10\%, 5) = 1.2638A$$

$$A = \$91.25$$

3.63(a)

3.64(b)

3.65(b)

$$\$25,000 + \$30,000(P / F, 10\%, 6)$$

$$= C(P / A, 10\%, 12) + \$1,000(P / A, 10\%, 6)(P / F, 10\%, 6)$$

$$\$41,935 = 6.8137C + \$2,458.43$$

$$C = \$5,794$$

## Solving for an Unknown Interest Rate of Unknown Interest Periods

3.66

$$2P = P(1+i)^5$$

$$2^{1/5} = 1+i$$

$$i = 14.87\%$$

3.67

Establishing equivalence at  $n = 0$ 

$$\$2,000(P/A, i, 6) = \$2,500(P/A_1, -25\%, i, 6)$$

By Excel software,  $i = 92.36\%$ 

3.68

$$\$35,000 = \$10,000(F/P, i, 5) = \$10,000(1+i)^5$$

$$i = 28.47\%$$

3.69

$$\$1,000,000 = \$2,000(F/A, 6\%, N)$$

$$500 = \frac{(1+0.06)^N - 1}{0.06}$$

$$31 = (1+0.06)^N$$

$$\log 31 = N \log 1.06$$

$$N = 58.93 \approx 59 \text{ years}$$

3.70

$$\text{Option 1: } \$100,000(F/A, 7\%, 7)(F/P, 7\%, 13) = 2,085,484.95$$

$$\text{Option 2: } \$100,000(F/A, 7\%, 13) = 2,014,064.29$$

$$\$100,000(F/A, i, 7)(F/P, i, 13) = \$100,000(F/A, i, 13)$$

$$i = 6.6\%$$

3.71

Assuming that annual renewal fees are paid at the beginning of each year,

(a)

$$\$15.96 + \$15.96(P/A, 6\%, 3) = \$58.62$$

It is better to take the offer because of lower cost to renew.

(b)

$$\$57.12 = \$15.96 + \$15.96(P / A, i, 3)$$

$$i = 7.96\%$$

## Short Case Studies

### ST 3.1

(a)

$$P = 280,000(P / A, 8\%, 19) = 2,689,007.78$$

(b)

$$280,000(P / A, i, 19) = 5,600,000 - 283,770$$

$$i = 0.00709\%$$

### ST 3.2

(a)

$$\begin{aligned} P_{\text{Contract}} &= \$5,600,000 + \$7,178,000(P / F, 6\%, 1) \\ &\quad + \$11,778,000(P / F, 6\%, 2) + \dots \\ &\quad + \$17,778,000(P / F, 6\%, 9) \\ &= \$97,102,826.86 \end{aligned}$$

(b)

$$\begin{aligned} P_{\text{Bonus}} &= \$5,000,000 + \$5,000,000(P / A, 6\%, 5) + \$778,000(P / A, 6\%, 9) \\ &= \$31,353,535.52 > \$23,000,000 \end{aligned}$$

It is better stay with the original plan.

### ST 3.3

(a) Compute the equivalent present worth (in 2006) for each option at  $i = 6\%$ .

$$\begin{aligned} P_{\text{Deferred}} &= \$2,000,000 + \$566,000(P / F, 6\%, 1) + \$920,000(P / F, 6\%, 2) + \dots \\ &\quad + \$1,260,000(P / F, 6\%, 11) = \$8,574,491 \end{aligned}$$



$$P_{Non-Deferred} = \$2,000,000 + \$900,000(P / F, 6\%, 1) + \$1,000,000(P / F, 6\%, 2) + \dots + \$1,975,000(P / F, 6\%, 5) = \$7,431,562$$

$\therefore$  At  $i = 6\%$ , the deferred plan is a better choice.

- (b) Using either Excel or Cash Flow Analyzer, both plans would be economically equivalent at  $i = 15.72\%$ .

ST 3.4 Assuming that premiums paid at the end of each year, the maximum amount to invest in the prevention program is

$$P = \$14,000(P / A, 12\%, 5) = \$50,467.$$

If the premiums paid at the beginning of each year, the solution changes to

$$P = \$14,000 + \$14,000(P / A, 12\%, 4) = \$56,523.$$

ST 3.5

- Compute the required annual net cash profit to pay off the investment and interest.

$$\$70,000,000 = A(P / A, 10\%, 5) = 3.7908A$$

$$A = \$18,465,824$$

- Decide the number of shoes,  $X$

$$\$18,465,824 = X(\$100)$$

$$X = 184,658.24$$

ST 3.6

Establish the following equivalence equation:

$$\$140,000 = \$32,639(P / A, i, 9).$$

The interest rate makes two options equivalent is  $i = 18.10\%$  by Excel.  
So, if her rate of return is over 18.10%, it is a good decision.