

## 3

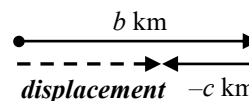
## Linear Motion

## Solutions

3-1. (a) Distance hiked =  $b + c$  km.

(b) Displacement is a *vector* representing Paul's change in position. Drawing a diagram of Paul's trip we can see that his displacement is  $b + (-c)$  km east =  $(b - c)$  km east.

(c) Distance = 5 km + 2 km = **7 km**; Displacement = (5 km - 2 km) east = **3 km east**.



3-2. (a) From  $\bar{v} = \frac{d}{t} \Rightarrow \bar{v} = \frac{x}{t}$ .

(b)  $\bar{v} = \frac{x}{t}$ . We want the answer in m/s so we'll need to convert 30 km to meters and 8 min to seconds:

$$30.0 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 30,000 \text{ m}; 8.0 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 480 \text{ s}. \text{ Then } \bar{v} = \frac{x}{t} = \frac{30,000 \text{ m}}{480 \text{ s}} = \mathbf{63 \frac{m}{s}}.$$

$$\left( \text{Alternatively, we can do the conversions within the equation:} \right)$$

$$\bar{v} = \frac{x}{t} = \frac{30.0 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}}}{8.0 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}}} = \mathbf{63 \frac{m}{s}}.$$

In mi/h:

$$30.0 \text{ km} \times \frac{1 \text{ mi}}{1.61 \text{ km}} = 18.6 \text{ mi}; 8.0 \text{ min} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.133 \text{ h}. \text{ Then } \bar{v} = \frac{x}{t} = \frac{18.6 \text{ mi}}{0.133 \text{ h}} = \mathbf{140 \frac{mi}{h}}.$$

$$\text{Or, } \bar{v} = \frac{x}{t} = \frac{30.0 \text{ km}}{8.0 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{1 \text{ mi}}{1.61 \text{ km}} = \mathbf{140 \frac{mi}{h}}. \text{ Or, } \bar{v} = \frac{x}{t} = \frac{30.0 \text{ km} \times \frac{1 \text{ mi}}{1.61 \text{ km}}}{8.0 \text{ min} \times \frac{1 \text{ h}}{60 \text{ min}}} = \mathbf{140 \frac{mi}{h}}.$$

There is usually more than one way to approach a problem and arrive at the correct answer!

3-3. (a) From  $\bar{v} = \frac{d}{t} \Rightarrow \bar{v} = \frac{L}{t}$ .

$$(b) \bar{v} = \frac{L}{t} = \frac{24.0 \text{ m}}{0.60 \text{ s}} = \mathbf{40 \frac{m}{s}}.$$

3-4. (a) From  $v = \frac{d}{t} \Rightarrow v = \frac{x}{t}$ .

$$(b) v = \frac{x}{t} = \frac{0.30 \text{ m}}{0.010 \text{ s}} = \mathbf{30 \frac{m}{s}}.$$

3-5. (a)  $\bar{v} = \frac{d}{t} = \frac{2\pi r}{t}$ .

$$(b) \bar{v} = \frac{2\pi r}{t} = \frac{2\pi(400 \text{ m})}{40 \text{ s}} = \mathbf{63 \frac{m}{s}}.$$

3-6. (a)  $t = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow t = \frac{h}{\bar{v}}$ .

(b)  $t = \frac{h}{\bar{v}} = \frac{508 \text{ m}}{15 \frac{\text{m}}{\text{s}}} = \mathbf{34 \text{ s.}}$

(c) **Yes.** At the beginning of the ride the elevator has to speed up from rest, and at the end of the ride the elevator has to slow down. These slower portions of the ride produce an average speed lower than the peak speed.

3-7. (a)  $t = ?$  Begin by getting consistent units. Convert 100.0 yards to meters using the conversion factor on the inside cover of your textbook:  $0.3048 \text{ m} = 1.00 \text{ ft}$ .

Then  $100.0 \text{ yards} \times \left(\frac{3 \text{ ft}}{1 \text{ yard}}\right) \times \left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 91.4 \text{ m}$ . From  $v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{\mathbf{91.4 \text{ m}}}{v}$ .

(b)  $t = \frac{d}{v} = \frac{91.4 \text{ m}}{\left(6.0 \frac{\text{m}}{\text{s}}\right)} = \mathbf{15 \text{ s.}}$

3-8. (a)  $t = ?$  From  $v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{L}{c}$ .

(b)  $t = \frac{L}{v} = \frac{1.00 \text{ m}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = 3.33 \times 10^{-9} \text{ s} = \mathbf{3.33 \text{ ns.}}$  (This is  $3\frac{1}{3}$  *billionths* of a second!)

3-9. (a)  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t$ .

(b) First, we need a consistent set of units. Since speed is in m/s let's convert minutes to seconds:

$5.0 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 300 \text{ s}$ . Then  $d = \bar{v}t = 7.5 \frac{\text{m}}{\text{s}} \times 300 \text{ s} = \mathbf{2300 \text{ m.}}$

3-10. (a)  $\bar{v} = \frac{v_0 + v_f}{2} = \frac{v}{2}$ .

(b)  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \frac{vt}{2}$ .

(c)  $d = \frac{vt}{2} = \frac{\left(2.0 \frac{\text{m}}{\text{s}}\right)(1.5 \text{ s})}{2} = \mathbf{1.5 \text{ m.}}$

3-11. (a)  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \left(\frac{v_0 + v_f}{2}\right)t = \left(\frac{0 + v}{2}\right)t = \frac{vt}{2}$ .

(b)  $d = \frac{vt}{2} = \frac{\left(12 \frac{\text{m}}{\text{s}}\right)(8.0 \text{ s})}{2} = \mathbf{48 \text{ m.}}$

3-12. (a)  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \left(\frac{v_0 + v_f}{2}\right)t = \left(\frac{0 + v}{2}\right)t = \frac{vt}{2}$ .

(b) First get consistent units: 100.0 km/h should be expressed in m/s (since the time is in

seconds).  $100.0 \frac{\text{km}}{\text{h}} \times \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \times \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) = 27.8 \frac{\text{m}}{\text{s}}$ . Then,  $d = \frac{vt}{2} = \frac{\left(27.8 \frac{\text{m}}{\text{s}}\right)(8.0 \text{ s})}{2} = \mathbf{110 \text{ m.}}$

3-13. (a)  $a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t}$ .

(b)  $\Delta v = 40 \frac{\text{km}}{\text{h}} - 15 \frac{\text{km}}{\text{h}} = 25 \frac{\text{km}}{\text{h}}$ . Since our time is in seconds we need to convert  $\frac{\text{km}}{\text{h}}$  to  $\frac{\text{m}}{\text{s}}$ :

$$25 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 6.94 \frac{\text{m}}{\text{s}}. \text{ Then } a = \frac{\Delta v}{t} = \frac{6.94 \frac{\text{m}}{\text{s}}}{20 \text{ s}} = \mathbf{0.35 \frac{m}{s^2}}.$$

$$\left[ \text{Alternatively, we can express the speeds in m/s first and then do the calculation:} \right. \\ \left. 15 \frac{\text{km}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 4.17 \frac{\text{m}}{\text{s}} \text{ and } 40 \frac{\text{km}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 11.1 \frac{\text{m}}{\text{s}}. \text{ Then } a = \frac{11.1 \frac{\text{m}}{\text{s}} - 4.17 \frac{\text{m}}{\text{s}}}{20 \text{ s}} = \mathbf{0.35 \frac{m}{s^2}}. \right]$$

3-14. (a)  $a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t}$ .

(b) To make the speed units consistent with the time unit we'll need  $\Delta v$  in m/s:

$$\Delta v = v_2 - v_1 = 20.0 \frac{\text{km}}{\text{h}} - 5.0 \frac{\text{km}}{\text{h}} = 15.0 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 4.17 \frac{\text{m}}{\text{s}}. \text{ Then } a = \frac{v_2 - v_1}{t} = \frac{4.17 \frac{\text{m}}{\text{s}}}{10.0 \text{ s}} = \mathbf{0.417 \frac{m}{s^2}}.$$

$$\left[ \text{An alternative is to convert the speeds to m/s first:} \right. \\ \left. v_1 = 5.0 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 1.4 \frac{\text{m}}{\text{s}}; \quad v_2 = 20.0 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 5.56 \frac{\text{m}}{\text{s}}. \right. \\ \left. \text{Then } a = \frac{v_2 - v_1}{t} = \frac{(5.56 \frac{\text{m}}{\text{s}} - 1.4 \frac{\text{m}}{\text{s}})}{10.0 \text{ s}} = \mathbf{0.42 \frac{m}{s^2}}. \right]$$

(c)  $d = \bar{v}t = \frac{v_1 + v_2}{2} t = \left( \frac{1.4 \frac{\text{m}}{\text{s}} + 5.56 \frac{\text{m}}{\text{s}}}{2} \right) 10.0 \text{ s} = \mathbf{35 \text{ m}}. \text{ Or,}$

$$d = v_1 t + \frac{1}{2} a t^2 = \left( 1.4 \frac{\text{m}}{\text{s}} \right) (10.0 \text{ s}) + \frac{1}{2} \left( 0.42 \frac{\text{m}}{\text{s}^2} \right) (10.0 \text{ s})^2 = \mathbf{35 \text{ m}}.$$

3-15. (a)  $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{\Delta t} = \frac{0 - v}{t} = \frac{-v}{t}$ .

(b)  $a = \frac{-v}{t} = \frac{-26 \frac{\text{m}}{\text{s}}}{20 \text{ s}} = \mathbf{-1.3 \frac{m}{s^2}}.$

(c)  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \left( \frac{v_0 + v_f}{2} \right) t = \left( \frac{26 \frac{\text{m}}{\text{s}} + 0 \frac{\text{m}}{\text{s}}}{2} \right) 20 \text{ s} = \mathbf{260 \text{ m}}.$

$$\left( \text{Or, } d = v_0 t + \frac{1}{2} a t^2 = 26 \frac{\text{m}}{\text{s}} (20 \text{ s}) + \frac{1}{2} \left( -1.3 \frac{\text{m}}{\text{s}^2} \right) (20 \text{ s})^2 = \mathbf{260 \text{ m}}. \right)$$

(d)  $d = ?$  Lonnie travels at a constant speed of 26 m/s before applying the brakes, so

$$d = vt = \left( 26 \frac{\text{m}}{\text{s}} \right) (1.5 \text{ s}) = \mathbf{39 \text{ m}}.$$

3-16. (a)  $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{\Delta t} = \frac{0 - v}{t} = \frac{-v}{t}$ .

(b)  $a = \frac{-v}{t} = \frac{-72 \frac{\text{m}}{\text{s}}}{12 \text{ s}} = \mathbf{-6.0 \frac{m}{s^2}}.$

$$(c) d = ? \text{ From } \bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \left( \frac{v_0 + v_f}{2} \right) t = \left( \frac{72 \frac{\text{m}}{\text{s}} + 0 \frac{\text{m}}{\text{s}}}{2} \right) (12 \text{ s}) = \mathbf{430 \text{ m}}.$$

$$\text{Or, } d = v_0 t + \frac{1}{2} a t^2 = 72 \frac{\text{m}}{\text{s}} (12 \text{ s}) + \frac{1}{2} \left( -6.0 \frac{\text{m}}{\text{s}^2} \right) (12 \text{ s})^2 = \mathbf{430 \text{ m}}.$$

$$3-17. (a) t = ? \text{ From } \bar{v} = \frac{d}{t} \Rightarrow t = \frac{d}{\bar{v}} = \frac{L}{\left( \frac{v_f + v_0}{2} \right)} = \frac{2L}{v}.$$

$$(b) t = \frac{2L}{v} = \frac{2(1.4 \text{ m})}{15.0 \frac{\text{m}}{\text{s}}} = \mathbf{0.19 \text{ s}}.$$

$$3-18. (a) \bar{v} = \left( \frac{v_0 + v_f}{2} \right) = \frac{v}{2}.$$

$$(b) \bar{v} = \frac{350 \frac{\text{m}}{\text{s}}}{2} = \mathbf{175 \frac{\text{m}}{\text{s}}}. \text{ Note that the length of the barrel isn't needed—yet!}$$

$$(c) \text{ From } \bar{v} = \frac{d}{t} \Rightarrow t = \frac{d}{\bar{v}} = \frac{L}{\bar{v}} = \frac{0.40 \text{ m}}{175 \frac{\text{m}}{\text{s}}} = \mathbf{0.0023 \text{ s} = 2.3 \text{ ms}}.$$

$$3-19. (a) \text{ From } \bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \left( \frac{v_0 + v_f}{2} \right) t = \left( \frac{v_0 + v}{2} \right) t.$$

$$(b) d = \left( \frac{v_0 + v}{2} \right) t = \left( \frac{25 \frac{\text{m}}{\text{s}} + 11 \frac{\text{m}}{\text{s}}}{2} \right) (7.8 \text{ s}) = \mathbf{140 \text{ m}}.$$

$$3-20. (a) v = ? \text{ There's a time } t \text{ between frames of } \frac{1}{24} \text{ s, so } v = \frac{d}{t} = \frac{x}{\left( \frac{1}{24} \text{ s} \right)} = \left( 24 \frac{1}{\text{s}} \right) x. \text{ (That's 24x per second.)}$$

$$(b) v = \left( 24 \frac{1}{\text{s}} \right) x = \left( 24 \frac{1}{\text{s}} \right) (0.15 \text{ m}) = \mathbf{3.6 \frac{\text{m}}{\text{s}}}.$$

$$3-21. (a) a = ? \text{ Since time is not a part of the problem we can use the formula } v_f^2 - v_0^2 = 2ad \text{ and}$$

$$\text{solve for acceleration } a. \text{ Then, with } v_0 = 0 \text{ and } d = x, a = \frac{v^2}{2x}.$$

$$(b) a = \frac{v^2}{2x} = \frac{\left( 1.8 \times 10^7 \frac{\text{m}}{\text{s}} \right)^2}{2(0.10 \text{ m})} = \mathbf{1.6 \times 10^{15} \frac{\text{m}}{\text{s}^2}}.$$

$$(c) t = ? \text{ From } v_f = v_0 + at \Rightarrow t = \frac{v_f - v_0}{a} = \frac{\left( 1.8 \times 10^7 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}} \right)}{1.6 \times 10^{15} \frac{\text{m}}{\text{s}^2}} = \mathbf{1.1 \times 10^{-8} \text{ s} = 11 \text{ ns}}.$$

$$\left( \text{Or, from } \bar{v} = \frac{d}{t} \Rightarrow t = \frac{d}{\bar{v}} = \frac{L}{\left( \frac{v_f + v_0}{2} \right)} = \frac{2L}{(v + 0)} = \frac{2(0.10 \text{ m})}{\left( 1.8 \times 10^7 \frac{\text{m}}{\text{s}} \right)} = \mathbf{1.1 \times 10^{-8} \text{ s}}. \right)$$

3-22. (a)  $v_f = ?$  From  $\bar{v} = \frac{d}{t} = \left( \frac{v_0 + v_f}{2} \right) t$  with  $v_0 = 0 \Rightarrow v_f = \frac{2d}{t}$ .

(b)  $a_f = ?$  From  $d = v_0 t + \frac{1}{2} a t^2$  with  $v_0 = 0 \Rightarrow d = \frac{1}{2} a t^2 \Rightarrow a = \frac{2d}{t^2}$ .

(c)  $v_f = \frac{2d}{t} = \frac{2(402 \text{ m})}{4.45 \text{ s}} = \mathbf{181 \frac{m}{s}}$ ;  $a = \frac{2d}{t^2} = \frac{2(402 \text{ m})}{(4.45 \text{ s})^2} = \mathbf{40.6 \frac{m}{s^2}}$ .

3-23. (a)  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v} t = \left( \frac{v_0 + v_f}{2} \right) t = \left( \frac{v + V}{2} \right) t$ .

(b)  $d = \left( \frac{v + V}{2} \right) t = \left( \frac{110 \frac{m}{s} + 250 \frac{m}{s}}{2} \right) (3.5 \text{ s}) = \mathbf{630 \text{ m}}$ .

3-24. (a)  $t = ?$  Let's choose upward to be the positive direction.

From  $v_f = v_0 + at$  with  $v_f = 0$  and  $a = -g \Rightarrow t = \frac{v_f - v_0}{a} = \frac{0 - v}{-g} = \frac{v}{g}$ .

(b)  $t = \frac{v}{g} = \frac{32 \frac{m}{s}}{9.8 \frac{m}{s^2}} = \mathbf{3.3 \text{ s}}$ .

(c)  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v} t = \left( \frac{v_0 + v_f}{2} \right) t = \left( \frac{v + 0}{2} \right) \left( \frac{v}{g} \right) = \frac{v^2}{2g} = \frac{\left( 32 \frac{m}{s} \right)^2}{2 \left( 9.8 \frac{m}{s^2} \right)} = \mathbf{52 \text{ m}}$ .

We get the same result with  $d = v_0 t + \frac{1}{2} a t^2 = \left( 32 \frac{m}{s} \right) (3.3 \text{ s}) + \frac{1}{2} \left( -9.8 \frac{m}{s^2} \right) (3.3 \text{ s})^2 = \mathbf{52 \text{ m}}$ .

3-25. (a)  $v_0 = ?$  When the potato hits the ground  $y = 0$ . From

$d = v_0 t + \frac{1}{2} a t^2 \Rightarrow y = v_0 t - \frac{1}{2} g t^2 \Rightarrow 0 = t \left( v_0 - \frac{1}{2} g t \right) \Rightarrow v_0 = \frac{1}{2} g t$ .

(b)  $v_0 = \frac{1}{2} g t = \frac{1}{2} \left( 9.8 \frac{m}{s^2} \right) (12 \text{ s}) = 59 \frac{m}{s}$ . In mi/h,  $59 \frac{m}{s} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ mi}}{1.61 \text{ km}} \times \frac{3600 \text{ s}}{1 \text{ h}} = \mathbf{130 \frac{mi}{h}}$ .

3-26. (a)  $t = ?$  Choose downward to be the positive direction. From

From  $d = v_0 t + \frac{1}{2} a t^2$  with  $v_0 = 0$ ,  $a = g$  and  $d = h \Rightarrow h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$ .

(b)  $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(25 \text{ m})}{9.8 \frac{m}{s^2}}} = 2.26 \text{ s} \approx \mathbf{2.3 \text{ s}}$ .

(c)  $v_f = v_0 + at = 0 + gt = \left( 9.8 \frac{m}{s^2} \right) (2.26 \text{ s}) = \mathbf{22 \frac{m}{s}}$ .

(Or, from  $2ad = v_f^2 - v_0^2$  with  $a = g$ ,  $d = h$ , and  $v_0 = 0 \Rightarrow v_f = \sqrt{2gh} = \sqrt{2 \left( 9.8 \frac{m}{s^2} \right) (25 \text{ m})} = \mathbf{22 \frac{m}{s}}$ .)

3-27. (a)  $v_0 = ?$  Let's call upward the positive direction. Since the trajectory is symmetric,  $v_f = -v_0$ .

$$\text{Then from } v_f = v_0 + at, \text{ with } a = -g \Rightarrow -v_0 = v_0 - gt \Rightarrow -2v_0 = -gt \Rightarrow v_0 = \frac{gt}{2}.$$

$$(b) v_0 = \frac{gt}{2} = \frac{\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(4.0\text{s})}{2} = 20 \frac{\text{m}}{\text{s}}.$$

$$(c) d = ? \text{ From } \bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \left(\frac{v_0 + v_f}{2}\right)t = \left(\frac{v_0}{2}\right)t = \frac{\left(20 \frac{\text{m}}{\text{s}}\right)}{2}(2.0\text{ s}) = 20 \text{ m}.$$

We use  $t = 2.0\text{ s}$  because we are only considering the time to the highest point rather than the whole trip up and down.

3-28. (a)  $v_0 = ?$  Let's call upward the positive direction. Since no time is given, use

$$v_f^2 - v_0^2 = 2ad \text{ with } a = -g, v_f = 0 \text{ at the top, and } d = (y - 2\text{ m}).$$

$$\Rightarrow -v_0^2 = 2(-g)(y - 2\text{ m}) \Rightarrow v_0 = \sqrt{2g(y - 2\text{ m})}.$$

$$(b) v_0 = \sqrt{2g(y - 2\text{ m})} = \sqrt{2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(20\text{ m} - 2\text{ m})} = 18.8 \frac{\text{m}}{\text{s}} \approx 19 \frac{\text{m}}{\text{s}}.$$

3-29. (a) Taking upward to be the positive direction, from

$$2ad = v_f^2 - v_0^2 \text{ with } a = -g \text{ and } d = h \Rightarrow v_f = \pm\sqrt{v_0^2 - 2gh}. \text{ So on the way up}$$

$$v_f = +\sqrt{v_0^2 - 2gh}.$$

(b) From above, on the way down  $v_f = -\sqrt{v_0^2 - 2gh}$ , same magnitude but opposite direction as (a).

$$(c) \text{ From } a = \frac{v_f - v_0}{t} \Rightarrow t = \frac{v_f - v_0}{a} = \frac{-\sqrt{v_0^2 - 2gh} - v_0}{-g} = \frac{v_0 + \sqrt{v_0^2 - 2gh}}{g}.$$

$$(d) v_f = -\sqrt{v_0^2 - 2gh} = -\sqrt{\left(16 \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(8.5\text{ m})} = -9.5 \frac{\text{m}}{\text{s}}. \quad t = \frac{v_f - v_0}{a} = \frac{-9.5 \frac{\text{m}}{\text{s}} - 16 \frac{\text{m}}{\text{s}}}{-9.8 \frac{\text{m}}{\text{s}^2}} = 2.6\text{ s}.$$

3-30. (a)  $v_f = ?$  Taking upward to be the positive direction, from

$2ad = v_f^2 - v_0^2$  with  $a = -g$  and  $d = -h \Rightarrow v_f = -\sqrt{v_0^2 + 2gh}$ . The displacement  $d$  is negative because upward direction was taken to be positive, and the water balloon ends up *below* the initial position. The final velocity is negative because the water balloon is heading downward (in the negative direction) when it lands.

$$(b) t = ? \text{ From } a = \frac{v_f - v_0}{t} \Rightarrow t = \frac{v_f - v_0}{a} = \frac{-\sqrt{v_0^2 + 2gh} - v_0}{-g} = \frac{v_0 + \sqrt{v_0^2 + 2gh}}{g}.$$

(c)  $v_f = ?$  Still taking upward to be the positive direction, from

$$2ad = v_f^2 - v_0^2 \text{ with initial velocity } = -v_0, a = -g \text{ and } d = -h \Rightarrow v_f^2 = v_0^2 + 2gh \Rightarrow v_f = -\sqrt{v_0^2 + 2gh}.$$

We take the negative square root because the balloon is going downward. Note that the final velocity is the same whether the balloon is thrown straight up or straight down with initial speed  $v_0$ .

(d)  $v_f = -\sqrt{v_0^2 + 2gh} = -\sqrt{\left(5.0 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(1.8 \text{ m})} = -16 \frac{\text{m}}{\text{s}}$  for the balloon whether it is tossed upward or downward. For the balloon tossed upward,

$$t = \frac{v_f - v_0}{a} = \frac{-16 \frac{\text{m}}{\text{s}} - 5 \frac{\text{m}}{\text{s}}}{-9.8 \frac{\text{m}}{\text{s}^2}} = 2.1 \text{ s}.$$

3-31. (a) Call downward the positive direction, origin at the top.

$$\text{From } d = v_0 t + \frac{1}{2} a t^2 \quad \text{with } a = +g, d = h \quad y = +h \Rightarrow h = v_0 t + \frac{1}{2} g t^2 \Rightarrow \frac{1}{2} g t^2 + v_0 t - h = 0.$$

From the general form of the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  we identify

$$a = \frac{g}{2}, b = +v_0, \text{ and } c = -h, \text{ which gives } t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{g}{2}\right)(-h)}}{g} = \frac{-v_0 \pm \sqrt{v_0^2 + 2gh}}{g}.$$

To get a positive value for the time we take the positive root, and get

$$t = \frac{-v_0 + \sqrt{v_0^2 + 2gh}}{g}.$$

(b) From

$$2ad = v_f^2 - v_0^2 \text{ with initial velocity } v_0, a = g \text{ and } d = h \Rightarrow v_f^2 = v_0^2 + 2gh \Rightarrow v_f = +\sqrt{v_0^2 + 2gh}.$$

$$\text{Or you could start with } v_f = v_0 + at = v_0 + g \left( \frac{-v_0 + \sqrt{v_0^2 + 2gh}}{g} \right) = \sqrt{v_0^2 + 2gh}.$$

$$(c) t = \frac{-v_0 + \sqrt{v_0^2 + 2gh}}{g} = \frac{-3.2 \frac{\text{m}}{\text{s}} + \sqrt{\left(3.2 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(3.5 \text{ m})}}{9.8 \frac{\text{m}}{\text{s}^2}} = 0.58 \text{ s};$$

$$v_f = +\sqrt{v_0^2 + 2gh} = \sqrt{\left(3.2 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(3.5 \text{ m})} = 8.9 \frac{\text{m}}{\text{s}}$$

$$3-32. (a) \text{ From } d = v_0 t + \frac{1}{2} a t^2 \Rightarrow a = \frac{2(d - v_0 t)}{t^2}.$$

$$(b) a = \frac{2(d - v_0 t)}{t^2} = \frac{2(120 \text{ m} - 13 \frac{\text{m}}{\text{s}} \cdot 5.0 \text{ s})}{(5.0 \text{ s})^2} = 4.4 \frac{\text{m}}{\text{s}^2}.$$

$$(c) v_f = v_0 + at = 13 \frac{\text{m}}{\text{s}} + \left(4.4 \frac{\text{m}}{\text{s}^2}\right)(5 \text{ s}) = 35 \frac{\text{m}}{\text{s}}.$$

$$(d) 35 \frac{\text{m}}{\text{s}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ mi}}{1.61 \text{ km}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 78 \frac{\text{mi}}{\text{h}}. \text{ This is probably not a safe speed for driving in an environment that would have a traffic light!}$$

$$3-33. (a) \text{ From } x = \bar{v}t = \frac{v_0 + v_f}{2} t \Rightarrow v_f = \frac{2x}{t} - v_0.$$

$$(b) a = \frac{v_f - v_0}{t} = \frac{\left(\frac{2x}{t} - v_0\right) - v_0}{t} = \frac{\frac{2x}{t} - 2v_0}{t} = 2\left(\frac{x}{t^2} - \frac{v_0}{t}\right).$$

$$(c) \ v_f = \frac{2x}{t} - v_0 = \frac{2(95\text{m})}{11.9\text{s}} - 13 \frac{\text{m}}{\text{s}} = \mathbf{3.0 \frac{m}{s}}.$$

$$a = 2 \left( \frac{x}{t^2} - \frac{v_0}{t} \right) = 2 \left( \frac{95\text{m}}{(11.9\text{s})^2} - \frac{13 \frac{\text{m}}{\text{s}}}{11.9\text{s}} \right) = \mathbf{-0.84 \frac{m}{s^2}} \quad \text{or} \quad a = \frac{v_f - v_0}{t} = \frac{3.0 \frac{\text{m}}{\text{s}} - 13 \frac{\text{m}}{\text{s}}}{11.9\text{s}} = \mathbf{-0.84 \frac{m}{s^2}}.$$

3-34. (a) From  $2ad = v_f^2 - v_0^2$  with  $d = L \Rightarrow v_f = \sqrt{v_0^2 + 2aL}$ . This is Rita's speed at the bottom of the hill. To get her time to cross the highway: From  $\bar{v} = \frac{d}{t} \Rightarrow t = \frac{d}{\bar{v}} = \frac{d}{\sqrt{v_0^2 + 2aL}}$ .

$$(b) \ t = \frac{d}{\sqrt{v_0^2 + 2aL}} = \frac{25\text{m}}{\sqrt{(3.0 \frac{\text{m}}{\text{s}})^2 + 2(1.5 \frac{\text{m}}{\text{s}^2})(85\text{m})}} = \mathbf{1.54\text{s}}.$$

3-35. (a) Since  $v_0$  is upward, call upward the positive direction and put the origin at the ground. Then

$$\text{From } d = v_0 t + \frac{1}{2} a t^2 \quad \text{with } a = -g, \ d = h \quad y = +h \Rightarrow h = v_0 t - \frac{1}{2} g t^2 \Rightarrow \frac{1}{2} g t^2 - v_0 t - h = 0.$$

From the general form of the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  we identify

$$a = \frac{g}{2}, \ b = -v_0, \ \text{and } c = -h, \ \text{which gives } t = \frac{v_0 \pm \sqrt{v_0^2 - 4\left(\frac{g}{2}\right)(-h)}}{g} = \frac{v_0 \pm \sqrt{v_0^2 + 2gh}}{g}.$$

$$(b) \ \text{From } 2ad = v_f^2 - v_0^2 \text{ with } a = -g \text{ and } d = h \Rightarrow v_f^2 = v_0^2 - 2gh \Rightarrow v_f = \pm \sqrt{v_0^2 - 2gh}.$$

$$(c) \ t = \frac{v_0 \pm \sqrt{v_0^2 - 2gh}}{g} = \frac{22 \frac{\text{m}}{\text{s}} \pm \sqrt{(22 \frac{\text{m}}{\text{s}})^2 - 2(9.8 \frac{\text{m}}{\text{s}^2})(14.7\text{m})}}{9.8 \frac{\text{m}}{\text{s}^2}} = \mathbf{0.82\text{s} \text{ or } 3.67\text{s}}. \ \text{So}$$

Anthony has to have the ball leave his hand either 0.82s or 3.67s before midnight. The first time corresponds to the rock hitting the bell on the rock's way up, and the second time is for the rock hitting the bell on the way down.

$$v_f = \pm \sqrt{v_0^2 - 2gh} = \pm \sqrt{(22 \frac{\text{m}}{\text{s}})^2 - 2(9.8 \frac{\text{m}}{\text{s}^2})(14.7\text{m})} = \pm \mathbf{14 \frac{m}{s}}.$$

3-31. (a)  $v_1 = ?$  The rocket starts at rest and after time  $t_1$  it has velocity  $v_1$  and has risen to a height  $h_1$ . Taking upward to be the positive direction, from  $v_f = v_0 + at$  with  $v_0 = 0 \Rightarrow v_1 = at_1$ .

$$(b) \ h_1 = ? \text{ From } d = v_0 t + \frac{1}{2} a t^2 \text{ with } h_1 = d \text{ and } v_0 = 0 \Rightarrow h_1 = \frac{1}{2} a t_1^2.$$

(c)  $h_2 = ?$  For this stage of the problem the rocket has initial velocity  $v_1$ ,  $v_f = 0$ ,  $a = -g$  and the distance risen  $d = h_2$ .

$$\text{From } 2ad = v_f^2 - v_0^2 \Rightarrow d = \frac{v_f^2 - v_0^2}{2a} \Rightarrow h_2 = \frac{0 - v_1^2}{2(-g)} = \frac{v_1^2}{2g} = \frac{(at_1)^2}{2g} = \frac{a^2 t_1^2}{2g}.$$

(d)  $t_{\text{additional}} = ?$  To get the additional rise time of the rocket: From

$$a = \frac{v_f - v_0}{t} \Rightarrow t_{\text{additional}} = \frac{v_f - v_0}{a} = \frac{0 - v_1}{-g} = \frac{at_1}{g}.$$

(e) The maximum height of the rocket is the sum of the answers from (a) and (b) =

$$h_{\text{max}} = h_1 + h_2 = \frac{1}{2} a t_1^2 + \frac{a^2 t_1^2}{2g} = \frac{1}{2} a t_1^2 \left( 1 + \frac{a}{g} \right)$$



(f)  $t_{\text{falling}} = ?$  Keeping upward as the positive direction, now  $v_0 = 0$ ,  $a = -g$  and  $d = -h_{\text{max}}$ .

$$\text{From } d = v_0 t + \frac{1}{2} a t^2 \Rightarrow -h_{\text{max}} = \frac{1}{2} (-g) t^2$$

$$\Rightarrow t_{\text{falling}} = \sqrt{\frac{2h_{\text{max}}}{g}} = \sqrt{\frac{2 \left[ \frac{1}{2} a t_1^2 \left( 1 + \frac{a}{g} \right) \right]}{g}} = \sqrt{\frac{a t_1^2 (g + a)}{g^2}} = \sqrt{a(g + a)} \frac{t_1}{g}$$

$$(g) \ t_{\text{total}} = t_1 + t_{\text{additional}} + t_{\text{falling}} = t_1 + \frac{a t_1}{g} + \sqrt{a(g + a)} \frac{t_1}{g}.$$

$$(h) \ v_{\text{runs out of fuel}} = v_1 = a t_1 = \left( 120 \frac{\text{m}}{\text{s}^2} \right) (1.70 \text{ s}) = \mathbf{204 \frac{m}{s}}; h_1 = \frac{1}{2} a t_1^2 = \frac{1}{2} \left( 120 \frac{\text{m}}{\text{s}^2} \right) (1.70 \text{ s})^2 = \mathbf{173 \text{ m}}.$$

$$h_{\text{additional}} = h_2 = \frac{a^2 t_1^2}{2g} = \frac{\left( 120 \frac{\text{m}}{\text{s}^2} \right)^2 (1.70 \text{ s})^2}{2 \left( 9.8 \frac{\text{m}}{\text{s}^2} \right)} = \mathbf{2123 \text{ m}}.$$

$$t_{\text{additional}} = \frac{a t_1}{g} = \frac{120 \frac{\text{m}}{\text{s}^2} (1.70 \text{ s})}{9.8 \frac{\text{m}}{\text{s}^2}} = \mathbf{20.8 \text{ s}}.$$

$$h_{\text{max}} = 173 \text{ m} + 2123 \text{ m} = 2296 \text{ m} \approx \mathbf{2300 \text{ m}}.$$

$$t_{\text{falling}} = \sqrt{\frac{2h_{\text{max}}}{g}} = \sqrt{\frac{2(2300 \text{ m})}{9.8 \frac{\text{m}}{\text{s}^2}}} = \mathbf{21.7 \text{ s}}.$$

$$t_{\text{total}} = t_1 + t_{\text{additional}} + t_{\text{falling}} = 1.7 \text{ s} + 20.8 \text{ s} + 21.7 \text{ s} = \mathbf{44.2 \text{ s}}.$$

$$3-32. \ \bar{v} = \frac{\text{total distance}}{\text{total time}} = \frac{x + x}{t + 0.75t} = \frac{2x}{1.75t} = \mathbf{1.14 \frac{x}{t}}.$$

$$(b) \ \bar{v} = 1.14 \frac{x}{t} = 1.14 \left( \frac{140 \text{ km}}{2 \text{ hr}} \right) = \mathbf{80 \frac{km}{hr}}.$$

$$3-33. (a) \ \bar{v} = \frac{\text{total distance}}{\text{total time}}. \text{ From } v = \frac{d}{t} \Rightarrow d = vt.$$

$$\text{So } \bar{v} = \frac{d_{\text{walk}} + d_{\text{jog}}}{t_{\text{walk}} + t_{\text{jog}}} = \frac{v_{\text{walk}} t_{\text{walk}} + v_{\text{jog}} t_{\text{jog}}}{t_{\text{walk}} + t_{\text{jog}}} = \frac{v(30 \text{ min}) + 2v(30 \text{ min})}{30 \text{ min} + 30 \text{ min}} = \frac{3v(30 \text{ min})}{2(30 \text{ min})} = \mathbf{1.5 v}.$$

$$(b) \ \bar{v} = 1.5v = 1.5 \left( 1.0 \frac{\text{m}}{\text{s}} \right) = \mathbf{1.5 \frac{m}{s}}.$$

$$(c) \ d_{\text{to cabin}} = \bar{v} t_{\text{total}} = \bar{v} (t_{\text{walk}} + t_{\text{jog}}) = 1.5 \frac{\text{m}}{\text{s}} (30 \text{ min} + 30 \text{ min}) \times \frac{60 \text{ s}}{1 \text{ min}} = \mathbf{5400 \text{ m} = 5.4 \text{ km}}.$$

$$3-34. (a) \ \bar{v} = \frac{\text{total distance}}{\text{total time}}. \text{ From } v = \frac{d}{t} \Rightarrow d = vt.$$

$$\text{So } \bar{v} = \frac{d_{\text{slow}} + d_{\text{fast}}}{t_{\text{slow}} + t_{\text{fast}}} = \frac{v_{\text{slow}} t_{\text{slow}} + v_{\text{fast}} t_{\text{fast}}}{t_{\text{slow}} + t_{\text{fast}}} = \frac{v(1 \text{ h}) + 4v(1 \text{ h})}{1 \text{ h} + 1 \text{ h}} = \frac{5v(1 \text{ h})}{2 \text{ h}} = \mathbf{2.5 v}.$$

$$(b) \ \bar{v} = 2.5v = 2.5 \left( 25 \frac{\text{km}}{\text{h}} \right) = \mathbf{63 \frac{km}{h}}.$$

3-35. (a)  $\bar{v} = \frac{\text{total distance}}{\text{total time}}$ . From  $v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{x}{v}$ .

$$\text{So } \bar{v} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{2x}{\left(\frac{x}{v_1}\right) + \left(\frac{x}{v_2}\right)} = \frac{2x}{x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2}{\left(\frac{v_2 + v_1}{v_1 v_2}\right)}$$

$$= 2\left(\frac{v_1 v_2}{v_2 + v_1}\right) = 2\left(\frac{v(1.5v)}{1.5v + v}\right) = 2\left(\frac{1.5v^2}{2.5v}\right) = \mathbf{1.2v}.$$

Note that the average velocity is biased toward the lower speed since you spend more time driving at the lower speed than the higher speed.

(b)  $\bar{v} = 1.2v = 1.2\left(28 \frac{\text{km}}{\text{h}}\right) = \mathbf{34 \frac{\text{km}}{\text{h}}}$ .

3-36. (a)  $d_{\text{Atti}} = ?$  From  $V_{\text{Atti}} = \frac{d_{\text{Atti}}}{t} \Rightarrow d_{\text{Atti}} = Vt$ . The time that Atti runs = the time that Judy

walks, which is  $t = \frac{x}{v}$ . So  $d_{\text{Atti}} = V\left(\frac{x}{v}\right) = \left(\frac{V}{v}\right)x$ .

(b)  $X = \left(\frac{V}{v}\right)x = \left(\frac{4.5 \frac{\text{m}}{\text{s}}}{1.5 \frac{\text{m}}{\text{s}}}\right)(150 \text{ m}) = \mathbf{450 \text{ m}}$ .

3-37.  $\bar{v} = \frac{d}{t} = \frac{3 \text{ m}}{1.5 \text{ s}} = \mathbf{2 \frac{\text{m}}{\text{s}}}$ .

3-38.  $h = ?$  Call upward the positive direction.

From  $v_f^2 - v_0^2 = 2ad$  with  $d = h$ ,  $v_f = 0$  and  $a = -g$

$$\Rightarrow h = \frac{v_f^2 - v_0^2}{2a} = \frac{-v_0^2}{2(-g)} = \frac{v_0^2}{2g} = \frac{\left(14.7 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)} = \mathbf{11 \text{ m}}.$$

3-39.  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v}t = \left(\frac{v_0 + v_f}{2}\right)t = \left(\frac{0 + 27.5 \frac{\text{m}}{\text{s}}}{2}\right)(8.0 \text{ s}) = \mathbf{110 \text{ m}}$ .

3-40.  $t = ?$  Let's take down as the positive direction. From  $d = v_0 t + \frac{1}{2}at^2$  with  $v_0 = 0$  and  $a = g \Rightarrow d = \frac{1}{2}gt^2$

$$\Rightarrow t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(16 \text{ m})}{9.8 \frac{\text{m}}{\text{s}^2}}} = \mathbf{1.8 \text{ s}}.$$

3-41.  $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t} = \frac{12 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}}{3 \text{ s}} = \mathbf{4 \frac{\text{m}}{\text{s}^2}}$ .

3-42.  $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t} = \frac{75 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}}{2.5 \text{ s}} = \mathbf{30 \frac{\text{m}}{\text{s}^2}}$ .

3-43.  $d = ?$  With  $v_0 = 0$ ,  $d = v_0 t + \frac{1}{2}at^2$  becomes  $d = \frac{1}{2}at^2 = \frac{1}{2}\left(2.0 \frac{\text{m}}{\text{s}^2}\right)(8.0 \text{ s})^2 = \mathbf{64 \text{ m}}$ .

3-44.  $a = ?$  With  $v_0 = 0$ ,  $d = v_0 t + \frac{1}{2} a t^2$  becomes  $d = \frac{1}{2} a t^2 \Rightarrow a = \frac{2d}{t^2} = \frac{2(5.0 \text{ m})}{(2.0 \text{ s})^2} = \mathbf{2.5 \frac{m}{s^2}}$ .

3-45.  $d = ?$  With  $v_0 = 0$ ,  $d = v_0 t + \frac{1}{2} a t^2$  becomes  $d = \frac{1}{2} a t^2 = \frac{1}{2} \left( 3.5 \frac{\text{m}}{\text{s}^2} \right) (5.5 \text{ s})^2 = \mathbf{53 \text{ m}}$ .

3-46.  $v_0 = ?$  Here we'll take upwards to be the positive direction, with  $a = -g$  and  $v_f = 0$ .

$$\text{From } v_f^2 - v_0^2 = 2ad \Rightarrow v_0^2 = v_f^2 - 2(-g)d \Rightarrow v_0 = \sqrt{2gd} = \sqrt{2 \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) (3.0 \text{ m})} = \mathbf{7.7 \frac{m}{s}}.$$

3-47.  $t = ?$  We can calculate the time for the ball to reach its maximum height (where the velocity will be zero) and multiply by two to get its total time in the air. Here we'll take upward to be the positive direction, with  $a = -g$ .

$$\text{From } a = \frac{v_f - v_0}{t} \Rightarrow t = \frac{v_f - v_0}{a} = \frac{-v_0}{-g} = \frac{v_0}{g} = \frac{18 \frac{\text{m}}{\text{s}}}{9.8 \frac{\text{m}}{\text{s}^2}} = 1.84 \text{ s. This is the time to reach}$$

the maximum height. The total trip will take  $2 \times 1.84 \text{ s} = \mathbf{3.7 \text{ s}}$ , which is less than 4 s.

Alternatively, this can be done in one step with by recognizing that since the trajectory is symmetric  $v_f = -v_0$ .

$$\text{Then from } v_f = v_0 + at, \text{ with } a = -g \Rightarrow -v_0 = v_0 - gt \Rightarrow -2v_0 = -gt$$

$$\Rightarrow t = \frac{2v_0}{g} = \frac{2 \left( 18 \frac{\text{m}}{\text{s}} \right)}{9.8 \frac{\text{m}}{\text{s}^2}} = \mathbf{3.7 \text{ s}}.$$

3-48.  $v_0 = ?$  Since she throws and catches the ball at the same height,  $v_f = -v_0$ . Calling upward the positive direction,  $a = -g$ .

$$\text{From } v_f = v_0 + at \Rightarrow -v_0 = v_0 + (-g)t \Rightarrow -2v_0 = -gt \Rightarrow v_0 = \frac{gt}{2} = \frac{\left( 9.8 \frac{\text{m}}{\text{s}^2} \right) (3.0 \text{ s})}{2} = \mathbf{15 \frac{m}{s}}.$$

3-49. For a ball dropped with  $v_0 = 0$  and  $a = +g$  (taking downward to be the positive direction),

$$d_{\text{fallen, 1st second}} = v_0 t + \frac{1}{2} a t^2 = \frac{1}{2} \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) (1 \text{ s})^2 = 4.9 \text{ m. At the beginning of the 2nd second we have } v_0 = 9.8 \text{ m/s so}$$

$$d_{\text{fallen, 2nd second}} = v_0 t + \frac{1}{2} a t^2 = 9.8 \frac{\text{m}}{\text{s}} (1 \text{ s}) + \frac{1}{2} \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) (1 \text{ s})^2 = 14.7 \text{ m. The}$$

$$\text{ratio } \frac{d_{\text{fallen, 2nd second}}}{d_{\text{fallen, 1st second}}} = \frac{14.7 \text{ m}}{4.9 \text{ m}} = \mathbf{3}. \text{ More generally, the distance fallen from rest in a time}$$

$t$  is  $d = \frac{1}{2} g t^2$ . in the next time interval  $t$  the distance fallen is

$$d_{\text{from time } t \text{ to } 2t} = v_0 t + \frac{1}{2} a t^2 = (gt)t + \frac{1}{2} g t^2 = \frac{3}{2} g t^2. \text{ The ratios of these two distances is}$$

$$\frac{d_{\text{from time } t \text{ to } 2t}}{d_{\text{from rest in time } t}} = \frac{\frac{3}{2} g t^2}{\frac{1}{2} g t^2} = \mathbf{3}.$$

3-50.  $h = ?$  Call upward the positive direction. From  $v_f^2 - v_0^2 = 2ad$  with  $d = h$ ,  $v_f = 0$  and  $a = -g$

$$\Rightarrow h = \frac{v_f^2 - v_0^2}{2a} = \frac{-v_0^2}{2(-g)} = \frac{v_0^2}{2g} = \frac{\left(1,000 \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)} \approx \mathbf{51,000 \text{ m} > 50 \text{ km.}}$$

3-51.  $h = ?$  With  $d = h$ ,  $v_0 = 22 \frac{\text{m}}{\text{s}}$ ,  $a = -g$  and  $t = 3.5 \text{ s}$ ,  $d = v_0 t + \frac{1}{2} a t^2$  becomes

$$\Rightarrow h = (22 \frac{\text{m}}{\text{s}})(3.5 \text{ s}) + \frac{1}{2} \left(-9.8 \frac{\text{m}}{\text{s}^2}\right)(3.5 \text{ s})^2 = \mathbf{17 \text{ m}}$$

3-52.  $t = ?$  From  $v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{65 \text{ m}}{13 \frac{\text{m}}{\text{s}}} = \mathbf{5.0 \text{ s.}}$

3-53.  $t = ?$  From  $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t} \Rightarrow t = \frac{v_f - v_0}{a} = \frac{28 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}}{7.0 \frac{\text{m}}{\text{s}^2}} = \mathbf{4.0 \text{ s.}}$

3-54. (a)  $t = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow t = \frac{d}{\bar{v}} = \frac{d}{\left(\frac{v_f + v_0}{2}\right)} = \frac{2d}{v}$ .

(b)  $a = ?$  With  $v_0 = 0$  and  $v_f = v$ ,  $v_f^2 - v_0^2 = 2ad$  becomes  $a = \frac{v^2}{2d}$ .

(c)  $t = \frac{2d}{v} = \frac{2(140 \text{ m})}{28 \frac{\text{m}}{\text{s}}} = \mathbf{10 \text{ s}};$   $a = \frac{v^2}{2d} = \frac{\left(28 \frac{\text{m}}{\text{s}}\right)^2}{2(140 \text{ m})} = \mathbf{2.8 \frac{\text{m}}{\text{s}^2}}.$

3-55.  $d = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow d = \bar{v} t = \left(\frac{v_0 + v_f}{2}\right) t = \frac{\left(0 \frac{\text{m}}{\text{s}} + 25 \frac{\text{m}}{\text{s}}\right)}{2} (5.0 \text{ s}) = \mathbf{63 \text{ m.}}$

3-56.  $t = ?$  From  $\bar{v} = \frac{d}{t} \Rightarrow t = \frac{d}{\bar{v}} = \frac{2462 \cancel{\text{mi}} \times \frac{1 \cancel{\text{km}}}{0.621 \cancel{\text{mi}}}}{28,000 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1 \cancel{\text{h}}}{60 \text{ min}}} = \mathbf{8.5 \text{ min.}}$

3-57.  $a = ?$  With  $v_f = 0$ ,  $v_f^2 - v_0^2 = 2ad$  becomes

$$a = \frac{-v_0^2}{2d} = \frac{-\left(220 \frac{\text{mi}}{\text{h}} \times \frac{1 \text{ km}}{0.621 \text{ mi}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}}\right)^2}{2(800 \text{ m})} = -6.05 \frac{\text{m}}{\text{s}^2} \approx \mathbf{-6 \frac{\text{m}}{\text{s}^2}}.$$

3-58. From  $d = v_0 t + \frac{1}{2} a t^2 \Rightarrow \frac{1}{2} a t^2 + v_0 t - d = 0$ . From the general form of the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ we identify } a = \frac{1}{2} a, b = +v_0, \text{ and } c = -d, \text{ which gives}$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2} a\right)(-d)}}{a} = \frac{-v_0 \pm \sqrt{v_0^2 + 2ad}}{a}. \text{ To get a positive answer for } t \text{ we take}$$

$$\text{the positive root, which gives us } t = \frac{-45 \frac{\text{m}}{\text{s}} + \sqrt{\left(45 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(3.2 \frac{\text{m}}{\text{s}^2}\right)(440 \text{ m})}}{3.2 \frac{\text{m}}{\text{s}^2}} = \mathbf{7.7 \text{ s.}}$$

3-59.  $v_0 = ?$  The candy bar just clears the top of the balcony with height  $4.2\text{ m} + 1.1\text{ m} = 5.3\text{ m}$ .

With  $v_f = 0$ ,  $v_f^2 - v_0^2 = 2ad$  with  $v_0$  and  $y$  positive and  $a = -g \Rightarrow v_0^2 = v_f^2 - 2(-g)h$

$$\Rightarrow v_0 = \sqrt{2gh} = \sqrt{2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(5.3\text{ m})} = 10.19 \frac{\text{m}}{\text{s}} = \mathbf{10.2 \frac{m}{s}}. \text{ The total time is the time for the}$$

way to the top of the balcony rail plus the time to fall 1.1 m to the floor of the balcony.

$$t_{\text{up}} = ? \text{ From } d = v_f t - \frac{1}{2}at^2 \text{ with } v_f = 0 \text{ and } a = -g \Rightarrow d = \frac{1}{2}(-g)t^2 \Rightarrow t_{\text{up}} = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(5.3\text{ m})}{\left(9.8 \frac{\text{m}}{\text{s}^2}\right)}} = 1.04\text{ s}.$$

$$t_{\text{down}} = ? \text{ From } d = v_0 t + \frac{1}{2}at^2 \text{ with } v_0 = 0, a = -g \text{ and } d = -h \Rightarrow -h = \frac{1}{2}(-g)t^2 \Rightarrow t_{\text{down}} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.1\text{ m})}{\left(9.8 \frac{\text{m}}{\text{s}^2}\right)}} = 0.47\text{ s}.$$

So  $t_{\text{total}} = t_{\text{up}} + t_{\text{down}} = 1.040\text{ s} + 0.47\text{ s} = \mathbf{1.51\text{ s}}$ . An alternative route is: Since  $v_0$  is upward, call upward the positive direction and put the origin at the ground. Then

$$\text{From } d = v_0 t + \frac{1}{2}at^2 \text{ with } a = -g, d = 4.2\text{ m} \Rightarrow d = v_0 t - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - v_0 t + d = 0.$$

From the general form of the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  we identify

$$a = \frac{g}{2}, b = -v_0, \text{ and } c = d, \text{ which gives } t = \frac{v_0 \pm \sqrt{v_0^2 - 4\left(\frac{g}{2}\right)d}}{g} = \frac{v_0 \pm \sqrt{v_0^2 - 2gd}}{g}$$

$$= \frac{10.19 \frac{\text{m}}{\text{s}} \pm \sqrt{\left(10.19 \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(4.2\text{ m})}}{9.8 \frac{\text{m}}{\text{s}^2}} = 0.57\text{ s or } \mathbf{1.51\text{ s}}. \text{ The first answer}$$

corresponds to the candy reaching 4.2 m but not having gone over the top balcony rail yet. The second answer is the one we want, where the candy has topped the rail and arrives 4.2 m above the ground.

3-58. Consider the subway trip as having three parts—a speeding up part, a constant speed part, and a slowing down part.  $d_{\text{total}} = d_{\text{speeding up}} + d_{\text{constant speed}} + d_{\text{slowing down}}$ .

$$\text{For } d_{\text{speeding up}}, v_0 = 0, a = 1.5 \frac{\text{m}}{\text{s}^2} \text{ and } t = 12\text{ s, so } d = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}\left(1.5 \frac{\text{m}}{\text{s}^2}\right)(12\text{ s})^2 = 108\text{ m}.$$

$$\text{For } d_{\text{constant speed}} = vt. \text{ From the speeding up part we had } v_0 = 0, a = 1.5 \frac{\text{m}}{\text{s}^2} \text{ and } t = 12\text{ s}$$

$$\text{so } v = v_0 + at = \left(1.5 \frac{\text{m}}{\text{s}^2}\right)(12\text{ s}) = 18 \frac{\text{m}}{\text{s}} \text{ and so } d = \left(18 \frac{\text{m}}{\text{s}}\right)(38\text{ s}) = 684\text{ m}$$

$$\text{For } d_{\text{slowing down}}, v_f = 0, a = -1.5 \frac{\text{m}}{\text{s}^2} \text{ and } t = 12\text{ s, so } d = v_f t - \frac{1}{2}at^2 = -\frac{1}{2}\left(-1.5 \frac{\text{m}}{\text{s}^2}\right)(12\text{ s})^2 = 108\text{ m}.$$

$$\text{So } d_{\text{total}} = d_{\text{speeding up}} + d_{\text{constant speed}} + d_{\text{slowing down}} = 108\text{ m} + 684\text{ m} + 108\text{ m} = \mathbf{900\text{ m}}.$$

3-59. One way to approach this is to use Phil's average speed to find how far he has run during the time it takes for Mala to finish the race.

$$\text{From } v = \frac{d}{t} \Rightarrow d_{\text{Phil}} = \bar{v}_{\text{Phil}} t_{\text{Mala}} = \left(\frac{100.0\text{ m}}{13.6\text{ s}}\right)(12.8\text{ s}) = 94.1\text{ m}. \text{ Since Phil has only}$$

traveled 94.1 m when Mala crosses the finish line, he is behind by  $100\text{ m} - 94.1\text{ m} = 5.9\text{ m} \approx \mathbf{6\text{ m}}$ .

- 3-60.  $t = ?$  The time for Terrence to land from his maximum height is the same as the time it takes for him to rise to his maximum height. Let's consider the time for him to land from a height of 0.6 m. Taking down as the positive direction:

$$\text{From } d = v_0 t + \frac{1}{2} a t^2 \text{ with } v_0 = 0 \text{ and } a = g \Rightarrow d = \frac{1}{2} g t^2$$

$$\Rightarrow t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(0.6 \text{ m})}{9.8 \frac{\text{m}}{\text{s}^2}}} = 0.35 \text{ s.}$$

His total time in the air would be twice this amount, **0.7 s**.

$$3-61. \quad v = \frac{d}{t} = \frac{1 \text{ mi}}{\left(45 \text{ s} \times \frac{1 \text{ h}}{3600 \text{ s}}\right)} = \mathbf{80 \frac{mi}{h}}.$$

$$3-62. \quad \bar{v} = \frac{\text{total distance}}{\text{total time}}. \text{ If we call the distance she drives } d, \text{ then from } v = \frac{d}{t} \Rightarrow t = \frac{d}{v}.$$

$$\begin{aligned} \text{So } \bar{v} &= \frac{d_{\text{there}} + d_{\text{back}}}{t_{\text{there}} + t_{\text{back}}} = \frac{2d}{\left(\frac{d}{v_{\text{there}}}\right) + \left(\frac{d}{v_{\text{back}}}\right)} = \frac{2d}{d\left(\frac{1}{v_{\text{there}}} + \frac{1}{v_{\text{back}}}\right)} = \frac{2}{\left(\frac{v_{\text{back}} + v_{\text{there}}}{v_{\text{there}} v_{\text{back}}}\right)} = 2 \frac{v_{\text{there}} v_{\text{back}}}{v_{\text{back}} + v_{\text{there}}} \\ &= 2 \frac{\left(40 \frac{\text{km}}{\text{h}}\right)\left(60 \frac{\text{km}}{\text{h}}\right)}{\left(60 \frac{\text{km}}{\text{h}} + 40 \frac{\text{km}}{\text{h}}\right)} = 2 \left(\frac{2400 \left(\frac{\text{km}}{\text{h}}\right)^2}{100 \frac{\text{km}}{\text{h}}}\right) = \mathbf{48 \frac{km}{h}}. \end{aligned}$$

Note that the average velocity is biased

toward the lower speed since Norma spends more time driving at the lower speed than at the higher speed.