

## Chapter 2: Differentiation

1. Find the slope of the tangent line to the graph of the function below at the given point.

$$f(x) = 2x - 10, (3, -4)$$

- A) 2
- B) -2
- C) -10
- D) 12
- E) none of the above

Ans: A

2. Find the slope of the tangent line to the graph of the function at the given point.

$$f(x) = -5x^2 + 10, (-2, -10)$$

- A) 20
- B) -5
- C) -10
- D) -20
- E) none of the above

Ans: A

3. Find the slope of the tangent line to the graph of the function at the given point.

$$f(x) = 2x^2 + 6, (3, 24)$$

- A) 12
- B) 2
- C) -6
- D) 18
- E) none of the above

Ans: A

4. Use the limit definition to find the slope of the tangent line to the graph of

$$f(x) = \sqrt{4x + 29} \text{ at the point } (5, 7).$$

- A)  $\frac{2}{7}$
- B)  $-\frac{2}{7}$
- C)  $\frac{1}{7}$
- D)  $-\frac{1}{7}$
- E)  $\frac{1}{5}$

Ans: A

5. Find the derivative of the following function using the limiting process.

$$f(x) = -2x^2 - 9x$$

- A)  $-2$
- B)  $-4x - 9$
- C)  $-4x + 9$
- D)  $-4x$
- E) none of the above

Ans: B

6. Find the derivative of the following function using the limiting process.

$$f(x) = \sqrt{9x - 6}$$

- A)  $f'(x) = \frac{9}{2\sqrt{9x - 6}}$
- B)  $f'(x) = -\frac{9}{2\sqrt{9x - 6}}$
- C)  $f'(x) = \frac{9}{2}(9x - 6)^{1/2}$
- D)  $f'(x) = -\frac{9}{\sqrt{9x - 6}}$
- E) either B or D

Ans: A

7. Find the derivative of the following function using the limiting process.

$$f(x) = \frac{2}{x - 9}$$

- A)  $f'(x) = \frac{2}{(x - 9)^2}$
- B)  $f'(x) = -\frac{2}{(x + 9)^2}$
- C)  $f'(x) = -\frac{2}{(x + 9)}$
- D)  $f'(x) = -\frac{2}{(x - 9)^2}$
- E) none of the above

Ans: D

8. Find an equation of the line that is tangent to the graph of  $f$  and parallel to the given line.

$$f(x) = 5x^2, \quad 20x - y + 2 = 0$$

- A)  $y = 20x - 20$
- B)  $y = 20x + 20$
- C)  $y = -20x + 20$
- D)  $y = -20x - 20$
- E) none of the above

Ans: A

9. Find an equation of the a line that is tangent to the graph of  $f$  and parallel to the given line.

$$f(x) = 5x^3, \quad 135x - y + 6 = 0$$

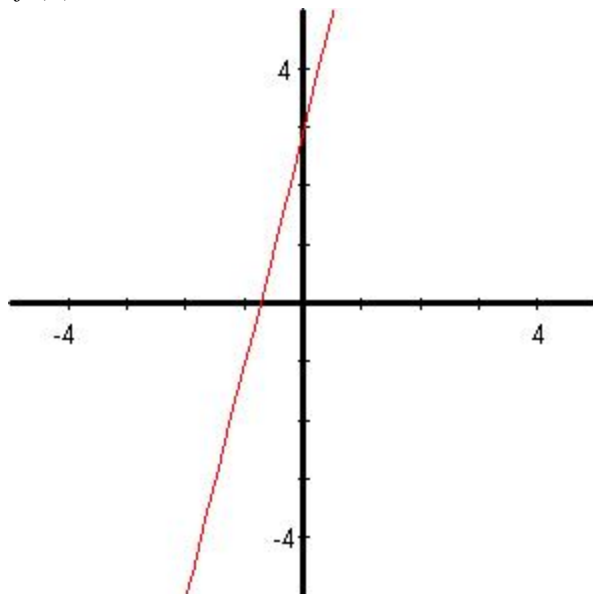
- A)  $y = -135x - 270$
- B)  $y = 135x + 270$
- C)  $y = -135x + 270$
- D)  $y = 135x - 270$
- E) both B and D

Ans: E

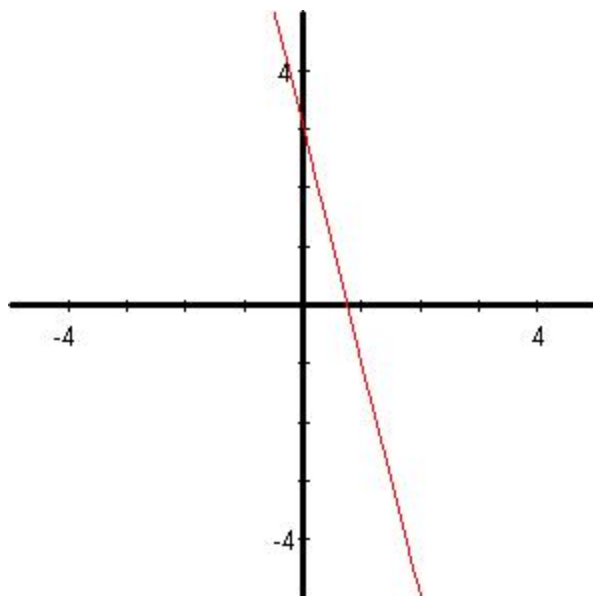
10. Identify a function  $f(x)$  that has the given characteristics and then sketch the function.

$$f(0) = 3; f'(x) = 4, -\infty < x < \infty$$

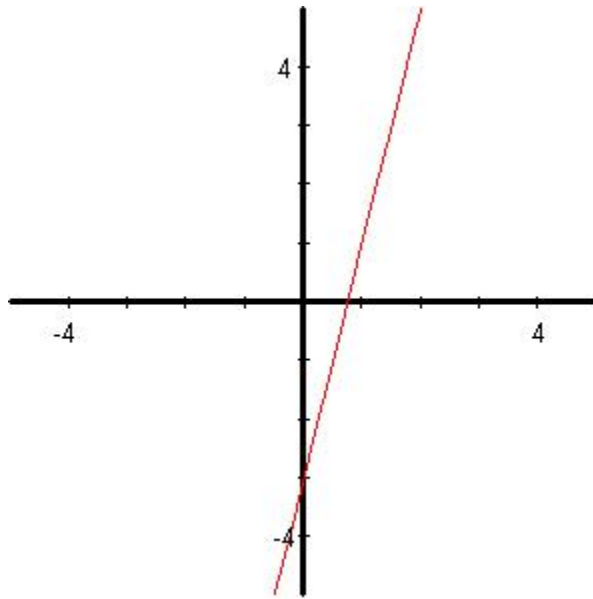
A)  $f(x) = 4x + 3$



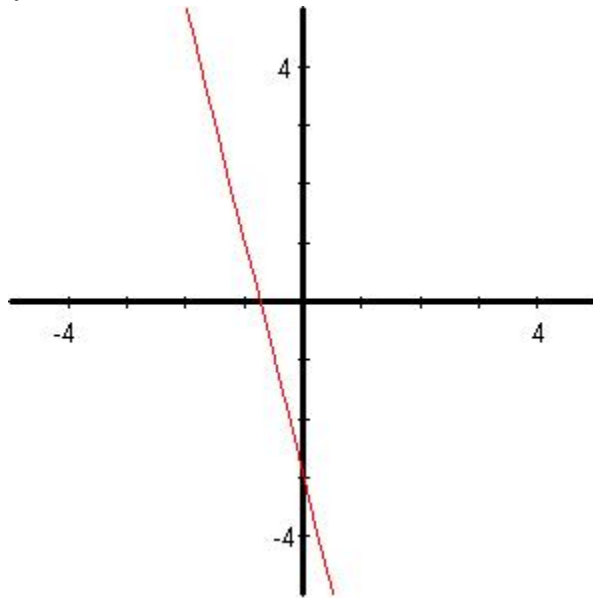
B)  $f(x) = -4x + 3$



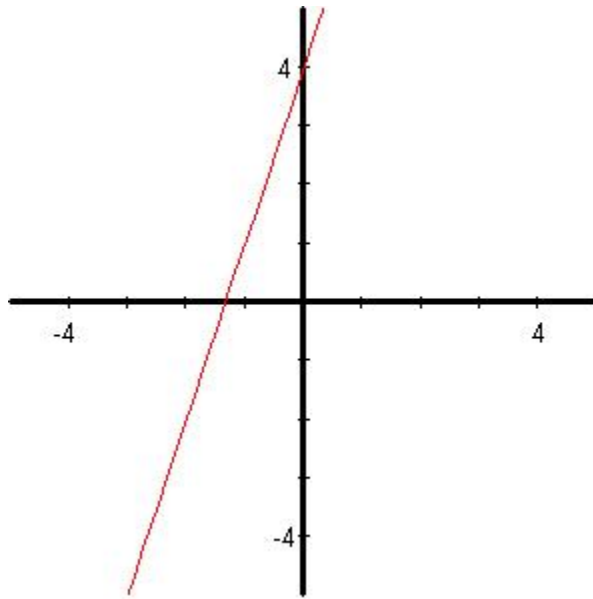
C)  $f(x) = 4x - 3$



D)  $f(x) = -4x - 3$



E)  $f(x) = 3x + 4$



Ans: A

11. Find the derivative of the function.

$$f(x) = x^7$$

- A)  $f'(x) = 7x^7$
- B)  $f'(x) = 7x^6$
- C)  $f'(x) = 6x^6$
- D)  $f'(x) = 6x^8$
- E) none of the above

Ans: B

12. Find the derivative of the function.

$$f(x) = 2x^3 - 3x^2 + 1$$

- A)  $f'(x) = 6x^2 - 6x$
- B)  $f'(x) = 4x^2 - 3x$
- C)  $f'(x) = 4x - 3x^2$
- D)  $f'(x) = 6x^2 - 6x + 1$
- E) none of the above

Ans: A

13. For the function given, find  $f'(x)$ .

$$f(x) = x^3 - 15x - 6$$

- A)  $x^2 - 15$
- B)  $3x^2 - 6$
- C)  $3x^2 - 15$
- D)  $3x^3 - 15x$
- E)  $x^3 - 15x - 6$

Ans: C

14. Find the derivative of the function.

$$h(x) = 15x^{23} + 11x^{13} - 4x^{10} + 3x - 7$$

- A)  $330x^{22} + 132x^{12} - 36x^9 + 3$
- B)  $345x^{23} + 143x^{13} - 40x^{10} + 3x$
- C)  $15x^{22} + 11x^{12} - 4x^9 + 3$
- D)  $345x^{22} + 143x^{12} - 40x^9 + 3$
- E)  $330x^{23} + 132x^{13} - 36x^{10} + 3x$

Ans: D

15. Find the derivative of the function  $h(x) = x^{5/3}$ .

- A)  $h'(x) = \frac{5}{3}x^{8/3}$
- B)  $h'(x) = -\frac{5}{3}x^{2/3}$
- C)  $h'(x) = \frac{5}{3}x^{2/3}$
- D)  $h'(x) = -\frac{5}{3}x^{8/3}$
- E)  $h'(x) = \frac{5}{3}x^{-2/3}$

Ans: C

16. Find the derivative of the function  $s(t) = 2x^{-2} + 8$ .

- A)  $s'(t) = \frac{4}{x^3}$
- B)  $s'(t) = -\frac{4}{x^3}$
- C)  $s'(t) = -\frac{4}{x^3} + 8$
- D)  $s'(t) = \frac{4}{x^3} + 8$
- E)  $s'(t) = 2x^{-3}$

Ans: B

17. Find the derivative of the function.

$$f(x) = \frac{1}{x^3}$$

A)  $f'(x) = -\frac{2}{x^4}$

B)  $f'(x) = -\frac{3}{x^2}$

C)  $f'(x) = -\frac{3}{x^4}$

D)  $f'(x) = -\frac{4}{x^4}$

E) none of the above

Ans: C

18. Differentiate the given function.

$$y = \frac{3}{4x^4}$$

A)  $-\frac{12}{x^5}$

B)  $-\frac{3}{x^4}$

C)  $-\frac{12}{x^4}$

D)  $-\frac{3}{x^5}$

E)  $-\frac{4}{x^5}$

Ans: D

19. Differentiate the given function.

$$y = \frac{5}{(4x)^4}$$

A)  $\frac{80}{(4x)^5}$

B)  $-\frac{20}{(4x)^5}$

C)  $-\frac{80}{(4x)^5}$

D)  $\frac{20}{(4x)^5}$

E)  $-\frac{20}{(4x)^3}$

Ans: C



20. Determine the point(s), (if any), at which the graph of the function has a horizontal tangent.

$$y(x) = x^4 - 32x + 1$$

- A) 0
- B) 0 and 2
- C) 0 and -2
- D) 2
- E) There are no points at which the graph has a horizontal tangent.

Ans: D

21. The graph shows the number of visitors  $V$  to a national park in hundreds of thousands during a one-year period, where  $t = 1$  represents January. Estimate the rate of change of  $V$  over the interval  $[5, 8]$ . Round your answer to the nearest hundred thousand visitors per year.



- A) 176.92 hundred thousand visitors per year
- B) 328.57 hundred thousand visitors per year
- C) 166.67 hundred thousand visitors per year
- D) 383.33 hundred thousand visitors per year
- E) 766.67 hundred thousand visitors per year

Ans: C

22. Find the marginal cost for producing  $x$  units. (The cost is measured in dollars.)

$$C = 205,000 + 9800x$$

- A) \$9800
- B) \$9850
- C) \$8800
- D) \$8850
- E) \$9750

Ans: A

23. Find the marginal revenue for producing  $x$  units. (The revenue is measured in dollars.)

$$R = 50x - 0.5x^2$$

- A)  $50 - x$  dollars
- B)  $50 + x$  dollars
- C) 50 dollars
- D)  $50 - 0.5x$  dollars
- E)  $50 + 0.5x$  dollars

Ans: A

24. Find the marginal profit for producing  $x$  units. (The profit is measured in dollars.)

$$P = -2x^2 + 72x - 145$$

- A)  $-4x + 72$  dollars
- B)  $4x + 72$  dollars
- C)  $-4x - 72$  dollars
- D)  $4x - 72$  dollars
- E)  $-4 + 72x$  dollars

Ans: A

25. The cost  $C$  (in dollars) of producing  $x$  units of a product is given by  $C = 3.6\sqrt{x} + 500$ . Find the additional cost when the production increases from 9 to 10.

- A) \$0.58
- B) \$0.36
- C) \$0.62
- D) \$0.12
- E) \$0.64

Ans: A

26. The profit (in dollars) from selling  $x$  units of calculus textbooks is given by  $p = -0.05x^2 + 20x - 3000$ . Find the additional profit when the sales increase from 145 to 146 units. Round your answer to two decimal places.

- A) \$5.45
- B) \$20.00
- C) \$5.55
- D) \$11.00
- E) \$10.80

Ans: A

27. The profit (in dollars) from selling  $x$  units of calculus textbooks is given by  $p = -0.05x^2 + 20x - 1000$ . Find the marginal profit when  $x = 148$ . Round your answer to two decimal places.

- A) \$34.80
- B) \$864.80
- C) \$5.20
- D) \$20.00
- E) \$859.55

Ans: C

28. The population  $P$  ( in thousands) of Japan from 1980 through 2010 can be modeled by  $P = -15.56t^2 + 802.1t + 117,001$  where  $t$  is the year, with  $t = 0$  corresponding to 1980.

Determine the population growth rate,  $dP/dt$ .

- A)  $dP/dt = -31.12t + 802.1$
- B)  $dP/dt = 31.12t + 802.1$
- C)  $dP/dt = -31.12t - 802.1$
- D)  $dP/dt = 31.12t - 802.1$
- E)  $dP/dt = -31.12 + 802.1t$

Ans: A

29. When the price of a glass of lemonade at a lemonade stand was \$1.75, 400 glasses were sold. When the price was lowered to \$1.50, 500 glasses were sold. Assume that the demand function is linear and that the marginal and fixed costs are \$0.10 and \$ 25, respectively. Find the profit  $P$  as a function of  $x$ , the number of glasses of lemonade sold.

- A)  $P = -0.0025x^2 + 2.65x - 25$
- B)  $P = 0.0025x^2 + 2.65x - 25$
- C)  $P = -0.0025x^2 + 2.65x + 25$
- D)  $P = 0.0025x^2 - 2.65x - 25$
- E)  $P = -0.0025x^2 + 2.65x + 25$

Ans: A

30. When the price of a glass of lemonade at a lemonade stand was \$1.75, 400 glasses were sold. When the price was lowered to \$1.50, 500 glasses were sold. Assume that the demand function is linear and that the marginal and fixed costs are \$0.10 and \$ 25, respectively. Find the marginal profit when 300 glasses of lemonade are sold and when 700 glasses of lemonade are sold.

- A)  $P'(300) = 1.15, P'(700) = -0.85$
- B)  $P'(300) = -0.85, P'(700) = 1.15$
- C)  $P'(300) = 1.15, P'(700) = 0.85$
- D)  $P'(300) = 0.85, P'(700) = -1.15$
- E)  $P'(300) = -1.15, P'(700) = -0.85$

Ans: A

31. Use the product Rule to find the derivative of the function  $f(x) = x(x^2 + 3)$ .

- A)  $f'(x) = 3x^2 + 3$
- B)  $f'(x) = 3x^2 + 1$
- C)  $f'(x) = x^2 + 3$
- D)  $f'(x) = 3x^2 - 3$
- E)  $f'(x) = 3x^2 - 1$

Ans: A

32. Find the derivative of the function  $f(x) = \frac{x^3 + 6x}{3}$ .

- A)  $f'(x) = x^2 + 2$
- B)  $f'(x) = x^2 + 6$
- C)  $f'(x) = x^2 + 2x$
- D)  $f'(x) = x^2 + x$
- E)  $f'(x) = x^2 - 2x$

Ans: A

33. Find the derivative of the function  $f(x) = \frac{x^2 - x - 20}{x + 4}$ . State which differentiation rule(s) you used to find the derivative.

- A) 1, Product Rule.
- B) 1, Quotient Rule
- C) 5, Product Rule.
- D) 5, Quotient Rule
- E)  $x+3$ , Product Rule.

Ans: A

34. Find the point(s), if any, at which the graph of  $f$  has a horizontal tangent line.

$$f(x) = \frac{x^2}{x-1}$$

- A)  $(0,0), (2,4)$
- B)  $(0,2), (0,4)$
- C)  $(4,0), (2,0)$
- D)  $(0,4), (2,0)$
- E)  $(0,0), (4,2)$

Ans: A

35. A population of bacteria is introduced into a culture. The number of bacteria  $P$  can be modeled by  $P = 500 \left( 1 + \frac{4t}{50 + t^2} \right)$  where  $t$  is the time (in hours). Find the rate of change of the population when  $t = 2$ .

- A) 31.55 bacteria/hr
- B) 29.15 bacteria/hr
- C) 33.65 bacteria/hr
- D) 32.75 bacteria/hr
- E) 30.25 bacteria/hr

Ans: A

36. Use the given information to find  $f'(2)$  of the function  $f(x) = g(x)h(x)$ .

$$g(2) = 3 \text{ and } g'(2) = -2, h(2) = -1 \text{ and } h'(2) = 4$$

- A)  $f'(2) = 14$
- B)  $f'(2) = -11$
- C)  $f'(2) = 17$
- D)  $f'(2) = -9$
- E)  $f'(2) = 12$

Ans: A

37. Find an equation of the tangent line to the graph of  $f$  at the given point.

$$f(s) = (s-4)(s^2-3), \text{ at } (1, 6)$$

- A)  $y = 8s + 14$
- B)  $y = 2s - 8$
- C)  $y = -8s - 2$
- D)  $y = -8s + 14$
- E)  $y = -8 + 14s$

Ans: D

38. Find an equation of the tangent line to the graph of  $f$  at the given point.

$$f(s) = (s-5)(s^2-6), \text{ at } (3, -6)$$

- A)  $y = 9s + 21$
- B)  $y = 33s - 9$
- C)  $y = -9s - 33$
- D)  $y = -9s + 21$
- E)  $y = -9 + 21s$

Ans: D

39. Use the demand function  $x = 325 \left( 1 - \frac{4p}{5p+4} \right)$  to find the rate of change in the demand

$x$  for the given price  $p = \$2.00$ . Round your answer to two decimal places.

- A) 26.53 units per dollar
- B) -6.63 units per dollar
- C) 6.63 units per dollar
- D) 36.11 units per dollar
- E) -26.53 units per dollar

Ans: E

40. A population of bacteria is introduced into a culture. The number of bacteria  $P$  can be modeled by  $P = 225\left(1 + \frac{4t}{45 + t^2}\right)$  where  $t$  is the time (in hours). Find the rate of change of the population when  $t = 4.00$ .

- A) 31.03 units per dollar
- B) 1.75 units per dollar
- C) 7.01 units per dollar
- D) 3.63 units per dollar
- E) 7.76 units per dollar

Ans: C

41. Find  $dy/du$ ,  $du/dx$ , and  $dy/dx$  of the functions  $y = u^2$ ,  $u = 4x + 7$ .

- A)  $dy/du = 2u$ ,  $du/dx = 4$ , and  $dy/dx = 32x + 56$
- B)  $dy/du = 2u$ ,  $du/dx = 2$ , and  $dy/dx = 16x + 49$
- C)  $dy/du = 4u$ ,  $du/dx = 4$ , and  $dy/dx = 32x + 56$
- D)  $dy/du = 4u$ ,  $du/dx = 2$ , and  $dy/dx = 32x + 56$
- E)  $dy/du = 2u$ ,  $du/dx = 4$ , and  $dy/dx = 16x + 49$

Ans: A

42. Find  $\frac{dy}{dx}$  of  $y = \sqrt{u}$ ,  $u = 7 - x^2$ .

- A)  $\frac{x}{\sqrt{7 - x^2}}$
- B)  $\frac{1}{2\sqrt{7 - x^2}}$
- C)  $\frac{-x}{\sqrt{7 - x^2}}$
- D)  $-\frac{1}{2\sqrt{7 - x^2}}$

- E) none of these choices

Ans: C

43. Find the derivative of the function.

$$f(t) = (1 + 3t)^{\frac{4}{7}}$$

A)  $f'(t) = \frac{1}{7}(1 + 3t)^{\frac{-3}{7}}$

B)  $f'(t) = \frac{12}{4}(1 + 3t)^{\frac{-3}{7}}$

C)  $f'(t) = \frac{12}{7}(1 + 3t)^{\frac{-3}{4}}$

D)  $f'(t) = \frac{3}{7}(1 + 3t)^{\frac{-3}{7}}$

E)  $f'(t) = \frac{12}{7}(1 + 3t)^{\frac{-3}{7}}$

Ans: E

44. Differentiate the given function.

$$y = \sqrt{5x^9 + 9x}$$

A)  $\frac{1}{2}(45x^8 + 9)^{-1/2}$

B)  $\frac{1}{2}(5x^9 + 9x)^{-1/2}$

C)  $\frac{1}{2}(45x^9 + 9x)^{-1/2}(5x^9 + 9)$

D)  $\frac{1}{2}(5x^9 + 9x)^{-1/2}(45x^8 + 9)$

E)  $-\frac{1}{2}(5x^9 + 9x)^{-3/2}(45x^8 + 9)$

Ans: D

45. Find the derivative of the function.

$$f(x) = x^8(7 + 6x)^4$$

A)  $f'(x) = x^3(7 + 6x)^7(56 + 72x)$

B)  $f'(x) = 6x^8(7 + 6x)^3(56 + 72x)$

C)  $f'(x) = x^7(7 + 6x)^4(56 + 72x)$

D)  $f'(x) = x^7(7 + 6x)^3(56 + 72x)$

E)  $f'(x) = x^7(7 + 6x)^3(56 + 6x)$

Ans: D

46. Find the derivative of the given function. Simplify and express the answer using positive exponents only.

$$c(x) = 3x\sqrt{x^7 + 5}$$

- A)  $\frac{3(9x^7 - 10)}{2(x^7 + 5)^{1/2}}$
- B)  $\frac{3(7x^7 - 10)}{2(x^7 + 5)^{1/2}}$
- C)  $\frac{3(7x^7 - 10)}{(x^7 + 5)^{1/2}}$
- D)  $\frac{3(9x^7 + 10)}{2(x^7 + 5)^{1/2}}$
- E)  $\frac{3(7x^7 + 10)}{(x^7 + 5)^{1/2}}$

Ans: D

47. Find the derivative of the function.

$$f(x) = x^8\sqrt{4 - 2x}$$

- A)  $f'(x) = \frac{x^7(64 - 34x)}{2\sqrt{4 - 2x}}$
- B)  $f'(x) = \frac{x^7(64 + 34x)}{2\sqrt{4 - 2x}}$
- C)  $f'(x) = \frac{x^7(4 - 34x)}{2\sqrt{4 - 2x}}$
- D)  $f'(x) = \frac{x^7(64 - 2x)}{2\sqrt{4 - 2x}}$
- E)  $f'(x) = \frac{x^7(4 + 2x)}{2\sqrt{4 - 2x}}$

Ans: A



48. Find the derivative of the function.

$$g(x) = \left( \frac{x+5}{x^2+5} \right)^5$$

- A)  $g'(x) = \frac{5(5-10x+x^2)}{(5+x)(5+x^2)} \left( \frac{(5+x)}{(5+x^2)} \right)^5$
- B)  $g'(x) = \frac{5(5+10x-x^2)(5+x)^4}{(5+x^2)^6}$
- C)  $g'(x) = \frac{5(5-10x-x^2)(5+x)^6}{(5+x^2)^4}$
- D)  $g'(x) = -\frac{5(5-10x-x^2)(5+x)^4}{(5+x^2)^6}$
- E)  $g'(x) = \frac{5(5-10x-x^2)(5+x)^4}{(5+x^2)^6}$

Ans: E

49. You deposit \$4000 in an account with an annual interest rate of change  $r$  (in decimal form) compounded monthly. At the end of 4 years, the balance is  $A = 4000 \left( 1 + \frac{r}{12} \right)^{48}$ .

Find the rates of change of  $A$  with respect to  $r$  when  $r = 0.13$ .

- A) 6709.32
- B) 318,595.99
- C) 559.11
- D) 26549.67
- E) 26,265.13

Ans: D

50. The value  $V$  of a machine  $t$  years after it is purchased is inversely proportional to the square root of  $t+5$ . The initial value of the machine is \$10,000. Find the rate of depreciation when  $t = 2$ . Round your answer to two decimal places.

- A) -603.68 per year
- B) -1889.82 per year
- C) 1767.77 per year
- D) 447.21 per year
- E) -1207.36 per year

Ans: A

51. Find the second derivative of the function.

$$f(x) = 5x^{\frac{3}{13}}$$

A)  $f''(x) = \frac{-150}{169} x^{\frac{10}{13}}$

B)  $f''(x) = \frac{3}{169} x^{\frac{-23}{13}}$

C)  $f''(x) = \frac{845}{169} x^{\frac{-23}{13}}$

D)  $f''(x) = \frac{-150}{169} x^{\frac{-23}{13}}$

E) None of the above

Ans: D

52. Find the third derivative of the function  $f(x) = x^5 - 3x^4$ .

A)  $60x^2 - 72x$

B)  $30x^2 - 36x$

C)  $60x^2 - 72x^2$

D)  $60x^2 - 36x$

E)  $30x^2 - 36x$

Ans: A

53. Find the  $f^{(6)}(x)$  of  $f^{(4)}(x) = (x^2 + 1)^2$ .

A)  $12x^2 + 4$

B)  $12x^2 + 2$

C)  $6x^2 + 4$

D)  $6x^2 + 2$

E)  $12x^2 + 1$

Ans: A

54. Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

If  $y = f(x)g(x)$ , then  $y' = f'(x)g'(x)$

A) True

B) False. The product rule is  $[f(x)g(x)]' = f(x)g'(x) + g(x)f'(x)$

Ans: B

55. Find the third derivative.

$$y = \frac{2}{x^5}$$

A)  $\frac{-420}{x^7}$

B)  $\frac{420}{x^8}$

C) 0

D)  $\frac{84}{x^7}$

E)  $\frac{-420}{x^8}$

Ans: E

56. Find the value  $g''(4)$  for the function  $g(t) = 3t^8 + 6t^6 + 1$ .

A) 734,208

B) 430,080

C) 221,185

D) 430,081

E) 3,403,776

Ans: A

57. Find the indicated derivative.

Find  $y^{(4)}$  if  $y = x^8 - 4x^3$ .

A)  $336x^5$

B)  $336x^4$

C)  $336x^4 - 24x$

D)  $1680x^5 - 24x$

E)  $1680x^4$

Ans: E

58. Find the second derivative for the function  $f(x) = 4x^3 + 12x^2 - 20x - 18$  and solve the equation  $f''(x) = 0$ .

A) -1

B) 4

C) 0

D) 18

E) 20

Ans: A

59. Find the second derivative for the function  $f(x) = \frac{5x}{5x+7}$  and solve the equation

$$f''(x) = 0.$$

- A) 0
- B) 7
- C) no solution
- D) -7
- E)  $-\frac{1}{7}$

Ans: C

60. A brick becomes dislodged from the Empire State Building (at a height of 1025 feet) and falls to the sidewalk below. Write the position  $s(t)$ , velocity  $v(t)$ , and acceleration  $a(t)$  as functions of time.

- A)  $s(t) = 16t^2 + 1025$ ;  $v(t) = 32t$ ;  $a(t) = 32$
- B)  $s(t) = -16t^2 - 1025$ ;  $v(t) = -32t$ ;  $a(t) = -32$
- C)  $s(t) = -16t^2 + 1025$ ;  $v(t) = -32t$ ;  $a(t) = -32$
- D)  $s(t) = 16t^2 - 1025$ ;  $v(t) = -32t$ ;  $a(t) = -32$
- E)  $s(t) = -16t^2 + 1025$ ;  $v(t) = -32$ ;  $a(t) = -32t$

Ans: C

61. Find  $y'$  implicitly for  $6x^9 - y^9 = 3$ .

- A)  $y' = \frac{6x^9}{y^9}$
- B)  $y' = \frac{y^9}{6x^9}$
- C)  $y' = \frac{6x^8}{y^8}$
- D)  $y' = \frac{y^8}{6x^8}$
- E)  $y' = \frac{x^8}{6y^8}$

Ans: C

62. Find  $\frac{dy}{dx}$  for the equation  $\frac{9x+7y}{5x-6y} = 4$ .

A)  $\frac{dy}{dx} = -\frac{11}{31}$

B)  $\frac{dy}{dx} = \frac{29}{31}$

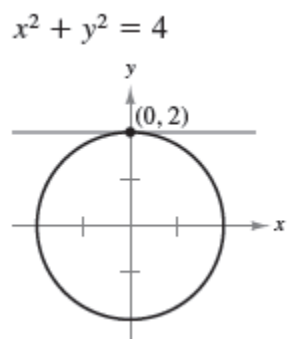
C)  $\frac{dy}{dx} = \frac{11}{31}$

D)  $\frac{dy}{dx} = -\frac{29}{31}$

E)  $\frac{dy}{dx} = 4$

Ans: C

63. Find the slope of the graph at the given point.



A) 0

B) 3

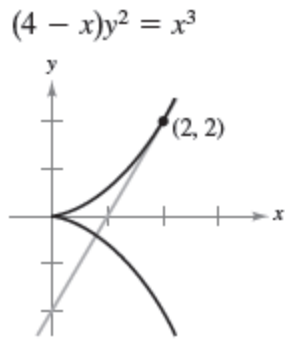
C) 5

D) 4

E) 7

Ans: A

64. Find the slope of the graph at the given point.



- A) 2
  - B) 0
  - C) 1
  - D) 3
  - E) 5
- Ans: A

65. Find the rate of change of  $x$  with respect to  $p$ .

$$p = \frac{2}{0.00001x^3 + 0.1x} \quad x \geq 0$$

- A)  $-\frac{2}{p^2(0.00003x^2 + 0.1)}$
  - B)  $-\frac{2}{p(0.00003x^2 + 0.1)}$
  - C)  $-\frac{2}{p^2x(0.00003x^2 + 0.1)}$
  - D)  $-\frac{2}{px(0.00003x^2 + 0.1)}$
  - E)  $-\frac{2x}{p^2(0.00003x^2 + 0.1)}$
- Ans: A

66. Find the rate of change of  $x$  with respect to  $p$ .

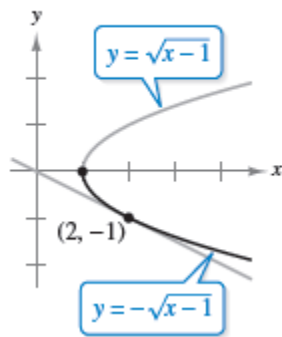
$$p = \sqrt{\frac{200-x}{2x}}, \quad 0 < x \leq 200$$

- A)  $-\frac{4xp}{2p^2+1}$
- B)  $\frac{4xp}{2p^2+1}$
- C)  $-\frac{4x}{2p^2+1}$
- D)  $\frac{4x}{2p^2+1}$
- E)  $-\frac{4xp}{2p+1}$

Ans: A

67. Find  $dy/dx$  implicitly and explicitly (the explicit functions are shown on the graph) and show that the results are equivalent. Use the graph to estimate the slope of the tangent line at the labeled point. Then verify your result analytically by evaluating  $dy/dx$  at the point.

$$x - y^2 - 1 = 0$$



- A)  $\frac{1}{2y}, -\frac{1}{2}$
- B)  $-\frac{1}{2y}, \frac{1}{2}$
- C)  $-\frac{1}{2y}, -\frac{1}{2}$
- D)  $\frac{1}{2y}, \frac{1}{2}$
- E)  $\frac{1}{2}, -\frac{1}{2}$

Ans: A

68. Let  $x$  represent the units of labor and  $y$  the capital invested in a manufacturing process. When 135,540 units are produced, the relationship between labor and capital can be modeled by  $100x^{0.75}y^{0.25} = 135,540$ . Find the rate of change of  $y$  with respect to  $x$  when  $x = 1500$  and  $y = 135,540$ .

A) -2  
B) 0  
C) 3  
D) -7  
E) 5

Ans: A

69. Find  $dy/dx$  for the following equation:

$$2x + y^2 - 5y + 9 = 0.$$

A)  $\frac{dy}{dx} = \frac{5}{2-2y}$   
B)  $\frac{dy}{dx} = \frac{2}{5-2y}$   
C)  $\frac{dy}{dx} = \frac{1}{5-y}$   
D)  $\frac{dy}{dx} = \frac{5}{5-y}$   
E)  $\frac{dy}{dx} = \frac{1}{2-y}$

Ans: B

70. Find  $\frac{dy}{dx}$  for the equation  $\sqrt{xy} = x - 20y$  by implicit differentiation and evaluate the derivative at the point  $(50, 2)$ .

A)  $-\frac{1}{25}$   
B)  $\frac{1}{25}$   
C)  $\frac{3}{25}$   
D)  $-\frac{3}{25}$   
E) 0

Ans: B



71. Assume that  $x$  and  $y$  are differentiable functions of  $t$ . Find  $dy/dt$  using the given values.

$$y = 4x^3 + 6x^2 - x \text{ for } x = 3, dx/dt = 2.$$

- A) 288
- B) 159
- C) 318
- D) 286
- E) 143

Ans: D

72. Given  $xy = 10$ , find  $\frac{dy}{dt}$  when  $x = -9$  and  $\frac{dx}{dt} = 3$ .

- A)  $\frac{dy}{dt} = \frac{260}{27}$
- B)  $\frac{dy}{dt} = \frac{10}{27}$
- C)  $\frac{dy}{dt} = -\frac{10}{27}$
- D)  $\frac{dy}{dt} = -\frac{27}{10}$
- E)  $\frac{dy}{dt} = -\frac{27}{260}$

Ans: C

73. Assume that  $x$  and  $y$  are differentiable functions of  $t$ . Find  $dx/dt$  given that  $x = 2$ ,  $y = 8$ , and  $dy/dt = 3$ .

$$y^2 - x^2 = 60$$

- A) 1.50
- B) 5.33
- C) 0.75
- D) 24.00
- E) 12.00

Ans: E

74. **Area.** The radius,  $r$ , of a circle is increasing at a rate of 5 centimeters per minute.

Find the rate of change of area,  $A$ , when the radius is 4.

- A)  $\frac{dA}{dt} = 20\pi$
- B)  $\frac{dA}{dt} = 160\pi$
- C)  $\frac{dA}{dt} = -160\pi$
- D)  $\frac{dA}{dt} = 40\pi$
- E)  $\frac{dA}{dt} = -40\pi$

Ans: D

75. **Volume and radius.** Suppose that air is being pumped into a spherical balloon at a rate of  $4 \text{ in.}^3 / \text{min.}$  At what rate is the radius of the balloon increasing when the radius is 7 in.?

- A)  $\frac{dr}{dt} = \frac{4}{49\pi}$
- B)  $\frac{dr}{dt} = \frac{1}{7\pi}$
- C)  $\frac{dr}{dt} = \frac{49}{4\pi}$
- D)  $\frac{dr}{dt} = \frac{7}{4\pi}$
- E)  $\frac{dr}{dt} = \frac{1}{49\pi}$

Ans: E

76. The radius  $r$  of a sphere is increasing at a rate of 3 inches per minute. Find the rate of change of volume when  $r = 8$  inches. Round your answer to one decimal place.

- A) 804.2 cubic inches per minute
- B) 2144.7 cubic inches per minute
- C) 6434.0 cubic inches per minute
- D) 2412.7 cubic inches per minute
- E) 7238.2 cubic inches per minute

Ans: D

77. **Profit.** Suppose that the monthly revenue and cost (in dollars) for  $x$  units of a product are  $R = 900x - \frac{x^2}{50}$  and  $C = 4000 + 30x$ . At what rate per month is the profit changing if the number of units produced and sold is 100 and is increasing at a rate of 10 units per month?

- A) \$86,960 per month
- B) \$8660 per month
- C) \$8960 per month
- D) \$260 per month
- E) \$89,960 per month

Ans: B

78. The lengths of the edges of a cube are increasing at a rate of 8 ft/min. At what rate is the surface area changing when the edges are 15 ft long?

- A) 384 ft<sup>2</sup>/min
- B) 1440 ft<sup>2</sup>/min
- C) 720 ft<sup>2</sup>/min
- D) 5760 ft<sup>2</sup>/min
- E) 120 ft<sup>2</sup>/min

Ans: B

79. A point is moving along the graph of the function  $y = 9x^2 + 2$  such that  $\frac{dx}{dt} = 3$  centimeters per second.

Find  $dy/dt$  for the given values of  $x$ .

(a)  $x = 4$

(b)  $x = 8$

A)  $\frac{dy}{dt} = 4$

$\frac{dy}{dt} = 432$

B)  $\frac{dy}{dt} = 216$

$\frac{dy}{dt} = 432$

C)  $\frac{dy}{dt} = 432$

$\frac{dy}{dt} = 216$

D)  $\frac{dy}{dt} = 8$

$\frac{dy}{dt} = -216$

E)  $\frac{dy}{dt} = 8$

$\frac{dy}{dt} = 432$

Ans: B

80. A point is moving along the graph of the function  $y = \frac{1}{7x^2 + 2}$  such that  $\frac{dx}{dt} = 5$  centimeters per second.

Find  $dy/dt$  when  $x = 3$ .

- A)  $\frac{dy}{dt} = -\frac{42}{4225}$   
 B)  $\frac{dy}{dt} = -\frac{42}{845}$   
 C)  $\frac{dy}{dt} = \frac{42}{845}$   
 D)  $\frac{dy}{dt} = \frac{42}{4225}$   
 E)  $\frac{dy}{dt} = -\frac{42}{13}$

Ans: B

81. **Boat docking.** Suppose that a boat is being pulled toward a dock by a winch that is 21 ft above the level of the boat deck. If the winch is pulling the cable at a rate of 23 ft/min, at what rate is the boat approaching the dock when it is 28 ft from the dock? Use the figure below.



- A) 28.75 ft/min  
 B) 23.00 ft/min  
 C) 38.33 ft/min  
 D) 17.25 ft/min  
 E) 13.80 ft/min

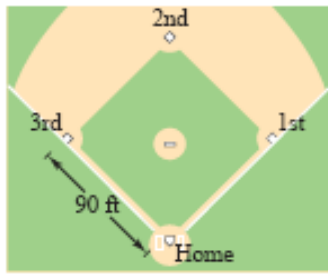
Ans: A

82. An airplane flying at an altitude of 5 miles passes directly over a radar antenna. When the airplane is 25 miles away ( $s = 25$ ), the radar detects that the distance  $s$  is changing at a rate of 250 miles per hour. What is the speed of the airplane? Round your answer to the nearest integer.

- A) 255 mi/hr  
 B) 236 mi/hr  
 C) 510 mi/hr  
 D) 128 mi/hr  
 E) 118 mi/hr

Ans: A

83. A baseball diamond has the shape of a square with sides 90 feet long (see figure). A player running from second base to third base at a speed of 30 feet per second is 80 feet from third base. At what rate is the player's distance  $s$  from home plate changing? Round your answer to one decimal place.



- A)  $-58.2$  feet/second
- B)  $-0.2$  feet/second
- C)  $-0.7$  feet/second
- D)  $-19.9$  feet/second
- E)  $-1.9$  feet/second

Ans: D

84. A retail sporting goods store estimates that weekly sales and weekly advertising costs are related by the equation  $S = 2270 + 60x + 0.35x^2$ . The current weekly advertising costs are \$1700, and these costs are increasing at a rate of \$130 per week. Find the current rate of change of weekly sales.

- A) 162,500 dollars per week
- B) 164,770 dollars per week
- C) 87,420 dollars per week
- D) 85,150 dollars per week
- E) 1,021,570 dollars per week

Ans: A